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Nonlinear Stability of Magnetically Focused Particle Beams

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FUNDACAO DE APOIO DA UNIVERSIDADE FEDERAL DO RGS.

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14. ABSTRACT
This project investigates several circumstances of the interaction of charged particles and electromagnetic fields. To mention some of the topics: (i) Stability of intense electron beams in magnetic focusing systems; (ii) Extreme acceleration of electrons by modulated electromagnetic carrier waves; (iii) Stability of warm beams in relativistic magnetrons. In all cases we are mostly concerned with the investigation of regions of efficiency and stability of the respective processes.

15. SUBJECT TERMS
Magnetic focusing of intense beams, warm relativistic cross field devices, laser accelerators, long-range interaction, free-electron lasers

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• Introduction and Objectives

Our main program involves the study of nonlinear properties of intense beams of charged particles transported under the action of magnetic focusing fields. This kind of system finds a very wide range of applications, such as particle acceleration, RF generation, communication technologies, and controlled nuclear fusion.

In previous years, considerable amount of attention has been dedicated to the analysis of emittance growth in coasting beams initially far from equilibrium. Emittance growth is related to particles ejected from the beam core and is a limiting factor for beam performance. A number of papers by our and other groups show that beam inhomogeneities are frequently behind particle ejection and the subsequent process of emittance growth. Ensuing works by our group then showed how judiciously chosen initial mismatches of the beam envelope and very small levels of initial emittance can largely help to postpone wave breaking and large emittance growth in the case of inhomogeneous profiles.

As mentioned in the preceding paragraph, in past years we have used the well established model of the coasting beam both in our numerical and analytical investigations. The coasting beam model is physically relevant, simplifies the analysis since it is basically a 2D view of the problem, but fails in regimes of pulsed electronics where transverse and longitudinal beam dimensions become comparable.

More recently we therefore started to examine the dynamics of initially spherical beams and our latest works considers pulsed beams with arbitrary elliptical shapes. The importance of these new geometries relies on the fact they create new regions of stability for beam propagation, which is a feature of high relevance in the subject of controlled transport.

In another but related research line, new results on the operations of magnetrons were also obtained. In the subject of magnetron stability, we describe in detail the transition from stationary to non-stationary regimes of operation as the magnetron temperature is varied in space-charge dominated settings. We see in particular that cold and space-charge dominated magnetrons are always unstable. On the other hand, in relativistic models we showed that relativistic and associated diamagnetic effects tend to push the electron beam more towards to the cathode, as compared with nonrelativistic cases. This feature has important implications in the operational range of powerful long-range radars.

A new line has recently been opened to examine the problem of extreme particle acceleration by electromagnetic modes. With the advent of powerfull lasers working in the Terawatt and Petawatt ranges, the possibility of electron acceleration up to nearly the speed of light has been seen as very promising. Our interest here is on the development of appropriate methods to reach these high levels of acceleration. In recent works we examined a particular technique that combines a pre-acceleration stage provided by a slowly modulated high-frequency electromagnetic wave, and a final highly accelerating stage that takes place when particles are resonantly trapped by the very same modulated carrier. In order to slow down the electromagnetic carrier an inverse Free-electron laser procedure has been proposed with basis on our previous and current experience in the subject of FELs that are part of our overall body of work.

• Research Effort

Our group works with theoretical tools and self-consistent methods of numerical simulations. Particle dynamics is studied with help of paraxial equations which are solved simultaneously with the relevant Poisson and vector potential equations. In inhomogeneous and relativistic systems, fluid models with Lagrange coordinates are also employed to describe particle dynamics prior to the wave breaking phenomenon discussed earlier. The machinery of Lagrangian average methods is then set to work to estimate the size of resonances covered by test particles. With that knowledge, emittance can be estimated, which has been verified to be in very good agreement with full numerical simulations discussed in recent works listed later.

Studies with free-electron lasers make use of similar techniques. We have recently seen that the phenomenon of wave breaking can also be present in the ponderomotive bucket wells of the system, provided space-charge effects are not neglected. Space-charge is currently recognized as an important factor in this type of system. Related effects, like wave breaking, can be crucial in governing relaxation and gain saturation.

We have also investigated with a little more care the problem of relaxation in mismatched homogeneous beams. Both in theoretical and computational approaches, violent relaxation was seen in the transport of matched and mismatched beams.

The newly opened area of study on particle acceleration is under current investigation with help of particle simulations and analytical methods based on ponderomotive theories. Ponderomotive approximations work well under the condition of slow carrier modulations and have so far been in agreement with nunerical results.

All these results have been published in refereed journals and presented in major conferences in the area.

- **Group Members and Group Activities During the Period**

The core of the Beam Physics group is currently formed by Profs. Felipe Barbedo Rizzato, Renato Pakter and Yan Levin, the latter presently acting as the leader of the Statistical Mechanics group. All are Full Professors at the Institute of Physics of Universidade Federal do Rio Grande do Sul and all are supported by the AFOSR grant. In addition we count with 5 PhD candidates and 2 pos-doctoral collaborators.

- **Recent Publications and Conferences (2016/2019)**

(i) We now proceed to list some of the 2016/2019 publications of the group more directly related to the AFOSR project:

1. T. Marques, F.B. Rizzato, R. Pakter, Y. Levin; Nonlinear stability in the transport of intense bunched beams. Physics of Plasmas, **23**, p. 113102, 2016.

2. S. Marini, F. B. Rizzato, R. Pakter; Stationary to nonstationary transition in crossed-field devices. Physics of Plasmas, **23**, p. 033107, 2016.

3. E. Peter, S. Marini, R. Pakter, F. B. Rizzato; Triplet and beam interaction in a plasma. Physics of Plasmas, **24**, p. 102124, 2017.

4. S. Marini, E. Peter, G. I. Oliveira, F .B. Rizzato; Breakdown of the ponderomotive approximation as an acceleration mechanism in wave-particle nonlinear dynamics. Physics of Plasmas, **24**, p. 093113, 2017.

5. E. Peter, A. Endler, F. B. Rizzato; Application of nonlinear models to estimate the gain of one-dimensional free-electron lasers. Journal of Plasma Physics, **83**,, p. 905830302, 2017.

6. S. Marini, R. Pakter; Single electron dynamics in a Hall thruster electromagnetic field profile. Physics of Plasmas **24**, p. 053507, 2017.

7. R. Pakter, Y. Levin; Entropy production in systems with long-range interactions. Journal of Statistical Mechanics: Theory and Experiment **26**, p.1, 2017.

8. I. Almansa, D. A. Burton, R.A. Cairns, S. Marini, E. Peter, F. B. Rizzato, F. B. Russman; Uphill acceleration in a spatially modulated electrostatic field particle accelerator. Physics of Plasmas, **25**, p. 113107, 2018.

9. S. Marini, F. B. Rizzato, R. Pakter; Space-charge and thermal effects in relativistic crossed-field devices. Physics of Plasmas, **25**, p. 063111, 2018.

10. F. B. Russman, S. Marini, E. Peter, G. I. Oliveira, F. B. Rizzato; Self focusing in a spatially modulated electrostatic field particle accelerator. Physics of Plasmas **25**, p. 023110, 2018.

11. R. Pakter, Y. Levin; Nonequilibrium statistical mechanics of two-dimensional vortices. Physical Review Letters **121**, p. 020602, 2018.

12. I. Almansa, F. B. Russman, S. Marini, E. Peter, G. I. Oliveira, R. A. Cairns, F. B. Rizzato; Ponderomotive and resonant effects in the acceleration of particles by electromagnetic modes. Physics of Plasmas **26**, p. 033105, 2019.

(ii) As mentioned earlier, we presented our results, among other conferences, at:

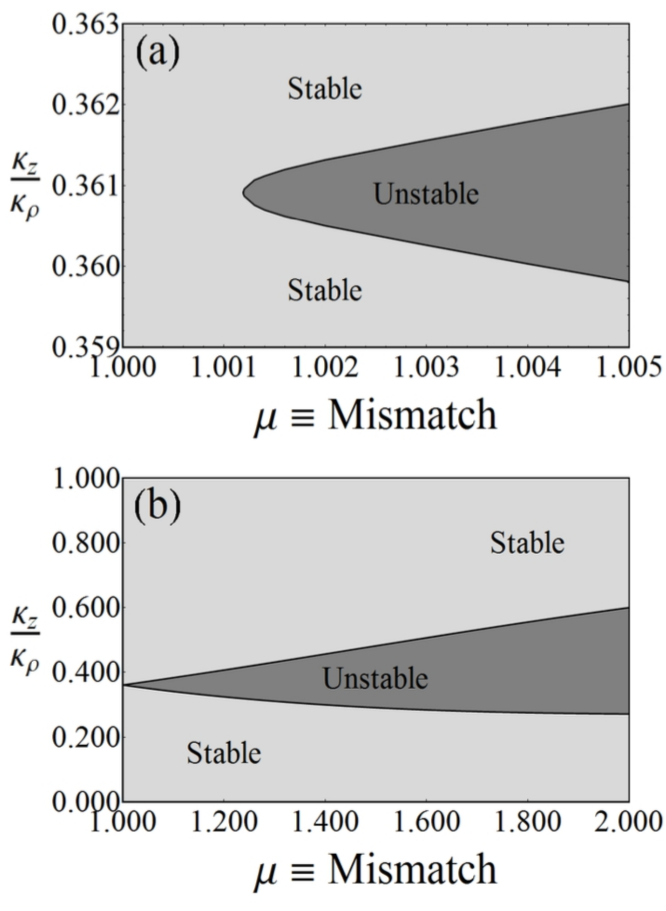
- 2017 EPS Meeting, Division of Physics of Plasmas, Belfast, Northern Ireland;
- 2018 EPS Meeting, Division of Physics of Plasmas, Prague, Czech Republic;
- 2019 EPS Meeting, Division of Physics of Plasmas, Milan, Italy;

• Description of the Current Group Projects Sponsored by AFOSR Grant

1. Instabilities in the dynamics of pulsed charged beams

This research subject investigates the nonlinear coupling of envelope modes of oscillation for intense bunched beams. Extending previous results on 2D beam geometries, the analysis now concentrates on the case of 3D spherically symmetric beams for which longitudinal and transverse focusing forces are assumed to be the same. It is investigated how externally induced spherically symmetric breathing oscillations may nonlinearly drive the growth of ellipsoidal modes which can break the spherical beam symmetry. Next, a more general case in which the focusing forces are not symmetric such that the matched beam already presents an ellipsoidal shape is studied. It is found that depending on the parameters of the system, even a very small mismatch amplitude can drive an instability, which leads to an effective coupling of longitudinal and transversal envelope oscillations by means of the space-charge forces. Use is made of Poincaré plots and the stability index of periodic orbits to perform a detailed analysis of the location of the instability in the parameter space and how it affects the beam transport. Self-consistent numerical simulations are performed in order to verify the onset of the nonlinear instability and its effect on the evolution of the RMS size and emittance of the beam.

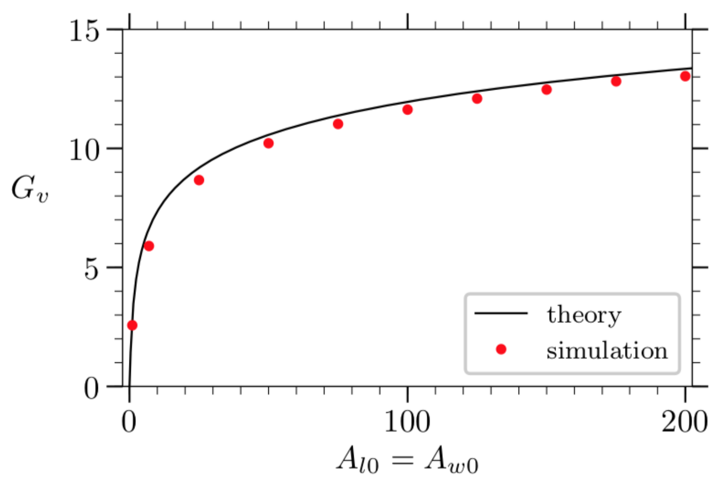
Many of our results can be synthesized in the following plot of the stability regions in terms of the focusing parameters k_z and k_ρ, and the initial mismatch μ of the beam profile with respect to the equilibrium distribution.



Stability diagram of the mismatch μ vs. α ≡ κz/κρ. The symmetric mode of oscillation is stable in the light gray areas and unstable in the dark gray. This instability is a period-doubling bifurcation of the mode. We point out that (a) is a zoom of (b) near μ ≈ 1. The remainder parameters are β = 1 and ζ = 5.

2. Particle acceleration by slowly-modulated localized laser pulses

In the present analysis we study the dynamics of charged particles under the action of slowly modulated electromagnetic carrier waves. With the use of a high-frequency laser mode along with a modulated static magnetic wiggler, we show that the ensuing total field effectively acts as a slowly modulated high-frequency beat-wave field typical of inverse free-electron laser schemes. This effective resulting field is capable of accelerating particles in much the same way as space-charge wake fields do in plasma accelerators, with the advantage of being more stable than plasma related methods. Acceleration occurs as particles transition from ponderomotive to resonant regimes, so we develop the ponderomotive formalism needed to examine this problem. The ponderomotive formalism includes terms that, although not discussed in the usual applications of the approximation, are nevertheless of crucial importance in the vicinity of resonant capture. The role of these terms is also being analyzed in the context of generic laser-plasma interactions. The figure below illustrates our main results on the gain as a function of the inverse FEL's laser and wiggler amplitudes.

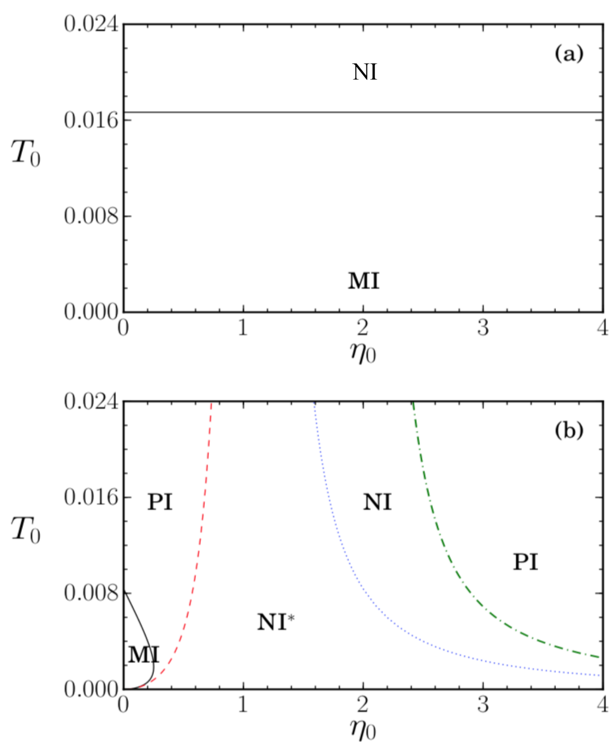


The graphic above compares our full simulations with analytical results based on the model we propose. One observes good agreement and high levels of velocity gain for achievable laser and wigglers' current and future amplitudes.

3. Relativistic cross-fields devices (magnetrons)

In the present line of work, a fully kinetic theory for the relativistic electron flow in a crossed-field device is developed and analyzed. The relevance of powerfull devices like this (magnetrons, for instance) lies on their application to extreme discrimination radars among others.

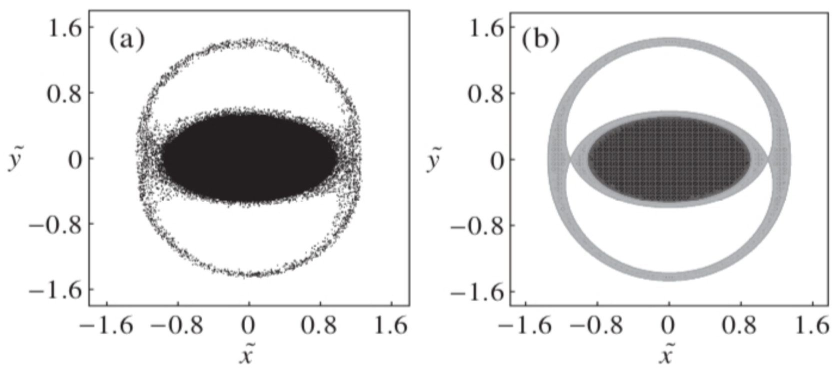
The theory takes into account self-electric, self-magnetic, and thermal effects, and allows to determine the final stationary state achieved by the electrons in phase-space. A number of different possible stationary modes are identified and described in detail. Particular attention is given to the study of how space charge and thermal effects affect the magnetic insulation when the external magnetic field exceeds the Hull cutoff field. In the nonrelativistic limit it is found that there is only a single mode transition that leads to the loss of the magnetic insulation. This transition is completely independent of the electron density and occurs for relatively large injection temperatures. On the other hand, in a moderate relativistic regime it is found a much richer scenario with the onset of a series of stationary state mode transitions as both electron density and injection temperature are varied. In particular, it is found that the transitions and the consequent loss of magnetic insulation may occur even at very low injection temperatures. Self-consistent numerical simulation results are also presented and used to verify the theoretical findings.



Parameter space showing the stationary solution mode predicted by the theory as a function of η0 (beam density) and T_0 (beam temperature). In (a) we consider a nonrelativistic case with a small relativistic parameter ζ_0 ($\equiv e\,Voltage / mc^2$) = 0.0002. The only transition found is a MI (magnetically insulated) to NI (non-insulated) that occurs as the normalized temperature exceeds $T_0 \approx 0.017$. On the other hand, for the moderately relativistic case with $\zeta_0 = 0.2$ shown in (b), it is a found a much richer scenario with many mode transitions appearing as the temperature and charge intensity parameters are varied. Of particular importance is the transition from MI to NI as the density varies at the same temperature, which is non-existent in non-relativistic settings.

4. Vorticity and the dynamics of electrons in ultra-strong crossed-fields

It has been observed empirically that two-dimensional vortices tend to cluster, forming a giant vortex. To account for this observation, Onsager introduced the concept of negative absolute temperature in equilibrium statistical mechanics. In line of research, we show that in the thermodynamic limit a system of interacting vortices does not relax to the thermodynamic equilibrium but becomes trapped in a nonequilibrium stationary state. We show that the vortex distribution in this nonequilibrium stationary state has a characteristic core-halo structure, which can be predicted a priori. All the theoretical results are compared with explicit molecular dynamics simulations. We discuss the relevance of this type of dynamics for the case of ion and electron dynamics in strong crossed-fields formed by magnetic and electric components. The application to confinement machines is investigated.



The figure shows the final equilibrium state of either vortices or electron-ion systems under strong crossed-field field conditions. Simulations (left) and theory based on long-range interaction considerations (right) agree to minimum details.

5. Entropy production in systems with long range "Coulombic" interactions

On a fine grained scale the Gibbs entropy of an isolated system remains constant throughout its dynamical evolution. This is a consequence of Liouville's theorem for Hamiltonian systems and *appears* to contradict the second law of thermodynamics. In reality, however, there is no problem since the thermodynamic entropy should be associated with the Boltzmann entropy, which for non-equilibrium systems is different from Gibbs entropy. The Boltzmann entropy accounts for the microstates which are not accessible from a given initial condition, but are compatible with a given macrostate. In a sense the Boltzmann entropy is a coarse grained version of the Gibbs entropy and will not decrease during the dynamical evolution of a macroscopic system. In this sub-project connected with interparticle forces in beam-like systems, we will explore the entropy production for systems with long range interactions. Unlike for short range systems, in the thermodynamic limit, the probability density function for these systems decouples into a product of one particle distribution functions and the coarse grained entropy can be calculated explicitly. We find that the characteristic time for the entropy production scales with the number of particles as N^{α}, with $\alpha > 0$, so that in the thermodynamic limit entropy production takes an infinite amount of time.

• Conclusions and Acknowledgements

During this first year of support by the AFOSR/SOARD grant FA9550-16-1-0280, we maintained our research effort in the area of waves and beams. Papers were published and presented at major conferences, and a number of students obtained their PhD degrees. All in all, we considered the group performance as positive, and again express our gratitude to the continuing help provided by the Scientific Office and the South America division.