



Three-dimensional Vortex Dynamics in Unsteady, Separated Flows

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07/15/2019
Final Report

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REPORT DOCUMENTATION PAGE					Form Approved OMB No. 0704-0188	
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1. REPORT DATE (DD-MM-YYYY) 07/03/2019		2. REPORT TYPE Final			3. DATES COVERED (From - To) 01 Feb 2016 - 31 Jan 2019	
4. TITLE AND SUBTITLE Three-dimensional Vortex Dynamics in Unsteady, Separated Flows				5a. CONTRACT NUMBER		
				5b. GRANT NUMBER FA9550-16-1-0107		
				5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) Buchholz, James, H, J Thurow, Brian, S Wabick, Kevin, J Johnson, Kyle, C Berdon, Randall, L, Jr.				5d. PROJECT NUMBER		
				5e. TASK NUMBER		
				5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) The University of Iowa Division of Sponsored Programs 2 Gilmore Hall The University of Iowa, Iowa City, IA 52242					8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Office of Scientific Research 875 North Randolph Street, Suite 325 Arlington, VA 22203					10. SPONSOR/MONITOR'S ACRONYM(S) AFOSR	
					11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Distribution A. Approved for public release: distribution unlimited.						
13. SUPPLEMENTARY NOTES Part of this work was completed through a subcontract to Auburn University, Auburn AL.						
14. ABSTRACT Plenoptic particle image velocimetry was used to acquire volumetric flow field measurements on a transient maneuvering wing of aspect ratio 2, articulated in pitch, roll, and combined roll and pitch. A vorticity transport framework was adapted to accommodate the non-inertial reference frame attached to the wing, and used to quantify the sources and sinks of vorticity governing the development of the leading-edge vortex on the wing. Two new parameters were developed for classifying the flow topology and evolution with varying kinematics. Passive bleed was implemented, locally on a rolling wing, which significantly altered the global flow structure and underlying transport mechanisms.						
15. SUBJECT TERMS Maneuvering wing, vortex dynamics, vorticity transport, plenoptic particle image velocimetry, flow control, unsteady aerodynamics, separated flow.						
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON	
a. REPORT	b. ABSTRACT	c. THIS PAGE			James Buchholz	
U	U	U	UU	118	19b. TELEPHONE NUMBER (Include area code) 319-335-5935	

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Final Report for AFOSR Award Number FA9550-16-1-0107

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Executive Summary

Leading-edge vortices are ubiquitous in aerodynamic problems, and are responsible for the enhancement of aerodynamic loads in a broad array of applications such as maneuvering aircraft and blades on helicopter rotors and gas turbine engines. In some cases, such as dynamic stall on rotor blades or wing response to gusts at high angle of attack, the vortex forms and interacts momentarily with the aerodynamic body, augmenting forces and moments for a brief period of time. In other cases, such as in the vortex flow over a leading-edge extension or a delta wing, the interaction is perpetual.

The present study investigates the mechanisms governing vortex growth and evolution on a wing of aspect ratio 2, articulated in pitching, rolling, or combined pitching and rolling motions in a free stream at a Reynolds number of 10,000, to elucidate the effects of rotational accelerations and other attributes of the kinematics. Revolving wings in quiescent flow have been studied extensively, and to a limited extent within a uniform stream. Rotation has been shown to promote vortex attachment under certain conditions, and over limited regions of the span. The present study significantly expands the investigated parameter space of revolving wings in uniform flow, and addresses pitching of a revolving wing to further isolate the effects of wing revolution. The early evolution of the flow – after initiating the motion of the wing – was the primary focus of this study, as it is anticipated that the vortex would be most susceptible to exogenous perturbations to control aerodynamic loads.

Data were acquired using a novel particle image velocimetry (PIV) method, developed in Prof. Thurow's laboratory at Auburn University, which employs a plenoptic camera to render three-dimensional images using a single camera and lens. The comparatively simple hardware configuration of plenoptic PIV is ideally suited to the present study in which a wing is articulated in complex motions, creating highly three-dimensional velocity fields. The flow field data were supplemented with surface pressure measurements, and direct measurement of aerodynamic forces. Data analysis was conducted using a vorticity transport framework that was extensively expanded within this project, to take into account the rotational accelerations of a non-inertial reference frame attached to the maneuvering wing.

The formation of an arch vortex from the leading edge of the wing was observed in all cases; however, the spanwise extent of the arch, its topology, and the timing of its development and shedding varied with the type of motion and rotation rates. The sources of circulation supporting the growth and decay of the vortex were analyzed within a planar control region on the surface of the wing, and consisted of convective vorticity fluxes due to the separating shear-layer, spanwise flow, diffusion from the surface of the wing, and artifacts related to vortex tilting and the non-inertial accelerations. In most cases, vortex growth was regulated through highly-correlated shear-layer and surface-diffusive fluxes; however, a dominant spanwise convective sink of circulation gradually developed and replaced the diffusive flux where a stable, conical vortex was established on the wing.

In the case of pure pitch, a multiple vortex system evolved at low pitch rate, whereas, for high pitch rate, the vortex remained more compact. In both cases, the influence of pitch rate on vortex strength was limited to within the first few degrees of motion; subsequent increments to the strength of the vortex were dependent only on pitch angle, regardless of the pitch rate.

For a rolling wing at a constant angle of attack, the flow structure was strongly-dependent on the advance ratio, J and dimensionless radius of gyration of the wing, R_g/c (where c is the wing chord). For a fast-rotation case, in which $J = 0.54$ and $R_g/c = 3.25$, the span of the wing was divided into three regions exhibiting distinct flow and transport behavior, with clear boundaries. A single *rotation parameter* was found to govern the transition in flow state, with varying J and R_g/c . Force measurements exhibited strong correspondence to events in the evolution of the leading-edge flow, with mean lift coefficients collapsing across a broad parameter range when calculated using the relative flow velocity at the wing radius of gyration.

In the case of combined pitch and roll, a modified reduced pitch rate parameter, k_{rg} , was defined, that

was found to strongly govern the rate of vortex development. Despite rotational accelerations induced by the roll motion, vortex development, and the underlying transport mechanisms were found to be similar to those observed in pure pitch, with reduced rate of development.

Passive bleed, applied locally at the root of a rolling wing, caused a significant change in the structure of the LEV through disruption of the spanwise convective flux, without varying wing kinematics. This demonstration of the link between transport mechanisms and flow structure, as well as the robust nature of the relationship between the surface-diffusive flux (which can be determined through limited surface measurements) and the shear-layer flux (which is the primary source of vortex growth) suggests that the vorticity transport framework can form a useful basis for aerodynamic load estimators and control models on operational aircraft.

Publications and Presentations Related to this Grant

Published in Peer-Reviewed Journals

- Akkala, J. M. and Buchholz, J. H. J. (2017) Vorticity transport mechanisms governing the development of leading-edge vortices, *J. Fluid Mech.* 829, pp. 512–537.

In Preparation

Chapters 4 through 8 each form the basis for a manuscript in preparation to be submitted to a peer-reviewed journal. Each paper will acknowledge this grant. The anticipated authorship, titles, and journals are given below.

1. Johnson K.C., Thurow B.S., Wabick K.J., Berdon R.L., and Buchholz J.H.J. Vortex topology of a pitching and rolling wing in forward flight. To be submitted: *Journal of Fluid Mechanics*
2. Berdon R.L., Wabick K.J., Buchholz J.H.J., Johnson K.C., and Thurow B.S. Flow structure and forces on a revolving wing in uniform flow. To be submitted: *Physical Review Fluids*
3. Wabick K.J., Berdon R.L., Buchholz J.H.J., Johnson K.C., and Thurow B.S. Transient leading-edge vortex development on a rolling wing. To be submitted: *Journal of Fluid Mechanics*
4. Wabick K.J., Buchholz J.H.J., Johnson K.C., and Thurow B.S. Leading-edge vortex development on a pitching wing: effects of concurrent roll. To be submitted: *TBD*
5. Wabick K.J., Berdon R.L., Buchholz J.H.J., Johnson K.C., and Thurow B.S. Passive bleed on a rolling wing in uniform flow. To be submitted: *TBD*

Conference Papers and Presentations

1. Wabick K, Berdon R, Buchholz J, Johnson K, and Thurow, B (2019) Characterization and Manipulation of Vorticity Transport on a Rolling Wing, AIAA Scitech 2019 Forum, 7-11 January, 2019, San Diego, CA. AIAA Paper 2019-2326.
2. Thurow B.S., Johnson K., Wabick K.J., Berdon R.L. and Buchholz J.H. (2018) Three dimensional visualization of leading edge vortex structure of a pitching and rolling wing in forward flight, 71st Annual Meeting of the APS Division of Fluid Dynamics. Volume 63, no. 13, Nov. 18-20, 2018, Atlanta, GA.

3. Berdon R.L., Wabick K.J., Buchholz J.H., Johnson K., Thurow B.S. (2018). Passive bleeding applied to a wing in a roll maneuver, 71st Annual Meeting of the APS Division of Fluid Dynamics. Volume 63, no. 13, Nov. 18-20, 2018, Atlanta, GA.
4. Wabick K.J., Berdon R.L., Buchholz J.H., Johnson K., Thurow B.S. (2018). Transport mechanisms governing initial leading-edge vortex development on a rolling wing, 71st Annual Meeting of the APS Division of Fluid Dynamics. Volume 63, no. 13, Nov. 18-20, 2018, Atlanta, GA.
5. Wabick KJ, Berdon RL, Buchholz JHJ, Johnson KC, and Thurow, BS (2018) Characterization of vorticity transport in the leading-edge vortex of a rolling wing using plenoptic PIV, AIAA SciTech Forum, 8-12 January, 2018, Kissimmee, Florida, AIAA Paper 2018-1292.
6. Wabick K, Berdon R, Buchholz J, Johnson K, and Thurow, B (2017) Transport mechanisms governing leading edge vortex development on a pitching wing, 70th Annual Meeting of the APS Division of Fluid Dynamics. Vol. 62, no. 14, Nov. 19- 21, 2017, Denver, CO.
7. Berdon R, Wabick K, Buchholz J, Johnson K, and Thurow, B (2017) Parametric dependence of initial LEV behavior on maneuvering wings, 70th Annual Meeting of the APS Division of Fluid Dynamics. Vol. 62, no. 14, Nov. 19-21, 2017, Denver, CO.
8. Johnson K, Thurow B, Berdon R, Wabick K, and Buchholz J (2017) Vortex topology of rolling and pitching wings, 70th Annual Meeting of the APS Division of Fluid Dynamics. Vol. 62, no. 14, Nov. 19-21, 2017, Denver, CO. *****Best student poster*****
9. Buchholz, J. and Akkala, J. (2016). Manipulation of applied leading-edge vortex development by applied suction. 69th Annual Meeting of the American Physical Society Division of Fluid Dynamics, Nov. 20-22, 2016, Portland, OR.
10. Wabick, K. and Buchholz, J. (2016) Characterization of vortex development on a pitching plate. 69th Annual Meeting of the American Physical Society Division of Fluid Dynamics, Nov. 20-22, 2016, Portland, OR.

Theses and Dissertations

- Berdon, R. L. Flow Structures and Aerodynamic Loads of a Rolling Wing in a Free Stream, M.S. Thesis, The University of Iowa, December, 2018.
- Wabick, K. J. The Dynamics of Spanwise Vorticity on a Rotating Flat Plate in a Starting Motion, Ph.D. Dissertation, The University of Iowa, May 2019.
- Johnson, K. C. Vortex Topology of a Rolling and Pitching Wing in Forward Flight, Ph.D. Dissertation, Auburn University, Fall 2019 (expected completion).

Graduate Student Participation

Three graduate students were supported by this work and contributed extensively to the research outcomes of this project.

- Randall L. Berdon, University of Iowa (M.S. December 2018)
- Kyle C. Johnson Auburn University (Ph.D. study ongoing – expected graduation 2019)
- Kevin J. Wabick, University of Iowa (Ph.D., May 2019)

Nomenclature

.	Differentiation with respect to time
a	Acceleration of wing surface at location of interest (m/s ²)
<i>A</i>	Planform area of wing (m ²)
<i>b</i>	Wing span (m)
<i>c</i>	Wing chord length (m)
$C_{D_{rel}} = 2D/(\rho U_{rel}^2 bc)$	Drag coefficient based on relative velocity at the wing radius of gyration
$C_{L_{rel}} = 2L/(\rho U_{rel}^2 bc)$	Lift coefficient based on relative velocity at the wing radius of gyration
$C_{D_{\infty}} = 2D/(\rho U_{\infty}^2 bc)$	Drag coefficient based on free-stream velocity
$C_{L_{\infty}} = 2L/(\rho U_{\infty}^2 bc)$	Lift coefficient based on free-stream velocity
<i>D</i>	Drag (N)
$J = \Omega_x rc / U_{\infty}$	Advance ratio
$k = \dot{\alpha} c / (2U_{\infty})$	Reduced pitch rate
$k_{rg} = \dot{\alpha} c / (2U_{rel}) = k / (J^{-2} + 1)^{0.5}$	Modified reduced pitch rate
<i>L</i>	Lift (N)
LEV	Leading-edge vortex
<i>p</i>	Pressure (Pa)
<i>r</i>	Radial position measured from roll axis (m)
$Re_c = U_{\infty} c / \nu$	Reynolds number
R_g	Wing radius of gyration (m)
<i>t'</i>	Dimensionless time
$U_{rel} = ((U_{R_g})^2 + U_{\infty}^2)^{\frac{1}{2}}$	Relative velocity of approach flow at the wing radius of gyration (m/s)
$U_{R_g} = \Omega_x R_g$	Wing velocity induced by roll motion at the wing radius of gyration (m/s)
U_{∞}	Free-stream velocity (m/s)
<i>x, y, z</i>	Coordinates in non-inertial reference frame attached to the wing (m)
<i>X, Y, Z</i>	Coordinates in inertial ground reference frame (m)
α_{geo}	Geometric pitch angle (rad, deg.)
$\alpha_{eff} = \alpha_{geo} + \arctan \frac{1}{J}$	Effective angle of attack (rad, deg.)
$\dot{\alpha}$	Wing pitch rate (rad/s)
ϕ	Wing roll angle (rad)
Γ	Circulation (m ² /s)
λ_{ci}	Swirl strength criterion
ν	Kinematic viscosity (m ² /s)
$\pi_{rot} = \Omega_x^2 rc / U_{rel}^2 = (c/R_g) / (1 + J^2)$	Rotation parameter
ρ	Fluid density (kg/m ³)
$\sigma_u, \sigma_v, \sigma_w$	Uncertainties in measured flow velocity components (% of free stream)
$\tau = c / U_{rel}$	Convective time constant (s)
ω	Vorticity (s ⁻¹)
Ω	Wing angular velocity (rad/s)
$\dot{\Omega}$	Wing angular acceleration (rad/s ²)

Chapter 1

Introduction

The interaction of a vortex with the surface of an aerodynamic body is a ubiquitous phenomenon in unsteady and high-angle-of-attack aerodynamics, and is observed in a wide range of aerodynamic structures, including aggressively maneuvering aircraft, blades on helicopter rotors and gas turbine engines, and wind turbines (Yilmaz and Rockwell, 2012; Mulleners et al., 2012; Bridges, 2010; Ol and Gharib, 2003). Robust models for prediction of aerodynamic loads or the application of flow control in massively-separated flows require a better understanding of the underlying nonlinear and complex dynamics and transport processes, which are currently not well-understood. A vorticity transport framework therefore provides a useful conceptual framework to understand and quantify the factors governing vortex evolution, and thereby also aerodynamic loads.

The present study aims to dissect the formation process of leading-edge vortices by exploring how variations in kinematics and, therefore, non-inertial accelerations alter the flow structure and underlying transport processes, and to investigate how these processes are manipulated through perturbations to the boundary conditions. A wing of aspect ratio 2, articulated in pure pitch, pure roll, and combined pitch and roll starting rotations is used as a model platform for this study, as the variation of rotation rates and radius of gyration of the wing provide a means to systematically investigate the factors that influence vortex development on a wing, and inform novel strategies to perturb its development. Volumetric particle image velocimetry is employed, using a novel plenoptic imaging approach with a single high-resolution camera (plenoptic PIV), to provide quantitative measurements of the complex, three-dimensional flow fields. Vorticity fluxes contributing to the development and decay of the vortex are examined, while varying advance ratio and the radius of gyration of the wing, to provide insight into the fundamental mechanisms governing vortex development. To enable this analysis, an existing vorticity transport framework is extensively augmented to accommodate non-inertial rotational accelerations that are present in a reference frame fixed to the articulated wing.

The field of rotating wings is rich with documented non-linear and intricate processes (recently reviewed by Eldredge and Jones (2019)), yet the 3D structures resulting from the combination of rotations about multiple wing axes remains insufficiently understood to robustly predict the aerodynamic loads resulting from complex maneuvers or exogenous inputs. Since the physical implications of such flow fields range from natural fliers to high-performance aircraft, this study seeks to define new dimensionless classifiers for these flows. These parameters seek to relate the flow morphology and underlying flow transport processes to the body kinematics. Such an understanding will contribute to strategies for perturbing the flow to alter the transport processes and aerodynamic loads. Thus, the study presented here first documents the vortex topology formed by articulating an aspect ratio 2 wing in specific pitching and rolling motions, while subjected to a $Re_c = 10,000$ freestream flow. Additionally, the element of forward flight creates a competition of forces between: (1) the freestream velocity that convects the vortices away and (2) the rotational wing kinematics that have been shown to sustain LEV attachment. To explore the opposition between these physical factors,

the parameter space encompasses a range of pitch and roll rates, guided by past work (Bross et al., 2013) and our own dye visualizations. The ranges were selected with the purpose of capturing two different classes of vortex structure as described by Garman et al. (2013): stable leading-edge vortices and shedding arch vortices. Keeping in mind the goal of subsequent research to control and perturb these flows to achieve desired results, the present study focuses specifically on the early development of flow phenomena, which is most susceptible to control in the early stages of wing motion. After a brief review of pitching and rolling wings, results from an expansive data set of quantitative and volumetric plenoptic PIV measurements are presented here to understand the roles of two new parameters proposed in the study, and draw conclusions on the physical implications of the pitch rate and roll rate on the formation of vortex structures.

Leading-edge vortex formation and evolution on rotating wings will be examined through the lens of vorticity transport, in a non-inertial reference frame, which is attached to the wing. Using this framework, the effects of rotational accelerations can be studied, for various rotating wings in a free-stream. Once understood, the kinematics will be combined into a pitching and rolling wing in a free-stream, to examine how robust these rotational accelerations are. Since vorticity transport is being used to quantify the flow, this gives insight into which mechanisms of vorticity transport can be manipulated to augment the flow.

The following is a brief summary of this report. Chapter 2 briefly reviews relevant literature, describing the current state of knowledge and provides context for the study. Chapter 3 describes the experimental and analysis methods used in the study, including a derivation of the non-inertial elements of the vorticity transport framework that were developed for this work. Results of the study are discussed in detail in Chapters 4 through 8. In chapter 4, renderings of the flow fields for pitching, rolling, and combined pitching and rolling wings are used to understand how pitch and roll maneuvers control the topology of a developing leading-edge vortex. A new parameter, the modified reduced pitch rate k_{rg} is examined, which takes into account both pitch and roll rates. Chapters 5 and 6 report in-depth investigations on a wing articulated in pure roll. In Chapter 5, a broad range of roll rates and radii of gyration is explored, using dye visualization to characterize leading-edge vortex structure. A dimensionless rotation parameter, π_{rot} is derived from a description of the spanwise gradient in approach velocity of a rolling wing, and used to define a parametric boundary in the classification of vortex structures. Chapter 6 quantifies the vorticity transport mechanisms within the leading-edge vortex, to provide a comprehensive understanding of the factors governing vortex growth and evolution. A similar analysis is applied to wings articulated in pure pitch, and in combined pitch and roll, in Chapter 7, to extend the understanding of these maneuvers established in Chapter 4 by revealing how the pitch and roll parameters control the transport processes and how the transport processes correlate with observed vortex evolution. Finally, in Chapter 8, we return to the pure-roll case and – with guidance from these studies, perturb vortex formation with a strategically-placed bleed hole near the root of the wing, to achieve a significantly altered flow field. The relationship between flow fields, transport processes, and aerodynamic loads of the baseline and perturbed cases is examined. Conclusions and future outlook are discussed in Chapter 9.

Chapter 2

Background

The formation of the leading-edge-vortex is due to the instability in the shear layer and roll-up of the separated shear layer. This roll up can alter the dynamics around the wing, which, then affects the separated flow emanating from the leading edge. The conditions in which the leading-edge vortex (LEV) forms govern the vorticity fluxes contributing to its growth or decay. An example of this is the surface pressure gradients established by the vortex interacting with the boundary leading to the generation of opposite-sign vorticity Lighthill (1963); Wu and Wu (1993), which can cause significant decay of the vortex through cross-cancellation Wojcik and Buchholz (2014b). This chapter serves to review our current understanding of the formation, evolution and aerodynamic effect of the leading-edge vortex.

2.1 LEV Formation Process

The formation of an LEV is preceded by the formation of a boundary layer on the surface of an aerodynamic body, due to the no-slip condition at the boundary. As the angle of attack increases, the stagnation point on the surface moves away from the leading edge and an adverse pressure gradient (APG) develops near the leading edge (Doligalski et al., 1994; Hammerton and Kerschen, 2005). The incipient formation of the LEV begins as the APG of sufficient strength is forced upon the boundary layer, triggering an ejection of vorticity into the flow from the surface (van Dommelen and Shen, 1980; Doligalski et al., 1994). van Dommelen and Shen (1980) showed that the formation of a stagnation point on the surface of the airfoil would spontaneously generate a singularity in the laminar boundary layer equation. The separation process has since been characterized as the formation of reversed flow at the surface, and once the boundary layer separates, the structure rolls up to create the leading-edge vortex. During its growth, the LEV remains connected to the leading edge via a shear layer that provides a continued flux of vorticity into the LEV to increase its circulation. As the LEV continues to grow, it begins to impose large pressure gradients on the surface which, due to the no-slip condition, lead to the formation of a layer of opposite-signed, or secondary, vorticity beneath the LEV (Acharya and Metwally, 1992; Hondan and Andreopoulos, 1997). The leading-edge shear layer can entrain this secondary vorticity, carrying it into the LEV, where it interacts via cross-cancellation, thus weakening the LEV and secondary vortex (Eslam Panah et al., 2015). Thus, the interaction with the surface provides an important regulatory function during vortex formation.

The LEV is considered detached when the connection to the feeding shear layer is broken, preventing the main source of circulation to cease, and halting the additional accumulation of circulation. The process can be described using ideas from turbulent boundary layer theory, where the approach of the LEV towards the surface of the airfoil results in a “sweep event” that causes an eruption of opposite-signed vorticity (Doligalski et al., 1994; Andreopoulos and Agui, 1996). In most cases the phenomenon is characterized as the layer of opposite-signed vorticity accumulating enough circulation for it to break the connection between

the LEV and its feeding shear layer. This process was first described by Acharya and Metwally (1992) when they showed opposite-sign vorticity cutting off a dynamic-stall vortex from the leading-edge shear layer. Similarly, in their study of a plunging airfoil, Lewin and Haj-Hariri (2003) also found the evolution of the secondary vorticity to play a significant role in the detachment of the LEV.

While it has been accepted that the detachment of the LEV is caused by the interaction between the LEV and secondary vortex, there are multiple ways in which secondary vorticity can be introduced into the flow. Widmann and Tropea (2015) describe two fundamental mechanisms by which the detachment of the LEV can occur: bluff body detachment and boundary layer eruption. They used topological analyses to characterize these mechanisms.

In bluff body detachment, the LEV separates from the airfoil but maintains a reattachment point that proceeds to move downstream. When the reattachment point reaches the trailing edge, the LEV detaches and allows reverse flow to extend up the airfoil from the trailing edge. On the other hand, the boundary layer eruption detachment mechanism occurs when the viscous vortex-wall interaction causes an eruption of secondary vorticity from beneath the LEV before the reattachment point of the LEV reaches the trailing edge. This breaks the topology between the shear layer and the LEV by forming a counter-rotating region between the two, the division of which incites detachment. As the advection of the reattachment point of an LEV has been shown to be dependent on reduced frequency (Choi et al., 2015), this transition to the boundary layer eruption mechanism is associated with high frequency motions.

2.1.1 LEV Scaling

Interest in the LEV primarily stems from its ability to produce large lift values, a byproduct of the high kinetic energy within the LEV generating low-pressure regions on the upper surface of the airfoil. It has long since been established that the total lift produced by an airfoil is related to the total circulation of the flow (Wu et al., 1999) and the fact that most of the circulation about an airfoil is contained within the LEV for vortex-dominated flows (Pitt Ford and Babinsky, 2012) results in the maximum lift values being achieved just before detachment of the LEV Jantzen et al. (2014). Thus, developing a non-dimensional circulation-threshold for the detachment of the LEV provides a useful means for characterizing the maximum lift of an airfoil. Although many attempts have been made to develop a universal scaling, no such scaling has been found, which suggests that the governing mechanisms are still not well-understood. The following paragraphs will highlight some general ways in which scaling has been applied to specific flows.

Since the bluff body detachment mechanism is characterized by the reattachment point of the LEV arriving at the trailing edge, some studies have demonstrated success in using the chord length to scale the circulation of an LEV (Rival et al., 2013; Jones and Babinsky, 2011). However, Widmann and Tropea (2015) showed that for a plate in a plunging motion, this was an insufficient scaling parameter in the case of the eruption-based detachment, as it had no way to account for the viscous effects near the leading edge that inevitably cause the eruption of secondary vorticity. Instead, they found the best way to scale the circulation of the LEV was to use the thickness of the shear layer. Alternatively, both Rival et al. (2009) and Kriegseis et al. (2013) found the effective shear-layer velocity to be a suitable scaling parameter for the LEV circulation. Akkala et al. (2014) proposed yet another scaling parameter for leading-edge vortex circulation using a kinematically-based velocity and length scale, which was found to be particularly effective for flexible plunging airfoils. Buchholz et al. (2011) demonstrated success in scaling the total circulation shed by finite aspect-ratio pitching panels based on a simple model of surface pressure gradients. Yet in spite of the many scaling parameters that have been proposed, no robust predictor of LEV circulation has been identified. This is primarily because the boundary layer eruption mechanism is still poorly understood, and without sufficient knowledge of the pertinent flow physics it becomes very difficult to identify any sort of unifying principle.

2.2 Wing Kinematics and Leading Edge Vortex Formation

Basic research on maneuvering wings has focused on simple canonical motions to isolate physical mechanisms present in unsteady aerodynamics of engineered vehicles and biological locomotion. The basic kinematics are forward translation, pitching, plunging and rotation (rolling). Since the formation of an LEV is a non-linear process, these base kinematics can not be superimposed to construct the flow field created by a more complex maneuver, but can give insight into fundamental mechanisms that are governing LEV development.

2.2.1 Translating Wings

The simplest maneuver is that of a pure translation, which is defined by a wing at fixed angle of attack, starting a linear motion from rest in a quiescent flow. A leading-edge vortex is usually created and shed within the first few chord lengths of travel, with a commensurate rise and rapid drop in lift.

By increasing the wing velocity (or relative free-stream velocity) the evolution and circulation of the LEV will be modified. This, in turn, affects the shedding and force history. The velocity of the plate will dictate the strength, and structure, of the LEV. Increasing the velocity will lead to a more compact, stronger LEV, while a slower velocity will lead to weaker vortices and lower forces (Chen et al., 2010; Mancini et al., 2015). Once the LEV has shed, the trajectory of the LEV is relatively robust and independent of kinematics (Mancini et al., 2015; Manar et al., 2015), and the startup transient will persist for multiple convective times until a fully developed wake is established.

While the force history is driven by the LEV, it is also a direct function of the added mass (Pullin and Wang, 2004; Pitt Ford and Babinsky, 2013). When the added mass component is removed, it can be shown that the remaining circulatory force, is not as susceptible to starting motions Chen et al. (2010). Early in the motion, the trailing-edge vortex (TEV) is still rolling up and all of the circulation is contained in the LEV and TEV (Pitt Ford and Babinsky, 2013), making the bounded circulation contribution small. The total lift force on the plate can be attributed to added mass and LEV (Pullin and Wang, 2004; Pitt Ford and Babinsky, 2014). Recently, Corkery and Babinsky (2018) showed that added mass could be accounted for by surface vorticity fluxes during transient motions.

Finite wings have more three-dimensionality which derives partially from the tip vortex, which affects both the formation of the LEV and, by extension, the shedding process. With increasing aspect ratios, these three-dimensional effects decrease, but still can exhibit spanwise arch vortices and stall cells. Through both numerical simulations and flow visualization experiments, it has been shown that, at early times in the motion, the flow development is relatively insensitive to aspect ratio ($AR=1, 2$, and 4). Flow separates at the leading and trailing edges and forms a closed loop vortex system. The wake of the wing becomes more sensitive to aspect ratio as tip vortices gain strength. For instance, in the $AR=1$ case, the tip vortices cover nearly the entire surface, which causes the vortex sheet to detach. At higher aspect ratios this effect is not seen. For the $AR=2$ and 4 case the shear layer separates from the wing and forms an arch (Taira and Colonius, 2009; Kim and Gharib, 2010).

The overall dynamics of the LEV on a translating rectangular wing have distinct features, such as, a tip vortex that increases spanwise flow. In higher aspect ratio cases, vorticity transport in these two-dimensional kinematics is not great enough to allow for an LEV to remain attached for longer period of times (Taira and Colonius, 2009; Mancini et al., 2015).

2.2.2 Pitching Wings

A pitching maneuver occurs when a wing or airfoil changes its angle of attack in time, in the presence of a free stream. This motion is governed by the pitch rate, pitch axis location, and wing aspect ratio. It is

important to note that the Reynolds number has a relatively unimportant role in the studies conducted in the range of $Re_c = 100$ to $100,000$, in which they all share a common behavior (OL et al., 2010). The LEV dynamics are also seemingly unaffected by the wing cross-sectional geometry.

Many have studied this problem through flow visualizations (Walker and Chou, 1987; Shih et al., 1992; Yilmaz and Rockwell, 2012; Granlund et al., 2013) or numerical investigations (Jantzen et al., 2014; Visbal and Shang, 1989; Eldredge et al., 2009; Visbal, 2017), to shed light on the dynamics of a pitch up maneuver, all of which have shown that the flow follows a characteristic sequence of events. At low angles of attack, the separation point starts near the trailing edge, and a recirculation zone forms just aft of this point. As the angle increases, the separation point moves toward the leading edge until it reaches the leading edge, at which point the flow departs from the quasi-steady behavior observed in the translating wing. At this point a closed loop of circulation forms, and the boundary layer begins to roll up and a LEV begins to form.

By increasing the dimensionless pitch rate, k , all of these processes are observed but two changes occur. First, the LEV becomes more compact, and stronger, with larger pressure gradients induced by the rotation. Second, many of the features (recirculation zone, LEV roll up) are delayed in the development, and shedding is delayed to higher angles of attack (Walker and Chou, 1987; Visbal and Shang, 1989; Granlund et al., 2013; Buchner et al., 2012; Jumper et al., 1987; Strickland and Graham, 1987).

Delays in these processes can also be seen by moving the pitch axis aft (Visbal and Shang, 1989; Yu and Bernal, 2016). The normal component to velocity at the leading edge, with respect to the incident flow is $U(\sin\alpha - 2kx_p/c)$, in which a change in either the pitch axis or reduced pitch rate, k , will change this component. However, a change in the pivot location has been shown to not increase the LEV strength. It will merely shift the processes in time. It was found that the peak suction increases with reduced pitch rate k but decreases with increasing x_p/c (Visbal and Shang, 1989).

2.2.3 Revolving and Rolling Wings

Compared to translating and pitching wings, a revolving or rolling wing, which is rotating, at high angle of attack, about an axis perpendicular to the span, and parallel to the free-stream (if present), creates a conical LEV which, under certain conditions can remain attached to the wing. Dickinson et al. (1999) have shown that the aerodynamics of flapping flight relies significantly on the generation of a large, conical LEV, which is formed in the presence of rotational accelerations. The initial development of this LEV occurs in a similar manner to that of the translating wing. The vortex lift generated during the rotation remains throughout the motion. This is distinguished from the LEV that forms on a translating or pitching wing, in which the LEV is shed into the wake, and followed by periodic shedding (Taira and Colonius, 2009). The works of Lentink and Dickinson, and Jardin and David (Lentink and Dickinson, 2009b; Jardin and David, 2014, 2015), suggest that the Coriolis acceleration that accompanies rotation plays a key role in the robust attachment of the LEV.

Lentink and Dickinson (Lentink and Dickinson, 2009a,b) performed a non-dimensionalization of the Navier-Stokes equation in a non-inertial reference frame for a purely rotating wing at a constant angular velocity to determine the effects of rotation:

$$\frac{D\mathbf{u}}{Dt} + \frac{1}{A^*} \frac{d\mathbf{\Omega}}{dt} \times \mathbf{r} + \frac{1}{Ro} \cdot \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) + \frac{1}{Ro} \cdot 2\mathbf{\Omega} \times \mathbf{u} = -Eu \cdot \nabla \mathbf{p} + \frac{1}{Re} \cdot \nabla^2 \mathbf{u}. \quad (2.1)$$

The first term on the left hand side is the unsteady term, the second term is the angular acceleration term over the stroke amplitude, A^* , for a given angular acceleration, $d\mathbf{\Omega}/dt$. The third term is the centripetal acceleration term, and the fourth and final term is the Coriolis acceleration term. On the right hand side, the first term is the pressure term and the second term is the viscous term, where Eu is the Euler number, defined as $Eu = p/\rho u^2$, where p is the pressure, ρ is the density, and u is the fluid velocity. The Reynolds number, Re , is defined as $Re = \rho u c / \mu$, where ρ is the density, u is the free-stream, c is the chord length and

μ is the dynamic viscosity. From the non-dimensionalization, it was shown that the centripetal and Coriolis accelerations (Equations 2.2 and 2.3, respectively) are governed by the inverse of the Rossby number ($Ro = R/c$; where R is the wingtip radius and c is the chord length).

$$a_{cent} = \frac{1}{Ro} \cdot \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) \quad (2.2)$$

$$a_{Cor} = \frac{1}{Ro} \cdot 2\boldsymbol{\Omega} \times \mathbf{u} \quad (2.3)$$

In the case of translating wings, the Rossby number tends to infinity which makes a null contribution for the Coriolis and centripetal accelerations. However, the analysis shows that the centripetal and Coriolis accelerations could not be neglected in a rotation case since the Rossby number will be relatively low, and would influence LEV development.

The strength of the LEV is highly dependent on the angle of attack (Ozen and Rockwell, 2012; Wojcik and Buchholz, 2014a), but interestingly the Reynolds number does not have a significant effect on the LEV strength (DeVoria and Ringuette, 2012; Ozen and Rockwell, 2012; Wojcik and Buchholz, 2014a). It has been shown by Wojcik and Buchholz (2014a) that the circulation of the LEV on a revolving wing in quiescent flow, at a given spanwise location, exceeded the bound circulation predicted by thin airfoil theory (for equivalent angle of attack); however, Pitt Ford and Babinsky (2012) showed that it still agreed well with the measured forces, supporting the claims that lift on the wing is the result of the LEV and not bound circulation.

Lentink and Dickinson observed that, for $Re < 14,000$, over 300 species of flying animals can be associated with a Rossby number of three (Lentink and Dickinson (2009a)). In this range of Reynolds number the wings rotate about their root, and the Rossby number is proportional to the aspect ratio. For wings revolving about their root, detailed analysis of the flow structure revealed an overall loss of coherency of the LEV as the Rossby number increases (Lentink and Dickinson, 2009b; Wolfinger and Rockwell, 2014, 2015). Likewise, Harbig et al. (2013), Garmann and Visbal (2014), and Carr et al. (2015) showed, for wings undergoing small amplitude revolution, that there are two different flow regions along the span of the wing. The first is an inboard region where the LEV is stable and generates high lift, and the second is an outboard region where the LEV lifts away from the surface, resulting in a local loss in flow structure.

For most of the purely-revolving wing cases, LEV attachment has been attributed to spanwise flow because it is hypothesized to carry circulation out of the LEV, maintaining a bounded LEV strength and size. However, Birch and Dickinson (2001) performed a study on a revolving *Drosophila melanogaster* robotic wing in which fences and baffles were installed along the span of the wing to impede the spanwise flow. By using digital particle image velocimetry, it was observed that the LEV remained attached while the spanwise flow was mitigated. It was noted that the circulation of the LEV drops in most cases when the fences and baffles were installed. However, in a particular case, when a cylindrical wall was added just on the edge of the wingtip radius to avert spanwise flow due to formation of a tip vortex, the LEV circulation increased by 14%, demonstrating that the wing tip interaction does indeed have destructive relationship with the LEV. Even though axial flow was not observed, a large outboard velocity was seen just behind the LEV. It was hypothesized that the attached flow was maintained by the tip vortex inducing a downwash on the LEV, pushing the LEV into the surface. This also been observed by Shyy et al. (2009) on a flapping flat plate (aspect ratio 4) where the tip vortex was found to create a low pressure region near the tip of the wing, in turn, anchoring the LEV.

When a free stream is introduced, Bross et al. (2013) note that the vorticity concentration is relatively independent of advance ratio ($J = 0$ to 0.537) in the early to intermediate stages of rotation ($\phi < 45^\circ$). A caveat of this observation is that it results only from measurements made at the midspan plane. Representations of the 3D vortex structure are created by stitching 38 stereoscopic PIV imaging planes together, specifically

for the $J = 0.537$ case at $\phi = 36^\circ$, useful for comparison to the present study. Bross et al. (2013) describe a dual-vortex system at the LE, that extends from about 40 percent of the span to the tip. Inboard of this point, the LEV deflect up and away from the wing root. Additionally, the TiV is well-defined and shown to have a helical instability about its periphery. It is hypothesised that adding a freestream velocity may shift the timing of formation of a loop vortex connecting the TiV to the root. Furthermore, Bross et al. (2013) note that the streamline topology of the flow over the wing shows generic features over $J = 0$ to 0.537, but the roll angle ϕ at which the features occurred did depend on the value of J . While not explicitly defined as a time-shift in the sense of Yilmaz et al. (2010), this shifting of flow features over roll angle is similar in nature.

A simultaneously pitching and rolling wing produces flow regimes that are a function of all of the previously mentioned factors: reduced pitch frequency, advance ratio, aspect ratio, radius of gyration, and centre of rotation. Combining the pitching motion with a rolling motion has primarily been studied with the motivation of understanding biological flight, almost exclusively in the hovering scenario. Most relevant to the present study, Phillips et al. (2015) utilized high-speed, stereoscopic PIV to study the 3D vortex structure from an insect-like flapping machine in a quiescent room, finding that across a range of aspect ratios, the flow fields developed nearly identically, qualitatively characterised by a conical LEV that arches at the outboard end of the wing. As the stroke progressed, the LEV would eventually detach at about 70 percent of the span, moving progressively inboard as the stroke continues. The remainder of the body of work is focused on low-Reynolds-number flow of insects or animals (van den Berg and Ellington, 1997; Birch et al., 2004; Aono et al., 2007; Sun and Tang, 2002; Usherwood and Ellington, 2002; Borazjani and Daghooghi, 2013), including rotating actual animal wings (Usherwood, 2009; Thomas et al., 2004), each with substantial variation from the work presented herein.

2.3 The Role of Spanwise Flow

Under steady conditions, the effect of sweep on lift enhancement through leading-edge vortex (LEV) stabilization is well understood, particularly in the context of delta wings, as discussed by Polhamus (1971). With increasing degrees of sweep, one finds a proportionally stronger corewise flow drawing vorticity away from the leading edge vortex, which in turn is sufficient to maintain LEV attachment.

When allowing for the possibility of spanwise vortex stabilization on rotating wings, Ellington et al. (1996) were the first to observe this phenomenon for the case of a flapping model-hawkmoth wing ($Re = 100$). Ellington et al. postulated that spanwise flow through the core of the vortex, causing a conical spiral vortex, was responsible for the redirection of momentum toward the wing tip. This would then allow for the LEV circulation to remain indefinitely bounded, similar to the quasi-steady stabilization experienced in the delta-wing LEV arrangement. Beem et al. (2012) showed that the spanwise flow, in isolation, was not responsible for vortex stability, which corroborated previous findings by Birch and Dickinson (2001). However, *nominally two-dimensional* spanwise flow, such as that investigated by Beem et al. (2012), cannot result in three-dimensional effects because the convection of spanwise-oriented vorticity depends also on a gradient of vorticity magnitude in order to change the circulation, as will be discussed in the next section.

As the nature of LEV growth and detachment remains poorly understood, reduced-order models have been developed to account for LEV growth on simple geometries. These reduced-order models are often based on potential flow, such as those described by Ansari et al. (2006), and Hammer (2011). Xia and Mohseni (2012) extended a discrete vortex method to include quasi-three-dimensional effects by including a potential sink to simulate spanwise flow. However, in a computational study by Garmann and Visbal (2012) that compared LEV stability of a rotating wing to that of a plunging twisted wing, it was demonstrated that LEV stability was not an effect of a spanwise variation of effective incidence. The results of Garmann and Visbal (2012) and Beem et al. (2012) showed that neither spanwise variation in effective incidence nor

uniform spanwise flow was responsible for LEV stability. Therefore, a fully three-dimensional model for LEV growth that accounts for both spanwise flow and spanwise variation in effective incidence may be required to capture the mechanism of LEV stability.

On a nominally two-dimensional geometry, spanwise flow alone is not sufficient to produce vortex stabilization. Rather there must be spanwise gradients in vorticity or spanwise flow, such that vorticity can be convected and stretched. Planforms such as delta wings drain vorticity with spanwise flow to produce a stable LEV, such as that shown by Gursul et al. (2005). However, it is not clear what specific three-dimensional features, such as wing kinematics or shape, would result in an appropriate balance of vortex convection and stretching on planforms with a spanwise variation in effective incidence, such as for a flapping wing. The question of what form of maneuvering-wing kinematics can result in an appropriate balance of vorticity convection and stretching ultimately motivates the development of reduced-order modeling tools, such as those described in the following section.

2.4 Analyses of Vorticity Transport

With the advent of high resolution imaging instrumentation and high-fidelity computational capabilities, flow variables can be accurately quantified in highly three-dimensional, unsteady flows. The analysis of vorticity transport has thus been possible. For example, Wojcik and Buchholz (2014b) were able to quantify the sources and sinks of vorticity, in a rotating blade in quiescent fluid, by using equation 2.4 and integrating around a closed loop that encases the LEV.

$$\frac{d\Gamma_z}{dz} = \int_{A_z} U_z \frac{\partial \omega_z}{\partial z} dA_z + \int_{A_z} \left(\omega_y \frac{\partial U_z}{\partial y} + \omega_x \frac{\partial U_z}{\partial x} \right) dA_z + \int_{L-y} U_x \omega_z dL - \text{Annihilation.} \quad (2.4)$$

The term on the left side of the equation is the time rate of change of circulation in the control region. The first term on the right hand side is the spanwise convective flux over a control region that encases the LEV. The second term is vortex tilting. The third term is the in-plane convective flux. Finally, the annihilation term represents the cross cancelation of negative sign vorticity from the opposite sign vorticity, which was deduced, assuming a balanced budget. Wojcik and Buchholz (2014b) showed that the diffusive flux of secondary, opposite-sign vorticity from the surface of the blade was necessary, but not sufficient for LEV stability, under at least some cases of rotating wings, due to its regulating effect on the strength of the LEV. Spanwise flux was found to be negligible in some attached LEVs, demonstrating that it is not always necessary for LEV stability.

Eslam Panah et al. (2015) further expanded the vorticity flux analysis, by expanding the vorticity transport equation and computing the diffusive flux term in equation 2.5, from surface pressure measurements.

$$\frac{d\Gamma}{dt} = \int_A u_z \frac{\partial \omega_z}{\partial z} dA + \int_A \left(\omega_x \frac{\partial u_z}{\partial x} + \omega_y \frac{\partial u_z}{\partial y} \right) dA - \oint_{\partial A} (\mathbf{u} \cdot \mathbf{n}_{\partial A}) \omega_z ds + \frac{1}{\rho} \int_{\text{boundary}} \frac{\partial p}{\partial x} dx \quad (2.5)$$

By applying this equation, Eslam Panah et al. (2015) were able to bring new insight to the plunging plate problem. For a rectangular plate in a pure plunge motion, during the downstroke, the surface vorticity flux due to the pressure gradient is consistently approximately half that due to the leading-edge shear layer for all parameter values investigated, demonstrating that production and entrainment of secondary vorticity is also an important mechanism regulating LEV strength in periodic plunging. A small but non-negligible vorticity source was also attributed to spanwise flow toward the end of the downstroke. Aggregate vortex tilting is notably more significant for higher plunge frequencies, suggesting that the vortex core is more three-dimensional Panah (2014).

When rotation was introduced to the model with a sweep-back leading edge, Wong and Rival (2015) demonstrated that the ‘relative’ stability of the LEV (defined as the convective speed of the LEV relative

to the wing) can be improved by regulating the LEV growth via the enhancement of the spanwise vorticity transport and vortex stretching, in which the latter helps to keep the vortex size under the critical length scale of one chord length which, once breached, the vortex must inevitably separate, as suggested by Widmann and Tropea (2015). The finding of Wong and Rival (2015) reinforced the importance of rotation in the stabilization of the LEV.

More recently, Onoue and Breuer (2016) characterized the LEV dynamics on a rapidly pitching plate over a wide range of flow conditions, and demonstrated, through a vorticity transport analysis, that the LEV circulation and formation time scale uniformly with the feeding shear-layer velocity at the leading edge shown in Equation 2.6:

$$U_{SL} = \frac{4fc\Delta\alpha}{2 + U_{\infty}\sin(\alpha(t))} \quad (2.6)$$

where f , c and $\Delta\alpha$ denote pitching frequency, plate's chord length, and maximum pitching amplitude in radians, respectively. The vortex scaling proposed in Onoue and Breuer (2016) conformed very well with the concept of optimal vortex formation number originally introduced in the seminal work of Gharib et al. (1998). This offered an appealing framework for to think about how we can potentially control and predict the strength and formation time of the LEV.

The fluid dynamic field says that the prolonged attachment from a rotating wing is due to Coriolis accelerations. Jardine and David Jardin and David (2015) have artificially tuned the Coriolis term in the Navier Stokes equation and showed that the LEV remained attached longer for a higher values. Eldredge and Jones (2019) said the Coriolis is a a tilting term, which acts similar to tilting and stretching and then combine all three of the terms into a single term called Coriolis tilting term. Garmann and Visbal (2012) claimed that the Coriolis term can act as a stabilizing force if the vector is point to the surface of the wing. The one thing all these arguments have in common, is that the Coriolis accelerations stabilize the LEV. Once the Coriolis term is understood on a rolling wing in a freestream, this work will extend he knowledge of rotational augmentation. It will examine a plate that is actively pitching during a rolling maneuver in a free stream. Once the foundation is laid out for rotational augmentation, a strategy to manipulate the flow to alter aerodynamic loads and flow topology will be employed.

2.5 Overview of the Present Work

The preceding discussion illustrates that flow development on revolving or rolling wings can be substantially different than for their translating counterparts. However, flow development and the governing physics on wings that are revolving in forward flight has received relatively little attention in the literature, and it is not well-understood how the flow fields, and thus also the aerodynamic loads, evolve over a broad range of kinematic parameters. The systematic study of more complex motions, such as active pitch articulated on a forward-moving, rolling wing, is essentially unexplored. However, understanding how these types of maneuvers govern transport mechanisms in the flow, and thus influence the flow structure and resulting aerodynamic loads can provide critical insight into the physics governing aerodynamic loads in transient conditions, and thus forms an important building block in the development of robust aerodynamic models and control strategies for unsteady, separated flows.

The study presented in this report is constructed such as to systematically isolate the effects of pitch and roll rotations on flow structure development, and investigate their interaction in combined pitch and roll maneuvers. The application of plenoptic particle image velocimetry has facilitated the acquisition of volumetric data sets covering a broad set of parameter values, and dye visualization has been used to guide its application and further expand the parameter space. The existing vorticity transport framework has been significantly expanded to accommodate the rotational accelerations on maneuvering wings so that the underlying sources and sinks of LEV circulation can be quantified over a broad range of kinematics, and

the spatial and temporal variation in transport mechanisms underlying vortex development and aerodynamic loads can be identified.

Chapter 3

Methods

3.1 Overview

All of the studies presented in this report consider a rectangular flat plate of aspect ratio 2 articulated in a uniform free stream in maneuvers consisting of either a) pitching about the leading edge, b) rolling with constant angle of attack about an axis parallel to the free stream, or c) simultaneous pitch and roll maneuvers. In all cases, the Reynolds number based on the free-stream velocity and wing chord length was 10,000. Dye visualizations, plenoptic particle image velocimetry, and direct force measurements were used to characterize flow development on the wing and aerodynamic loads. Finally, one of the pure roll cases was investigated with a modified wing containing a small through-hole near the root, creating a passive bleed perturbation that altered the development of the leading-edge vortex. For this case, the Shake the Box (Schanz et al., 2016) particle tracking velocimetry method was applied to characterize flow development. The following sections describe, in detail, the experimental setup and application of measurement methods used to study this problem. In Section 3.8, the investigated parameter space is defined, and cases are identified and labeled.

3.2 Experimental Setup

A flat-plate wing of aspect ratio 2 was manufactured from clear colorless acrylic to minimize laser reflections, and had a chord length, $c = 76.3$ mm, span $b = 152.6$ mm, and thickness of 3.3% of the chord, with both the leading and trailing edges rounded with a constant radius of half the plate thickness to eliminate separation caused by edges. To ensure negligible deflection, the root of the wing was inserted into a stainless steel bracket extending the full chord length, and supported by a 3.2 mm diameter rod centered at the leading edge. Plate features and dimensions are presented in figure 3.1.

Figure 3.2 illustrates the motion mechanism and wing. The wing was articulated in a roll maneuver about an axis aligned with the free stream, and coincident with the surface of a skim plate extending approximately 0.45 m upstream and 0.78 m downstream of the wing mid-chord. The motion was achieved using an Emerson XVM-8020-TONS-000 DC servo motor driven by an Emerson EP-204B servo drive and Galil DMC-4040 motion controller. A second servo motor was used to maintain static pitch angles during experiments. Experiments were conducted in the University of Iowa’s free-surface water channel with test section width of 0.61 m, and water depth of 0.33 m. The water channel has flow conditioning consisting of an 8:1 contraction ratio, honeycomb, and five screens to maintain free stream turbulence intensity below 0.3%.

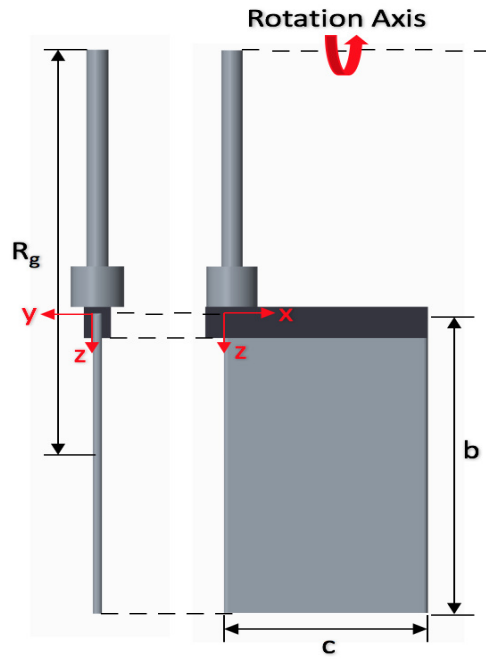


Figure 3.1: Plate Geometry.

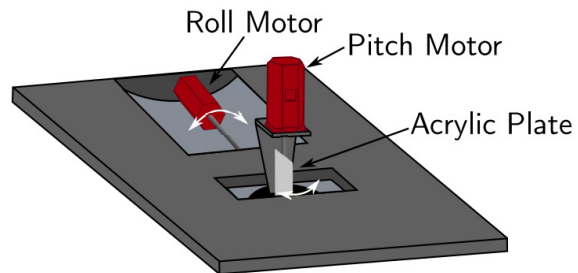


Figure 3.2: Experimental set-up for the pitching and rolling flat plate.

3.3 Parameters

A wing Reynolds number of 10,000 was maintained, based on the free-stream velocity, U_∞ and chord length of the wing, c :

$$Re_c = \frac{U_\infty c}{\nu} \quad (3.1)$$

where ν is the kinematic viscosity of the water. The roll, pitch, and combined roll/pitch motions each have unique parameters describing the kinematics. They are described below.

3.3.1 Parameters related to Pure Rolling Motions

The radial position of the wing is varied in the roll motions, and is characterized by the radius of gyration R_g of the wing planform, defined as:

$$R_g = \sqrt{\frac{I_x}{A}} \quad (3.2)$$

where the second moment of the area about the roll axis is:

$$I_x = I'_x + Ad^2 \quad (3.3)$$

and I'_x is the second moment of the planform area of the wing about a chordwise axis through its centroid, A is the planform area of the wing, and d is the distance between the roll axis and the centroid of the wing.

An important velocity scale is the relative flow velocity magnitude at the radius of gyration, U_{rel} , which results from the vector addition of the transverse velocity of the wing due to the rolling velocity at the radius of gyration, and the free stream velocity:

$$U_{rel} = ((U_{R_g})^2 + U_\infty^2)^{\frac{1}{2}} \quad (3.4)$$

where $U_{R_g} = \Omega_x R_g$ is the rotation-induced wing velocity at the radius of gyration and Ω_x is the roll rate (rad/s).

Since the sectional angle of attack on a rolling, planar wing varies with spanwise position, the effective angle of attack at the radius of gyration of the wing planform is used to specify the wing state. The effective angle of attack is dependent on the geometric pitch angle α_{geo} and the advance ratio J of the wing, where:

$$J = \frac{U_\infty}{\Omega_x R_g} \quad (3.5)$$

The construction of α_{eff} is shown in Figure 3.3 and is calculated by:

$$\alpha_{eff} = \alpha_{geo} + \arctan \frac{1}{J} \quad (3.6)$$

For the wing articulated in pure roll, most cases employed $\alpha_{eff} = 33^\circ$, although values of $\alpha_{eff} = 23^\circ$ and 43° were also considered.

3.3.2 Parameters related to Pure Pitching Motions

In all cases of pitching, the pitch axis is located at the leading edge of the wing. The motion is initiated with the wing chord aligned with the free stream (i.e. $\alpha_{geo} = 0$), and the wing is pitched to a final pitch angle of approximately 45° . The only parameter that was varied for pure pitch cases is the reduced pitch rate:

$$k = \frac{\dot{\alpha}c}{2U_\infty} \quad (3.7)$$

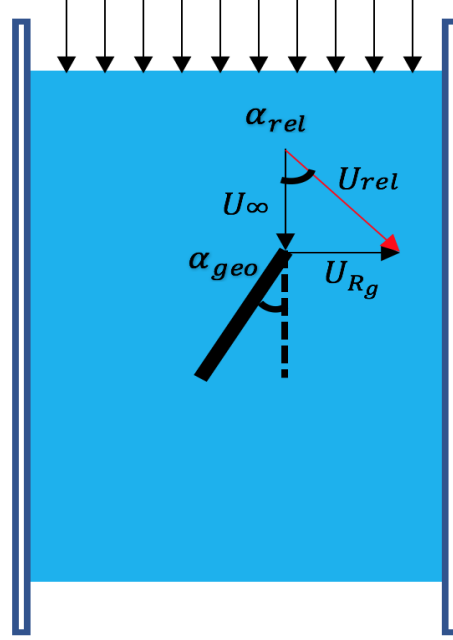


Figure 3.3: Demonstration of the effective angle of attack (α_{eff})

where $\dot{\alpha}$ (rad/s) is the pitch rate. The parameter, which is frequently used to characterize pitching wings, is defined such that for $k = 1$, a wing pitching about the mid-chord would have a leading- or trailing-edge tangential velocity of U_∞ . In the present case, since the wing is pitching about the leading edge, the trailing edge velocity is equal to U_∞ for $k = 0.5$.

3.3.3 Parameters related to Combined Roll and Pitch Motions

For the combined roll and pitch motion, the parameters of Section 3.3.1 and 3.3.2 are relevant. However, the reduced pitch rate k , for the pure pitching wing, depends on the free-stream velocity, U_∞ , whereas the flow approach velocity in the combined roll/pitch case is the relative velocity, Equation 3.4. Therefore, a new dimensionless parameter, k_{rg} , is defined to help normalize the incident velocity modification due to roll, and isolate the effects of pitch rate in the rolling reference frame. The parameter is similar to the reduced pitch rate k :

$$k_{rg} = \frac{\dot{\alpha}_{eff} C}{2U_{rel}} \quad (3.8)$$

$$(3.9)$$

where U_{rel} is given by Equation 3.4. Given the definitions of the reduced pitch rate ($k = \dot{\alpha} c / 2U_\infty$) and the advance coefficient ($J = U_\infty / \dot{\Omega} R_g$), equation 3.8 simplifies to

$$k_{rg} = \frac{k}{\sqrt{J^{-2} + 1}}. \quad (3.10)$$

Thus, the effect of a rotating reference frame on the vortex topology caused by a pitch-up maneuver is found by matching k and k_{rg} . It is noted that for a purely pitching wing, $k = k_{rg}$.

3.4 Dye visualization

Dye visualizations were conducted using a mixture of Elmer's water soluble white glue mixed with fluorescein disodium salt. The dye-glue mixture was applied in an approximately 0.5 cm wide strip on the suction surface of the wing adjacent to the leading-edge, as shown in Figure 3.4. Videos were recorded with a Nikon D3100 DSLR camera with a 105 mm Nikon f/2.8 macro-lens. An f-stop of f/8 was used with an ISO of 3200. The shutter speed was set to 1/80 s.

3.5 Plenoptic PIV

This study employs the unique measurement capabilities of plenoptic particle image velocimetry (PIV), which allows for volumetric measurements of all three components of velocity with a single camera. A brief summary is provided here, and the reader is referred to Fahringer et al. (2015) for more details on the construction of a plenoptic camera and tomographic reconstruction techniques. Furthermore, Hall et al. (2017) provide the details on the 3D calibration procedure that was used in this study.

Plenoptic PIV is conducted using a plenoptic camera, which is capable of measuring both the spatial and angular information of light rays entering the camera aperture. This is achieved by modifying an off-the-shelf 29 megapixel (MP) Imperx Bobcat ICL-B6640 conventional PIV camera to house a microlens array, which encodes the angular information onto the image sensor. Each lens in the microlens array has a focal length of 308 μm and a pitch of 77 μm . A hexagonally-packed grid of 471×362 microlenses covers the $6600 \times 4400 \text{ pixel}^2$ image sensor which recorded light scattered from silver-coated, hollow glass spheres with a mean diameter of 10 μm seeded in the flow field. In this experiment a 200 mJ dual-cavity, pulsed Nd:YAG laser illuminated a $102 \times 68 \times 68 \text{ mm}$ volume, which included the entire chord of the wing. Particle volumes were reconstructed onto a $527 \times 357 \times 357 \text{ voxel}^3$ grid of which corresponding pairs are cross-correlated to produce velocity fields. The plenoptic camera was fixed on a rotational stage below the water tunnel, viewing the wing along the span from tip to root. A schematic of the arrangement is shown in figure 3.5.

3.5.1 Definition of Inertial and Non-inertial Coordinate Systems

Plenoptic images were recorded when the wing was vertically aligned with the plenoptic camera, which required that the initial position of the wing was pre-set to the negative roll angle of a given phase. For example, when images were taken at $\phi = 12^\circ$, the initial position of the wing was set to $\phi = -12^\circ$, so when the wing crossed the vertical plane aligned with the camera, it had rolled 12° . Additionally, the camera was rotated on the rotational stage to match the geometric angle of attack of the wing at each phase for each kinematic case, such that the wing position in each recorded image was consistent from phase to phase. Thus, the data is presented in a coordinate system fixed to the wing. This coordinate system originates on the LE of the wing, on the surface of the suction side, and at the root, shown in figure 3.6a. When $\alpha_{\text{geo}} = 0^\circ$ and $\phi = 0^\circ$ the wing fixed coordinate system is equal to the lab coordinate system. The lab coordinate system is shown by XYZ in the orange box in 3.6b, where U , V and W are the corresponding velocities in each direction. The freestream velocity U_∞ is in the X direction. When the wing is rolled and pitched, the wing-fixed coordinate system, given by xyz , is displaced from the XYZ ground reference system by Euler angles ϕ about the roll axis, and α_{geo} about the pitch axis, shown in figure 3.6b. In the wing fixed coordinate system, velocity components are given by u , v and w , where u is, in general, not in the same direction as the freestream velocity U_∞ .

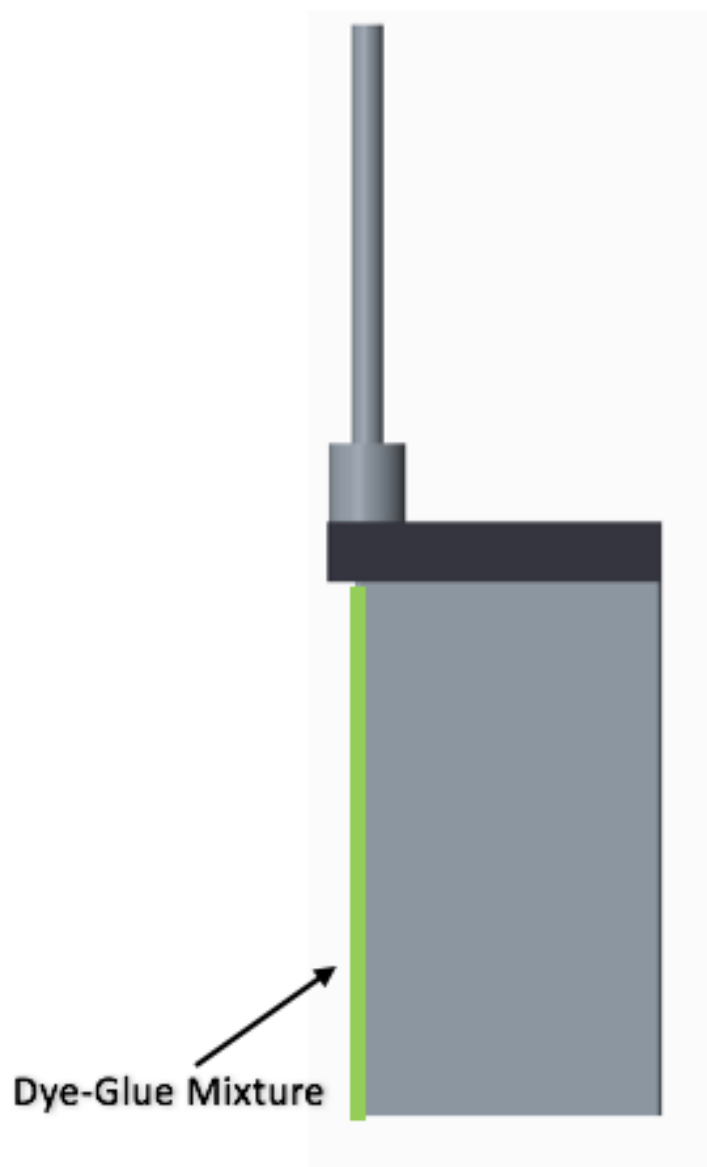


Figure 3.4: Application of the dye visualization

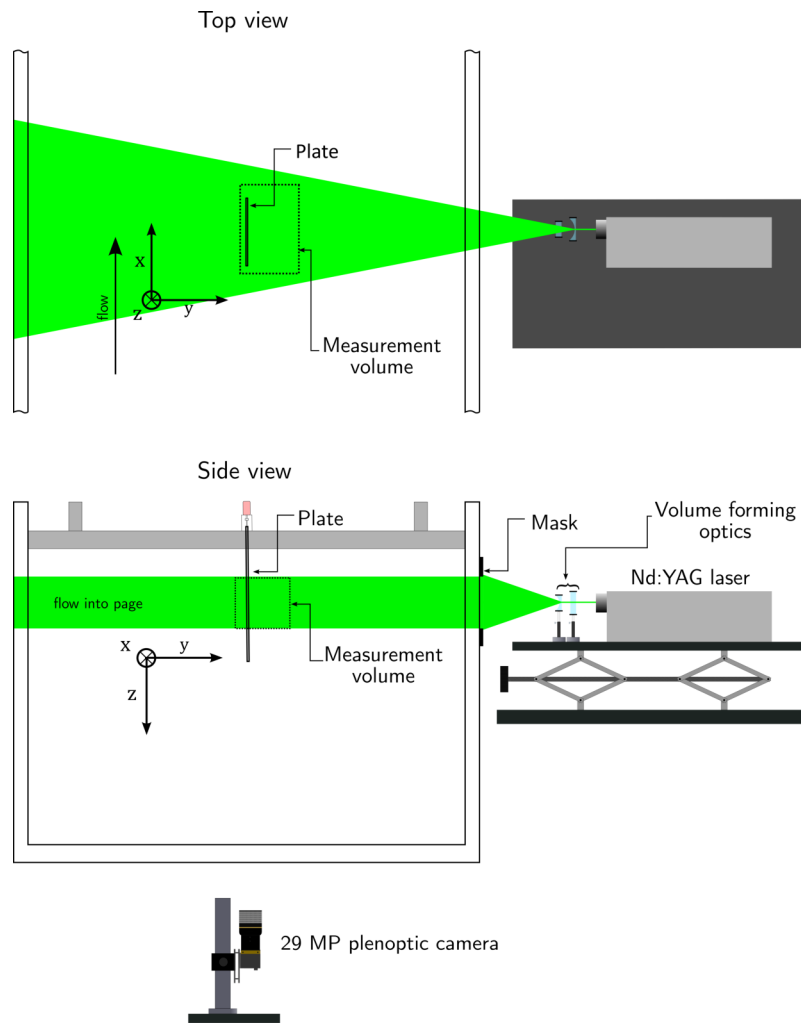


Figure 3.5: Experimental schematics: a top and side view of the experimental arrangement

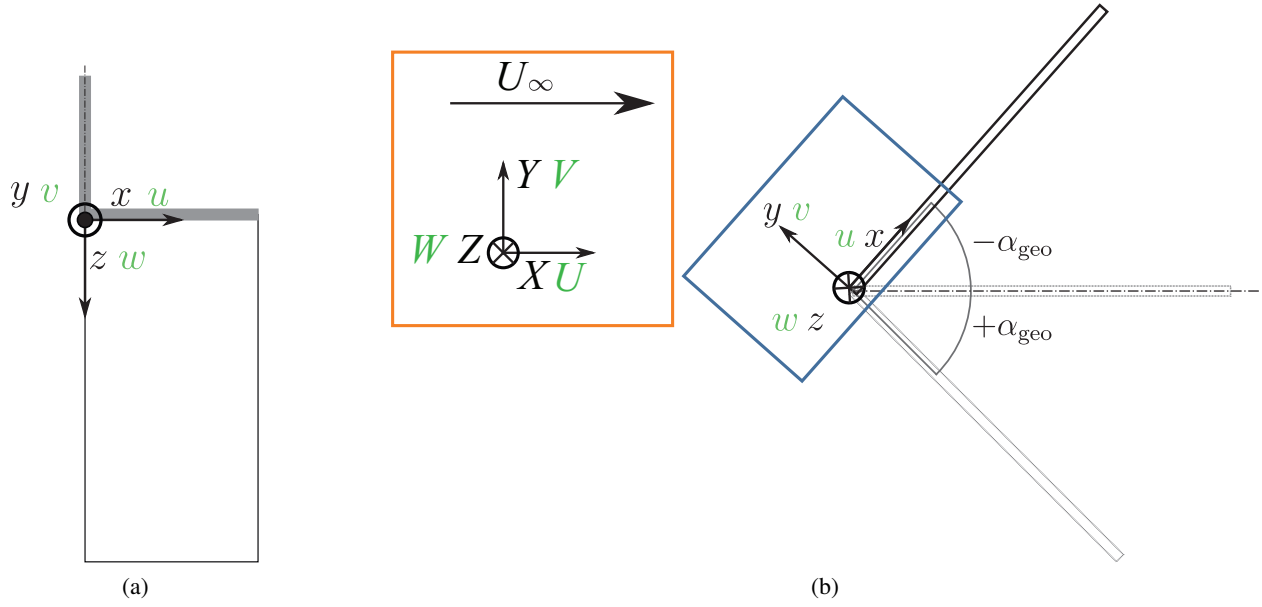


Figure 3.6: The wing-fixed reference frame illustrated, (a) plan view and (b) top view, where the velocity labels are shown in green and the physical space labels in black.

3.5.2 Measurement Volumes

The entire span of the wing was imaged in 3 overlapping regions, illustrated in figure 3.7. Each coloured region represents a different measurement volume and the dashed line marks the focal plane of each volume, which was also the centre of each volume. The imaging parameters are identical for each measurement volume, as the plenoptic camera and laser was only shifted along the z axis. Thus, each volume has the same dimensions, extending 68 mm in the z direction. The measurement volumes were centred at 20, 55, and 89 percent of the total span which allowed for an overlap of 14.7 mm and 16.2 mm for the root-centre and centre-tip interfaces, respectively. The volumes were precisely located using a 2 stage dot card that was translated in the z direction. A volumetric de-warping function, detailed by Hall et al. (2017), was created by imaging 15 z planes for each measurement volume, including locations outside the volume to help ensure better alignment when stitching volumes together. The translation stage on which the two-stage dot card was mounted was manually traversed, accurately located in the z direction to 0.05 mm using a vernier scale.

For every kinematic case, 100 image pairs were recorded except for $P_{.5}$, $R_{.54,33}$, and $S_{.46,.54}^{.22}$ (as defined in Section 3.8), at the centre volume, where 500 image pairs were initially utilized for determining convergence of the mean flow, but will also be fundamental in subsequent investigations.

3.5.3 Data reduction and analysis

The processing of the raw plenoptic particle image pairs began with a background subtraction algorithm based on the proper orthogonal decomposition. The reader is referred to Mendez et al. (2017) for the details of the algorithm, who used the technique on traditional PIV images of flow over a flapping wing. In the similar implementation here, the process typically resulted in 8–13 of the most energetic modes removed from each image, effectively acting as an intelligent high pass filter, removing all intensities but a sparse particle field.

Using the implementation of the MART algorithm described by Fahringer et al. (2015), the plenoptic

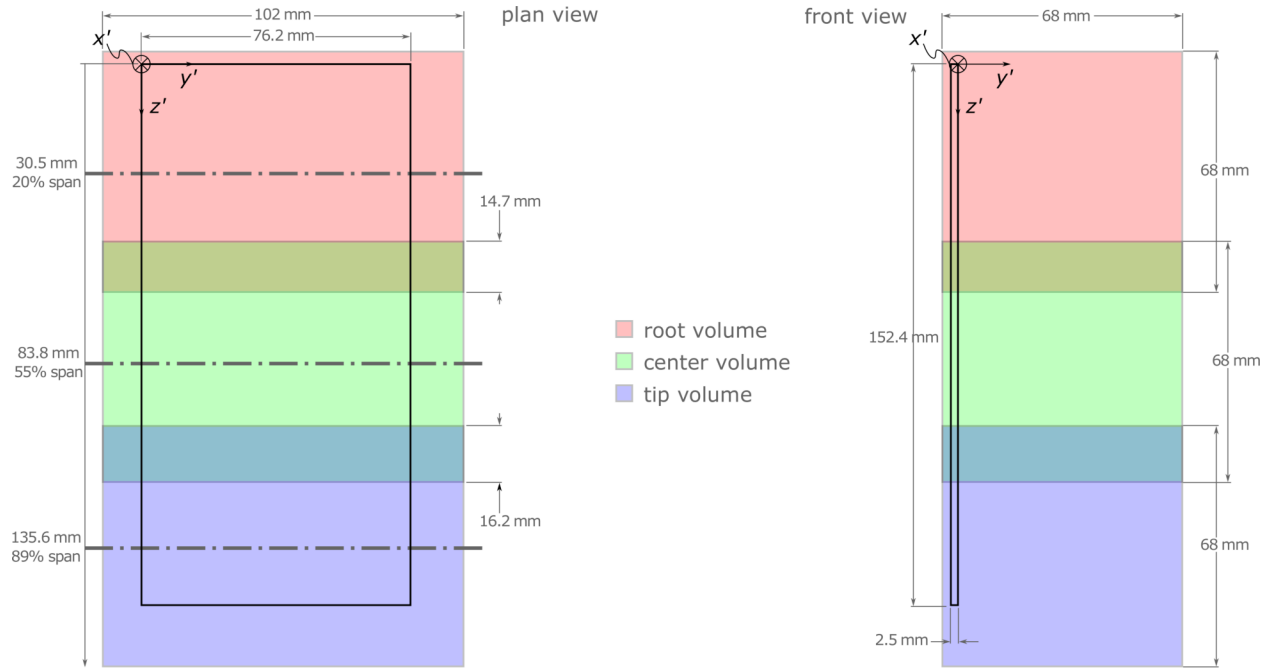


Figure 3.7: Plan and front view of the 3 different measurement volumes used during this study.

images were reconstructed into volumes using 3 iterations and a relaxation parameter $\mu = 1.0$. These values were selected by optimizing the normalized intensity variance, defined by Lynch (2015) as a criterion for quality of tomographic reconstruction utilized for tomographic PIV, and by minimizing the velocity field divergence to best satisfy continuity. The resulting volumes had dimensions of $527 \times 357 \times 357$ voxel³ in the x, y , and z directions, respectively. Each voxel is a cube of 0.19 mm side length. The volume pairs were then correlated using an in-house 3D cross-correlation algorithm consisting of 5 passes utilizing cubic windows of 64, 48, 32, 16, and 16 voxel sides for each pass, respectively. The algorithm is based on the VODIM (Scarano and Poelma, 2009), further detailed by Fahringer et al. (2015). Each pass had a window overlap of 50%; resulting in a final vector field of dimensions $63 \times 42 \times 42$ physically spaced by 1.5 mm.

For each phase, the leading edge was visually selected in the raw images, which was then used to set the x, y, z origin in every velocity field, ensuring proper alignment for averaging instantaneous fields. A phase-averaged field was then calculated, from which all of the ensuing quantities (vorticity, λ_{ci} -Criterion, etc.) were calculated. The swirling strength criterion (λ_{ci} -Criterion) (Zhou et al., 1999), which uses the imaginary component of the complex eigenvalue of the velocity gradient tensor to identify vortices, was used in this study. Multiple other vortex identification techniques were considered, each differing aesthetically, but the fundamental features and observations remained the same. Comparison between phases of various kinematic cases are made using normalized time $t' = t/\tau$, where t is the elapsed time since epoch for each phase and τ is the convective time defined as $\tau = c/V_{rel}$. Here, V_{rel} is the relative motion of the freestream to the wing, accounting for the motion associated with pitching and rolling manoeuvres.

The convergence of the chordwise velocity u is calculated for a $5 \times 5 \times 5$ vector³ region, far away from the wing surface. Using 100 image pairs, the converged value is within 0.4 percent of the converged value using 500 pairs. In the core of the vortex, where the chordwise velocity is small, the averaged chordwise velocity is 15 percent larger using 100 pairs than the when using 500 pairs. However, just outside of the vortex core, the velocity from the 100 pair average is about 1 percent different than the final averaged velocity. To aid in quantification of measurement uncertainty of this particular application of plenoptic PIV, 1000 image pairs of freestream flow were gathered with the identical experimental arrangement and imaging parameters,

with the wing removed. Using the same processing techniques, the vector fields were created and averaged. The standard deviation of each velocity component was calculated at every point in the volume, allowing for the uncertainty to be quantified spatially. The velocity components have uncertainty of approximately $\sigma_u = 2.3$, $\sigma_v = 1.8$, and $\sigma_w = 4.0$ percent of the freestream velocity. The most notable variation is along the z axis, where the standard deviation increases for each velocity component at the $\pm z$ edges of the volume to approximately $\sigma_u = 5.6$, $\sigma_v = 4.8$, and $\sigma_w = 8.4$ percent of the free stream. This is the region where phase-averaged results at each measurement volume on the wing have been stitched together, which can lead to slight misalignment and non-continuous structures. The primary method of presenting results in the next section is through use of isocontours of λ_{ci} , in which the stitched together regions are occasionally visible. The reduced accuracy of the w measurement, further detailed by Fahringer et al. (2015), is considered in the results presented herein.

3.6 Force measurements

Lift and drag force measurements were conducted for the pure rolling wing with a 6-axis ATI Mini-40 force transducer attached to the shaft of the pitch motor, and thus was fixed in the wing reference frame. Whereas the objective of the force measurements was to isolate the hydrodynamic loading on the wing, the total force recorded by the transducer during the wing motions included contributions due to buoyancy, static gravitational, and wing inertial forces, such that, for a given force component, i :

$$F_{total,i} = F_{H,i} - F_{G,i} + F_{B,i} \quad (3.11)$$

where $F_{total,i}$ is the total force component measured by the force transducer, $F_{H,i}$ is the (desired) hydrodynamic force acting on the wing, and $F_{G,i}$, $F_{B,i}$, and $F_{I,i}$ are the gravitational, buoyancy, and wing inertial forces. Isolation of the hydrodynamic forces required multiple force measurements with the wing articulated with the prescribed kinematics, as well as in a quasi-static manner, while submerged in water, and without water. Thus the measured quantities also included the force due to articulation of the wing in air with the prescribed kinematics F_{air} , the force due to quasi-static motion (through the roll range) in air $F_{static,air}$, and the force due to quasi-static motion in water $F_{static,water}$. These constituent quantities are comprised of F_H , F_G , F_B , and F_I as follows:

$$F_{air} = F_I + F_G, \quad (3.12)$$

$$F_{static,water} = F_G + F_B, \quad (3.13)$$

$$F_{static,air} = F_G, \quad (3.14)$$

such that:

$$F_H = F_{total} - F_{air} - F_{static,water} - F_{static,air}, \quad (3.15)$$

and thus:

$$F_H = (F_H + F_G + F_I + F_B) - (F_I + F_G) - (F_G + F_B) - (F_G). \quad (3.16)$$

The *static* forces were recorded on the wing in 5° roll-angle increments ranging from $\phi = 0^\circ$ to 60° .

3.7 Vorticity Transport Analysis

The objectives of the vorticity transport analysis are to identify the sources and sinks of circulation governing the strength of the leading-edge vortex, and to therefore provide a framework through which to understand the physical mechanisms influencing vortex growth and decay, and to ultimately control them. The analysis is conducted within a planar control region attached to the maneuvering wing, with surface normal parallel

to the z -axis (i.e. the spanwise direction) in a non-inertial reference frame attached to the wing, as shown in Figure 3.8. In constructing the control region this way, the local velocity in the reference frame is zero on the boundary adjacent to the wing and, as will be shown, it can be accurately estimated using surface pressure measurements.

3.7.1 The Vorticity Flux Equation in a Non-Inertial Reference Frame

For a fluid with constant properties, the rate of change of circulation within a control region can be expressed as in equation 3.17 by substitution of the Navier-Stokes equation (Potter and Foss, 1982):

$$\begin{aligned} \frac{d\Gamma}{dt} &= \oint \frac{\partial \mathbf{u}}{\partial t} \cdot d\mathbf{s} \\ &= - \int_A [\nabla \times (\boldsymbol{\omega} \times \mathbf{u})] \cdot \mathbf{n}_A dA - \oint_{\partial A} \frac{dp}{\rho} - \oint_{\partial A} \mathbf{a}_I \cdot d\mathbf{s} + \oint_{\partial A} \nu \nabla^2 \mathbf{u} \cdot d\mathbf{s} \end{aligned} \quad (3.17)$$

where, for the planar control region shown in Figure 3.8, $\mathbf{n}_A = \mathbf{e}_z$ is the surface normal to the plane of the control region, and $\mathbf{a}_I$ is the local acceleration in the inertial frame, which can be written as:

$$\mathbf{a}_I = -2\boldsymbol{\Omega} \times \mathbf{u} - \dot{\boldsymbol{\Omega}} \times \mathbf{r} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}), \quad (3.18)$$

where $\dot{}$ indicates differentiation with respect to time. The terms on the right side of equation 3.18 are expressed in terms of the rotation rate of the non-inertial coordinate system $\boldsymbol{\Omega}$, which is measured in a (stationary) inertial system with components $(\Omega_X, \Omega_Y, \Omega_Z)$, and the velocity $\mathbf{u}$ and position $\mathbf{r}$ vectors measured in the non-inertial system. The terms are, from left, to right, the Coriolis, angular, and centripetal accelerations, respectively. Evaluation of the non-inertial accelerations, thus requires transformation of the angular velocities and accelerations into components of the non-inertial system.

Rotational Velocities and Accelerations in the Non-Inertial System

The non-inertial system (xyz) is fixed to the wing, which is maneuvering within a stationary, inertial (XYZ) frame. In general, the vectors $\boldsymbol{\Omega}$ and $\dot{\boldsymbol{\Omega}}$ represent the angular velocity and acceleration of the non-inertial system relative to the inertial system. In the present study, the wing is articulated in pitching and/or rolling motions such that the roll axis is coincident with the X axis of the inertial reference frame. The pitch axis is fixed in the non-inertial frame – coincident with the z -axis – such that it is normal to and revolves around the stationary X -axis (i.e. within the inertial $Y - Z$ plane). Therefore, the rotational velocity and acceleration of

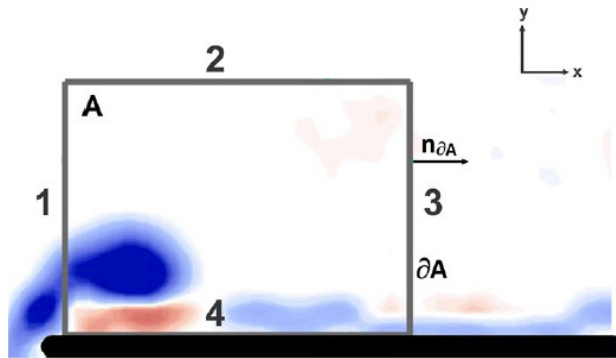


Figure 3.8: Control region in which the vorticity flux analysis will be computed. This is in the non-inertial, plate aligned coordinate system.

the roll motion is specified relative to the inertial reference frame, and the rotational velocity and acceleration of the pitch motion is specified relative to the non-inertial (rolling) reference frame.

The control region in which (3.17) will be evaluated is fixed to the rolling and pitching wing, as is the non-inertial reference frame. It is therefore necessary to first determine $\mathbf{\Omega}$ and $\dot{\mathbf{\Omega}}$ of the wing, relative to a fixed reference frame, and then map these vectors to the non-inertial system in order to compute the cross products in Equation (3.18). The following discussion elucidates these transformations for the rolling, pitching, and combined rolling and pitching motions.

For a purely rolling wing, the non-inertial coordinate system is revolving at constant angular velocity $\mathbf{\Omega} = (\Omega_X, 0, 0)$ in the inertial coordinate system. Beginning with coincident inertial and non-inertial coordinate systems, their relative orientations at a given point in time can be described as two sequential rotations (Euler angles):

1. a rotation of angle ϕ about the X -axis due to the rolling motion at angular velocity Ω , and
2. a static rotation of angle θ about the non-inertial z -axis in order to achieve an angle of attack α at the radius of gyration of the wing, through vector addition of the transverse velocity induced by the rolling motion and the free stream.

A vector $\mathbf{c} = (c_X, c_Y, c_Z)$ in the inertial system can be expressed as $\mathbf{c}'' = (c_x, c_y, c_z)$ in the non-inertial system through the product of two transformations:

$$\mathbf{c}'' = [T_{pitch}][T_{roll}]\mathbf{c} \quad (3.19)$$

where the $[T_{roll}]$ transformation is defined as:

$$\begin{bmatrix} c_{x'} \\ c_{y'} \\ c_{z'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} c_X \\ c_Y \\ c_Z \end{bmatrix} \quad (3.20)$$

and the transformation about the (revolving) z -axis, $[T_{pitch}]$ is given by:

$$\begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{x'} \\ c_{y'} \\ c_{z'} \end{bmatrix} \quad (3.21)$$

Thus, in (3.18), the components of $\mathbf{\Omega}$ used to compute the cross products can be calculated by¹:

$$\begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \Omega_X \\ \Omega_Y \\ \Omega_Z \end{bmatrix} \quad (3.22)$$

or:

$$\begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \cos \phi & \sin \theta \sin \phi \\ -\sin \theta & \cos \theta \cos \phi & \cos \theta \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \Omega_X \\ \Omega_Y \\ \Omega_Z \end{bmatrix} \quad (3.23)$$

Therefore, since the roll rate vector is given in the inertial (X, Y, Z) coordinate system as $\mathbf{\Omega} = (\Omega_X, 0, 0)$ the components of $\mathbf{\Omega}$ in the non-inertial coordinate system are given, for the pure roll case, by:

$$\Omega_x = \Omega_X \cos \theta, \quad (3.24)$$

$$\Omega_y = -\Omega_X \sin \theta, \quad (3.25)$$

$$\Omega_z = 0. \quad (3.26)$$

¹The transformation matrix can be applied to any vector, and will be used later also to transform the angular acceleration.

For pure roll, the pitch angle is static. Therefore, the orientation of $\mathbf{\Omega}$ does not change with time, from the perspective of the non-inertial system. Therefore, if the roll angular velocity is constant, $\dot{\mathbf{\Omega}} = 0$ from the perspectives of both the inertial and non-inertial systems.

For the pure pitch case, the transformation is fully-described by $[T_{pitch}]$ so that:

$$\begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Omega_X \\ \Omega_Y \\ \Omega_Z \end{bmatrix}. \quad (3.27)$$

Therefore, for $\mathbf{\Omega} = (0, 0, \Omega_Z)$ expressed in the non-inertial system, we can easily show that $\Omega_z = \Omega_Z$, so that the angular velocity does not need to be transformed. Similarly, if the plate is pitching at constant rate, then $\dot{\mathbf{\Omega}} = 0$.

The combined pitch-roll motion is considerably more complex since the pitch axis is revolving around the roll axis in the inertial system. The roll angular velocity, $\mathbf{\Omega}$ is therefore changing with time, even if the magnitudes of the individual pitch and roll rates are not, and thus $\dot{\mathbf{\Omega}} \neq 0$. In this case, we therefore have a non-zero non-inertial angular acceleration contribution, $\dot{\mathbf{\Omega}} \times \mathbf{r}$. Therefore to quantify the cross-product, we first need to determine the components of $\dot{\mathbf{\Omega}}$ in the inertial system, and then transform them to the non-inertial system using (3.20) and (3.21).

We will, again, assume a constant roll rate with the roll axis coincident with the X -axis. Thus, the pitch axis revolves in the $Y - Z$ plane, and remains orthogonal to the roll axis. At a given point in time, the pitch axis (i.e. z -axis) is displaced from the Z -axis by an angle ϕ . The angular velocity at which the non-inertial reference frame is rotating is, thus, given by:

$$\mathbf{\Omega} = \mathbf{\Omega}_{roll} + \mathbf{\Omega}_{pitch} \quad (3.28)$$

where the roll rate is expressed in the inertial frame as:

$$\mathbf{\Omega}_{roll} = (\Omega_{roll}, 0, 0), \quad (3.29)$$

and, as illustrated in Figure 3.9, the pitch angular velocity can be expressed as a vector with the following components in the (XYZ) system:

$$\mathbf{\Omega}_{pitch} = \begin{bmatrix} \Omega_{pitch,X} \\ \Omega_{pitch,Y} \\ \Omega_{pitch,Z} \end{bmatrix} = \begin{bmatrix} 0 \\ -\Omega_{pitch} \sin \phi \\ \Omega_{pitch} \cos \phi \end{bmatrix} \quad (3.30)$$

Therefore, the total rotation vector is given, in the inertial frame, as:

$$\mathbf{\Omega} = (\Omega_{roll}, \Omega_{pitch,Y}, \Omega_{pitch,Z}) \quad (3.31)$$

$$= (\Omega_{roll}, -\Omega_{pitch} \sin \phi, \Omega_{pitch} \cos \phi) \quad (3.32)$$

Transforming $\mathbf{\Omega}$, for the combined pitch-roll case to the non-inertial reference frame using (3.23), we arrive at:

$$\begin{aligned} \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} &= \begin{bmatrix} \cos \theta & \sin \theta \cos \phi & \sin \theta \sin \phi \\ -\sin \theta & \cos \theta \cos \phi & \cos \theta \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \Omega_{roll} \\ -\Omega_{pitch} \sin \phi \\ \Omega_{pitch} \cos \phi \end{bmatrix} \\ &= \begin{bmatrix} \Omega_{roll} \cos \theta - \Omega_{pitch} \sin \theta \sin \phi \cos \phi + \Omega_{pitch} \sin \theta \sin \phi \cos \phi \\ -\Omega_{roll} \sin \theta - \Omega_{pitch} \cos \theta \sin \phi \cos \phi + \Omega_{pitch} \cos \theta \sin \phi \cos \phi \\ \Omega_{pitch} \sin^2 \phi + \Omega_{pitch} \cos^2 \phi \end{bmatrix} \\ &= \begin{bmatrix} \Omega_{roll} \cos \theta \\ -\Omega_{roll} \sin \theta \\ \Omega_{pitch} \end{bmatrix} \end{aligned} \quad (3.33)$$

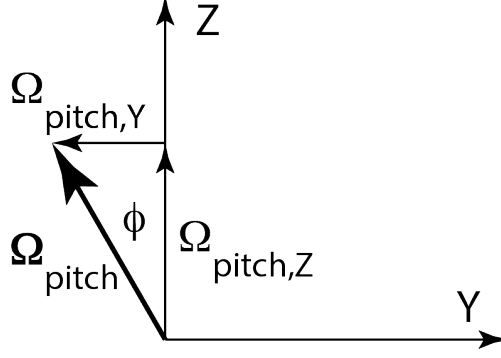


Figure 3.9: Components of the pitch-rate vector in the reference (XYZ) coordinate system. The X -axis, which is coincident with the roll axis, is out of the page.

As expected, this is simply equal to the roll angular velocity, described in equations (3.24) through (3.26), plus the pitch motion, which contributes only to the z -component of the angular velocity in the non-inertial frame.

Assuming that the roll- and pitch angular velocities are constant, the only contribution to $\dot{\mathbf{\Omega}}$ is due to the rotation of the pitch-rotation vector $\mathbf{\Omega}_{pitch}$ by the action of the roll motion, given by $\mathbf{\Omega}_{roll}$. Therefore:

$$\dot{\mathbf{\Omega}} = \mathbf{\Omega}_{roll} \times \mathbf{\Omega}_{pitch}, \quad (3.34)$$

where $\mathbf{\Omega}_{roll}$ is given by (3.29) and $\mathbf{\Omega}_{pitch}$ is given by (3.30). The angular acceleration can thus be expressed as components in the inertial system:

$$\begin{aligned} \begin{bmatrix} \dot{\Omega}_X \\ \dot{\Omega}_Y \\ \dot{\Omega}_Z \end{bmatrix} &= \begin{bmatrix} \mathbf{e}_X & \mathbf{e}_Y & \mathbf{e}_Z \\ \Omega_{roll} & 0 & 0 \\ 0 & -\Omega_{pitch} \sin \phi & \Omega_{pitch} \cos \phi \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -\Omega_{roll} \Omega_{pitch} \cos \phi \\ -\Omega_{roll} \Omega_{pitch} \sin \phi \end{bmatrix} \end{aligned} \quad (3.35)$$

Finally, for evaluation of (3.18), the components of the combined pitch-roll angular acceleration in inertial coordinates are transformed to the non-inertial system using the transformation matrix given in (3.23):

$$\begin{aligned} \begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \end{bmatrix} &= \begin{bmatrix} \cos \theta & \sin \theta \cos \phi & \sin \theta \sin \phi \\ -\sin \theta & \cos \theta \cos \phi & \cos \theta \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} 0 \\ -\Omega_{roll} \Omega_{pitch} \cos \phi \\ -\Omega_{roll} \Omega_{pitch} \sin \phi \end{bmatrix} \\ &= \begin{bmatrix} -\Omega_{roll} \Omega_{pitch} (\sin \theta \cos^2 \phi + \sin \theta \sin^2 \phi) \\ -\Omega_{roll} \Omega_{pitch} (\cos \theta \cos^2 \phi + \cos \theta \sin^2 \phi) \\ -\Omega_{roll} \Omega_{pitch} (-\sin \phi \cos \phi + \sin \phi \cos \phi) \end{bmatrix} \\ &= \begin{bmatrix} -\Omega_{roll} \Omega_{pitch} \sin \theta \\ -\Omega_{roll} \Omega_{pitch} \cos \theta \\ 0 \end{bmatrix}. \end{aligned} \quad (3.36)$$

By comparison with (3.35), it is easy to see that the transformation does not change the magnitude of the angular acceleration vector.

Computation of the Non-Inertial Transport Terms

Following Eslam Panah et al. (2015), who derived the transport equation in a non-rotating reference frame, equation 3.17 can be represented in terms of interpretable convective, diffusive, and tilting fluxes:

$$\begin{aligned} \frac{d\Gamma}{dt} = & - \int_{A_z} \left(u_z \frac{\partial \omega_z}{\partial z} \right) dA - \oint (\mathbf{u} \cdot \mathbf{n}_{\partial A}) \omega_z ds - \nu \int_{\text{boundary 4}} \frac{\partial \omega_z}{\partial y} dx \\ & + \int_{A_z} \left(\omega_x \frac{\partial u_z}{\partial x} + \omega_y \frac{\partial u_z}{\partial y} \right) dA - \oint_{\partial A} \mathbf{a}_I \cdot d\mathbf{s} \end{aligned} \quad (3.37)$$

Specifically, the terms on the right side of equation 3.37 are contributions to the growth or decay of circulation in the control region due to (left to right on the first line): the spanwise convective flux of vorticity through the plane of the control region; in-plane convective fluxes of vorticity through the boundaries of the control region, and the diffusive flux of vorticity from the surface of the wing. The second line of equation 3.37 contains *artifact* contributions due to tilting of vorticity² and the non-inertial accelerations of the wing. Since linear accelerations do not affect the representation or transport of vorticity in the control region, they are not represented in equation 3.18.

The angular acceleration term can be expressed in terms of the rotational acceleration of the non-inertial system, as given by (3.36) in the non-inertial system ($\dot{\mathbf{\Omega}} = (\dot{\Omega}_x, \dot{\Omega}_y, \dot{\Omega}_z)$):

$$- \oint_{\partial A} \mathbf{a}_{I,ang} \cdot d\mathbf{s} = \oint_{\partial A} (\dot{\mathbf{\Omega}} \times \mathbf{r}) \cdot d\mathbf{s} \quad (3.38)$$

$$= \oint_{\partial A} \left[\mathbf{e}_x (\dot{\Omega}_y r_z - \dot{\Omega}_z r_y) + \mathbf{e}_y (\dot{\Omega}_z r_x - \dot{\Omega}_x r_z) + \mathbf{e}_z (\dot{\Omega}_x r_y - \dot{\Omega}_y r_x) \right] \cdot d\mathbf{s}. \quad (3.39)$$

For the planar control region in the $x - y$ plane, shown in figure 3.8, $d\mathbf{s} = \mathbf{e}_x dx + \mathbf{e}_y dy$, so that the scalar product with the z -component of the vector in brackets vanishes. Therefore (3.39) reduces to:

$$- \oint_{\partial A} \mathbf{a}_{I,ang} \cdot d\mathbf{s} = \oint_{\partial A} (\dot{\mathbf{\Omega}} \times \mathbf{r}) \cdot d\mathbf{s} = \oint_{\partial A} \left[(\dot{\Omega}_y r_z - \dot{\Omega}_z r_y) dx + (\dot{\Omega}_z r_x - \dot{\Omega}_x r_z) dy \right]. \quad (3.40)$$

Applying Green's theorem converts the contour integral into the following area integral:

$$- \oint_{\partial A} \mathbf{a}_{I,ang} \cdot d\mathbf{s} = \iint_A \left[2\dot{\Omega}_z - \left(\dot{\Omega}_x \frac{\partial r_z}{\partial x} + \dot{\Omega}_y \frac{\partial r_z}{\partial y} \right) \right] dA. \quad (3.41)$$

As elucidated in the previous section, of the cases considered in this study, $\mathbf{\Omega}$ is non-zero only for the combined pitch-roll case. The $2\dot{\Omega}_z$ term represents the development of an apparent solid-body rotation in the presence of a time-varying pitch angular velocity ($\dot{\Omega}_z$); however, in the pitch-roll case, (3.36) shows that $\dot{\Omega}_z = 0$. Furthermore, since the control region exists in the $x - y$ plane, the derivatives of r_z also vanish, such that there is no contribution to the circulation budget due to the angular accelerations in the non-inertial system.

The centripetal acceleration can be examined by application of a vector identity to yield:

$$\begin{aligned} - \oint_{\partial A} \mathbf{a}_{I,cen} \cdot d\mathbf{s} &= \oint_{\partial A} \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) \cdot d\mathbf{s} \\ &= \oint_{\partial A} [\mathbf{\Omega}(\mathbf{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\mathbf{\Omega} \cdot \mathbf{\Omega})] \cdot d\mathbf{s}. \end{aligned} \quad (3.42)$$

²Vorticity tilting does not change the circulation of a vortex; however, the interaction of a continuous distribution of vorticity with the finite boundaries of the control region results in a contribution due to vortex tilting.

Again, evaluating and expanding the scalar products on the planar control region yields:

$$\begin{aligned}
-\oint_{\partial A} \mathbf{a}_{I, cen} \cdot d\mathbf{s} &= \oint_{\partial A} \left\{ [(\Omega_x r_x + \Omega_y r_y + \Omega_z r_z) \Omega_x - \Omega^2 r_x] dx \right. \\
&\quad \left. + [(\Omega_x r_x + \Omega_y r_y + \Omega_z r_z) \Omega_y - \Omega^2 r_y] dy \right\}
\end{aligned} \tag{3.43}$$

Applying Green's theorem to convert equation 3.43 to an area integral reveals that the centripetal acceleration makes no contribution to the rate of change of circulation within the control region:

$$-\oint_{\partial A} \mathbf{a}_{I, cen} \cdot d\mathbf{s} = \iint_A (\Omega_y \Omega_x - \Omega_x \Omega_y) dx dy = 0 \tag{3.44}$$

Evaluating the closed contour integral of the Coriolis acceleration about the control region depicted in figure 3.8 yields:

$$\begin{aligned}
-\oint_{\partial A} \mathbf{a}_{I, Cor} \cdot d\mathbf{s} &= -\oint_{\partial A} (2\boldsymbol{\Omega} \times \mathbf{u}) \cdot d\mathbf{s} \\
&= -\oint_{\partial A} \left[(\Omega_y w - \Omega_z v) \hat{i} - (\Omega_x w - \Omega_z u) \hat{j} + (\Omega_x v - \Omega_y u) \hat{k} \right] \cdot d\mathbf{s} \\
&= -\oint_{\partial A} \left\{ 2(\Omega_y u_z - \Omega_z u_y) dx + 2(\Omega_z u_x - \Omega_x u_z) dy \right\} \\
&= -\oint_{\partial A} 2\Omega_z (-u_y dx + u_x dy) - \oint_{\partial A} 2(\Omega_y u_z dx - \Omega_x u_z dy) \\
&= -\oint_{\partial A} 2\Omega_z (\mathbf{u} \cdot \mathbf{n}) ds - \oint_{\partial A} (\Omega_y u_z dx - \Omega_x u_z dy)
\end{aligned} \tag{3.45}$$

The first term is the integrated convective flux of apparent z -vorticity ($2\Omega_z$) due to the perceived solid body rotation of fluid within the rotating reference frame as a result of a pitch rotation. Thus it is a correction to the augmented ω_z measured in the rotating frame. The second term can be further manipulated, using Green's theorem:

$$-\oint_{\partial A} (\Omega_y u_z dx - \Omega_x u_z dy) = \iint_A \left(2\Omega_x \frac{\partial u_z}{\partial x} + 2\Omega_y \frac{\partial u_z}{\partial y} \right) dA. \tag{3.46}$$

This expression is identical in structure to the tilting term in equation 3.37, with the exception that the vorticity components ω_x and ω_y in equation 3.37 are replaced by the apparent vorticity components $2\Omega_x$ and $2\Omega_y$ in equation 3.46. Whereas the measured vorticity vector $\boldsymbol{\omega} = (\omega_x, \omega_y, \omega_z)$ is free to tilt, the *apparent* vorticity components, $2\Omega_x$ and $2\Omega_y$ are determined by the kinematics of the reference frame, and thus cannot provide any real contribution to the vorticity dynamics. Thus, this apparent tilting is, as with the $2\Omega_z$ term, a correction to fictitious contributions to the tilting term of equation 3.37 manifest in the measurement of ω_x and ω_y within the rotating frame.

Combining all the terms gives the following equation for a wing articulated in a general pitching and rolling motion:

$$\begin{aligned}
\frac{d\Gamma}{dt} &= - \int_{A_z} \left(u_z \frac{\partial \omega_z}{\partial z} \right) dA - \oint (\mathbf{u} \cdot \mathbf{n}_{\partial A}) \omega_z ds - \nu \int_{\text{boundary } 4} \frac{\partial \omega_z}{\partial y} dx \\
&\quad + \int_{A_z} \left(\omega_x \frac{\partial u_z}{\partial x} + \omega_y \frac{\partial u_z}{\partial y} \right) dA - \oint_{\partial A} 2\Omega_z (\mathbf{u} \cdot \mathbf{n}_{\partial A}) ds - \iint_A \left(2\Omega_x \frac{\partial u_z}{\partial x} + 2\Omega_y \frac{\partial u_z}{\partial y} \right) dA.
\end{aligned} \tag{3.47}$$

The last two terms in the second line are the only remaining influences from the rotational accelerations. Both are artifacts of the Coriolis acceleration. The first of the two is a correction to the in-plane fluxes measured in the non-inertial frame (the pitching motion creates an apparent solid body rotation of the surrounding fluid from the perspective of the non-inertial system), and a correction to the measured tilting fluxes. In a non-rotating frame, the surface-diffusive flux of vorticity due to its generation on boundary 4 of the control region can be accurately approximated by the streamwise component of the pressure gradient acting on the wing surface (Lighthill, 1963; Morton, 1984; Wu and Wu, 1993; Eslam Panah et al., 2015):

$$-v \frac{\partial \omega_z}{\partial y} \approx \frac{1}{\rho} \frac{\partial p}{\partial x}. \quad (3.48)$$

The surface-diffusive flux for the rotating system considered in the present case is discussed in the following section.

3.7.2 The Surface-diffusive Flux of Vorticity

The diffusive flux of vorticity from the surface of the wing depends on the wall-normal gradient of the vorticity at the surface. Direct measurement would require the velocity field near the surface of the airfoil to be very accurately resolved. This was achieved within the works of both Shih and Ho Shih and Ho (1994) and Kuo and Hsieh Kuo and Hsieh (2001) through the use of Laser Doppler Anemometry (LDA). The use of particle image velocimetry (PIV) in the present work means that measurements will have reduced accuracy near the wall due to laser reflections and other imaging artifacts near the wall. Furthermore, particle seeding considerations will limit the resolution at which the velocity field can be specified. It is therefore desirable to modify the diffusive flux integral over boundary 4 in Equation 3.47 so that the diffusive vorticity flux can be characterized without the use of the near-wall velocity field.

The generation of vorticity on a boundary has been studied extensively by Wu et al. (Wu and Wu, 1993, 1998; J.-Z. Wu and Zhou, 2007); however, their analyses do not take into account the rotational accelerations given in (3.18). The following derivation follows an approach similar to that used by Wu et al. (Wu and Wu, 1993, 1998), but taking into account the non-inertial accelerations.

Starting with the Navier-Stokes equation, Wu and Wu (1993) developed the following relation:

$$\mathbf{a} + \frac{\nabla p}{\rho} + \nu \nabla \times \boldsymbol{\omega} = 0, \quad (3.49)$$

where $\mathbf{a}$ is the acceleration of the surface. In the present case, we substitute $\mathbf{a}_I$ from (3.18). Considering the constraints imposed by the control region shown in figure 3.8, the angular acceleration, centripetal, and Coriolis terms reduce to:

$$-\dot{\boldsymbol{\Omega}} \times \mathbf{r} = (\dot{\Omega}_z r_y - \dot{\Omega}_y r_z) \mathbf{e}_x + (\dot{\Omega}_x r_z - \dot{\Omega}_z r_x) \mathbf{e}_y + (\dot{\Omega}_y r_x - \dot{\Omega}_x r_y) \mathbf{e}_z \quad (3.50)$$

$$\begin{aligned} -\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) = & -(\Omega_x \Omega_y r_y + \Omega_x \Omega_z r_z - \Omega_y^2 r_x - \Omega_z^2 r_x) \mathbf{e}_x \\ & -(\Omega_x \Omega_y r_x + \Omega_y \Omega_z r_z - \Omega_x^2 r_y - \Omega_z^2 r_y) \mathbf{e}_y \\ & -(\Omega_x \Omega_z r_x + \Omega_y \Omega_z r_y - \Omega_x^2 r_z - \Omega_y^2 r_z) \mathbf{e}_z \end{aligned} \quad (3.51)$$

$$-2\boldsymbol{\Omega} \times \mathbf{u} = 2[(\Omega_z u_y - \Omega_y u_z) \mathbf{e}_x + (\Omega_x u_z - \Omega_z u_x) \mathbf{e}_y + (\Omega_y u_x - \Omega_x u_y) \mathbf{e}_z] \quad (3.52)$$

Wu and Wu (1993) asserted that vorticity is generated at a boundary when a force is applied to a surface-bound fluid element so that the principal axes of the local strain-rate tensor are rotated out of the wall-normal direction. This definition was proposed, noting that the no-slip condition prevents a wall-bound fluid element from rotating, and that a vorticity-containing fluid element only develops an angular velocity after it has been

diffused into the flow. However, the majority of studies are content with characterizing the process as if the wall-bounded fluid element were actually permitted to rotate Lighthill (1963); Morton (1984); Wu et al. (1993); Brons et al. (2014).

We consider the surface of the wing, with the origin of the non-inertial coordinate system coincident with the (fixed) roll axis. Therefore, the (thin) wing has the surface-normal unit vector $\mathbf{e}_y$ and surface-tangent unit vectors $\mathbf{e}_x$ (chordwise) and $\mathbf{e}_z$ (spanwise), such that $\mathbf{e}_z = \mathbf{e}_x \times \mathbf{e}_y$. The surface of the wing is therefore defined by $r_y \approx 0$.

The generation of vorticity on the surface can be quantified by taking the cross product of the surface-normal unit vector ($\mathbf{e}_y$) and Equation 3.49. For the control region of figure 3.8, only the z -component is needed to get the relevant vorticity flux. This yields:

$$\begin{aligned} 0 &= \left[(\mathbf{e}_y \times \mathbf{a}) + (\mathbf{e}_y \times \frac{\nabla p}{\rho}) + \nu(\mathbf{e}_y \times (\nabla \times \vec{\omega})) \right] \cdot \mathbf{e}_z \\ &= \dot{\Omega}_y r_z + \Omega_x \Omega_z r_z - (\Omega_y^2 + \Omega_z^2) r_x - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial \omega_y}{\partial z} - \frac{\partial \omega_z}{\partial y} \right) \end{aligned} \quad (3.53)$$

The diffusive flux of vorticity of relevance to the analysis is $-\nu(\partial \omega_z / \partial y)$, which can be obtained by rearranging equation 3.53:

$$-\nu \frac{\partial \omega_z}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} - \nu \left(\frac{\partial \omega_y}{\partial z} \right) - \dot{\Omega}_y r_z - \Omega_x \Omega_z r_z + (\Omega_y^2 + \Omega_z^2) r_x. \quad (3.54)$$

The second term on the RHS of equation 3.54 has been associated with vorticity generation mechanisms that occur on curved surfaces (Wu and Wu, 1993; Wu et al., 2005), and can thus be ignored for the present analysis, where the control region intersects a flat boundary. Thus, for the problems investigated in the present work, the relevant surface diffusive flux is governed by the surface pressure gradient ($\partial p / \partial x$), and modified by the non-inertial accelerations due to the rotational velocities and accelerations of the plate:

$$-\nu \frac{\partial \omega_z}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} - \dot{\Omega}_y r_z - \Omega_x \Omega_z r_z + (\Omega_y^2 + \Omega_z^2) r_x. \quad (3.55)$$

It is necessary to consider the non-inertial terms in equation 3.55 in the context of the components of angular velocities and accelerations developed earlier in this chapter. In the case of pure roll, the angular acceleration vanishes, for constant roll velocity, and the angular velocity components are given by equations 3.24 through 3.26, where it is shown that $\Omega_z = 0$. Therefore, the non-inertial contribution to the surface-diffusive flux reduces to $-\Omega_y^2 r_x$ for pure roll. For the pure-pitch case, there are, similarly, no angular accelerations and the only non-zero component of the angular velocity is Ω_z . Therefore, the non-inertial contribution reduces to $\Omega_z^2 r_x$. Finally, in the combined pitch and roll cases, the angular velocity is given by equation 3.33 and the angular acceleration is given by equation 3.36. Thus, all of the terms in equation 3.55 are relevant.

3.8 Description of Wing Configuration Cases and Measurements

Wing motions were prescribed as linear ramps in pitch and/or roll angle, smoothed using the Eldredge function (Eldredge and Wang, 2010) with a smoothing parameter of 1.2. Sample wing kinematics, for the pitch, roll, and combined pitch-roll cases, are shown in figure 3.10. During the pure pitch manoeuvre, the wing is initially set to $\alpha_{\text{geo}} = 0^\circ$ and pitches to $\alpha_{\text{geo}} = 45^\circ$ at the prescribed pitch rate. At the end of the motion, the wing is static, followed by the reverse maneuver (without measurements) to return to $\alpha_{\text{geo}} = 0^\circ$, where the wing remains static for approximately 5 s to allow any flow perturbations caused by the wing motion to diminish. Then the motion is repeated. For the roll kinematics, the wing was initially set to a

geometric angle of attack (in figure 3.10b $\alpha_{\text{geo}} = -28.4^\circ$) and began rolling until it reached a constant roll rate, in which the induced angle of attack α_{ind} becomes constant, resulting in $\alpha_{\text{eff}} = \alpha_{\text{geo}} + \alpha_{\text{ind}} = 33^\circ$. When the wing is pitched and rolled, shown in figure 3.10c, the pitching motion does not begin until $\alpha_{\text{eff}} = 0^\circ$ and $\frac{d\alpha_{\text{eff}}}{dt} = 0^\circ$. Thus, the wing is set to an initial $\alpha_{\text{geo}} = -\alpha_{\text{ind}}$ such that once the rolling motion reached a constant rate $\dot{\phi}$, the effective angle of attack was zero ($\alpha_{\text{eff}} = 0^\circ$). The roll motion continues, keeping $\alpha_{\text{eff}} = 0$, for about 0.4 s, at which point the pitching motion begins. The epoch for the simultaneous pitching and rolling maneuver is defined when the pitching motion begins, here at $t = 0.88$ s.

3.8.1 Parameter Values and Case Naming Protocols

The following chapters contain a series of studies based on different subsets of the data acquired. Whereas Chapter 4 compares and contrasts flow structures investigated in all three of the categories of motion, the other chapters focus on either pure roll, or pitch and combined pitch and roll. In the context of each of these investigations, case naming conventions were established that are compact, yet sufficiently descriptive to differentiate the cases discussed in the chapter. These conventions, and the chapters in which they are used are described here, along with the parameter values considered in each of the studies.

The dye visualizations of the pure-roll cases – presented in Chapter 5 – were conducted over a broad range of values of J and R_g/c to characterize the behavior of the LEV. Combinations of 6 different J values and 4 different non-dimensional radii of gyration R_g/c were used, consisting of $J = 0.325, 0.542, 0.813, 1.08, 1.36$, and 1.54 along with $R_g/c = 2.0, 2.5, 3.25$, and 3.5 . A summary of all the flow visualization cases that are used in the absence of passive bleed are provided in table 3.2. These cases are discussed in Chapter 5, where the roll cases will be identified as: $J_{R_g/c} = J$. For example, for the case in which $J = 0.54$ and $R_g/c = 3.25$, is specified as $J_{3.25} = 0.54$. This convention is also used in Chapters 6 and 8.

The complete parameter space of wing kinematics investigated using plenoptic PIV is shown in table 3.1, along with a case name of which each case will be referred to in Chapter 4, where all three categories of motion are considered. The naming convention is comprised of a letter, adorned with a subscript and, in some cases, a superscript. Case $P_{.5}$ has a P , indicating it is a pure pitching maneuver, and a subscript of 0.5, corresponding to the reduced pitch frequency of the case. The convention is similar for $R_{.54,33}$, a purely rolling case with an advance coefficient of 0.54, where the additional subscript is the effective angle of attack of the case $\alpha_{\text{eff}} = 33^\circ$. For the simultaneously pitching and rolling case, $S_{.46,.54}^{.22}$, the subscripts indicate the values of k and J , respectively, and the superscript is the k_{rg} value.

Force measurements were also conducted, for pure-roll cases, over the advance ratios $J = 0.54$ and 1.36 with $R_g/c = 2.5$ and 3.25 . The direction of the lift was defined as perpendicular to the free stream while drag was parallel to the free stream.

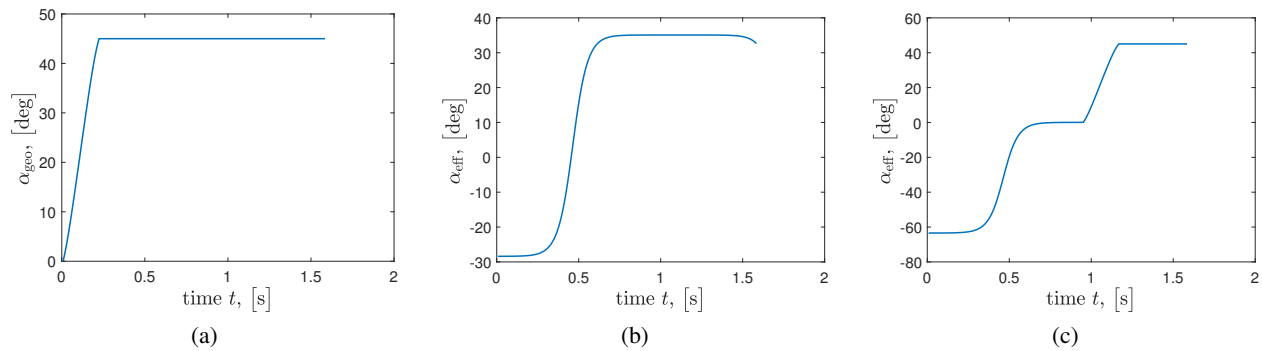


Figure 3.10: Wing kinematics for (a) pure pitch, (b) pure roll, and (c) simultaneous pitch & roll maneuvers

Case Name	k	J	k_{rg}	α_{eff}	ϕ
$R_{.54,23}$		0.54		23°	12°–49°
$R_{.54,33}$		0.54		33°	6°–49°
$R_{.54,43}$		0.54		43°	12°–49°
$R_{1.36,33}$		1.36		33°	3°–49°
$R_{1.36,43}$		1.36		43°	3°–49°
$P_{.5}$	0.50			15°–45°	0°
$P_{.20}$	0.20			15°–45°	0°
$S_{.46,.54}^{.22}$	0.46	0.54	0.22	13°–43°	30°–49°
$S_{.42,.54}^{.20}$	0.42	0.54	0.20	13°–43°	31°–49°
$S_{1.05,.54}^{.50}$	1.05	0.54	0.5	13°–43°	25°–40°
$S_{.46,1.36}^{.37}$	0.46	1.36	0.37	13°–43°	24°–38°

Table 3.1: Parameter space of wing kinematics

α_{eff}	α_{gem}	R_g/c	J
33°	-28.3 °	2	0.54
33 °	-17.9 °	2	0.81
33 °	-9.8 °	2	1.08
33 °	-3.3 °	2	1.36
33 °	0.3 °	2	1.56
33 °	-39.0 °	2.5	0.33
33 °	-28.3 °	2.5	0.54
33 °	-17.9 °	2.5	0.81
33 °	-9.8 °	2.5	1.08
33 °	-3.3 °	2.5	1.36
33 °	0.3 °	2.5	1.56
33 °	-28.3 °	3.25	0.54
33 °	-3.3 °	3.25	1.36
33 °	-39.0 °	3.5	0.33
33 °	-28.3 °	3.5	0.54
33 °	-17.9 °	3.5	0.81
33 °	-9.8 °	3.5	1.08
33 °	-3.3 °	3.5	1.36
33 °	0.3 °	3.5	1.56

Table 3.2: Flow visualization parameter space (Chapter 5).

Passive bleeding was applied in this study to perturb the LEV structure and further understand the mechanisms that govern the LEV evolution and behavior. Passive bleeding in this case was applied by adding a through hole to the wing at different locations. Passive bleed was applied to the $J_{3,25} = 0.54$ case because a conical structure was observed and remained attached throughout the motion. Through holes were added at 2 different locations and hole sizes. A diagram of the through hole locations is given in figure 3.11, with hole locations and sizes summarized in table 3.3. The naming convention for the passive bleeding cases will be p_{xy} where x is the spanwise location as indicated in the prior section by i (inboard) or o (outboard) and y is the hole size designated by s (small) or l (large). Only one passive bleed hole was used in each case.

Force measurements were conducted for the P_{il} case – because drastic flow changes were observed – and compared to those measured for the baseline case. This comparison was useful for identifying the impact of changing the flow structure while keeping the same kinematics and geometry.

Name	Advance Ratio (J)	Hole Spanwise Location (z/b)	Hole Chordwise Location (x/c)	Hole Diameter (mm)	Cases
Baseline	0.54	N/A	N/A	N/A	X
p_{is}	0.54	0.1	0.065	1.6	X
P_{os}	0.54	0.16	0.065	1.6	X
p_{il}	0.54	0.1	0.065	3.2	X
P_{ol}	0.54	0.16	0.065	3.2	X

X = Flow Visualization

 = Force Measurement

Table 3.3: Passive Bleed Hole Locations

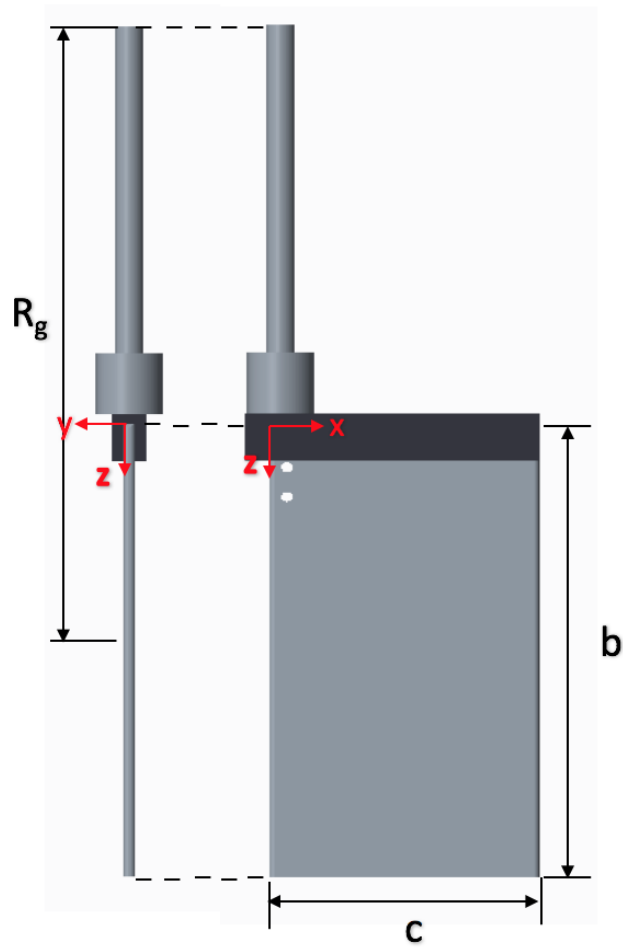


Figure 3.11: Passive Bleed Hole Locations

Chapter 4

Topology of a Pitching and Rolling Wing using Plenoptic PIV

This chapter investigates, in detail, the flow structures produced by the wing articulated in pitch, roll, and combined pitch and roll. Variability in the LEV structure, and in the LEV development rate (as referenced to the non-dimensional time, t' , as defined in section 3.5.3) are investigated and compared for different values of pitch and roll rate. Furthermore, the influence of the parameter, k_{rg} , defined in section 3.3.3, is examined for the combined pitch-roll cases. The cases and case names investigated here are summarized in table 3.1.

4.1 Pitching

Two kinematic cases were examined for the pure pitching maneuver: $P_{.5}$ and $P_{.20}$, having reduced pitch rates of 0.50 and 0.19, respectively. For each case, phase-averaged results are visualized using isocontours of normalized swirling strength ($\frac{\lambda_{ci}^2}{\max \lambda_{ci}^2}$) and stream ribbons of chordwise velocity normalized by the freestream velocity (u/U_∞). Two isocontour levels are shown: 10% and 15% of the maximum value in each volume, following the guidelines given by Zhou et al. (1999). Stream ribbons were calculated using a two-step Runge-Kutta method with a step size of 0.25 times the vector spacing. The stream ribbons are seeded in 3 rakes of 20 streamlines spanning from root to tip of the wing, all within the range of 6 to 10 mm from the leading edge and 6 to 12 mm off of the wing surface, and propagated in both directions. These rakes were designed to best seed the vortices produced at the leading edge.

The evolution of vortex structures created by kinematic case $P_{.5}$ is characterized by the formation of a coherent LEV in the earlier stages, which begins to lift up from the wing surface at $t' = 0.59$ ($\alpha_{geo} = 30^\circ$). As the motion continues, the middle region of the LEV lifts further from the wing surface while the ends of the LEV near the root and tip stay pinned to the surface. From $t' = 0.68$ ($\alpha_{geo} = 35^\circ$) to $t' = 0.88$ ($\alpha_{geo} = 45^\circ$) the LEV remains a distinct arch structure, closely matching the descriptions of Yilmaz and Rockwell (2012) and Jantzen et al. (2014). This arching structure is associated with a large influx of reverse flow ($u < 0$), which is shown at $t' = 0.78$ ($\alpha_{geo} = 40^\circ$) in Figure 4.1a. It is noted that the LEV has slightly left the surface at the root, however, this is hypothesised to be an effect of the support structure.

A similar development of the LEV is seen at the slower pitch rate, $P_{.20}$, where the LEV lifts up from the surface of the wing creating an arch vortex. By the end of the motion, the arch vortex has risen to a y value nearly twice that of the arch vortex in $P_{.5}$. In the larger void left by the arch vortex, two additional LEVs form, creating a triple-vortex system by the end of the motion. The vortex system appears to remain pinned throughout the motion, despite that data are not available near the wing root, since the visible portion of the vortex structure is highly symmetric in the vicinity of the mid-span of the wing. Two points have nearly the

same dimensionless time: $t' = 0.68$ ($\alpha_{\text{geo}} = 35^\circ$) for $P_{.5}$ and $t' = 0.70$ ($\alpha_{\text{geo}} = 15^\circ$) for $P_{.20}$. Here, the vortex structure is similar; however, as each case progresses, the qualitative evolution of each structure diverges significantly, such that the differences do not appear to be simply due to different rates of LEV development. Comparing $\alpha_{\text{geo}} = 40^\circ$ for both cases effectively illustrates the differences, shown in figure 4.1. Separated by slightly more than one convective time constant, the vortex topology for the $k = 0.2$ case shows a triple-vortex system, which remains pinned at the tip. Additionally, a strong tip vortex has formed, which is much more apparent in the λ_{ci} isosurface than for $P_{.5}$. This comparison is further visualized in Figure 4.1, where more of the streamlines exiting the LEV in figure 4.1b are entrained into the tip vortex. It is unlikely that further evolution of $P_{.5}$ would resemble later phases of $P_{.5}$ since the development of a multi-vortex system in $P_{.20}$ occurs when the arch shape of the LEV in $P_{.20}$ is much shallower than final stage of $P_{.5}$, where a single vortex is still present.

Nevertheless, it is possible to compare the early vortex development, for the $P_{.5}$ and $P_{.20}$ cases, to the time-shift described by Yilmaz et al. (2010), who used an $AR = 2$ wing at $Re = 10,000$, articulated in a pitch-up motion to $\alpha_{\text{geo}} = 45^\circ$ immediately followed by a pitch-down motion. Following the trend described by Yilmaz et al. (2010), an increase of k from 0.2 to 0.5 should advance the LEV time by a factor of 1.43. Thus, the structure of $P_{.20}$ at $t' = 0.638$ should correlate well with $P_{.5}$ at $t' = 0.470$; however, there exist significant differences. In fact, the LEV of $P_{.20}$ at $t' = 0.70$ more closely resembles the LEV of $P_{.5}$ at $t' = 0.68$ suggesting that – at least for this stage of development – t' dimensionless time is adequate to track vortex development without any additional pitch-rate-dependent delay.

4.2 Rolling

For the rolling maneuver, two advance ratios were prescribed at various effective angles of attack. Here, both $R_{.54,33}$ and $R_{1.36,33}$ are visualized in Figures 4.2 and 4.3, respectively, using the previously described methodology for the purely pitching cases.

The intricate evolution of the vortex structure formed by $R_{.54,33}$ begins with a nascent LEV at $t' = 0.39$ ($\phi = 6^\circ$), that is nearly cylindrical across the leading edge of the wing. As the motion progresses, a localized arch is formed at about 80 percent of the span. By $t' = 1.17$ ($\phi = 17^\circ$), the arch has grown in the spanwise direction and shifted downstream. The chordwise shifting allows another LEV to form at the leading edge, where both vortices are connected near the root and tip, well-established at $t' = 1.35$ ($\phi = 21^\circ$). This flow evolution is a manifestation of the dual-vortex system described by Bross et al. (2013). At $t' = 1.68$ ($\phi = 26^\circ$), the dual-vortex system becomes unpinned from the tip, such that the isocontour of each vortex in the system terminates well before the wing surface. Meanwhile, a third LEV forms. Through the rest of the motion, the unpinned vortices tangle and eventually convect down the chord, while a new dual-vortex system is established, which begins a similar process.

At the slower roll rate, $R_{1.36,33}$, the evolution of the vortex structure begins in a similar manner to $R_{.54,33}$, but quickly diverges. From $t' = 2.01$ ($\phi = 21^\circ$) and beyond, $R_{1.36,33}$ is characterized by a consistently shed LEV. It is apparent that the slower roll rate does not entrain enough energy into the LEV to overcome the convective forces of the bulk flow for any notable amount of time. Compared to $R_{.54,33}$, $R_{1.36,33}$ produces and sheds LEVs at a faster rate with respect to both t' and roll angle.

The results shown here suggest that changing the advance ratio may induce a time-shift and/or time scaling of the vortex structure and processes. Both $R_{.54,33}$ and $R_{1.36,33}$ form a dual-vortex system that becomes unpinned and convects down the chord, allowing the formation of another vortex or dual-vortex system in a cyclic manner. For $R_{1.36,33}$, the process has completed one cycle by $t' = 2.01$, whereas 1 cycle for $R_{.54,33}$ is complete after $t' = 2.58$. This suggests that there may be a better scaling or non-dimensionalized parameter for comparison of these kinematic cases. However, these measurements do agree well with the observations made by Bross et al. (2013), concluding that similar streamline topology features appear at different roll

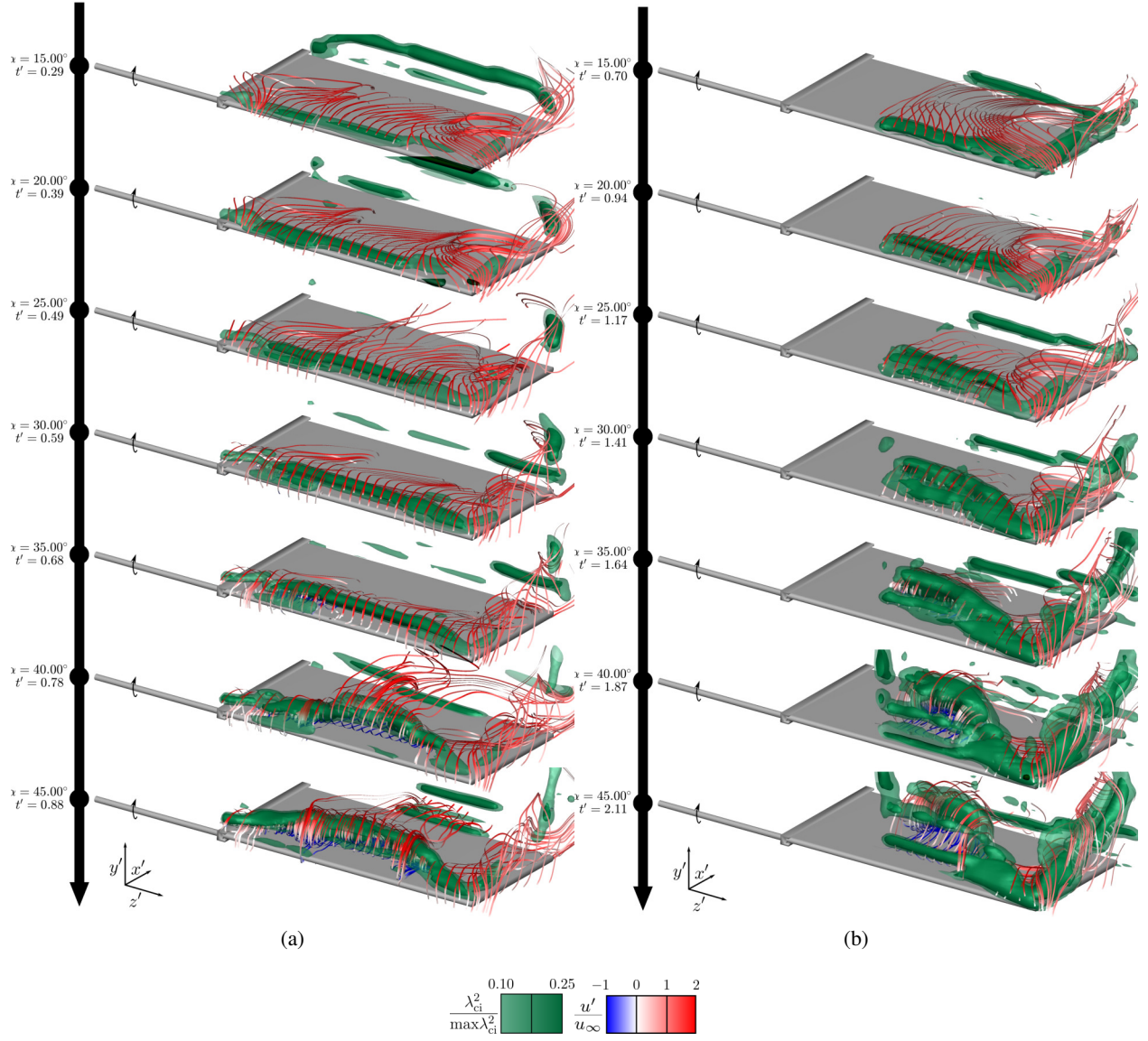


Figure 4.1: Phase-averaged results shown with isocontours coloured by normalized swirling strength and stream ribbons coloured by normalized chordwise-velocity u for (a) $P_{.5}$ and (b) $P_{.20}$

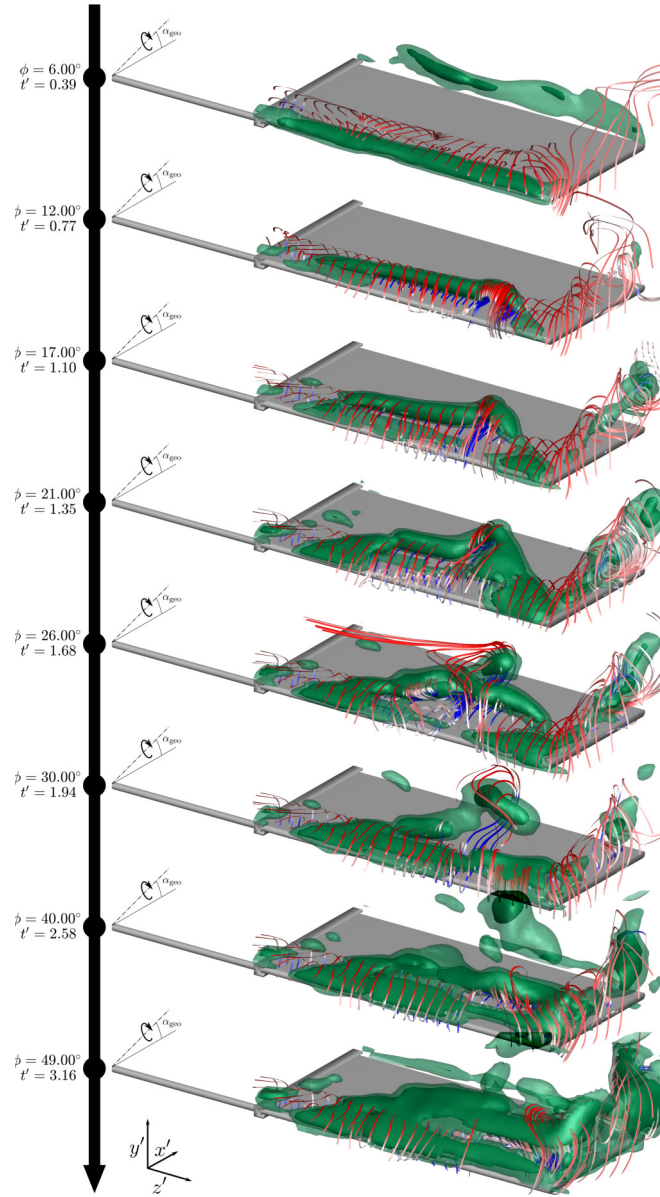


Figure 4.2: A sequence of images showing vortex evolution for case $R_{54,33}$ ($J = 0.54$, $R_g/c = 3.25$, $\alpha_{\text{eff}} = 33^\circ$).

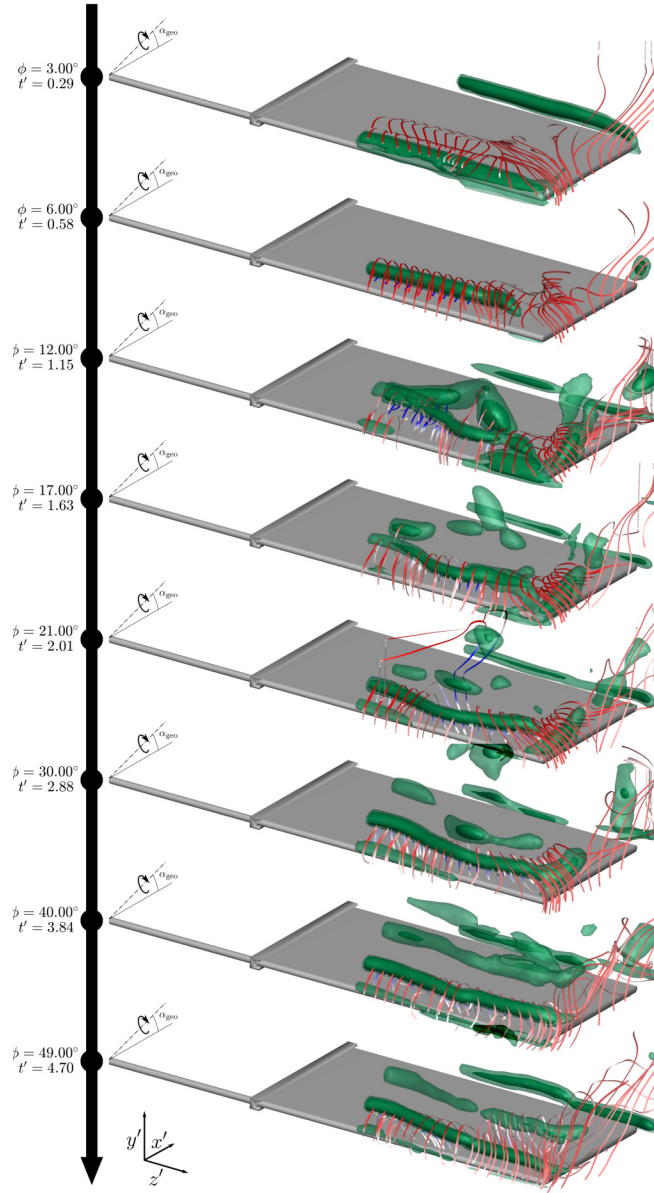


Figure 4.3: A sequence of images showing vortex evolution for case $R_{1.36,33}$ ($J = 1.36$, $R_g/c = 3.25$, $\alpha_{\text{eff}} = 33^\circ$).

angles when J is varied. Figure 4.4 further illustrates the similarities between these cases. The comparison here shows $R_{.54,.33}$ at $t' = 1.17$ ($\phi = 17^\circ$) (a) and $R_{1.36,.33}$ at $t' = 1.15$ ($\phi = 12^\circ$) (b). Both of these instances are the first to exhibit a dual-vortex system in their respective cases, at nearly the same t' . The structure of $R_{1.36,.33}$ in figure 4.4b appears to be a bit further along in the evolution, but the similarities are clear.

By altering the pitch angle of the wing before the rolling motion begins, the effect of α_{eff} was explored. Overall, the vortex structure developed similarly across the range of α_{eff} for each J value, with minor differences. Two sets of phase-averaged results are shown in Figures 4.5 and 4.6, for $R_{.54,.23}$ and $R_{.54,.43}$, where $\alpha_{\text{eff}} = 25^\circ$ and 45° , respectively. In the development of $R_{.54,.23}$, it is seen that the dual-vortex system appears to split closer to the tip (almost out of the centre measurement volume) when compared to $R_{.54,.33}$. Following this trend, at even greater α_{eff} the bifurcation point of the dual vortex system is not visible in the phases of $R_{.54,.43}$, suggesting that it is located in the root volume (lacking measurements). It is likely that the bifurcation point occurs near some critical *local* effective angle of attack, here between 24° and 30° .

4.3 Simultaneously pitching and rolling

Four different combinations of pitch rate and roll rate were investigated, each yielding a different k_{rg} value, as defined in equation 3.8. The first, presented in Figure 4.7, is a combination of $k = 0.46$ and $J = 0.54$. For this case, phases were taken every 5° from $\alpha_{\text{eff}} = 15^\circ$ to 45° , corresponding to roll angles between $\phi = 30^\circ$ to 49° . Early in the motion, the LEV is nearly cylindrical across the LE, but slowly transforms into a more conical shape while still connected to the wing tip. By $t' = 1.43$ ($\alpha_{\text{eff}} = 35^\circ$), the conical isocontour marking the LEV appears to have a slight bifurcation beginning around mid span up to about 90 percent span. In the next phase, $t' = 1.64$ a dual-vortex system has developed, which persists until the end of the motion. The dual-vortex system exhibits a more distributed arch, compared to $R_{.54,.33}$.

For $S_{1.05,.54}^{.50}$, where $k_{rg} = 0.50$, a relatively quick prescribed motion does not allow the LEV to evolve as drastically as previous cases. The phase-averaged results are shown in Figure 4.8. Here, a comparatively smaller LEV is formed (by size of swirling strength isocontour). By the end of the motion, a very shallow localized arch has formed, yet the LEV as a whole has remained tighter and closer to the wing surface than $S_{.46,.54}^{.22}$. In Figure 4.9, case $S_{.42,.54}^{.20}$ is shown, where $k_{rg} = 0.20$. Here, the prescribed kinematics are nearly the same as $S_{.46,.54}^{.22}$, and thus the development of the LEV is similar. Interestingly, at $t' = 1.78$ ($\alpha_{\text{eff}} = 40^\circ$), the dual vortex system closely resembles the phase of $S_{.46,.54}^{.22}$ that is at the closest t' value as opposed to the identical α_{eff} value. This helps validate the choice of using the V_{rel} when normalizing time by the convective time. Lastly, Figure 4.10 shows the phase-averaged results of $S_{.46,1.36}^{.37}$, where $k_{rg} = 0.37$. This case is unique

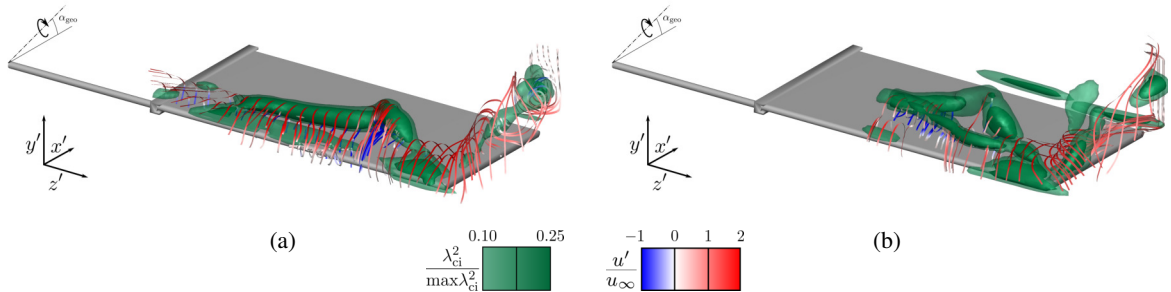


Figure 4.4: Isocontours coloured by normalized swirling strength and stream ribbons coloured by chordwise velocity u for (a) $R_{.54,.33}$ $t' = 1.17$ ($\phi = 17^\circ$) and (b) $R_{1.36,.33}$ $t' = 1.15$ ($\phi = 12^\circ$)

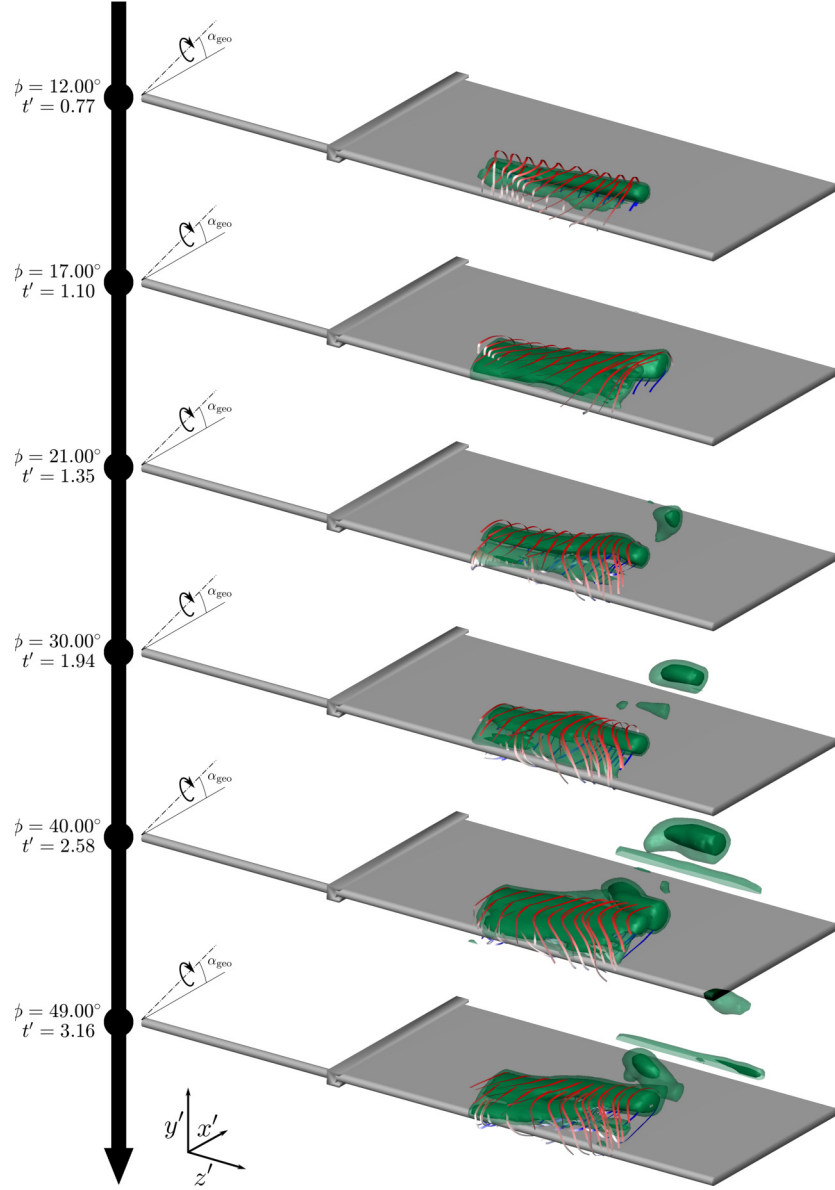


Figure 4.5: A sequence of images showing vortex evolution for case $R_{54,23}$ ($J = 0.54$, $R_g/c = 3.25$, $\alpha_{\text{eff}} = 23^\circ$).

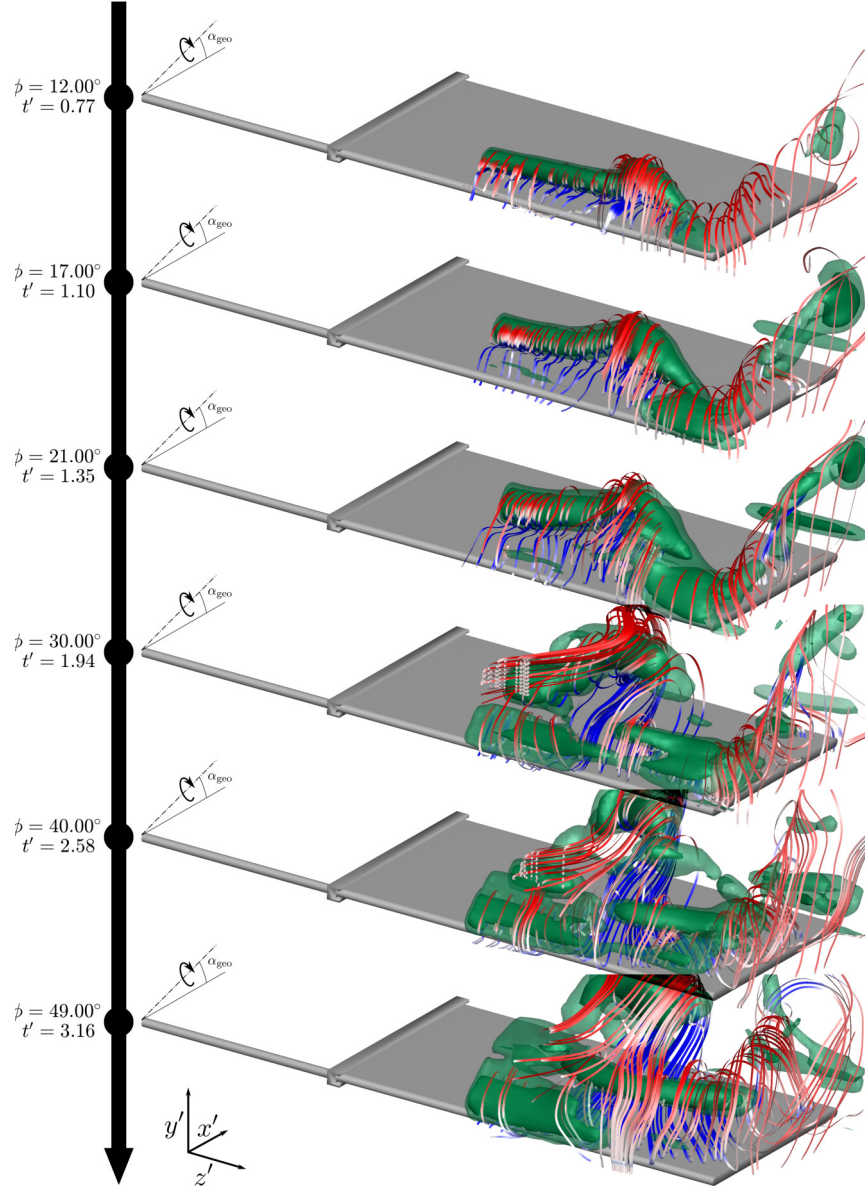


Figure 4.6: A sequence of images showing vortex evolution for case $R_{54,43}$ ($J = 0.54$, $R_g/c = 3.25$, $\alpha_{\text{eff}} = 43^\circ$).

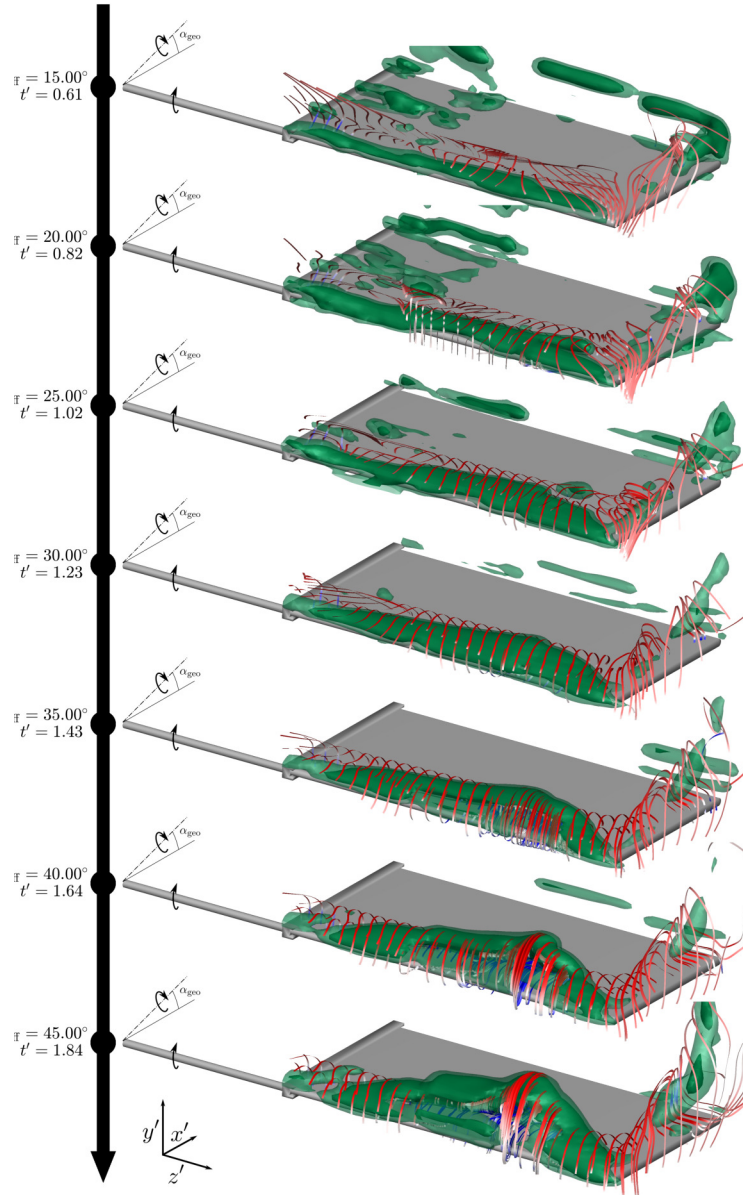


Figure 4.7: A sequence of images showing vortex evolution for case $S_{.46,.54}^{.22}$ ($k = 0.46$, $J = 0.54$).

in that it is the only simultaneously pitching and rolling case measured here to use the slower of the two roll rates. Here, a dual-vortex system may develop, but it is unclear since the isocontours remain unconnected through the centre volume, and the root volume was not measured for this case. Overall, the isocontours remain cylindrical, and by the end of the motion, a distinct gap is present between the first and second LEV, beginning to resemble the shedding behaviour in $R_{1.36,33}$. However, all the isocontours remain attached at the wing tip. While this k_{rg} value is nearly of the previous results, the evolution of the LEV does not appear to be an in-between of the evolution of $S_{.46,.54}^{.22}$ and $S_{1.05,.54}^{.50}$, thus suggesting that the vortex evolution is dependent on not only the k_{rg} value but *how* the k_{rg} value is formed.

For two of the kinematic cases, $S_{.46,1.36}^{.37}$ and $S_{1.05,.54}^{.50}$, measurements were made after the pitching motion was complete and the rolling motion was allowed to continue, shown in figure 4.11. In each case, the LEV began to progress down the chord, each convecting about 10 percent of the chord after about half of a convective time since the pitching motion had stopped. This suggests that the pitching motion plays a major role in stabilizing the LEV, keeping it present at the leading edge. Interestingly, in both cases, a new LEV has yet to form, despite the elapsed time and comparatively, ample space to do so.

4.4 The k_{rg} Parameter

The combined pitch-roll maneuver allows comparison of the flow structure between a wing articulated in a pure pitch maneuver vs. a pitch maneuver articulated on a rolling wing. Thus, comparison of a pitching wing in an inertial reference frame and a pitching wing in a non-inertial frame can be achieved. In this regard, the k_{rg} parameter isolates these effects, to some degree, by accounting for the increment in approach velocity added by the roll motion. Thus, the motivation for selecting $S_{1.05,.54}^{.50}$ and $S_{.42,.54}^{.20}$ becomes clear: to match the reduced pitch rates of the wing in the inertial frame of $P_{.5}$ and $P_{.20}$. A consequence of changing from an inertial frame to a non-inertial rolling frame of reference is that the approach flow along the leading edge is no longer uniform. Thus, the resulting vortex structures lose symmetry about the half span, clearly illustrated in figures 4.12 and 4.13, which compare $P_{.5}$ to $S_{1.05,.54}^{.50}$ and $P_{.20}$ to $S_{.42,.54}^{.20}$, respectively. In both of these figures, the dimensionless time t' and the effective pitch rate k_{rg} are, or nearly are, identical (recall that $k_{rg} = k$ for purely pitching kinematics). After the same non-dimensional time, the cases in the non-inertial frames (figures 4.12b and 4.13b) exhibit vortex structures that are much closer to the wing surface and the leading edge than their purely pitching counterparts. Furthermore, in the case of $k_{rg} = 0.2$ (figure 4.13b), the dual-vortex system has only begun to form, whereas $k = 0.19$ (figure 4.12a), a triple vortex system exists. This comparison illustrates the second notable effect of the non-inertial reference frame: the impediment to the vortex evolution. This impediment can also be thought of as a time-shift of the 3D evolution of the LEV is similar to the time-shift described by Bross et al. (2013), observed when they added a free stream to a rolling wing. It is noted that the closest resemblance to the structures formed by $S_{1.05,.54}^{.50}$ and $S_{.42,.54}^{.20}$, occur in $P_{.5}$ and $P_{.20}$ between $0.3-0.4 \tau$ earlier.

Building on the observations of the formation of dual-vortex systems, a correlation is found between the time taken for the dual-vortex system to develop and the k_{rg} value for a pitching motion in a non-inertial reference frame. Figure 4.14 shows the value of t' for the initial formation of the dual-vortex system plotted against k_{rg} for each of the simultaneously pitching and rolling cases. At each point, a small snippet of the initial formation is shown. There is a clear correlation that shows increasing k_{rg} decreases the t' required for the dual vortex system to form. This implies that the formation of LEVs from a more rapidly maneuvering wing feeds energy into the LEV faster than a slower maneuver, even when scaled by the relative fluid velocity. A linear fit is shown for local prediction of nearby k_{rg} and t' values, however, it is noted that this relationship cannot possibly be linear on a grander scale, as it must asymptote, or simply end, at more extreme values of k_{rg} .

To further explore the effect of the non-inertial reference frame, a simultaneously pitching and rolling

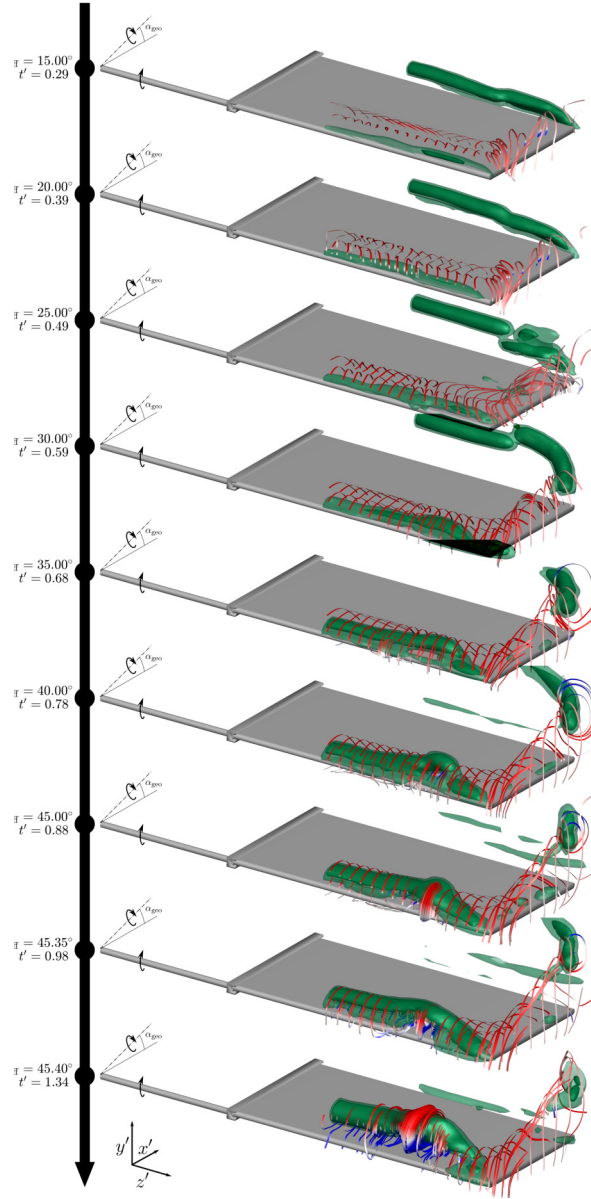


Figure 4.8: A sequence of images showing vortex evolution for case $S_{1.05,.54}^{.50}$ ($k = 1.05$, $J = 0.54$).

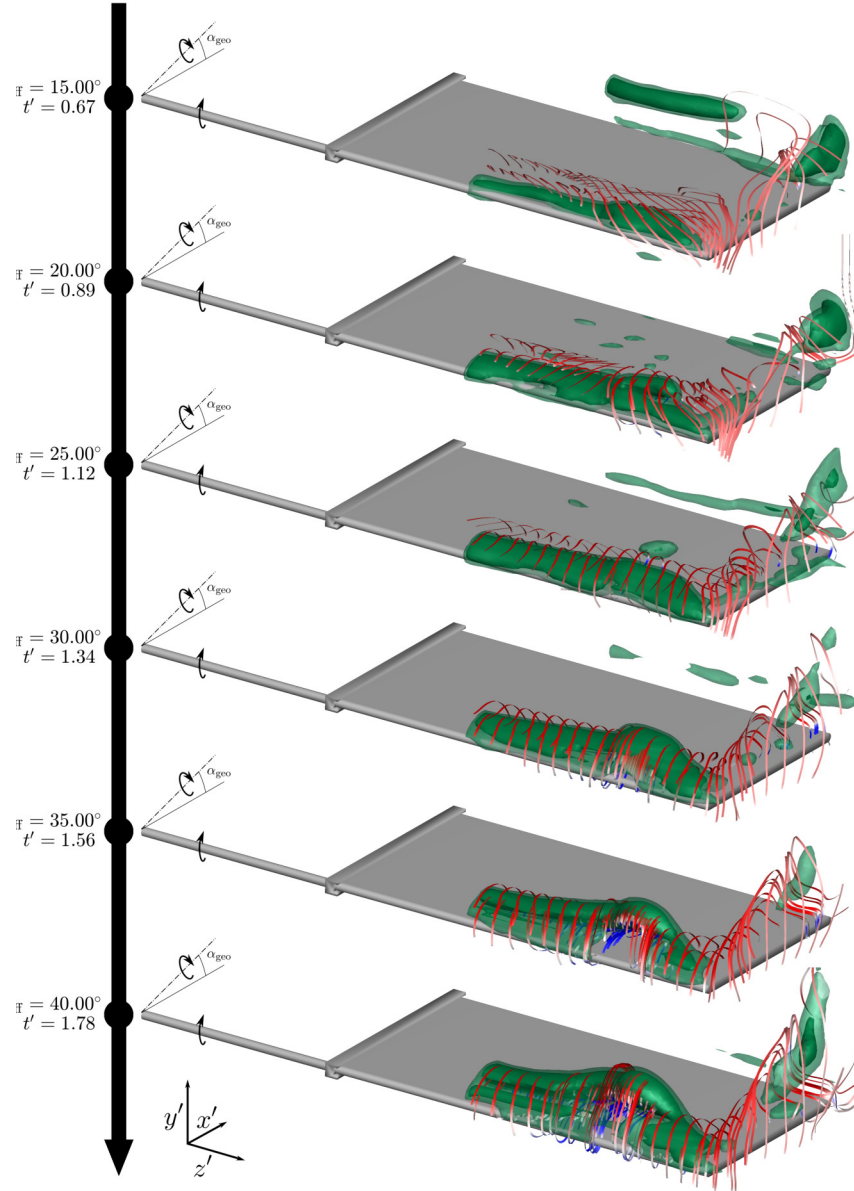


Figure 4.9: A sequence of images showing vortex evolution for case $S_{.42,.54}^{.20}$ ($k = 0.42$, $J = 0.54$).

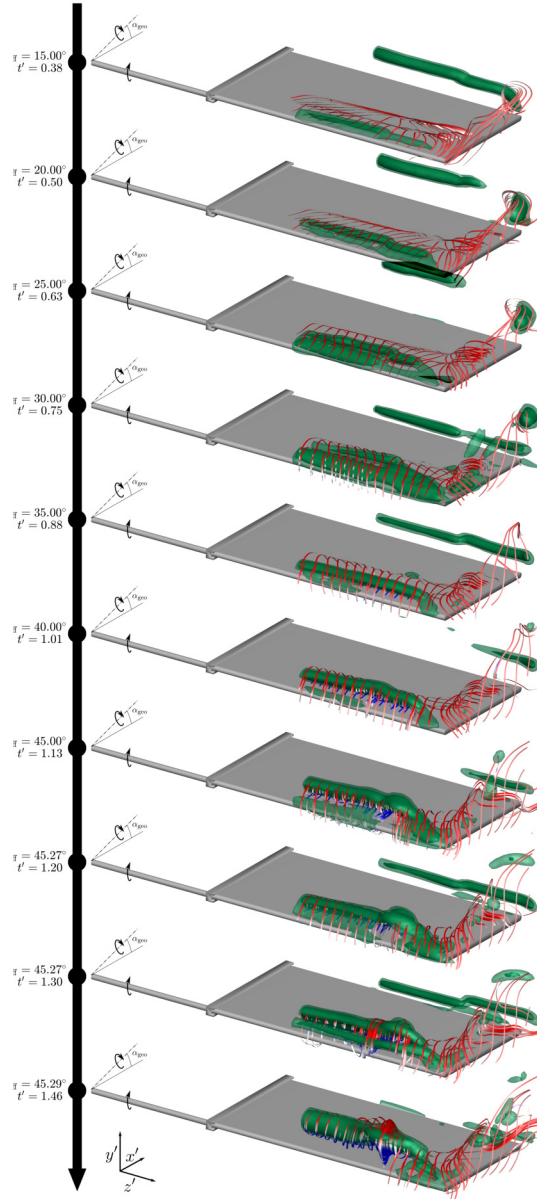


Figure 4.10: A sequence of images showing vortex evolution for case $S_{.46,1.36}^{.37}$ ($k = 0.46$, $J = 1.36$).

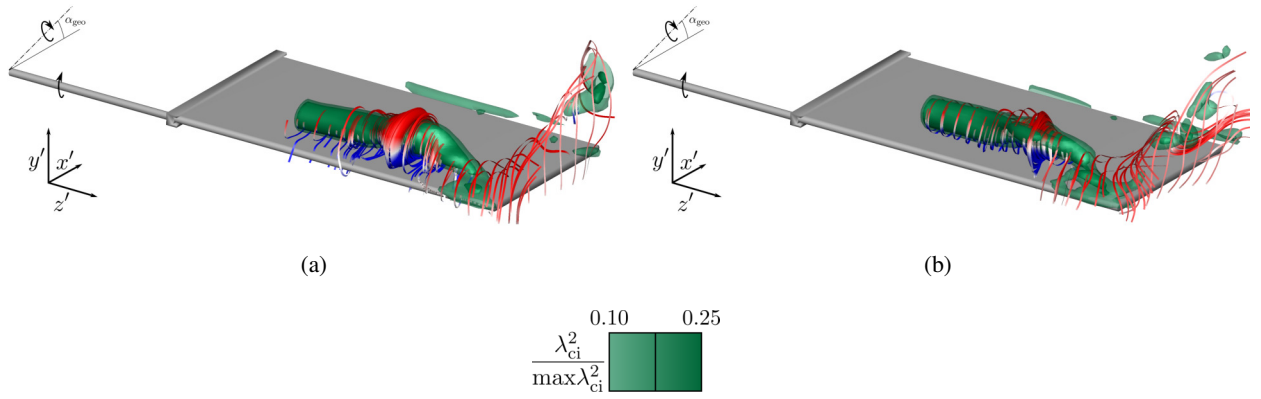


Figure 4.11: Comparison of vortex structure from post-pitch rolling motion: (a) $S_{1.05,.54}^{.50}$ $t' = 1.34$ and (b) $S_{.46,1.36}^{.37}$ $t' = 1.46$

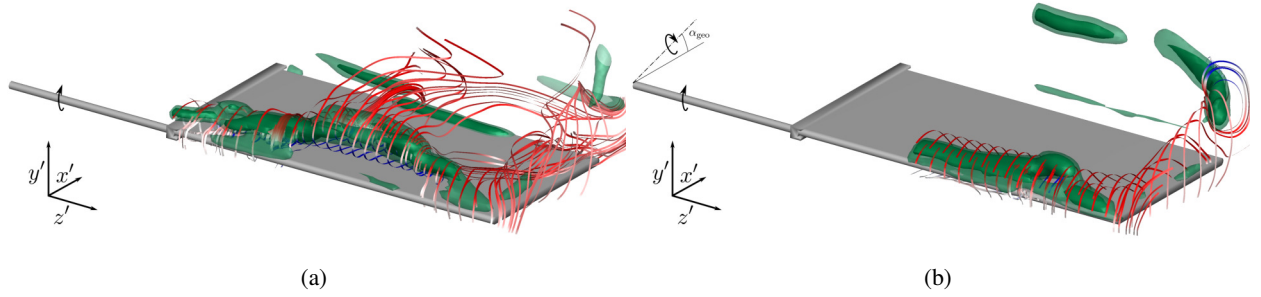


Figure 4.12: Phase-averaged comparison at $\alpha_{\text{eff}} = 40^\circ$ of (a) $P_{.5}$, where $t' = 0.88$, and (b) $S_{1.05,.54}^{.50}$, where $t' = 0.88$

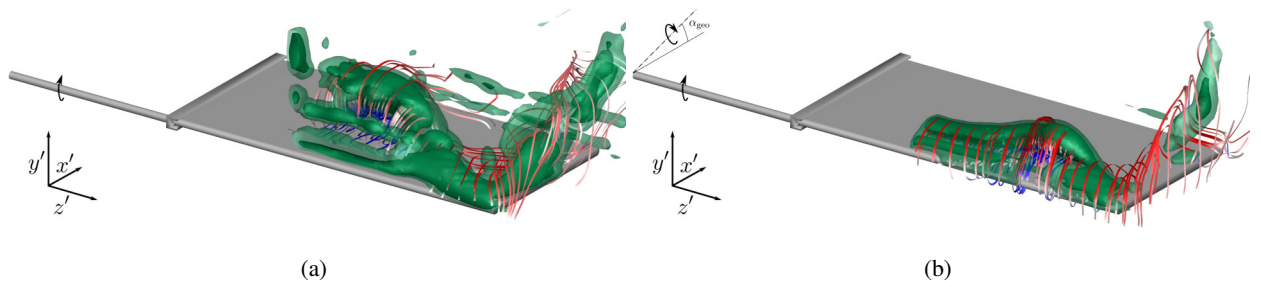


Figure 4.13: Phase-averaged comparison at $\alpha_{\text{eff}} = 40^\circ$ of (a) $P_{.20}$, where $t' = 1.87$, and (b) $S_{.42,.54}^{.20}$, where $t' = 1.87$

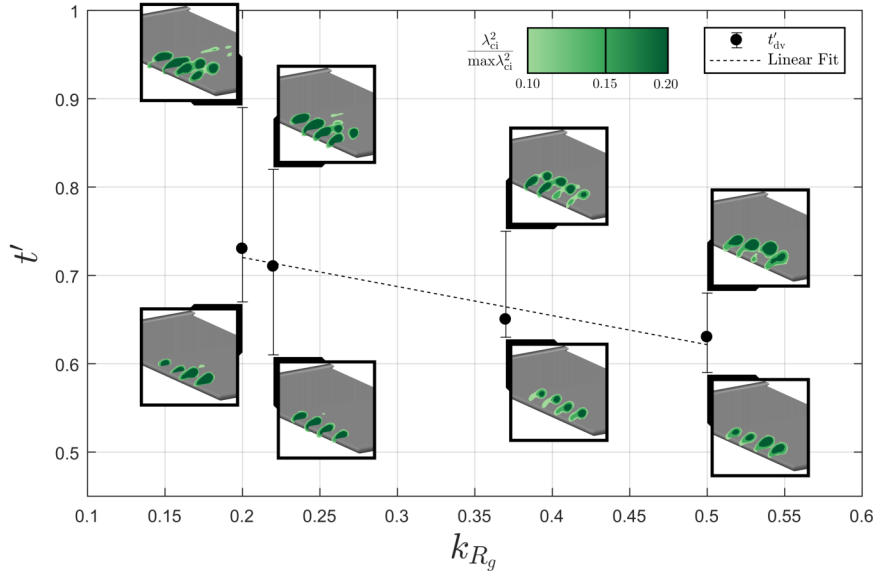


Figure 4.14: Dimensionless time for a dual-vortex system to develop verses k_{rg} , shown with a linear fit and 95% confidence interval

case using the same roll rate can be compared. In Figures 4.2 and 4.7, the phase-averaged results of $R_{.54,.33}$ and $S_{.46,.54}^{.22}$ are shown. After a similar start up, the final 3 phases of each kinematic case are where the vortex evolution between the cases begins to diverge. After $t' = 1.35$ ($\phi = 21^\circ$) for $R_{.54,.33}$, the dual-vortex system comes unpinned at the tip, and later sheds. Most closely resembling this structure, the flow at $t' = 1.84$ ($\alpha_{\text{eff}} = 45^\circ$ and $\phi = 49^\circ$) for $S_{.46,.54}^{.22}$ also contains a well defined dual-vortex system that is pinned at the wing tip. Thus, it can also be concluded that the addition of a pitching motion to a rolling motion delays the vortex evolution in both t' (by $\approx 0.5\tau$) and roll angle ϕ (by $\approx 24^\circ$).

4.4.1 Helicity Analysis

Another avenue for examining and predicting vortex behavior is the helicity or helical density of the LEVs. Moffatt and Tsinober (1992) describe the importance of helicity at a fundamental level in relation to flow kinematics because it admits topological interpretation in relation to the linkage or linkages of vortex lines of the flow, specifically citing the leading-edge vortex as a prime example of a helical structure, in this case a vortex having a non-zero axial component of velocity. Wolfinger and Rockwell (2015) are among the few to have quantified the helicity value, calculated as $H = \mathbf{v} \cdot \boldsymbol{\omega}$, for the LEV of a rolling wing. The authors found that structures that exhibited alternating signs of helicity rapidly degenerated. A vortex exhibiting a singular sign of helicity is helical in nature, thus, the findings of Wolfinger and Rockwell (2015) suggest that a more helical LEV is more stable than an LEV without a helical component.

For comparison, helicity was calculated for the present study shown in figure 4.15. Beginning with the purely rolling case, $R_{.54,.33}$, figure 4.15a, shows isocontours for λ_{ci} -Criterion at $t' = 0.754$, colored by helicity. The volume has been rotated such that the LEV is viewed from the trailing edge, similar to the presentation by Wolfinger and Rockwell (2015). A banding pattern is seen here, where the midspan region of the LEV is dominated by a negative helicity and the vortex arch and region near the root exhibit helicity of the opposite sign. In agreement with the observations of Wolfinger and Rockwell (2015), this structure ultimately degenerated. In figure 4.15b, negative helicity (blue) dominates the LEV, however, where the vortex has lifted from the wing surface, opposing-sign helicity is found on the bottom of the vortex. The

helicity in this case is arguably more uniform than the previous one, for which the LEV stays intact for the entire recorded motion here.

The simultaneous cases $S_{.46,.54}^{.22}$ and $S_{1.05,.54}^{.50}$ are shown in figures 4.15c and 4.15d, which support the previous findings. The case of $S_{.46,.54}^{.22}$ exhibits similar helicity patterns to $R_{.54,.33}$, foreshadowing the degradation of the vortex structure, although it is not observed in the recorded motion. The vortex structure formed by $S_{1.05,.54}^{.50}$ does appear more uniformly blue than $S_{.46,.54}^{.22}$, which may be an effect of the faster pitch rate. It has been previously noted that the addition of the pitching motion to the rolling motion helped stabilize and delay the evolution of the vortex structure. From the helicity plots presented here, it follows that the pitching motion may help to create a more established helical LEV, characterized by a more dominant sign of helicity, thus keeping the LEV intact, and thus near the leading edge for a prolonged time.

4.5 Conclusion

Examination of the vortex topology across a multi-variable parameter space revealed how variations in kinematics change vortex topology for early development of the LEV on a pitching and rolling wing, both simultaneously and independently, while subjected to a uniform flow. Vortex structure was quantitatively visualized through isocontours of swirling strength computed from 3D-3C vector fields produced from plenoptic PIV measurements. Through visual inspection of the results, the evolution of the vortex topology was described in detail. The purely rolling cases studied here were characterised by vortex structures that were ultimately convected from the leading edge. However, convection was delayed for the LEV formed by the faster roll rate, remaining pinned to the wing tip in early phases. The vortex topology of the faster roll rate was defined by a localized arch preceding the formation of a dual-vortex system. The bifurcation point of the dual-vortex system was observed to translate along the span of the wing when the effective angle of attack was changed, shifting towards the root when the effective angle of attack was increased. For the purely pitching cases, it was found that the vortex topology is characterized by the development of a pinned, arching LEV, that did not form a dual-vortex system.

This study also included various cases of simultaneously pitching and rolling kinematics of a wing in a free stream, of which very little data exists in the literature. These kinematics were formulated as a pitching wing in a non-inertial (rotating) reference frame. In order to make comparisons between pitching motions in inertial and non-inertial reference frames, a parameter was introduced, k_{rg} , which is similar to the dimensionless pitch rate k , yet it accounts for the velocity induced by a rotating reference frame. It was found that the presence of the rotating reference frame significantly impeded the development of vortex structures through comparison of matching k and k_{rg} values, in the realm of 0.3–0.4 τ . Similar time-shifts from kinematic changes have been described by Yilmaz and Rockwell (2012) and, to some degree, Bross et al. (2013). When comparing a simultaneously pitching and rolling case to a purely rolling case with the same roll rate, a similar time-shift is observed, on the order of 0.5 τ .

The k_{rg} parameter was found to have a correlation with the time of formation of a dual-vortex system, such that a larger k_{rg} produced the dual-vortex system sooner (i.e. at smaller t'). However, as described by comparing $S_{.46,.54}^{.22}$, $S_{.46,1.36}^{.37}$, and $S_{1.05,.54}^{.50}$, k_{rg} is limited in its ability to account for the effects of roll on the main features of the LEV topology of a pitching wing in a rolling frame. Thus, using the parameter as a predictive technique for flow features is cautioned. For comparison, the helicity analysis conducted by Wolfinger and Rockwell (2015), was repeated in the present study, where it was found that vortices of opposing signs of helicity would ultimately break down, in agreement with their results. While the method proved accurate, it requires the flow field to be measured to make a prediction, thus an *a priori* classification method is still desired, similar to the k_{rg} parameter.

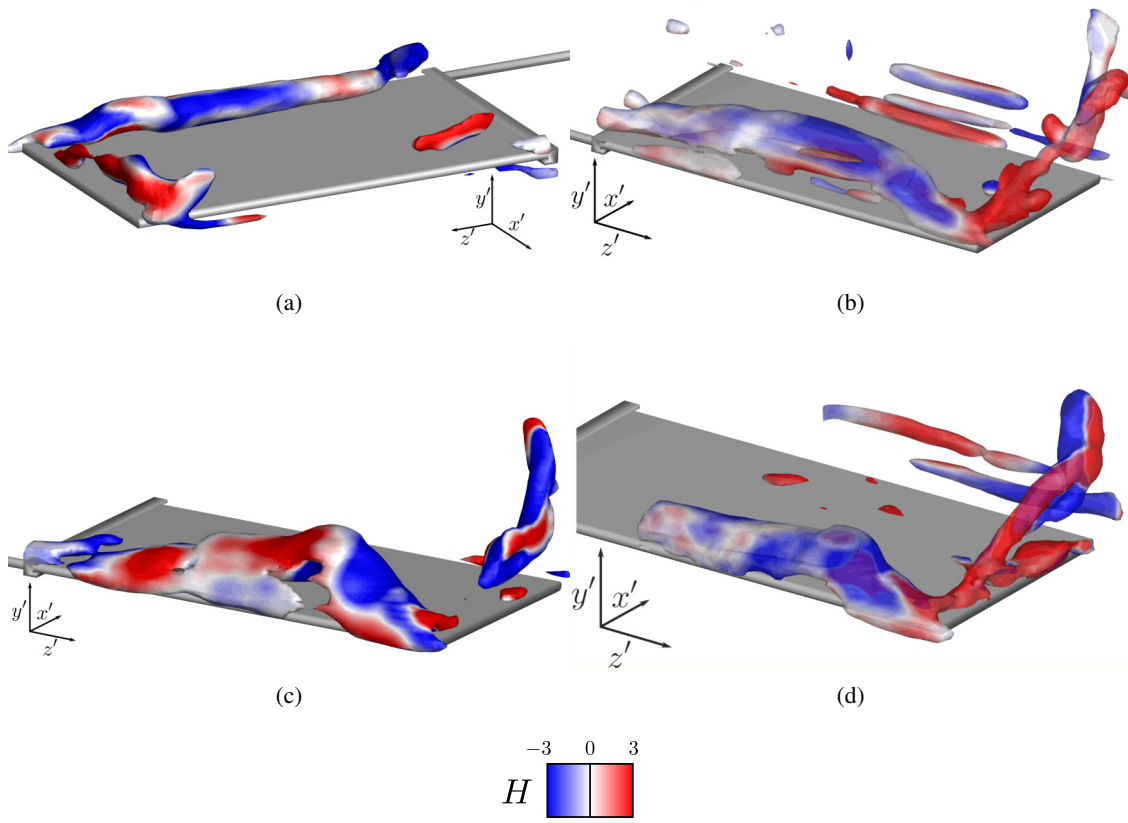


Figure 4.15: Isocontours of λ_{ci} -Criterion set to 5% of the maximum value, coloured by helicity H , depicting alternating signs of helicity in the LEV: (a) $R_{.54,.33}$ at $t' = 0.77$ (b) $P_{.5}$ at $t' = 0.88$ (c) $S_{.46,.54}^{.22}$ at $t' = 1.84$ (d) $S_{1.05,.54}^{.50}$ at $t' = 0.88$

Chapter 5

Leading-edge Vortex Structure and Forces on a Rolling Wing

This chapter focuses exclusively on the pure roll motion, using dye visualization to explore a larger range of advance ratios and radii of gyration than that investigated using plenoptic PIV in Chapter 4. A classification is defined based on LEV behavior near the root of the wing, and a dimensionless parameter is proposed by non-dimensionalizing the inflow velocity gradient. Its significance to flow behavior is examined by mapping its value within the parameter space of the vortex evolution classifications. The dye visualizations are validated by comparing to plenoptic PIV data, and the lift and drag forces are measured and analyzed to further understand the impact of changes in the flow to aerodynamic loads. As summarized in section 3.8, the pure-roll cases examined in this chapter follow the naming convention, $J_{R_g/c} = J$ (e.g., $J_{3.25} = 0.54$).

5.1 Qualitative Characterization of LEV Development using Flow Visualization

Figures 5.1a and 5.1b contain dye visualizations for the $J_{3.25} = 0.54$ and $J_{3.25} = 1.36$ cases respectively. The two cases are archetypical examples of LEV behavior for fast and slow roll motions, which were examined using plenoptic PIV in Chapter 4. Whereas dye visualization lacks the precise quantitative characterization of the flow field, in which plenoptic PIV excels, the time-resolved imaging, broader parameter space, and different visual perspective of the flow provide complementary insights to expand our understanding of the flow structure and dynamics. The $J_{3.25} = 0.54$ case maintains a relatively symmetric LEV across the span until approximately $\phi = 16^\circ$. At this point, an energetic tip vortex has developed. By $\phi = 32^\circ$, a conical leading-edge vortex has developed over the inboard region of the wing, extending to about the 60% spanwise position. In the outboard region, an arch structure has formed. The tip vortex persists and remains coherent as well as highly energetic. At $\phi = 42^\circ$ and 54° , the arch structure has shed, but the LEV maintains its conical shape in the inboard region of the wing, as the plenoptic data also revealed.

In contrast, the $J_{3.25} = 1.36$ case evolves much differently than the $J_{3.25} = 0.54$ case. The LEV for the $J_{3.25} = 1.36$ case has rolled up into a large, highly-symmetric arch structure by $\phi = 16^\circ$ and remains pinned at the outboard and inboard corners of the leading edge. Similar to the $J_{3.25} = 0.54$ case, an energetic tip vortex has formed. By $\phi = 32^\circ$, multiple vortices can be identified, shed from the leading edge. While the dye pattern does not clearly exhibit a conical, attached LEV as in the case of the $J_{3.25} = 0.54$ case, effects of rotation are apparent as the inboard edge of the dye plume is swept outboard as it traverses the chord of the wing.

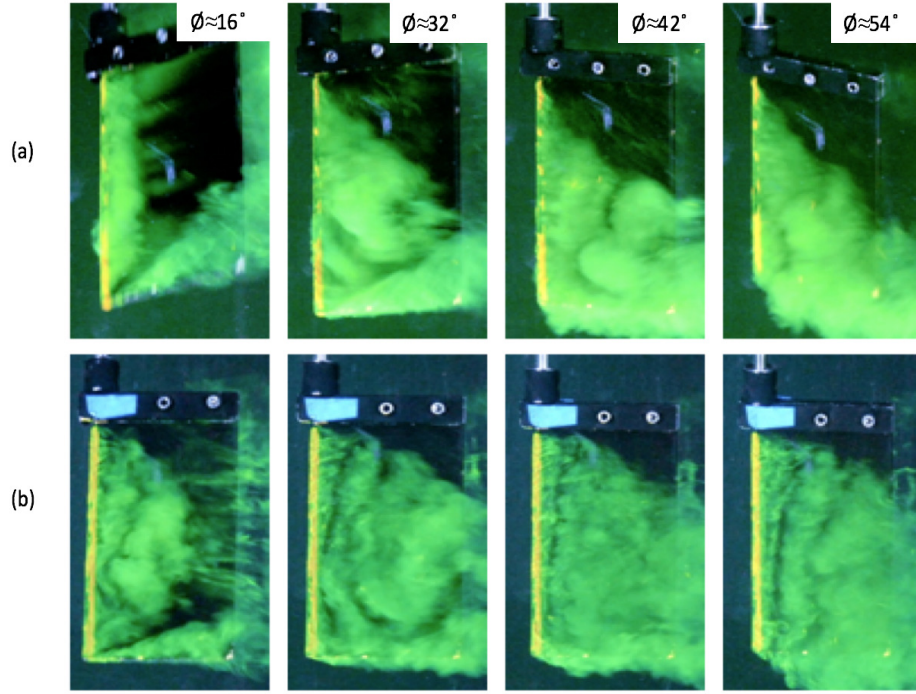


Figure 5.1: LEV evolution for (a) $J_{3.25} = 0.54$, $\alpha_{eff} = 33^\circ$ (b) $J_{3.25} = 1.36$, $\alpha_{eff} = 33^\circ$

5.2 Classification of the LEV

To objectively characterize the structure of the LEV by varying J and R_g/c , a binary classification was established which identified the LEV as *conical* or *non-conical*. The *conical* vortex classification includes a conical-shaped LEV initiating at the root of the wing, remaining coherent over at least 40% of the wing span, and persisting for a sweep angle of at least $\phi = 40^\circ$. The *non-conical* classification includes any cases that do not conform to the *conical* case, and typically includes arch-type LEV structures, as well as the shedding of multiple LEVs at a much higher frequency than that of the *conical* LEV.

The $J_{3,25} = 0.54$ and $J_{2,50} = 1.08$ cases shown in Figure 5.2 are examples of *conical* LEVs. The $J_{3,25} = 0.54$ case is an example of the most extreme *conical* classification. For this case, the LEV initially starts off symmetric in the spanwise direction. Subsequently, a distinct conical structure forms that spans approximately 60% of the total wing span and persists until an angle of rotation of $\phi = 45^\circ$. Outboard, the dye pattern loses coherence due to the high 3-dimensionality and interactions between the LEV and the tip vortex. The $J_{2,50} = 0.108$ case is marginally classified as *conical*. This case follows a similar evolution to the $J_{3,25} = 0.54$ case with the distinct difference being an arch structure appears to form outboard and is followed by the shedding of multiple LEVs. The coherent conical LEV occupies only approximately 40% of the span.

Figure 5.3 shows a *transitional* LEV structure and corresponds to $J_{2,50} = 1.56$. A *transitional* structure lies in between a *conical* and *non-conical* classification and is difficult to classify as one or the other because the inboard conical region cannot objectively be determined to exist up to the 40% spanwise location, based on dye visualizations. This case evolves nearly identically to the marginally conical case however, the conical feature inboard is smaller in size.

Figure 5.4 shows the $J_{3,50} = 1.08$ and $J_{3,50} = 1.36$ cases and are non-conical LEVs. The $J_{3,50} = 1.08$ case has an arch structure that spans approximately 60% of the wing while the conical structure spans only about 25% of the wing. This vortex starts off forming similar to that of the conical cases; however, a prominent conical vortex structure inboard does not form as large in comparison to the conical case. The arch structure is much closer to the midspan and the overall LEV remains relatively symmetric. A series of LEVs are shed throughout the motion at a relatively high frequency. $J_{3,50} = 1.36$ shows similar behavior to the case previously discussed with an even smaller conical structure and further exaggerated arch structure. This is the most extreme *non-conical* case within the parameter space examined.

Using this classification criterion, the LEVs in the parameter space were classified for all dye visualization cases listed in table 3.2. The distribution of classifications throughout the space of J and R_g/c values investigated here are shown in figure 5.5. There is a clear division in the parameter space, with conical structures existing for lower values of J and R_g/c , suggesting the existence of a parameter, dependent on J and R_g/c , whose value determines the classification, or the spatial extent and duration of the conical region. Such a parameter is investigated in the next section.

5.3 A Rotation Parameter

As Figure 5.5 reveals, LEV behavior varies such that conical structures are generally observed for lower values of J and R_g/c , and non-conical structures for larger values of the parameters, with an apparent continuous transition between the two states. It should be noted that the spanwise gradient of relative approach velocity to the wing is determined by both the roll rate and the radius of gyration of the wing, and therefore a dimensionless inflow shear parameter may govern this transition. Such a parameter is especially of interest given the discovery by Jardin and David (2014) that an attached conical vortex could be created on a wing placed in a linear shear flow, in the absence of rotational accelerations. The parameter may take

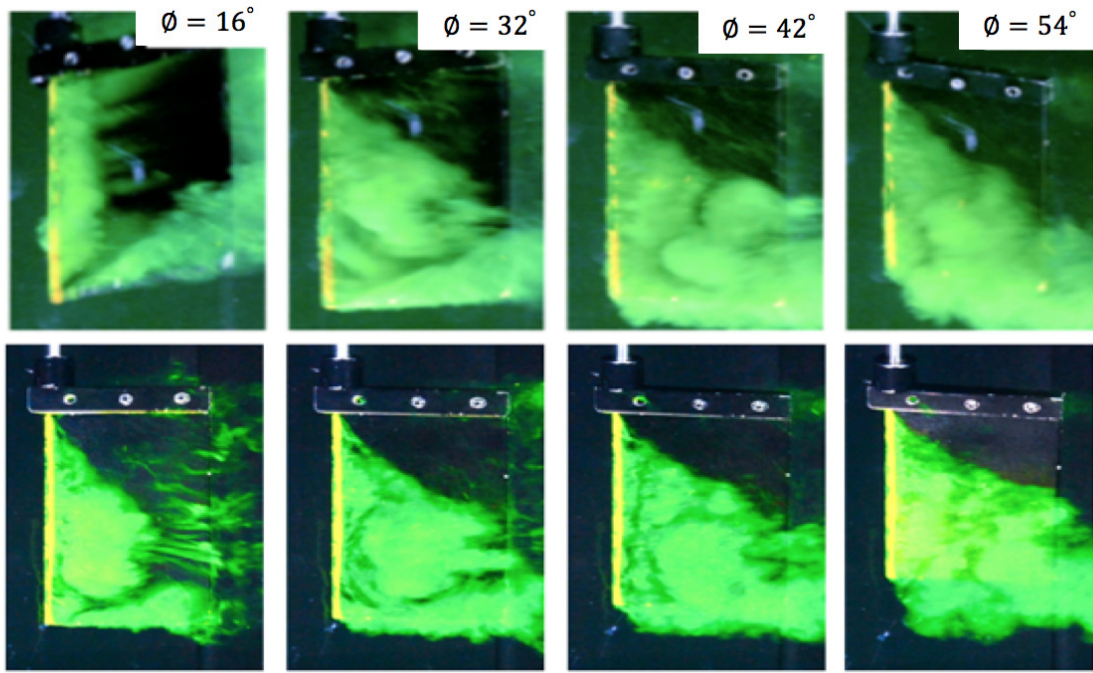


Figure 5.2: Visualization of LEV structures classified as *conical*. Top: $J_{3.25} = 0.54$; bottom: $J_{2.50} = 1.08$.

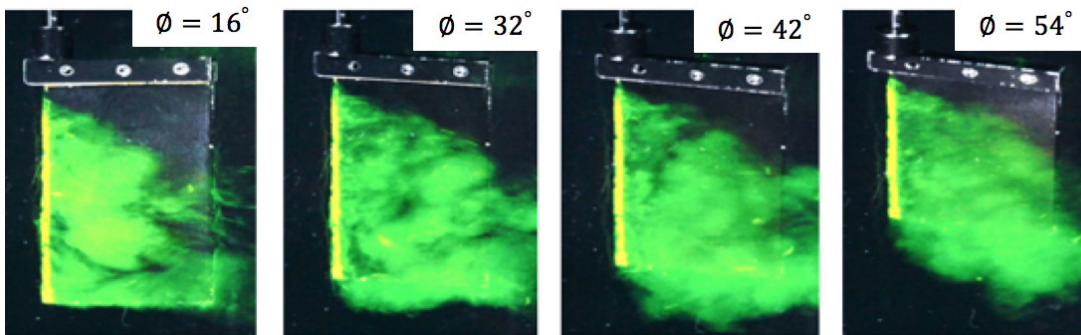


Figure 5.3: Visualization of an LEV structure classified as *transitional* ($J_{3.50} = 0.81$).

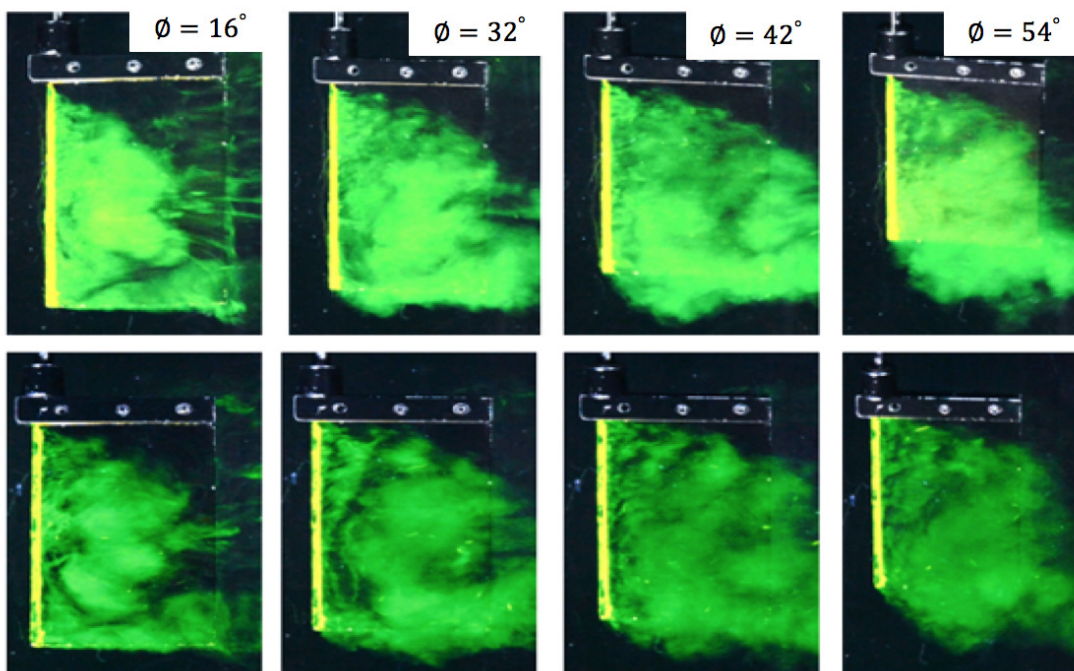


Figure 5.4: Visualization of LEV structures classified as *non-conical*. Top: $J_{3.50} = 1.08$; bottom: $J_{3.50} = 1.36$.

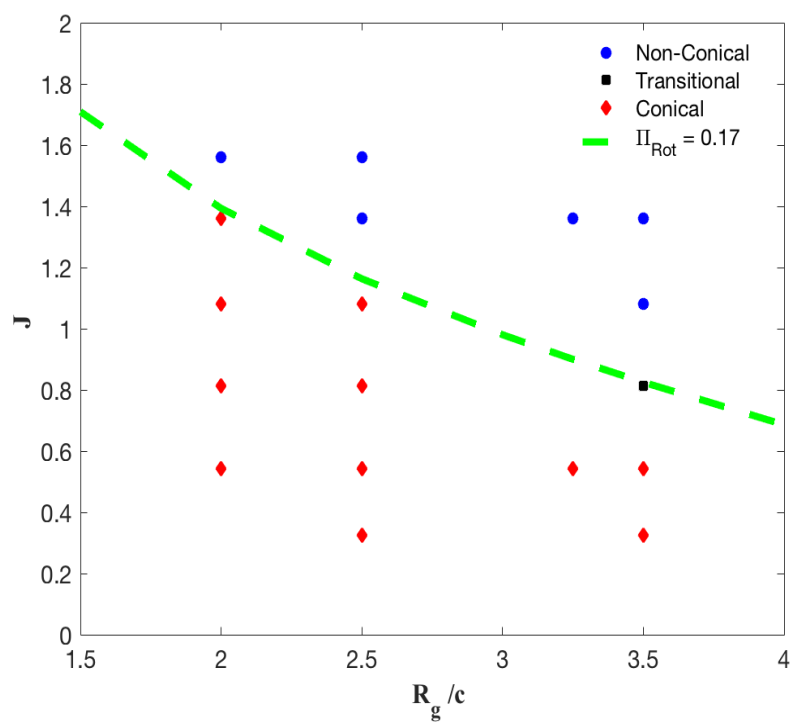


Figure 5.5: Classifications of LEV structures with varying J and R_g/c .

the following form:

$$\pi_{rot} = \frac{c}{U_{loc}} \frac{\partial U_{loc}}{\partial r} \quad (5.1)$$

where U_{loc} is determined from the vector addition of the free stream and the local relative velocity induced by wing rotation at a given radial position, r , calculated in the same manner as Equation (3.4):

$$U_{loc} = [(\Omega_x r)^2 + (U_\infty)^2]^{\frac{1}{2}} \quad (5.2)$$

In Equations 5.1 and 5.2, c is the wing chord and U_{loc} is the local velocity at a given radial position r . If Equation 5.1 is written in terms of the roll rate, Ω_x , the resulting expression matches the acceleration number (or inverse of the Rossby number) obtained by Lentink and Dickinson (2009a) through non-dimensionalization of the Navier-Stokes equation.

$$\pi_{rot} = \frac{\Omega_x^2 r c}{U_{loc}^2} \quad (5.3)$$

With further manipulation, Equation 5.3 can be expressed exclusively as a function of J and R_g/c :

$$\pi_{rot} = \frac{c/R_g}{1 + J^2} \quad (5.4)$$

A curve of $\pi_{rot} = 0.17$ is superimposed onto Figure 5.5, which appears to divide the conical and non-conical regions of the parameter space, as determined from flow visualization. Thus, the non-dimensional velocity gradient represented by Equation 5.1 is an alternate interpretation of the centripetal acceleration number, which is also demonstrated here to have utility in governing LEV behavior for a wing rolling in a free stream. Thus, for the revolving wing, a conical LEV is observed for $\pi_{rot} \gtrsim 0.17$ and a non-conical LEV is observed for $\pi_{rot} \lesssim 0.17$, under the definitions of conical and non-conical given in Section 5.2.

5.3.1 Force Measurements

Force measurements were conducted as described in Section 3.6, for the $J_{3.25} = 1.36$, $J_{2.50} = 1.36$, $J_{3.25} = 0.54$, and $J_{2.50} = 0.54$ cases. The aerodynamic lift and drag forces were computed from the measured forces according to Equation 3.16. The lift and drag forces were non-dimensionalized using the free-stream velocity and the relative velocity (given by Equation (3.4)), to produce two different sets of lift and drag coefficients:

$$C_{L_\infty} = \frac{2L}{\rho U_\infty^2 b c} \quad (5.5)$$

$$C_{D_\infty} = \frac{2D}{\rho U_\infty^2 b c} \quad (5.6)$$

$$C_{L_{rel}} = \frac{2L}{\rho U_{rel}^2 b c} \quad (5.7)$$

$$C_{D_{rel}} = \frac{2D}{\rho U_{rel}^2 b c} \quad (5.8)$$

Equations 5.5 and 5.6 non-dimensionalize the measured force using the free-stream velocity, U_∞ ; whereas equations 5.7 and 5.8 employ the relative velocity, U_{rel} , defined in equation 3.4. It is anticipated that the latter two force coefficients will better accommodate effects of the flow velocity relative to the wing, to help isolate the impact of flow structure on aerodynamic loads.

Figure 5.6 shows the lift and drag coefficients for the four cases, plotted as functions of roll angle. Figure 5.6a reveals distinctly different behaviors between the lift coefficients (equation 5.5) for the fast-roll

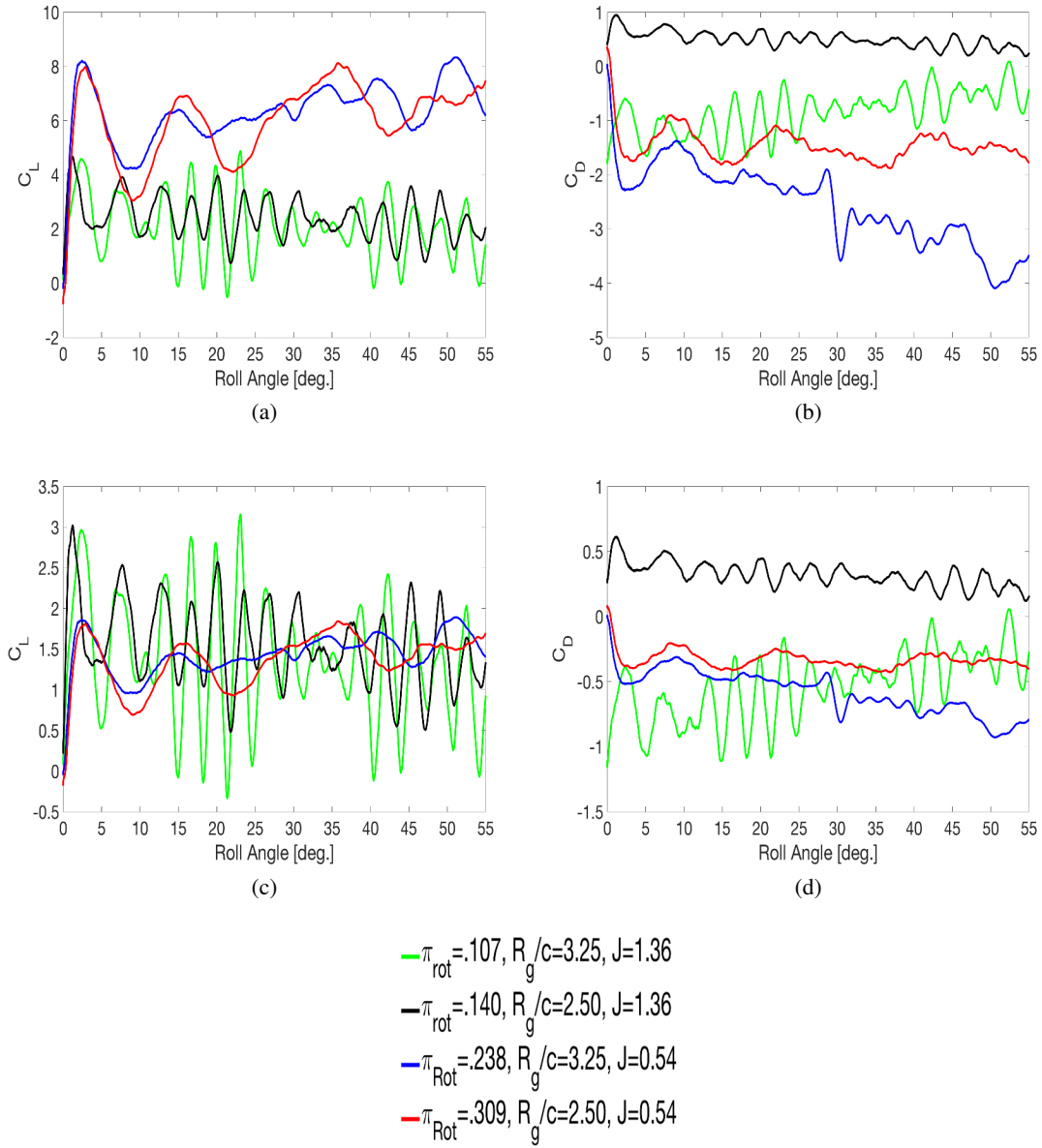


Figure 5.6: Lift and drag coefficients computed according to Equations 5.5 through 5.8. (a,b) Coefficients calculated using U_∞ (Equations 5.5 and 5.6). (c,d) Coefficients calculated using U_{rel} (Equations 5.7 and 5.8).

($J = 0.54$) and slow-roll ($J = 1.36$) cases. The $J = 1.36$ cases exhibit a high oscillation frequency consistent with the high-frequency shedding of vortices from the leading edge, as well as a beat frequency of which the origin is not clear. In contrast, the lift coefficients for the lower advance ratio have an inertial peak at the beginning of the motion, followed by irregular undulations concurrent with events occurring in the formation and evolution of the LEV. The fast-roll lift coefficients computed using Equation 5.5 are also significantly greater than the slow-roll lift coefficients due to the larger relative velocity of the approach flow, which is not taken into account in computing $C_{L_{\infty}}$.

The observed fluctuations in the lift are related to events that are observed in the flow via the isocontours in figure 5.7 for both $J_{3.25} = 0.54$ and $J_{3.25} = 1.36$ cases. In figure 5.6, it can be seen that around $\phi = 17^\circ$ the lift coefficient decreases while the drag coefficient increases. This change in forces coincides with the onset and formation of the arch structure in the outboard region as mentioned in section 3.5. By $\phi = 30^\circ$ another LEV has rolled up which creates an increase in lift along with an increase in thrust. At $\phi = 40^\circ$, there is a local maximum in the lift coefficient and a local minimum for the drag coefficient. This is related to the shedding of the outboard portion of the initial LEV and the presence of the dual vortex structure introduced in Chapter 4. The formation of the second vortex in the dual vortex structure increases suction near the surface of the wing and increases the overall lift as well as reduces the drag. A further increase in lift and decrease drag occurs at $\phi = 49^\circ$. This change in the forces is most likely due to the increase in vorticity at the tip of the wing.

The $J = 1.36$ cases exhibit similar ringing in the drag coefficients, as shown in figure 5.6b. However, unlike the lift coefficients, the drag coefficient magnitudes are highly disparate for the 4 cases. The $J_{3.25} = 0.54$, $J_{2.5} = 0.54$, and $J_{3.25} = 1.36$ cases produce thrust over the range of measured roll angles; however, the $J_{2.5} = 1.36$ case is slightly drag-producing. Additionally, whereas the $R_g/c = 2.50$ cases appear to fluctuate about a relatively constant value, the $R_g/c = 3.25$ cases exhibit either increasing or decreasing trends.

As Figures 5.6c and 5.6d demonstrate, the force coefficients based on relative velocity (Equations 5.7 and 5.8) exhibit much better collapse than those computed according to Equations 5.5 and 5.6, despite that the $J = 0.54$ cases are classified as conical and the $J = 1.36$ cases are non-conical. This is in contrast with numerous prior studies that attribute enhanced lift to the presence of an attached leading-edge vortex on a rolling wing. There may be multiple factors contributing to the similarity observed here, including that the conical LEV is constrained only to the inboard region of the wing, the slow-roll cases may still be governed by early transient effects that similarly enhance their lift, or that high-lift effects attributed to the LEV presence are actually related to another, concomitant, rotational influence. However, it will be shown in the next chapter that the circulation associated with the conical part of the LEV is actually much smaller in magnitude than that of the LEV further outboard, and this is likely a significant factor in limiting the lift of the fast-roll cases.

Whereas the drag coefficients computed using Equation 5.8 exhibit significant overlap for the $J_{3.25} = 0.54$, $J_{2.5} = 0.54$, and $J_{3.25} = 1.36$ cases, the $J_{2.5} = 1.36$ case, which is drag-producing, remains distinct. It should be noted that collapse of this case with the others is impossible due to the difference in sign of the drag force; however, the magnitude of $C_{L_{rel}}$ for the $J_{2.5} = 1.36$ case is similar to the others.

5.4 Conclusion

Dye visualizations were performed over a wide range of advance ratio and dimensionless radius of gyration to further understand the evolution of LEV under the given condition. Three structures were observed in visualizations: A conical structure that forms and persists at the inboard region, an arch structure that remains pinned at the tip and inboard region, and an energetic tip vortex. A classification system was prescribed based on the existence and extent of a conical inboard region of the LEV. Conical structures were generally observed for smaller values of J and R_g/c , and non-conical structures for larger values. The π_{rot}

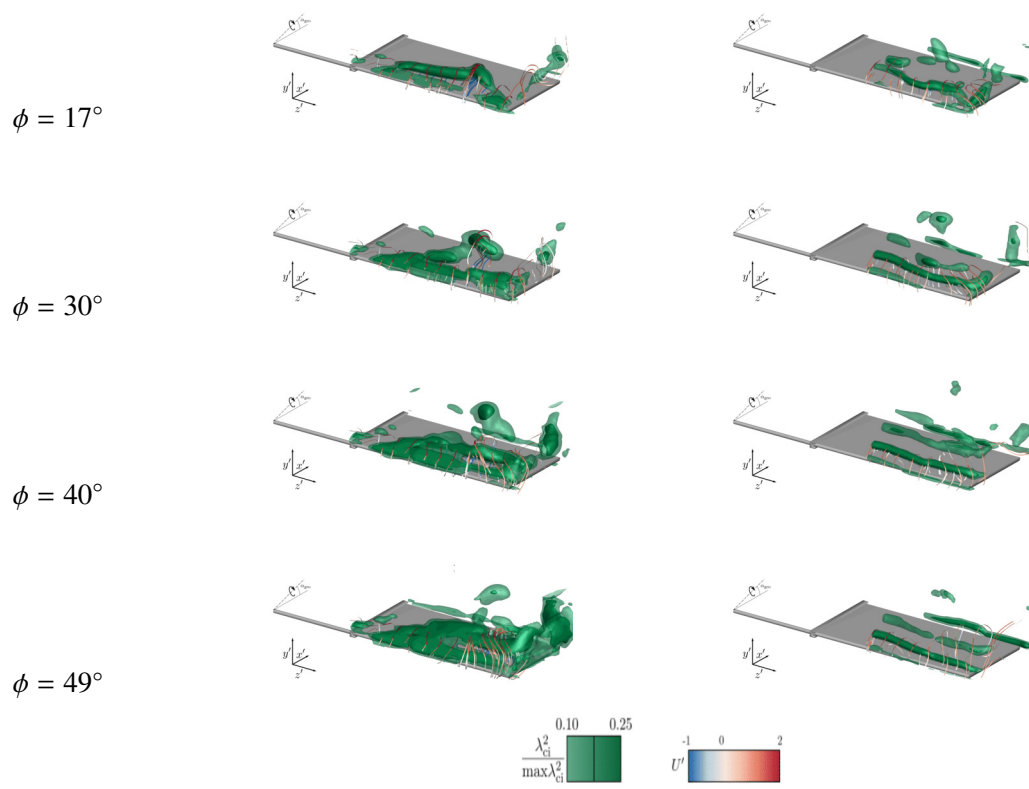


Figure 5.7: Contours of swirl strength, computed using plenoptic PIV, for $J_{3.25} = 0.54$ (left) and $J_{3.25} = 1.36$ (right).

parameter, initially defined as a non-dimensional velocity gradient, was proposed, and it was shown that a constant value of $\pi_{rot} = 0.17$ separated the conical and non-conical structures. Force measurements were conducted on 4 cases (2 conical, 2 non-conical) to understand the impact of the presence of an attached LEV structure at the inboard region. Fluctuations in forces mirrored vortex shedding and other events observed in the LEV evolution, with the conical and non-conical cases exhibiting distinct behaviors. Computing non-dimensional force coefficients using the relative approach velocity at the radius of gyration was found to significantly collapse the force coefficients for the four investigated cases.

Chapter 6

Vorticity Transport Mechanisms on a Rolling Wing

In this chapter, we examine the vorticity transport processes governing LEV formation, for four different cases of a wing in pure roll, using the framework established in section 3.7. The discussion begins with a brief review of salient flow structure developments, and then identifies three spanwise regions of the flow, which manifest distinct behaviors of the LEV, corresponding to markedly different transport processes in each case. The LEV structure and evolution on a rolling wing can thus be understood as a combination of flows observed in different studies with disparate geometries, kinematics, and external flow characteristics.

6.1 Flow Structure on a Wing in Pure Roll

Table 6.1 summarizes the four pure-roll cases that were considered in this chapter. The $J = 0.54$, $R_g = 3.25$ case is considered the baseline case, and will be examined extensively. Deviations from the baseline behavior will be examined for the other cases. Figure 6.1 shows the evolution of the LEV for all four cases. Plenoptic PIV data were acquired over the entire span for the $J_{3.25} = 0.54$ case, and subsets of the span for the others.

J	R_g/c	Name
0.54	3.25	$J_{3.25} = 0.54$
0.54	2.5	$J_{2.5} = 0.54$
1.36	3.25	$J_{3.25} = 1.36$
1.36	2.5	$J_{2.5} = 1.36$

Table 6.1: All the cases that were considered, within this paper. The naming convention that will be used hereafter, is denoted in the last column.

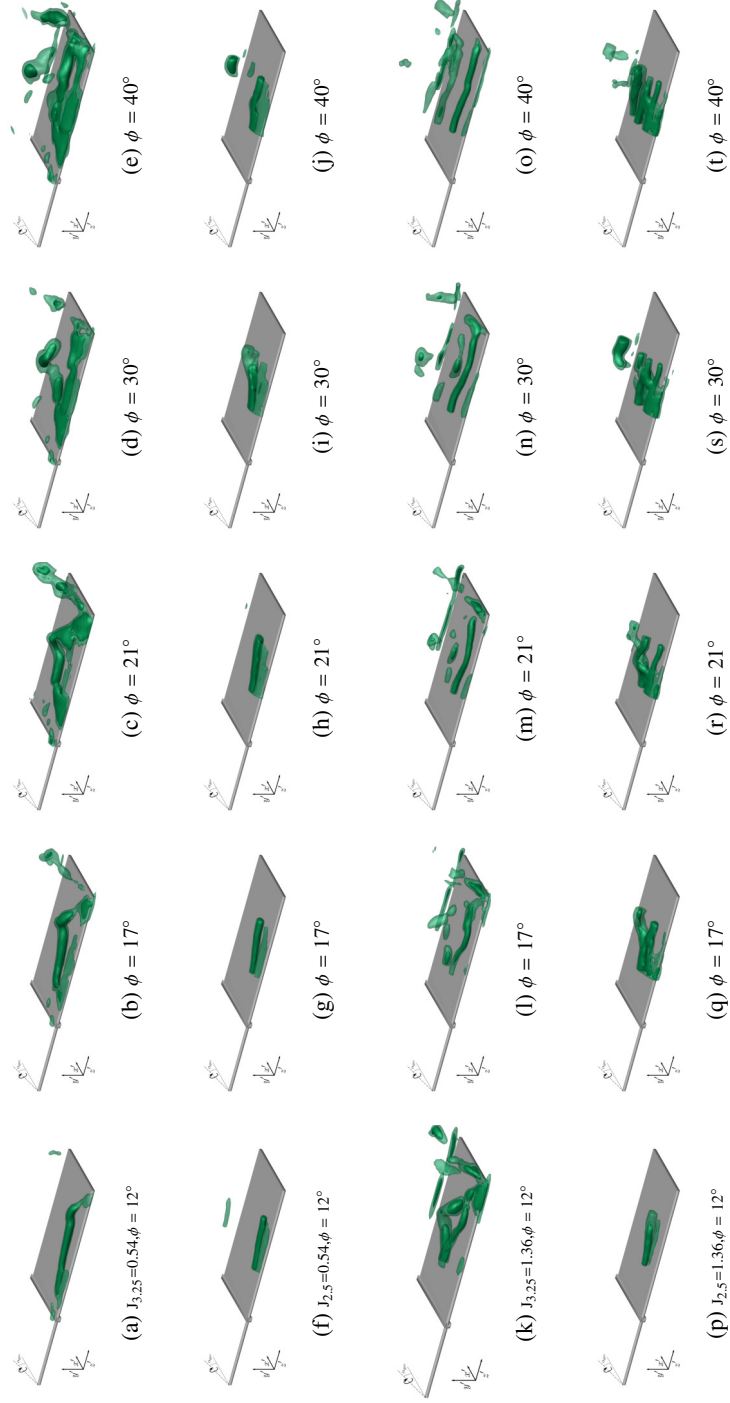


Figure 6.1: LEV formation shown with isosurfaces of normalized swirling strength, for $J=0.54$, $J=1.36$, $\alpha_{eff} = 35^\circ$ for $R_g/c=3.25$ and 2.5 .

As shown in previous chapters, both of the $J = 0.54$ cases ($R_g/c = 3.25, 2.5$) exhibit a conical LEV on the inboard region of the wing. Furthermore, for the baseline case ($J_{3.25} = 0.54$) three distinct behaviors can be observed across the span, as shown in figure 6.2. In Region 1 ($z/b < 0.4$, where the origin of the coordinate system is considered, here to be at the root of the wing), the LEV is *attached*, and essentially stationary on the wing for the duration of the starting roll maneuver. In Region 2 ($0.4 < z/b < 0.75$), the LEV separates from the wing, forming a large arch structure, followed by the creation of a second vortex structure to form a dual vortex system, as observed by Bross et al. (2013), and discussed in Chapter 4. In Region 3 ($z/b > 0.75$), tip effects dominate. A large tip vortex is formed, and the arch structure appears to be confined by the boundary between Regions 2 and 3.

For the cases with $R_g/c = 2.5$ (figures 6.1f through 6.1j and 6.1p through 6.1t), data were acquired only along the central region of the wing $0.35 \leq z/b \leq 0.75$. In the $J_{2.5} = 0.54$ case, it is evident that the LEV remains stationary at the inboard extent of the measurement domain, suggesting, again, a conical vortex structure in the inner region. However, unlike the $J_{3.25} = 0.54$ case, the LEV structure appears to evolve smoothly across the measurement domain as, by $\phi = 30^\circ$, it appears to be lifting into an arch in the outer region of the wing.

At $\phi = 12^\circ$ the $J = 1.36$ cases exhibit structures similar to the $J = 0.54$ cases at later times in their evolution. The measured flow structure for $J_{3.25} = 1.36$ in figure 6.1k exhibits strong similarities to the $J_{3.25} = 0.54$ case at $\phi = 21^\circ$ (figure 6.1c). Similarly, the central region of the $J_{2.5} = 1.36$ case visible in figure 6.1p bears a strong resemblance to the $J_{2.5} = 0.54$ case at $\phi = 21^\circ$.

6.1.1 Circulation Distributions

Figure 6.3 shows the variation in LEV circulation – measured within the control region illustrated in figure 3.8 for the four rolling wing cases. The circulation values are rendered non-dimensional using U_{rel} , defined in equation 3.4) and the wing chord length. For the baseline case ($J_{3.25} = 0.54$), the circulation within Region 1 grows throughout the stroke (figure 6.3a). A sharp transition in circulation is observed at the boundary between Regions 1 and 2. In Region 2, the maximum circulation observed is approximately double that in Region 1, and exhibits two notable peaks at approximately $\phi = 21^\circ$ and 30° , associated with the development of the first and second vortices in the dual vortex system. In contrast, in Region 3, the circulation remains relatively constant after approximately $\phi = 25^\circ$. Comparing the baseline case to the $J_{2.5} = 0.54$ case (figure 6.3b), it is evident that the circulation growth is more uniform in time for the case with the smaller radius of gyration.

For the advance ratio $J = 1.36$ the circulation distribution is being aliased since there are multiple shedding events during the motion, but only finite realizations of the flow field itself. For the smaller R_g/c case, $R_g/c = 2.5$ there is an increase of circulation through the motion and then a subsequent dip in circulation, near $\phi = 40^\circ$, which is due to an instance where a shedding event occurred and was captured during one of the realizations. When comparing the $R_g/c = 3.25$ case to the $R_g/c = 2.5$ case, the larger R_g/c has larger circulation values.

In Chapter 5, it was shown that the fast-rolling cases with attached conical LEVs over a substantial part of the inboard extent of the wing did not, in general, have larger force coefficients than the slow-rotating cases, when the force coefficients were defined using the relative velocity, U_{rel} . It is relevant to note that the part of the LEV that is described as conical has a markedly lower circulation than the portions further outboard. Thus, the effect of the conical LEV on aerodynamic loads may not be apparent in the fast-roll cases due to the diminished strength of the LEV.

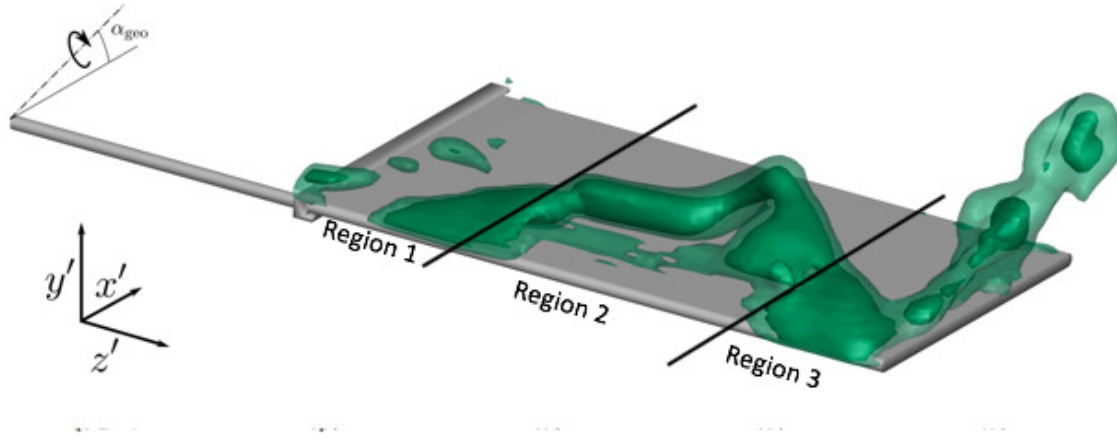
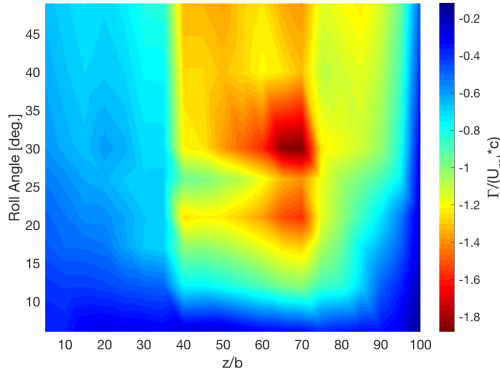
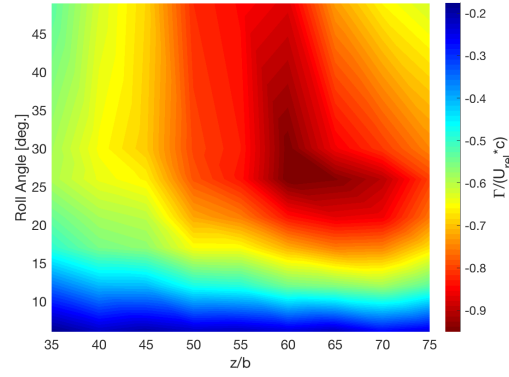


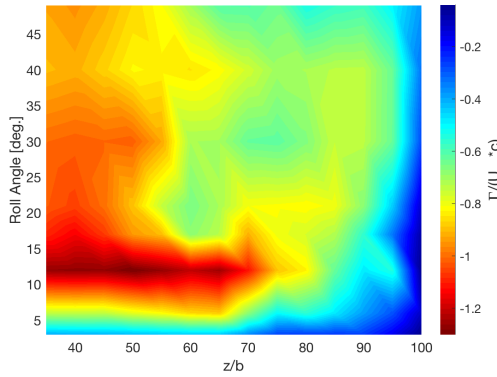
Figure 6.2: Three distinct topological regions shown in the baseline case.



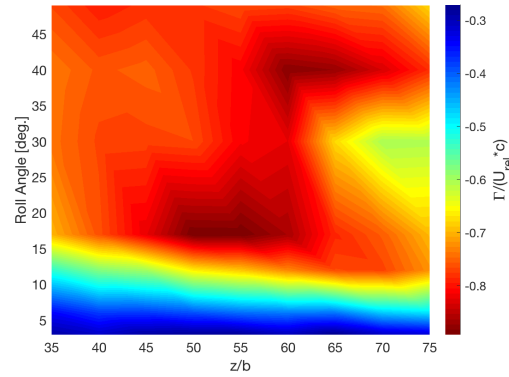
(a) $J_{3,25}=0.54$



(b) $J_{2,5}=0.54$



(c) $J_{3,25}=1.36$



(d) $J_{2,5}=1.36$

Figure 6.3: Spatial and temporal distribution of circulation ($\Gamma/U_{rel}c$).

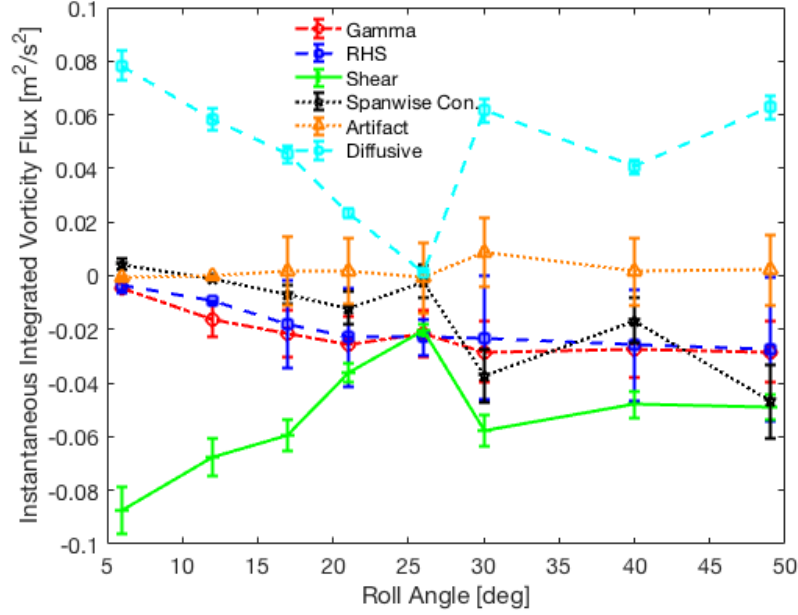


Figure 6.4: Vorticity flux distributions for $J_{3.25}=0.54$ at $z/b=0.5$ (mid-span location).

6.1.2 Circulation Budget Closure and Non-inertial Contributions

Closure of the circulation budget implies equality between the left and right sides of equation 3.47. Each of the terms can be approximated by numerical integration using the velocity field derived from plenoptic PIV, except for the integral on boundary 4 of the control region, which requires knowledge of the pressure on the surface of the wing. Pressure measurements were acquired at $z/b = 0.5$ to determine the surface diffusive flux on boundary 4. For the calculations in equation 3.47, a central difference scheme is used to calculate derivatives. Plotted in figure 6.4 is $J_{3.25} = 0.54$ computed at the mid-span location ($z/b = 0.50$), which shows the contributions of each term of the vorticity transport equation 3.47 discussed in chapter 3. The computed rate of change of circulation agrees well with the sum of the terms on the right hand side of equation 3.47, indicating that the transport mechanisms governing control-region circulation are adequately described by equation 3.47. Based on this agreement, the diffusive flux on boundary 4 will be computed directly from equation 3.47, using plenoptic PIV data to obtain the remaining terms, and assuming equality.

Figure 6.4 shows that the dominant contributions to the circulation within the control region are the leading-edge shear layer and the surface diffusive flux on boundary 4. It should be noted that the sign of the vorticity in the shear layer and LEV is primarily negative; hence the negative circulation contribution of the shear layer. The diffusive flux is opposite in sign, and thus strongly regulates the strength of the vortex as the shear layer contribution continues throughout the stroke. In the initial part of the roll maneuver ($\phi \leq 26^\circ$), the spanwise convective flux contributes little to circulation growth, but grows to magnitudes on the order of the shear layer contributions later in the stroke. This will be discussed in detail. Interestingly, the tilting and Coriolis terms remain small throughout the stroke.

As discussed in chapter 2, Coriolis accelerations have been shown to play a role in governing LEV attachment on revolving wings. Possible effects of Coriolis accelerations can be divided into the categories of those that affect circulation directly, and those that otherwise affect flow dynamics, such as redistribution of existing vorticity. We consider the former possibility in the present work. In this regard, it should be noted that the free vorticity in the flow can be tilted by velocity gradients; however, the Coriolis tilting effect described in equation 3.46 acts on apparent vorticity present in the non-inertial coordinate system,

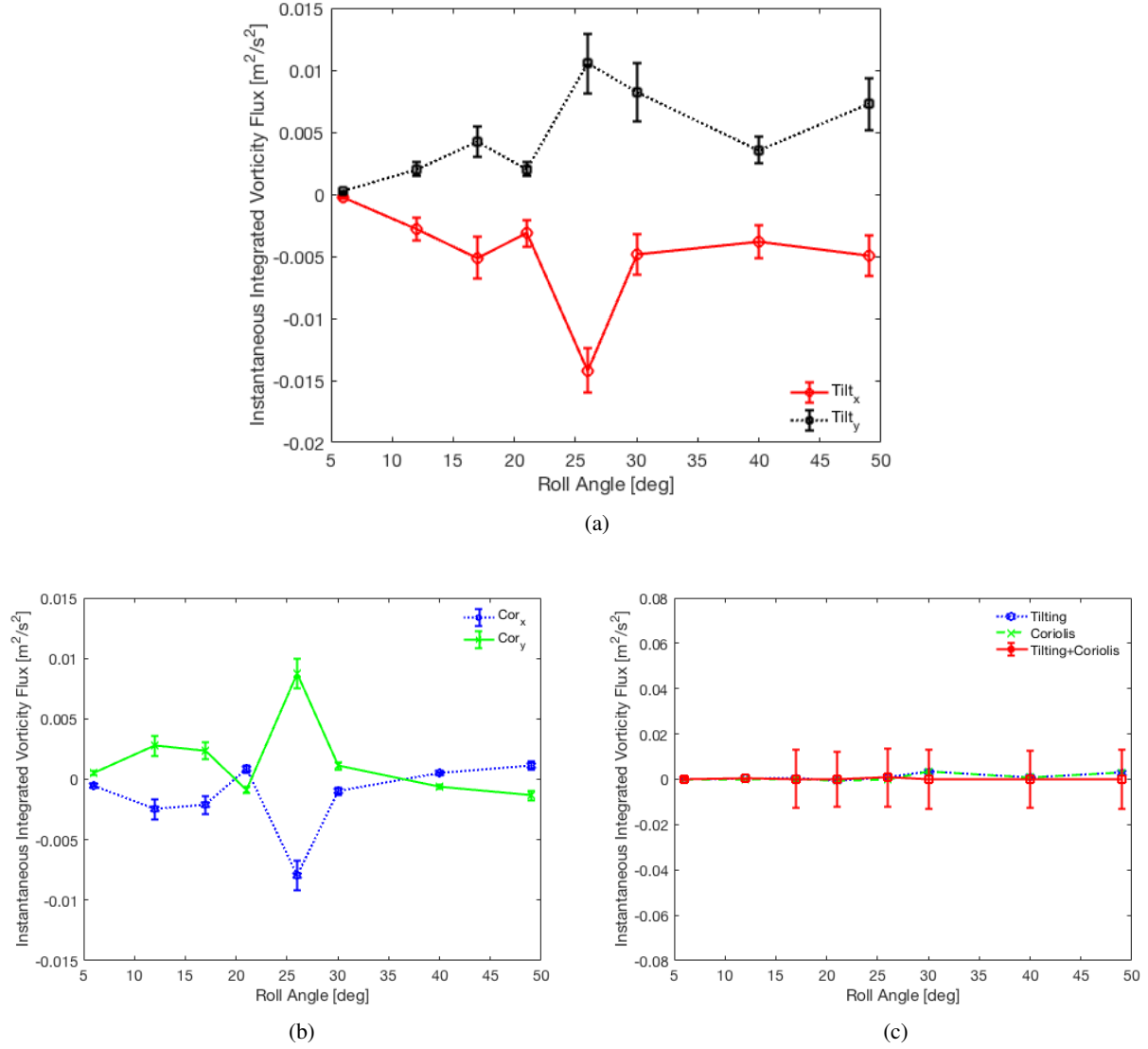


Figure 6.5: Coriolis and tilting fluxes, integrated over the control region, for $J_{3,25} = 0.54$ at $z/b = 0.5$. a) x- and y- tilting terms ($\omega_x \partial u_z / \partial x$, and $\omega_y \partial u_z / \partial y$), b) corresponding Coriolis terms ($2\Omega_x \partial u_z / \partial x$, and $2\Omega_y \partial u_z / \partial y$), and c) a comparison between tilting and Coriolis contributions, as well as the net values of combined tilting and Coriolis contributions.

due to solid-body rotations. Since these rotations are determined only by the wing kinematics, velocity gradients in the flow cannot tilt them, and we therefore assert that the Coriolis contributions, as presented in equation 3.46, cannot produce any real effect on circulation, but rather provide a correction to the tilting terms computed based on measurements in the non-inertial frame.

The x- and y-components of the physical and Coriolis tilting contributions are examined in figure 6.5. Figure 6.5a shows that the spatial integral of x- and y-tilting terms within the control region exhibit a highly symmetric behavior as the roll angle progresses, resulting in significant cancellation of their contributions to the circulation within the control region. This was also observed by Wojcik and Buchholz (2014a) for a wing revolving in quiescent fluid. The Coriolis tilting, shown in figure 6.5b, behaves in a similar manner with similar amplitude as the physical tilting term. The net tilting and Coriolis contributions, as well as their sum is presented in figure 6.5c. It is evident that the net Coriolis contribution is less than that of the physical tilting contribution. As will be shown in the next section, these values are small in comparison to those due to the other fluxes represented in equation 3.47, and therefore do not have a significant effect on the circulation within the control region.

6.2 Vorticity Transport Analysis on the $J_{3.25} = 0.54$ Case

Figure 6.6 shows the circulation contributions for each of the vorticity transport fluxes, throughout the roll maneuver, and over the majority of the span of the wing, for the $J_{3.25} = 0.54$ case. The sum of tilting and Coriolis contributions in each case is shown as a single contribution, since their values are small, and not a significant part of the circulation budget. Similarly, the rate of change of circulation within the control region is small compared to the dominant flux contributions, indicating slow growth of the vortex, due to an approximate balance of fluxes throughout the measured roll angles. Figures 6.6a through 6.6c contain data for planes within Region 1, figures 6.6d through 6.6f describe Region 2, and figures 6.6g to 6.6i describe Region 3. Juxtaposition of the transport rates for each of the regions in this way reveals distinct characteristics associated with each region.

Region 1 is characterized by a slow but relatively steady circulation growth due to the shear-layer flux, that is initially balanced by surface diffusion of circulation with similar magnitude. The diffusive flux diminishes over time and is overtaken by spanwise convection of circulation in the latter half of the maneuver.

Since the spanwise convective flux is frequently attributed to the regulation of LEV strength on rotating wings, its origin and distribution within Region 1 is of particular interest. Figure 6.7 shows the sectional distributions of vorticity, and the constituent quantities forming the growing spanwise convective contributions – spanwise velocity (u_z), vorticity gradient ($\partial\omega_z/\partial z$), as well as the spanwise convective flux itself ($u_z\partial\omega_z/\partial z$) – within a chordwise plane at $z/b = 0.25$. Note that the contribution to circulation, in equation 3.47 and figure 6.6 is the negative integral of this quantity, so blue (negative) regions in figures 6.7p to 6.7t produce positive contributions in figure 6.6c

Figure 6.6b reveals an increase in control-region circulation magnitude until $\phi \approx 17$ deg. During that time, the contributions due to the spanwise convective flux are small compared to the dominant shear-layer and diffusive fluxes, and the spanwise vorticity distribution (figures 6.7a and 6.7b) reveals a vortex growing in size. As the spanwise convection of circulation increases – beginning at $\phi = 21$ deg., the vorticity distribution exhibits a regression in LEV development, as the LEV vorticity distribution becomes slightly more diffuse, reconnects to the downstream boundary layer, and collapses down toward the surface of the wing. In figure 6.7e, a weak vortical feature is apparent above the boundary layer, near the downstream edge of the frame. This is likely an artifact of the wing mounting bracket.

At $\phi = 12$ deg., figure 6.7f reveals a negative (tip-to-root) spanwise flow through much of the LEV cross-section. It should be noted that, before initiation of the roll maneuver, the pitch angle of the wing is such that the wing has a negative angle of attack, and the initial negative velocity is probably a remnant

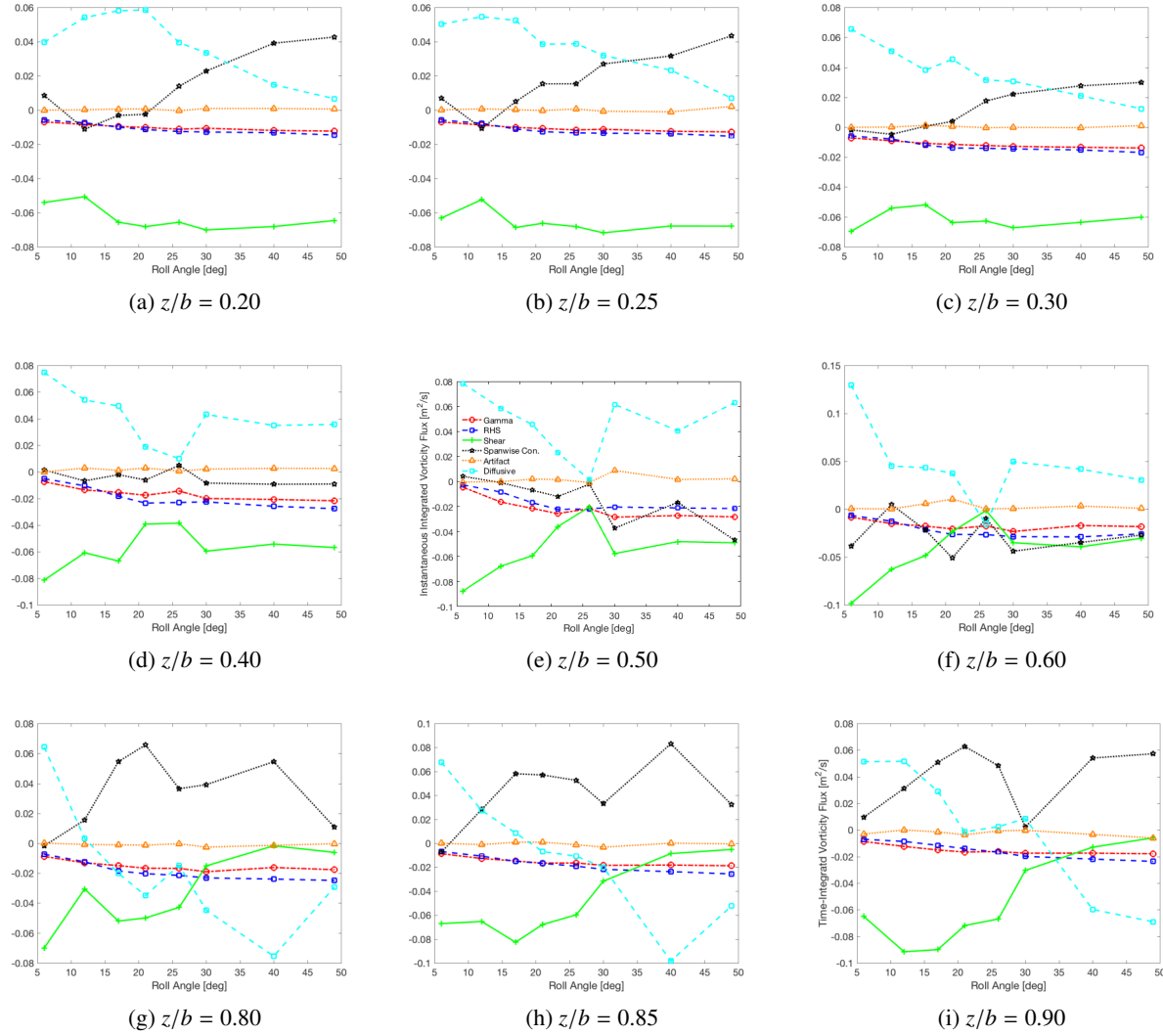


Figure 6.6: Vorticity transport budget for the attached region (top row), separated region (middle row) and tip induced region (bottom row).

of three-dimensional root effects before the motion started. By $\phi = 17$ deg. (figure 6.7g), the spanwise velocity is predominantly positive, and remains positive throughout the recorded motion. The spanwise vorticity gradient shown in figures 6.7k through 6.7o is predominantly negative, with greatest magnitude in the vicinity of the downstream, upper portion of the LEV, because of the expanding cross-section of the conical LEV with increasing spanwise position. The outboard convection of this gradient distribution, by the velocity fields shown in figures 6.7g through 6.7j, results in the classical spanwise convective sink often attributed to LEV regulation on revolving wings (Maxworthy, 1979; Lentink and Dickinson, 2009a; Wong and Rival, 2015). Interestingly, the weak vortical feature observed in figure 6.7j produces a local, strong, negative (red) spanwise convective flux, as shown in figure 6.7t due to the large concurrent spanwise velocity magnitude.

The flow evolution is distinctly different in Region 2. The inboard portion of Region 2 exhibits a quasi two-dimensional LEV. While the outboard portion develops an arch similar to a low aspect ratio plunging plate, as shown by Visbal (2012), and a tip to root flow is induced by this arch. In the case of the plunging plate, the arch vortex is symmetric about the midspan; whereas, for the rolling wing, it initiates at $z/b \approx 0.70$. The shear layer is, again, the largest source of LEV circulation in Region 2. Similar to a plunging plate, the primary sink of circulation in Region 2 (figures 6.6d through 6.6f) is provided by the diffusive flux (Eslam Panah and Buchholz, 2014; Akkala and Buchholz, 2017). However, spanwise convection – driven by the formation of the three-dimensional arch structure – actually strengthens the vortex. This is particularly true at $z/b = 0.60$ (figure 6.6c, for $\phi > 30$ deg.), where the spanwise-convective and shear-layer contributions are similar.

Specifically considering the mid-span location ($z/b = 0.5$), where figure 6.6e shows the vorticity transport budget, figure 6.8 depicts the evolution of the spanwise vorticity distribution, the spanwise convective contribution to the circulation, and its constituent parts at this location. In contrast to Region 1 (figure 6.7, the spanwise velocity is not directed uniformly outboard, but exhibits region of inboard and outboard flow since this plane cuts through the inboard leg of the inclined arch structure. Combined, also, with a more complex distribution of $\partial\omega_z/\partial z$, this results in a rather complicated distribution of spanwise convective flux containing regions of strong positive and negative contributions.

Figure 6.6e shows that the diffusive flux begins to decrease more rapidly at $\phi = 17$ deg. This is contemporaneous with the increase of secondary vorticity beneath the LEV, as shown in figure 6.8b. The shear layer subsequently reaches a minimum contribution by $\phi = 26$ deg., as the LEV begins to pinch off, and second LEV begins to roll up and form the second vortex in the dual vortex system, as described in Chapter 4. After the formation of this dual vortex system the shear layer and diffusive flux recover.

For much of the roll maneuver, figures 6.6g through 6.6i show that the circulation budget in Region 3 is dominated by the spanwise convective and diffusive contributions. The shear layer contribution is relatively weak due to pressure relief near the wing tip. Figure 6.9 shows the LEV evolution, the spanwise convective flux and its constituent parts from $\phi = 12$ deg. to $\phi = 30$ deg. at $z/b = 0.8$. At $\phi = 21$ deg., the shear layer flux is rapidly decreasing, with a concurrent spanwise velocity from the tip to root (figure 6.9h) induced by the tip vortex in the upper region of the LEV. Since the spanwise velocity is in the inboard direction, this acts to weaken the LEV (fig. 6.9r). As the motion continues, an outboard flow develops (fig. 6.9j), causing the spanwise convective contribution to decrease. The spanwise convective contribution reaches a minimum at $\phi = 30$ deg., while the LEV lifts off the surface at this spanwise location. Outboard of this location the LEV remains pinned.

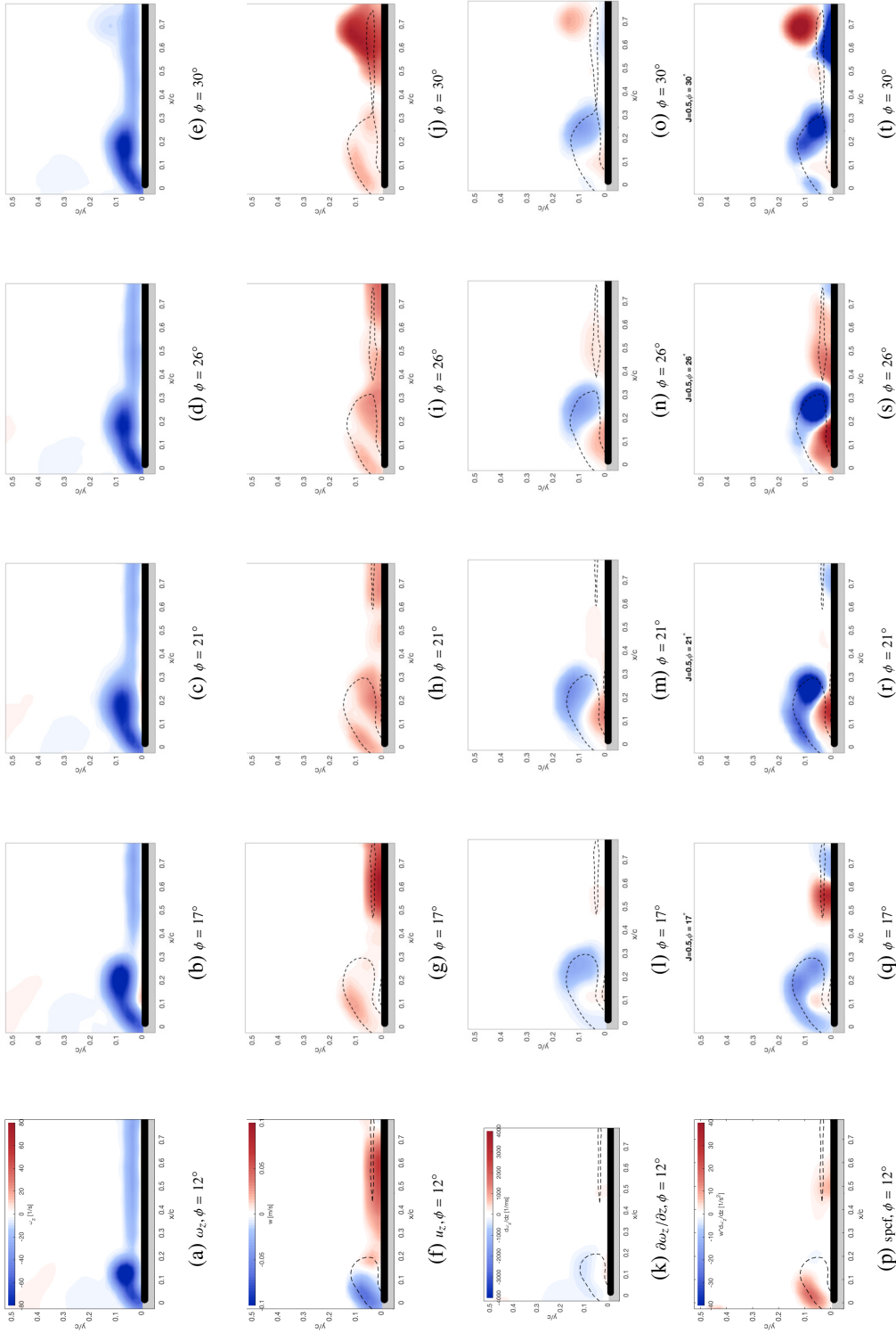


Figure 6.7: The LVE formation and components of the spanwise convective flux (spcf) at $z/b=0.25$. (a-e): evolution of the vorticity field, (f-j): spanwise velocity (u_z), (k-o): spanwise vorticity gradient ($\partial\omega_z/\partial z$), (p-t): the product of spanwise velocity and vorticity gradient ($u_z\partial\omega_z/\partial z$). Note that the actual spanwise convective flux in equation 3.47 within the control region is the opposite sign of $u_z\partial\omega_z/\partial z$.

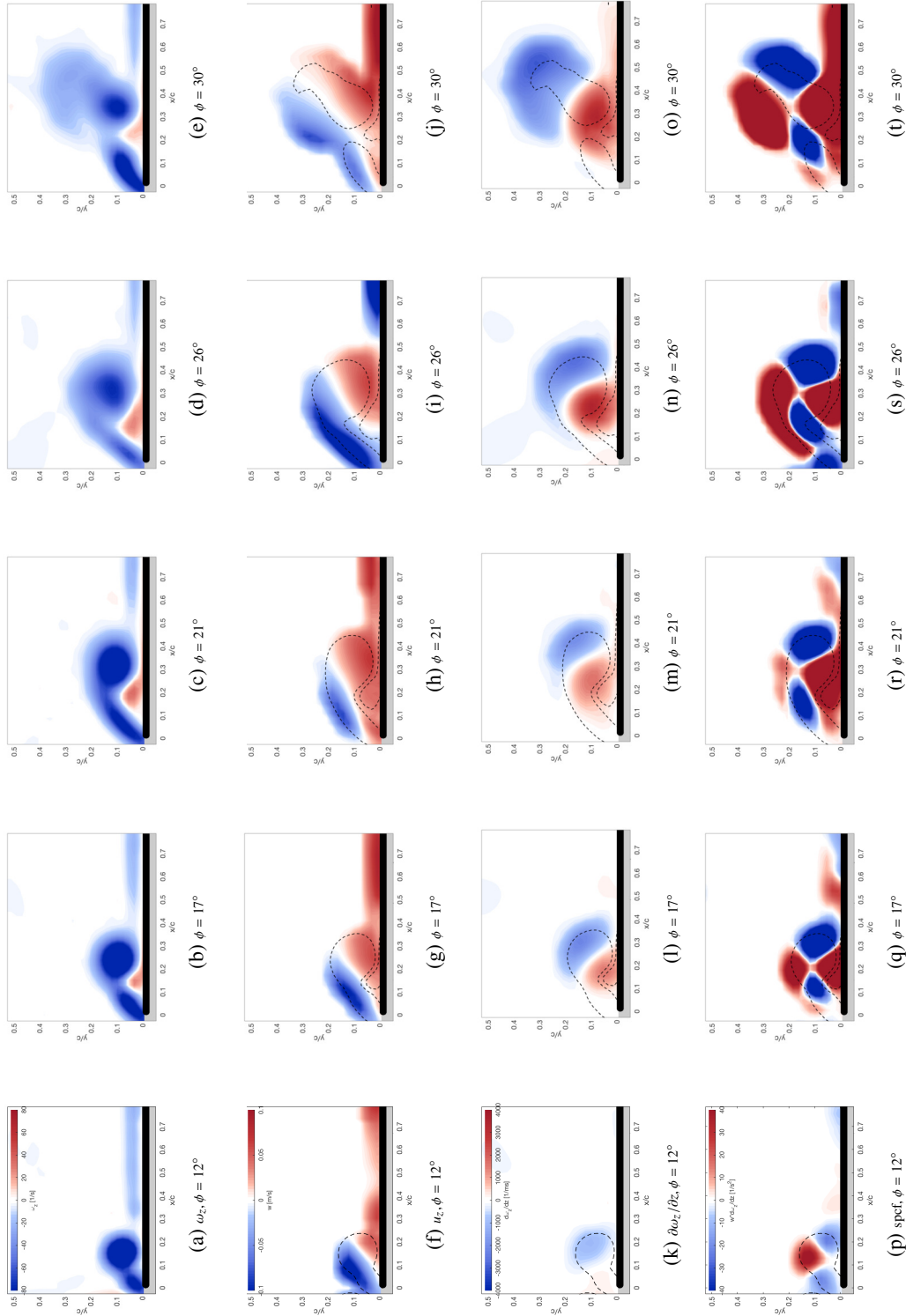


Figure 6.8: The LEV formation and components of the spanwise convective flux (spf) at $z/b=0.50$. (a-e): evolution of the vorticity field, (f-j): spanwise velocity (u_z), (k-o): spanwise vorticity gradient ($\partial\omega_z/\partial z$), (p-t): the product of spanwise velocity and vorticity gradient ($u_z\partial\omega_z/\partial z$). Note that the actual spanwise convective flux in equation 3.47 within the control region is the opposite sign of $u_z\partial\omega_z/\partial z$.

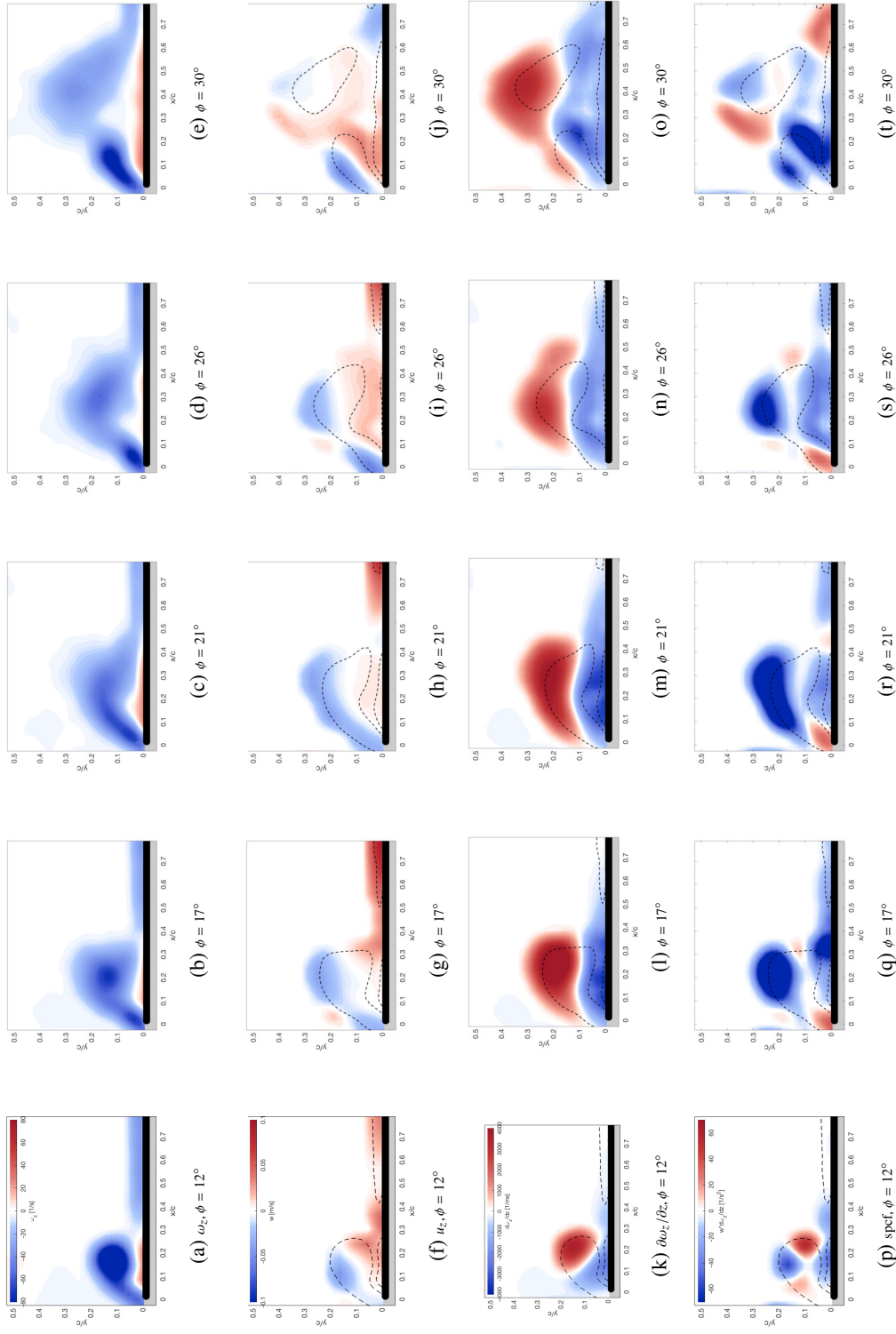


Figure 6.9: The LLEV formation and components of the spanwise convective flux (spcf) at $z/b=0.80$. (a-e): evolution of the vorticity field, (f-j): spanwise velocity (u_z), (k-o): spanwise vorticity gradient ($\partial\omega_z/\partial z$), (p-t): the product of spanwise velocity and vorticity gradient ($u_z\partial\omega_z/\partial z$). Note that the actual spanwise convective flux in equation 3.47 within the control region is the opposite sign of $u_z\partial\omega_z/\partial z$.

6.2.1 The Role of Radius of Gyration

Figure 6.3 shows that the boundaries of the three regions identified on the rolling wing mark distinct transitions in measured circulation. It also suggests that decreasing radius of gyration pushes the boundaries, and therefore the circulation distributions further outboard. Circulation for the baseline case shown in figure 6.3a, has a strong gradient line delineating Region 1 and Region 2, where Region 2 contains the strongest circulation. The maximum circulation is observed near the $z/b = 0.62$ for the baseline case. For the $J_{2.5} = 0.54$ case (fig. 6.3b) the maximum circulation occurs near $z/b = 0.68$. A similar behavior can be seen in the $J = 1.36$ cases. For the larger R_g/c , $J_{3.25} = 1.36$ the maximum circulation is near $z/b = 0.62$ (fig. 6.3c), while for $R_g/c = 2.5$ the maximum occurs $z/b = 0.68$ (fig. 6.3c).

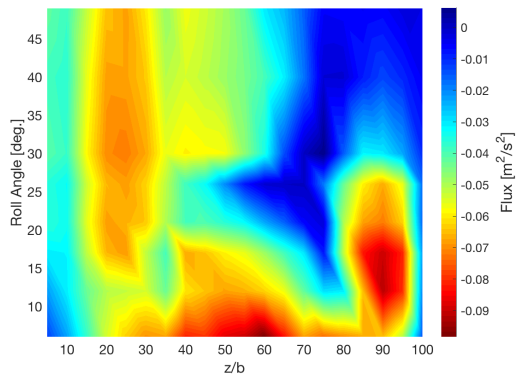
6.2.2 The Role of Advance Ratio

Figure 6.3 also shows that the maximum circulation is strongly influenced by advance ratio. An increase in the advance ratio will decrease the maximum dimensionless circulation observed on the wing for a given R_g/c . Considering the $J_{3.25} = 0.54$ and $J_{3.25} = 1.36$ cases, figure 6.3 shows that $J_{3.25} = 0.54$ has a stronger maximum dimensionless circulation with maximum magnitude of $\Gamma^* = 1.89$, while for $J_{3.25} = 1.36$ the maximum magnitude is $\Gamma^* = -1.26$. The dimensionless circulation is non-dimensionalized by the chord, c , and the relative velocity at the radius of gyration, u_{rel} . This accounts for the velocity induced by the rotation. A similar behavior is seen at the lower R_g/c , $J_{2.5} = 0.54$ which has a maximum circulation magnitude of $\Gamma^* = 0.99$ and for $J_{2.5} = 1.36$ which has a maximum $\Gamma^* = 0.87$.

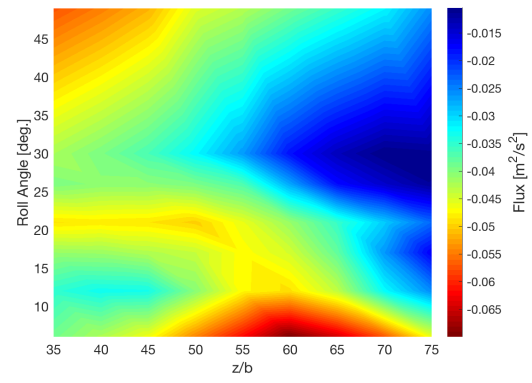
The advance ratio also dictates the shear layer flux distribution in time and with spanwise position. For the cases in which $J = 1.36$, figures 6.10c and 6.10d show that the shear layer varies less along the span, and lacks the large fluctuations observed for the $J = 0.54$ cases shown in figures 6.10a and 6.10b. However, it is noted that the $J = 1.36$ cases are not sampled with sufficiently high frequency to capture all of the fluctuations that may be associated with the generation and shedding of the vortices from the leading edge.

6.3 Conclusion

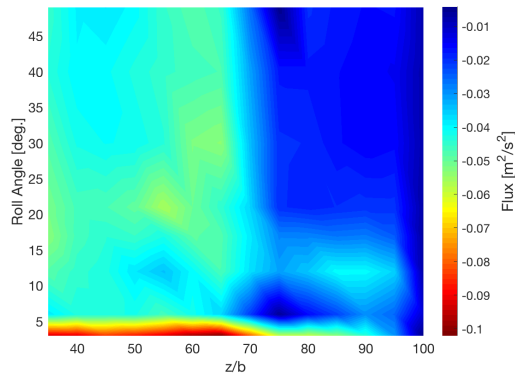
Four cases were considered, in which the wing was articulated in a pure roll maneuver at $\alpha = 33^\circ$. Two different advance ratios and two values of R_g/c were studied. The vorticity transport framework developed in section 3.7 was used to quantify the fluxes of circulation, and underlying mechanisms, governing LEV development. It was found that the contribution attributed to the Coriolis acceleration did not contribute significantly to the strength of the LEV. The baseline case ($J_{3.25} = 0.54$) exhibited three distinct regions encompassing a broad range of LEV behaviors previously observed on maneuvering wing. Region 1, in which an attached, conical LEV was observed during the measured trajectory, was governed by the shear layer flux that was nearly constant in time. The shear-layer flux was initially balanced by the diffusive flux early in the motion, but spanwise convection gradually overtook the diffusive flux as the spanwise flux grew in strength and the diffusive flux concomitantly weakened. Region 2 consisted of a quasi two dimensional LEV, which evolved into a dual vortex system and an arch structure. The shear layer contribution was the dominant source of vorticity, while in some spanwise locations, the spanwise convective contribution driven by the arch contributed as a source. These sources were balanced by the diffusive contribution, similar to a plunging wing. Thus, rotational effects did not appear to dominate the behavior of the LEV as they did in Region 1. Region 3 was governed by tip effects, where the LEV wing was pinned to the surface, and exhibited a *reverse* evolution previously seen in a plunging plate. The circulation budget was dominated by a balance between the diffusive flux and the spanwise convective contribution which reversed the LEV evolution process to reattach the LEV to the downstream boundary layer. Varying radius of gyration was found to govern the spatial location at which the region boundaries were located on the wing, with smaller



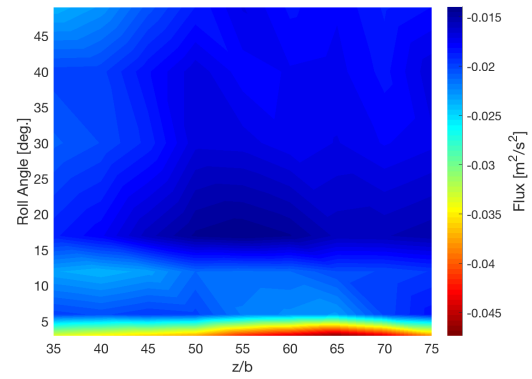
(a) $J_{3,25}=0.54$



(b) $J_{2,5}=0.54$



(c) $J_{3,25}=1.36$



(d) $J_{2,5}=1.36$

Figure 6.10: Spatiotemporal distribution of shear layer flux.

R_g/c resulting in the boundaries moving outboard. The advance ratio was found to determine the strength of the vortex, and also affected the overall evolution of the LEV. As the advance ratio decreased there was an increase in the maximum non-dimensional circulation.

Chapter 7

Vorticity Transport Mechanisms on a Pitching Wing in Inertial and Rolling Reference Frames

In this chapter, the vorticity transport mechanisms governing LEV strength will be examined for the wing articulated in pure pitch, as well as a pitch maneuver articulated on a rolling wing. Chapter 4 describes the flow structure and evolution in detail for the cases examined here. For completeness, key attributes of flow topology will be summarized prior to examining the LEV circulation and transport mechanisms for each case. Kinematics for these cases are described in Chapter 3.

For the pure pitch maneuver, two values of reduced pitch rate were investigated: $k = 0.2$ and 0.5 , where k is defined in equation 3.7. For the combined pitch and roll maneuver, the wing rolling motion is first initiated, with a wing pitch angle such that the effective angle of attack at the radius of gyration is zero. The pitch motion is subsequently initiated. It was shown in Chapter 4 that the parameter k_{rg} , defined in equation 3.8, plays a critical role in governing the structure and evolution of the LEV; however, it is not a similarity parameter, so that the reduced pitch rate, k and radius of gyration, R_g/c govern characteristics of the flow topology and development time scales that cannot be fully accounted for by k_{rg} .

Table 7.1 summarizes the combined pitch-roll cases examined here. Articulating a pitching motion on a rolling wing can help to isolate the effects of the roll motion on transport within the LEV, when compared to the pure pitch case. Also, by comparing the pure roll and pure pitch cases, we can determine whether the strengths of the LEVs and underlying transport mechanisms in the combined motions are preferentially governed by pitch or roll influences. The radius of gyration was held constant at $R_g/c = 3.25$ for all cases examined in this chapter.

k_{Rg}	J	k	R_g/c
0.22	0.54	0.46	3.25
0.37	1.36	0.46	3.25
0.50	0.54	1.05	3.25

Table 7.1: Constituent parameters composing k_{Rg}

7.1 The Pitching Wing

7.1.1 Flow topology of a Purely Pitching Plate

Figure 7.1 summarizes the flow field topologies for the $k = 0.2$ and $k = 0.5$ cases, and juxtaposes them with the pure roll $J_{3,25} = 0.54$ case. For the pure pitch maneuver an arch vortex forms across the span and begins to lift off, and eventually sheds. As discussed in Chapter 4, a triple vortex system forms for the slower pitch case, and both pitch cases appear to be highly symmetric. This is in contrast to the baseline roll case in which three distinct spanwise regions were identified, leading to a highly asymmetric evolution of the vortex. Nevertheless, in Regions 2 (mid-span) and 3 (tip) of the rolling wing – where rotational effects are diminished – the dual arch system, and tip-induced pinning of the LEV to the surface of the wing, respectively, do bear some resemblance to the structures observed on the pitching wings.

7.1.2 LEV Circulation for the Pitching Maneuver

Figure 7.2 shows the non-dimensional circulation for both of the pure-pitch cases at the mid-span location, where the scaling factor $c * U_\infty$ is the same for both cases. Despite that the values at an angle of attack of 5° deviate by a factor of 50%, it is notable that the rate at which circulation subsequently grows is similar in both cases despite that the pitch rates vary by a factor of 2.5. Figure 7.3 shows the dimensionless circulation data with the initial value subtracted. The dimensionless circulation is plotted as functions of dimensionless time, as well as angle of attack. A clear collapse of the data is evident when plotting the modified, dimensionless circulation against angle of attack. Thus, LEV circulation at the mid-span location depends only on the initial transient at the beginning of the motion, and the angle of attack.

7.1.3 Vorticity Transport on a Wing In Pure Pitch

Figure 7.4 shows the contributions of the shear-layer (Shear), surface-diffusive (Diffusive), spanwise convective (SPCF), and tilting + Coriolis (Tilt. & Cor.) fluxes to LEV circulation growth. It should be noted that surface pressures were not measured in this case, due to the strong pressure impulse present at the start of the motion (responsible for the very rapid growth in circulation implied in the previous section), so the diffusive flux presented here is derived based on the assumption that the circulation budget (equation 3.47) closes. These quantities are shown at the $z/b = 0.35$ and 0.50 cases for $k = 0.5$, and at $z/b = 0.5$ only for $k = 0.2$. It is noteworthy that, in all cases, the rate of growth of circulation (labeled Circulation) is quite small in comparison to the dominant shear-layer and diffusive fluxes. The shear-layer contribution in both cases ($k=0.2$ and $k=0.5$) saturates by an angle of attack of $\alpha = 25^\circ$.

Figure 7.5 compares the shear layer contributions, rendered non-dimensional by a scaling factor of $U_\infty * c * k$ for the two pitch rates at $z/b = 0.5$, allowing comparison of the saturation values. Despite that the scaling factor includes k , the dimensionless saturation values differ by a factor of approximately 40%, indicating that the scaling over-compensates for the value of k , and that the final value is not strongly-dependent on k . Taking into consideration the cross-sectional distributions of the spanwise vorticity component ω_z at $z/b = 0.5$, presented in figure 7.6, it is evident that the saturation of the shear-layer contribution is approximately concomitant with the emergence of secondary (opposite-sign) vorticity on the surface of the wing beneath the LEV, which is clearly evident in both cases by $\alpha = 30^\circ$. It has been shown, for a plunging plate (Eslam Panah et al., 2015), that the strength of the secondary vortex (which is included in the circulation measurements reported in the previous section) is regulated by entrainment of secondary vorticity by the leading-edge shear layer, leading to ingestion of the secondary vorticity into the LEV, and ultimately cross-cancellation. The observation of figure 7.6 thus indicates that, at this point in the LEV development, the entrainment rate is no-longer keeping up with generation of secondary vorticity. From a control perspective, this may provide an opportunity to perturb – and possibly reinvigorate – the entrainment

process, modifying the secondary vortex strength, and thus also possibly the trajectory and evolution of the LEV.

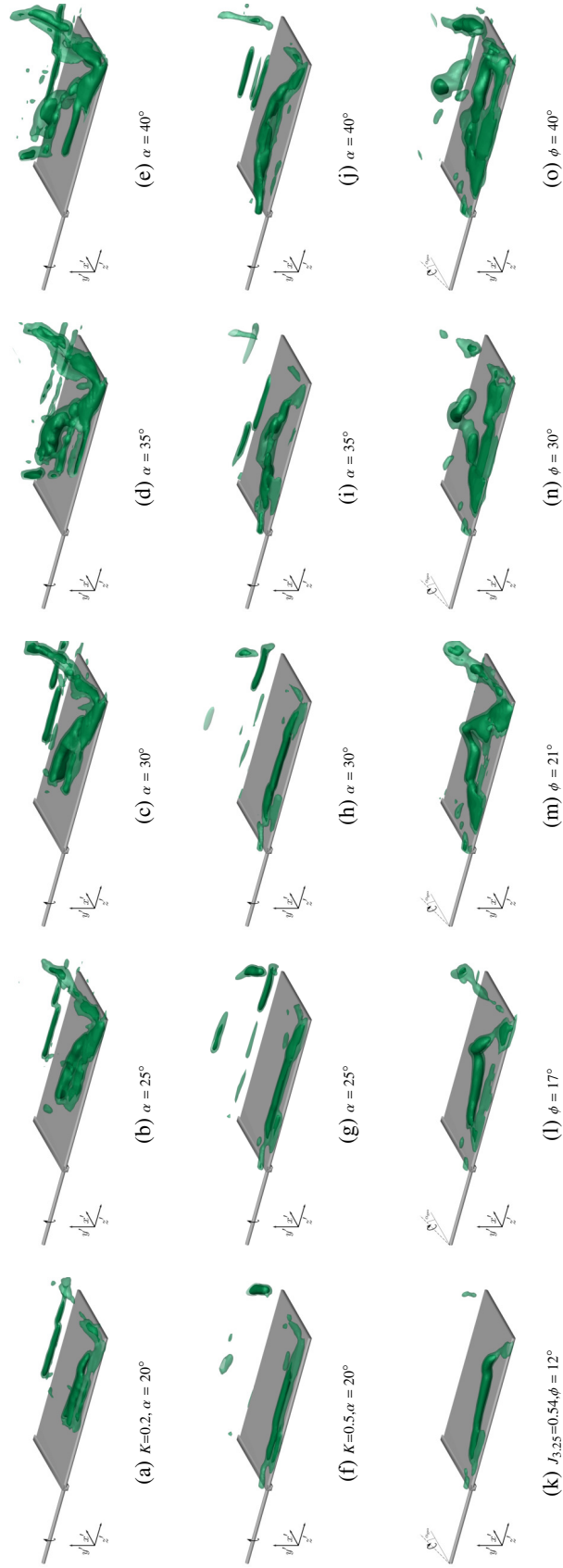


Figure 7.1: LEV formation shown with isosurfaces of normalized swirling strength, for $k = 0.2$ (a-e) and $k = 0.5$ (f-j) from $\alpha = 20^\circ$ to $\alpha = 40^\circ$, and for $J_{3,25} = 0.54$ (K-o) over the roll angles of $\phi = 12^\circ$ to $\phi = 40^\circ$

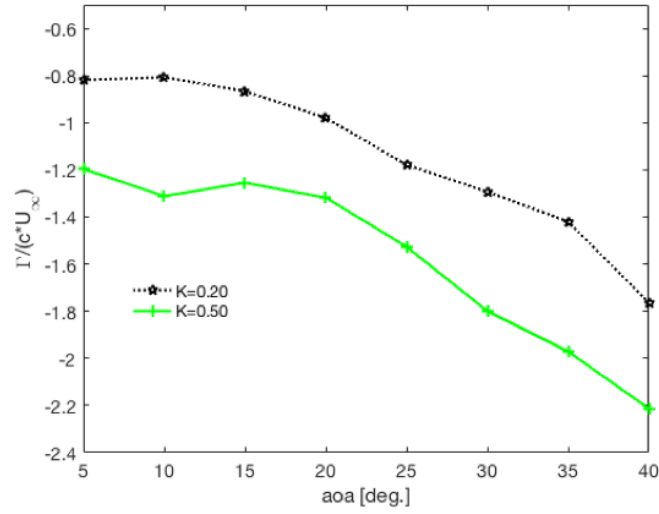


Figure 7.2: Circulation for $k=0.2$, and $k=0.5$

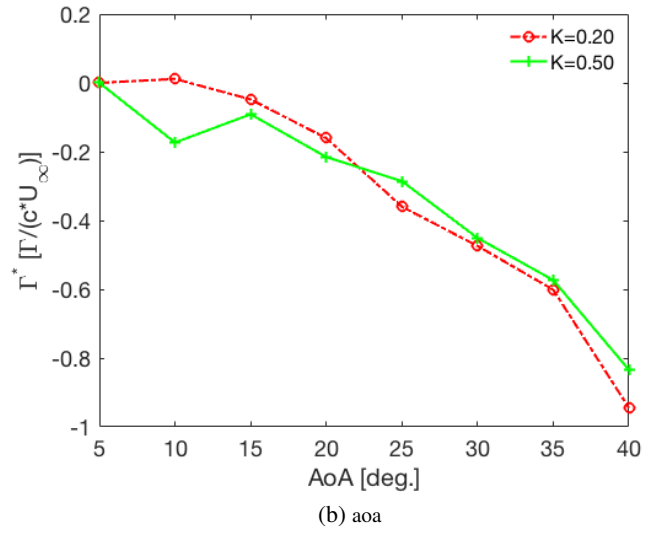
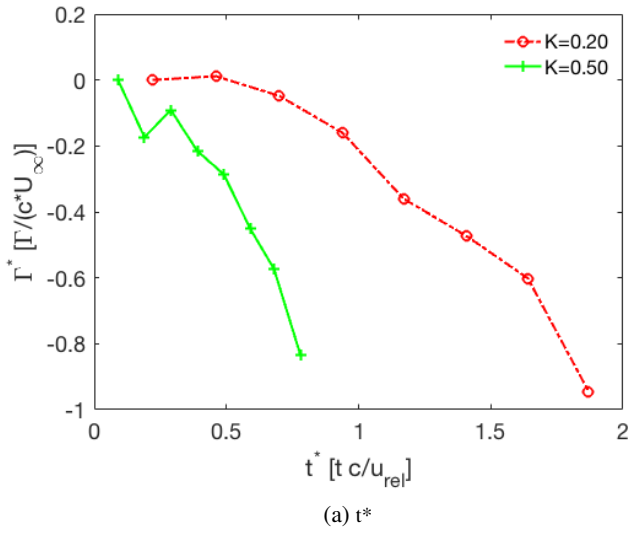
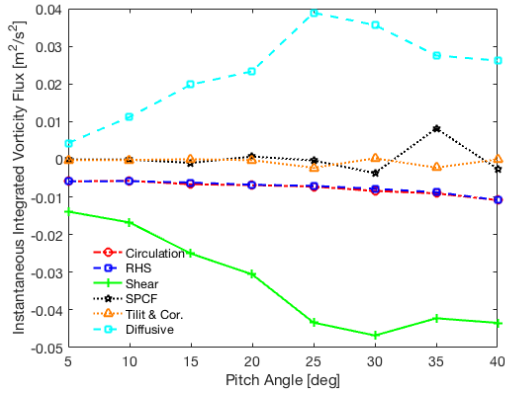
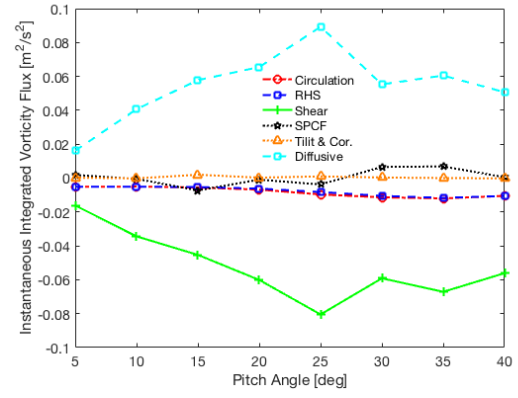


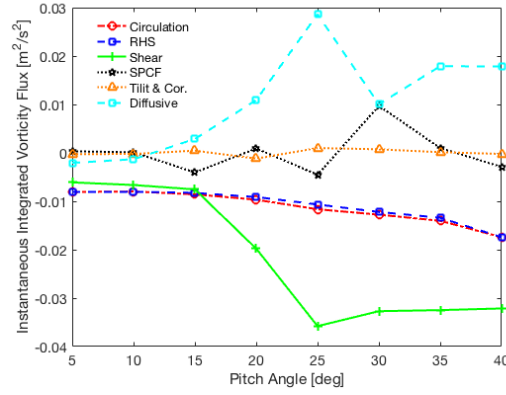
Figure 7.3: Circulation for $k=0.2$, and $k=0.5$, with values at 5-degree pitch angle subtracted.



(a) $k=0.5, z/b=0.35$



(b) $k=0.5, z/b=0.50$



(c) $k=0.2, z/b=0.50$

Figure 7.4: Flux Distributions for pure pitch $k=0.2$ and $k=0.5$

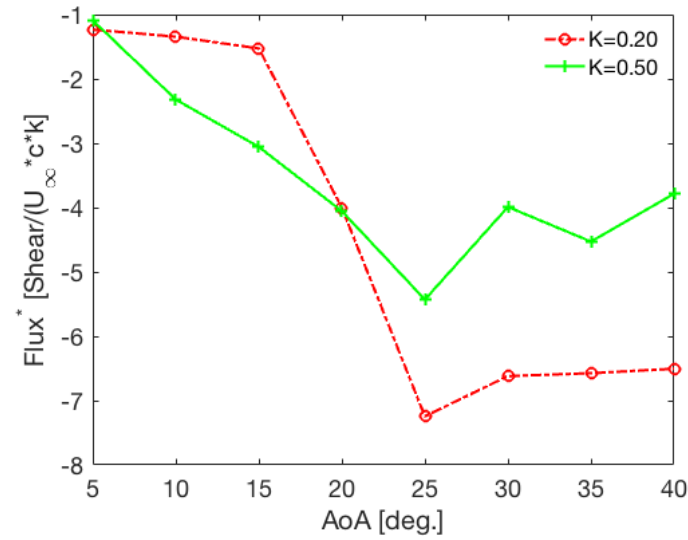


Figure 7.5: Shear layer contribution for $k=0.2$ and $k=0.5$ at $z/b=0.5$, highlighting the saturation at an angle of attack of $\alpha = 25^\circ$.

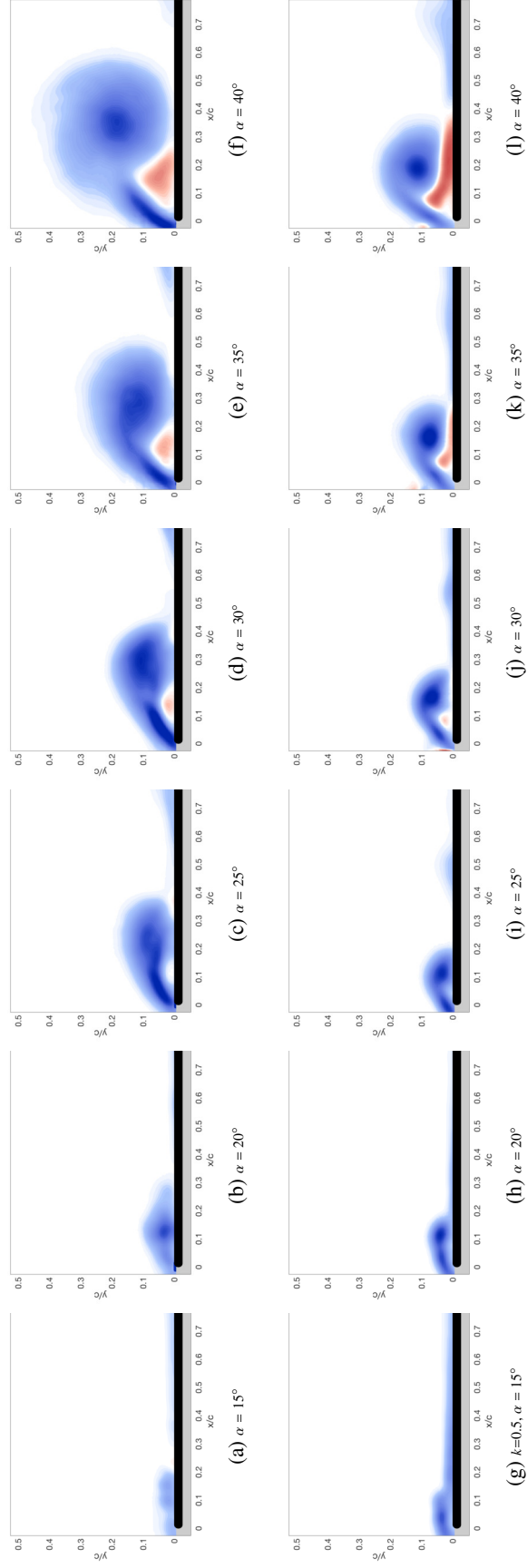


Figure 7.6: Vorticity at $z/b=0.5$ for a reduced pitch rate of $k=0.2$ (figs. 7.6a-7.6f) and $k=0.5$ (figs. 7.6g-7.6l), over the range $\alpha = 10^\circ - \alpha = 40^\circ$.

7.2 Dynamic Pitching of a Rolling Wing

7.2.1 Flow Structure due to Dynamic Pitching of a Rolling Wing

Figure 7.7 shows the evolution of the LEV for the three different k_{rg} values. Plenoptic PIV data were acquired over the entire span for the $k_{rg} = 0.22$ case, and $z/b \geq 0.34$ for all other cases considered. The $k_{rg} = 0.22$ case is shown in the first column of figure 7.7, in which its final state ($\alpha=45$ deg.) looks similar to $J_{3.25}=0.54$ at $\phi=21$ deg. shown in figure 6.1c. However, as discussed in Chapter 4, these two cases do not evolve in a similar manner. The pitching maneuver appears to keep the LEV attached and near the surface, similar to a pure pitching maneuver, and in contrast with the corresponding pure-roll case ($J_{3.25} = 0.54$). However, unlike the pure pitching kinematics, where the arch structure is symmetric about the mid-span, in this case it forms asymmetrically, outboard of the mid-span, akin to the baseline roll case.

The evolution of the $k_{rg}=0.37$ case, which employs the slow-roll kinematics of the $J_{3.25} = 1.36$ case, is shown in the second column of figure 7.7. The effect of roll appears to be minimal here, as the LEV evolution bears a strong similarity to the $k = 0.5$ pure-pitch case. The LEV remains near the surface of the plate for the duration of the motion, and does not form an outboard compact arch akin to the baseline roll case. Furthermore, when comparing this to the $J_{3.25}=1.36$ case, in which the LEV sheds rapidly, this LEV remains attached.

The $k_{rg} = 0.5$ case employs the fast-roll kinematics ($J = 0.54$), as with the $k_{rg} = 0.22$ case; however, the pitch rate is approximately doubled, achieving a comparatively very high pitch rate of $k = 1.05$. Evolution for the $k_{rg} = 0.50$ case is shown in the third column of figure 7.7. This case exhibits features of both of the prior pitch-roll cases early in the motion, as discussed in Chapter 4. Again, the LEV remains near the surface for the duration of the motion, which is in-line with higher reduced pitch rates. However, an asymmetric arch appears to form in the outboard portion of the wing indicating the influence of roll. The arch structure in this case is the most compact of any of the cases considered in this study, due to the strong pitch influence. In fact, as the motion continues beyond an angle of attack of 40 deg., the arch structure regresses, and the appearance of the LEV evolves toward that of the pure pitch case. This can be understood by considering that J initially governs the flow structure early in the motion, since the initial motion is pure roll. The subsequently-added pitch motion then appears to eventually dominate the LEV evolution.

7.2.2 Circulation due to Dynamic Pitching of a Rolling Wing

Measured, dimensional, circulation values are shown, throughout the motion, in figure 7.8, for all three cases, at $z/b = 0.35, 0.50$, and 0.65 . At all three spanwise locations, and all three cases, the circulation grows monotonically for the duration of the motion. This behavior is consistent with the pure-pitch maneuver. For the pure roll maneuver, the circulation initially grew rapidly then remained relatively constant. Also, for a given spanwise location, all three cases exhibit similar rates of circulation growth; however, these growth rates increase with increasing spanwise position. The former observation is consistent with that of pure pitch; however, the spanwise asymmetry in the growth rates suggests a roll influence.

The same data are shown in non-dimensional form, in figure 7.9, scaled by $U_{rel}c$, where U_{rel} is defined in equation 3.4. It is noteworthy that cases do not collapse similar to the pure pitch case. This is not surprising, since U_{rel} differentiates the cases based on roll-rate, and the slopes of the dimensional curves are clearly not systematically dependent on roll rate.

7.2.3 Vorticity Transport due to Dynamic Pitching of a Rolling Wing

To better understand the mechanisms governing the circulation growth rates discussed in the previous section, the underlying vorticity transport mechanisms are investigated. Figures 7.10 to 7.12 contain the in-

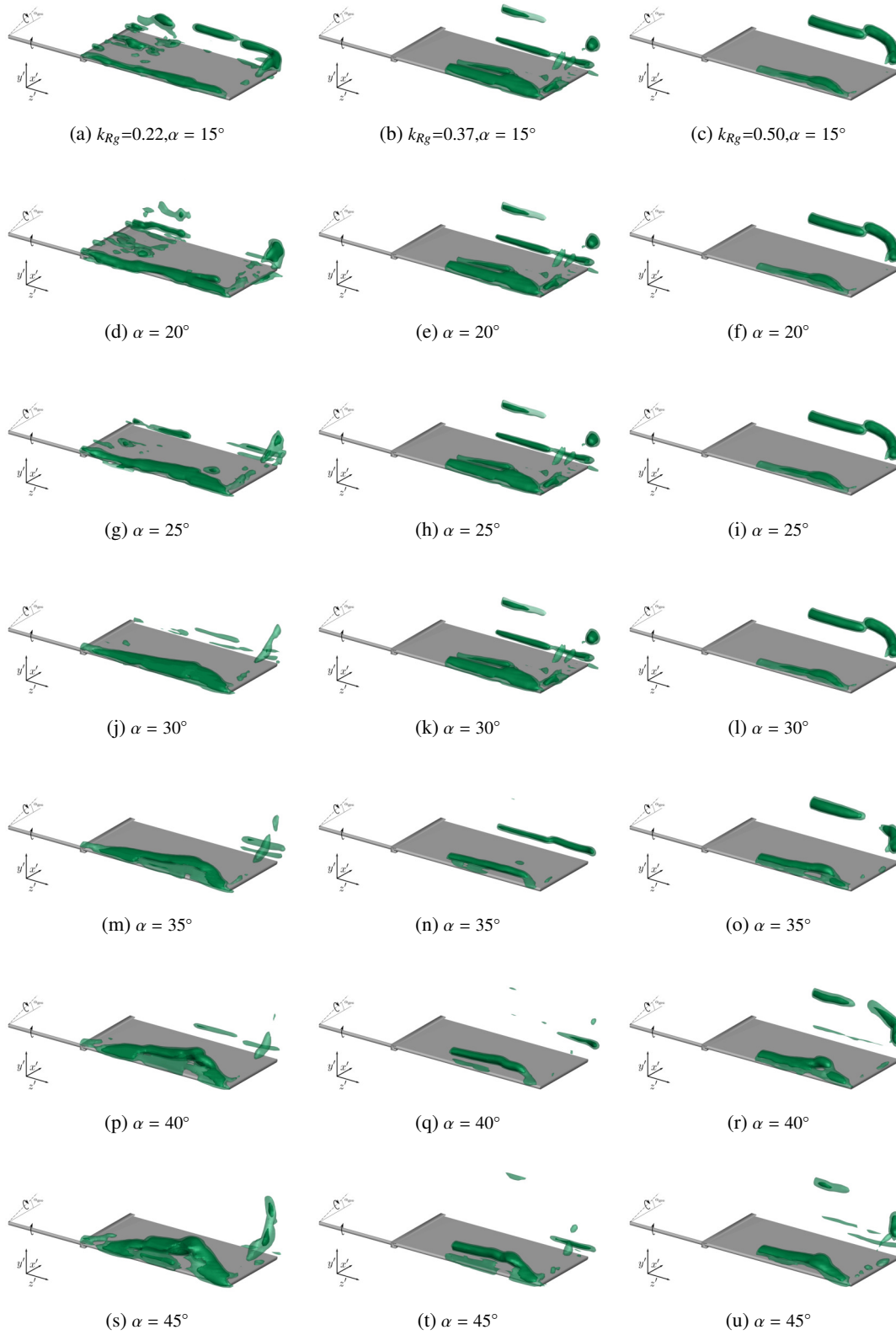


Figure 7.7: LEV formation shown with isosurfaces of normalized swirling strength, for $k_{Rg}=0.22$ (figs. 7.7a-7.7s), $k_{Rg}=0.37$ (figs. 7.7b-7.7t), and $k_{Rg}=0.50$ (7.7c-7.7u)

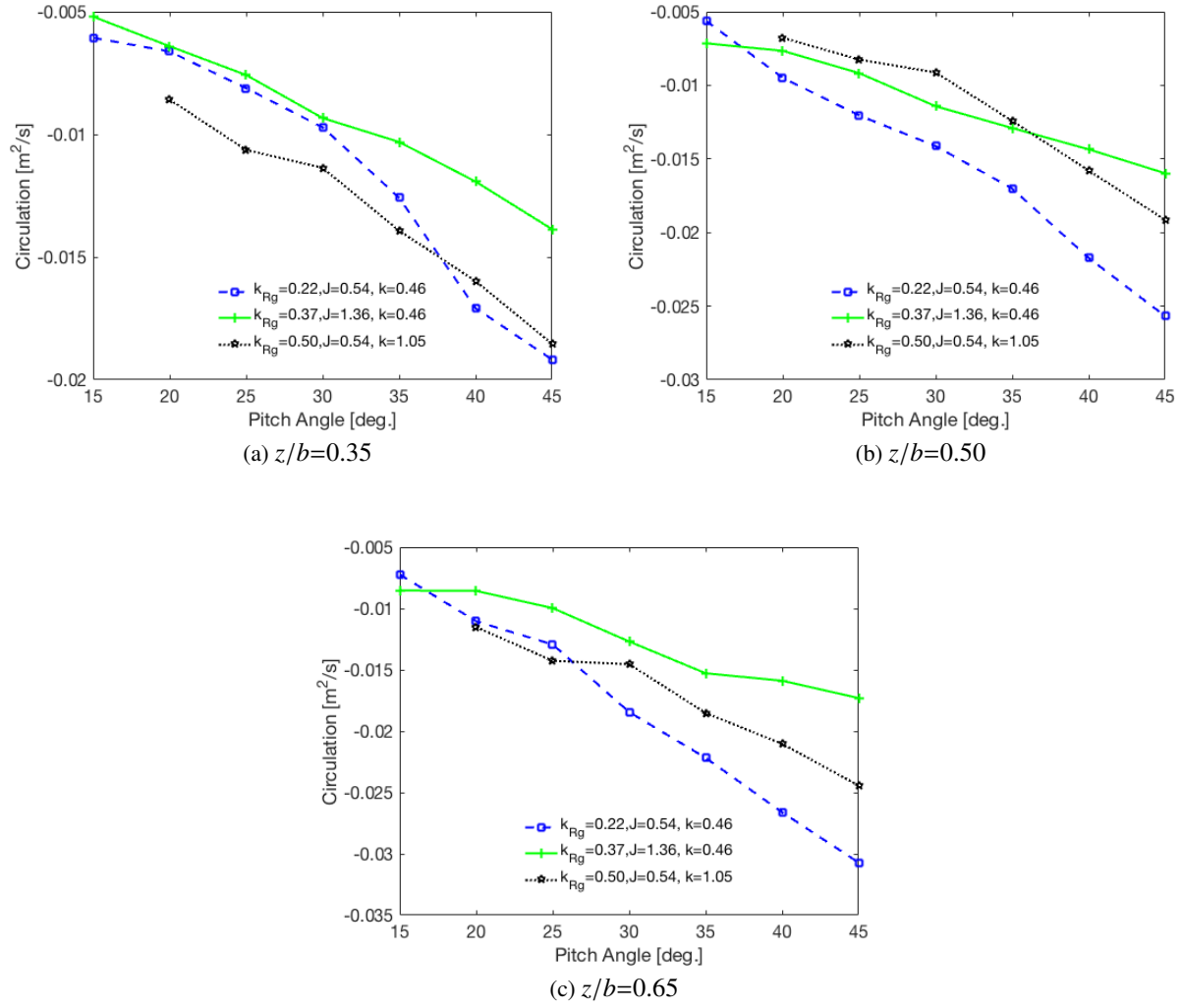


Figure 7.8: Circulation comparison for the simultaneous case at $z/b=0.35$, $z/b=0.50$ and $z/b=0.65$

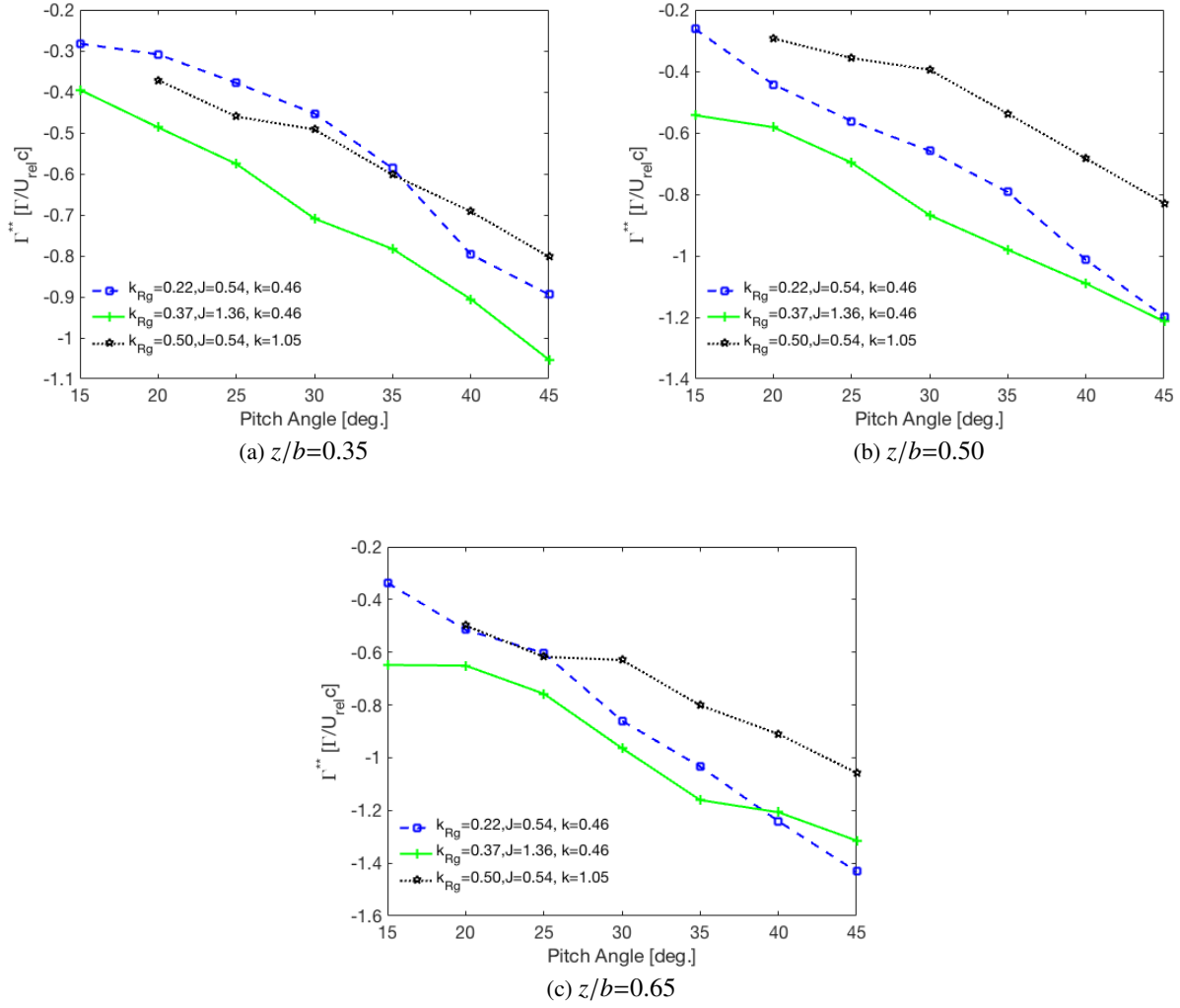


Figure 7.9: Circulation comparison for the simultaneous case non-dimensionalized by u_{rel}^*c at $z/b=0.35$, $z/b=0.50$ and $z/b=0.65$

stantaneous integrated vorticity fluxes at $z/b=0.35$, $z/b=0.5$, and $z/b=0.65$ for the three combined pitch-roll cases.

Considering, first, the measurements at $z/b = 0.35$, shown in figure 7.10, it is evident that the shear-layer and surface diffusive fluxes for $k_{rg} = 0.22$ and 0.37 are very similar throughout the motion. This is expected, since these two cases have identical reduced pitch rates, and the evolution of measured circulation was shown, in the previous section, to be dominated by the pitch motion. The $k_{rg} = 0.50$ case, in which k is a factor of 2.3 greater than the other cases, exhibits shear-layer and diffusive fluxes that also greater by a similar factor. These quantities also appear to saturate, and possibly also diminish late in the motion (evident in the $K_{rg} = 0.22$ and 0.37 cases); however, the saturation clearly occurs at a higher angle of attack than in the pure-pitch case, and especially so for the $k_{rg} = 0.50$ case. The spanwise convective flux is negligible in all cases, which, again, bears strong similarity to the pure-pitch cases.

The shear layer and diffusive contributions grow during the motion, which is the behavior that is observed in the pure pitching cases. The three cases with similar pitch rate $k_{rg}=0.22$ and 0.37 values are similar as well, showing that the dominant maneuver is the pitching maneuver, which governs the flux contributions at this location. It is important to note, that for the $k_{rg}=0.22$ case the flux contributions behaved in a similar manner as this location.

Similar conclusions can be made about the flux contributions measured at $z/b = 0.50$ and 0.65 , which are shown in figures 7.11 and 7.12. However, with increasing spanwise position, the shear-layer and diffusive fluxes exhibit fluctuations of increasing magnitude for the $k_{rg} = 0.5$ case. This is reminiscent of the oscillations observed for the $J_{3,25} = 0.54$ pure-roll case, in Chapter 6, which was attributed to the development of the dual-vortex system. However, for this case, figure 7.7 doesn't reveal any such LEV evolution. It is likely that the oscillations are related to trailing-edge shedding, which are not well-captured in the data.

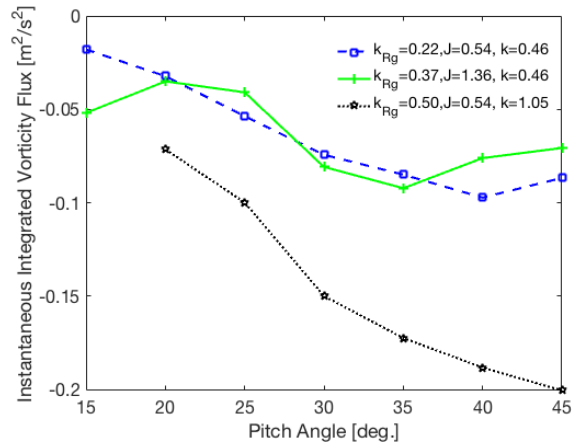
7.3 Conclusion

The wing was articulated in pure-pitch, and combined pitch-roll maneuvers, and the time-evolution of the LEV circulation and governing vorticity fluxes was investigated. For the pure pitch cases, with $k = 0.2$ and 0.5 , it was shown that the LEV develops in a highly symmetric manner across the span, and that LEV circulation grows at essentially the same rate for both cases, when referenced to the wing angle of attack. However, large differences in LEV growth were inferred very early in the motion, prior to the first measurement at an angle of attack of 5 deg. This rapid growth in circulation upon the initiation of motion reflects the conclusion by Corkery and Babinsky (2018), that inertial forces due to transient maneuvers may be attributed to surface vorticity fluxes.

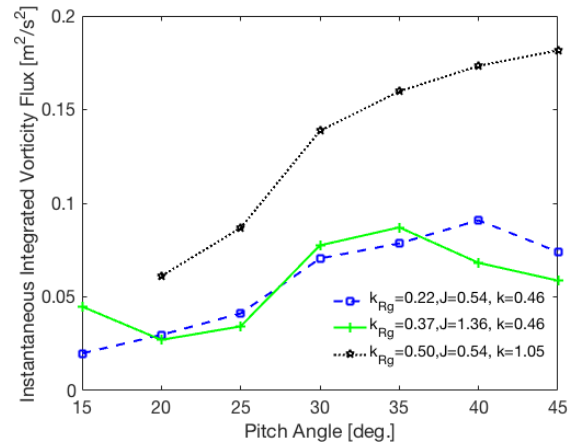
Despite the insensitivity of LEV growth rate to the pitch rate, the diffusive flux and the shear layer contributions were found to vary significantly for the two different pitch rates; however, their highly-coupled magnitudes resulted in substantial cancelation of these effects. Both the shear layer contribution and diffusive flux were found to saturate at an angle of attack of 25 deg, which is concurrent with the formation of a visible opposite-signed, secondary vortex beneath the primary vortex.

The combined pitch-roll cases included the two values of advance ratio J that were investigated in Chapter 6, in combination with two pitch rates. The flow topology associated with the fast-roll cases, $k_{rg} = 0.22$ and 0.50 , exhibited a compact outboard arch similar to that of the baseline rolling case (with matching advance ratio, J) investigated in Chapter 6. However, the LEV remained near the surface and more compact, similar to that of the pitching plate. In the case of $k_{rg}=0.50$ the arch development began to regress after $\alpha=40$ deg., evolving toward the flow structure produced in the pure-pitch motion.

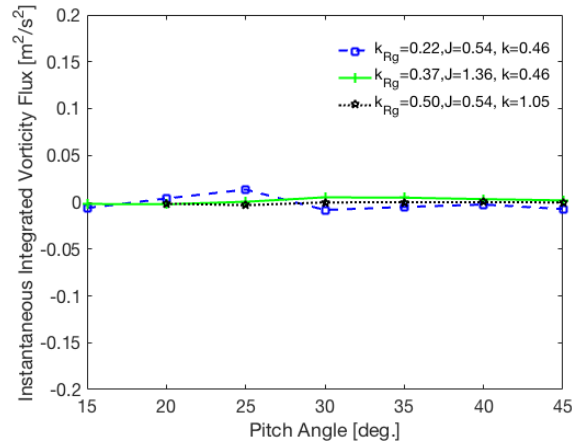
There was no systematic relationship found between the measured circulation and forcing parameters k_{rg} , k , or J . Circulation was found to vary similarly for all three combined pitch-roll cases. However, the shear-layer and diffusive fluxes, which dominated the circulation budget, were shown to be primarily



(a) Shear Layer, $z/b = 0.35$

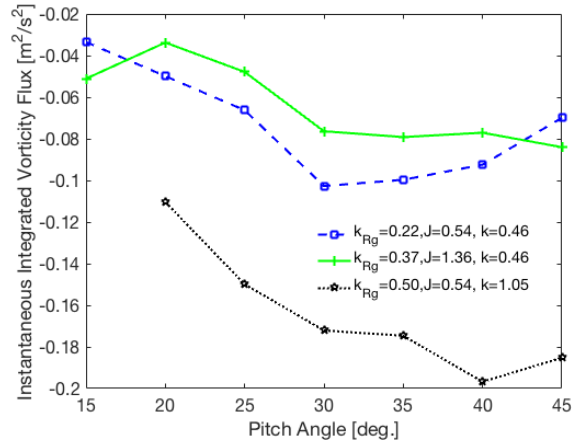


(b) Diffusive

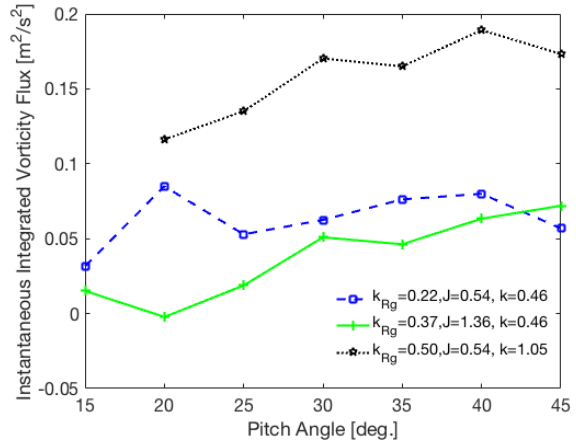


(c) SPCF

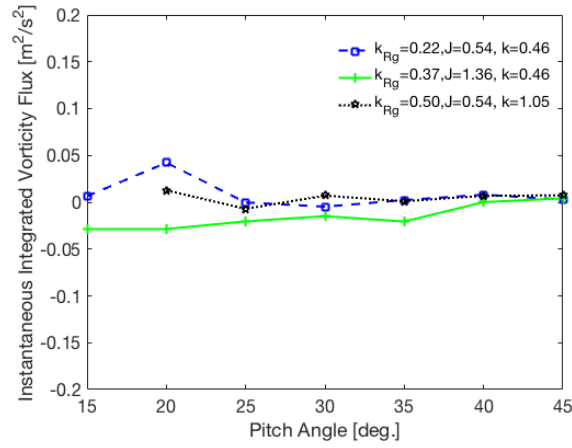
Figure 7.10: Flux Contributions for $z/b=0.35$



(a) Shear Layer

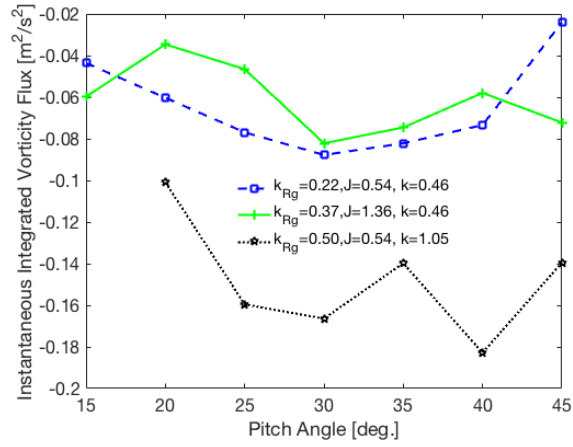


(b) Diffusive

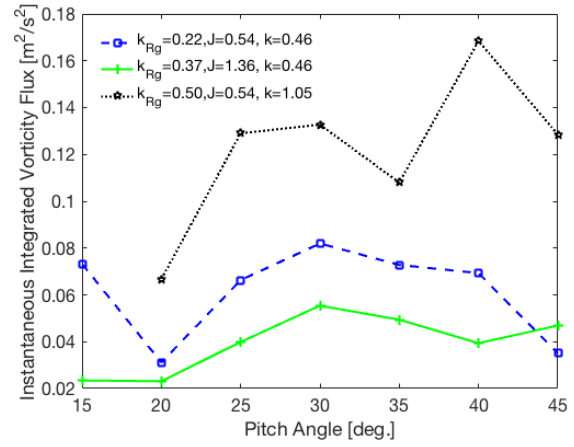


(c) SPCF

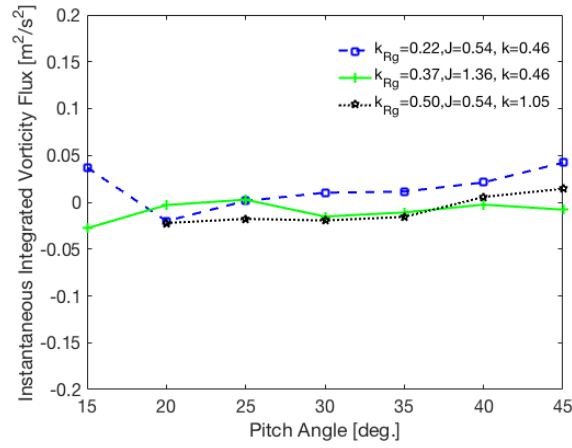
Figure 7.11: Flux Contributions for $z/b=0.50$



(a) Shear Layer



(b) Diffusive



(c) SPCF

Figure 7.12: Flux Contributions for $z/b=0.65$

governed by the reduced pitch rate, k , and not k_{Rg} . Specifically, for the $k_{Rg} = 0.22$ and 0.37 cases, for which $k = 0.46$, these flux contributions were very similar; whereas for the $k_{Rg} = 0.50$ case, in which $k = 1.05$, the shear-layer and diffusive flux contributions were much larger in magnitude.

Chapter 8

Effects of Passive Bleed

In the previous chapters, the early evolution of leading-edge vortices was investigated for a pitching wing, a rolling wing, and a wing articulated in a combination of pitch and roll, in order to understand the underlying physics governing LEV development. The vorticity transport framework was employed to understand the mechanisms underlying the growth of the vortex. The pure roll cases, with $J = 0.54$ were found to be unique in that a conical LEV structure was observed over the inboard portion of the wing that was sustained by a spanwise convective flux of circulation that gradually grew over time. In this chapter, passive bleed will be applied to the baseline ($J_{3.25} = 0.54$) case examined in detail in Chapter 6. The inboard portion of the LEV will be perturbed by a passive-bleed jet formed by drilling a small hole near the root of the wing. Flow structure, vorticity transport, and aerodynamic loads will be compared between the passive bleed and baseline cases.

While this study provides some insights into how to use the vorticity transport framework to inform and understand flow control strategies, it also provides a means for understanding the flow mechanisms that govern LEV formation and evolution by fundamentally changing the flow while maintaining the same wing kinematics and geometry.

8.1 Flow Topology of Baseline and Perturbed Flows

Dye visualizations were performed for the rolling maneuver, using the technique described in section 3.4. Preliminary visualizations with the absence of flow control were recorded at advance ratio values $J = 0.54$ and 1.36 at radius of gyration $R_g/c = 3.25$. This corresponds to $J_{3.25}=0.54$ and $J_{3.25}=1.36$, which were examined in detail in Chapter 6. In the preliminary visualizations, a maximum roll angle of $\phi = 85^\circ$ and effective angle of attack $\alpha_{eff} = 33^\circ$ during the constant velocity phase was maintained. Flow visualizations were conducted for both $J_{3.25} = 0.54$ and $J_{3.25} = 1.36$ without passive bleed for comparison. All passive bleed cases that were conducted can be seen in table 8.2.

8.1.1 Qualitative Characterization of LEV Development using Flow Visualization

Preliminary flow visualizations were performed at $J_{3.25} = 0.54$ and $J_{3.25} = 1.36$ to identify important topological features and differences between the variation in advance ratio. Additional visualizations were performed with passive bleed for $J = 0.54$ and $R_g/c = 3.25$ with bleed holes of different sizes and spatial locations to help understand the qualitative impact of each applied flow control configuration. Qualitative visualizations can help narrow the parameter space for quantitative measurements, vorticity transport analysis and help identify altered flux quantities.

Figure 8.1 shows a comparison of the evolution for a rolling wing with an advance ratio $J_{3.25} = 0.54$ and

Table 8.1: Flow control locations and hole sizes

Passive Flow Control Configurations				
Name	Advance Ratio (J)	Hole Spanwise location (z/b)	Hole chordwise location (x/c)	Hole Diameter(mm)
Baseline	0.54	N/A	N/A	N/A
p_{is}	0.54	0.1	0.065	1.6
p_{os}	0.54	0.16	0.065	1.6
p_{il}	0.54	0.1	0.065	3.2
p_{ol}	0.54	0.16	0.065	3.2

Table 8.2: Locations and sizes of passive bleed holes.

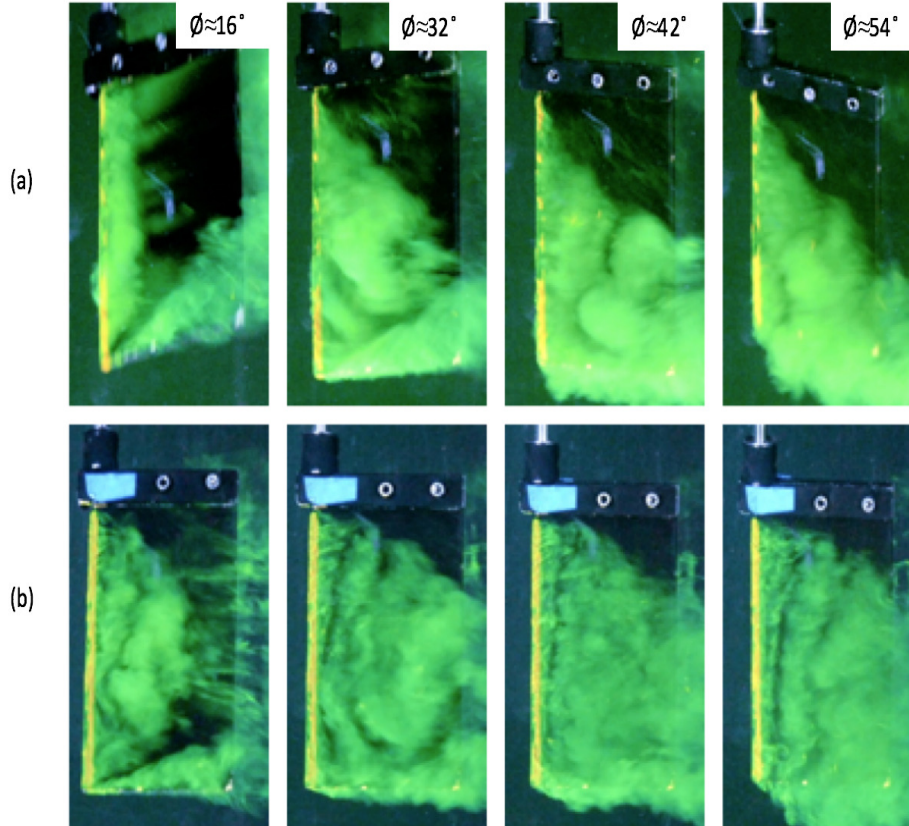


Figure 8.1: LEV evolution for (a) $J = 1.36$, $Rg/c = 3.25$, $\alpha_{eff} = 33^\circ$ (b) $J = 1.36$, $Rg/c = 3.25$, $\alpha_{eff} = 33^\circ$.

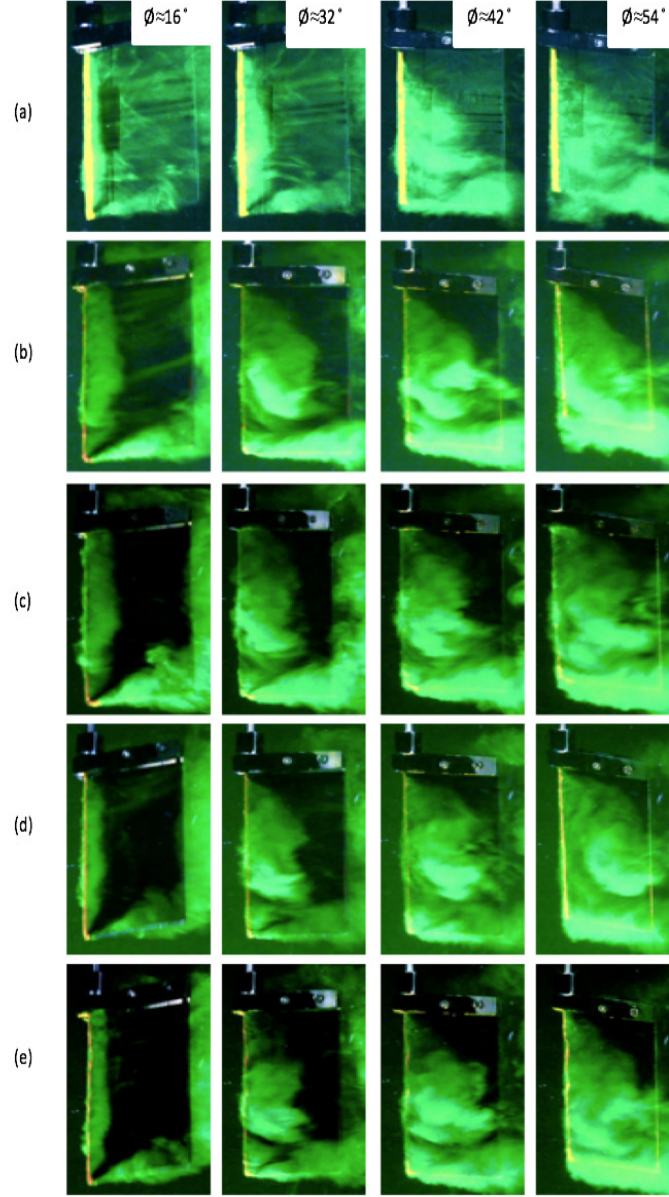


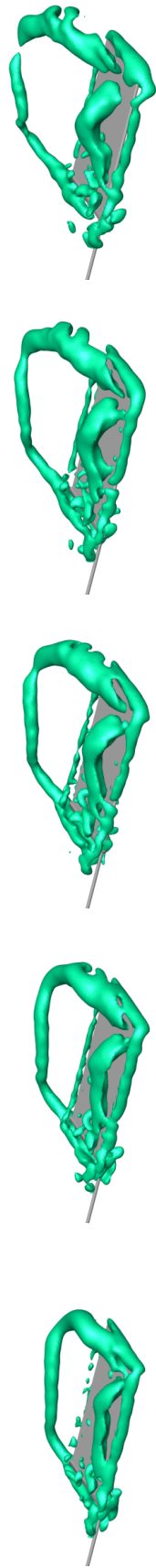
Figure 8.2: LEV evolution for $J = 0.54$, $R_g/c = 3.25$, $\alpha_{eff} = 33^\circ$ with (a) No bleeding (b) Passive bleeding at $z/b \approx 0.1$, $d = 1.6$ mm (p_{is}) (c) Passive bleeding at $z/b \approx 0.16$, $d = 1.6$ mm (p_{os}) (d) Passive bleeding at $z/b \approx 0.1$, $d = 3.2$ mm (p_{il}) (e) Passive bleeding at $z/b \approx 0.16$, $d = 3.2$ mm (p_{ol}).

$J_{3.25} = 1.36$ respectively. As discussed in previous chapters, at $\phi = 16^\circ$ the flow features are significantly different. For $J_{3.25} = 0.54$, an LEV has rolled up across the span of the wing and remains attached. However, the $J_{3.25} = 1.36$ case has generated an arch structure pinned at the root and the tip similar to that of a purely translating wing. As the wing progresses to $\phi = 32^\circ$, the $J = 0.54$ wing has a coherent conical structure at the inboard position that extends to approximately the 50% spanwise position. This conical structure persists for the entirety of the motion. For the $J_{3.25} = 1.36$ case at $\phi = 32^\circ$, arch structure has shed and convects downstream. A subsequent LEV has formed. This subsequent LEV is shed by $\phi = 54^\circ$. This suggests that, for $J_{3.25} = 0.54$, rotational effects substantially alter LEV development.

Visualizations at different roll angles are shown in Figure 8.2 for a wing in the absence of passive bleeding in comparison to a wing with passive bleed at various configurations. The overall flow evolution for the different instances vary with the hole diameter and spatial location. This behavior can range from the baseline case behavior to an arch structure forming across the span similar to that of a translating wing at an angle of attack. Thus, it appears that the application of passive bleed can significantly mitigate roll-induced effects. The most effective configuration was the largest bleed at the most inboard location (i.e. $z/b = 0.1$ and $d = 3.2$ mm). This case will be examined in detail throughout the rest of this chapter. It will be designated $J_{PB} = 0.54$.

8.1.2 Comparison of Flow Topologies using Quantitative Flow Field Measurements

Figure 8.3 compares the evolution of the LEV for the flow control $J_{PB} = 0.54$ and baseline $J_{3.25} = 0.54$ cases, using 3D particle tracking velocimetry, and plenoptic PIV measurements, respectively. Significant differences can be seen as early as $\phi = 6^\circ$, where an arch structure is observed for the flow control case (figure 8.3a). As the motion progresses, the inboard portion of the LEV is broken down into smaller shedding structures, leaving a broad arch structure over the remainder of the span, which contrasts with the localized arch that forms in the $J_{3.25} = 0.54$ case. Furthermore, the dual vortex system found in Region 2 of the $J_{3.25} = 0.54$ case (discussed in Chapter 6) is not observed in the flow control case.



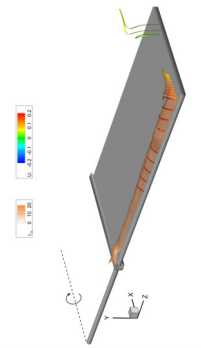
(a) $\phi = 6^\circ$

(b) $\phi = 12^\circ$

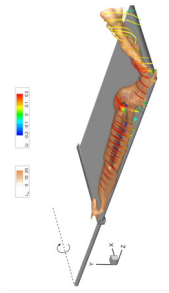
(c) $\phi = 17^\circ$

(d) $\phi = 21^\circ$

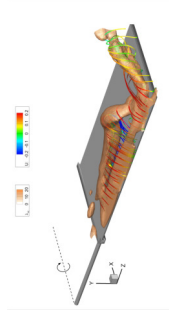
(e) $\phi = 26^\circ$



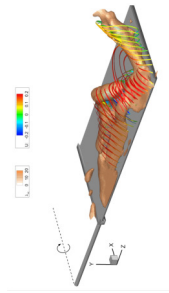
(f) $\phi = 6^\circ$



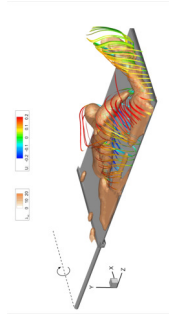
(g) $\phi = 12^\circ$



(h) $\phi = 17^\circ$



(i) $\phi = 21^\circ$



(j) $\phi = 26^\circ$

Figure 8.3: LEV formation shown with isosurfaces of normalized swirling strength, for the flow control case, $J_{PB} = 0.54$ (figs. 8.3a-8.3e), and the baseline case, $J_{3.25} = 0.54$ (figs. 8.3f-8.3j). Isosurfaces colored by $\lambda_{ci}/\lambda_{max}$

8.2 Vorticity Transport in the Passive Bleed Case

Figure 8.4 compares the transport contributions to LEV circulation for the baseline ($J_{3,25} = 0.54$), and slow-roll ($J_{3,25} = 1.36$) cases, as well as the passive bleed case ($J_{PB} = 0.54$) at $z/b = 0.35$, within Region 1, where a conical LEV structure was observed for the baseline case. Since the passive-bleed case exhibits a flow structure that resembles the $J_{3,25} = 0.54$ case, it is useful to include this case in the comparison. However, recall that the slow-roll case contains high-frequency vortex shedding from the leading edge which is not resolved in time by the PIV measurements. Important differences between the baseline and passive-bleed cases are evident that provide insight into the development of the leading-edge vortices in each case. Assessment of the circulation budget closure (i.e. comparison between the measured circulation and the right hand side (RHS) of Equation 3.47) suggests that the PTV has a larger error than the plenoptic PIV data – perhaps due, in part, to the smaller number of measurements in the averages; however, the deviation remains small with respect to the values of the dominant fluxes.

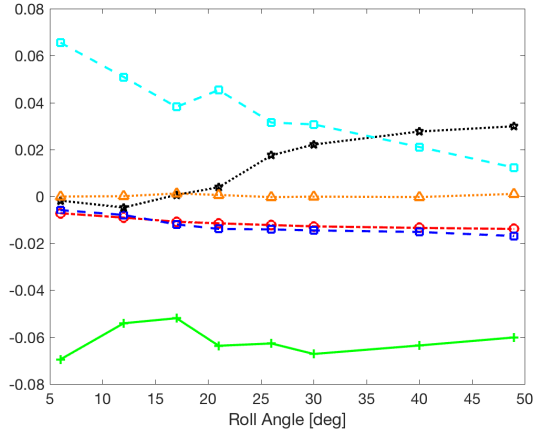
Qualitative differences are apparent in the behaviors of the shear-layer and diffusive fluxes between the fast and slow roll cases (Figs 8.4a and 8.4b, respectively). Whereas the slow roll case ($J_{3,25} = 1.36$) is affected by negligible spanwise convective flux, and the shear layer flux becomes approximately constant after a roll angle of $\phi = 30^\circ$, the fast roll case ($J_{3,25} = 0.54$) exchanges a mostly monotonically decreasing diffusive flux for a complementarily increasing spanwise convective flux as the roll angle increases. In both cases, the diffusive flux is on the same order of magnitude as the shear layer flux, and approximately mirrors the shear layer flux.

The passive bleed case provides an unique opportunity to perturb the fluxes in order to provide some insight into how changes in vorticity transport correlate with differences in LEV evolution. Comparing the fluxes at $z/b=0.35$ for the passive bleed case (Fig. 8.4) to the fast and slow roll cases, it appears that the transport phenomena are more representative of the slow-roll case, despite having fast-roll kinematics. Specifically, the shear layer flux is relatively constant throughout the motion, and the spanwise flux does not form a significant portion of the budget. Thus, the application of passive bleed has disrupted the establishment of the spanwise flux.

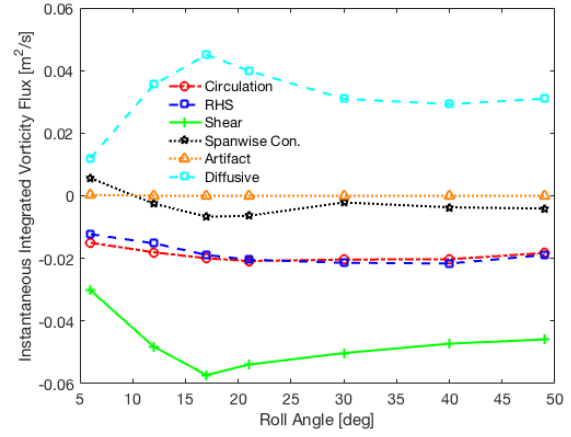
Figure 8.5 shows the vorticity transport measurements further outboard, at $z/b = 0.50$. For the baseline ($J_{3,25} = 0.54$) case, the shear layer contribution is balanced by the diffusive contribution at this spanwise location with a minor source of circulation due to the spanwise convection. The formation of the dual-vortex system causes the shear layer contribution and the diffusive contribution to oscillate. For the slow-roll case the shear layer contribution initially decreases rapidly and then remains relatively constant, while there are oscillations in the diffusive contribution and the spanwise convective contribution, representing the aliased shedding events. The passive bleed case is distinctly different. In this case, the shear layer contribution and diffusive contribution are nearly constant, with a near-zero spanwise convective contribution due to the absence of the highly-three-dimensional localized arch structure that was evident in the baseline case.

8.3 Force Measurements

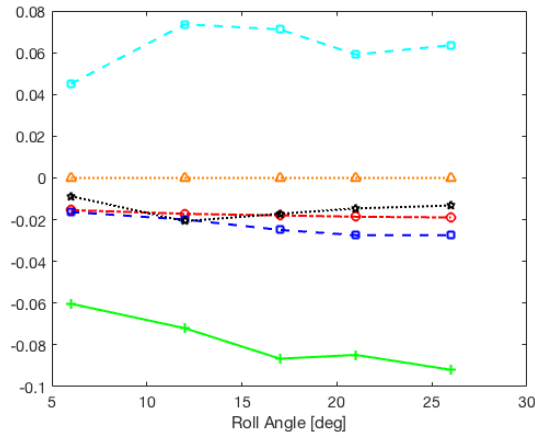
The lift and drag forces were measured using the method described in section 3.6, and are presented in coefficient form for the passive bleed and baseline cases in figure 8.6. The coefficients were computed using U_{rel} , according to equations 5.7 and 5.8. The lift coefficients of both the passive-bleed case and the baseline case are broadly similar in magnitude and features. The lift coefficient curves both include an initial transient peak followed by the frequency oscillations. This behavior was correlated with events in the evolution of the LEV for the baseline case in section 5.3.1. For the passive bleeding case, even though the conical portion of the LEV is not present, shedding occurs at a low frequency, representative of $J_{3,25} = 0.54$, and not rapidly, as observed for $J_{3,25} = 1.36$. Thus, despite the topological resemblance to the early development of



(a) $J_{3.25} = 0.54$

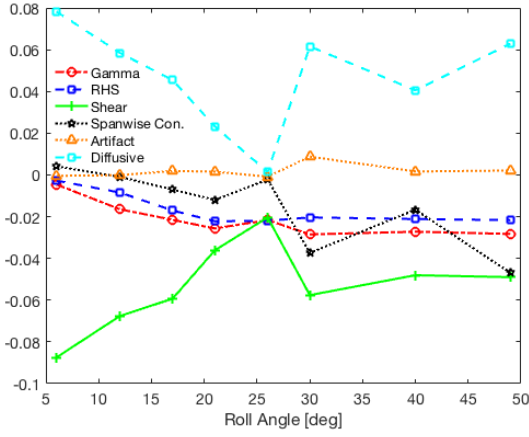


(b) $J_{3.25} = 1.36$

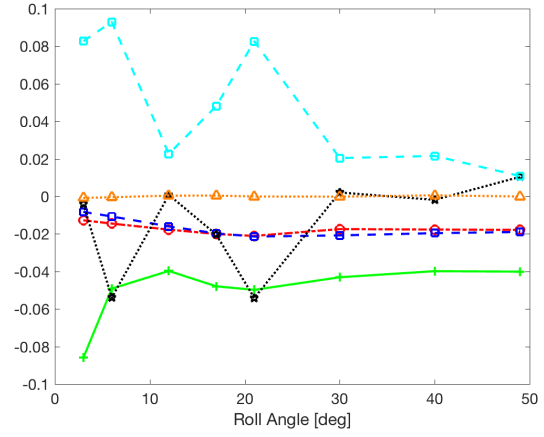


(c) $J_{PB} = 0.54$

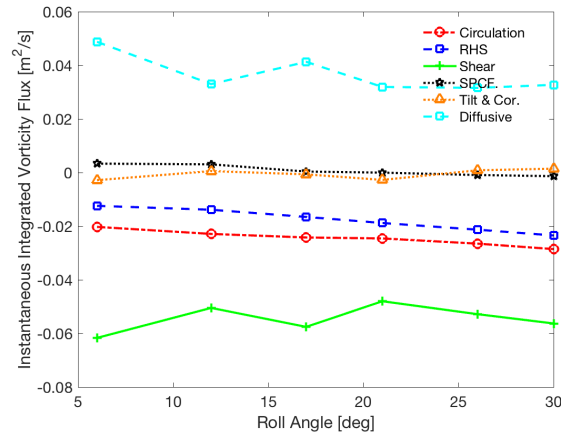
Figure 8.4: Flux analysis at $z/b = 0.35$ for $J_{3.25} = 0.54$, $J_{PB}=0.54$ and $J_{3.25} = 1.36$



(a) $J_{3.25} = 0.54$



(b) $J_{3.25} = 1.36$



(c) $J_{PB} = 0.54$

Figure 8.5: Flux analysis at $z/b = 0.50$ for $J_{3.25} = 0.54$, $J_{PB}=0.54$ and $J_{3.25} = 1.36$

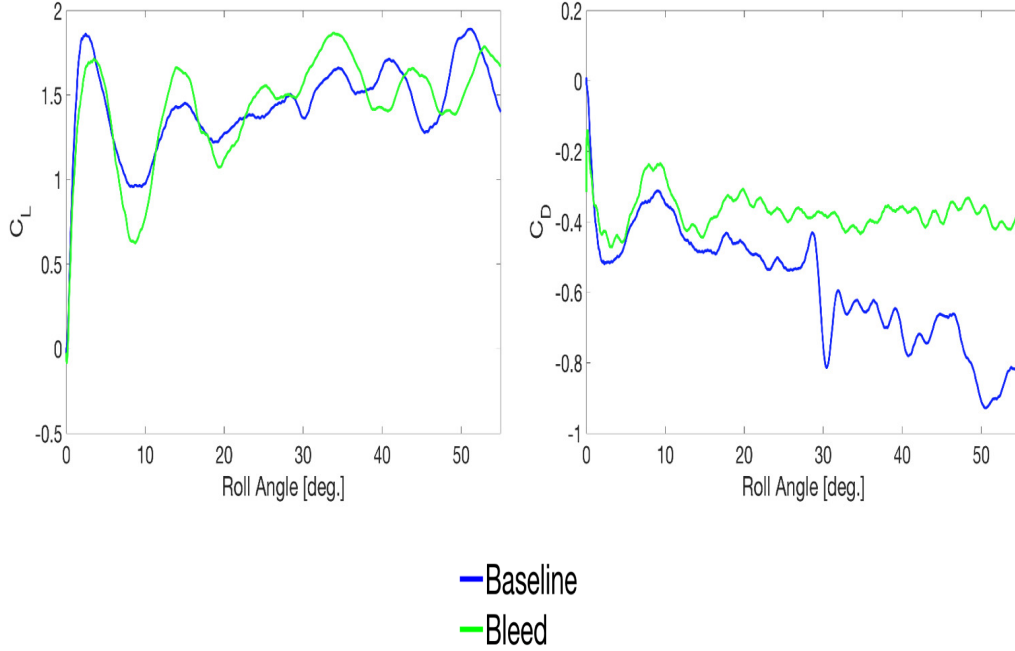


Figure 8.6: LEV evolution for $J_{3.25} = 0.54$, $\alpha_{eff} = 35^\circ$ with (a) No flow control (b) Passive flow control at $z/b \approx 0.1$, $d = 1.6$ mm (p_{is}) (c) Passive flow control at $z/b \approx 0.16$, $d = 1.6$ mm (p_{os}) (d) Passive flow control at $z/b \approx 0.1$, $d = 3.2$ mm (p_{il}) (e) Passive flow control at $z/b \approx 0.16$, $d = 3.2$ mm (p_{ol})

the $J_{3.25} = 1.36$ case, the high frequency fluctuations observed in the slow-roll cases of figure 5.6 are not evident.

However, the drag coefficients do exhibit notable differences. The baseline and passive-bleed cases both have negative drag coefficients throughout their respective measurement periods (i.e. they are thrust-producing). However, whereas the thrust is monotonically increasing in the baseline case, it is relatively constant in the passive bleed case, and the baseline thrust eventually becomes much greater. This is somewhat surprising because, for a separated flow on a flat plate, one would expect the lift and drag forces to vary proportionally. However, it is possible that leading-edge suction produced by the inboard conical LEV contributed exclusively to the thrust in the baseline case, causing the deviation.

8.4 Conclusions

The baseline pure-roll case investigated in detail in Chapter 6 was modified such as to perturb the conical LEV using a small passive bleed hole near the root of the wing. The effect of passive bleed on the structure of the LEV was to break down the conical LEV on the inboard region of the wing, while a broad arch vortex formed over the outboard portion of the wing, in contrast to the localized arch observed for the baseline case. In addition, the dual vortex structure that formed in the mid-span region for the baseline case was not observed in the passive bleed case. Analysis of vorticity transport on the passive bleed case showed that the growing spanwise convective flux, observed in the inboard region for the baseline case, was not present in the passive-bleed case. Rather, the shear-layer flux was predominantly balanced by the surface-diffusive flux of vorticity. The resulting lift force was not significantly affected by the restructuring of the LEV; however, the thrust was significantly reduced in the passive bleed case, suggesting that the conical LEV present in the baseline case contributed to the thrust through leading-edge suction.

Chapter 9

Conclusions

9.1 Conclusions

The leading-edge vortex structure and evolution, strength, underlying transport mechanisms, and aerodynamic loads were examined for a rectangular wing of aspect ratio 2, articulated in three maneuvers: pure roll, pure pitch and combined pitch and roll. Whereas the pure-pitch and pure-roll cases were independently of interest, the systematic construction of the study, in which the individual pitch and roll kinematics were ultimately superimposed, facilitated deeper investigation of the effects of rotational accelerations on vortex development.

A plenoptic camera and associated PIV algorithm, both developed in Prof. Thurow's laboratory at Auburn University, was the primary means of quantitative flow field measurements. Since plenoptic PIV utilizes a single camera and lens, configuration is simpler than most other volumetric flow measurement methods, making it an ideal measurement method to investigate the wing undergoing complex pitching and rolling motions. This study is the first application of plenoptic particle image velocimetry to investigate the flow physics on a maneuvering wing. The PIV data were used to make qualitative assessments about the evolution of the leading-edge vortex, and vorticity transport mechanisms governing the strength of the vortex were quantified by applying the data to a vorticity transport framework extended to take into account the non-inertial rotational accelerations on the maneuvering wing. For the analysis conducted here, Coriolis accelerations were found to influence measured convective sources of LEV circulation, but were insignificant in comparison to other fluxes. In contrast, centripetal accelerations could modify the diffusive flux from the surface of the wing.

9.1.1 Pure Pitch

Pure pitching of the wing about its leading edge was investigated for two reduced pitch rate values of $k = 0.2$ and 0.5 , with pitch angles up to 45° . For both pitch rates, a spanwise symmetric LEV structure was formed, which was pinned to the leading-edge corners at the root and tip, and convected away from the wing through the mid-span region of the wing. However, significant differences in topology were observed. For the slower pitch rate, a triple vortex system rapidly evolved, which was not apparent in the fast-roll case. A vorticity transport and circulation analysis in the vicinity of the leading-edge vortex revealed that a significant amount of circulation was established within the first 5° of motion for both cases, and the initial LEV strength was strongly dependent on k . Subsequent increments to the measured circulation were dependent only on angle of attack, with minimal further influence of k or elapsed time.

9.1.2 Pure Roll

The LEV structure for pure roll was investigated quantitatively, using plenoptic PIV, and qualitatively, using dye visualization while varying the advance ratio J and the dimensionless radius of gyration of the wing, R_g/c . A conical leading-edge vortex was observed over the inboard portion of the wing for certain values of J and R_g/c . The LEV structures were classified as *conical* or *non-conical* based on dye visualizations, and a parameter, π_{rot} , was proposed, based on non-dimensionalizing the gradient in relative velocity along the span of the wing that was induced by the roll motion. A constant value of $\pi_{rot} \approx 0.17$ formed a curve in the space of J vs. R_g/c that corresponded well to the transition between conical and non-conical states.

Extensive quantitative analyses were conducted for $J = 0.54$ and 1.36 , in combination with $R_g/c = 2.5$ and 3.25 . Three regions, over the span of the wing, were identified for the case in which $J = 0.54$ and $R_g/c = 3.25$. In Region 1 – the inboard region – a conical vortex was observed, and the shear layer was the primary source of circulation sustaining the vortex. Early in the roll motion, the shear layer flux was balanced by a strong source of opposite-signed secondary vorticity created by the diffusive flux from the surface of the wing. At later times, a spanwise convective flux of vorticity was established, which drained circulation from the LEV as it remained stably attached to the wing surface. In Region 2 – the mid-span region – the magnitude of the measured circulation was substantially greater than Region 1. Early in the roll motion, the vortex had little spanwise variation over much of the region, except for a localized arch structure that developed near the outboard boundary of the region. In time, the initial vortex shed, and a second vortex was created, forming a dual-vortex system. The circulation of the vortex was dominated by an interaction between the shear layer and the diffusive flux of secondary vorticity from the surface of the wing, with a minor influence of spanwise convective flux due to the three-dimensional flow established by induction of the distorted arch structure. Region 3 was strongly influenced by the tip, and consisted of the outboard portion of the arch structure, which reattached and was pinned to the surface of the wing at its tip. The vortex was sustained by a balance between the spanwise convective flux, caused by induction of the tip vortex, which acted as a source of LEV vorticity, and the surface diffusive flux, which acted as a sink. The leading-edge shear layer was weak in Region 3 due to the pressure relief in the tip region.

Lift and drag forces were measured on the rolling wings, for the four cases comprising combinations of $J = 0.54$ and 1.36 , and $R_g/c = 2.5$ and 3.25 . The fast-roll cases exhibited relatively low-frequency undulations, corresponding to events in the development of the LEV. The slow-roll cases exhibited high-frequency oscillations, which were consistent with the high-frequency vortex shedding from the leading edge. When lift coefficients were computed using the relative velocity (U_{rel}) as the velocity scale, the magnitudes of the fast and slow-roll cases were similar, suggesting that the presence of the LEV did not enhance the lift. This observation may be reconciled – at least in part – with other observations in the literature suggesting that the LEV is an important high-lift mechanism – by recognizing that the strength of the conical LEV is weak in comparison to the LEV structure in Region 2.

A passive bleed hole was introduced near the root of the wing for the baseline roll case in an attempt to alter the flow field, while observing the corresponding changes in vorticity transport mechanisms. The experiment provided a unique opportunity to examine the relationship between transport processes and flow structure across cases that maintain the same geometry and kinematics. Passive bleed caused the flow to transition from the state with three distinct regions, including an attached conical LEV, to a broad arch structure over most of the wing span, with no conical vortex. The vorticity fluxes were found to be vastly different from the baseline roll case. In the inboard region of the wing, the spanwise convective flux was essentially eliminated, causing the shear layer flux to be primarily balanced by the surface diffusive flux. Lift was not significantly modified by the change in flow structure. Given that the conical structure was not present in the passive-bleed case, this observation is in contradistinction with many other studies in which large lift forces are attributed to attached leading-edge vortices. However, it is noted that the LEV only exists over the inboard region of the wing in the baseline case, and its circulation is, in general, significantly lower

than the vortex system in Region 2. In contrast, passive bleed prevented a gradual increase in thrust that was observed for the baseline case, such that the thrust produced by the baseline case eventually became significantly greater than the passive bleed case. This suggests that the differences in forces are not only due to altered pressure distributions on the top and bottom surfaces of the wing, but possibly also due to leading-edge suction, in the vicinity of the LEV, contributing to the thrust.

9.1.3 Combined Pitch and Roll

The combined pitch and roll kinematics were formulated as a pitching wing in a non-inertial (rotating) reference frame – roll was initiated prior to pitch. The addition of the roll rotation was found to significantly impede the development of the leading-edge vortex for similar values of reduced pitch rate, k . To compensate for the relative flow velocity induced by the roll motion, a modified reduced pitch rate, k_{rg} was introduced, which employed the relative velocity at the radius of gyration as the velocity scale. The k_{rg} parameter was found to correlate strongly with the time at which a dual-vortex system formed on the wing, such that the formation of the dual vortex system formed at earlier dimensionless times for larger values of k_{rg} . However, as varying pitch and roll rates resulted in qualitative changes in flow topology, it was not possible to relate all cases simply in terms of a k_{rg} -related time shift. Rather, flow topology was found to be a strong function of the reduced pitch rate, k .

The vorticity transport analysis also showed that the fluxes contributing to the strength of the LEV behaved similar to a pitching wing, where the shear layer contribution grows in time, and is balanced by the diffusive contribution. Circulation was also shown to be a primary function for the reduced pitch rate, k , and not k_{rg} .

9.2 Synopsis and Future Outlook

Whereas the pure pitch case has been investigated extensively in prior work, the investigation of flow development on a rolling wing within a free stream has received relatively little attention in the literature. The combination of pitch and roll has been essentially unexplored in a systematic manner. The range of problems and parameters has revealed a range of flow structures and transport phenomena, often with distinct behaviors in different parts of the wing. These distinctions, as well as the identification of similar flow structures and behaviors, as the type of motion and parameter values were varied, has further revealed the conditions in which particular behaviors are found, and thereby helped to elucidate the underlying physics governing flow development.

For example, in all cases presented in this work, early vortex development was found to be regulated by counter-acting shear-layer and surface-diffusive fluxes of vorticity. Under conditions of sufficiently small advance ratio and radius of gyration, a conical vortex was found to form on the inboard portion of the wing, and the surface-diffusive flux of vorticity was concurrently replaced by a spanwise-convective flux, which ultimately balanced the shear-layer contribution to stabilize growth. The predominance of the surface diffusive flux across the kinematics investigated in this study and other studies – which can generally be accurately determined from a small number of surface pressure measurements and acceleration data that can be obtained from an onboard inertial measurement unit – and its correlation with the shear-layer flux, suggests that the salient flow phenomena contributing to vortex development (and thus, also, aerodynamic loads) may be obtained from relatively few sensors on an operational aircraft. Thus, the vorticity transport framework can form the basis of an estimator for aerodynamic load prediction or control, when coupled with an appropriate aerodynamic model, as illustrated in figure 9.1.

Two new parameters were discovered to be of utility in this study: the modified reduced pitch rate, k_{rg} and the rotation parameter, π_{rot} . Both exhibited clear utility in predicting the evolution or structure of the

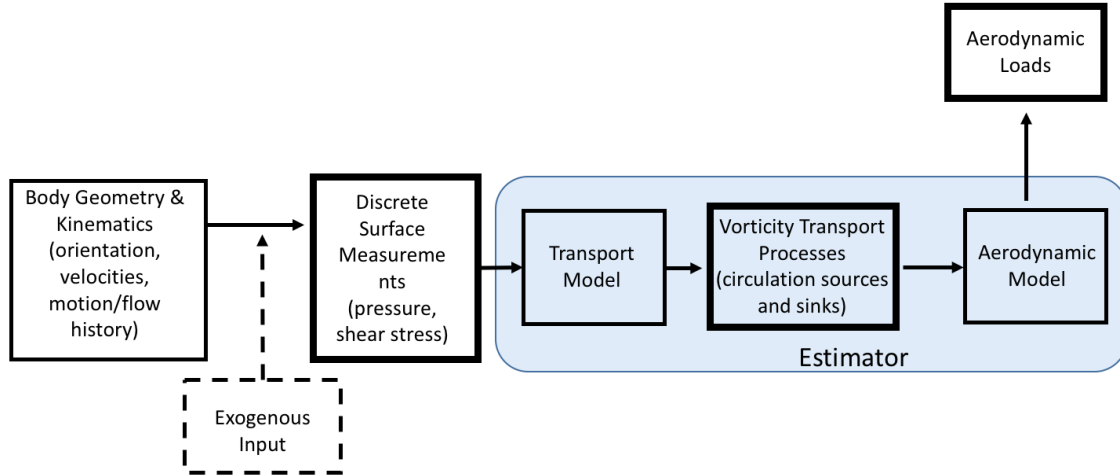


Figure 9.1: Flow chart depicting the use of discrete measurements to predict vorticity transport processes underlying aerodynamic loads.

leading-edge vortex. The bifurcation of the conical vortex into a dual (or triple) vortex system, is an event that can be localized in both time and spanwise position, suggesting a bifurcation in the dynamics of the LEV. The spatial location at which the vortex branches appears to be coincident with the (apparently sharp) boundary between an attached vortex and one that is shedding. It was shown that the regions on either side of that boundary exhibit vast differences in the behaviors of the transport mechanisms; however, the specific causes of these differences are still not well-understood. Similarly, the elapsed time that marks the initiation of the transition is a strong function of k_{rg} in the case of the simultaneous pitching and rolling wing; however, it is still unclear what processes or conditions the parameter represents, that lead to this transition.

The π_{rot} parameter is complementary, in that it predicts when, or to what extent, the conical vortex will persist on the inboard portion of the wing. Whereas it was derived through non-dimensionalization of the spanwise variation of inflow velocity – and thus representative of the shear flow to which the wing is subjected – it is also representative of the centripetal acceleration in the non-inertial coordinate system. Notably, the analysis in Chapter 3 showed that centripetal acceleration did not contribute to the convective fluxes of vorticity governing LEV circulation; although this characterization was given for a specific orientation of the control region, and the surface-diffusive fluxes did (in principle) have some dependence on the centripetal acceleration for a purely-rolling wing. So, while π_{rot} appears to be an important predictor of vortex evolution on a rolling wing, understanding its specific relation to underlying the underlying processes requires further investigation. Consideration of how the parameter may evolve to account for key flow structures in the case of combined pitch and roll may provide an additional perspective that may also help to elucidate its role.

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