REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
01-07-2016	Final Report	14-Sep-2012 - 13-Sep-2015

4. TITLE AND SUBTITLE
Final Report: Theoretical and Computational Studies of Tipping Points and Community Detection in Social Networks (Research Area 4.3.3)

5a. CONTRACT NUMBER
W911NF-12-1-0546

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

6. AUTHORS
Chjan Lim

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
Rensselaer Polytechnic Institute
110 8th Street

Troy, NY 12180 -3522

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
62449-NS.23

12. DISTRIBUTION AVAILABILITY STATEMENT
Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT
The work proposed has been completed with the following major conclusions in the context of an idealized model for social interaction: (1) the Tipping fraction for rapid consensus of committed minority opinion drops from 9.8 percent for complete graphs to 5% for regular degree 4 graphs, and also when the number of initial opinions is large. An important class of 2-Urns models were solved exactly by diagonalization of an infinite family of banded matrices to support this work but is of independent theoretical interests.

15. SUBJECT TERMS
Social Opinion dynamics, Tipping points, Naming games

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Chjan Lim
UU	UU	UU	UU		19b. TELEPHONE NUMBER 518-276-6904

Report Title

Final Report: Theoretical and Computational Studies of Tipping Points and Community Detection in Social Networks (Research Area 4.3.3)

ABSTRACT

The work proposed has been completed with the following major conclusions in the context of an idealized model for social interaction: (1) the Tipping fraction for rapid consensus of committed minority opinion drops from 9.8 percent for complete graphs to 5% for regular degree 4 graphs, and also when the number of initial opinions is large. An important class of 2-Urns models were solved exactly by diagonalization of an infinite family of banded matrices to support this work but is of independent theoretical interests.

A total of 8 peer reviewed has been published in the duration of this project (the paper app in ARO reads wrong!)
A total of 5 conference proceedings.
One postdoc supported for 3 yrs.
Two GRAs supported for 2 yrs.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
03/04/2013	5.00	Chjan Lim, B. Szymanski, Weituo Zhang. Analytic Treatment of Tipping Points for Social Consensus in LargeRandom Networks, Phys Review E, (12 2012): 0. doi:
07/31/2013	9.00	F. Molnár, S. Sreenivasan, B. K. Szymanski, G. Korniss. Minimum Dominating Sets in Scale-Free Network Ensembles, Scientific Reports, (04 2013): 0. doi: 10.1038/srep01736
07/31/2013	10.00	F. Molnár,, S. Sreenivasan,, B. K. Szymanski, , G. Korniss. Threshold-limited spreading in social networks with multiple initiators, Scientic reports, (04 2013): 0. doi:
08/05/2014	13.00	Weituo Zhang, Chjan C. Lim, G. Korniss, Boleslaw K. Szymanski. Opinion Dynamics and Influencing on Random Geometric Graphs, Scientific Reports, (07 2014): 0. doi: 10.1038/srep05568
08/16/2014	18.00	Weituo Zhang, Chjan Lim. The Concentration and Stability of the Community Detecting Functions on Random Networks, INTERNET MATH, (11 2013): 360. doi:

TOTAL: 5

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received		Paper
07/31/2013	11.00	W. Zhang, C. Lim. Large deviation results for optimal partitioning problems in random networks, INTERNET MATH, (04 2013): 0. doi:
TOTAL:		**1**

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Number of Presentations: 3.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper
TOTAL:	

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received		Paper
02/25/2013	1.00	Matthew Dippel, Chjan C. Lim. A Coauthorship Network and Preliminary Results of Binary Agreement Model Simulation, IEEE NSW West Point May 2013. 01-MAY-13, . : ,
02/25/2013	2.00	Yosef Treitman, Chjan Lim, Andrew Thompson, Weituo Zhang. Naming Game with Greater Stubbornness and Unilateral Zealots, IEEE NSW Westpoint May 1 2013. 01-MAY-13, . : ,
TOTAL:		**2**

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
02/25/2013	3.00	F. Molnar Jr., S. Sreenivasan, , B. Szymanski, G. Korniss. Minimum Dominating Sets in Scale Free Networks , arXiv:1106.1022 (02 2013)
02/25/2013	4.00	Weituo Zhang, Chjan Lim. The Concentration and Stability of the Community Detecting Functions on Random Networks, INTERNET MATH (10 2012)
07/31/2013	7.00	Chjan Lim, Weituo Zhang. Asymptotic Behavior of Coarse-grained Models for Opinion Dynamics on Large Networks, SIAM J on Applied Dynamical Systems (07 2013)
07/31/2013	12.00	Yosef Treitman , Chjan Lim. Effect of multiple degrees of ambivalence on the Naming Game, Phys Review E (08 2013)
07/31/2013	8.00	Chjan Lim. Information sharing with strong neutrals- exact solutions for expected times to multi-consensus, Phys Review E (05 2013)
08/05/2014	14.00	Andrew Thompson, B Szymanski, C Lim. Propensity and noncommitment in the naming games: Tipping fractions ofminorities, Physical Review E (06 2014)

TOTAL: **6**

Number of Manuscripts:

Books

Received	Book

TOTAL:

Received Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

RPI DIAZ prize for William Pickering 2016

RPI DiPrima Research Award 2016 Pickering

Research on the balance of diversity of opinions and social stability was picked up by Inside Rensselaer, a newsmagazine that reports only significant achievements.

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
W. Zhang	0.10	
W. Pickering	0.50	
A. Thompson	0.10	
G. Karapoulitis	0.30	
FTE Equivalent:	**1.00**	
Total Number:	**4**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
W. Zhang	1.00
FTE Equivalent:	**1.00**
Total Number:	**1**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Chjan Lim	0.65	
B. Szymanski	0.15	
G. Korniss	0.20	
FTE Equivalent:	**1.00**	
Total Number:	**3**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED	Discipline
T. Bodiksaksang	0.05	Math
FTE Equivalent:	**0.05**	
Total Number:	**1**	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 1.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 1.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 1.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 1.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 1.00

Names of Personnel receiving masters degrees

NAME	
A. Thompson	
G. Karapoulitis	
Total Number:	**2**

Names of personnel receiving PHDs

NAME	
W. Zhang	
G. Karapoulitis	
W. Pickering	
Total Number:	**3**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Exact solutions of the two and Multi Urns models in support of complex systems analysis of social opinion dynamics. Results on the bifurcation of Tipping points of committed fractions generalized to incomplete / sparse graphs including random graphs of fixed degree sequence, and to societies with high discord as measured by the number of initial opinions.

It was shown that the Tipping fractions decreased with increasing sparcity of underlying networks and also with increase entropy (number) of opinions.

Technology Transfer

Project Title
Proposal Number (e.g. XXXXX-CI)
Name (e.g. Professor Rube Goldberg, Affiliation (e.g. State University of Minnesota)

Theoretical and Computational Studies of Tipping Points and Community Detection in Network Science

Award W911NF-12-1-0546

PI: Professor Chjan C. Lim,
CoPI: Profs B. Szymanski and G. Korniss
Rensselaer Polytechnic Institute

http://www.rpi.edu/~limc

FR July 8 2016

OUTLINE

In collaboration with Dr. Gyorgy Korniss (Physics RPI) and Dr. Bolek Szymanski (CS RPI) at the NeST Center at RPI, the PI offers this final report which documents our concrete accomplishments in computational/theoretical network science, as well as recent progress on the mathematical and fundamental conceptual aspects of mathematical sociology /network science from 2012 to 2015. Our group including postdoc, undergrad and grad students have recently completed and submitted a total of 4 papers, 1 new paper have been published, several conference proceedings and invited talks. On the significance and broad relevance of this work, to the Army and in academia, there are numerous citations of our papers. In particular, we mention here the recent paper by Bruce West et al in Science Reports 2013 and the paper co-authored by several members of the National Academy and a Nobel Laureate: AP Kinzig, PR Ehrlich, LJ Alston, K Arrow, S Barrett… Social Norms and Global Environmental Challenges: The Complex Interaction of Behaviors, Values, and Policy, BioScience, 2013. There are two main parts:

(A) Those that are dominated by deterministic drift where the expected values of the change of macrostates (or coarse-grained quantities) is nonzero almost everywhere in phase space – they lead to mean field ODE models for which Monotonicity and Center Manifold properties of the resulting Dynamical Systems play a key role (cf. A. Thompson, B. Szymanski, C. Lim, "Propensity and Stubbornness in the Naming Games - Tipping Fractions of

Mavens", Phys Rev E final version 2014 and W. Zhang, C. Lim, G. Korniss, B. Szymanski, "Opinion Dynamics and Social Influencing in Random Geometric Graphs", Scientific Reports 4, 5568 (2014) and doi: 10.1038/srep05568)

(B) Those that are diffusion dominated such as the Voter models generalized to more than two but finite number of opinion types and to random networks – they lead to Martingale properties in their modeling by stochastic methods (cf. William Pickering and Chjan Lim, "Solution of the Voter Model by Spectral Analysis", PRE August 2014, on such models and their expected times to consensus. http://arxiv.org/abs/1408.2130).

The main themes overarching the objectives bulleted below, spans network science and mathematical sociology; they are: the effects of (I) local interaction rules / protocols (between two agents say) and (II) underlying network topology on the asymptotic and averaged properties of large dynamic social networks. The main tools, besides simulation methods, are applied probability and combinatorial modeling and analysis. This year, as part of (I), we focus on the robustness of the Tipping fraction of committed minority by (a) embedding the Naming game into a two parameter family of signaling games to show that for an open set of parameter values around the original NG, the Tipping phenomenon is preserved. In other words, in this large family, we show that the same underlying mathematical objects, namely the 1-dim center manifold and the saddle-node bifurcation, play the key roles in the Tipping points of signaling multi-agent models on both complete and random networks. Also as part of (I), we show that (b) generically, the deterministic drift part of the stochastic models for signaling multi-agent games like the Naming game do not give rise to chaotic dynamics. This is because of the existence of an ordering property called Monotonicity in these games (cf. C. Lim and W. Zhang, "Asymptotic Behavior of Coarse-grained Models for Opinion Dynamics on Large Networks", submitted July 2013).

OBJECTIVES

(1) Non Diffusion Driven Stochastic Dynamics: Monotonicity properties of ODEs for Binary Signaling Networks
(2) Diffusion- Driven Opinion Dynamics and Information Sharing: Exact Spectral Analysis of the Voter models
(3) Center Manifolds and Reduction of Dimensions in Opinion Dynamics: Tipping Points and Saddle-Node Bifurcations
(4) Exploration of Signaling Dynamics in Continuous PDE models on Random Geometric Graphs
(5) Threshold-Limited Spreading in Social Networks with Multiple Initiators
(6) Minimum Dominating Sets in Scale-Free Network Ensembles: Optimal placement of committed agents

DETAILED HIGHLIGHTS

Building on previous work by our group in network science and nonequilibrium statistical physics which focused on the above highlight by way of Coarse-graining and Random walk models in analyzing and simulating the effect of committed agents and central agency on time to consensus in a class of agent based network models (Naming Games, Voter Models) - Phys Rev E 2011 and Chaos 2011, the specific results uncovered this year by the PI's team at RPI are:

(I) **Exact Spectral Solutions for Diffusion-Dominated Signaling Games:**

This method is based on the generating function approach to solving the spectral problem of the Markov Propagator in the Voter models, and was inspired by Kac's work in the 1940s on the Ehrenfest Urn Problem. Complete knowledge of the leading eigenvalues and eigenvectors of these problems yield a trove of bulk or averaged quantities that are sought after in the nonequilibrium statistical physics of this class of models. They include expected times to consensus and all their moments, the latter providing valuable information on the fluctuations or variance of consensus times under different runs of the same models from the same initial data. Furthermore, these moments of consensus times are all solved for the whole range of possible initial data for the models (such as starting fractions of opinions etc).

Local times are the expected number of time steps or visits to each macrostate in the model. They are key quantities to be calculated in the nonequilibrium and computational physics of these opinion models.

(II) **Naming Games on Random Geometric Graphs – PDE methods**

Studying this model advances our understanding of the spatial distribution and propagation of opinions in social dynamics. A main feature of this model is the spontaneous emergence of spatial structures called opinion domains which are geographic regions with clear boundaries within which all individuals share the same opinion. We provide the mean-field equation for the underlying dynamics and discuss several properties of the equation such as the stationary solutions and two-time-scale separation. For the evolution of the opinion domains we find that the opinion domain boundary propagates at a speed proportional to
its curvature. Finally we investigate the impact of committed agents on opinion domains and find the scaling of consensus time.

(III) Monotonicity properties of ODEs for Binary Signaling Networks:

In new submitted work for 2013, PI and his postdoc W. Zhang ["Asymptotic Behavior of Coarse-grained Models for Opinion Dynamics on Large Networks", submitted July 2013] propose a general mathematical framework to represent many multi-agent signaling systems in recent works. Our goal is to apply previous results in monotonicity to this class of systems and study their asymptotic behavior. Hence we introduce a suitable partial order for these systems and prove nontrivial extensions of previous results on monotonicity. We also derive a convenient sufficient condition for a signaling system to be monotone and test our condition on the Naming Games, NG and K-NG on complete networks both with and without committed agents. We also give a counter example which fails to satisfy our condition. Next we further extend our conclusions to systems on sparse random networks. Finally we discuss several meaningful consequences of monotonicity which narrows down the possible asymptotic behavior of signaling systems in mathematical sociology and network science. It remains to be seen the many implications that Monotonicity Property will have as a game-changing idea and method in mathematical sociology and network science. Already we have concrete evidence and mathematical proofs that a very large class of signaling systems on networks (that are NOT diffusion-driven) have Tipping point properties and other socially and sociologically important effects just from this new type of partial order monotonicity in its mean field ODEs. This means that in the future we can avoid the tedious and expensive simulations of each separate multi-agent signaling networks to discover again and again its Tipping points.

(IV) **Diffusion- Driven Opinion Dynamics, Neutrals and Information Sharing:**

In new work, the PI and his graduate assistant William Pickering, started to analyze the nuances of the root concept of *neutral in sociology*. We focus on three related interpretations of neutrality and suggest corresponding mathematical models for each of them from the class of information-sharing multi-agents network games known as the Naming Games (NG). The subsequent line of approach is to embed each of these three models in their own one-parameter families of models, to better understand the consequences of incremental changes in the local interaction protocols on global statistical quantities such as expected times to consensus. A family of two-opinions Naming Games, the so-called M models for the interpretation of neutral which is applicable to the scenario of middle-roaders in a social forum or blog, is solved exactly for expected times to multiple consensus. These multi-opinions models represent a significant departure from the linguistic roots of the original NG and are constructed primarily to improve the modeling of the interacting agents' realistic cognitive traits on a large social network. The neutrals in the M models play the role of *Devil's Advocate* when they speak – that is, they speak only to pure A or pure B opinion agents and send the opinion opposite to that of the listener. The main results we obtained are that this family of NG are Martingale models, that is, purely diffusion-driven, and have expected times to multi-consensus which are O(N^2 log N) compared to the much shorter O(N log N) times for the Listener Only-NG models in the case of erratic neutrals.

(V) **The Impact of Heterogeneous Threshold Distribution on Cascades in the Threshold Model with Multiple Initiators**

We have investigated the threshold model (TM) for opinion spreading (Granovetter, 1978; Watts, 2002, [SSSK13]) for various distributions of the individuals' threshold. According to the TM, an individual changes its opinion only if a critical fraction of its neighbors have already adopted the new opinion. This required fraction of new adoptees in the neighborhood is designated the *adoption threshold* ϕ. We have found that the TM with uniform threshold for all individuals does not capture the complex nature of social influencing when multiple initiators are present. We found that for sufficiently large spread in the threshold distribution, the tipping point in the social influencing process (i.e. the fraction of initiators needed for global cascades) disappears and crosses over to a smooth transition governed by the size of initiators. Specifically, we studied cascades in the TM when nodes are assigned a threshold value drawn from the normal distribution with varying mean threshold $\bar{\phi}$ and standard deviation σ. We analyzed both synthetic and empirical networks using different sizes of initiators. We observed a non-monotonic change in the cascade size for varying σ that for small initiator sizes follows Watt's cascade condition. In addition, we found that, unlike the case of uniform thresholds, for large enough σ, a critical initiator size beyond which cascades become global ceases to exist. Thus, there is an important qualitative change from a sharp transition to a continuous change in the behavior of the cascade size vs. the fraction of initiators.

(VI) Dominating Scale-Free Networks Using Generalized Probabilistic Methods

It is a critical task in network science and its applications to find methods to efficiently detect, monitor and control the behavior of nodes in networks. Finding small dominating sets on static or slowly evolving networks is an effective approach in achieving these objectives. A dominating set of a network *G* with node set *V* is a subset of nodes , such that every node not in *S* is adjacent to at least one node in *S*, while the minimum dominating set (MDS) is the smallest cardinality dominating set. Dominating sets provide key solutions to various critical problems in networked systems, such as network controllability, social in influence propagation, optimal sensor placement for disease outbreak detection, and finding high-impact optimized subsets in protein interaction networks. The effective use of dominating sets in these problems demands profound understanding of the behavior of dominating sets with respect to various network features, as well as developing effective methods for finding different types of dominating sets that are optimal solutions for different problems.

In our recent investigations of methods for finding small or minimum dominating sets (MDS), we generalized the classic probabilistic method (Alon and Spencer, 2000) to heterogeneous (or scale-free) graphs. This method provides an upper bound and realizable candidates for small dominating sets. Further, we implemented and analyzed it for graph ensembles with tunable assortativity and empirical networks.

APPROACH

(1)Simulations Methods:
New methods for the analysis and simulations of social networks with respect to the above objectives are developed in terms of working models such as multi-agent systems. This mainly stochastic-theoretic approach will be combined with novel Markov chain methods to study the dynamical properties of social networks. A particularly relevant idea is to use the family of agent-based models known as the Naming Games to clarify and explore the properties of social networks. In the past year, we have already begun an intensive study of the Naming Games, leading to simulation algorithms that are efficient and can be implemented on new Intel desktops. This earlier work forms the basis on which further research along the lines discussed in objectives will be based.

(2) **Applied Probability and Exact Solution of a class of Diffusion-Dominated models:**

By mapping the original Voter model on a complete graph to a version of Ehrenfest's Urn Problem, we extended Mark Kac's classical spectral solution of the original Urn problem to the Voter model. This method has wide applications because of the versatility of its underlying Generating Functions Approach, which we adapted to incomplete networks such as bipartite complete and then uncorrelated fixed degree-sequence random graphs.

(3) **Monotonicity and Types of Equilibria and Their Bifurcations in Networked Opinion Dynamics:** In new work submitted "Asymptotic Behavior of Coarse-grained Models for Opinion Dynamics on Large Networks", 2015, we discovered that beyond the above conjecture on the role of the Center Manifold, Monotonicity properties of the mean field coarse-grained ODEs play a key role in the Tipping points of committed minority agents as well as the expected times to consensus by substantially limiting the asymptotic or long time behavior of the associated dynamical systems. This discovery suggests that Monotonicity or Preservation of Partial Order in phase space arises frequently in the dynamics of opinions and information sharing networked agents, and offers an easier way to calculate expected times to consensus and Tipping points phenomena than the more detailed and technically difficult method of finding the center manifold and then calculating explicitly the bifurcation points. In the case of the original NG with two opinions, the phase space is 2D and hence, the center manifold and its saddle-node bifurcation under the influence of a committed minority of agents, was easily discovered and reported in the 2011 papers in PRE and Chaos by us. Usable progress beyond these so-called lower hanging fruits will likely depend on ramifications and extensions of the Monotonicity methods begun here.

(4) **Threshold-Limited Spreading in Social Networks with Multiple Initiators**

We studied the above threshold model with uniformly distributed threshold - triggered by *multiple initiators*, in the context of opinion formation on synthetic and empirical social network. We considered different strategies of selecting a fraction p of initiator nodes to obtain a large and efficient spread of opinion. As is well-known from prior works, for a single initiator or small initiating clique, spread is be restricted by an individual's high threshold and if the threshold is sufficiently high, spread becomes impossible [$\phi_c \sim 1/\langle k \rangle$, where $\langle k \rangle$ is the average degree of the network (Watts, 2002)]. However, we found that for an arbitrary high threshold, there exists a critical fraction p_c of initiators (unique for a given threshold) that is needed to trigger a *global* cascade. Network structure, in particular local clustering, plays an important role in this scenario. Similar to the case of single-node or single-clique initiators, we observed that locally well-connected clusters facilitate the opinion spread as opposed to a homogeneous random network.

We found that for any given local threshold $\phi_c < 1$ there exists a critical value of the fraction of initiators p_c, above which global cascades can be triggered. We obtained the dependence p_c on ϕ which turns out to be a smooth curve separating the two phases, one in which cascades are observed and the other where cascades cannot be triggered. This finding provides an important insight into how *local* neighborhood-level thresholds give rise to the emergence of tipping points for cascades on *global scales* on sparse social graphs. [P. Singh, S. Sreenivasan, B.K. Szymanski, and G. Korniss, "Threshold-limited spreading in social networks with multiple initiators", *Scientific Reports* (in press, 2013); http://arxiv.org/abs/1304.7034].

(5) **Minimum Dominating Sets in Scale-Free Network Ensembles**

We studied the scaling behavior of the size of minimum dominating set (MDS) in scale-free networks, with respect to network size N and power-law exponent γ, while keeping the average degree fixed. We studied ensembles generated by three different network construction methods, and we use a greedy algorithm to approximate the MDS. With a structural cutoff imposed on the maximal degree ($k_{max}=\sqrt{N}$) we found linear scaling of the MDS size with respect to N in all three network classes. Without any cutoff ($k_{max}=N$-1) two of the network classes display a transition at $\gamma\approx1.9$, with linear scaling above, and vanishingly weak dependence below, but in the third network class we find linear scaling irrespective of γ. We found that the partial MDS, which dominates a given z<1 fraction of nodes, displays essentially the same scaling behavior as the MDS. [F. Molnár Jr., S. Sreenivasan, B.K. Szymanski, and G. Korniss, "Minimum Dominating Sets in Scale-Free Network Ensembles", *Scientific Reports* **3**, 1736 (2013); http://dx.doi.org/10.1038/srep01736].

SCIENTIFIC BARRIERS

The barriers in the work in Network Science are many-fold. In addition to the availability of real world data sets that are moreover relevant to the Army, many of the concepts used in network science are not well-defined and lack mathematical rigor. Finding suitable and computable measures of these concepts is one of the tough barriers that we hope to overcome by novel applications of probability theory, statistical physics and theoretical computer science. An approach that is likely to be successful is to sharpen the statistical notion of variance so that minimizing the appropriate measure of variance in a system is a computable methodology towards the aim of maximizing the trustworthiness of the information flows involved.

Moreover, the high costs of detailed numerical simulations of each separate networked system of agents has to be avoided in order to achieve useful progress beyond the lower hanging successes reported in our papers of 2010, 2011 and early 2012. Generalized NG models, Center Manifolds of higher dimensional opinion dynamics and more recently, the Monotonicity properties of coarse-grained mean field opinion dynamics in information sharing networks of agents, are ideas and methods that allow us to break this log-jam. Most significant of the past year's achievements are the formulation of the spectral method and its applications to exact solutions of the Voter models on a range of network topologies.

SIGNIFICANCE

The significance of the work on network science is already clear from the priority and attention that the federal funding agencies have given network science groups. There are however many angles for studying network science that could benefit from a more in-depth and rigorous theoretical approach that we have taken in the current ARO grant. It is clear from some recent accomplishments that the detailed Markov chain analyses, multi-agents model simulations and probabilistic analyses of dominating sets in social networks outlined above will yield powerful general results that will have lasting impact on the Army's research portfolio on information processing and network science.

This impact is already showing up in the 100 plus citations (Google scholar – Chjan Lim) that the PI's joint papers in Phys Rev E and Chaos have collected in 2011, 2012 through 2014. Moreover the significance of the latest phase of this project as outlined in detailed highlights section is underscored by recent global events such as the Arab Spring and the recent push-backs by moderates against militant groups, and by the military against the Islamic Brotherhood in Egypt.

Geo-political events underscore the need for rigorous theories on social dynamics, influencing, and community detection and evolution where for instance, the role of committed agents in social influencing is explored by constructing and analyzing multi-agents models where the individuals' complexity ranges from simple to feature-rich. The optimal placement of committed minority/ leaders in a social network to influence the final outcomes such as faster total consensus, is a computably hard problem, that requires the type of combinatorial and probabilistic analysis for efficient resolution.

Detailed Significance:

Center Manifolds, Reduced Bifurcations, and Robust Tipping Fractions:

We expect that in generalized NG, like the original NG, the Tipping point arises from a saddle-node bifurcation as the fraction of committed minority increases through a critical value. In this bifurcation, the ubiquitous saddle fixed point of the symmetric NG merges with a stable node on the center manifold that is associated with the symmetry-breaking effect of the committed minority on the social dynamics. As a result, a hyperbolic degenerate fixed point known as the saddle-node is born which is attractive in the hyperplanes normal to the center manifold but along the center manifold, it is attractive on the side of the B consensus and repulsive towards the A consensus when the committed minority has the A opinion.

Andrew Thompson with Szymanski and the PI completed and submitted a paper that embedded the original Naming game within a two parameter family with the specific intention to verify mathematically the robustness of the Tipping fraction phenomenon of committed minorities in signaling multi-agent games. In mathematical terms, we showed that the saddle-node bifurcation on a simple center manifold (first reported in Xie,.., Lim,.. et al PRE 2011) persists for a relatively large open set of parameters in this family of generalized Naming game models.

Spectral Solutions of Diffusion-Dominated Opinion models:

One of the most impactful advances of this grant is the PI's execution of his program for the generating-functions based spectral analysis of an iconic class of opinion and epidemic models on a range of network topologies. Previous works calculated expected times to consensus. Our spectral method give a more complete understanding of the non-equilibrium processes and timescales in the approach to consensus in these models. Since the Voter models have been studied for their applications in epidemiology and social opinion dynamics, as well as their pure mathematical properties related to coalescing random walks on lattices, our method clearly represent a significant advance in the technology that can be applied to social networks.

Minimum Dominating Sets in Scale-Free Network Ensembles:

The recent progress in the work of Szymanski, Korniss, including postdocs and students previously supported by the ARL – NSCTA, have made a dent in the problem of optimal placement of the committed agents or leader nodes in a social network that are sparse and complex such as scale-free ones. Their work involves the computationally hard graph-theoretical problem of characterizing and finding the dominating sets in a network graph.

ACCOMPLISHMENTS

(feel free to use a bulleted list here)
As discussed in detailed highlights section.

One of the most important accomplishments of the PI is the recruitment and completion of the PhD training of William Pickering, an excellent energetic math graduate student at RPI to network science. The PI will apply for a renewal or new ARO / DOD grant specifically to transition Pickering to a postdoc in his group after the defense of PhD in october 2016. The achievements of this graduate student in his applications of the PI's spectral generating-functions approach to signaling games in the study of social opinion dynamics, cannot be overstated (cf. the manuscript uploaded).

Papers submitted, in adjudication and published in the three years of this grant that are relevant to this project:

Area (1): New Mean Field Methods, Monotonicity and Robustness of Tipping – Saddle Node Bifurcations

W. Zhang and C.C. Lim, "The Concentration and Stability of the Community Detecting Functions on Random Networks", Internet Mathematics 9 (4), 360-383, 2013.

W. Zhang, C.C. Lim and B. Szymanski, "Analytic Treatment of Tipping Points for Social Consensus in Large Random Networks", Phys Rev E 86 (6), 061134, 2012

A. M. Thompson, B. Szymanski, C. Lim "Propensity and Stubbornness in the Naming Games - Tipping Fractions of Mavens", Phys Rev E 90, 042809, oct 15 2014

Area (2): Exact Spectral Analysis of Voter models and Crowds:

W. Pickering and C. Lim, "Solution of the Voter Models by Spectral Analysis", Physical Review E (Vol.91, No.1): DOI: 10.1103/PhysRevE.91.012812 http://link.aps.org/doi/10.1103/PhysRevE.91.012812

W. Pickering and Chjan Lim, "Solution of the multistate voter model and application to strong neutrals in the naming game", 1 March 2016 issue of Physical Review E (Vol.93, No.3): DOI: 10.1103/PhysRevE.93.032318

W. Pickering, Bolek Szymanski and Chjan Lim, "Analysis of the high-dimensional naming game with committed minorities", May 1 2016 issue of Physical Review E, http://link.aps.org/doi/10.1103/PhysRevE.93.052311 DOI: 10.1103/PhysRevE.93.052311

Area (3): Network Topologies:

W. Zhang, Korniss, Szymanki, C.C. Lim, "Spatial Propagation of Opinions: Naming games on random geographic graphs", Sci Reports 2014, Scientific Reports 4, 5568 (2014) and doi: 10.1038/srep05568 Sci Reports

Area (4): **Threshold-Limited Spreading in Social Networks with Multiple Initiators**

F. Molnár Jr., S. Sreenivasan, B.K. Szymanski, and G. Korniss, "Minimum Dominating Sets in Scale-Free Network Ensembles", *Scientific Reports* **3**, 1736 (2013); http://dx.doi.org/10.1038/srep01736.

P. Singh, S. Sreenivasan, B.K. Szymanski, and G. Korniss, "Threshold-limited spreading in social networks with multiple initiators", *Scientific Reports* (in press, 2013); http://arxiv.org/.

Citations and Public Interest: Selected and most recent few

1. Citation by a policy paper co-authored by Nobeller Kenneth Arrow and several members of the National Academy

Social Norms and Global Environmental Challenges: The Complex Interaction of Behaviors, Values, and Policy

AP Kinzig, PR Ehrlich, LJ Alston, K Arrow, S Barrett… - BioScience, 2013 - JSTOR
Government policies are needed when people's behaviors fail to deliver the public good. Those policies will be most effective if they can stimulate long-term changes in beliefs and norms, creating and reinforcing the behaviors needed to solidify and extend the public **...**

2. Citation in a Nature article by the CEO of Advantis Cambridge

Better connected

M Fishman, R Cross, B Tadmor - Nature, 2013 - nature.com

Research-and-development companies are constantly changing their organizational structures to nurture innovation and increase productivity. Yet no single organizational model has emerged as the best option in either academia or industry.

3. Citation by Princeton University group on Math Ecology

Decision Accuracy and the Role of Spatial Interaction in Opinion Dynamics

CJ Torney, SA Levin, ID Couzin - Journal of Statistical Physics, 2013 - Springer
Abstract The opinions and actions of individuals within interacting groups are frequently determined by both social and personal information. When sociality (or the pressure to con- form) is strong and individual preferences are weak, groups will remain cohesive until a consensus **...**

4. Citation by Bruce West et al in Scientific Reports 2013

Role of committed minorities in times of crisis

M Turalska, BJ West, P Grigolini - Scientific reports, 2013 - nature.com
The surprising social phenomena of the Arab Spring and the Occupy Wall Street movement posit the question of whether the active role of committed groups may produce political changes of significant importance. Under what conditions are the convictions of a minority **...**

PRESENTATIONS: Conferences attended, invited talks, posters and proceedings papers relevant to this project

[1] C. Lim, "Social Networks and Opinions Dynamics on Random Graphs", Invited Talk, Workshop on Socio- Physics, Sigma-Phi 2014, Rhodes, Greece, July 7-11, 2014

[2] C. Lim, " Math Models of Communications and Leadership", Invited Panel Member, Satellite Workshop on Team Networks, NetSci 2014, Berkeley, June 2014, organized by C. Arney and K. Coronges

[3] W. Zhang and Chjan Lim, "Growth of Social Networks based on Relevance and Importance Attachment", SIAM Workshop on Network Science, Chicago, July 7 -9, 2014

[4] W. Zhang and Chjan Lim, “Growth of Social Networks based on RIPA”, NetSci 2014, Berkeley, June 4-8, 2014

[5] W. Zhang and Chjan Lim, “Social Opinion Dynamics on RGG”, NetSci 2014, Berkeley, June 2014

[6] W. Zhang and Chjan Lim, “Social Opinion Dynamics on Random Geometric Graphs”, SIAM Workshop on Network Science, Chicago, July 2014

[7] Scaling of Various Dominating Sets in Scale-Free and Empirical Networks, N. Derzsy, F. Molnar Jr. E. Czabarka, L. Szekely, B. Szymanski, G. Korniss, Bulletin of the American Physical Society. APS March Meeting, Volume 59, Number 1, 2014

[8] Stability of Dominating Sets in Complex Networks against Random and Targeted Attacks, F. Molnar Jr., N. Derzsy, B. Szymanski, G. Korniss, Bulletin of the American Physical Society. APS March Meeting, Volume 59, Number 1, 2014

[9] Cascades in the Threshold Model with Multiple Initiators and Heterogeneous Threshold Values, P. Karampourniotis, S. Sreenivasan, B. Szymanski, G. Korniss, Bulletin of the American Physical Society. APS March Meeting, Volume 59, Number 1, 2014

[10]A network approach in analysis of the matching hypothesis, Tao Jia, Robert Spivey, B. Szymanski, G. Korniss, Bulletin of the American Physical Society. APS March Meeting, Volume 59, Number 1, 2014

[11]Competing effects of social balance and influence, P. Singh, S. Sreenivasan, B. Szymanski, G. Korniss, Bulletin of the American Physical Society. APS March Meeting, Volume 59, Number 1, 2014

[12] Scaling of Various Dominating Sets in Scale-Free Network Ensembles, F. Molnar Jr., N. Derzsy, E. Czabarka, L. Szekely, B. K. Szymanski, G. Korniss, June 4, NetSci 2014 Conference, Berkeley, CA.

[13]Stability of Dominating Sets in Complex Networks against Random and Targeted Attacks, F. Molnar Jr., N. Derzsy, B. K. Szymanski, G. Korniss, June 4, NetSci 2014 Conference, Berkeley, CA.

[14]Competing effects of social balance and influence, Pramesh Singh, Sameet Sreenivasan, Boleslaw Szymanski and Gyorgy Korniss, June 4, NetSci 2014 Conference, Berkeley, CA

[15]Cascades in the Threshold Model with Multiple Initiators and Heterogeneous Threshold Values in Social Networks, P. Karampourniotis, S. Sreenivasan, B. Szymanski and G. Korniss, June 4, NetSci 2014 Conference, Berkeley, CA.

Invited talks by BKS

16. Spreading and Opinion Dynamics in Social Networks, keynote at Social Network Analysis and Applications at ASONAM 2013, Niagara Falls, ON, Canada, August 25, 2013

17. Influence Spreading and Opinion Dynamics in Social Networks, keynote at the BRICS Countries Congress (BRICS-CCI), Recify, Brazil, September 10, 2013.

18. IEEE Lecture on Spreading and Opinion Dynamics in Social Networks, at Temple University, Philadelphia, PA, September 24, 2013.

19. Spread of Opinions Dynamics in Social Networks, invited seminar at the Department of Computer Science, University of California, Davis, CA, January 22, 2014.

20. Tipping Points and Cascades of Opinion Spread in Social Networks, The ACM WSDM 2014 Workshop: Diffusion Networks and Cascade Analytics, New York, NY, February 28, 2014.

21. Spread of Opinions Dynamics in Social Networks, Distinguished Lecture Series, Queen's University, Kingston, ONT, Canada, March 6, 2014.

22. Spread of Opinion Dynamics in Social Networks: Tipping Points and Cascades, Central European University, Budapest, Hungary, April 1st, 2014.

23. Influence of culturally based features of social networks on spread of opinions, CODYM Spring Workshop, Wroclaw Technological University, Wroclaw, Poland, April 7, 2014.

24. Dynamics of Opinion Spread in Social Networks, Invitted talk at the Sociophysics Workshop, International Conference of Statistical Physics, Rhodes, Greece, July 10, 2013.

Collaborations and Leveraged Funding
(feel free to use a bulleted list here)

Collaborations with mathematical ecologists at Princeton University through S. Levin's group is actively cultivated. Meanwhile, the spring 2013 visit and talk by the PI at Dr. Mubarak's Vision Research Center at U. Central Florida promises to inject fresh new ideas and methods into the PI's initial work on swarms dynamics and crowds.

Funding from DARPA, Department of Homeland Security and the ONR are actively pursued by the PI to supplement the funds to support more postdocs and energetic graduate and undergrad students at RPI and to encourage them to join and remain in STEM fields that will ultimately prove beneficial to the nation's scientific and military capabilities.

Conclusions

See above.

Technology Transfer

This is given in more details above in Citations and Public Interest, where the high level policy papers citing our works provide concrete evidence that the rigorous discoveries in network science at RPI have entered the world of government, policies, industry such as workplace efficiencies in a pharmaceutical research laboratory, and academia.

Most importantly, it has begun to be incorporated into aspects of military science and programs through the citations of our works by ARL scientists such as B. West, Grigolini and A. Swami.

Future Plans

In Area (2) above, we have started to work on Swarming Dynamics and applications to crowds as a direct result of the enlightening workshop organized by Drs. P. Iyer and M. Tambe at U Southern California in September 2012. Together with an energetic undergrad and the PI's postdoc, Dr. W. Zhang, we have found a new mechanism for the aggregation of a flock of bird-like agents, namely a velocity averaging algorithm that complements those already studied.

As a result of the satellite workshop, Teams in Networks, organized by Kate Congores and Chris Arney at the NetSci 2014 conference in Berkeley, and in particular, the late afternoon panel in which the PI participated, we are opening a new line of study involving game-theoretic concepts in the multi-agent signaling games that our group has so far explored in depth.

Expenditure:

The PI graduated PhD student W. Zhang, who is now a postdoc supported 100 percent by this grant within the PI's group and has recruited and started support for graduate students, William Pickering and Andrew Thompson, partially on this ARO grant. Another graduate student, P. Singh, partially supported by this grant has successfully defended his PhD thesis. Moreover, the PI leads a weekly working seminar on Network Science within the math department, which is attended by his new postdoctoral fellow Dr Weituo Zhang, these two grad students. Andrew is in the RPI accelerated PhD program where he expects to finish a B.S and PhD at RPI in 7 years. He will be a fifth year student, making excellent progress on a two-parameter family of generalized Naming Games that he came up with to model some subtle aspects of social – linguistic interactions. Drs. Korniss and Szymanski have partially supported their grad students F. Molnar, P. Singh, and Panagiotis, as well as two undergrads at RPI. The second year's allotment will be fully expended by early November 2014, and complete funding of the third year of this grant will allow us to complete the tasks proposed as well as make inroads into new areas.The third year budget has been fully expended.