



Direct Numerical Simulation of Hypersonic Transition Delay over Carbon/Carbon Ultrasonically Absorptive Coatings

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14. ABSTRACT Dr. Wagner's group at DLR has successfully pioneered the use of carbon-carbon (C/C) ultrasonically absorptive coatings (UAC) to delay hypersonic transition on a slender cone (Wagner et al., AIAA 2012-5865). C/C UACs are a very attractive passive hypersonic flow control technology due to their compatibility with practical thermal protection systems already found in hypersonic vehicles. Direct numerical simulations replicating the German experiments has been carried out by modeling the C/C UACs with the novel time-domain impedance boundary condition (TD-IBC) formulation developed by Scalo et al., Phys. Fluids (2015). Dr. Scalo's technique allows for the exact imposition in a Navier-Stokes solver of any physically admissible IBC. The required IBCs will be derived from a new campaign of C/C UACs acoustic absorption measurements. Their acoustic response will therefore be accurately modeled without the need to resolve or over-simplify the porous structure.					
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Summary of Achievements:

The three papers attached to this progress report represent the main achievements during this period of performance. The three papers are:

1. Sousa, Patel, Chapelier, Wartemann, Wagner, Scalo, “Numerical Investigation of Second Mode Attenuation over Carbon/Carbon Porous Surfaces”, *Journal of Spacecraft and Rockets*. 2018 **invited

This is an invited contribution from the NATO Transition group. This is our first contribution where we run direct numerical simulations of second-mode waves with impedance boundary conditions modeled to match what we expect the impedance of the porous carbon-carbon to be.

2. Wagner, Schramm, Dittert, Sousa, Patel, Scalo, “Experimental and numerical acoustic characterization of ultrasonically absorptive porous materials”, *AIAA-Aviation 2018*

This paper for the first time presents bench-test acoustic measurements of the carbon surfaces carried out in Germany and compared against various acoustic models. This paper is of seminal importance since it validates the acoustic models to predict the impedance of porous carbon-carbon materials and it enables the set up and execution of a direct numerical simulation of Dr. Wagner’s experiments.

3. Sousa, Batista, Kuehl, Scalo, “Nonlinear Dynamics of Second-Mode Waves on a Hypersonic Flared Cone”, *AIAA-Aviation 2018*

This paper initiates a collaboration with Prof. Joe Kuehl on spectral broadening of second-mode waves. We bring our expertise in nonlinear acoustics and direct numerical simulations to assist the development and validation of nonlinear PSE models.

4. Sousa, Wartemann, Wagner, Scalo, “Hypersonic Transition Delay Over Complex Wall Impedance”, *in preparation for Journal of Fluid Mechanics*

This paper will contain the final set of results replicating the experiments of hypersonic transition delay by Alex Wagner. This paper is a significant step forward with respect to paper (1) (which is limited to axisymmetric results) since it contains for the first time data from 3D transitional runs.



Numerical Investigation of Second-Mode Attenuation over Carbon/Carbon Porous Surfaces

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Axisymmetric direct numerical simulations of a spatially developing hypersonic boundary layer over a sharp 7 deg half-angle cone at $M_\infty = 7.5$ at Reynolds numbers $Re_m = 1.46 \cdot 10^6$, $2.43 \cdot 10^6$, and $4.06 \cdot 10^6 \text{ m}^{-1}$ have been carried out. The streamwise extent of the simulated domain is 0.9 m. The boundary layer is excited with the one-time application of a broadband pulse that is advected downstream over an impermeable wall and a complex impedance boundary condition (IBC) modeling the acoustic response of carbon/carbon (C/C) ultrasonically absorptive porous surfaces. The complex IBCs are derived as an output of a pore-scale aeroacoustic analysis: the inverse Helmholtz solver (IHS) and algebraic low-order models. The IHS estimate of the C/C impedance is compared to other methods for porous acoustic absorbers, revealing uncertainties in the applicability of analytical models for porous absorbers. The introduction of the IBCs in all cases leads to a significant attenuation of the instability waves, up to one order of magnitude reduction in pressure perturbation amplitude for the $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ case.

Nomenclature

a	=	speed of sound, m/s
C/C	=	carbon/carbon
f	=	frequency, kHz
H	=	depth, m
$\Im$	=	imaginary
L	=	distance from the leading edge, m
M	=	Mach number
Pr	=	Prandtl number
p	=	pressure, Pa
p_k	=	poles
$\hat{R}$	=	complex reflection coefficient
$\Re$	=	real
Re	=	Reynolds number
$\hat{S}$	=	complex absorption coefficient
T	=	temperature, K
u or U	=	streamwise velocity, m/s
x	=	streamwise direction, m
y	=	wall-normal direction, m
Z	=	impedance, (Pa · s)/m
β	=	absorption coefficient

γ	=	ratio of specific heats
θ	=	angle of incidence
κ	=	structure factor
λ	=	wavelength, m
μ_k	=	residues
Ξ	=	resistivity, (MPa · s)/m ²
ρ	=	density, kg/m ³
ϕ	=	porosity
ψ	=	zenith angle in spherical coordinate system
ω	=	frequency, rad/s

Subscripts

c	=	cone
e	=	boundary-layer edge
w	=	wall
∞	=	freestream
$*$	=	nondimensional
0	=	base state

I. Introduction

THE design of hypersonic vehicles is constrained by the considerable heat transfer and shear-stress loads imparted on the vehicles surface by the boundary layer. While being already critical in the laminar regime, such loads are significantly enhanced by the transition to turbulence. Reed et al. [1] highlighted the differences in fully turbulent and fully laminar trajectories for a hypersonic flight vehicle, reporting that the vehicle would experience a heat flux approximately five times higher under turbulent conditions and therefore would require at least twice the weight in thermal protection systems (TPS). Recent studies in the framework of the national Aerospace Plane Project led by the Defense Science Board committee [2] quantify the drop in performance of high-speed vehicles due to transition, reporting that the payload-to-weight ratio for fully laminar conditions is three times larger than for fully turbulent. These studies emphasize the importance of laminar-to-turbulent transition delay for sustainability of high-speed flight and the need for any realizable technology for boundary-layer control to be symbiotic with TPS.

Hypersonic vehicles that fly at small angles of attack and with high lift-to-drag ratios tend to have predominantly symmetric geometries

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and slender shapes. Under these conditions, second-mode waves become the main mechanism driving transition to turbulence [3]. The second mode was discovered by Mack [4] and consists of traveling ultrasonic acoustic waves trapped within the boundary layer; they are found to be amplified by wall-cooling conditions, that is when the wall temperature is below adiabatic temperatures, $T_w < T_{ad}$, which is commonly the case. Mack's studies on the evolution of these and higher modes [5] stressed the importance of second-mode attenuation in any attempt to increase the transitional Reynolds number in high-speed flows.

Malmuth et al. [6] proved through linear stability theory (LST) the capability of ultrasonically absorbing coatings (UACs) to mitigate the second mode, confirmed by experiments that followed. The first experimental validation of this principle is due to Fedorov et al. [7], who constructed a cone with a half-angle of 5 deg with two different surfaces: one smooth and impermeable, and the other perforated with regularly spaced micro holes. They reported that the porous surface was capable of doubling the transitional Reynolds number in comparison with the smooth surface in experiments at Mach 5 at the T-5 hypersonic wind tunnel located at the California Institute of Technology.

The development of UACs continued, with the goal of combining the acoustic absorption and TPS characteristics into one solution. Knowing that an irregular porous structure is naturally found in TPS materials used for hypersonic vehicles, Fedorov et al. [8,9] conducted experiments in the Mach 6 wind tunnel T-326 at the Institute of Theoretical and Applied Mechanics (ITAM) in Novosibirsk, Russia, over a felt-metal surface with an irregular porous structure (as well as another surface with regular microstructure). These experiments were the first to produce data capable of quantitatively demonstrating the attenuation of the second-mode waves via UACs, confirmed by stability analysis, but also showed a small destabilization of the first mode.

Chokani et al. [10] performed a bispectral analysis on this same data with the goal of identifying nonlinear mechanisms that could be triggered by the use of porous coatings; this was motivated by the observation of a weak enhancement of first-mode waves. The experiments revealed the occurrence of nonlinear phase locking, which is the most efficient method for a mode to feed itself or to transfer energy between modes, involving only the first mode, in the presence of porous walls and not over an impermeable surface. This effect was also shown to be small, not impacting the overall stabilizing role of the porous coating. More recently, Lukashevich et al. [11], performed new experiments in ITAM's tunnel, where it was shown that placing the porous insert in the region where the second mode is stable leads to an increase in the disturbance signal amplitude; the opposite is observed when the insert is placed in regions of second-mode growth.

Inspired by some of these results, Wagner et al. [12–14] pioneered the use of carbon/carbon (C/C), an intermediate state of C/C silicon carbide (C/C-SiC) already employed on hypersonic vehicles [15,16], to control second-mode waves. Experiments at Mach 7.5 have been conducted in the DLR High Enthalpy Shock Tunnel Göttingen (HEG), and the stabilization of the second mode as well as an increase in the laminar portion of the boundary layer over the porous surface have been observed. For these flow conditions, Wartemann et al. [17] have performed an analysis based on the parabolized stability equations, confirming the effectiveness of porous surfaces based on C/C in achieving transition delay. The present paper is the first step toward performing a full direct numerical simulation of a spatially developing boundary layer over a conical surface with assigned complex broadband wall-impedance representative of the distributed random porosity of C/C surfaces.

Most of previous high-fidelity numerical studies of attenuating, canceling, or reinforcing second-mode instabilities in high-speed flow

over porous walls have focused on uniformly spaced and/or geometrically regular porosity in temporally developing boundary layers [18–22]. More recently, Wang and Zhong [23] have performed direct numerical simulations of a hypersonic boundary layer over a flat plate modeling the effects of the irregular felt-metal surface used in the experiments conducted by Fedorov et al. [9]. Their studies concluded that the modeled felt-metal porous surface destabilized the first mode and attenuated the second, consistent with the experiments.

The current paper will focus on axisymmetric flow over a sharp 7 deg half-angle cone, analyzing the three lowest Reynolds numbers investigated in the experiments by Wagner et al. [12,13] (see Table 1). The novelty of the adopted computational approach lies in the time-domain impedance boundary conditions (TDIBC) technique, as implemented by Scalo et al. [24] in the context of subsonic compressible near-wall turbulence, allowing the exact application of any complex impedance boundary conditions in direct numerical simulations.

II. Physical Model and Computational Approach

The numerical study performed in this work is based on tests conducted by Wagner [13] in the DLR High Enthalpy Shock Tunnel Göttingen (HEG). In the present study, test conditions of $Re_m = 1.46 \cdot 10^6 \text{ m}^{-1}$, $Re_m = 2.43 \cdot 10^6 \text{ m}^{-1}$, and $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ are chosen; all freestream conditions and the parameters of these runs are reported in Table 1. Simulations of spatially developing boundary layer are carried out over a 0.95-m-length cone, starting at 0.05 m from the tip. A sharp cone with a half-angle of 7 deg is the geometry of choice, being an idealization of Wagner's 2.5 mm round-tip cone model actually employed for the transition delay experiments over C/C.

High-order structured compact-finite-difference simulations are carried out with the CFDSU solver (more details are given in Sec. III.A) with the objective of capturing the perturbation evolution with minimum numerical dissipation and high resolving power for a given amount of points per wavelength. Simulations are performed with inlet flow conditions to the high-order simulations starting at $x = 0.05 \text{ m}$ from the tip (Fig. 1) and informed by combining the Taylor–Maccoll [25] inviscid solution with a viscous solution for the boundary layer. The latter is derived by applying the Mangler [26] transformation for bodies of rotation to the compressible boundary-layer similarity solution for a perfect gas over a flat plate [27], as done by Lees [28] for the flow over a cone.

Using this transformation, one can describe the spatial evolution of a boundary layer through the use of the following similarity variables:

$$\xi = \int_0^x \rho_e U_e \mu_e r_\perp^2 dx \quad (1)$$

$$\eta = \frac{U_e r_\perp}{\sqrt{2\xi}} \int_0^y \rho dy \quad (2)$$

where x is the streamwise direction (tangential to the wall), y the direction normal to the wall, and $r_\perp(x)$ is the cone radius as measured perpendicularly to its axis of rotational symmetry.

Noting that, for a cone of half-angle ψ_c , the radius of cross section of a body of revolution can be written as $r_\perp = x \sin \psi_c$ and that the flow properties after a conical shock are constant along straight lines originating from the tip of the cone, it is assumed that the properties at the boundary-layer edge are independent of x , and hence the similarity variables 1 and 2 can be simplified to

Table 1 Selected flow parameters from Wagner's experiments [13]

Re_m, m^{-1}	M_∞	p_∞, Pa	T_∞, K	$\rho_\infty, \text{kg/m}^3$	$u_\infty, \text{m/s}$	p_e, Pa	T_e, K	$\rho_e, \text{kg/m}^3$	$u_e, \text{m/s}$
1.46×10^6	7.3	789	267	0.0102	2409	1944	345	0.0187	2376
2.43×10^6	7.4	1453	285	0.0177	2480	3443	368	0.0324	2446
4.06×10^6	7.4	2129	268	0.0276	2422	5076	347	0.0508	2388

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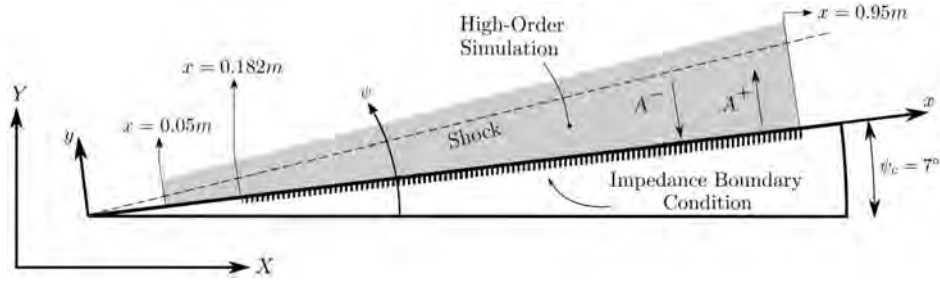


Fig. 1 Computational setup of DNS of second-mode waves over complex impedance boundary conditions.

$$\xi = \frac{\rho_e U_e \mu_e x^3 \sin^2 \psi}{3} \quad (3)$$

$$\eta = \sqrt{\frac{3U_e}{2x\rho_e\mu_e}} \int_0^y \rho dy \quad (4)$$

This allows to numerically solve the boundary-layer equations as a nonlinear ordinary differential equation (ODE) with η as the only dimensionless coordinate over which to perform the integration. In practice, the solution is generated on a very fine grid in the η space and then mapped back to the (x, y) space and interpolated bilinearly on the grid used to perform the Navier–Stokes calculations. As shown in Fig. 2, excellent matching is observed between the thus-obtained semi-analytical flow and steady Navier–Stokes calculations.

Simulations are first carried out under quiet conditions (no perturbations), allowing the mean flow to be fully established and all residual perturbations (artifacts of the flow initialization, retained by the low-dissipation numerics) to be convected out of the domain. The solution was allowed to relax on the Navier–Stokes grid for approximately four flow-through cycles, based on the freestream velocity and the domain extent, before perturbations were applied.

The grid used to discretize the conical domain is a structured spherical grid with the origin at the tip of the cone. The version of CFDSU under development at Purdue University is able to perform a trivariate coordinate system transformation, mapping a general curvilinear coordinate system onto a uniform and orthogonal spatial domain. The choice of spherical coordinates guarantees perfect orthogonality (reduced numerical errors in the grid transformation step) between gridlines and is advantageous because it can be described very trivially analytically. The disadvantage of this grid is that the resolution in the wall-normal direction (y) decreases as we move away from the origin; this issue is mitigated by the streamwise

boundary-layer growth. The spherical grid is stretched in the wall-normal (angular) direction with a half-tangent hyperbolic law to resolve not only the boundary layer of the mean flow but also the acoustic-wave-induced Stokes boundary-layer thickness.

The inflow plane of the high-order Navier–Stokes simulations is located under the shock, at a streamwise distance of $x = 0.05$ m from the tip (Fig. 1). Here, hardly imposed Dirichlet boundary conditions are used, informed from the precalculated semi-analytical steady laminar viscous solution described previously. Outlet conditions are homogeneous Neumann for all flow quantities. At the wall, no-slip, no-penetration and (when and where active) suction and blowing conditions are imposed. At the top boundary (above the shock), steady freestream conditions are imposed.

Inlet and outlet sponge layers of approximately 0.045 m in thickness are used to control spurious oscillations due to the introduction of the shock in the domain at the inlet and to prevent the formation of upstream traveling disturbances near the outlet, in both cases arising in the subsonic portion of the boundary layer. These layers relax the instantaneous flow to the aforementioned semi-analytical steady laminar solution.

A shock-capturing scheme based on implementations by Cook [29] and Kawai and Lele [30] was used to thicken the shock and make it resolvable on the computational grid. This is done by applying artificial viscosity and conductivity in the regions where the shock is detected. To ensure that no artificial dissipation is introduced inside the boundary layer, the shock-capturing scheme is deactivated in the region $\psi - \psi_c < 1$ deg.

Because of the short duration of the test run, the surface of the model does not have time to heat up and thus can be assumed to be isothermal with a temperature of 300 K. For runs modeling the porous carbon/carbon surfaces, impedance boundary conditions [see Eq. (8)] were applied, controlling the transpiration velocity as a function of the wall pressure fluctuations. The value of the impedance is extracted from a pore-scale acoustic analysis called the inverse

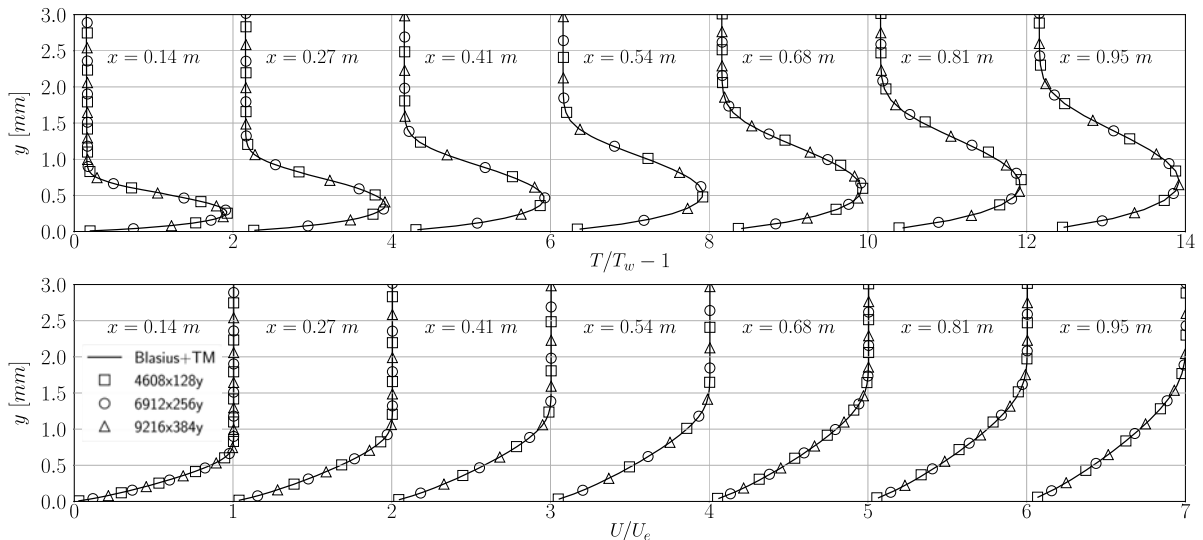


Fig. 2 Temperature and streamwise velocity profiles from the unperturbed DNS (symbols); analytical approximation combining Mangler-transformed Blasius and Taylor–Maccoll solutions (lines).

Helmholtz solver (Sec. III.B) and compared to low-order algebraic models (Sec. V.B). This approach allows to model the acoustic response of the porous surface without the need to resolve its complex geometrical structure. This strategy allows to retain high resolution on the flow side by removing the grid resolution requirements that would be needed to capture the acoustic wave propagation in the pores.

III. Computational Tools

A. High-Order Structured Compact Finite Difference Solver in Curvilinear Coordinates: CFDSU

CFDSU solves the fully compressible Navier–Stokes equations on a structured curvilinear grid using a sixth-order compact and staggered finite difference scheme [31]. Compact methods outperform conventional finite difference schemes based on spatially local discretization at high wave numbers without restricting the geometry and boundary conditions of the problem demanded by a spectral method [32,33].

The governing flow equations are solved in a curvilinear coordinate system for the contravariant components of velocity. The full set of transformed equations can be found in Nagarajan et al. [34]. Recent developments of the code at Purdue University have enabled the use of fully three-dimensional, nonorthogonal grid transformations in the code.

CFDSU has also been equipped with time-domain impedance boundary conditions (TDIBCs) based on the formalism of Fung and Yu [35] and the implementation in fully compressible Navier–Stokes codes by Scalo et al. [24]. The TDIBC formulation belongs to the class of characteristic boundary conditions [36] and is implemented based on the complex wall softness coefficient $\hat{S}(\omega)$, which is related to the reflection coefficient $\hat{R}(\omega)$ via

$$\hat{S}(\omega) = \hat{R}(\omega) + 1 \quad (5)$$

and is evaluated using the complex poles and residues that describe the IBC [see Eq. (12)] being ultimately imposed in the time domain in the Navier–Stokes calculations. Following, $\hat{R}(\omega)$ relates the incident ($\hat{A}^-$) and reflected ($\hat{A}^+$) waves (Fig. 1) in the frequency domain via

$$\hat{R} = \frac{\hat{A}^+}{\hat{A}^-} \quad (6)$$

where the time-domain definition of the incident and reflected waves is

$$\begin{cases} A^-(t) = v'(t) + \frac{p'(t)}{\rho_0 a_0} \\ A^+(t) = v'(t) - \frac{p'(t)}{\rho_0 a_0} \end{cases} \quad \text{and} \quad \begin{cases} \hat{A}^-(\omega) = \hat{v}(\omega) + \frac{\hat{p}(\omega)}{\rho_0 a_0} \\ \hat{A}^+(\omega) = \hat{v}(\omega) - \frac{\hat{p}(\omega)}{\rho_0 a_0} \end{cases} \quad (7)$$

where v' and p' are the fluctuating values of wall-normal velocity and pressure, respectively, and ρ_0 and a_0 are the base density and speed of sound of the fluid.

Adopting a harmonic convention to transform the variables to frequency domain, the specific acoustic impedance $Z_*(\omega)$ (made dimensionless via the base impedance $\rho_0 a_0$) is defined as a function

of the perturbed pressure $p'(t)$ and its induced normal velocity component $v'(t)$ into a porous surface, satisfying the relation

$$\hat{p}(\omega) = \rho_0 a_0 Z_*(\omega) \hat{v}(\omega) \quad (8)$$

Fung and Yu [35] demonstrated that it is possible to evaluate the outgoing wave by a causal convolution of the incident wave. The wall-normal velocity can then be recovered and imposed as a Dirichlet boundary condition at each time step:

$$v' = \frac{1}{2}[A^- + A^+] \quad (9)$$

The staggered variable arrangement does not place pressure nodes at the boundary and, as such, only requires condition 9 to be imposed to continue the calculations.

B. Pore-Cavity Inverse Ultrasonic Solver: Inverse Helmholtz Solver

The inverse Helmholtz solver (IHS) is a novel computational methodology that allows the evaluation of the spatial distribution of acoustic impedance at the open surface of an arbitrarily shaped cavity for a given frequency. This technique spatially integrates the linearized Navier–Stokes equations transformed into the frequency domain

$$j\omega \frac{\rho_0}{p_0} \hat{p} - j\omega \frac{\rho_0}{T_0} \hat{T} + \rho_0 \frac{\partial \hat{u}_k}{\partial x_k} = 0 \quad (10a)$$

$$j\omega \rho_0 \hat{u} + \frac{\partial \hat{p}}{\partial x} - \mu \frac{\partial}{\partial x_k} \left(\frac{\partial}{\partial x_k} \hat{u} \right) = 0 \quad (10b)$$

$$j\omega \rho_0 \hat{v} + \frac{\partial \hat{p}}{\partial y} - \mu \frac{\partial}{\partial x_k} \left(\frac{\partial}{\partial x_k} \hat{v} \right) = 0 \quad (10c)$$

$$j\omega \rho_0 c_v \hat{T} + p_0 \frac{\partial \hat{u}_k}{\partial x_k} = \kappa \frac{\partial}{\partial x_k} \left(\frac{\partial}{\partial x_k} \hat{T} \right) \quad (10d)$$

on an unstructured grid for a broad range of frequency values (where ω is a real-valued input to the solver), from the cavity walls (with assigned no-slip, isothermal conditions) up to the open surface, where the local value of impedance for each frequency is retrieved as a result of the calculation (see Patel et al. [37,38] and Wagner et al. [39] for more details).

Multiple instances of the IHS can be concurrently executed to reconstruct the full, broadband acoustic impedance at the open surface of any given geometry. Such an impedance can then be implemented as a time-domain impedance boundary condition (IBC) in flow-side-only simulations as discussed in Sec. III.A).

To apply this technique to a material composed of arbitrarily distributed geometrically complex cavities, such as a C/C block, high-resolution images are needed. The descriptive images, such as the ones shown in Fig. 3, are read as two-dimensional arrays containing grayscale values of each pixel using an image processing code written in Python. A filter is then applied wherein each pixel with a grayscale value under a given threshold is considered to be part of a pore. The images are then scanned for connected regions that



Fig. 3 Reflected-light microscopy generated images of classic (untreated) C/C (left), optimized C/C (middle), and optimized C/C-SiC (right) [50].

represent each pore, and an automatic approximation to a rectangular slot or a cylindrical hole is made based on the pore's extent in each direction. Finally, pixels are converted to micrometers and used to recover the significant dimensions of each pore, which can be used to determine the porosity and eventually the impedance of the C/C surface.

Once the surface porosity and significant dimensions of each surface pore are known, the inverse Helmholtz solver (IHS) [38] is used to evaluate the broadband surface-averaged specific acoustic impedance of each pore. These impedances are then combined, assuming that the hard walled portion of the surface has zero admittance, such that the total surface-averaged impedance of the sample is given by

$$Z_*(\omega) = \left(\sum_{i=1}^{N_{\text{pores}}} \frac{1}{Z_{i,*}(\omega)} \frac{A_i}{A_{\text{total}}} \right)^{-1} \quad (11)$$

where Z_i is the specific acoustic impedance of the i th pore, and A_i its surface area.

Time-domain impedance boundary condition (TDIBC) application requires that the acoustic impedance be specified as a set of complex poles and residues representing a superposition of causal second-order oscillators. These poles and residues are obtained by fitting the wall softness coefficient corresponding to the evaluated broadband acoustic impedance from the IHS:

$$\hat{S}(\omega) = \frac{2}{1 + Z_*(\omega)} \approx \sum_{k=1}^{n_o} \left[\frac{\mu_k}{s - p_k} + \frac{\tilde{\mu}_k}{s - \tilde{p}_k} \right] \quad (12)$$

where $\hat{S}$ is the wall softness coefficient, n_o is the number of oscillators, μ_k is the residues, p_k is the poles, and $s = j\omega$. In Eq. (12), the superscript $\sim$ denotes the complex conjugate.

The fit is performed following the procedure described in Lin et al. [40], where the values and quantity of poles and residues are varied iteratively while minimizing the difference between the wall softness evaluated using the impedance obtained from the IHS for a discrete set of frequencies and the one evaluated using the poles and residues obtained in the previous iteration.

IV. Analytical Models of Surface Impedance for Porous Materials

A. Homogeneous Absorber Theory

The homogeneous absorber theory (HAT) [41] models the normal impedance of a porous absorber as

$$Z_{\text{HAT}} = Z_\infty \frac{1 + e^{-j2k_a H}}{1 - e^{-j2k_a H}} \quad (13)$$

which corresponds to the effective impedance of the absorber when the wave propagates normal to its surface (see discussion in Sec. IV. C). The other parameters in Eq. (13) are

$$k_a = k\sqrt{\kappa} \sqrt{1 - j\frac{\omega_k}{\omega}}, \quad Z_\infty = Z_0 \frac{\sqrt{\kappa}}{\phi} \sqrt{1 - j\frac{\omega_k}{\omega}}$$

and ω_k , the breaking frequency, is given by

$$\omega_k = \frac{\Xi\phi}{\rho_0\kappa} \quad (14)$$

where κ is the structure factor, Ξ is the flow specific resistivity, ϕ is the volume porosity, k and $\omega = 2\pi f$ are the wave number and angular frequency of incident wave, H is the total depth of the absorber, and $Z_0 = \rho_0 a_0$ is the base impedance. The breaking frequency of the absorber can be interpreted as the (approximate) demarcation between two distinct behaviors of wave propagation inside the porous media; for $\omega \ll \omega_k$, the propagation is dispersive, and for $\omega \gg \omega_k$, there is no frequency dependency on the propagation speed of the waves.

B. Fedorov's Model for Porous Absorber Impedance

Fedorov et al. [42] used the following expression to estimate the impedance due to wave propagation through a regular porous media composed of cylindrical holes with axes oriented normal to the surface (and aligned with the wave propagation direction):

$$Z_{\text{fedorov}} = Z_0 \left(\frac{\phi}{Z_{\infty,*}} \tanh(\Lambda H) \right)^{-1} \quad (15)$$

where Λ is the propagation constant, whose dimensions are the inverse of a length. Fedorov et al. [8] extended the model to porous media with random porous structure through the use of the nondimensional dynamic density ρ_{dyn}^* and the nondimensional dynamic compressibility C_{dyn}^* based on previous theoretical work performed by Johnson et al. [43] and Allard and Champoux [44]. In his 2003 publication [8], Fedorov et al. nondimensionalized the resulting expression for the impedance of the metal felts in a manner consistent with his LST framework. Because we are interested in comparing his model with the current IHS and HAT predictions, we recast some of equations and parameters of his model in the following form:

$$\begin{aligned} Z_{\infty,*}(\omega) &= \sqrt{\rho_{\text{dyn}}^*(\omega)/C_{\text{dyn}}^*(\omega)}, \quad \Lambda(\omega) = j\omega \sqrt{\rho_{\text{dyn}}(\omega)/K_{\text{dyn}}(\omega)}, \\ \rho_{\text{dyn}}^*(\omega) &= \kappa \left[1 + \frac{g^*(\lambda_1^*)}{\lambda_1^*} \right], \quad C_{\text{dyn}}^*(\omega) = \gamma - \frac{\gamma - 1}{1 + g^*(\lambda_2^*)/\lambda_2^*}, \\ \rho_{\text{dyn}}^* &= \rho_{\text{dyn}}(\omega)/\rho_w, \quad C_{\text{dyn}}^* = \gamma P_w/K_{\text{dyn}}(\omega), \\ g^*(\lambda) &= \sqrt{1 + \frac{4\kappa\mu\lambda^*}{\Xi\phi r_p^2}}, \quad \lambda_1^* = \frac{j\kappa\rho_w\omega}{\phi\Xi}, \\ \lambda_2^* &= 4Pr\lambda_1^*, \quad \text{and} \quad r_p = s_h \sqrt{\frac{8\mu\kappa}{\Xi\phi}} \end{aligned}$$

where the symbol $*$, being a superscript or a subscript, refers to nondimensional quantities (and not to the complex conjugate). All of the parameters present in these equations variables have been previously defined except for $K_{\text{dyn}}(\omega)$, which is the dynamic bulk modulus relating the divergence of the average molecular displacement of the gas to the average pressure variation; r_p , which is the characteristic size of the pores; and s_h , which is a shape factor that accounts for the anisotropy of the pores. As a way of estimating the shape factor for the current C/C samples, and consistently with Fedorov et al. [8], we treat r_p as a hydraulic radius (i.e., the ratio of the doubled pore area to the pore perimeter at the surface). With this assumption, we use Fig. 3 to visually estimate the shape factor for the classical C/C sample obtaining $s_h \approx 1.26$. The Prandtl number Pr is taken to be 0.704.

In Fedorov et al. [8], the propagation constant multiplied by the depth ΛH (made dimensional here) was expressed as proportional to $j\omega H \sqrt{\rho_{\text{dyn}}(\omega)K_{\text{dyn}}(\omega)}$, whereas in Allard and Champoux [44], it was expressed as $\Lambda H = j\omega H \sqrt{\rho_{\text{dyn}}(\omega)/K_{\text{dyn}}(\omega)}$. The latter is the expression adopted herein because we believe that what was reported in Fedorov et al. [8] is a mere typographical error. For porous absorbers with high flow resistivity Ξ , coincidentally both expressions yield similar overall impedances due to the asymptotic nature of the hyperbolic tangent function.

C. Wave Angle Effects on Acoustic Absorption

The wave angle is an important factor significantly affecting the overall acoustic performance of the absorber. This is typically taken into account, with the assumption of a locally reacting surface, as a correction to the value of the normal impedance, yielding the effective impedance

$$Z_{\text{eff}}(\omega) = Z(\omega) \cos(\theta_i) \quad (16)$$

ultimately used in linear acoustic relations to predict the absorption coefficient, given by

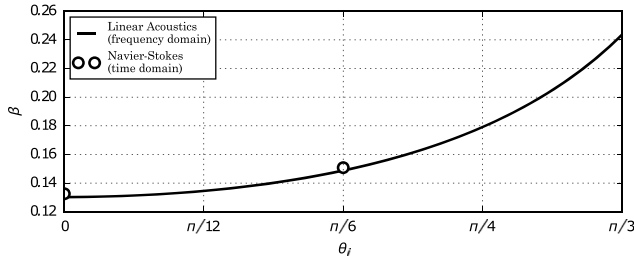


Fig. 4 Influence of incidence angle θ_i in the absorption coefficient β for $f = 300$ kHz, $p_0 = 10$ kPa, and $T_0 = 300$ K.

$$\beta(\omega) = 1 - \left(\frac{|\hat{A}^+|}{|\hat{A}^-|} \right)^2 = 1 - \left| \frac{1 - Z_{\text{eff},*}(\omega)}{1 + Z_{\text{eff},*}(\omega)} \right|^2 \quad (17)$$

which is evaluated by comparing the amplitude of the incident wave $|\hat{A}^-|$ to the reflected wave $|\hat{A}^+|$, representing a figure of merit of the absorber.

The cosine term in Eq. (16) accounts for the wave incidence angle θ_i , intended as zero when the direction of wave propagation is normal to the surface of the absorber; $Z(\omega)$ is hence intended as the normal impedance and is the only one that should be imposed in Navier–Stokes calculations with time-domain impedance boundary conditions (TDIBC). To verify this, we have performed two-dimensional Navier–Stokes calculations, mimicking Wagner et al.’s bench-test experiments [39], where a quasi-planar acoustic wave was introduced through a source term controlling the main propagation direction angle. Although the experimental setup was constructed to measure waves with angle of incidence of $\theta_i = 30$ deg, we also performed a simulation of a wave normal reflection.

Figure 4 shows a comparison between the absorption coefficient predicted via the cosine correction [see Eq. (17)], that is using the effective value of impedance [Eq. (16)] and the Navier–Stokes calculations, showing very good agreement. It is important to stress that the IBCs imposed in the Navier–Stokes calculations 1) only affect the inviscid wall-normal flux, and 2) only impose the normal impedance, i.e., values of impedance as predicted by Eq. (13) or Eq. (15) and not the effective value, that is Eq. (16).

V. Results

The goal of this section is to reproduce and analyze the disturbance amplification mechanism in a hypersonic flow over a sharp cone with impermeable smooth walls through the excitation of the boundary layer with a broadband frequency pulse. Then, we model the acoustic response of real C/C materials through the IHS technique (Sec. III.B), compare it against impedance estimates from low-order acoustic models as well as bench-test data from the ultrasonic measurements by Wagner et al. [39], and ultimately test its attenuation capability by assigning the predicted impedance as a boundary condition to Navier–Stokes calculations, which allows a comparison with the tests conducted in HEG with a C/C porous insert.

An approximate steady laminar solution obtained by blending of the compressible Blasius boundary-layer solution applied to a supersonic sharp cone [26,28] with the inviscid Taylor–Maccoll [25] flow is introduced in the Navier–Stokes calculation both as an inlet and initial condition. After initialization, the inputted solution

needs to adapt to the discretized full Navier–Stokes equations. The boundary-layer profiles at various locations after this (merely numerical) transient adjustment are shown in Fig. 4 for $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$; excellent matching between the approximate solution and the steady Navier–Stokes calculations is observed. The same degree of agreement was also achieved for the lower-Reynolds-numbers cases (not shown). As shown in Table 2, the highest Reynolds number is the most stringent case in terms of grid requirements. Given the long streamwise extent of the computational domain, the accuracy of the boundary-layer profiles toward the end of the cone is very sensitive to the degree of numerical dissipation. In fact, preliminary runs with more dissipative settings of numerical filtering have shown to overpredict the boundary-layer thickness for large values of x .

A. Broadband Pulse Disturbance Introduction and Amplification over Smooth/Impermeable Wall

After the establishment of an unperturbed steady base state, controlled velocity disturbances with an amplitude of 1 m/s are imposed via suction and blowing at the wall for a finite amount of time ($1.67 \mu\text{s}$) and in a finite-length interval ($x = 0.14 \pm 0.0036$ m) following the expression (in meters per second):

$$v(x, y = 0, t) = \cos(\pi\xi)^3 \sin(2\pi ft), \quad \text{for } 0 \leq t \leq 1/f \quad (18)$$

where ξ is a variable with values ranging in $[-1, 1]$, which is mapped to the actual spatial interval of application of the pulse, and $f = 600$ kHz is the frequency around the which the pulse spectrum is centered. This perturbation aims at mimicking a natural transition scenario as well as helping identifying the most-amplified frequency inside the boundary layer as a function of the streamwise position on the surface of the cone. It was first used by Gaster and Grant [45] in incompressible boundary-layer transition simulations and by Sivasubramanian and Fasel [46] in hypersonics.

Figure 5 shows the pressure time series at the wall for different streamwise locations. A rapid amplification of the overall pressure signal is observed initially, from $x = 0.15$ m up to approximately $x = 0.25$ m, followed by a later gentler growth. However, the total disturbance amplitude contains energy at various frequencies, and although the pulse excitation inputs a similar amount of energy in all the frequencies range relevant to the boundary-layer instability, some of the components experience a long region of decay before reaching their unstable region (Fig. 6). In the end, the overall amplitude of the signal depends on the balance between the attenuation in the stable region and the overall integrated growth in the unstable region of each frequency component. In fact, consistent with amplification dynamics of instability waves in (canonical) hypersonic boundary layers, higher-frequency modes experience growth earlier (i.e., where the boundary layer is thinner and, simultaneously, the remaining frequency components are attenuated).

The early rapid overall pressure signal amplitude increase is hence due to the high-frequency component, in the range of 800 kHz for $Re_m = 4.0 \times 10^6$, of the inputted broadband disturbance pulse. After this initial phase, a rapid decay is also observed due to the fast evolution of the boundary-layer thickness in its early stages, which changes the quickly unstable frequency band. At last, the later gentler growth shown in Fig. 5 happens because, as the signal moves downstream, the boundary-layer thickness evolution becomes

Table 2 Grid resolution metrics for the three Reynolds numbers chosen in this paper

Re_m, m^{-1}	1.46×10^6			2.43×10^6			4.06×10^6		
n_x	4608	6912	9216	4608	6912	9216	4608	6912	9216
n_y	128	256	384	128	256	384	128	256	384
$(\Delta x/\delta)_{0.14m}$	0.1560	0.1080	0.0809	0.2354	0.1637	0.1225	0.2746	0.1814	0.1383
$(\Delta x/\delta)_{0.95m}$	0.0591	0.0403	0.0304	0.0922	0.0605	0.0451	0.1006	0.0689	0.0506
$(\Delta y_{\min}/\delta)_{0.14m}$	0.0137	0.0070	0.0046	0.0207	0.0106	0.0070	0.0242	0.0117	0.0079
$(\Delta y_{\min}/\delta)_{0.95m}$	0.0363	0.0183	0.0121	0.0566	0.0273	0.0180	0.0618	0.0311	0.0202

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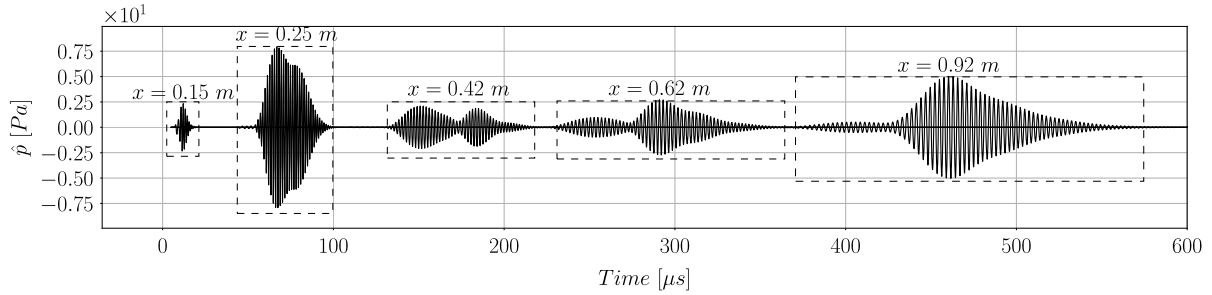


Fig. 5 Time series of the pressure perturbation at different streamwise locations for $Re_m = 4.06 \times 10^6 \text{ m}^{-1}$ with a $n_x = 9216$ and $n_y = 384$.

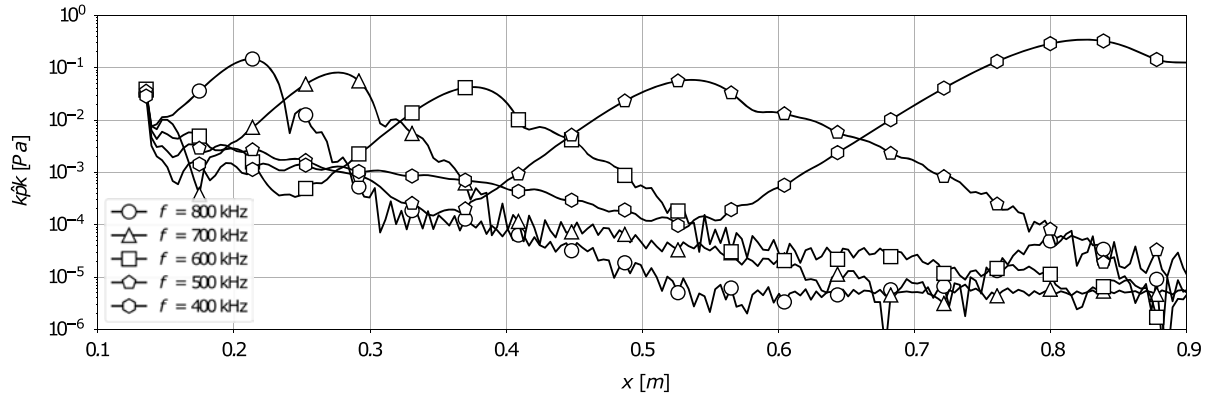


Fig. 6 Amplitude of pressure disturbance at the wall for different frequency components with $n_x = 6912$, $n_y = 256$, and $Re_m = 4.0 \times 10^6 \text{ m}^{-1}$.

slower, and the overall integrated growth becomes steadily bigger than the initial attenuation.

A Fourier analysis of such times series is shown in Fig. 7, demonstrating that, for all Reynolds numbers considered, the initial broadband characteristic of the signal is very rapidly lost, and the power spectrum concentrates in a narrow band of ultrasonic frequencies, consistent with second-mode amplification mechanisms for each flow conditions considered. Experimental studies conducted by Stetson [47] confirmed the validity of the following estimation for the second-mode frequency:

$$f = \frac{U_e}{2\delta} \quad (19)$$

where U_e is the boundary-layer edge velocity, and δ is the boundary-layer thickness, defined as the wall-normal distance where $u \approx 0.999U_e$. Predictions based on Eq. (19) are plotted in Fig. 7, showing good agreement with the DNS results. A detailed phase speed analysis reveals that both slow and fast modes [48] are observed in the current DNS; however, most of the disturbance energy and growth are carried by the slow mode (not shown),

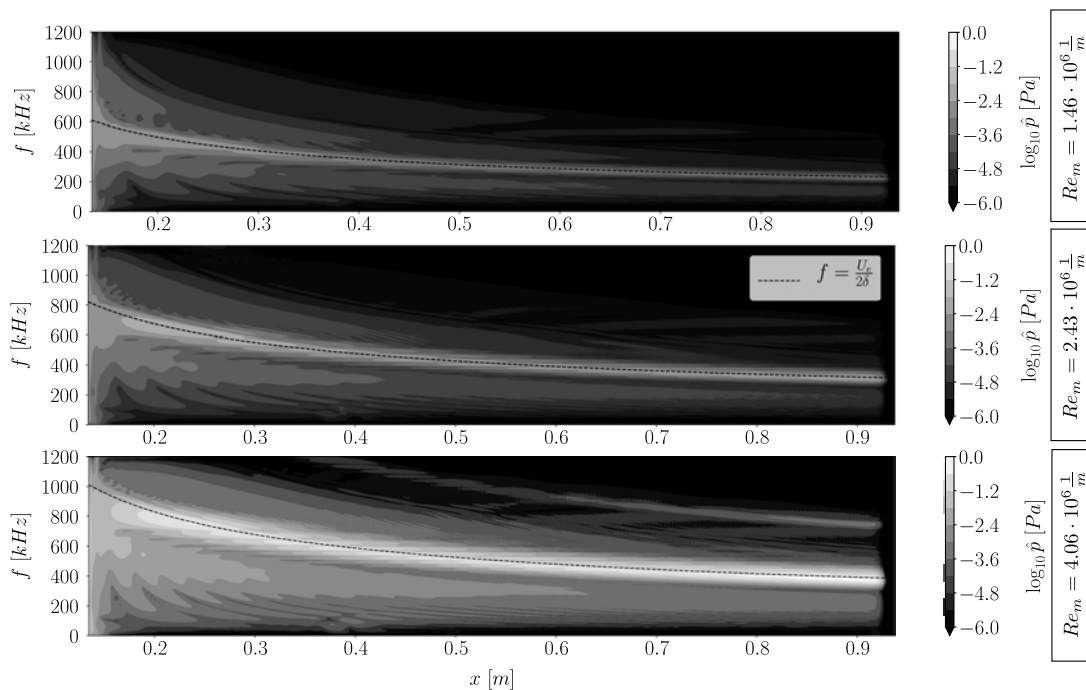


Fig. 7 Contours of Fourier-mode amplitude as a function of frequency and space for the three Reynolds numbers considered and $n_x = 6912$ and $n_y = 256$.

corresponding to Mack's second mode after the phase speed synchronization point (see discussion on terminology of instability modes in Egorov et al. [21]).

The broadband pulse excitation is able to capture the correlation between the most-amplified frequencies in the boundary layer and its thickness revealing information similar to stability analysis. It is observed that the second-mode frequency follows closely the boundary-layer thickness; the higher the Reynolds number is, the thinner the boundary layer is and the higher the excited frequencies are. Figure 8 shows pressure isosurfaces near the surface of the cone, reminiscent of acoustically channeled waves with some power

transmitted (or lost) outward through the sonic line, characteristic of the second mode (Egorov et al. [21]).

Another phenomenon that occurs as the imposed pulse advects downstream is that its magnitude becomes large enough to excite higher harmonics. For the high Re_m case, specifically, the appearance of the first harmonic of the second mode is visible after $x = 0.7$ m, shown in Fig. 9. At the very end of the computational domain, a sponge layer, starting around $x = 0.94$ m, damps the imposed disturbances before they reach the outflow boundary condition. No sign of spurious upstream disturbances has been identified in the subsonic part of the boundary layer near the outflow.

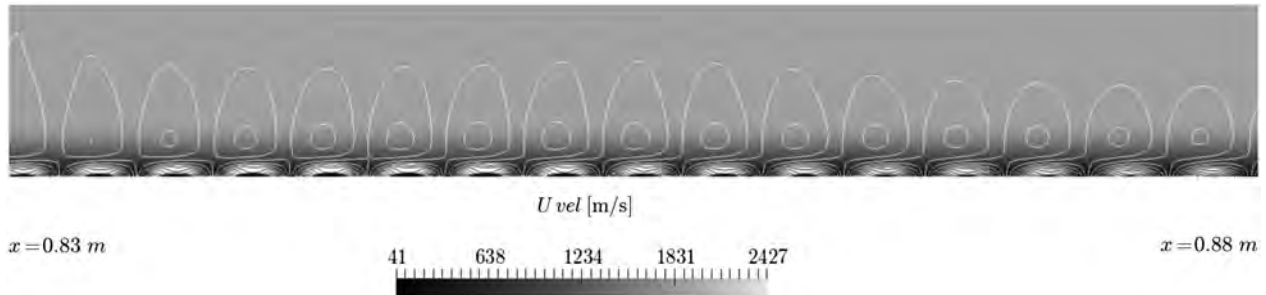


Fig. 8 Pressure oscillation isosurfaces in the boundary-layer region for $n_x = 4608$, $n_y = 128$, and $Re_m = 4.0 \times 10^6 \text{ m}^{-1}$.

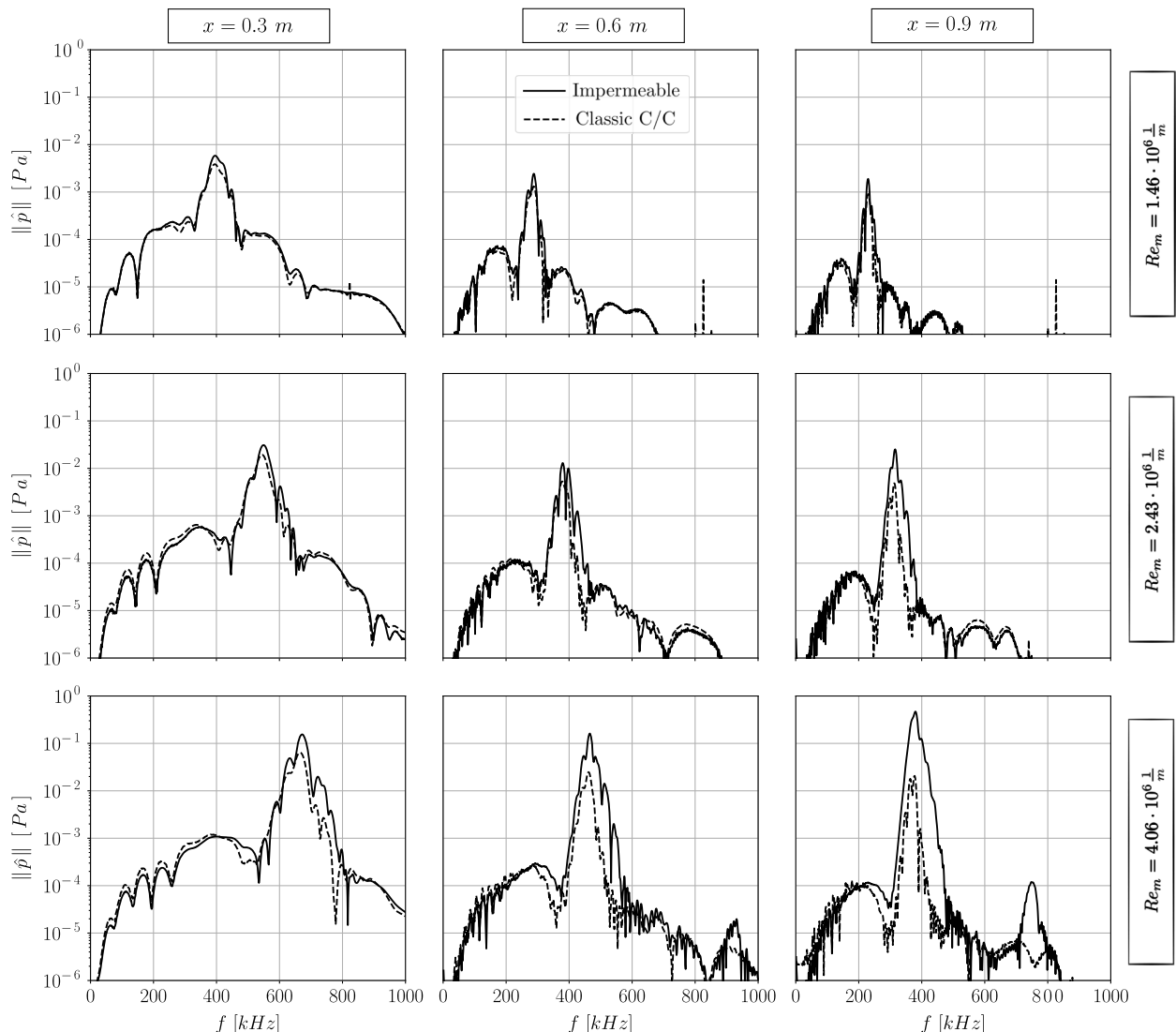


Fig. 9 Frequency domain distribution of the power spectrum at the wall for different streamwise locations in the advection of broadband pulse simulations over porous and impermeable walls.

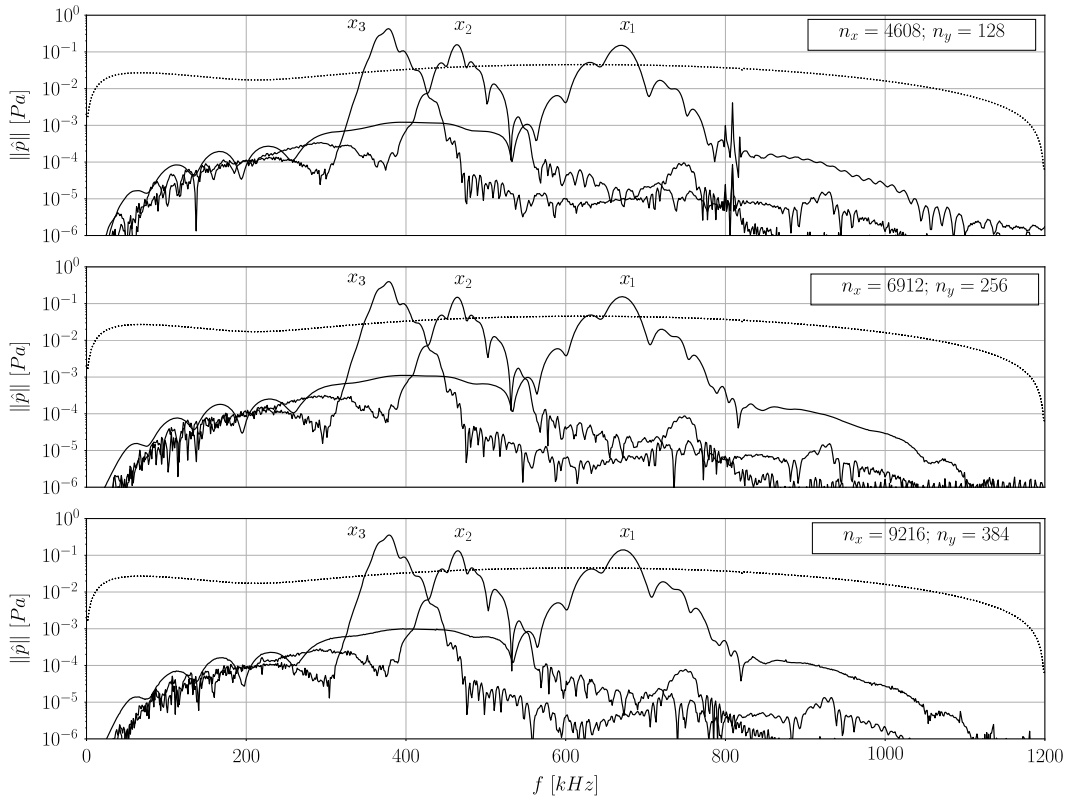


Fig. 10 Grid convergence analysis for the perturbation spectrum at $x_1 = 0.3$ m, $x_2 = 0.6$ m, and $x_3 = 0.9$ m in an axisymmetric simulation of broadband pulse propagation at $Re_m = 4.06 \times 10^6 \text{ m}^{-1}$.

The spatial resolution and the numerical discretization strategy play a crucial role in the quality of the prediction of the evolution dynamics of high-frequency perturbations, such as second-mode waves. The grid convergence study in Fig. 10 demonstrates adequate spectral resolution on the intermediate grid adopted ($n_x = 6912$, $n_y = 256$); results on the coarser grid ($n_x = 4608$, $n_y = 128$) are also adequate, with the exception of spurious energy accumulation at $f = 800$ kHz. The latter is actually due to a numerical disturbance originating from the interaction of the inflow sponge layer and the coarse grid as well as insufficient resolution of the propagating waves. This spurious wave source propagates into the domain and is not damped due to the low artificial diffusion of the scheme. Although the dynamics of the disturbance evolution inside the conical hypersonic boundary layer are not altered by this spurious perturbation, only intermediate or high resolutions ($n_x \geq 6912$, $n_y \geq 256$) of such full cone axisymmetric computations will be considered.

The finest grid resolution available ($n_x = 9216$, $n_y = 384$) is capable of adequately resolving wavelengths of 0.8 mm, assuming a minimum requirement of eight points per wavelength for compact finite difference schemes. With the boundary-layer thickness toward the end of the cone of the order of 2 mm, this resolution is not adequate to capture the nonlinear breakdown to turbulence. The effects of porous walls on the latter will be investigated in future computations focused on (inevitably) shorter sections of the cone.

B. Impedance Characterization of Carbon-Fiber-Reinforced Carbon Ceramics (C/C) Materials

Carbon/carbon silicon carbide (C/C-SiC) is a material that has already been used as a thermal protection system (TPS) in hypersonic flight [15,16]. C/C represents an intermediate state of its manufacturing process. The porosity of C/C materials is a result of thermal stresses that appear in the cooldown process of the material after pyrolyzation of the matrix. This material offers excellent thermal resistance, low expansion, and specific weight as well as high temperature stability in nonoxidizing atmospheres. However, C/C cannot be used when the oxidizing effects are important in the flow without a protective

treatment. Resistance against oxidation is acquired by the infiltration of a liquid phase of silicon into the porous carbon, followed by a reaction to SiC. If this infiltration is done in the untreated C/C, its microstructural gaps will be filled and the acoustic absorption properties of the material lost.

To solve this problem, DLR Stuttgart developed a technique that is able to selectively insert cavities into the C/C structure by replacing carbon fibers with a nonstable material that will degrade after the pyrolyzation step in the desired locations, which allowed to nearly double the porosity [49]. The existence of larger gaps in this optimized C/C (Fig. 3) allows it to retain a certain porosity degree after the SiC reaction. This results in the optimized C/C-SiC, which combines the properties of oxidation resistance with acoustic absorption [50].

The complex acoustic impedance of the classic C/C sample shown in Fig. 3 was estimated via various methodologies with results shown in Fig. 11. The acoustic analysis of the optimized C/C and C/C-SiC are deferred to future work.

Experimentally determined and theoretically predicted values of β Eq. (17) are plotted in Fig. 12. The experimentally determined absorption coefficient by Wagner et al. [39] was obtained by comparing the amplitude of a wave reflected off an acoustically absorptive surface to the same wave reflecting off a purely reflective (impermeable) boundary. This absorption process is a function not only of the porous absorber but also of the flow properties themselves; for example, it will depend on the angle of incidence, which will change the effective impedance $Z_{\text{eff},*}(\omega)$, which should be used to evaluate β . To enable comparison between the experiments performed by Wagner et al. [39], all the results presented in Figs. 11 and 12 consider a angle of incidence of $\theta_i = 30^\circ$.

The predictions of acoustic impedance evaluated using the inverse Helmholtz solver (IHS) are compared with experimental results of bench-test acoustic characterization of ultrasonically absorptive porous samples [39] and two different theoretical models: 1) the homogeneous absorber theory (HAT) detailed in Möser [41] and used by Wagner [13], and 2) the model for acoustic impedance of a porous surface with random porosity used by Fedorov et al. [8] in his LST

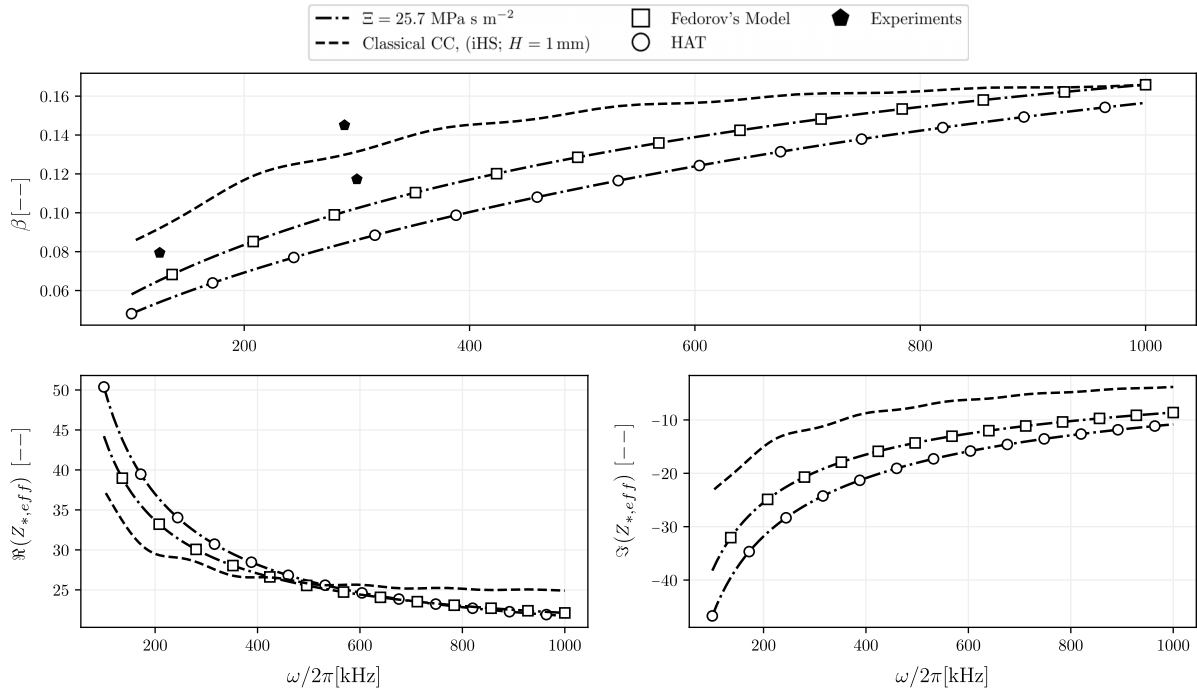


Fig. 11 Comparison of real $\Re(Z_{*,\text{eff}})$ and imaginary $\Im(Z_{*,\text{eff}})$ parts of the specific impedance $Z_{*,\text{eff}}(\omega)$ and absorption coefficient β obtained from various models at $\theta_i = 30^\circ$ deg and wall conditions corresponding to $Re_m = 4.06 \times 10^6 \text{ m}^{-1}$.

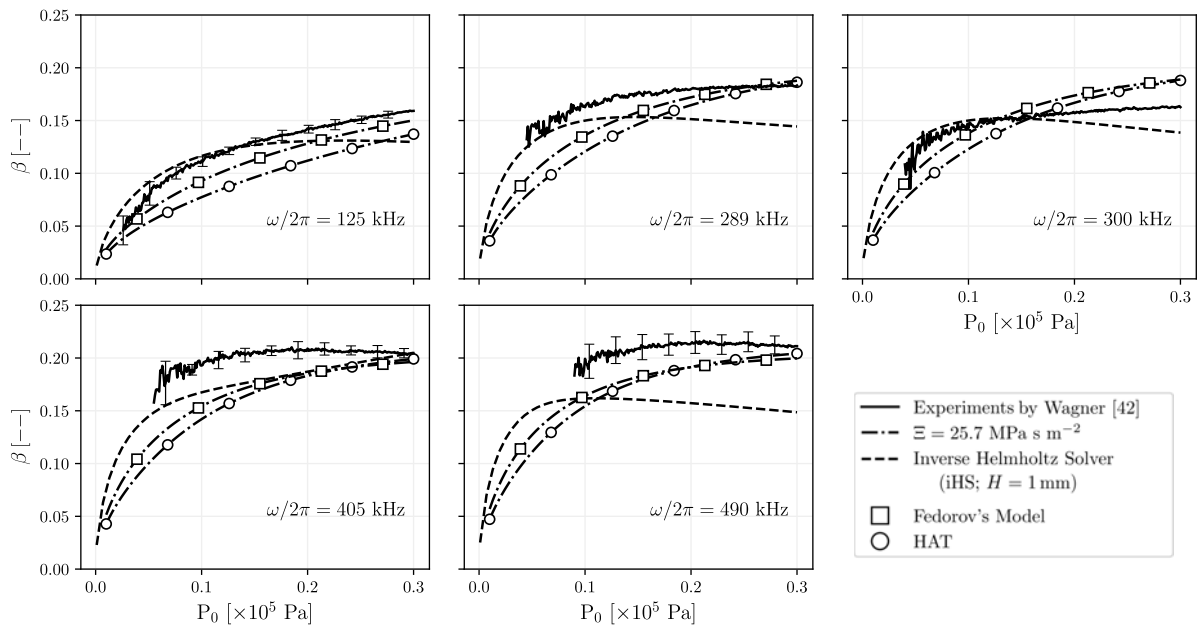


Fig. 12 Comparison of measured and predicted absorption coefficient β for the classical C/C vs base pressure.

calculations of second-mode attenuation over a felt metal coatings. Both these models have been briefly summarized in Sec. IV.

The use of these models relies on the determination of physical parameters that describe some characteristics of the flow through the porous medium such as: the porosity ϕ , which is the ratio of the pore volume to the total volume; the flow resistivity Ξ , which represents the resistance (pressure difference) to the flow of air through the medium; and the structure factor κ , which accounts for the presence of some “blind” and tortuous paths in the absorber. Wagner [13] in 2014 reported for the first time the values that characterize the classical C/C material, which are summarized in Table 3. Although ϕ and Ξ can be experimentally obtained [51], κ cannot be measured or calculated directly for randomly structured porous materials, and it was inferred from the experimentally measured reflection coefficient. In 2018 DLR has remeasured the flow resistivity Ξ of the classical C/C

sample, reporting a value almost double what was previously published. The reason for the difference in the measured Ξ was attributed to the better sealing of the samples (the edge of the samples was galvanized with copper to avoid leakage) in the more recent experiments. Results from these new measurements are also reported in Table 3 and will be the one used in the present analysis to inform the choice of parameters in the HAT and Fedorov’s model.

Table 3 Properties of the porous absorber investigated

Experiment	ϕ	κ	Ξ , MPa $\cdot$ s $\cdot$ m $^{-2}$	Depth H , m
Wagner (2014) [13]	0.15	8.0	13.3	5.0×10^{-3}
DLR (2018)	0.15	8.0	25.7	5.0×10^{-3}

The acoustic characterization experiments by Wagner et al. [39] provide wave absorption data for a discrete set of frequencies because a different transducer receiver pair is needed for testing different frequencies. However, to execute the implementation of TDIBC, a continuous reconstruction of the complex impedance in the frequency domain is required. Wagner's data are hereby compared against various acoustic absorption models to assess their predictive capability both in the form of complex impedance versus frequency at one pressure corresponding to flow conditions of $Re_m = 4.06 \times 10^6 \text{ m}^{-1}$ (Fig. 11) and in the form of absorption coefficient β at various frequencies versus pressure (Fig. 12).

For the predictions carried out numerically by the IHS methodology, a blind-hole porosity assumption was used. A surface (and hence also volume) porosity of 0.078 is used, as obtained from the processing of the high-resolution C/C surface images. A nominal depth of $H = 1 \text{ mm}$, common to all pores, has been assumed based on visual inspection of depthwise slices of a C/C sample provided by DLR Stuttgart [38].

For the chosen value of flow resistivity $\Xi = 25.7 \text{ MPa} \cdot \text{s} \cdot \text{m}^{-2}$, both the HAT and Fedorov's model underestimate the absorption coefficient at low base pressures and relative to the IHS predictions. As the pressure increases, the absorption coefficient prediction based on the IHS starts to deviate from the experimental data, and both low-order models provide better (but still not satisfying) predictions. An overall underprediction of the magnitude of the absorption coefficient is observed by all models.

The discrepancy of IHS results is attributed to the underestimated volume porosity given that only a limited selection of the largest visible pores in Fig. 3 (left) were used for the direct impedance estimate. This leads us to conclude that the smaller pores, which were ignored in the selection surface porosity estimate step (Sec. III.B), play an important role in the absorption. The latter contribution is, on the other hand, captured by low-order models for porous acoustic absorbers, especially at higher pressures. In the end, because the experiments performed in the HEG at DLR go up to a maximum pressure of $0.1 \times 10^5 \text{ Pa}$, the IHS is a better predictive tool than the theoretical models limited to the range of pressures reproduced by the DNS results in the present manuscript.

It is also worth noting that, although the acoustic impedance estimate by the use of the theoretical models presented by Fedorov and the HAT explicitly accounts for the influence of the thickness of the absorber, the high magnitude of flow resistivity Ξ of this material, in the order of $10 \text{ MPa} \cdot \text{s} \cdot \text{m}^{-2}$, results in unchanged absorption characteristics for $H \geq 1 \text{ mm}$ (i.e., the absolute thickness of the absorber for such high values of Ξ becomes irrelevant for thicknesses larger than 1 mm). This also implies that the absorbed wave is evanescent with respect to the depth of the absorber (infinitely thick absorber model), and wave reflection from the impermeable backing of the absorber is not important. Another observation is that the use of a lower flow resistivity Ξ value, such as the (now obsolete) value measured by Wagner [13] in 2014, would lead to a higher absorption coefficient and hence an unexpected better agreement with the measured absorption coefficient than the more recently obtained value.

Figure 11 compares complex impedance prediction of various models versus frequency at one given base pressure $p_0 = 5076 \text{ Pa}$. Because of the difficulty of measuring the propagation and reflection of an ultrasonic wave at low pressures and because of the need for a separate transducer/receiver pair for each frequency, only a few experimental data points could be included in these plots. These points are not sufficient to make a statement on which is the best way of predicting the complex impedance trend versus frequency, but they reiterate the previous observations that the IHS is more accurate in this low-pressure regime than the other low-order models and that a lower flow resistivity Ξ value would lead to a better agreement of the low-order acoustic models with the experimental data.

A word of caution regarding the adoption of the various models analyzed herein is that none of them take into account impedance changes due to grazing flow effects; the latter, however, are expected to be negligible due to the small size of the pores. Minimal-unit pore-resolved simulations are currently ongoing to assess this effect. It is also important to note that the evaluated specific impedance Z_* , despite being dimensionless, is a function of the thermodynamic base state. The impedance estimates presented here were obtained for the pressure conditions at the surface of the cone model (i.e., the conditions after the conical shock, at the wall as predicted by the inviscid Taylor–Maccoll solution) and depend on the Reynolds numbers as seen in Table 1 (see the p_e value, which corresponds to the pressure at the wall, and hence within the C/C pores). In conclusion, the effective impedance of the C/C surface is a function of the thermodynamic state of the gas that fills the C/C pore space and is hence directly relatable to the external flow conditions.

C. Porous Walls and Second-Mode Attenuation

Once we have modeled the acoustic energy absorption at the wall for the different C/C samples, we are able to input this information in the DNS as a complex impedance boundary condition. Only the impedance estimates from the IHS methodology are taken into considerations for the current sets of runs. Simulations with IBCs are run quiescent first, and then, after spurious transient waves due to initialization errors are convected out of the domain, a broadband pulse is applied via suction and blowing at the wall [Eq. (18)], with results shown in Fig. 13.

The effects of frequency and Reynolds numbers on the evolution of the Fourier mode of pressure are analyzed. It is noted that, for all cases considered, the disturbance energy for the impermeable wall case and for the porous case start at the same level and keep the same amplitude relative to each other before the impedance strip starts (denoted by the vertical dotted line). Once the pulse is advecting over the impedance boundary, acoustic energy is being extracted from the signal, and its amplitude falls below the impermeable-wall baseline until the end of the domain. It is observed that the second mode experiences higher attenuation rates due to the C/C in the regions where the mode is already stable and lower growth rates in the regions of instability for the advection over the porous walls in comparison with the run over solid walls. The combination of these phenomena leads to the decrease in the maximum power content for each frequency throughout the domain and depicts the capability of ultrasonic absorbing coating to dampen the second-mode instability in a hypersonic boundary layer. Another noticeable behavior is that the location of the maximum pressure amplitude moves upstream due to the porous walls.

Low-amplitude short-wavelength oscillations are observable at high Reynolds numbers at the later stages of the second-mode evolution (in Fig. 13). Such spatial variations in the amplitude of the Fourier modes of pressure at the wall, $\|\hat{p}\|$, may indicate the presence of a short-wavelength standing wave or spatial modulation of that particular temporal frequency, or nonlinear spectral broadening. These oscillations cannot be attributed to lack of grid resolution (see Fig. 10), and their origin will be scrutinized in more detail in future work.

Figure 9 shows the power spectrum at a given streamwise location on the surface of the cone to see how it is distributed over the frequency domain, for all the Reynolds numbers considered in this study. In this plot, we observe that the effectiveness of the porous walls in absorbing energy starts to become relevant for frequencies above 200 kHz and that the maximum amplitude of the power spectrum attenuated at the three different probed locations for all Reynolds numbers occurs beyond this frequency. The effectiveness of the attenuation of the spectrum induced by the porous surfaces increases with Reynolds numbers. The reason for this interesting behavior is believed to be because of the shift of the spectrum to higher frequencies, and because all acoustic models presented predict that the absorption coefficient of the C/C-based materials increase with frequency, the UAC becomes more effective.

Wagner [13] reported a similar behavior in his transition delay experiments in the HEG tunnel at DLR Göttingen but for a different range of Re_m . He observed that, as the Reynolds numbers increased from $4.0 \cdot 10^6$ to $9.8 \cdot 10^6 \text{ m}^{-1}$, the relative transition delay on the porous surface increased from 0.15 to 0.29. With the same reasoning, we speculate that the reason the experiments conducted by Willems et al. [52] were not able to measure any attenuation of the second-mode waves with the same C/C sample from DLR is that the

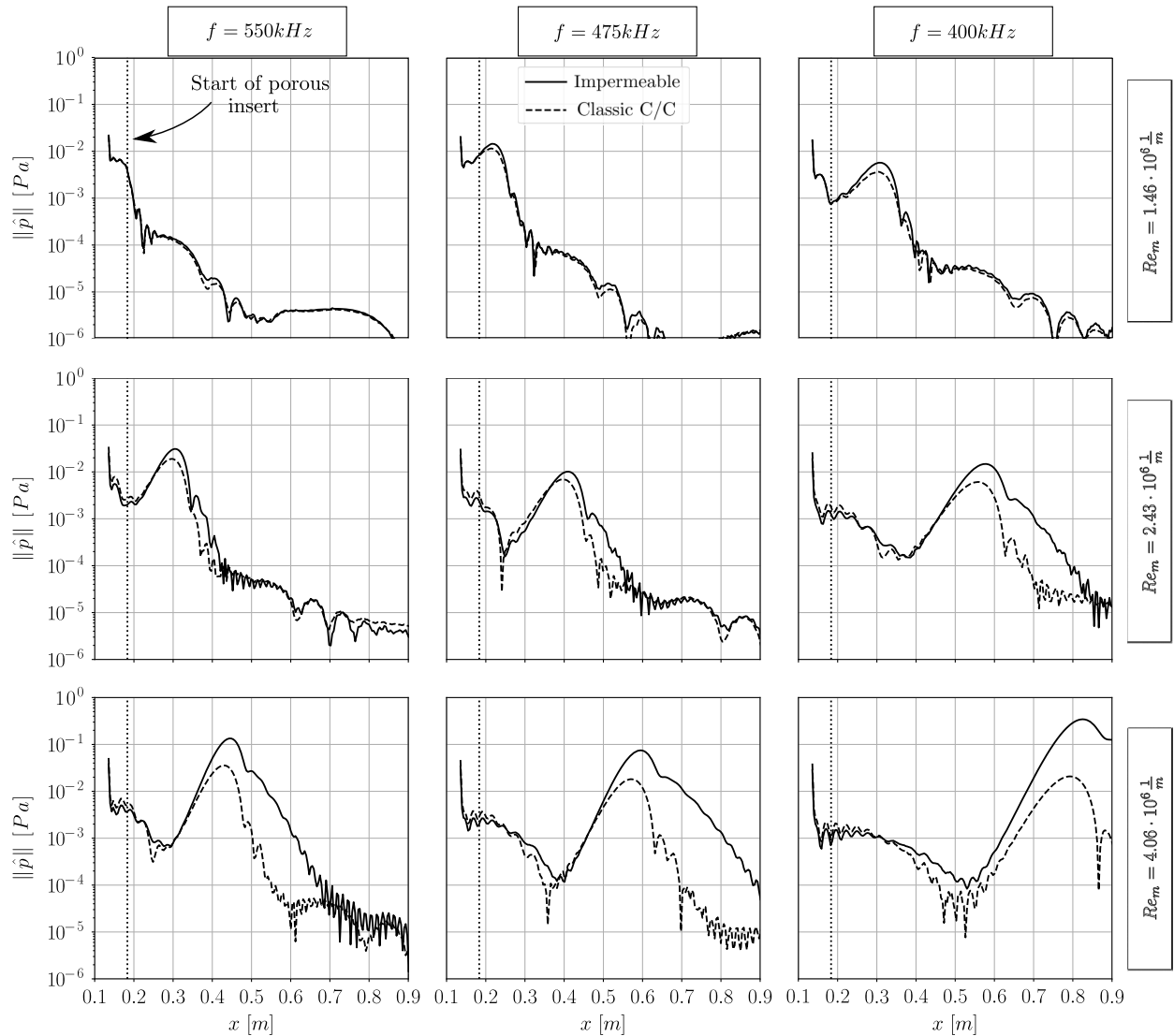


Fig. 13 Spatial distribution of the Fourier transform of pressure fluctuations at the wall for different frequencies in the broadband-pulsed simulations over porous and impermeable walls.

frequency of the second-mode waves were approximately 100 kHz at the flow conditions chosen for that study. Another factor that could have influenced the results are the very low pressures (from 0.003×10^5 to 0.008×10^5 Pa) at which the tests by Willems et al. were carried. These pressure levels lead to low absorption coefficients (shown in Fig. 12).

Other evidence of the hypersonic boundary-layer transition delay capability of acoustic energy absorption can be found at the farthest downstream probed location of the highest Reynolds number. At this position, the advection of the broadband pulse over impermeable walls was amplified to the point that started to excite the first overtone (or harmonic) of the second-mode waves. However, the presence of porous walls dampens such high-frequency modes more effectively than the second mode itself (which cascades energy onto the higher modes due to nonlinear effects, when present). This means that the disturbance with the same initial amplitude would need to propagate farther downstream to reach the necessary strength to trigger transition into turbulence effectively delaying transition.

VI. Conclusions

The effect of realistic acoustically absorptive surfaces on the second mode in a hypersonic boundary layer over a sharp slender cone was numerically studied in the present work. High-order simulations informed by the analytical relations of that describing the supersonic viscous flow of a perfect gas over a sharp cone were

performed, and the high-order simulations were capable of holding the solution and matching accurately the shape and growth of the boundary-layer profiles as they moved downstream of the cone. In addition, a novel technique (IHS) was used to solve the acoustic response at the mouth of the cavities present in porous materials and to model their collective influence through a surface averaging. The complex impedance obtained as a result was used to model the impact of the ultrasonically absorptive surface in a spatially developing boundary-layer simulation without having to resolve the intricate porous structure. The result of this simulation was then compared with previously published theoretical models, and a better agreement with experimental data for the low-pressure domain is achieved.

Artificial disturbances were introduced in the boundary layer, and their evolution with downstream advection was observed. With the introduction of a broadband pulse and its advection over a solid surface, it was possible to identify the second-mode dynamics over an impermeable wall, used as a baseline, and over materials currently being developed to be used in hypersonic transition control such as the “classical” C/C. In conclusion, for the frequencies and flow condition tested, the C/C was found capable of attenuating the second-mode and delaying boundary-layer transition into turbulence.

All results shown in the present contribution are for a sharp-tip cone. The presence of a nonnegligible tip, bluntness increases the boundary-layer thickness (for the same freestream conditions) and hence lowers growth rates and the frequency of second-mode waves. Although this will lower the effectiveness of the acoustic absorption

properties of the porous walls, resulting in a less-effective attenuation of second-mode waves, we would be still in a suitable region for the tested C/C porous material.

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Experimental and numerical acoustic characterization of ultrasonically absorptive porous materials

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The paper addresses the experimental and numerical acoustic characterization of ultrasonically absorptive porous materials with random microstructure such as carbon fiber reinforced carbon ceramic C/C or C/C-SiC. The present study builds upon previous efforts by the authors, improving and extending the established experimental method, complemented by a numerical analysis based on linear acoustics. The latter includes a blind-hole porosity approximation, only accounting for the larger cracks in the C/C with complex acoustic impedance given by the inverse Helmholtz Solver approach, and a highly parametrized homogeneous acoustic absorber model, accounting for the complete volumetric structure of the porous absorber albeit with lower fidelity. The experimental approach is complemented by high-speed Schlieren visualization and Mach-Zehnder Interferometer measurements to qualitatively and quantitatively assess the interaction between an ultrasonic wave packet and a porous surface. It is found that neglecting the smaller pores and only accounting for the surface porosity, as done in the blind-hole porosity approximation, leads to the underestimation of the acoustic energy absorption coefficient. Phase shifts were found to be experimentally assessable, but remain to be corroborated by a numerical analysis. The comparisons carried out in this paper will pave the way for accurate determination of impedance boundary conditions to be applied in direct numerical simulations of hypersonic transition delay over C/C. The main emphasis of the paper is to assess the potential and the limitations of the experimental methods and the comparison of the experimental results to the numerically obtained absorption characteristics.

I. Introduction

In flow fields with small free-stream disturbances, negligible surface roughness and predominantly two-dimensional hypersonic boundary layers second-mode instabilities are known to be the dominant mechanism leading to transition. Due to the acoustic nature of the instability ultrasonically absorptive coatings have the potential to damp the instabilities and thus delay laminar-turbulent transition [1]. Preceding studies in hypersonic wind tunnels indeed revealed that porous surfaces, such as metal sheets with cylindrical blind holes, felt metal and carbon fiber reinforced carbon ceramic (C/C) can be used to damp hypersonic boundary layer instabilities and thus passively control boundary layer transition.[2–9]

However, to achieve practical relevance the ultrasonically absorptive materials need to withstand the high shear stresses and heat loads present in hypersonic sustained flight. Such a material is for instance C/C-SiC which has been used successfully as TPS on hypersonic vehicles, [10, 11], and which can be manufactured with a comparable porosity and microstructure as found on C/C.[12] To assess the potential of such a material with respect to transition control the requirement arises to determine the ultrasonic absorption characteristic at the surface. For this purpose a test rig was set up at DLR Göttingen to measure the reflection coefficient at varying static pressures as reported in Wagner et al. [13].

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The improved setup uses ultrasonic sound transducers covering a frequency range of 125 kHz to 490 kHz and thus include a wide range of the frequency domain of interest with respect to the second mode instabilities.

On the numerical side, the inverse Helmholtz solver methodology [14, 15] is used to directly evaluate the broadband acoustic impedance associated with the C/C approximated as drilled-hole (or blind) porosity. The iHS methodology is hence only applied to a limited number of cracks in the C/C (the larger ones). Comparisons with experiments show that this simplification results in the underestimation of the overall absorption coefficient, especially at higher frequencies; it is therefore concluded that, especially in the latter case, even the smaller, more randomly distributed cavities contribute to the acoustic absorption. Results from the homogeneous absorber theory [16] are also included, which demonstrate a superior matching with the experiments, showing that the blind-hole porosity approximation (relying on surface porosity only) is insufficient to correctly account for the absorption capabilities of the C/C.

II. Method

A. Experimental Setup

1. Transducer Based Setup

Ultrasonic air-coupled PZT* sound transducers with specific resonance frequencies are arranged pairwise to transmit ultrasonic wave packets towards a material of interest and to record the wave packet after its reflection from the surface. Sound transducers with resonance frequencies of 125 kHz, 289 kHz, 300 kHz, 405 kHz and 490 kHz are used in the present study at an angle of incidence of 30° and a distance of 40 mm such that the transducer axes intersect at the sample surface. Figure 1 shows a schematic of the setup while table 1 provides the geometric parameters introduced in the figure. The angle of incidence was chosen based on experimental work performed by Fedorov et al. [17]. Further, it

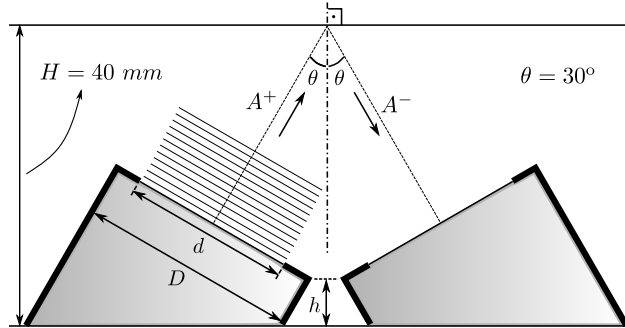


Fig. 1 Schematic of the experimental setup using ultrasonic sound transducers - transmitter on the left, receiver on the right (not to scale).

is justified by a numerical investigation conducted by Brès et al. [18] showing that in all cases relevant for ultrasonically absorptive coatings the angle of incidence of second mode waves is smaller than 26° . Further information on the experimental setup is provided in Wagner et al. [13].

2. Qualitative Schlieren Measurements

A high-speed Schlieren setup (in a z-type shaped arrangement) is used to visualize the generated sound waves as they propagate towards the test sample from which they are reflected. The optical setup is illuminated by a Cavitar Cavilux Smart laser system, which is operated with a pulse width of 20 ns. A Phantom v2012 camera is used with a resolution of 640px x 480px at a frame rate of 70 kHz to capture the development of the sound waves. No other light, other than that from the laser system, exposes the camera chip. This means that the laser pulse duration determines the exposure time, limiting motion blur to about $7 \mu\text{m}$. Quantitative measurements beyond the Schlieren visualisation are planned using an interferometric method, which is introduced below.

*Plumb-Zirconate-Titanate piezoelectric ceramic

f [kHz]	d [mm]	D [mm]	h [mm]
125	19	30	1
223	11	17	0
289	7	17	0.7
300	7	17	1
405	9	17	0
490	8	17	0

Table 1 Geometric parameters of the transducer/receiver pairs.

3. Quantitative Mach-Zehnder-Interferometer Measurements

In order to obtain complementary measurements to the one dimensional temporally resolved recording with the sound transducers an optical setup based on a Mach-Zehnder-Interferometer (MZI) built by Carl Zeiss in 1926 was implemented. The sound waves created by the PZT transducers are visualized and quantitatively evaluated based on interferometric measurements of the wave distribution. The experimental setup and measurements obtained with it will be described and the possibilities and challenges of this measurement technique when applied to the current experiments are discussed. A high speed camera in combination with the MZI is used to image the sound waves as they propagate towards the test sample. A schematic of the optical setup is given in figure 2. A green diode-pumped solide-state cw-laser (532nm ,230mW) is used to illuminate the system. A non-resonant mode cleaner consisting of a focusing lens L1 = 20 mm and a pinhole P having a diameter of 20 μm is used to optimize and shape the beam profile of the laser. A second lens L3 = 40 mm expands the laser beam onto the forming lens L3 = 500 mm after which is becomes parallel. The parallel laser beam is then fed into the MZI. Here the laser beam is split into a reference path and the object path by means of two half mirrors (HM) and reflecting it with two full mirrors (FM). Both beams are collimated through the lens L3 = 500 mm and focused on the chip of the high speed camera, where they interfere. The high speed camera capturing the images is a Phantom v2012, equipped with the FAST option, and is used at a resolution of 640x384px² and frame rate of 77kHz. The exposure time is set to the lowest possible value, which is 0.285 μs for the Phantom v2012. The time shift between the recording of consecutive images is 12.98 μs .

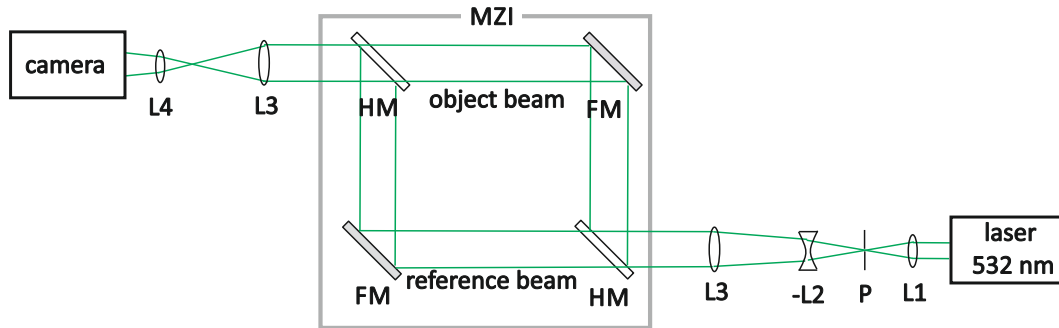


Fig. 2 Schematic of the optical setup using a Mach-Zehnder-Interferometer (MZI). Non resonant mode cleaner (L1, P, -L2), main lenses (L3,L4), HM: half mirror and FM: full mirror are parts of the MZI.

The optical resolution of the system is determined with a 1951 USAF resolution test chart conforming to MIL-STD-150A standard (set by US Air Force in 1951) to be 198.43 μm (see figure 3b)). Transforming the frequency of $f = 125$ kHz used for the sound transducers at the current tests, we observe a wavelength of 2.74 mm. Therefore, we are able to resolve one wavelength with approximately 14 data points or pixels on the chip. A better resolution is not feasible, because of the limit of minimal exposure time of the camera, which introduces a motion blur of approximately 87 μm . Quantitative measurements of the pressure waves observed in the object beam of the MZI are based on the interference pattern formed by the superposition of light waves which originate from the coherent source of the laser but traverse different paths (object beam and reference beam, see figure 2). The fringe pattern indicates the local phase shifts arising from the difference in the optical paths traversed by the interfering beams.

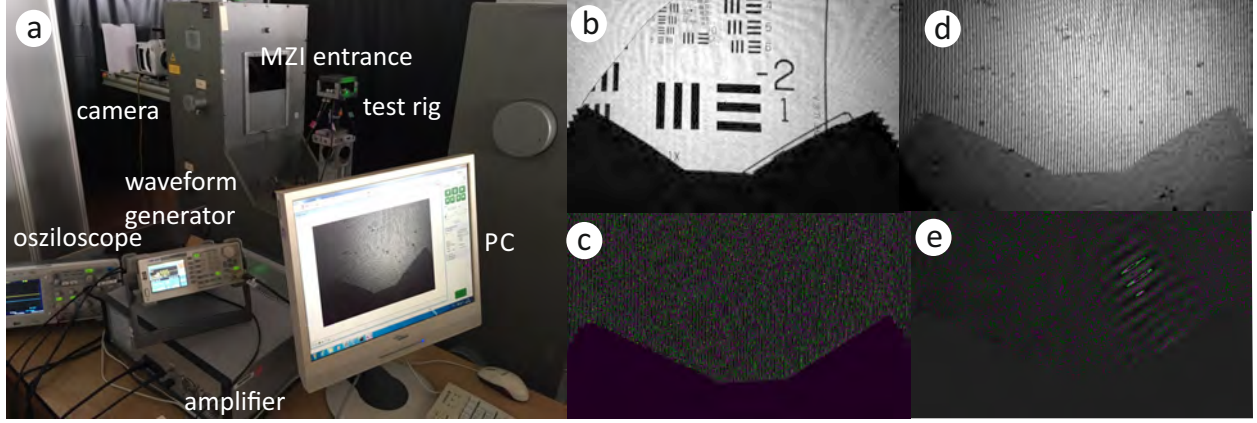


Fig. 3 A photograph of the experimental setup is shown in a), a camera image of object beam with the optical resolution target is given in b). A typical interference pattern between the object and the reference beam without any phase change in the object beam is shown in d). The reconstructed interference pattern with a 2D-FFT is given in c) and the evaluated phase shift distribution of a wave packet in e).

The resulting interference pattern without any introduced phase change in the object beam can be controlled by adequate adjustment of the half and full mirrors of the MZI. In order to use a two-dimensional Fast-Fourier-Transformation (2D-FFT) to evaluate the phase distribution from the interferograms, we create spatial carrier fringes as shown in figure 3d). The procedure is standard and well documented in the literature ([19–23]), therefore only a short summary is provided. The spatial intensity distribution of the fringe pattern, determined by the object beams phase shift Φ and the spatial carrier phase, can be written as

$$I(\vec{r}) = I_0(\vec{r}) + m(\vec{r}) \cos(2\pi\nu_0(\vec{r})\vec{r} + \Phi(\vec{r})),$$

where I_0 and m are the background and contrast functions, ν_0 is the carrier frequency vector with components for the x- and y-direction and $\Phi(x, y)$ is the required phase function. If the largest gradient from the object phase is less than the spatial carrier phase and ν_0 is less than half of the sampling frequency Nyquist condition), then the phase distribution can be determined in both magnitude and sign. The above equation can be transformed into

$$I(\vec{r}) = I_0(\vec{r}) + c(\vec{r})e^{(2\pi i\nu_0\vec{r})} + c^*(\vec{r})e^{(-2\pi i\nu_0\vec{r})}$$

where

$$c(\vec{r}) = \frac{1}{2}m(\vec{r})e^{i\Phi}.$$

The superscript (*) denotes the complex conjugate. The Fourier transform of $I(x, y)$ yields

$$\widehat{I}(\vec{r}) = \widehat{I}_0(\nu) + \widehat{c}(\nu - \nu_0) + \widehat{c}^*(-\nu - \nu_0).$$

This clearly reveals the purpose of generating the fringe carrier system: Disturbing changes in background intensity I_0 are of lower frequencies. By applying a comparable high carrier frequency ν_0 , the information of the fringe system is separated from the disturbing low frequencies in the Fourier domain, shifting it to the vicinity of ν_0 . The determination of the phase distribution $\Phi(r)$ is straight forward. First the domain $c(\nu - \nu_0)$ of the interferogram around the carrier frequency is determined via the FFT, then all frequencies outside this domain are set to zero. Then the domain centered around ν_0 is transferred towards the origin (zero-frequency), which removes the carrier. This resulting frequency field is the inverse Fourier transformed yielding $c(r)$. Finally the phase modulo 2π (see figure 3e)) is calculated by

$$\Phi(\vec{r}) = \arctan \left(\frac{\text{Im}(c(\vec{r}))}{\text{Re}(c(\vec{r}))} \right).$$

B. Numerical Setup

The methodology used to determine the complex acoustic impedance of the blind-hole porosity approximation of the C/C is the inverse Helmholtz Solver (iHS) [14, 15].

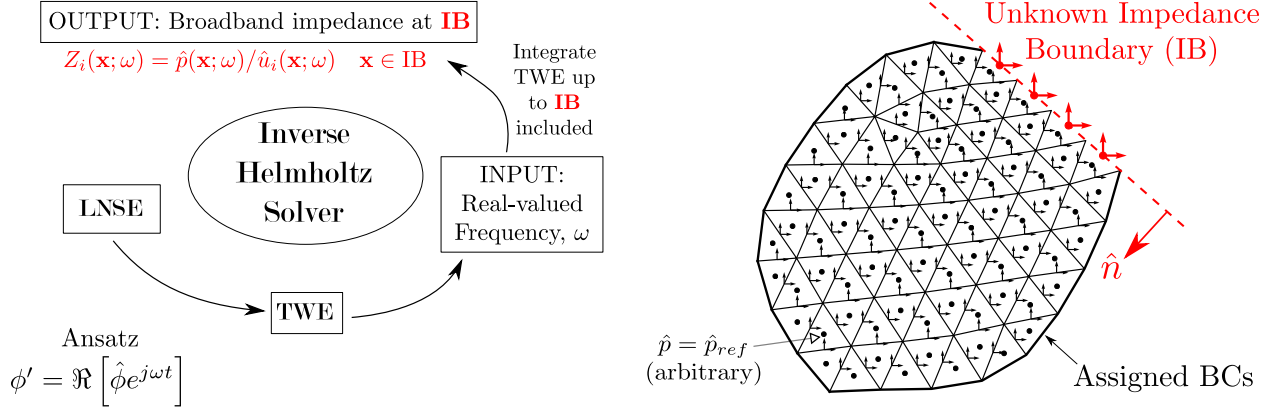


Fig. 4 Inverse Helmholtz solver procedure used to calculate the impedance of cavities typical of blind-hole porous surfaces.

This code reconstructs, at a given frequency, the complex acoustic waveform in the cavity up to the open surface or unknown impedance boundary (IB), hereby providing the complete spatial distribution of impedance as a result of the calculation. Multiple instances of the iHS can be executed in parallel for different frequencies, allowing for a rapid determination of the full broadband impedance at every point of the IB.

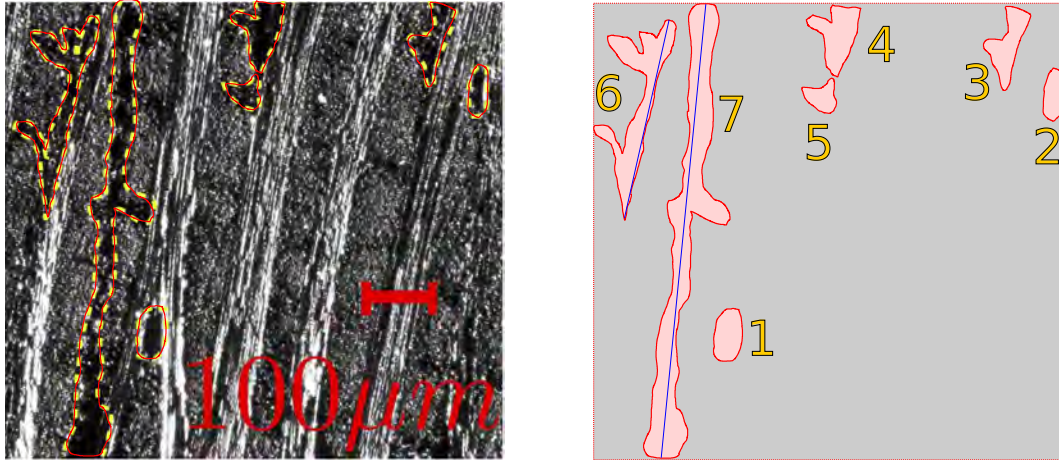


Fig. 5 Inverse Helmholtz solver procedure used to calculate the impedance of cavities typical of blind-hole porous surfaces.

From a high definition image of the C/C block (figure 5), large cavity edges are detected and meshed to be fed as input to the iHS. The impedance for various cavities are then evaluated by the solver and combined together—noting that the admittance of the surrounding hard surface is zero—to yield the effective, surface averaged impedance of the block.

III. Results

A. Experimental Results

1. Qualitative Schlieren Visualization

The high speed schlieren setup described in section A was used to visualize the ultrasonic waves generated by the sound transducers in the above introduced setup. Although the signal to noise ratio is low, waves at 125 kHz, 300 kHz and 490 kHz were successfully visualized at ambient pressures as shown in figure 6. The schlieren images resemble the schematic in figure 1 in which the ultrasonic wave packets are generated by the transducer on the left. Subsequently the waves propagate along the transducer axis towards the investigates sample surface at the top of the image and are reflected back towards the receiver. The schlieren visualizations provide vital qualitative information, such as the dispersion angle of the sound waves, the point of interaction at the sample and the receiver and the wave dimensions. These information are required to numerically reconstruct the experiment. Unfortunately, the signal to noise ratio is too low to allow a quantitative evaluation of the wave amplitudes for instance.

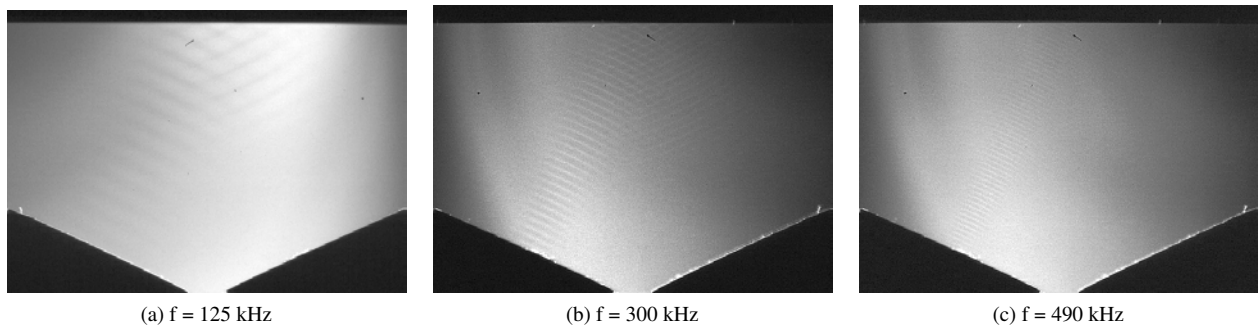


Fig. 6 Schlieren visualizations of ultrasonic waves at 125 kHz, 300 kHz and 490 kHz at ambient pressure.

2. Absorption Coefficient Measurements by means of Sound Transducers

Figure 7 provides representative results of the measured absorption coefficient of a C/C sample ([24]) using an optimized experimental setup, compared to the setup introduced in Wagner et al. [13]. The absorption coefficient is provided as a function of pressure in a frequency range of 125 kHz to 490 kHz. The optimized setup now allows conducting tests at pressures as low as approximately 2500 Pa. Note that the lowest pressure at which absorption measurements can be conducted depends on the sound wave frequency. The now accessible low pressure range is of particular interest regarding possible applications of ultrasonically absorptive materials for transition control on hypersonic vehicles.

Figure 7 shows some important trends which were repeatably observed on various investigated porous materials:

- a dependency on ambient pressure: the absorption coefficient decreases with decreasing ambient pressure, in particular in the low pressure range,
- a dependency on the frequency: the absorption coefficient increases with increasing frequency and
- with regard to the measurement technique: increasing uncertainties with decreasing ambient pressure.

The provided measurement errors are derived by evaluating a series of independent measurements with an acoustically hard surface. Since the reflection coefficient of that test case is known to be unity a mean, ambient pressure dependent, measurement error could be determined as indicated in figure 7. Note that this error does not account for locally changing surface properties.

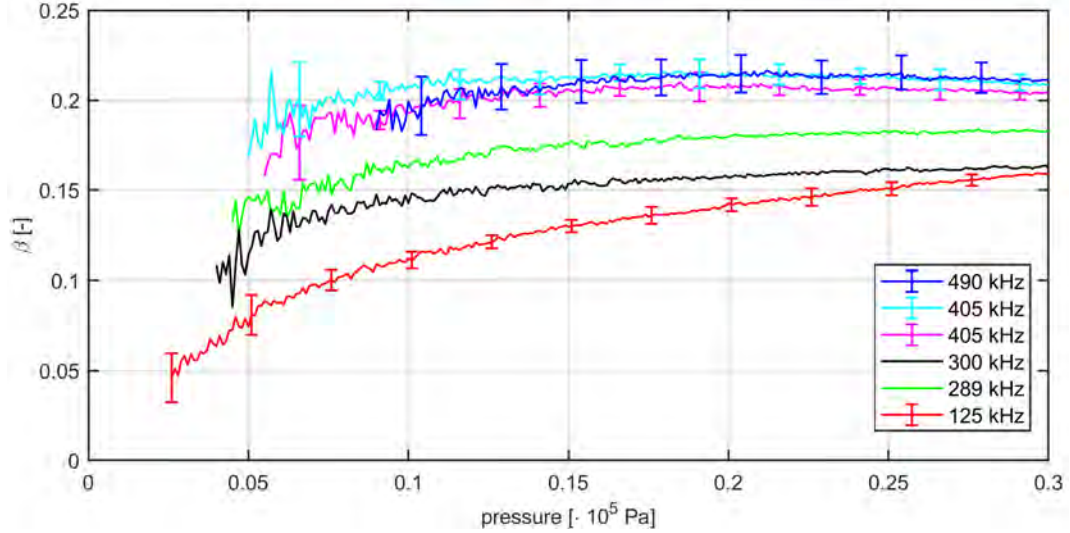


Fig. 7 Absorption coefficient experimentally obtain on a C/C sample using sound transducers at different frequencies.

3. Sound Transducer Phase Shift Measurements

In the course of the experimental study the question was raised whether the setup could be used to measure the expected and numerically predicted phase shift resulting from the reflection of the sound wave at a porous wall. This would allow to fully acoustically characterize the sample of interest by determining the reflection coefficient and the phase shift and thus the real and imaginary part of the surface impedance. To address this issue the setup described in section A was used in combination with an acoustically hard surface positioned with a sub-wavelength offset from the reference position. The offset was realized using metal sheets of 0.3 mm thickness. Figure 8 shows the time resolved

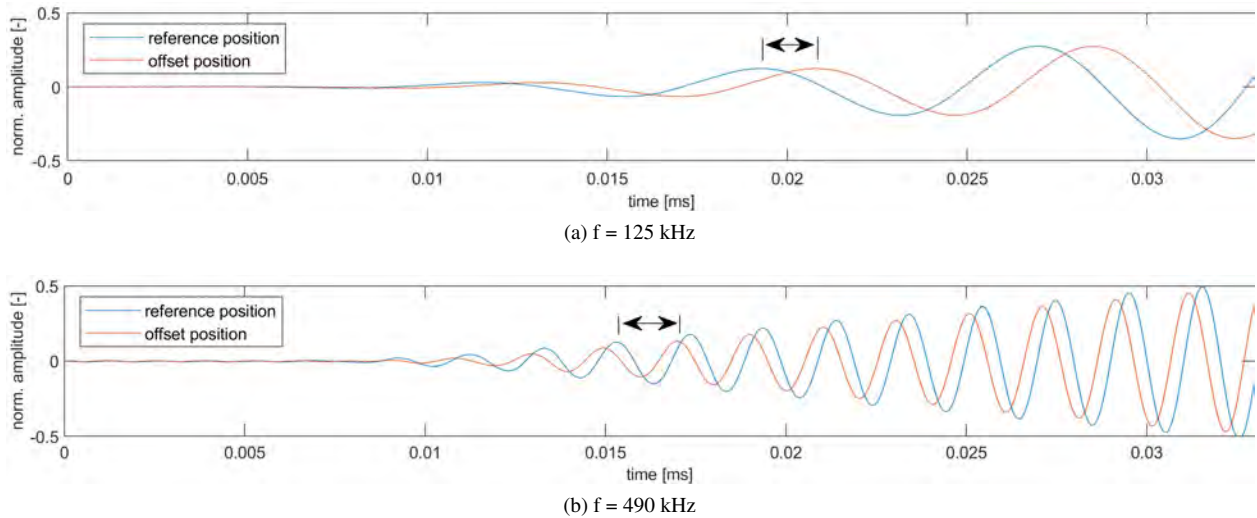


Fig. 8 Time resolved reflected sound waves with 125 kHz and 490 kHz. The indicated phase shift results from a position offset of the reference sample.

normalized pressure measurements at the receiver after the sound wave reflection at the wall. Tests were conducted using two different frequencies; 125 kHz shown in figure 8a and 490 kHz shown in figure 8b. Both figures depict the first periods of the wave packets after reflection at an acoustically hard surface at the reference position (shown in blue) and the surface positioned with an offset (shown in red). It can be seen that the latter shows a phase shift compared to the reference case. This phase shift corresponds to the change of the path length experienced by the acoustic wave due

to the altered position of the reflection surface. An evaluation of the phase shift of multiple wave packets allows to determine the distance of the reflection surface is shown in figure 9. The speed of sound in the nitrogen test atmosphere

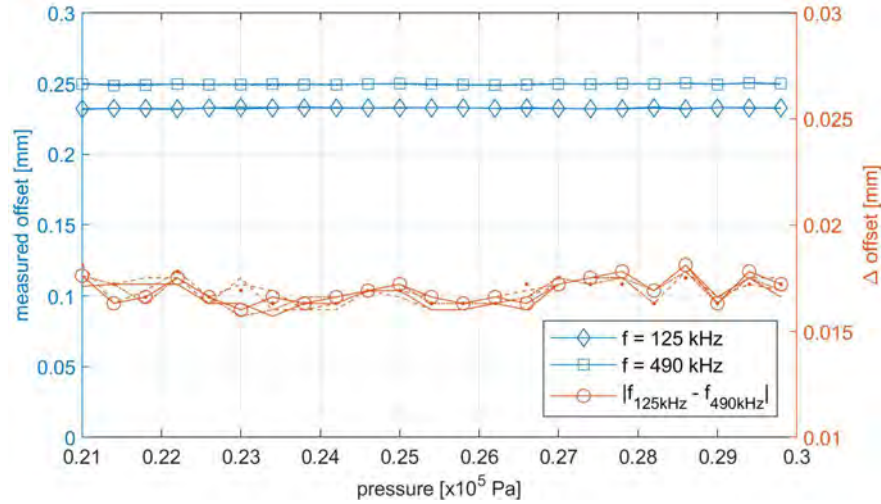


Fig. 9 Measured offset from the reference position (left) and approximate resolution limit (right).

was estimated with 349 m/s assuming a test gas temperature of 293 K. However, the gas temperature was not monitored during the tests adding to the uncertainties of the measurements. Note that the precise value of the offset is also subject to a number of uncertainties linked to the installation of the metal sheets. However, the distance between the reflection surface and the transducer pairs is expected to be identical. Thus, a comparative measurement between the 125 kHz and the 490 kHz transducers provides a first estimate of the achievable precision. Figure 9 shows that the independent measurements differ by approximately 0.017 mm which corresponds to 2.2 degree and 8.6 degree of the 125 kHz and the 490 kHz sound wave. These values provide an estimate of the smallest resolvable phase shift. Further improvements of the setup are necessary to reduce the positioning uncertainty of the investigated sample before assessing possible phase shifts due to the interaction of the sound wave with a porous surface.

4. Preliminary Results of the Wave Propagation determined by Interferometric Measurements

A first result of the observed waves is given in figure 10. The test rig was installed without a reflective surface in the object beam of the MZI. A wave packet with a frequency of $f = 125$ kHz was generated with 4 periods. The image sequences given in 10 show the temporal and spatial two dimensional evolution of the wave packet generated by the sound transducers. The propagation of the wave packet can clearly be observed and the spatial broadening perpendicular to the direction of motion can be determined. The first recording at $t = 12.98 \mu\text{s}$ shows the appearance of two waves ($1/f = 1/125 \text{ kHz} = 6 \mu\text{s}$), one image later, the complete packet (4 periods) is visible. As the packet travels along the propagation direction, it can be observed that additional weak waves are emitted behind the wave packet. This is due to the decay of the transducer.

In figure 11 the phase data along the symmetry line of the propagation direction is shown for different instants in time. The data is extracted from the images shown in 10. For reference purposes, the data at $t = 0 \mu\text{s}$ is added to every graph as a red line. The propagation and damping of the wave packet over time can be observed until it leaves the field of view. We note at this point, that the signal to noise ratio is approximately 5 in average over all data obtained. To improve this ratio, the measurements can only be performed in a different gas than air, which would yield stronger signals. To investigate how far the analysis of the data can be driven, a set of 10 consecutive recordings of a continuous generated waves of $f = 125$ kHz are spatially averaged. We compare the averaged data of the incoming and the reflected waves in figure 12 for a solid wall and a porous wall.

The data in figure 12a) compares the recorded waves approaching the walls (incoming waves). The repeatability for the different experiments is very good. The comparison for the waves which return from the walls after reflection (reflected waves) in figure 12b) reveals the differences of both waves after the reflection at the solid and the porous wall. The amplitude is obviously lower, but a distinct phase shift cannot be observed.



Fig. 10 Temporal and spatial evolution of the sound waves. The test rig is used without a reflective wall.

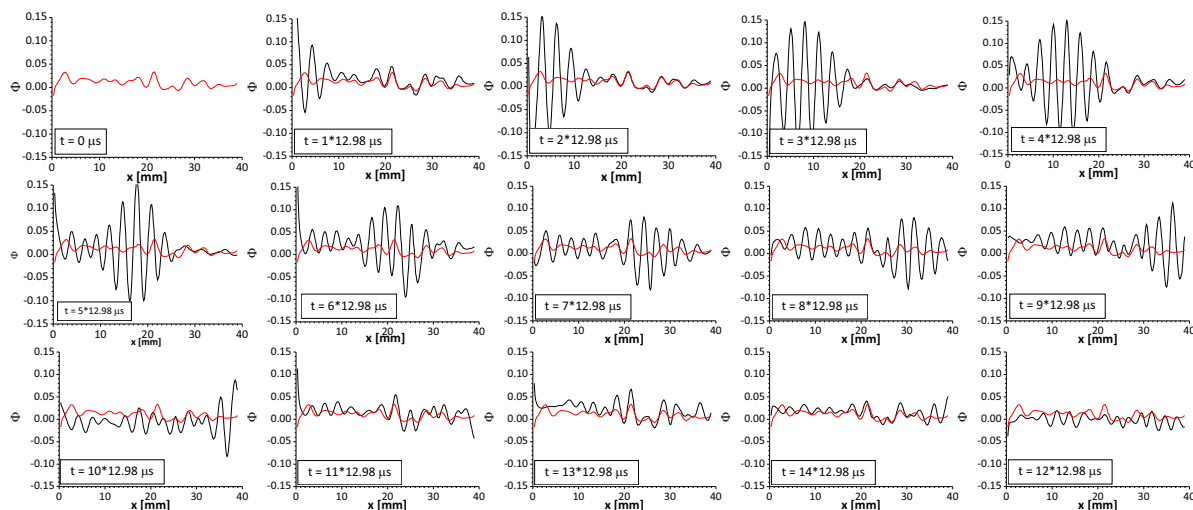


Fig. 11 Temporal evolution of the sound waves along the symmetry line of the propagation direction. The red curve is the reference recording at $t = 0 \mu s$. The test rig is installed without a reflective plate.

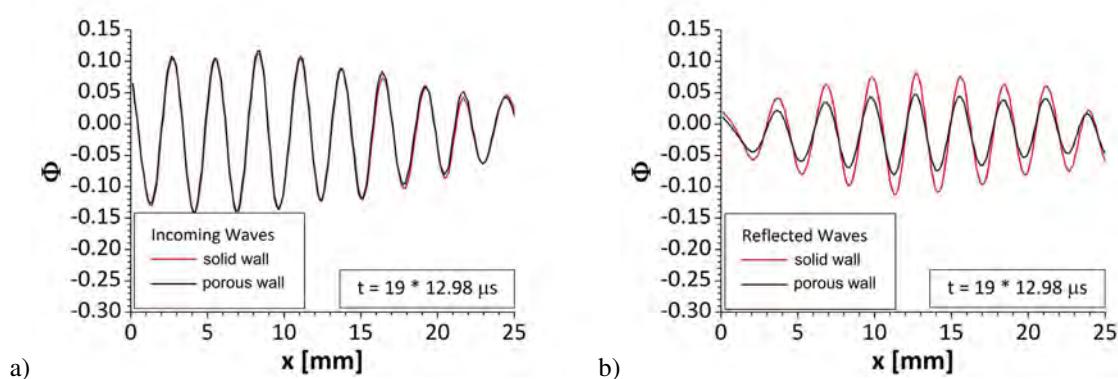


Fig. 12 Wave distribution of incoming waves and reflect waves for different surfaces.

B. Numerical Results

The iHS was used to evaluate the effective surface-averaged specific acoustic impedance of the C/C surface approximated as blind-hole porosity and only composed of the larger pores, as shown in figure 5. This yields a surface porosity of 0.078 which is extruded in the depth, hence yielding the same assumed volumetric porosity. A broad range of conditions is covered, bracketing experimental conditions.

Comparison were then made between numerical and experimental results as well as with the homogeneous absorber theory, a theoretical model that calculates the acoustic impedance of a homogeneous porous absorber accounting for the whole volumetric distribution of the pores. A value of volumetric porosity equal to 0.15 is used in this approach, which is almost twice the porosity used in the blind-hole approximation of the C/C.

The variable of choice for these comparisons is the acoustic absorption coefficient, β , which is a quantitative estimate of the amount of acoustic energy absorbed at the C/C surface due to the presence of microporous cracks. The absorption coefficient relates to the acoustic impedance via,

$$\beta(\omega) = 1 - \left| \frac{1 - Z_*(\omega)}{1 + Z_*(\omega)} \right|^2, \quad (1)$$

where $Z_*(\omega)$ is the broadband surface averaged specific acoustic impedance of the C/C absorber.

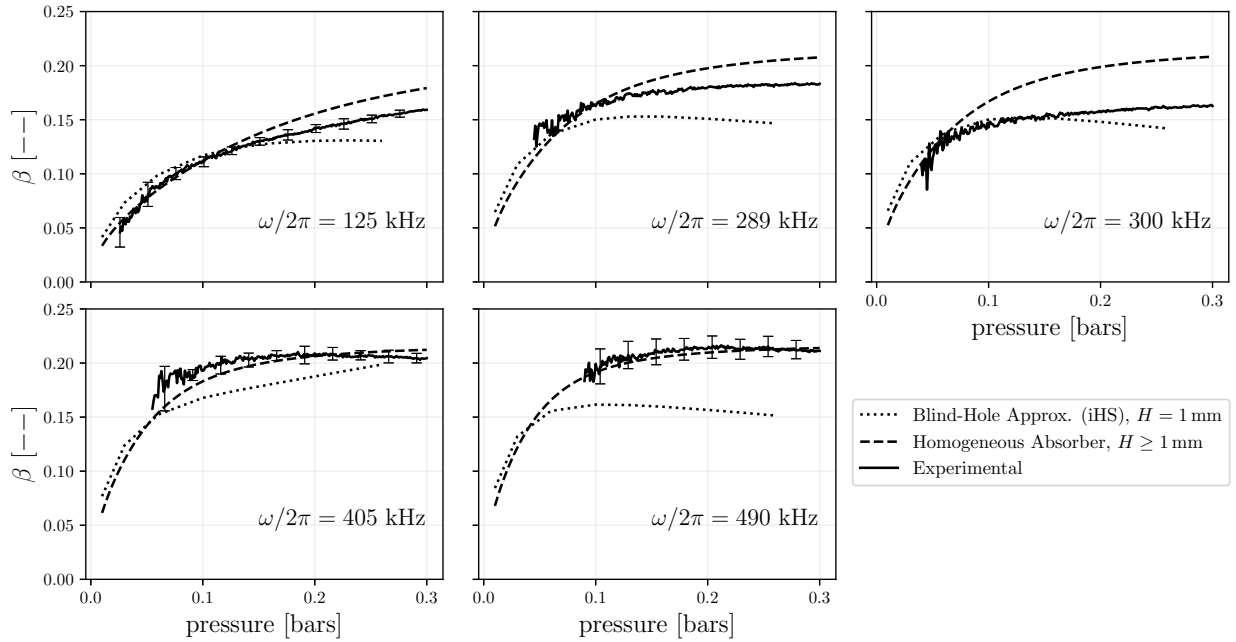


Fig. 13 Comparisons in the variation of numerically, theoretically, and experimentally evaluated absorption coefficients, β , with ambient pressure.

Homogeneous Absorber Theory

The homogeneous absorber theory [16] defines the impedance of a volumetric porous absorber subject to wave incidence at the angle θ_i as,

$$Z_{\text{abs}} = Z_{\infty} \frac{1 + e^{-j2k_a H}}{1 - e^{-j2k_a H}} \cos(\theta_i) \quad (2)$$

with,

$$k_a = k\sqrt{\kappa} \sqrt{1 - j\frac{\omega_k}{\omega}}, \quad Z_{\infty} = Z_0 \frac{\sqrt{\kappa}}{\phi} \sqrt{1 - j\frac{\omega_k}{\omega}}, \quad \text{and} \quad \omega_k = \frac{\Xi\phi}{\rho_0\kappa}$$

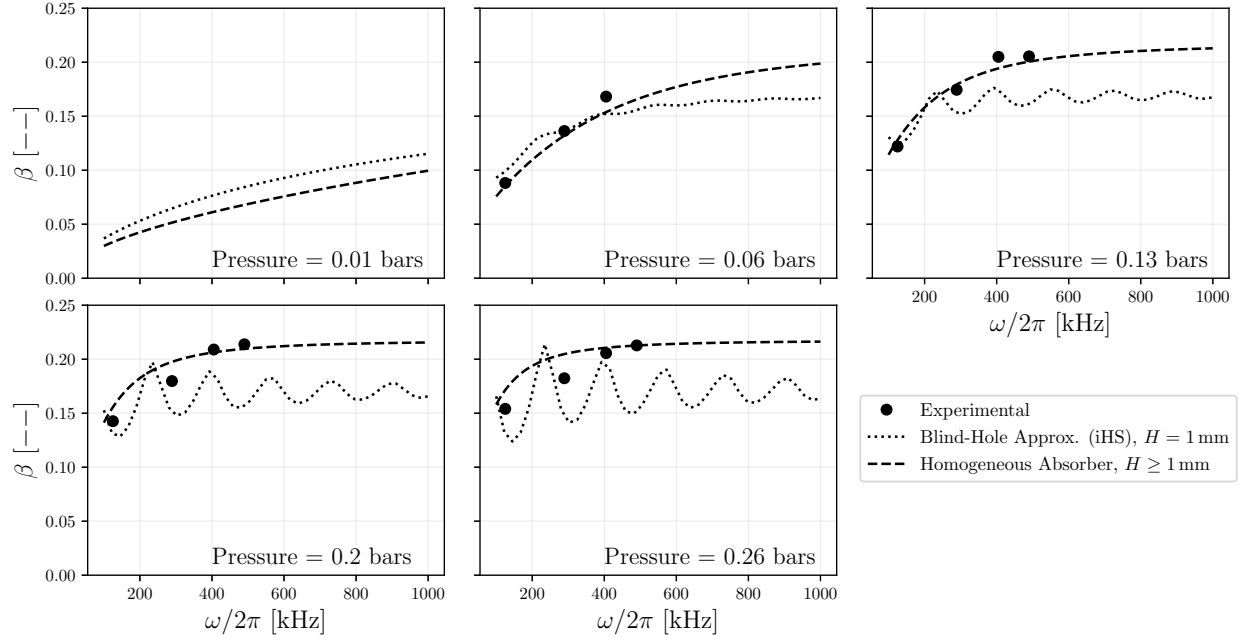


Fig. 14 Comparisons of broadband absorption coefficient behaviors evaluated using experimentally, numerically, and theoretically measured acoustic impedance.

where, κ is the structure factor, Ξ the flow specific resistivity, ϕ the volume porosity, k the wave number, $\omega = 2\pi f$ the angular frequency of incident wave, θ_i the angle of incidence, H the depth of the absorber, and $Z_0 = \rho_0 a_0$ the base impedance measured using ambient temperature T_w . The parameter ω_k is referred to as the breaking frequency of the absorber, and specifies the frequency below which the absorber is expected to perform poorly. A value of $\theta_i = 30^\circ$ has been assumed for the angle of incidence to emulate the benchtest setup in these calculations. Relevant values for various parameters in equation (2) have been summarized in table 2.

It is worth noting that the acoustic impedance determined this way does not vary much for assumed absorber depths over $H \geq 1$ mm (curves in Figure 14 from the homogeneous absorber theory would be indistinguishable for $H \geq 1$ mm).

For the predictions carried out with a blind-hole porosity assumption, a surface (and hence also volume) porosity of 0.078 is used, as obtained from the processing of the high-resolution C/C surface images. A nominally chosen depth of $H = 1$ mm has been assumed based on visual inspection of depth-wise slices of a C/C sample provided by DLR Stuttgart [25].

Table 2 Parameters used with the absorber theory

Porosity, ϕ	Structure Factor, κ	Resistivity, Ξ	Depth, H or h^*	T_w [K]
0.15 [-]	8.0 [-]	13 [MPa s m ⁻²]	5×10^{-3} [m]	295

Figure 13 compares the results obtained experimentally with: (1) an analytical model for volumetric porous absorbers, the Homogeneous Absorber Theory (HAT), and (2) the numerically obtained results with the iHS methodology applied to the blind-hole approximation of the C/C (only considering the 7 cracks in figure 5). It is possible to observe that both absorber theory and the iHS predict similar absorption coefficient trends for low base pressures. These are in good agreement with the lowest base pressure cases explored in the experiments.

As the pressure increases the absorption coefficient predicted based on the blind-hole porosity approximation starts to deviate from both the experimental data and the HAT predictions. We first observe an overall under-prediction of the magnitude of the absorption coefficient, which is attributed to the underestimated volume porosity (by a factor of two) – a natural outcome of the blind-hole approximation. This leads us to conclude that the smaller cracks (ignored in the selection done in figure 5) play an important role in the absorption.

Another important difference observed is the appearance of resonance-like peaks in the absorption predicted by the blind-hole approximation with the iHS formulation, due to the assumed finite depth of the cavities. This behavior is apparent also in the HAT results but only for $H < 1$ mm (not shown). The experimental data is not spaced out finely enough in the frequency domain to be able to address this. A different transducer/receiver would have to be manufactured for different frequencies making it very cumbersome to address this point experimentally. However, especially for higher pressures (above 0.2 bars), the experimental data falls below the HAT predictions for frequencies below 400 kHz. It is not possible to establish with the present data if this is due to a resonant-like, blind-hole porosity variation of the absorption coefficient or just a decreased accuracy of the HAT model for higher pressures. Given the broadband nature of perturbations in hypersonic boundary layers, accounting for the resonant-like behavior of the wall-impedance (were it to be truly present in C/Cs) is expected to be critical towards accurate predictions of the transition dynamics over porous surfaces.

IV. Summary

A joint experimental and numerical effort was undertaken to measure and predict the absorption characteristics of ultrasonically absorptive porous materials with random microstructure under zero-mean-flow conditions. Various measurement techniques were applied such as high speed Schlieren visualization to obtain qualitative information on the acoustic waves. Optical interferometer measurements were conducted to assess the phase shift of the waves due to their interaction with the porous walls. The interferometer measurements were found to be promising. The first tests provided important information on the optical resolution required and the temporal demands on the recording system. Future experiments are planned with a pulsed light source with pulse time in the range of a few nanoseconds to avoid the motion blur and to allow increasing the optical resolution. The transducer based measurements represent the bulk part of the experimental activities. The setup was further refined and now provides reliable information on the absorption coefficient down to 2500 Pa which is of particular interest for applications in the hypersonic flight regime. In the scope of the study the setup was evaluated with respect to the resolvable phase shift. Future measurements will incorporate minor changes to the setup to conduct phase shift measurements. Furthermore, the inverse Helmholtz Solver was used to approximate the complex surface impedance based on a blind-hole model of the C/C only based on the larger cavities extracted from high resolution images of the porous surface. This choice resulted in an underestimated volume porosity by a factor of two with respect to the experimentally determined one via mercury porosimetry. Results were compared to the experiments and to the Homogeneous Absorber Theory. A particularly good agreement was obtained at low pressures, approximately below 0.2 bar, and low frequencies, approximately up to 400 kHz. The discrepancy of the blind-hole approximation is attributed to the underestimated volume porosity, in particular regarding the neglected smaller pores. That indicates the need of a refined model of the surface porosity. Future plans involve impedance predictions in the presence of a shear layer overlying the acoustic absorber with the overall goal to numerically replicate experiments using porous surfaces for transition control.

Acknowledgments

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Nonlinear Dynamics of Second Mode Waves on a Hypersonic Flared Cone

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A numerical analysis of spectral broadening of second mode waves over a hypersonic flared cone, designed to maintain a constant thickness boundary layer, was conducted through a comparison between direct numerical simulations (DNS) and nonlinear parabolized stability equations (NPSE). The current simulations show that the constant thickness focuses the spectrum of the second mode on one frequency band centered around 300 kHz; its amplitude grows by approximately 8 orders of magnitude over 0.45 m of cone length. This growth is shown to trigger nonlinear mechanisms such as mean flow distortion and spectral broadening, as expected. The DNS and NPSE exhibit very good agreement for the growth rates estimated for the principal frequency and the first two harmonics although the spatial location of the beginning of the growth differs for the higher frequency harmonics. It is also shown that the “rope waves” observed in Schlieren images in previous experiments of hypersonic transition are connected to the region of high nonlinear interaction of the second mode, where the pressure fluctuation amplitude is in the order of 50 Pa. This work is an intermediate step towards studying the relative importance of the different nonlinearities present in this flow and the energy source of the second mode waves at very high amplitudes.

I. Introduction

One major obstacle in the design of hypersonic vehicles is laminar-turbulent transition uncertainty. Transition significantly affects the values of heat transfer and shear stress in a hypersonic boundary layer. Reed et al¹ assumed a fully turbulent and a fully laminar flow for a low-Earth orbit hypersonic flight vehicle and reported that the vehicle would experience 5 times more heating in the turbulent condition and would also need at least 2 times the weight in thermal protection systems. Another problem that arises is that transition leads to a local heat transfer flux that exceeds fully turbulent flow calculations because the newly transitioned boundary layer is thinner and with higher gradients than the fully turbulent model. On the other hand, there are some scenarios in which turbulent flow and mixing is desired, such as the flow over control surfaces and the inlet of a scramjet engine. This context leads to the conclusion that the optimum design of such vehicles is intrinsically associated with a fundamental understanding of the laminar-turbulent transition process.

Mack^{2,3} described the instability mechanisms in compressible boundary layers as a function of free stream velocity. As the speed increases from subsonic to supersonic and beyond, Tollmien-Schlichting waves starts to become three-dimensional, traveling at approximately 45 degrees to the free stream, and to decrease in relative importance to transition. This mode was named “first mode” and it is the principal disturbance

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for Mach numbers up to 4. At higher speeds and for canonical geometries, two dimensional acoustic modes take over, namely “second mode” waves. Such modes are expected to be predominant for transition in flight conditions with small angles of attack, on vehicles with a high lift to drag ratio, which tend to have predominantly 2D shapes and sharp tips, and in the presence of cooled walls.⁴ Second mode disturbances are acoustic waves trapped between the solid wall and the relative sonic line.⁵ Their resonance-like behavior is in accordance with experimental results that show that second-mode frequencies are strictly linked to the boundary layer thickness.⁶

The path to transition, especially under low disturbance conditions, is described by the modal growth of the primary modes, such as the second mode.⁷ As their amplitude grow, the nonlinear interactions start to become important and effects such as spectral broadening of resonant modes, harmonic generation and saturation becomes important before the final breakdown to turbulence occurs. Experiments conducted by Berridge et al.⁸ of hypersonic flow over a cone were able to capture all these transition stages in different wind tunnels under noisy flow conditions. Demetriades⁹ performed transition experiments over a sharp cone at Mach 8 and reported the appearance of “rope waves” (Figure 1) in the region preceding transition and this was attributed to the effects of nonlinearities in the flow.

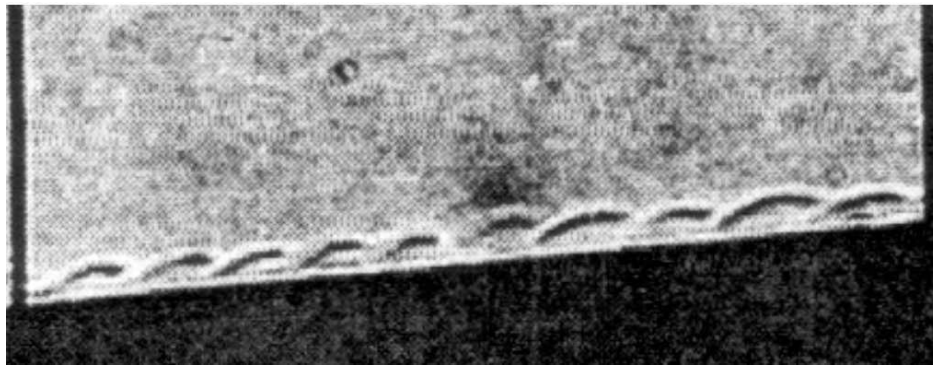


Figure 1: Schlieren images of a sharp cone at zero angle of attack in a Mach 8 free stream flow that show rope-like features at the edge of the boundary layer.⁹

Although the linear regime of transition is well understood, the investigation of the influence of nonlinearities in the flow deserves more attention. Achieving high amplitude second mode waves is in fact challenging in experiments under quiet conditions on straight cones due to size restrictions.¹⁰ This problem, in the study of the second mode, can be overcome by the design of a compression cone specifically to maintain the boundary layer thickness constant along the length of the cone.^{11,12}

This strategy relies on the observations that the frequency of the most amplified waves in a hypersonic boundary layer is closely related to the boundary layer thickness.⁶ Therefore, a geometry that is capable of maintaining a constant boundary layer allows the tuning of a specific band of frequencies over a larger spatial span and, in the end, allows for higher disturbance amplitudes and the study of nonlinear interactions. Since this configuration highlights the second mode characteristics it is the optimum configuration for a fundamental study of this instability mechanisms for both experiments and simulations.

Lachowitz et al.¹¹ performed experiments of transition on a flared cone in the NASA Langley Nozzle Test Chamber Facility quiet tunnel and reported the existence of higher harmonics. It was proven that the existence of the higher harmonics in the flow is attributed to an energy cascade instead of being a consequence of noisy flow conditions existent in previous experiments.⁹ He also reported “rope-like” waves with a wavelength of approximately two times the boundary layer thickness in the Schlieren images in the region right before transition. Pruett and Chang¹³ performed a DNS of the transition on a flared cone replicating the experiments performed by Lachowitz and were able to replicate the most unstable frequency in the experiments and also show the presence of rope-like waves in their numerical simulations. More recently Sivasubramanian and Fasel¹⁴ performed a DNS of hypersonic transition over a flared cone replicating experiments performed in the BAM6QT at Purdue University¹² and were able to identify that both fundamental (K-type) and oblique breakdown are viable paths for transition on this geometry.

This work intends to replicate the nonlinear mechanisms of the flow over the Purdue compression cone with NPSE and DNS to provide a basis for a future theoretical studies of energy transfer mechanisms from

the mean flow to the disturbance in the linear and nonlinear regime from a nonlinear acoustics perspective.

II. Computational Tools

A. Parabolized Stability Equations Solver: JoKHeR

The JoKHeR code was developed by Kuehl^{15–17} and will be utilized in this work. Herbert and Bertolotti, during a critical review of Gaster’s early nonparallel work, originally identified the parabolized stability equations (PSE) method as an efficient and powerful tool for studying the stability of advection-dominated laminar flows. An excellent introduction to the PSE method and summary of its early development are provided by Herbert.¹⁸

The stability analysis begins by considering a generic disturbance and decomposing the flow variables into a steady basic state plus a disturbance (1) and assuming the basic state is a solution to the governing equations of motion, which represents the flow that exists in the absence of any environmental disturbances or forcing. Assuming, without loss of generality, a 2-D basic state and 2-D disturbances, substitution into the governing set of equations yields the equations describing the evolution of the disturbances.

$$\begin{aligned}\phi(x, y, t) &= \underbrace{\bar{\phi}(x, y)}_{\text{basic state}} + \underbrace{\phi'(x, y, t)}_{\text{disturbance}} \\ \phi &= [u, v, T, \rho]^T\end{aligned}\tag{1}$$

Here x, y are the wall-parallel and -normal coordinates, t is time, u, v are the wall-parallel and -normal velocity components, ρ is the density and T is the temperature. Different stability methods solve the disturbance equations under different sets of assumptions. Formally, Linear Stability Theory (LST) imposes the assumptions of infinitesimal (linear) disturbances and parallel flow upon which separable solutions may be found. That is, the basic state is assumed to be only a function of y and $\phi' = \hat{\phi}(y)e^{i(\alpha x - \omega t)}$. Here α and ω are the streamwise wavenumber and frequency, respectively, of the disturbance. Substitution of this form into the disturbance equations yields a generalized eigenvalue problem.

PSE analysis is similar to the Fourier/Laplace transform in that it considers an initial-value problem. However, an assumption is made for PSE based on the observation that the basic flow state is slowly varying in the streamwise direction, an assumption the Laplace transform is not capable of accommodating because it assumes a fixed wave number decomposition and, in order to capture the disturbance evolution downstream, it is needed to allow the wave number to be variable.

Instead a method-of-multiple-scales approach is taken in which a disturbance is assumed of the form $\phi' = \underbrace{\tilde{\phi}(\bar{x}, y)}_{\text{shape}} \underbrace{\Phi(x, t)}_{\text{wave}}$ where the wave part satisfies $\frac{\partial \Phi}{\partial x} = i\alpha(\bar{x})\Phi$ and $\frac{\partial \Phi}{\partial t} = -i\omega\Phi$. Notice the multiple-scales introduction of a slow variable $\bar{x} = \frac{x}{Re}$ into the shape function and streamwise (x) complex exponential $\alpha(\bar{x})$, where $Re = \frac{U_e \delta_r}{\nu_e}$ is a Reynolds number based on characteristic values of edge velocity (U_e), edge kinematic viscosity (ν_e), and reference boundary-layer length scale ($\delta_r = \sqrt{\frac{\nu_e x}{U_e}}$). Formally, this is the result of a Fourier transform. Thus, PSE considers disturbances of the form

$$\phi' = \int_{-\infty}^{\infty} \underbrace{\tilde{\phi}(\bar{x}, y, \omega)}_{\text{shape}} \underbrace{A(\bar{x}, \omega)e^{-i\omega t}}_{\text{wave}} d\omega\tag{2}$$

where $A(\bar{x}, \omega) = e^{i \int \alpha(\bar{x}, \omega) d\bar{x}}$ and the dependence of the shape function ($\tilde{\phi}$) and amplitude function (A) on ω has been made explicit. The shape and amplitude functions are essentially the Fourier transform of the disturbance. Upon expansion of the streamwise derivatives it is found that the second spatial derivative $\frac{\partial^2 \tilde{\phi}}{\partial \bar{x}^2}$ is of highest order and a perturbation expansion may be consistently truncated resulting in the neglect of this term. This leaves the disturbance equation nearly parabolized and an efficient marching solution may be sought. Recently, Kuehl¹⁶ has extended to NPSE methodology to consider gaussian wave packets, which allows for spectral broadening, low frequency generation¹⁹ and more physically consistent representation of the instability physics than the traditional NPSE formulation.,

B. High-order structured compact finite-difference solver in curvilinear coordinates: **CFDSU**

CFDSU solves fully-compressible Navier-Stokes equations on a structured curvilinear grid using sixth-order compact finite difference scheme.²⁰ The staggered variable arrangement used in this code in particular was shown to retain near spectral accuracy at very high wavenumbers,²¹ which is an indispensable characteristic to solve the ultrasonic waves present at hypersonic flight conditions.

The discretized Navier stokes equations are solved in an uniform and orthogonal spatial domain and, to do that, a coordinate system transformation of a general curvilinear coordinate system is executed. It is possible to write the governing equations for of the flow in a curvilinear coordinate system (x^1, x^2, x^3) based on the contravariant velocity terms $v^i = dx^i/dt$ but we also need to define the cartesian coordinate system (x_1, x_2, x_3) where the Navier stokes equation is originally derived. The equations shown here were presented by Nagarajan et al.²²

$$\frac{\partial J\rho}{\partial t} + \frac{\partial}{\partial x^j}(J\rho v^j) = 0 \quad (3)$$

$$\frac{\partial J\rho v^i}{\partial t} + \frac{\partial}{\partial x^j}(J\rho v^i v^j + Jpg^{ij} - J\sigma^{ij}) = -J\Gamma_{qj}^i(\rho v^q v^j + pg^{qj} - \sigma^{qj}) \quad (4)$$

$$\frac{\partial JE}{\partial t} + \frac{\partial}{\partial x^j}(J(E+p)v^j + Jq^j) = \frac{\partial}{\partial x^k}(J\sigma^{ij}g_{ik}v^k) \quad (5)$$

In this frame of reference the total energy, the viscous stress tensor and the heat flux vector are described by slightly modified relations described below.

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho g_{ij}v^i v^j \quad (6)$$

$$\sigma^{ij} = \mu(g^{jk}\frac{\partial v^i}{\partial x^k} + g^{ik}\frac{\partial v^j}{\partial x^k} - \frac{2}{3}g^{ij}\frac{\partial v^k}{\partial x^k}) \quad (7)$$

$$q_i = -kg^{ij}\frac{\partial T}{\partial x^i} \quad (8)$$

In the notation used the superscripts indicate the contravariant tensors and the subscripts the covariant ones. Following the convention, g^{ij} is the contravariant metric tensor and the covariant is g_{ij} . Following, Γ_{qj}^i is the Christoffel symbol of the second kind, which is the representation of a second order derivative of a vector or a first order derivative of a second order tensor and J is the Jacobian of this transformation, which is the determinant of the Jacobi matrix ($J_{ij} = \partial x_i / \partial x^j$).

These metrics components are responsible for translating the curvilinear coordinates into its cartesian component and accounting for the effects of using a non-inertial reference frame.

$$g^{ij} = \frac{\partial x_i \partial x_j}{\partial x^k \partial x^k} \quad (9)$$

$$g_{ij} = \frac{\partial x^k \partial x^k}{\partial x_i \partial x_j} \quad (10)$$

$$\Gamma_{qj}^i = \frac{\partial x_i}{\partial x^l} \frac{\partial^2 x^l}{\partial x_q \partial x_j} \quad (11)$$

For this work, the three general cartesian coordinate system directions are named as $x_1 = X$, $x_2 = Y$, $x_3 = Z$ and the three general curvilinear directions are related to a cylindrical grid transformation where $x^1 = X$, $x^2 = r$, $x^3 = \theta$. In addition, the simulations performed were limited to axissymmetric, therefore the third direction is discarded from this point on. Another grid transformation step is introduced in order to display results in the streamwise (x) and wall normal (y) directions.

Table 1: Flow parameters from Purdue’s compression cone experiments¹²

Re_m	ρ_e [kg/m ³]	U_e [m/s]	T_e [K]	T_w [K]
10.8×10^6	0.0417	864.31	51.65	300

III. Physical Model and Computational Approach

The BAM6QT tunnel conditions are used as the basis of this numerical study (Table 1). These flow conditions are applied over the flared cone geometry used in the Purdue University experiments.¹² The cone was modeled as a smooth surface with a 3-meter-radius flare. The initial half angle is $\theta_i = 1.32$ degrees, the total axial length is $L = 516.89$ mm, the half base is $b = 57.15$ mm, and the nose-tip radius is $r_{tip} = 0.15$ mm, as shown in figure 2.

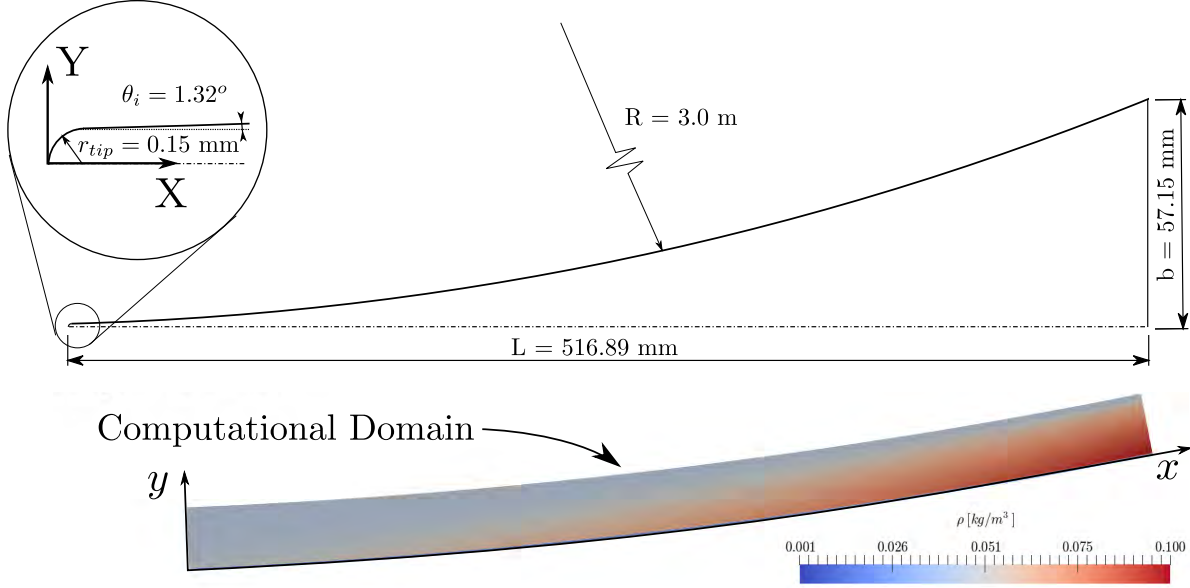


Figure 2: Flared cone geometry and computational domain arrangement.

The grid used for the boundary layer computations is a structured polar grid with the origin at the center of the circle that describes the surface of cone. The choice of polar coordinates guarantees orthogonality of gridlines, which leads to a well-conditioned grid transformation step, and is advantageous since it can be simply described analytically. However, using this grid generation strategy has the disadvantage of retaining an unnecessary high resolution in the streamwise direction away from the wall. This concern is mitigated by observing that the radius of curvature is large compared to the height of the computational domain, their ratio being of the order of 100. The polar grid is stretched in the wall-normal direction with a half-tangent hyperbolic law to resolve the flow gradients in the boundary layer.

DNS of hypersonic flows typically^{23,24} use low-order precursor simulations, capturing the laminar flow around the geometry, to inform inlet and initial conditions for high-order structured simulations. The latter are designed to solve all the scales involved in the transition dynamics of the boundary layer. The high-order simulations are first carried out with no perturbations, allowing the mean flow to be fully established and all residual perturbations that arise from flow initialization and retained by the low-dissipation numerics to be convected out of the domain.

The left and the top boundary of the high-order axisymmetric simulations are treated as inflow, i.e. with Dirichlet boundary conditions imposing the flow conditions established by a precursor calculation. At the wall, no-slip and no penetration conditions are imposed everywhere except when suction and blowing is applied to generate perturbations. The wall is also assumed to be isothermal at the temperature of 300 K justified by the small duration of the experimental runs, which do not allow sufficient time for the

model to change its temperature. Neumann boundary conditions are imposed at right boundary. To prevent oscillations that could appear in the subsonic portion of the domain and to ensure the numerical stability of the simulations, the inlet and outlet boundaries are coupled with a buffer layer.

IV. Disturbance Evolution over a Hypersonic Flared Cone

This section aims to simulate and analyze the perturbation growth mechanism in a hypersonic flow over a flared cone through the excitation of the boundary layer with a broadband frequency pulse. In the future, we aim to use this canonical setup with an almost constant boundary layer thickness to study the principal energy source and dissipation mechanisms for second mode waves at very large amplitudes.

Precursor simulations of the Purdue's compression cone flow conditions (Table 1) were obtained using **US3D**^{25–27} and they were applied as initial condition to a high order simulation in **CFDSU**. The initialization step relies on the interpolation of the precursor solution onto a new grid. This triggered a transient adjustment phase of the numerical solution by **CFDSU**. Figure 3 shows the boundary layer profiles obtained after an unperturbed state is reached over the flared cone geometry. An excellent agreement for different grid refinement levels is shown. It is important to highlight that the adverse pressure gradient, induced by the flared geometry, effectively keeps the boundary layer thickness constant along the majority of the length of the cone.

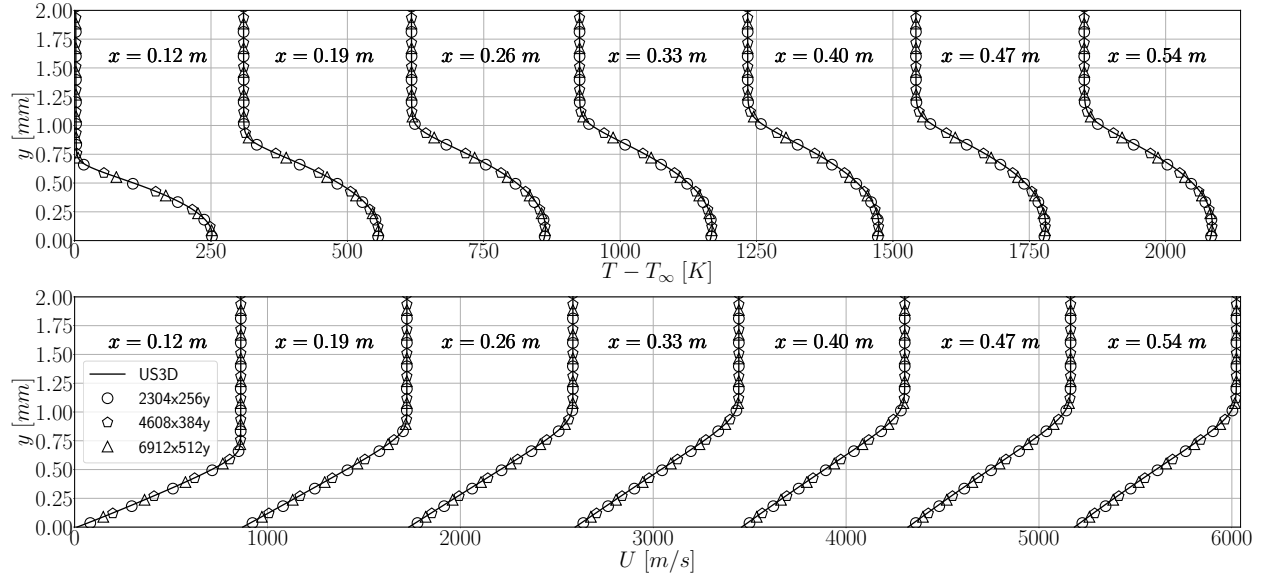


Figure 3: Boundary layer profiles for temperature and streamwise velocity as a function of distance from the wall at different locations at the cone surface for the precursor and high order calculations. Symbols are from **CFDSU** simulations and the solid line is **US3D**.

A. Comparison with NPSE

Initially a pulse with a duration of approximately $1.67 \mu\text{s}$ and maximum amplitude of $1.0 \times 10^{-6} \text{ m/s}$ was applied via suction and blowing at the wall near to the beginning of the computational domain, after the inlet buffer layer becomes negligible. The region of excitation was centered around $X = 0.075 \text{ m}$ and had a length of 2 cm , with the following expression for the wall normal velocity at the wall:

$$v = \cos(\pi x_l)^3 \sin(2\pi f t), \quad \text{for } 0 \leq t \leq 1/f \quad (12)$$

where x_l is a variable with values between $[-1, 1]$, which is mapped to the actual spatial interval of application of the pulse, and $f = 600\text{kHz}$ is the frequency around the which the pulse spectrum is centered.

A Fourier analysis of the wall pressure disturbance at different streamwise spatial locations as the pulse advects downstream is shown in figure 4. The initial pressure wave amplitude effectively received by the

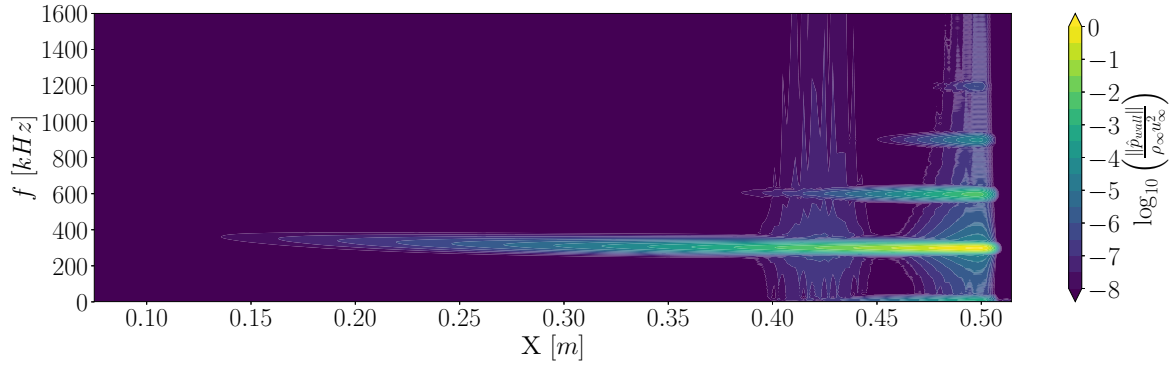


Figure 4: Amplitude spectrum of the pressure disturbance at the wall due to advection of a broadband pulse with amplitude of $1.0 \times 10^{-6} \text{ m/s}$ over a flared cone.

boundary layer from the disturbance imposed at the wall is of the order of $1.0 \times 10^{-4} \text{ Pa}$ and, therefore, only linear mechanisms of growth are expected to be excited initially and any subsequent excitation of higher harmonics can be attributed to nonlinear effects developed within the flow, and not to the external forcing.

Initially, only one narrow frequency band, centered around 296 kHz is observed. This result is consistent with previous experiments performed by Wheaton et al,¹² who reported a maximum amplitude at 295 kHz. As the waves propagate downstream along the flared cone two trends can be highlighted. First, there is a small shift towards lower frequencies consistent with a small growth in the boundary layer thickness, not fully counteracted by the adverse pressure gradient effects, which establish themselves in full later. Second, the continuous energy injection focused on a narrow band of frequencies makes them increase rapidly in amplitude, i.e. by 8 orders of magnitude in less than half a meter of streamwise evolution and triggering nonlinear effects, such as the appearance of higher harmonics. A disturbance with the same initial amplitude evolving over a straight cone at similar flow conditions will have a much lower cumulative growth, since, as the waves propagate into regions of higher boundary layer thicknesses, the most amplified frequencies shift to lower values.

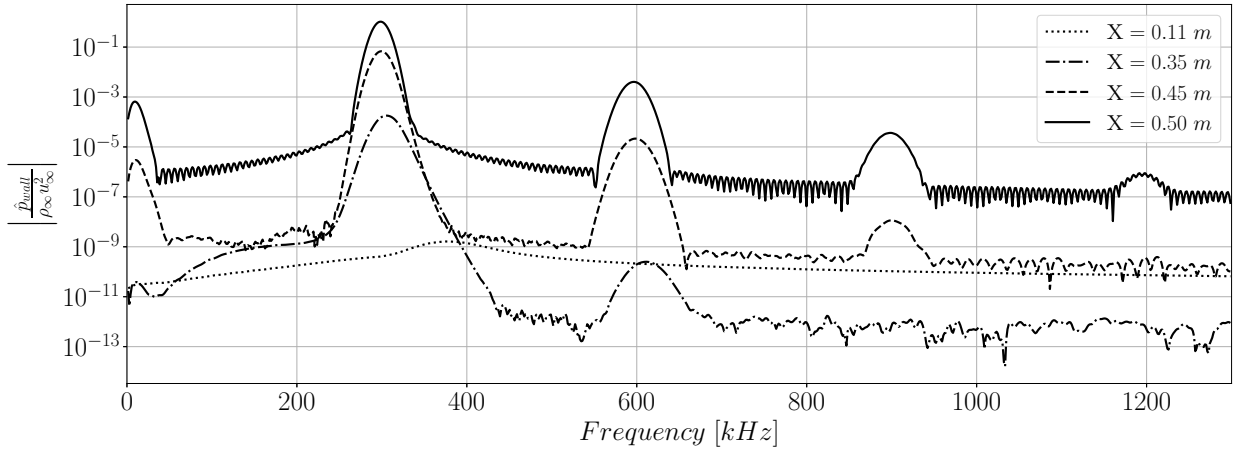


Figure 5: Amplitude distribution over frequency at different spatial locations of the pressure signal at the wall cause by a pulse disturbance with amplitude of $1.0 \times 10^{-6} \text{ m/s}$.

In order to depict better some characteristics of the perturbation advection and spectral broadening, the power spectrum of the pressure signal at the wall at specific locations is show in figure 5. At $X = 0.11 \text{ m}$, one can observe the pressure perturbation near the excitation location and its broadband characteristic and low amplitude. At this location, it is possible to observe the beginning of the energy concentration in the

unstable frequency band of the primary perturbation. At $X = 0.35 \text{ m}$ the first evidence of nonlinear dynamics starts to appear as the first harmonic rises from the round-off noise floor. In the following two locations the excitation of the next two harmonics is observed. Another interesting fact that can be outline from this plot is the growth of a zero frequency perturbation. This component is associated with the mean flow distortion caused by the relatively high amplitude perturbations and it is another evidence of the presence of nonlinearities in the flow.

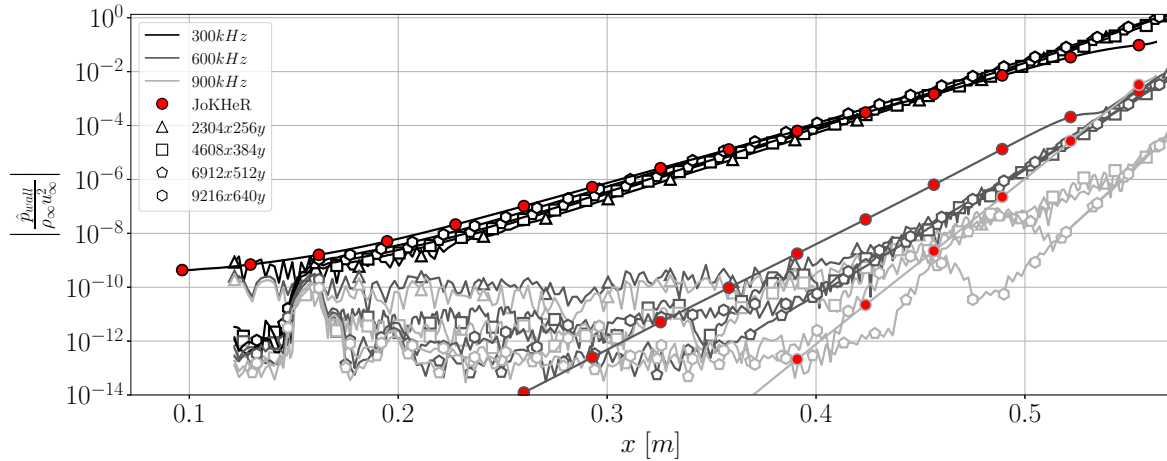


Figure 6: Comparison of the current DNS simulations with results obtained from nonlinear PSE.

We compared the results obtained in the current DNS simulations with NPSE calculations made in the JoKHeR (see section II.A) and these are gathered in figure 6. The results shows that, for all grids simulated, the growth rate of the primary frequency is in good agreement. As the grid is refined the estimated growth rates for the higher harmonics also start to exhibit agreement with NPSE but there is disagreement in the spatial location where the harmonics start to become important and in the region where saturation the wave is reached. This is consistent with the fact that disturbances of higher frequencies need more points to be accurately resolved with minimum numerical dissipation and therefore it is expected that increasing the grid resolution will lead to better matching of the results. It is noteworthy to say that the noise floor decreases monotonically as the grid is refined. To capture the growth rates of the 900 kHz mode, a grid resolution of at least $n_x = 9216$ and $n_y = 640$ was required.

B. Spectral Broadening and Physical Implications

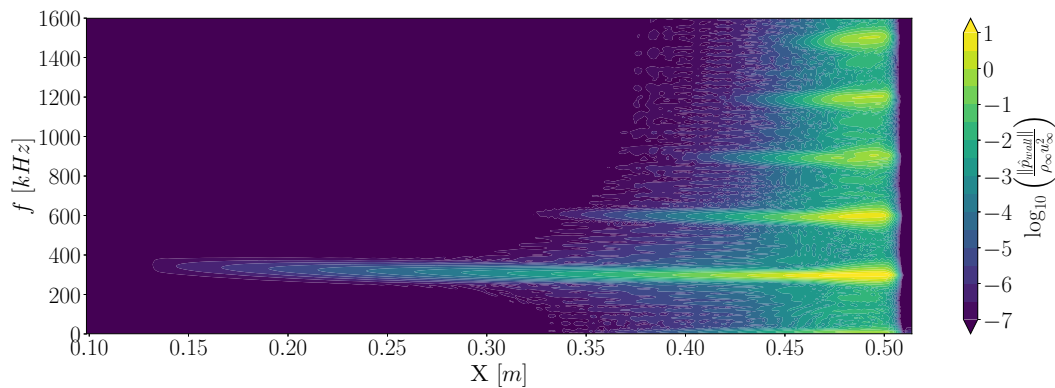


Figure 7: Amplitude spectrum of the pressure disturbance at the wall caused by advection of a relatively high amplitude broadband pulse with amplitude of $1.0 \times 10^{-4} \text{ m/s}$ over a flared cone.

The results shown in the previous section focus on the development of the perturbation from linear to nonlinear and its comparison against another prediction methods. Because of that, the imposed disturbance only reaches a weakly nonlinear region in the limited spatial domain simulated. In order to show interesting phenomena occurring when highly nonlinear effects are present in the boundary layer, a pulse with the same period and spatial distribution but a higher amplitude of $1.0 \times 10^{-4} \text{ m/s}$ was applied.

Figure 7 gathers all the information about power distribution for different frequencies and for different locations at the surface of the flared cone. This figures depicts all the trends of power concentration on specific frequency bands, higher harmonics and mean flow distortion discussed previously with the difference that it acts in a different amplitude range. This increase in amplitude enhance the importance of the nonlinear effects to the point that very high frequency harmonics are fully developed and have similar amplitudes, indicating the presence of saturation.

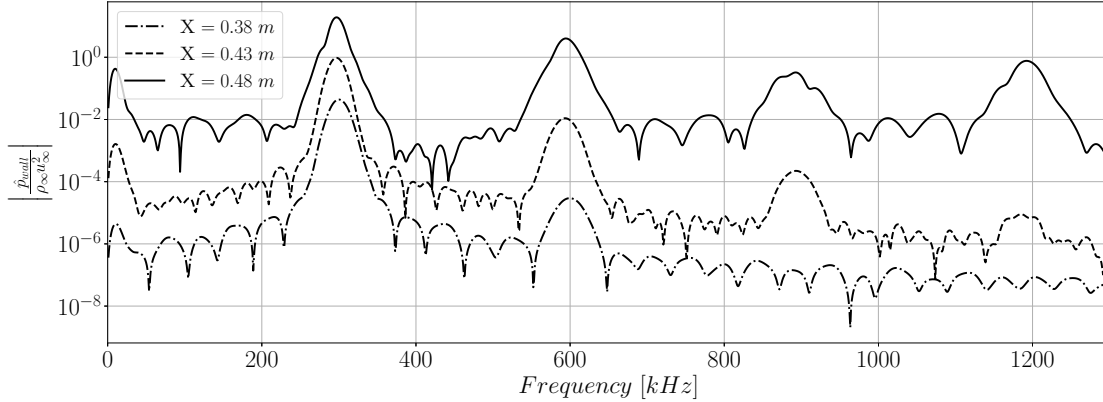


Figure 8: Amplitude distribution over frequency at different spatial locations of the pressure signal at the wall caused by advection of a relatively high amplitude broadband pulse with amplitude of $1.0 \times 10^{-4} \text{ m/s}$.

Towards the end of the computational setup it is also observed, in figure 7, that the frequency bands corresponding to the principal wave and its harmonics becomes wider, an evidence of spectral broadening. The amplitude distribution over frequency for the advected disturbance is shown for specific spatial locations in order to better depict the described phenomenon in figure 8. In this plot it is possible to observe that the excited frequency bands stay concentrated up to $X = 0.43 \text{ m}$ but, at the downstream probed location ($X = 0.48 \text{ m}$), the disturbance is spreads out and spans a larger portion of the frequency domain. This is an intermediate step before the final breakdown into turbulence, not captured in the current axysymmetric simulations, where the energy is distributed into all the scales until viscous dissipation allows.

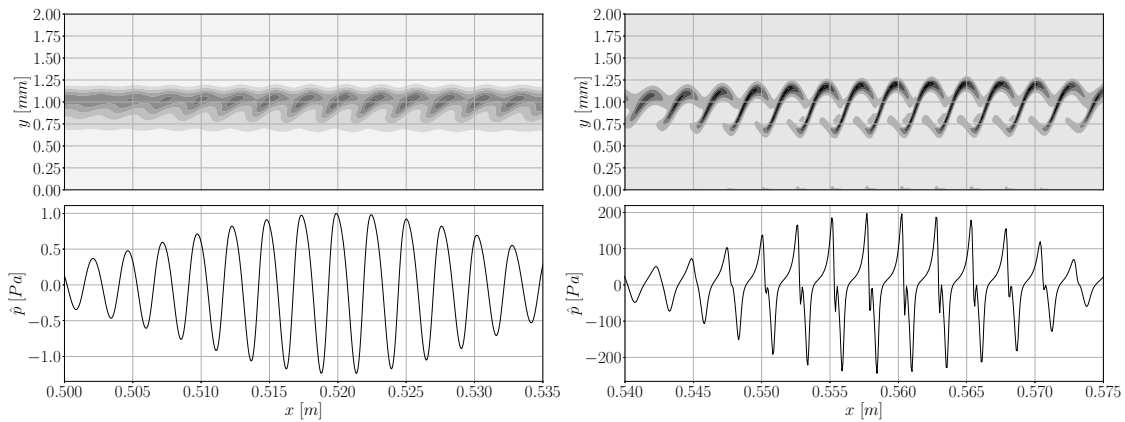


Figure 9: Appearance of “rope waves” in the numerical Schlieren is connected to non-linear interactions between second mode waves of high amplitude

The increase in amplitude and the highly nonlinear interactions that happen in the aft region of the flared cone lead to the appearance of entangled structures at the edge of the boundary layer when the magnitude of the density gradient becomes significant (figure 9). Since the optical technique of measuring second mode waves based on Schlieren relies on density gradients, a numerical Schlieren rendering is used to create a parallel with experimental results. It is interesting to highlight that these structures of fluctuations is only present in highly nonlinear regions and this is consistent with the experimental and numerical observations that the “rope waves” appear in the region preceding transition to turbulence.^{9,11,13} The simulated “rope waves” are also in agreement with the observations made by Stetson⁶ that the second mode has a wavelength of approximately two times the boundary layer thickness.

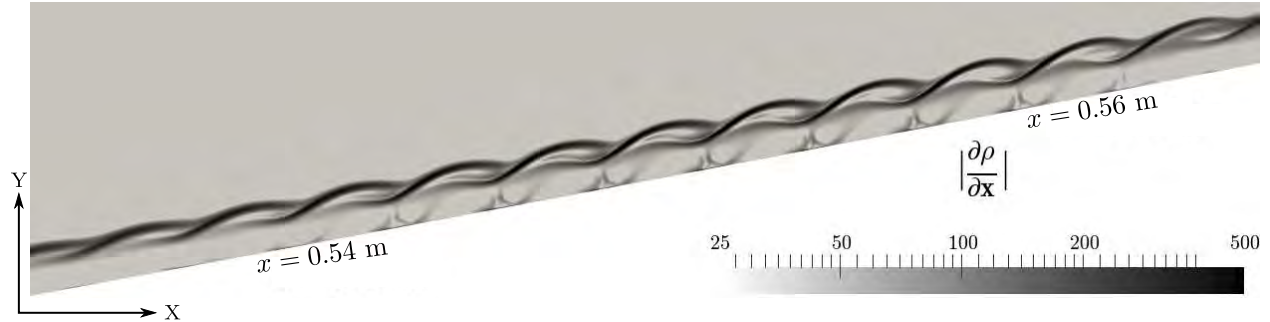


Figure 10: “Rope waves” form at boundary layer edge at the end of the computational domain in physical coordinates.

Since the physical domain is transformed into wall normal and streamwise coordinates in figure 9, the numerical Schlieren is replotted in figure 10 in physical coordinates. In this figure, the entanglement of the spatial variation of amplitude is more visible and, in addition, regions of compression and rarefaction inside the boundary layer are observed.

V. Summary

The spectral broadening of the second mode over a hypersonic flared cone was addressed in this work. A high order compact finite difference scheme was initialized from the laminar mean flow solution of a precursor simulation and was used to numerically predict the perturbation’s evolution downstream. It was observed that the flared cone designed was able to create an adverse pressure gradient that maintained the boundary layer thickness approximately constant over the entire cone. This effect concentrated the growth mechanisms of the second mode wave in a single frequency band, up to triggering nonlinear effects such as mean flow distortion and higher harmonics. The results of linear growth and nonlinear interaction was compared with a nonlinear parabolized stability equations (NPSE) solver and a good agreement was achieved. A pulse of higher amplitude was then introduced to visualize the phenomena of spectral broadening and that the so called “rope waves”, which develop only in the region of highly nonlinear interactions.

In conclusion, this work represents an intermediate step towards the understanding the relevant contributions of the nonlinearities present in a hypersonic flow and the principal energy source mechanism for second mode waves when they reach very high amplitudes. In the end, we demonstrate that we have developed the necessary tools for analyzing this in a future contribution.

Acknowledgments

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Hypersonic Transition Delay Over Complex Wall Impedance

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We have carried out numerical simulations of hypersonic transition delay on a spatially developing hypersonic boundary layer over a 7° -half-angle cone at $M_\infty = 7.5$ for both sharp and 2.5mm-round-tip geometries due to porosity representative of carbon fiber reinforced carbon ceramics. The acoustic absorption of carbon/carbon (C/C) porous surfaces was experimentally measured and this information was used to reconstruct the dependence of the real and imaginary parts of impedance in the frequency domain via low-order models. Ultimately, a complex impedance boundary condition (IBC) is used to introduce the porosity effects in the computational domain. The flow conditions simulated are at Reynolds numbers $Re_m = 2.43 \cdot 10^6 \text{ m}^{-1}$ and $4.06 \cdot 10^6 \text{ m}^{-1}$ for the sharp cone and $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ and $6.40 \cdot 10^6 \text{ m}^{-1}$ for the round tip with isothermal walls at $T_w = 300 \text{ K}$. First, axisymmetric simulations are performed over impermeable walls and IBC, in which the second mode is excited via the application of a broadband pulse. The spatial amplification of individual frequency modes compares favorably against linear stability analysis. The matching obtained supports the correct numerical implementation of the IBC. Following, three-dimensional simulations are performed and the boundary layer is excited with a grid independent filtered pseudo-random pressure perturbations, these are received and amplified ultimately leading to transition onset and turbulent breakdown. [Victor: Discovered things after this is done.](#) Transition delay is confirmed but with different dynamics.

1. Introduction

1.1. Background and Relevance

Laminar-to-turbulent transition has critical implications on the design, performance and safe operation of hypersonic vehicles. At such high speeds, aerodynamic and thermal loads, which are already significant under laminar flow conditions, can increase dramatically in the presence of turbulence. For example, ? made a comparison between fully turbulent and fully laminar flow conditions for a low-Earth orbit hypersonic flight vehicle concluding that doubling the weight in thermal protection systems (TPS) would be necessary due to order of 5 increase in heat flux into the vehicle under turbulent conditions. A different study, developed in during the National Aerospace Plane project and led by the Defense Science Board committee (?), reached similar conclusions regarding the operation performance a such a vehicle, reporting three-time payload-to-weight ratio decrease comparing fully laminar and fully turbulent conditions. These studies emphasize

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the importance of laminar-to-turbulent transition delay for sustainability of high-speed flight.

Considering a low disturbance environment such as the flight conditions, transition to turbulence over smooth surfaces happen due to the incorporation of the free stream disturbances by the boundary layer, i.e. receptivity, followed by the modal growth of unstable waves, whose behavior is accurately described by linear stability theory (LST), these then trigger nonlinear interactions and, ultimately, turbulence breakdown (?). In supersonic and hypersonic flows, the dynamics of the first and second modal instabilities are of great importance for transition. Their contribution is even higher considering the slender and two dimensional geometries used in the majority of designs of high speed vehicles, over which the effects of other flow instabilities such as Görtler vortices and crossflow are minimized.

The first mode is the high-speed correspondent of the Tollmien-Schlichting unstable waves that govern transition in low speed flow. Their difference relies on the fact that, at low speeds, the T-S waves are predominantly viscous instabilities with vortical nature but, as the free stream velocity increases and compressibility effects progressively gain importance, the presence of a generalized inflection point in compressible boundary layer profiles (?) leads to a change in the instability characteristic from viscous to inviscid, which dominates at high speeds. Moreover, the second mode is an acoustic instability propagating inside the boundary layer and driven by a region of supersonic mean flow relative to the disturbance phase velocity (?). As the Mach number increases, the relative importance of the second mode to the turbulence transition increases and surpasses the first mode. For adiabatic wall conditions, this happens around $M = 4$ (?). Additionally, the presence of cooled walls ($T_w < T_{adb}$) is known to destabilize the second mode while having the opposite effect to the first mode's growth rate. Commonly the case for hypersonic vehicles, the second mode dominance can happen at even lower speeds. Mack's studies on the evolution of these and higher modes (?) in two dimensional boundary layers stressed the importance of second-mode attenuation in any attempt to increase the transitional Reynolds number in high speed flows.

1.2. High Speed Transition

After the growth of unstable frequencies, the following step towards turbulence breakdown is the nonlinear interaction between the disturbances present inside the boundary layer. The principal mechanisms of nonlinear interaction preceding turbulence were classified [Victor: you asked here by whom and I could not find an initial paper, it seems to be something embedded in the community](#) as oblique breakdown, when two oblique instability waves with equal but opposite wave angles interact, and resonance, when a two-dimensional or axisymmetric wave reaches an amplitude level sufficient to excite a secondary oblique instability at the same frequency or half of the main mode frequency. These were named, respectively, fundamental and subharmonic resonance. [Victor: you commented here about the possibility of wave steepening being a third mechanism and I think it could be true since these initial studies were mostly incompressible. The high mach number flows studies of nonlinear interaction followed the lead of the low speed community. Also, isn't wave steepening the appearance of higher harmonics in the sense of a spectral burgers equation solution?.. what you want to say here is that we think that at high mach numbers this could be mainly a 2d phenomenon?..](#)

The oblique breakdown path was first seen as a viable path to reach transition by ? in direct numerical simulations of a flat-plate boundary layer transition at Mach 1.6. The oblique breakdown path was also reported as feasible by the subsequent study of

?, who performed a nonlinear parabolic stability equations assessment of flows at low supersonic Mach numbers. In a temporal evolution simulation of a Mach 4.5 boundary layer, ? investigated the transition mechanisms and reported subharmonic resonance had the strongest interaction before breakdown. Since the simulation was performed with temporally evolving parallel flow setup, a constant growth of the second mode was enforced, and ultimately the behavior of a spatially developing boundary layer was not accurately represented. ? performed a DNS of a flat plate boundary layer that developed in space and concluded that fundamental resonance was much stronger than subharmonic resonance and that both of those needed a higher spatial extent to lead to transition than the first mode oblique breakdown path.

Since cones are simple and useful prototypical shapes (?) for a flight vehicle's nose region, the investigation of high speed transition followed to study the flow over conical geometries. Early experiments of the transition over slender conical shapes (??) reported the second mode as the most important unstable wave. Following, ? performed wind tunnel transition experiments at Mach 8 over a 7-degree-half-angle cone and reported the existence of higher harmonics but no evidence of a subharmonic instability. The same flow conditions were used in a DNS performed by ?. They investigated different possible transition routes and concluded that subharmonic resonance was indeed unlikely and suggested second mode oblique breakdown as a viable path.

More recently, ? used a broadband pulse to study a natural transition scenario in hypersonic conical boundary layers and concluded that both fundamental resonance and second mode oblique breakdown are viable transition paths at such flow conditions. **Victor: you comment here that we should say that we use the same technique to validate our code against LST but this is said in the axisymmetric section and much more deeply, so I would rather not repeat it..** A controlled disturbance simulation of fundamental resonance under the same flow conditions was performed by ? and a detailed description of the nonlinear interactions observed during this path to complete breakdown is given. Some discrepancies between controlled disturbance transition simulations and experiments were observed and, later, ? proposed using a pseudorandom forcing as a tool to investigate nonlinear breakdown without prioritizing any specific mechanism. The proposed strategy was shown to confirm the relevance of fundamental resonance as a nonlinear breakdown path when compared against controlled disturbance simulations and experimental data of transition over a flared cone acquired by ?. Moreover, the pseudorandom perturbation technique also led to an improved matching against the mean wall heat flux measured experimentally in comparison to what was predicted by introducing controlled disturbances.

1.3. Transition Delay over Porous Walls

? proved through Linear Stability Theory (LST) the capability of ultrasonically absorbing coatings (UACs) to mitigate the second mode, confirmed by experiments that followed. The first experimental validation of this principle is due to ?, who constructed a cone with a half angle of 5 degrees with two different surfaces: one smooth and impermeable, and the other perforated with regularly spaced micro holes. They reported that the porous surface was capable of doubling the transitional Reynolds number in comparison with the smooth surface in experiments at Mach 5 at the T-5 hypersonic wind tunnel located at Caltech.

The development of UACs continued, with the next goal of combining the acoustic absorption and TPS characteristics into one solution. Knowing that an irregular structure is typical for TPS materials used for hypersonic vehicles, (??) conducted experiments in a Mach 6 wind tunnel, named T-326 and located at the Institute of Theoretical and Applied

Mechanics (ITAM) in Novosibirsk, first over a porous surface with regular microstructure, and then over a felt-metal surface. These experiments were the first to produce data capable of quantitatively demonstrating the attenuation of the second mode waves via UACs, also confirmed by stability analysis.

? performed a bispectral analysis on this same data with the goal of identifying nonlinear mechanisms that could be triggered by the use of porous coatings; this was motivated by the observation of a weak enhancement of first-mode waves. The experiments revealed the occurrence of nonlinear phase locking involving first mode waves, in the presence of porous walls and not over an impermeable surface. This effect was also shown to be small, not impacting the overall stabilizing role of the porous coating. More recently, ?, performed new experiments in ITAM's tunnel, where it was shown that placing the porous insert in the region where the second mode is stable leads to an increase in the disturbance signal amplitude; the opposite is observed when the insert is placed in regions of second-mode growth. Angle of attack effects, with $\alpha < 1^\circ$, was studied experimentally in the same tunnel by ?, who reported that the porous coating was able to attenuate second mode disturbances on either windward or leeward sides of the cone.

Previous high-fidelity numerical studies of attenuating, canceling, or reinforcing second-mode instabilities in high-speed flow over porous walls have focused on uniformly spaced and/or geometrically regular porosity in both temporally developing boundary layers (???). Two-dimensional simulations of the influence regular and irregular porous surfaces on the disturbances of spatially developing boundary layers over a flat plate (???) and over a axisymmetric cone (?) where also performed based on the analytical relation developed by ?, which couples the vertical velocity disturbance with the wall pressure disturbance multiplied by the real part of the porous wall admittance combined with the time derivative of the wall pressure disturbance multiplied by the imaginary part of the admittance and divided by the angular frequency. Being the admittance defined as the inverse of the impedance, this is effectively a time domain impedance boundary condition but, differently then the formulation using in the current and previous studies, this implementation is limited to predicting the effect of porous walls on single frequency disturbance spectra. Previous numerical simulations on the effects of porous walls on boundary layers were limited to the investigation of the stability and receptivity for individual frequencies tested in different simulations. ? pioneered using a different technique of implementing a time domain impedance boundary condition that is capable of imposing the acoustic effects of a porous surface on all the frequency components simultaneously and showed the attenuation effect on a broadband second mode disturbance advecting over an axisymmetric conical boundary layer. Being a broadband IBC, this new technique is capable of modeling the porous walls effects on the whole frequency spectrum of the boundary layer transition and also in the fully turbulent regime.

1.4. Carbon/Carbon Ultrasonic Absorbing Coatings

Inspired by some of these results and the need for any realizable technology for boundary layer control to be symbiotic with TPS, ?? established the use of carbon-carbon (C/C), an intermediate state of C/C Silicon Carbide (C/C-SiC) already employed on hypersonic vehicles (??), in controlling second-mode waves. Experiments at Mach 7.5 have been conducted in the DLR High Enthalpy Shock Tunnel Göttingen (HEG), and the stabilization of the second mode as well as an increase in the laminar portion of the boundary layer over the porous surface have been observed. For these flow conditions, ? has performed an analysis based on the parabolized stability equations, confirming

numerically the effectiveness of porous surfaces based on C/C in achieving transition delay.

A similar C/C surface was used in a 3° half angle sharp cone on different experimental runs in a conventional and a quiet wind tunnel at Mach 6. ? performed experiments in the Boeing/AFOSR Mach-6 Quiet Tunnel (BAM6QT) at Purdue University and in the hypersonic wind tunnel (H2K) of the German Aerospace Center (DLR) in Cologne and reported no significant attenuation nor transition delay over the porous surfaces. ? infers that the reason for this this comes from the flow conditions chosen for the study led to very low pressures at the model's surface and low-frequency second mode waves (lower than 150 kHz) and, at these conditions the carbon/carbon surface has poor acoustic absorption capabilities. In appendix A.4, we perform an LST analysis of the flow conditions reported on the experiments in order to confirm the previously made claim.

1.5. Manuscript Outline

The current work aims capturing the effects of second-mode propagation in a spatially developing boundary layer over assigned complex broadband wall-impedance representative of a distributed random porosity based on a C/C material, in order to investigate its influence in the full transition path from linear growth, nonlinear interactions and turbulent breakdown in an attempt to replicate numerically the experiments performed by (?) and get further insight to the flow physics present in the setup.

In order to do this, the computation methodology is first presented in section 2. This includes the computational domain discretization, the governing equations solved and the numerical scheme used to solve them as well as the boundary conditions used in the simulations. Special attention is given to the time-domain impedance boundary conditions (TDIBC) used to introduce the acoustic effects of the C/C surfaces without the need to solve all its intricate geometric structure. In Section 3, the characterization of the C/C's surface acoustic properties are described and low order models are used to represent the impedance of this surface in the frequency domain. In section 4 we perform axis symmetric simulation of the base flows around the sharp and blunt conical geometries chosen for this study and also compare the results from of second mode linear growth computed by the simulations with the one predict by the linear stability equations. Ultimately, In section 5 we perform three-dimensional simulations of the full transition path for different cone tip bluntness and Reynolds numbers and assess the effects of wall porosity on the transitional and ultimate turbulent structures.

2. Problem Formulation

2.1. Computational Setup

The numerical study performed in this work is based on tests conducted by ? in the DLR High Enthalpy Shock Tunnel Göttingen (HEG). In the present study, test conditions of, $Re_m = 2.43 \cdot 10^6 \text{ m}^{-1}$, $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ and $Re_m = 6.40 \cdot 10^6 \text{ m}^{-1}$ are chosen to represent this parameter space. Since tip-bluntness is know to delay transition, we choose to run the lowest Reynolds number per meter only over a sharp cone with a half angle of 7 degrees, and the highest only to simulate a 2.5mm-round-tip conical model with the same half angle. The intermediate value of Re_m will be used over the two geometrical choices. This choice is made in order to keep the computational domain necessary to simulate the full transition path at a minimum extent in order to decrease the computational cost. With these runs, it will be possible to numerically reproduce the transition delay

Color/Symbol	Re_m [1/m]	M_∞ [–]	p_∞ [Pa]	T_∞ [K]	ρ_∞ [kg/m ³]
★	$1.46 \cdot 10^6$	7.3	789	267	0.0102
◄	$2.43 \cdot 10^6$	7.4	1453	285	0.0177
●	$4.06 \cdot 10^6$	7.4	2129	268	0.0276
◆	$6.40 \cdot 10^6$	7.4	3083	248	0.0432
▲	$9.80 \cdot 10^6$	7.4	5174	265	0.0678

Table 1: Nominal flow parameters from Wagner’s experiments (?). Throughout the paper, each of these flow conditions will be represented by the color here assigned.

experiments caused by the C/C porous surface that were performed with a $R_n = 2.5$ mm. All free stream conditions and the parameters of these runs are reported in table 1.

In the case of the sharp tipped conical model, the initial flow conditions and inflow parameters are informed by combining the Taylor-Maccoll (?) inviscid solution with a viscous solution for the boundary layer. The latter is derived by applying the Mangler transformation for bodies of rotation (?) to the compressible boundary layer similarity solution for a perfect gas over a flat plate (?), as done by ? for the flow over a cone.

On the other hand, the blunt nosed conical model is initialized in the high order structured simulation domain informed by a precursor calculation performed by the DLR FLOWer code, which is a three-dimensional parallel hybrid multi-grid code and has been validated for hypersonic flows (?). In section 4, the matching between the steady Navier-Stokes calculations performed in this study and the semi-analytical flow for the sharp cone and the precursor calculation for the blunt cone will be discussed.

To avoid the computational costs associated with the (infinitesimally) thin boundary layer near the leading edge of the cone, the inlet of the computational domain in this study is located at $x_i = 0.045$ m downstream from the sharp cone tip, while the outlet is located L_x m (see table 2) further downstream, as shown in figure 1. The stream wise extent of the computational domain (L_x) is chosen based on previous experimental results as being the smallest distance needed to be able to capture the full transition path.

The authors, have previously performed two dimensional simulations of the advection of a broadband disturbance over a solid and porous wall on a similar, but not identical, computational setup (?). In such previous work, the grid used to discretize the conical domain is a structured spherical grid with the origin at the tip of the cone; this allowed to concentrate points in the regions of interest by decreasing the amount of points in the free stream above the shock. That grid design, however, leads to an increase in grid spacing normal to the wall as one moves downstream and, in in order to accurately solve the boundary layer near the outflow region the grid in the region of the left boundary had to be over refined. In the end, that strategy induced a high limitation of the time step require for a stable explicit time integration and was changed in favor of the current approach, which decreased considerably the time step allowed by the Courant-Friedrichs-Lewy (CFL) condition, while yielding insufficient resolution in the boundary layer in the transition region.

At the inlet and top boundaries, a Dirichlet boundary condition is used to input the flow velocity calculated by the analytical procedure or by the precursor simulations previously described. Sponge layers that span a region equal to 3% of the total computational domain extent in the streamwise extent (L_x) for the inflow and outflow sponge and equal to 7.5%

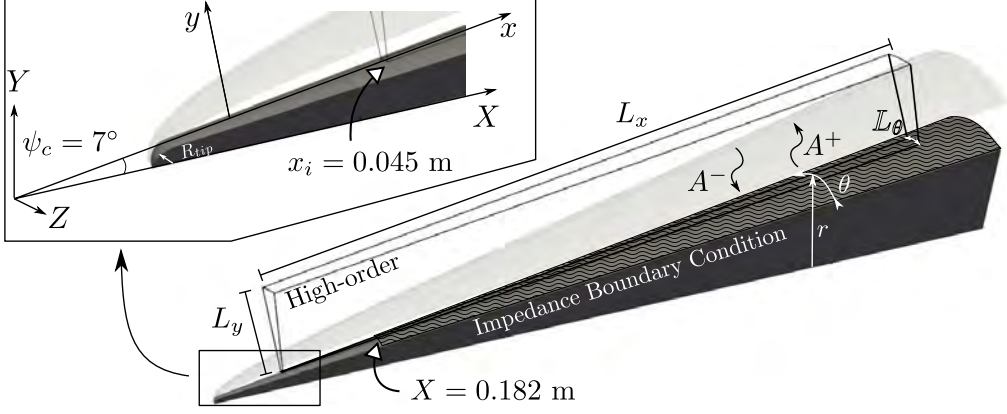


Figure 1: Computational setup of DNS of transition delay over a blunt cone.

$L_x \times L_y \times L_\theta$		Re_m		
		$2.43 \cdot 10^6 \text{ m}^{-1}$	$4.06 \cdot 10^6 \text{ m}^{-1}$	$6.40 \cdot 10^6 \text{ m}^{-1}$
R_{tip}	0.0 mm	$0.8 \text{ m} \times 0.07 \text{ m} \times 8^\circ$	$0.6 \text{ m} \times 0.05 \text{ m} \times 8^\circ$	—
	2.5 mm	—	$0.9 \text{ m} \times 0.07 \text{ m} \times 8^\circ$	$0.8 \text{ m} \times 0.06 \text{ m} \times 8^\circ$

Table 2: Computational domain geometrical parameters for the different flow conditions and cone tips considered. Actual nose tip radius for sharp experimental results is 0.1 mm

the normal extent (L_y) for the top wall sponge, are used at the inlet to control spurious oscillations due to the introduction of the shock in the domain and, at the outlet, to suppress the oscillations in the flow before reaching the computational boundary and to prevent the formation of upstream traveling disturbances arising from the boundary in the subsonic portion of the boundary layer.

Outlet conditions are homogeneous Neumann for all flow quantities. At the wall, Dirichlet conditions are used to impose no-slip, no penetration and, when active, the time domain impedance boundary conditions (TDIBC) used to model the carbon/carbon porous surface. This approach allows to model the acoustic response of the porous surface without the need to resolve its complex geometrical structure and is a way to retain high resolution on the flow side by removing the grid resolution requirements that would be needed to capture the acoustic wave propagation in the pores. Due to the short duration of the test run, the surface of the model does not have time to heat up and thus can be assumed to be isothermal with a temperature of 300 K.

2.2. Governing Equations In Curvilinear coordinates

High-order structured compact-finite-difference simulations are carried out with the objective of capturing the perturbation evolution with minimum numerical dissipation and high resolving power for a given amount of points per wavelength.

The compressible Navier-Stokes equations are discretized in space using a sixth-order accurate compact finite difference method with a staggered grid arrangement (?) and in time using a third order accurate 3-stage explicit Runge-Kutta method. The numerical strategy used yields spectral-like resolution properties, being able to solve the perturbation evolution inside the boundary layer with minimum numerical dissipation

and with high accuracy for a given amount of points per wavelength (?). The scheme implemented is based on Padé finite difference methods and, since these are defined in an equispaced domain, the curved domain is mapped into a rectangle by the use of a covariant grid transformation. The governing equations, after the transformation to the a curvilinear coordinate system (x^1, x^2, x^3) , are shown hereafter.

$$\frac{\partial J\rho}{\partial t} + \frac{\partial}{\partial x^j}(J\rho v^j) = 0 \quad (2.1)$$

$$\frac{\partial J\rho v^i}{\partial t} + \frac{\partial}{\partial x^j}(J\rho v^i v^j + Jpg^{ij} - J\sigma^{ij}) = -J\Gamma_{qj}^i(\rho v^q v^j + pg^{qj} - \sigma^{qj}) \quad (2.2)$$

$$\frac{\partial JE}{\partial t} + \frac{\partial}{\partial x^j}(J(E+p)v^j + Jq^j) = \frac{\partial}{\partial x^k}(J\sigma^{ij}g_{ik}v^k) \quad (2.3)$$

In this frame of reference the total energy, the viscous stress tensor and the heat flux vector are described by slightly modified relations described below:

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho g_{ij}v^i v^j \quad (2.4)$$

$$\sigma^{ij} = \mu \left(g^{jk} \frac{\partial v^i}{\partial x^k} + g^{ik} \frac{\partial v^j}{\partial x^k} \right) + \left(\beta - \frac{2}{3}\mu \right) g^{ij} \frac{\partial v^k}{\partial x^k} \quad (2.5)$$

$$q_i = -kg^{ij} \frac{\partial T}{\partial x^i} \quad (2.6)$$

where μ is the shear viscosity, β is the bulk viscosity and k is the conductivity. In the notation used the superscripts indicate the contravariant tensors and the subscripts the covariant ones. Following the convention, g^{ij} is the contravariant metric tensor and the covariant is g_{ij} . Γ_{qj}^i is the Christoffel symbol of the second kind, which is the representation of a second order derivative of a vector or a first order derivative of a second order tensor and J is the Jacobian of this transformation, which is the determinant of the Jacobi matrix ($J_{ij} = \partial y^i / \partial x^j$). These metrics components are responsible for translating the curvilinear coordinates into its Cartesian component and accounting for the effects of using a non-inertial reference frame.

$$g_{ij} = \frac{\partial y^i \partial y^j}{\partial x^k \partial x^k}, \quad g^{ij} = \frac{\partial x^k \partial x^k}{\partial y^i \partial y^j}, \quad \Gamma_{qj}^i = \frac{\partial x^i}{\partial y^l} \frac{\partial^2 y^l}{\partial x^q \partial x^j}. \quad (2.7)$$

The generic curvilinear coordinate system transformation applied in the scope of this work is shown in figure 1, where the physical domain represented in cartesian coordinates ($y^1 = X, y^2 = Y, y^3 = Z$) is mapped into the cylindrical coordinate system ($x^1 = X, x^2 = r, x^3 = \theta$), where the equations are ultimately solved. In figure 1, a wall-normal (y) and stream wise (x) reference system is also introduced as it will be used as a natural way of displaying results of the flow over a conical geometry.

2.3. Local Artificial Diffusivity

A local artificial diffusivity (LAD) scheme based on implementations by ? and Kawai *et al.* (??) is used to diffuse regions of high magnitude gradients where there is presence of a shock or intense turbulent events and make them resolvable on the computational grid. This is done by adding artificial dynamic viscosity, $\mu = \mu_f + \mu_{\text{art}}$, artificial bulk viscosity,

$\beta = \beta_f + \beta_{\text{art}}$, and artificial conductivity, $k = k_f + k_{\text{art}}$, in the regions where the shock is detected on top of the nominal flow diffusion parameters, shown in the aforementioned relations with the subscript f . The ultimate objective is to circumvent the manifestation of non-physical spurious oscillations originated from solving steep gradients with high-order methods that can lead to numerical instability. The LAD relations used in the current work is summarized hereafter:

$$\mu_{\text{art}} = C_\mu \overline{\left[\rho f_{\text{CvP}} \Sigma_{l=1}^3 \frac{\partial^4 S}{\partial^4 x^l} (\Delta x^l)^4 \Delta_{l,\mu}^2 \right]} \quad (2.8)$$

$$\beta_{\text{art}} = C_\beta \overline{\left[\rho f_{sw} \Sigma_{l=1}^3 \frac{\partial^4 \nabla \cdot \mathbf{u}}{\partial^4 x^l} (\Delta x^l)^4 \Delta_{l,\beta}^2 \right]} \quad (2.9)$$

$$k_{\text{art}} = C_k \overline{\left[\rho \Sigma_{l=1}^3 \frac{\partial^4 e}{\partial^4 x^l} (\Delta x^l)^4 \Delta_{l,k} \right]} \quad (2.10)$$

where the overbar represents an approximate truncated-Gaussian filter (?).

The derivatives of the magnitude of the strain rate tensor, $S = \sqrt{S_{ij} S_{ij}}$, of the dilatational field ($\nabla \cdot \mathbf{u}$) and of the internal energy (e) are taken in the curvilinear domain (x^1, x^2, x^3) but they are scaled back with the computational grid spacing (Δx^l), a known function of the mapping. The grid spacing ($\Delta_{l,\cdot}$) in the direction of each derivative is used to scale the artificial diffusivity properties, being defined as:

$$\Delta_{l,\mu} = |\Delta y^i|, \quad \Delta_{l,\beta} = \left| \Delta y^i \cdot \frac{\nabla \rho}{|\nabla \rho|} \right|, \quad \Delta_{l,k} = \left| \Delta y^i \cdot \frac{\nabla e}{|\nabla e|} \right|, \quad (2.11)$$

where Δy^i is the physical local displacement vector. Since a mapping function is defined from the physical to computational space in order to solve the governing equations in the natural curvilinear coordinates of the problem, we define the physical local displacement in the cartesian coordinate system as:

$$\Delta y^i = \sqrt{g_{1i} \Delta x^i \Delta x^1 + g_{2i} \Delta x^i \Delta x^2 + g_{3i} \Delta x^i \Delta x^3}. \quad (2.12)$$

To ensure that artificial bulk dissipation is introduced only in regions of inviscid compressibility effects, the shock sensor (f_{sw}), described in ?, was used. This shock sensor function allows the activation of the local artificial diffusivity technique all the way to the near wall region making sure it is only active in compression regions where the turbulence induced fluctuations create shock-like gradients. f_{sw} is defined in equation 2.13, where a heaviside function (H) is coupled together with the Ducros sensor where ϵ is just a small tolerance value to prevent division by zero in regions where both the dilation and vorticity are zero (?).

$$f_{sw} = H(-\nabla \cdot \mathbf{u}) \times \frac{(\nabla \cdot \mathbf{u})^2}{(\nabla \cdot \mathbf{u})^2 + \|\nabla \times \mathbf{u}\|^2 + \epsilon} \quad (2.13)$$

Victor: begin of CvP sensor that we do not use anymore

A novelty is introduced in the LAD implementation in the current work. A sensor used to detect the presence of local vorticity at the small-scale level, (f_{CvP}), described by ?, is used in the artificial shear viscosity relation. The sensor is based on evaluation of the ratio of the test-filtered to resolved (or grid-filtered) enstrophy ($\boldsymbol{\omega} \cdot \boldsymbol{\omega}/2$), where $\boldsymbol{\omega}$ is the vorticity vector. This ratio is able to inform the presence of small-scale motion because the enstrophy's spectral content grows in the turbulence inertial range as the wavenumber increases and at the small-scales.

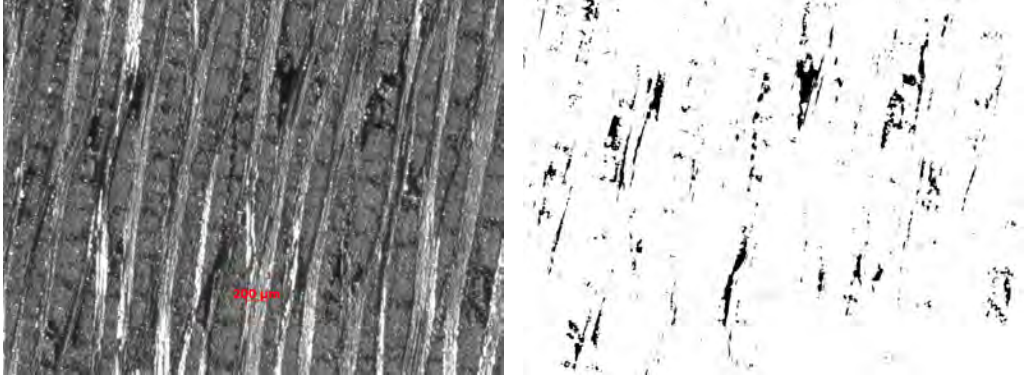


Figure 2: The figure on the left shows reflected-light microscopy generated images of classic C/C and on the right the highlighted pores that can be found in such a material. Large pores tend to be slit-like and smaller pores resemble circular patterns. [Victor: Alex asked: "Add Source" and you agreed but what source to cite?](#)

The used of this sensor ensures the application of μ_{art} only on the small unresolvable scales since it is deactivated in the regions dominated by large-scale motion and laminar vorticity regions. Hence, the use of f_{CvP} to scale the artificial dynamic viscosity deems unnecessary the use of a wall damping function when modeling high speed wall-bounded viscous flows, what was required for the simulations reported in ?.

[Victor: End of CvP sensor that we do not use anymore - maybe we could use it for fully turbulent simualtions](#)

In the simulations performed in this manuscript, we define the constants C_μ , C_β and C_κ as 0.00, 1.75 and 0.10, respectively.

2.4. Time-Domain Impedance Boundary Conditions (TDIBC)

In this work, porous walls are modeled via the imposition of a normal impedance at the boundary, $Z_n(\omega)$, the ratio of complex pressure oscillations and the velocity component normal to the boundary in the frequency domain non dimentionalized by the base impedance, $\rho_0 a_0$, yielding

$$Z_n(\omega) = \frac{1}{\rho_0 a_0} \frac{\hat{p}(\omega)}{\hat{u}_n(\omega)}. \quad (2.14)$$

[Carlo: Provide justification regarding linear assumption of the acoustics](#)

Reflections of an impedance boundary can be exactly imposed by the knowledge of its normal impedance when the assumption of a locally acting surface is valid (?). Since the C/C material studied in this work is highly anisotropic (figure 2), having the majority of the pore extent aligned with the wall normal direction, the amplitude of the normal disturbances component are much higher than the streamwise or azimuthal counterparts, what confirms the initial hypothesis validity. In addition, ? performed a parametric study to investigate the effects of the impedance components parallel to the surface and concluded that their effect is indeed small arguing that the inviscid nature of the second mode instability undermines the effects such components though they could be important for viscous dominated instabilities. Therefore, the acoustic effects of a C/C surface in the flow can be modeled using only the normal impedance as a boundary condition.

Imposing directly the normal impedance at the boundary represents an obstacle,

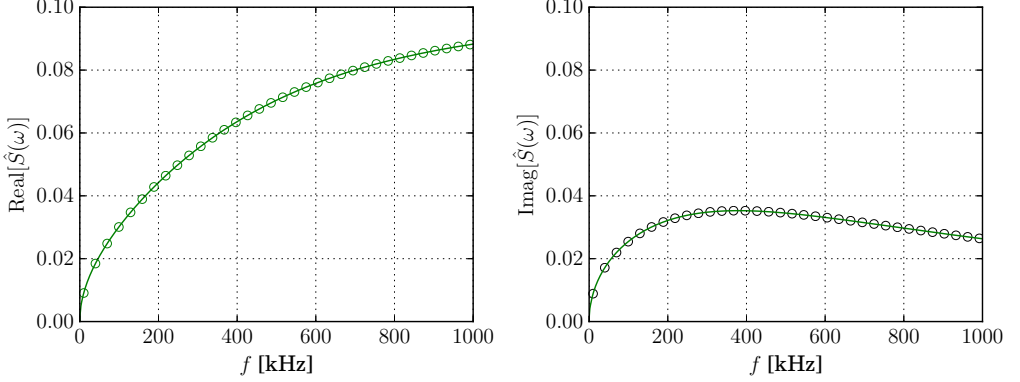


Figure 3: Real and imaginary components of the frequency domain wall softness ($\hat{S}(\omega)$) representative of the C/C surface at wall pressure conditions corresponding to $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$, shown in symbols, and the multipole reconstruction of the complex signal following the vector fitting procedure, shown in solid lines.

though, because it is unbounded. To overcome this, different formulations using either the reflection coefficient ($\hat{R}$) or the wall softness ($\hat{S}$) were developed.

$$\hat{R} = \frac{1 - Z_n(\omega)}{1 + Z_n(\omega)}; \quad \hat{S}(\omega) = \hat{R}(\omega) + 1, \quad (2.15)$$

TDIBC is implemented in the current numerical scheme as a function of the wall softness ($\hat{S}$) based on the formalism of ? and the numerical integration of the methodology in a compressible Navier-Stokes solver by ? and subsequent work by ? and ?.

To apply the derived complex impedance in a direct numerical simulation, one needs to transform it to the time domain. In the current numerical scheme, we fit the wall softness frequency response $\hat{S}(\omega)$ with a set of complex poles and residues in such a way that their frequency domain response is carefully constrained so that it satisfies the causality, passivity and reality constraints that make the TDIBC a physically admissible boundary condition (?). In the end, the fitted frequency response is easily transformed to time domain via an inverse Laplace transform and imposed as inviscid fluxes at the boundary.

$$\hat{S}(\omega) = \frac{2}{1 + Z_n(\omega)} \approx \sum_{k=1}^{n_o} \left[\frac{\mu_k}{s - p_k} + \frac{\mu_k^*}{s - p_k^*} \right] \quad \text{and} \quad S(t) \approx \sum_{k=1}^{n_o} \left[\mu_k e^{p_k t} + \mu_k^* e^{p_k^* t} \right], \quad (2.16)$$

where n_o is the number of oscillators, μ_k the residues, p_k the poles, and $s = j\omega$. In equation 2.16, the superscript (*) denotes complex conjugate.

The fit in the frequency domain is performed following the vector fitting procedure described by ?, where least square approximation is performed iteratively in order to reallocate the poles and residues and represent accurately the complex response of the function to be fitted in a given interval. In the current work, the complex wall softness representative of a C/C porous material at wall pressure conditions corresponding to $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ (see section 3) was fitted in the frequency interval between 10 and 1000 kHz and the signal reconstructed from the multipole superposition accurately represents the original behavior, as shown in figure 3.

The implementation of TDIBC in a Navier-Stokes solver uses the relation between the complex wall softness coefficient, $\hat{S}(\omega)$, with the incident, $\hat{A}^-$, and reflected, $\hat{A}^+$, waves from a given surface (see figure 1) in the frequency domain via

$$\hat{S} = \frac{\hat{A}^+ + \hat{A}^-}{\hat{A}^-}, \quad (2.17)$$

In order to update the normal velocity boundary condition at each time step, an inverse Fourier transform is performed. Respecting causality, the convolution integral is then taken in the interval $[0, t]$, as demonstrated by ?, via

$$A^+(t) = -A^-(t) + \int_0^\infty S(\tau)A^-(t - \tau)d\tau. \quad (2.18)$$

where the time-domain definition of the incident and reflected waves are defined with respect to v' and p' are the fluctuating values of wall normal velocity and pressure respectively, as in relation 2.19.

$$\begin{cases} A^-(t) = v'(t) + \frac{p'(t)}{\rho_0 a_0} \\ A^+(t) = v'(t) - \frac{p'(t)}{\rho_0 a_0} \end{cases} \quad (2.19)$$

In order to implement the convolution integral shown in 2.18 and integrate it with a Navier-Stokes solver, it needs to be evaluated at $t + \Delta t$, where Δt is a finite time interval used to advance the numerical solution in time, as

$$A^+(t + \Delta t) = -A^-(t + \Delta t) + \sum_{k=1}^{n_o} [s_k(t + \Delta t) + s_k^*(t + \Delta t)] \quad (2.20)$$

where the contribution of each pole and residue pair (s_k) to the convolution integral can be expressed as

$$s_k(t + \Delta t) = \int_0^\infty \mu_k e^{p_k t} A^-(t + \Delta t - \tau) d\tau. \quad (2.21)$$

Using the internal addition property of the integral in 2.21, ? shows that

$$s_k(t + \Delta t) = z_k s_k(t) + \int_0^{\Delta t} \mu_k e^{p_k t} A^-(t + \Delta t - \tau) d\tau, \quad (2.22)$$

where $z_k = e^{p_k \Delta t}$. Following ?, the integral in 2.24 is evaluated using a trapezoidal quadrature rule, which is second-order accurate in time. The resulting relation for the contribution of each pole and residue pair can be written as

$$s_k(t + \Delta t) = z_k s_k(t) + \mu_k \Delta t [w_{k0} A^-(t + \Delta t) + w_{k1} A^-(t)] \quad (2.23)$$

where

$$w_{k0} = \frac{z_k - 1}{p_k^2 \Delta t^2} - \frac{1}{p_k \Delta t} \quad \text{and} \quad w_{k1} = -\frac{z_k - 1}{p_k^2 \Delta t^2} + \frac{z_k}{p_k \Delta t}. \quad (2.24)$$

After the numerical evaluation of the convolution integral and after summing the contribution of each pole and residue pair, the outgoing wave (A^+) at time $t + \Delta t$ can be

determined and, ultimately, the wall normal velocity can then be recovered and imposed at each time step as

$$v'(t + \Delta t) = \frac{1}{2} [A^-(t + \Delta t) + A^+(t + \Delta t)]. \quad (2.25)$$

3. Impedance Characterization of C/C

The porous carbon fiber reinforced carbon ceramic (C/C) used in ? hypersonic transition delay experiments, depicted in figure 2, represents an intermediate state of the manufacturing process of a material previously used in hypersonic flight of the sharp edge flight experiment (SHEFEX) as a Thermal Protection System (TPS) (??). During the manufacturing process of such material, a green body composed by carbon fibers impregnated by a phenolic resin, is pyrolyzed and the resin is converted to isotropic carbon. The C/C inherent porosity appears after this step as, during the cool-down, contraction of the material is anisotropically hindered by the carbon fibers resulting in a pattern of small “slit-like” cracks, some of which propagate to the surface, lending the material its volumetric and surface porosity.

C/C’s porosity contributes to the desired properties this material has in favor to be used as an external Thermal Protection System (TPS) in hypersonic flight by adding acoustic absorption and passive transition delay to high thermal resistance, low expansion and density as well as high temperature stability in non oxidizing atmospheres. Resistance against oxidation is acquired by the infiltration of a liquid phase of silicon into the porous carbon, followed by a reaction to SiC. During the initial development of this material as a TPS, its microstructural gaps were intentionally filled and the acoustic absorption properties of the material lost. Currently, there is development at DLR towards optimizing the end product not only regarding the oxidation resistance but also acoustic absorption capability. [Victor: to be presented in June 2019 on IUTAM symposium, London.](#)

In order to model the effects of the porosity of the C/C sample in a cost-effective way inside a DNS, the acoustic properties of a surface composed by such a material is introduced as a surface averaged impedance in the frequency domain as a boundary condition. In classical acoustics, it is common practice to perform measurements of the pressure field after a reflection of a planar wave of a surface at two different locations and estimate its surface average impedance. However, the region of interest regarding the acoustic absorption of a C/C material is centered around high frequencies ($\simeq 100$ kHz) and low pressures ($\simeq 10$ kPa), what increases the difficulty of such measurements.

? has set up a test rig capable of performing acoustic absorption coefficient (β) estimation by measuring the modulus of the reflection coefficient ($\hat{R}$) at different base pressures and varying discrete frequencies . Since a separate transducer/receiver pair for each frequency was needed, few experimental points were acquired in the frequency domain. The previously mentioned coefficients are defined by the relations 3.1 and 3.2.

$$\hat{R}(\omega) = \frac{1 - Z_n(\omega)}{1 + Z_n(\omega)}, \quad (3.1)$$

$$\beta(\omega) = 1 - |\hat{R}|^2, \quad (3.2)$$

$\hat{R}$ relates the reflected and incident wave and, being a complex quantity carries the information of the amplitude and phase change that happens after the reflection at the

impedance surface and β is related to the fraction of incident energy is absorbed by the porous boundary, representing a figure of merit of the absorber. These coefficients will be useful to quantify how a wave that impinges on a C/C surface is influenced.

By measuring β , it is possible to estimate the modulus of the surface average impedance of a region of interest but information on the phase shift that happens due to the wave-surface interaction during the reflection is also needed to characterize the whole complex impedance. ? addresses the question of whether the setup could be used to measure the phase shift resulting from the reflection of the sound wave at a C/C porous wall. He reported that, the minimum phase shift ($\angle \hat{R}$) resolution that could be measurable by the technique of comparing the time signals of the transducers ranges from approximately 2.2° for 125 kHz to 8.4° for 490 kHz. Since, the phase shift predicted by theoretical models (figure 5) is in the same order of the limit of experimental resolution, the current experimental techniques available are not able to fully characterize the complex behavior of the C/C's impedance.

In the end, all of these difficulties are increased by the fact that a sweep in the frequency parameter space, needed to accurately describe the impedance, would be unfeasible since since each frequency measurement requires an specific transducer/receiver pair. This scenario leads to the conclusion that impedance analytical models, informed by the experimental data, are needed to ultimately characterize how the complex impedance varies in the frequency domain for the conditions of interest.

In this work, predictions of the surface averaged acoustic impedance are performed via the homogeneous absorber theory (HAT) described by ? and used previously by ? to predict the C/C's behavior. A summary of this model can be found in ?. The results obtained analytically are then compared with experimental results of experimental acoustic characterization of ultrasonically absorptive porous samples (?).

These models are informed by physical parameters that characterize porous media and its influence on the flow through them such as: the porosity (ϕ), which is the ratio of the pore volume to the total volume; the flow resistivity (Ξ), that represents the resistance (pressure difference) to the flow of air through the medium; and the structure factor (κ), that accounts for the presence of some "blind" and tortuous paths in the absorber. A complete description of these parameters can be found in ?.

In the current analysis Ξ and κ were used as optimization parameters that were chosen to minimize the difference between the modeled and measured values. The reason behind this is because κ cannot be measured or calculated directly for randomly structured porous materials. On the other hand, Ξ can be measured but, although measurements performed by DLR are accurate, the approach of using the flow resistivity as an optimization parameter was chosen here in order to facilitate the the generation of an impedance curve that fitted the experiments with minimum deviation. The goal is to reproduce numerically the real surface's impedance of the transition delay experiments as closely as possible.

Figure 4 shows the absorption coefficient behavior as the base pressure changes for the different frequencies and how the "curve-fitted" models approximate the experimentally measure results. The values obtained through the optimization procedure were $\Xi_{\text{HAT}} = 13.64 \text{ MPa s m}^{-2}$; $\kappa_{\text{HAT}} = 7.94$. The results of the optimization for the flow resistivity, when compared with previously published results of $\Xi = 13.3 \text{ MPa s m}^{-2}$ (?), agree well.

It is worth noting that both the theoretical model and experiments relatively predict high levels of Ξ related to the C/C porous material. This results to a fairly thin layer of material, with a depth (H) bigger than 1 mm, behave as an infinite thickness absorber, Z_∞ , i.e. the impinging wave is dissipated due to thermoviscous effects either before its reflection from the impermeable end of the absorber can significantly affect the wave at

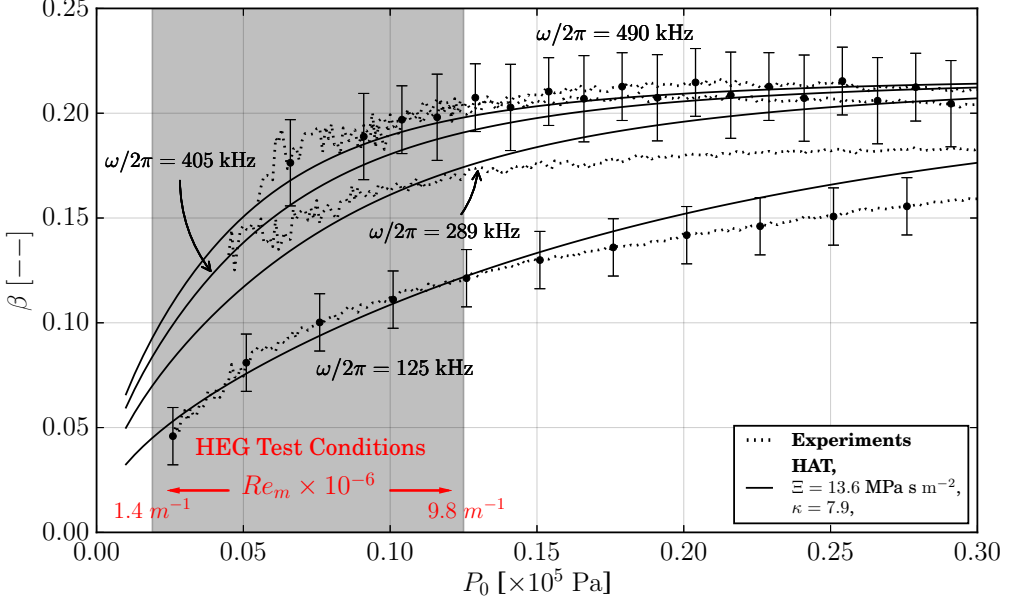


Figure 4: Comparison between experimentally measured data (dotted lines) for the absorption coefficient (β) of a C/C sample and the curve fit results based on the homogeneous absorber theory (—). The data here is shown for as a function of base pressure for the different frequencies tested experimentally and a shaded region is included to outline the region of interest regarding the surface pressure conditions of ? experiments. Error bars are included for the frequencies of 125, 405 and 490 kHz.

the surface. This statement is made in the scope of the homogenous absorber theory, that accounts for finite thickness effects with a correction term as,

$$Z_{\text{HAT}} = Z_{\infty} \frac{1 + e^{-j2k_a H}}{1 - e^{-j2k_a H}} \quad \text{and} \quad k_a = k\sqrt{\kappa} \sqrt{1 - j \frac{\Xi \phi}{\rho_0 \kappa \omega}}, \quad (3.3)$$

where ϕ is the volume porosity, k and $\omega = 2\pi f$ are the wave number and angular frequency of incident wave and ρ_0 the base density. Therefore, in the frequency range of interest, k_a is very large and the exponential terms of the correction factor decay rapidly to zero.

The region of interest of the experiments regarding the experiments conducted by ? is shown in figure 4 as a shaded region. This is done by approximating the inviscid recovery pressure for a sharp cone under the initial conditions using the Taylor-Maccoll ODE. In this region, characterized by low pressures, the model achieves a fairly good approximation with what is observed experimentally specially if one considers that the model lies within experimental error, which was measured for the frequencies of $\omega/2\pi = 125, 405$ and 490 kHz. Additionally, as pressure increases, the homogeneous absorber theory model loses some of its predictive accuracy.

Wave absorption data is first presented as a function of pressure at various frequencies (figure 4) and vice versa (figure 5). It is interesting to observe the acoustic behavior of the C/C surface in the frequency domain because, ultimately, this is the information that is introduced in the Navier-Stokes solver as the IBC. Figure 5 shows β and the phase

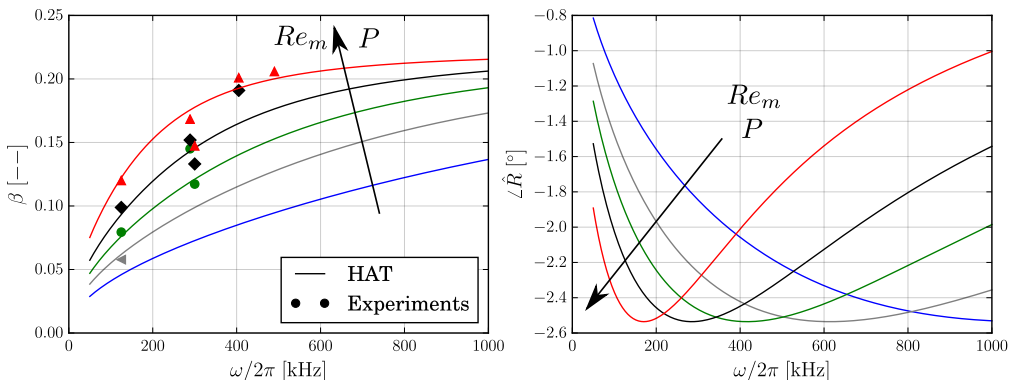


Figure 5: Absorption coefficient (β) and phase difference between the incoming and reflected waves ($\angle \hat{R}$) are shown as a function of frequency. The discrete frequencies tested experimentally are shown in symbols and, in solid lines, the results obtained through the modeling of surface impedance by the use of the HAT is shown. As pressure decreases less experimental data is available due to the difficulty of measuring high frequency waves in rarefied environments. The Re_m dependency is related to HEG experimental conditions (see table 1)

of the reflection coefficient ($\angle \hat{R}$) for 5 different pressures corresponding to the surface pressures that a sharp cone would experience under the 5 different Re_m reported in table 1. It shows that, as Re_m , and consequently pressure, increases, β also becomes higher. This correlates with the experimental findings of ?, who reported a higher transition delay capability of C/C ultrasonic absorbing coatings for higher Re_m . It is noteworthy to notice that, the behavior of the acoustic absorption coefficient for high frequencies seems to approach a value asymptotically but the experimental points obtained, shown in symbols, are not sufficient to make a statement on whether the asymptotic behavior of the absorption coefficient is accurately predicted. The prediction of the behavior at high frequencies is made even harder by the fact that at rarefied ambients such frequencies dissipate very rapidly, what makes its experimental measurement difficult.

4. Base Flow Computations and Stability Analysis

The goal of this section is to reproduce and analyze the disturbance amplification mechanism in a hypersonic flow over a cone with impermeable smooth walls and over porous walls with an acoustic absorption characteristic akin to a C/C surface. Two types of nose tip bluntness are analyzed, a sharp-nosed cone, and a cone with nose tip radius equal to $R_{tip} = 2.5$ mm. The initial conditions of the laminar base state for the sharp-tipped cone were obtained through coupling of the Blasius relations for a supersonic sharp cone and the inviscid Taylor-Maccoll relations. As for the blunt-nosed cone, a low order precursor simulation was performed using the DLR FLOWer code (?). A more detailed description of these calculations can be found in ?.

A numerical transient behavior is observed immediately after the initialization of the simulations in the discretized computational domains, which differ for each Re_m and nose tip radius, as reported in table 2. The simulation is run until it is relaxed and, after the primary adjustment the boundary layer profiles are shown at various locations as a function of the streamwise and wall normal coordinates in figures 6 and 7. For all

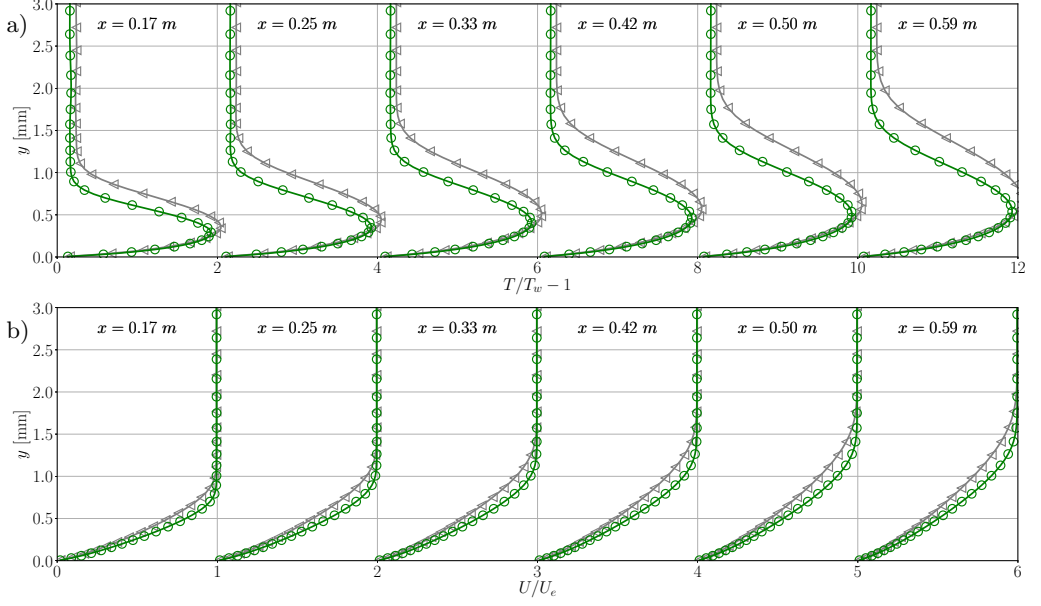


Figure 6: Temperature (a) and streamwise velocity (b) profiles are shown for blunt cone for the different Re_m considered, those being $Re_m = 2.43 \cdot 10^6 \text{ m}^{-1}$ and $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$. Results from the unperturbed DNS (—) are compared against the similarity profile obtained from Mangler-transformed Blasius (:), shown in symbols ($\triangle$; $\circ$), these being shown in crescent order of Re_m .

cases considered, an excellent agreement between the predicted and simulated conditions is observed. In the aforementioned figures, it is also possible to observe the differences in the laminar profiles for the conditions tested. It is noticeable that as the Reynolds numbers increases, the boundary layer thicknesses decreases. Additionally, the blunt tip induces an entropy layer that affects the near wall profiles leading to a higher thickness of the boundary layer near the leading edge. However, as the flow is advected downstream, the relative importance of the nose tip bluntness in the flow decreases and the profiles eventually converge to those predicted by the self similar conical Blasius. In the end, this leads to a region, near the tip, where the second mode is stable followed by a zone where the boundary layer develops spatially slower than for a the sharp tipped cone case.

Following the institution of an unperturbed laminar state over the different conical geometries and for the different Reynolds numbers, a broadband pulse disturbance is introduced in the domain via a low amplitude suction and blowing at the wall, as shown in figure 8. The pulse is imposed with a period equivalent to a signal with $f = 600 \text{ kHz}$, so that we excite all the frequencies related to the second mode instability in a finite-length streamwise region equivalent to $x = (0.05 \pm 0.005)L_x$ as following:

$$tv(x, y = 0, t) = \cos(\pi\xi)^3 \sin(2\pi ft), \quad \text{for } 0 \leq t \leq 1/f \quad (4.1)$$

where ξ is a variable with values ranging in $[-1, 1]$ which is mapped to the actual spatial interval of application of the pulse. This perturbation aims at mimicking a natural transition scenario due to its broadband characteristics. It was first used by ? in incompressible transition simulations and by ? to study its high speed counterpart. The same strategy was used by ? in the context of showing the capabilities of second

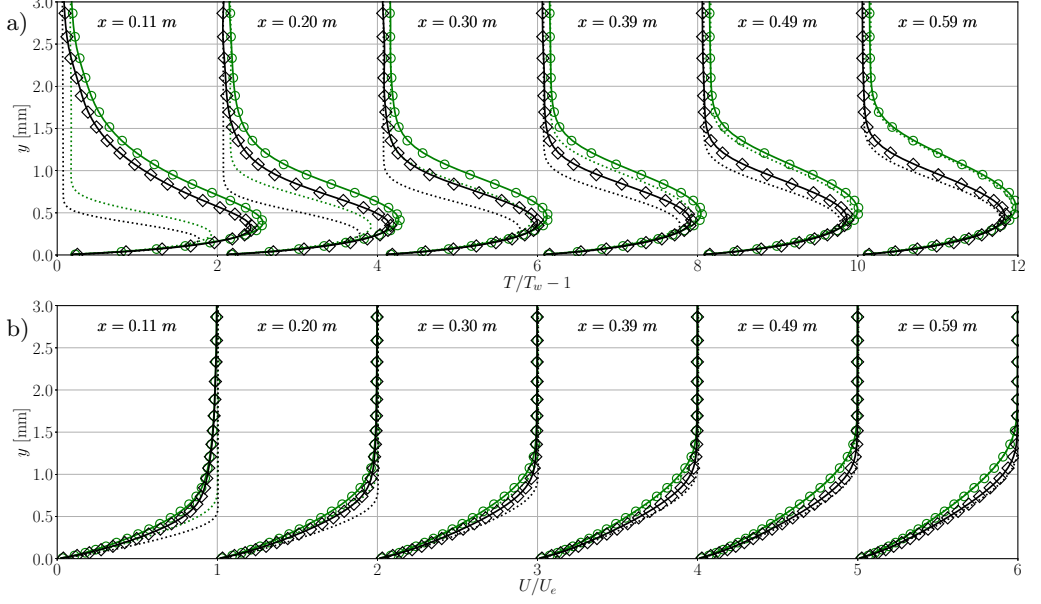


Figure 7: Temperature (a) and streamwise velocity (b) profiles are shown for blunt cone for the different Re_m considered, those being $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ and $Re_m = 6.40 \cdot 10^6 \text{ m}^{-1}$. Results from the unperturbed DNS (—) are compared against the similarity profile obtained from Mangler-transformed Blasius in the case of the sharp tipped cone (· · ·) and with the low order DLR FLOWer precursor simulation for a $R_{tip} = 2.5 \text{ mm}$, shown in symbols ($\circ$; $\diamond$), these being shown in crescent order of Re_m .

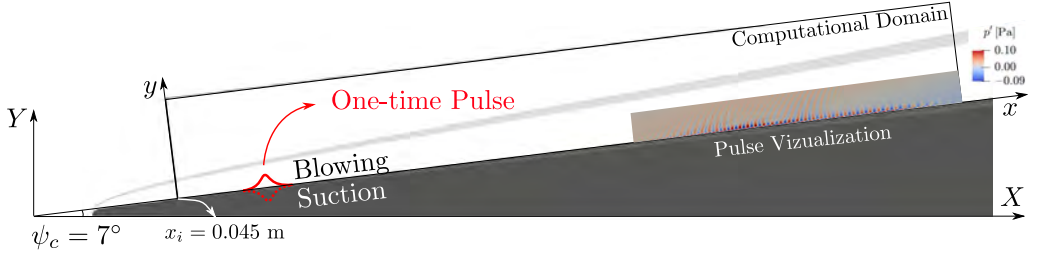


Figure 8: Broadband disturbance excitation through a one-time suction and blowing pulse at the wall.

mode attenuation by an impedance boundary mimicking a porous wall. We now extend the discussion performed previously to the effect on nose tip bluntness and comparing with results from an LST solver assuming a parallel flow. The implementation of the LST solver is discussed in appendix A.

In order to ensure that the high frequency perturbations evolution as they advect downstream is accurately captured, we perform the broadband test case over an impermeable wall for the two highest Reynolds numbers considered for each nose tip configuration. Results for the amplitude of the magnitude of wall pressure perturbations are shown for a progression of grid arrangements are shown in figure 9. The aforementioned plot shows that the second mode instability is not accurately captured on the coarser grid ($n_x = 4608, n_y = 128$), as the predicted amplitude for this arrangement falls behind of

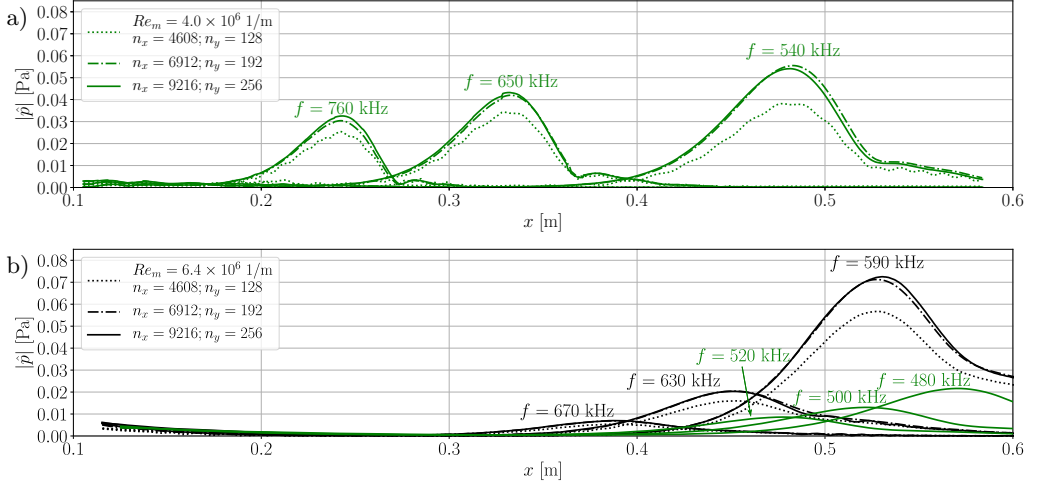


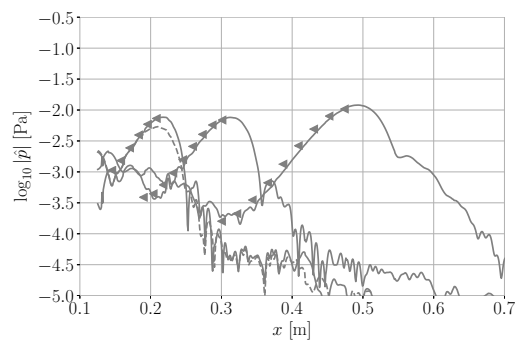
Figure 9: Grid sensitivity analysis for different frequency components of the wall pressure perturbation amplitude in an axisymmetric simulation of broadband pulse propagation over a sharp (a) and blunt (b) cone.

the predicted by finer grids. This effect happens because a numerical solution in a coarse grids leads to a non negligible numerical dissipation, what decreases to insignificant levels as the grid is refined. Ultimately, the intermediate ($n_x = 6912, n_y = 192$), and fine ($n_x = 9216, n_y = 256$) grid arrangements lead to a qualitatively converged solution.

By comparing the results of the second mode amplitude evolution for the sharp and blunt cases, it is seen that a much wider range of unstable frequencies is present in the laminar flow over a sharp cone than over a blunt cone. Moreover, although the unstable region for the blunt cone is shrunk due to the presence of the entropy layer, its spatial growth is more abrupt. Due to the relation of the boundary layer thickness and the unstable second mode, one can explain this behavior by observing the boundary layer profiles shown in figure 9. As the profiles evolve in the stream wise direction, the influence of the blunt tip decreases and the, eventually, the boundary layer thickness follows a self similar profile. In the end, this behavior means that there exists a transition region where the boundary layer of a blunt cone evolves spatially slower than the boundary layer over a sharp cone. The presence of this region yields a wider spatial zone where a specific frequency is unstable and, therefore, its integrated growth can even surpass that of a mode developing in a self similar boundary layer. This higher integrated growth, however, does not lead to a higher amplitude of the second mode in the same spatial location downstream due to the previous attenuation caused by entropy layer, as seen in figure 10.

? observed that, up to a nose Reynolds number ($Re_{nose} = R_{tip} Re_m$) of approximately $1.0 \cdot 10^5$, the bluntness is considered small and has a stabilizing effect on transition. This effect is reversed at $Re_{nose} \geq 1.0 \cdot 10^5$ and any increase of relative bluntness leads to an earlier transition location. At the flow conditions and cone geometries studied in the current work, the biggest Re_{nose} is around $1.6 \cdot 10^4$, therefore the stabilizing effect caused by bluntness observed in figure 9 is in agreement with previous experiments.

The effects of frequency, Reynolds number and nose tip bluntness on the evolution of a given Fourier mode of pressure are analyzed by studying its signal near the surface of the cone, shown in lines, and the results are compared against a linear stability solver,



reduction happens by increasing the attenuation rates in the region where it is stable and decreasing the growth rates in the regions of instability. The combination of both effects leads to the amplitude of the surface pressure oscillation signal over a porous surface be lower than over the baseline impermeable surface in all regions of the domain. Figure 10 also shows that overall attenuation effects are more pronounced for the blunt nosed geometry in comparison with the sharp tipped cone.

Comparing the results obtained from the direct numerical simulations of the advection of a broadband disturbance with the ones predicted by linear stability theory, one can infer that the implementation of the TDIBC technique in the Navier-Stokes solver is numerically consistent between different solvers. In the remainder of the manuscript the same numerical framework will be used to predict the full path to turbulence and how it will be affected by the presence of a porous surface.

Even though the numerical implementation of the solver and required boundary conditions are shown to be consistent, the question of whether the current model can be used to replicate the physics observed in the experiments performed by ? in the HEG wind tunnel was not yet addressed. In order to address that, a comparison against the frequencies of maximum amplitude of the wall pressure fluctuation signal at a given location measured experimentally in the HEG tunnel to the ones predicted by the numerical representation of the similar conditions for the hypersonic flow over a cone are shown in figure 11.

The results show that, for both sharp and blunt-nosed cones, the numerically predicted frequencies of maximum amplitude for the lowest Re_m considered fall within the experimental uncertainty while the frequency content of the disturbance signal for the higher Re_m simulations is overestimated in different degrees. It is important to highlight that $Re_m = 1.46 \cdot 10^6 \text{ m}^{-1}$ is the only case in which all the surface pressure probes are located outside the region where the experimentally measured heat flux has departed from laminar values for both sharp and blunt cones although the blunt $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ case for $x < 0.55 \text{ m}$ also has measurements in the laminar region, where intermittent transitional and turbulent structures do not contaminate the signal [Victor: CITE FIG.](#) Therefore, most of the measurements for the higher Re_m considered are made in regions where disturbance amplitudes are big enough to induce non linear interactions and to make their dynamics depart from the linear theory predictions and the frequency content overestimation simulated by the advection of a small amplitude pulse and by the LST solver for these cases are regarded as a divergence from linear results. Generally, the over prediction grows for higher Re_m and as one moves downstream for a given flow condition, what is in agreement with the hypothesis that the nonlinear dynamics is affecting the measurements since in both cases the effective Re is increasing. Another observable effect is that the over prediction in frequency is higher for a sharp cone case compared to the blunt nose case. Since, for the simulated cases, the nose tip bluntness of 2.5 mm is small (?), it leads to a transition delay, what is also in agreement with the hypothesis presented.

Additionally, although the frequency of maximum amplitude simulated for $Re_m = 1.46 \cdot 10^6 \text{ m}^{-1}$ lies within experimental uncertainty, the predicted values are consistently above the mean value of the measurements for both blunt and sharp cases, what is also true for the laminar measurements of the blunt $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ case. Now, comparing only laminar results, this type of over prediction in the frequencies with maximum amplitude at each spatial location was also observed by ? when comparing the results obtained from the DLR stability code, NOLOT (??), with the experimental results measured in the HEG wind tunnel running at $Re_m = 1.46 \cdot 10^6 \text{ m}^{-1}$ over a 7-degree half-angled cone with a finite nose tip bluntness of $R_{\text{tip}} = 2.5 \text{ mm}$. Moreover, in the study performed by ?, an assessment of the influence of inaccuracies in the measurements of the free

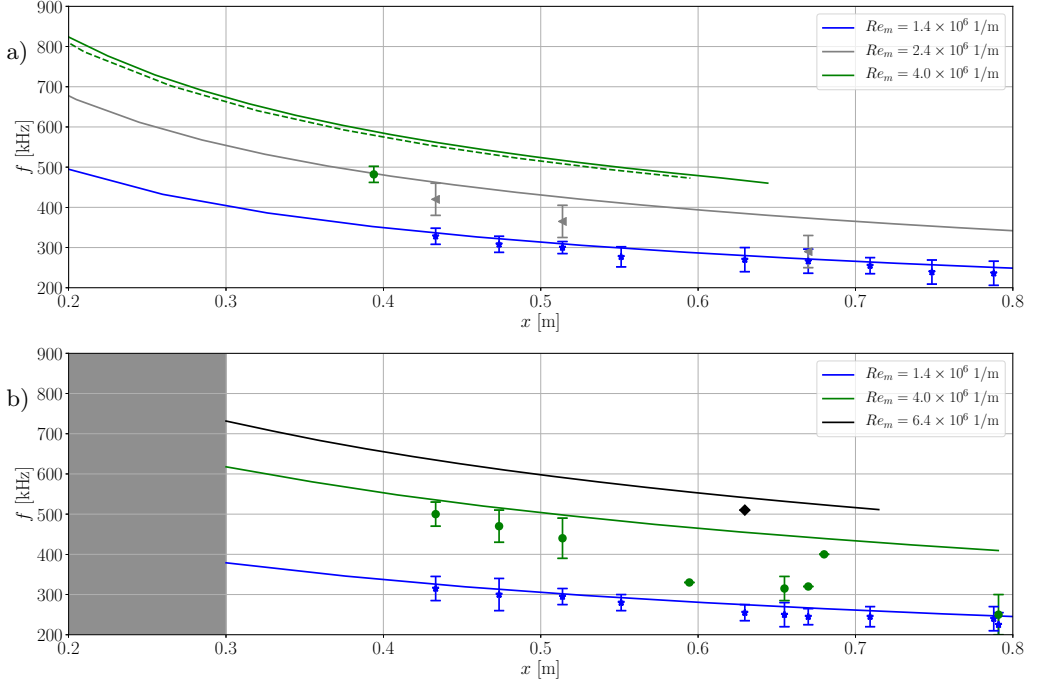


Figure 11: Comparison for the location of maximum amplitude for each frequency component for the wall pressure fluctuation signal between what is predicted by the mathematical model of the hypersonic flow over a sharp (a) and blunt (b) cones and the experiments performed by ? at the HEG wind tunnel. LST results are shown in solid lines and experimental results are shown in symbols with their respective error bars. The shaded region in the plot corresponding to the blunt-nosed cone represents the approximate region where the entropy layer effects are important, i.e. where the second mode is stable. To illustrate the effect of inaccuracies in the determination of the model's wall temperature, the dashed line in figure (a) shows the results obtained with $T_w = 330$ K, a 10% increase.

stream condition was performed by increasing the free stream Re_m by a factor of 5%, in the order of the uncertainties present in the HEG shock tunnel experimental runs. Ultimately, it was shown that a 5% uncertainty on Re_m can lead to variations of the maximum frequency distribution in the order of 10%. Wartemann *et al.* also looked into inaccuracies in the wall temperature determination and concluded that a variation of 10% in the wall temperature leads to an uncertainty of around 2% in the maximum frequency spatial profile. A similar result was obtained in the current simulations, shown in the dashed line on the top of figure 11, as a small decrease in the predicted frequency obtained by increasing the wall temperature from 300 K to 330 K. A different study performed by ? at the BAM6QT comparing experimental results and the stability theory predictions by STABL (?) also reported inaccuracies that ranged from 2% to 14%.

Ultimately, the hypersonic boundary layer transition delay experiments performed at the HEG were only performed using the blunt-nosed cone geometry and, therefore, the effect of the discrepancies in the comparison against the current numerical model are minimized. Moreover, the behavior of the acoustic absorption coefficient (β) in the frequency domain is asymptotic, as depicted in figure 5. This leads to the conclusion that,

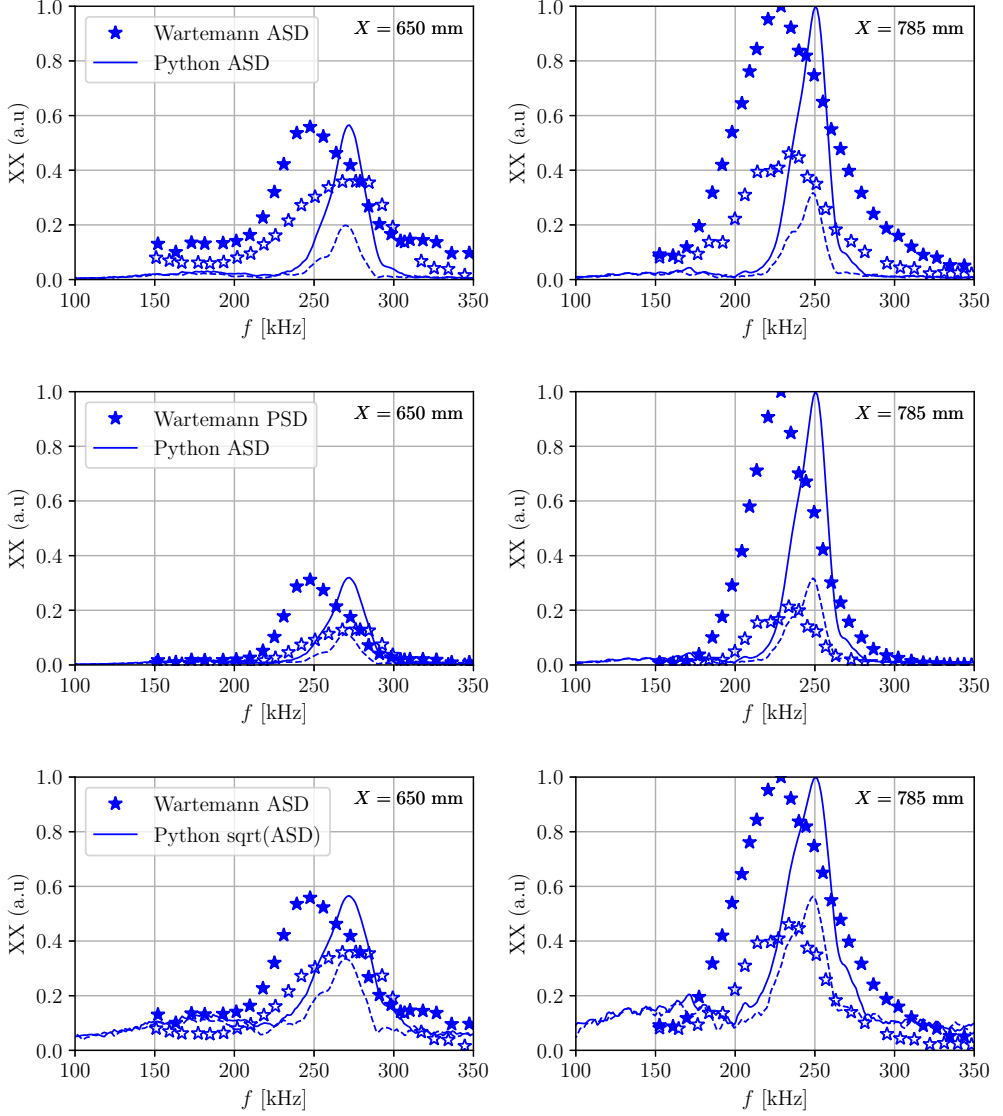


Figure 12: Measured amplitude spectral density (ASD) of the wall pressure fluctuations for different locations at the surface for both impermeable (—) and porous walls (---). The flow conditions correspond to a blunt nosed cone with $R_{\text{tip}} = 2.5 \text{ mm}$ with free stream conditions reported to be around $Re_m = 1.46 \cdot 10^6 \text{ m}^{-1}$. Experimental measurements are shown in symbols. (?)

due to the high frequency content of the boundary layer perturbations, any frequency over prediction leads only to a small increase in the imposed β .

5. Transitional Boundary Layer Simulations

In this section, the same flow conditions and cone tip geometries used for the axisymmetric simulations reported in the previous section are used with the addition of an 8-degree azimuthal extension to the computational domain of each simulation. Three dimensional simulations of the full transition path from laminar to turbulent with different Reynolds numbers and nose tip radius over impermeable and porous walls are performed. With this set of simulations the effects of distributed wall porosity on the transitional structures and the ultimate turbulence will be analyzed.

5.1. Three Dimensional Forcing

The laminar flow is perturbed with grid independent pressure (p') fluctuations and their isentropic density (ρ') counterpart around the steady state. Since the imposed perturbations do not have an intrinsic propagation velocity, they are initially advected with the base state velocity field triggering the unstable mechanisms inside a hypersonic boundary layer until only the modes present on a “natural” transition scenario survive.

$$\rho' = p'/a_0^2, \quad (5.1)$$

where a_0 is the local speed of sound of the base state. In order to impose a disturbance field that approximates what is observed in a natural transition scenario, a pseudorandom field ranging from -1 to 1 is initialized. A Fourier transform is then applied in three-dimensions and the high wavenumber region, not solvable in the coarsest grid considered, is discarded. Then an inverse FFT operation is performed, what leads to a three-dimensional field of complex coefficients (C) that can be use to reconstruct a grid resolved near “white” spatial noise for each frequency component as,

$$p'_i(\mathbf{x}, t, f_i) = A(f_i) \text{Real} \left(C(\mathbf{x}) e^{-j2\pi f_i t} \right), \quad (5.2)$$

where $A(f)$ is the amplitude of the disturbance signal as a function of frequency. ? has reported the amplitude spectra of the noise measure in the HEG hypersonic wind tunnel for different operating conditions and showed that the amplitudes of the pressure disturbance signal is concentrated at low frequencies and that, if the measurements are normalized by the pressure measured at the probe’s surface, the noise curves approximately collapse. Based on these results, we fitted an exponential curve through the noise measurements to model the disturbance amplitude behavior for the different frequencies (see figure 13).

Ultimately, we introduce the disturbance field as a linear combination of the individual disturbance fields for each frequency in the range of 10 kHz to 590 kHz at each 20 kHz. Moreover, the total perturbation field achieved through this method is normalized by its standard deviation (σ) estimated using a period equivalent to the four times the largest time scale of the imposed perturbation field, in this case equivalent to $1e - 4s$, and the root mean squared pressure amplitude (p_{rms}) is chosen as free parameter. Therefore the functional form of the introduce disturbance field can be written as,

$$p'(\mathbf{x}, t) = \frac{p_{rms}}{\sigma} \sum_{i=0}^{28} p'_i(\mathbf{x}, t, f_i), \quad (5.3)$$

where $f_i = 10 + 20i$ kHz. In the current simulations, p_{rms} is chosen as $0.08p_\infty$ and good agreement with the experimental results is achieved even though the noise amplitude

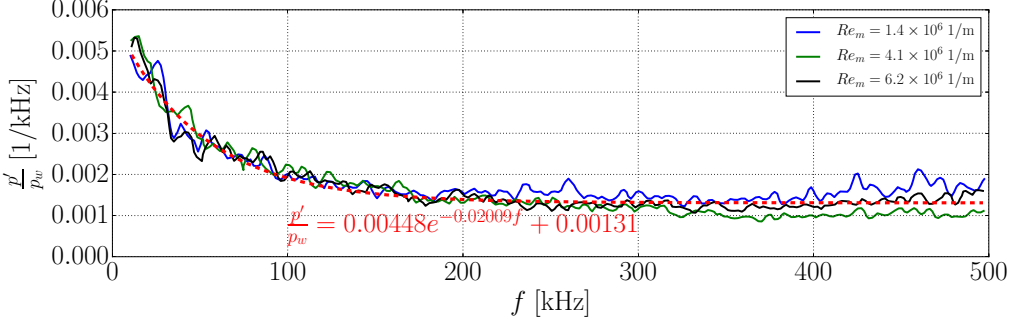


Figure 13: Reproduced results from ? for the amplitude spectra of the HEG tunnel for wall pressure fluctuations at the surface of a wedge probe normalized by its base surface pressure for different flow conditions are shown in solid lines (—). In dashed lines (---) the fitted curve of pressure fluctuation amplitude as a function of frequency. Although the flow conditions are not exactly the same as the ones reported in table 1, used in the current work, they are very close and the same color code is used.

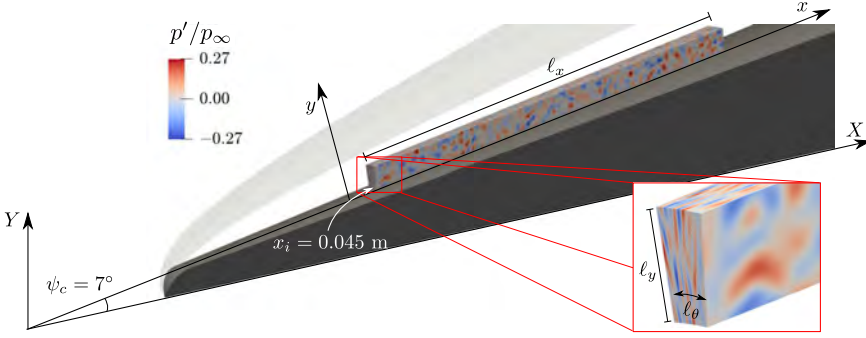


Figure 14: Volume of pressure fluctuations introduced in the simulation domain near the the inflow. The box size is $\ell_x = 0.075L_x$, $\ell_y = 0.002$ m and $\ell_\theta = 8.0$ degrees, being L_x the computational domain extent in the streamwise direction. Hyperbolic tangent functions are used to smoothly transition for the perturbed to the unperturbed regions and the transition occurs in one quarter of the noise volume's extent in both the streamwise and wall normal directions.

spectra collapse for different flow conditions reported in ? was with respect to the pressure at the surface of the model.

In the current set of simulations, the laminar base state was perturbed near the inflow and close to the wall in a volume of dimensions equal to $\ell_x = 0.075L_x$, $\ell_y = 0.002$ m and $\ell_\theta = 8.0$ degrees, where L_x is the simulation's extent in the streamwise direction (see table 2), as shown in figure 14. **Victor: Here I am still studying the possibility of increasing the wall normal extent of the domain in for the blunt cases.** Since the chosen pressure fluctuations is normalized by the standard deviation (σ), it is possible that the instantaneous maximum and minimum values exceed 3 times the imposed p_{rms} but, assuming a normal distribution, 99.7% of the values are expected to be below $3p_{rms} = 0.24p_\infty$ and, of those, 68.2% should be smaller than $p_{rms} = 0.08p_\infty$.

Victor: We decide for the start of the forcing br an order of magnitude lower than the second mode

5.2. Comparison against experiments

[Victor: St Plots](#)

[Victor: Rephrase and explain better + add laminar heat transfer from blunt cases](#)

To identify the transition between laminar to turbulent conditions, we plot the Stanton number (St) as a function of the stream wise coordinate. The Stanton number, as seen in relation 5.4, is directly proportional to the heat transfer at the wall (q_w) and it is possible to to define correlations that estimate its laminar and turbulent values.

$$St = \frac{q_w}{\rho_e C_p U_e (T_{aw} - T_w)} \quad (5.4)$$

The subscript e means that the values used in the normalization are taken at the edge of the boundary layer, the subscript w refers to values at the wall and, lastly, aw is used to indicate adiabatic wall conditions. The adiabatic wall temperature is estimated by the use of the recovery factor $r = Pr^{1/3}$, as indicated in White ?.

$$T_{aw} = \left(1 + r \frac{\gamma - 1}{2} M_e^2\right) T_e \quad (5.5)$$

The laminar correlation is obtained from the solution of the compressible Blasius similarity equations over a sharp cone. The friction coefficient (C_f) and for the the Stanton number (St) are given by

$$C_f = \frac{\mu_w \frac{\partial u}{\partial y}|_w}{\frac{1}{2} \rho U^2} = \sqrt{6} C_w Re_x^{-1/2} f''(0) \quad (5.6)$$

$$St = \frac{q_w}{\rho_e U_e C_p (T_{aw} - T_e)} = \frac{1}{\sqrt{2}} \frac{1}{Re_x^{1/2} Pr} \frac{T_e}{T_{aw} - T_w} g(0) g'(0) \quad (5.7)$$

where C is the Chapman-Rubesin parameter defined as $C = \rho \mu / \rho_e \mu_e$ and f' and g are the similarity functions that describe the profiles of the non-dimensional velocity and temperature, respectively. Since both of this quantities scale with $Re_x^{-1/2}$ one can compute the ratio and test if it is constant. This was called Reynolds analogy and can be shown to be true for Prandtl number equal to 1.

$$\frac{St}{C_f} \frac{T_{aw} - T_w}{T_e} = \frac{g'(0)}{2f''(0)Pr} \quad (5.8)$$

The turbulent St number values are estimated via the van Driest correlation, shown hereafter in equations 5.9 to 5.13.

$$a = \sqrt{r \frac{\gamma - 1}{2} M_e^2 \frac{T_e}{T_w}} \quad (5.9)$$

$$b = \left(\frac{T_{aw}}{T_w} - 1 \right) \quad (5.10)$$

$$A = \frac{2a^2 - b}{\sqrt{b^2 + 4a^2}} \quad (5.11)$$

$$B = \frac{b}{\sqrt{b^2 + 4a^2}} \quad (5.12)$$

$$\frac{\sin^{-1} A + \sin^{-1} B}{\sqrt{bC_f}} = 4.15 \log_{10} \left(\frac{Re_x C_f \mu_e}{\mu_w} \right) + 1.7 \quad (5.13)$$

From these relations, one can solve for the turbulent skin friction of a flat-plate iteratively. ? reports that turbulent skin frictions of a cone can be related to one of a flat plate through the relation shown in 5.14, with m between $\frac{1}{8}$ and $\frac{1}{4}$.

$$\frac{C_{f,cone}}{C_{f,plate}} = (2 + m) \left(\frac{m}{1+m} \right) \quad (5.14)$$

This leads to a 10 to 15 percent increase in turbulent heat transfer from a laminar flat plate to a cone. In this work, we chose the value of $m = 1/8$. After this step, the Reynolds analogy is again used to estimate the heat transfer values from skin friction.

5.3. Breakdown Mechanisms

[Victor: FFTs](#)

[Victor: maybe discuss streaks development](#)

[Victor: qualitative analysis of structures development](#)

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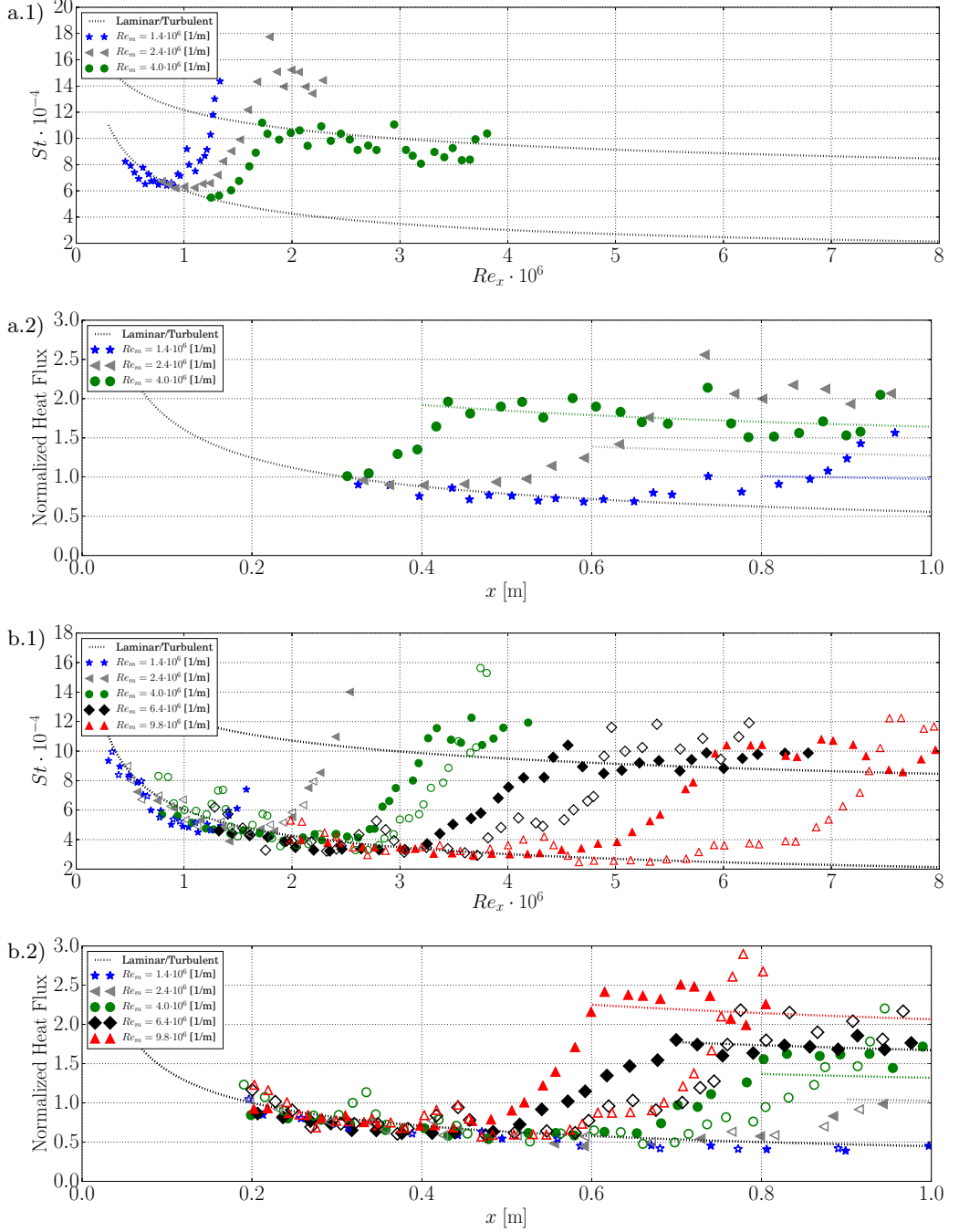


Figure 15: Heat flux experimental measurements, shown in symbols, and simulation results, shown in lines are gathered for both sharp-tipped (a) and blunt (b) cones. For each cone geometry the heat flux is plotted in two ways: 1) Stanton number against Re_x ; 2) Heat flux normalized by the laminar value corresponding to the first measurement location, being equal to $x = 0.31$ m for the sharp cone and $x = 0.2$ m for the blunt cone, versus the streamwise distance from the tip of the cone. Laminar and turbulent correlations are shown as dotted lines. Distribution A: distribution approved for public release.

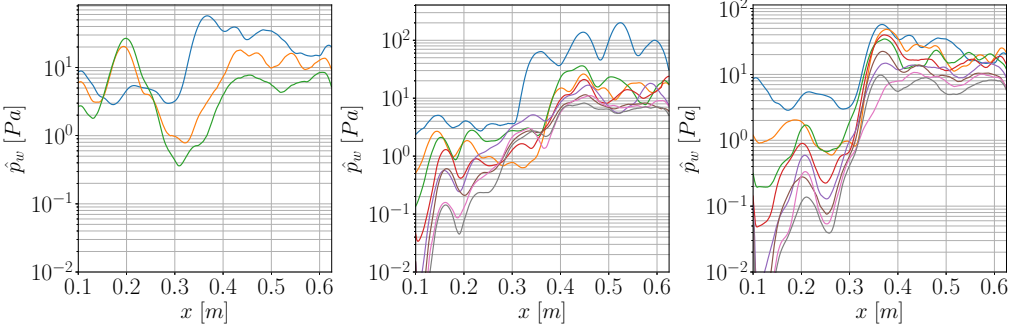


Figure 16: Caption

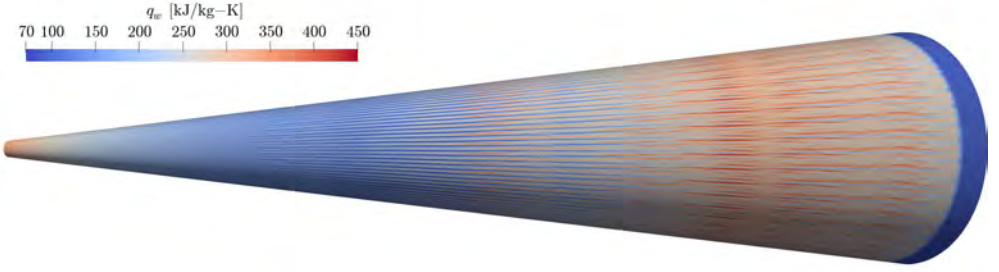


Figure 17: Caption

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In figure 18 we compare the difference in the transitional structures simulated with impermeable walls, shown in the left, and porous walls, shown in the right. Isosurfaces of the second invariant of the velocity gradient tensor, known as Q-criterion, are plotted as a way of visualizing the vortices produced during transition. A first conclusion that can be drawn is that the isosurfaces of the Q-criterion show the presence of smaller scales in the flow for the impermeable wall. The results presented in figure 18 also support the idea that the presence of a finite complex wall-impedance does not only affect the acoustics, decreasing the amplitude of the disturbances, but also the structure of the near-wall turbulence; from the visualizations in figure 18(right) it is possible to notice a crossing in the spanwise direction of transitional structures, as opposed to the classic hairpins found in low-speed boundary layers. This maybe be associated with intense transpiration events occuring at the wall, connected to local maxima in the acoustic flux, I_{ac} , as shown at the bottom of figure 18. Also, supporting previously shown results in figure ??, the heat transfer levels for the case of impermeable walls are much higher for the same location in space.

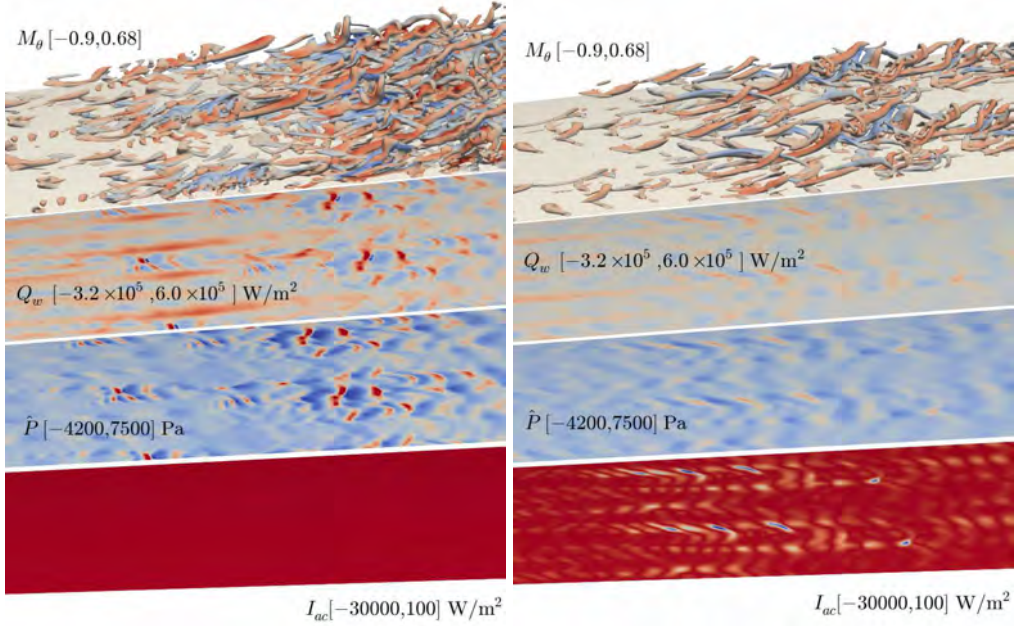


Figure 18: Comparison between flow quantities in the transitional region, from $x = 0.48$ m to $x = 0.52$ m, between the solid wall case, shown in the left, and the porous surface, shown in the right. From top to bottom it is shown: Q-criterion isosurfaces colored by the Mach number in the azimuthal direction, the instantaneous heat flux to the wall, the pressure oscillations and the acoustic flux.

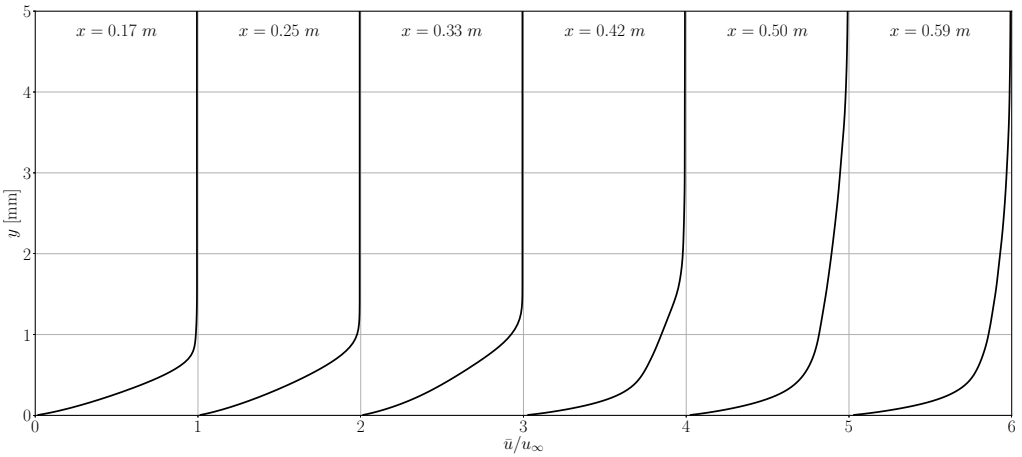


Figure 19: Caption

5.4. Turbulent flow evolution

[Victor:](#) mean U and mean T profiles evolution

[Victor:](#) profiles of rmach rms

[Victor:](#) profiles of reynolds shear stress and turbulent heat transfer

[Victor:](#) Turbulent energy vs wavenumber?

[Victor:](#) U+ VD and TL

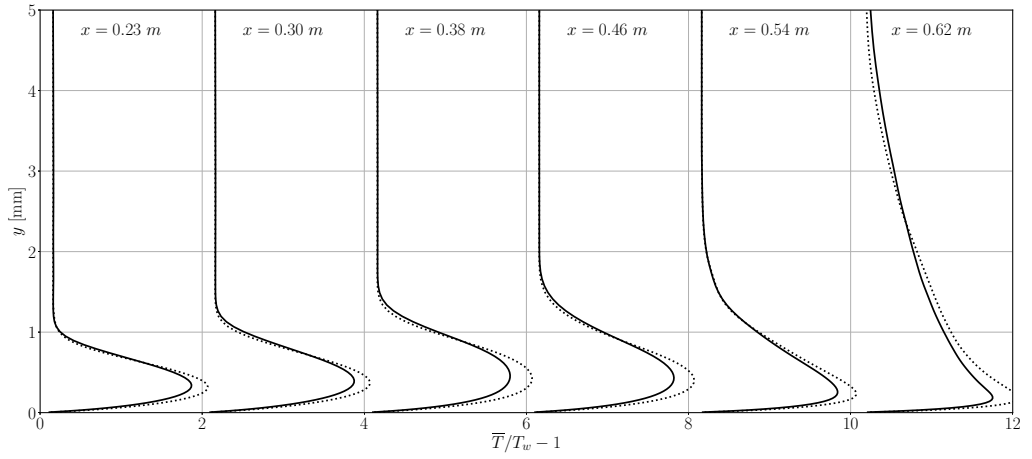


Figure 20: Caption

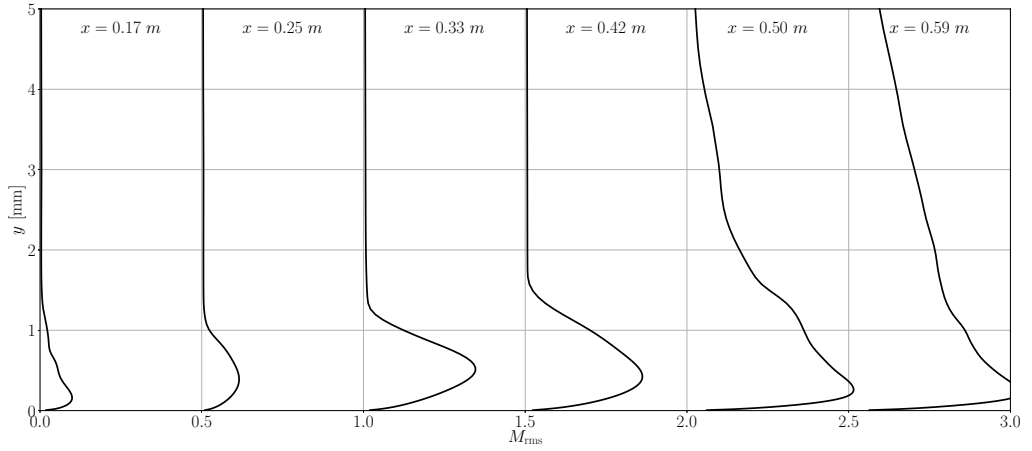


Figure 21: Caption

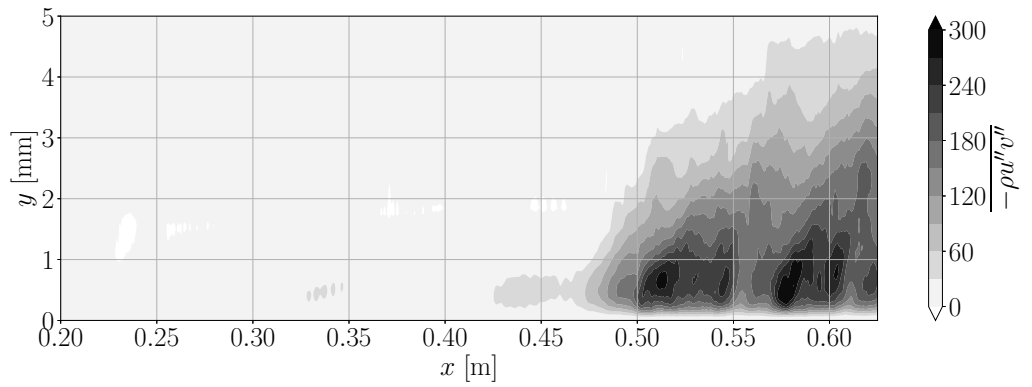


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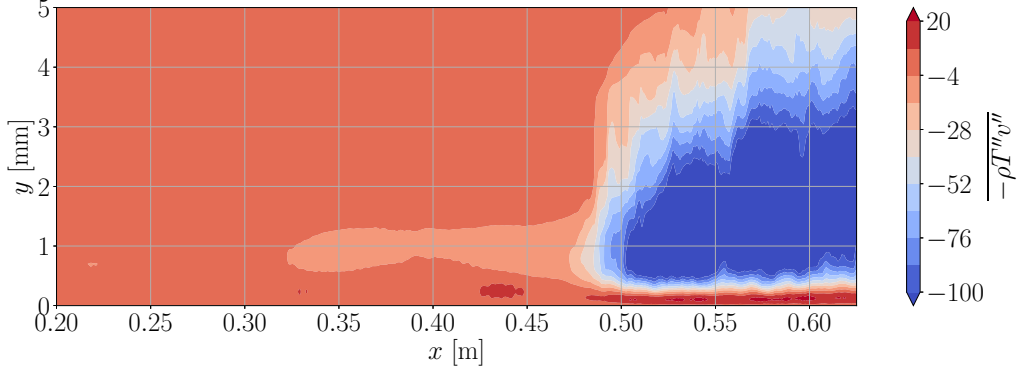


Figure 23: Caption

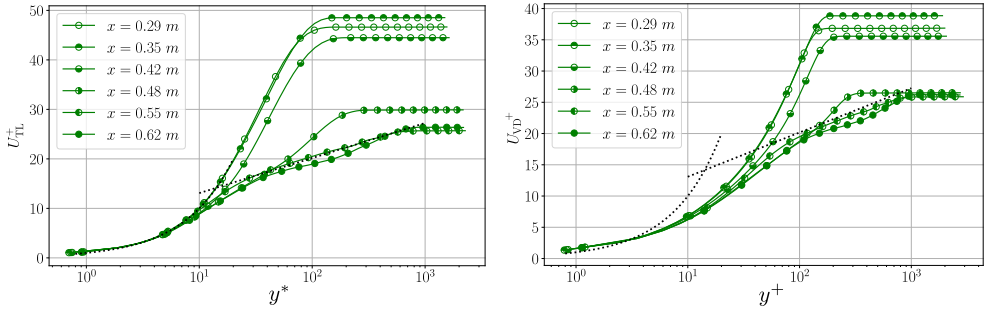


Figure 24: Caption

[Victor: analysis of intermittency and low frequencies in BL transition](#)

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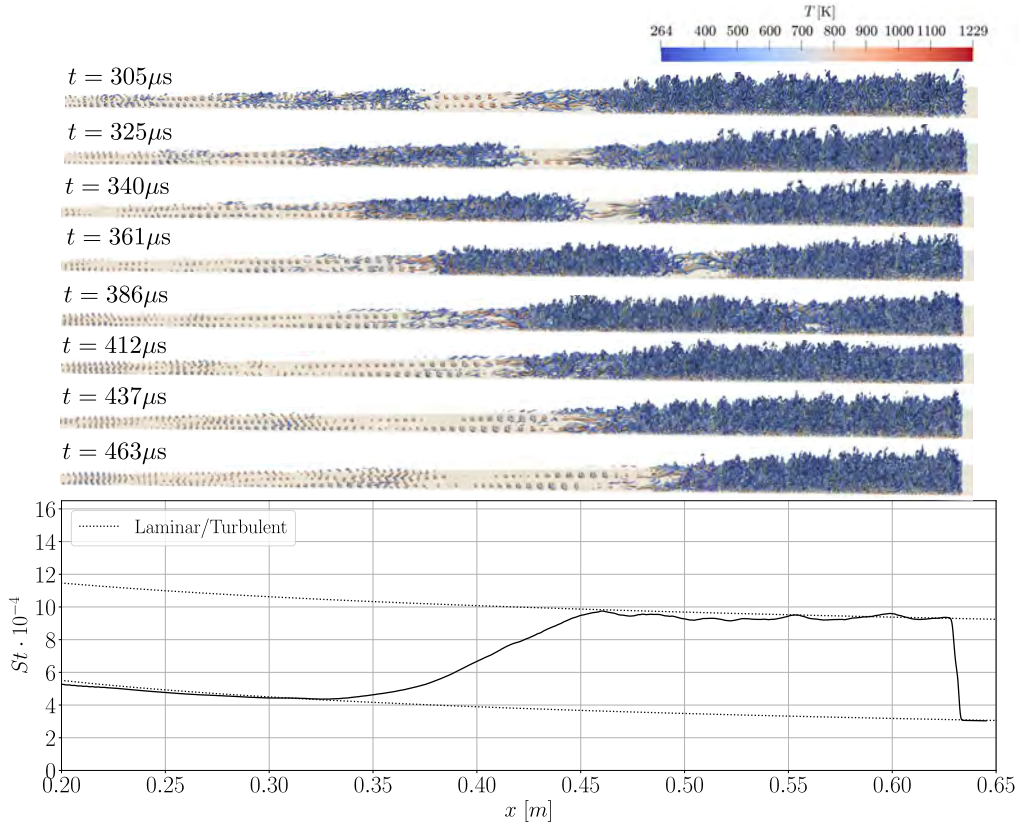


Figure 25: Caption

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6. Conclusion

We have carried out numerical simulations of hypersonic transition delay on a spatially developing hypersonic boundary layer over a 7° -half-angle cone at $M_\infty = 7.5$ for both sharp and 2.5mm-round-tip geometries due to porosity representative of carbon fiber reinforced carbon ceramics. The acoustic absorption of carbon/carbon (C/C) porous surfaces was experimentally measured and this information was used to reconstruct the dependence of the real and imaginary parts of impedance in the frequency domain via low-order models. Ultimately, a complex impedance boundary condition (IBC) is used to introduce the porosity effects in the computational domain. The flow conditions simulated are at Reynolds numbers $Re_m = 2.43 \cdot 10^6 \text{ m}^{-1}$ and $4.06 \cdot 10^6 \text{ m}^{-1}$ for the sharp cone and $Re_m = 4.06 \cdot 10^6 \text{ m}^{-1}$ and $6.40 \cdot 10^6 \text{ m}^{-1}$ for the round tip with isothermal walls at $T_w = 300 \text{ K}$. First, axisymmetric simulations are performed over impermeable walls and IBC, in which the second mode is excited via the application of a broadband pulse. The spatial amplification of individual frequency modes compares favorably against linear

stability analysis. The matching obtained supports the correct numerical implementation of the IBC. Following, three-dimensional simulations are performed and the boundary layer is excited with a grid independent filtered pseudo-random pressure perturbations, these are received and amplified ultimately leading to transition onset and turbulent breakdown. Transition delay is confirmed but with different dynamics.

[Victor: Abstract above](#)

Also, the presence of porous walls effectively decreased the heat flux to the wall for the same streamwise location, which is the ultimate design objective for the enhancement of hypersonic vehicles performance. Another result, shown in this manuscript, is that the finite impedance induced by the C/C random porosity not only decreases the magnitude of the waves during the linear regime but also changes the near-wall turbulent structures possibly due to acoustic fluxes induced by turbulent events.

As a final remark, the results here obtained assume that the length scales involved in the boundary layer disturbance growth and breakdown are much larger than the length scales involved in the random pore structure found in the C/C material, therefore it is possible to perform an spatial average of the admittance of the individual C/C pores as seen from the flow's perspective. Since the pores in a C/C surface are in the order of $1 - 10 \mu\text{m}$, this assumption is verified in spatial regions of modal growth, where the wavelengths involved are in the order of $1 - 10 \text{ mm}$; however, turbulent structures exhibit smaller scales, of the order of a fraction of a millimeter, which may require us to account for the larger-scale distributed surface porosity structure.

Acknowledgments

Victor Sousa and Carlo Scalo acknowledge the support of the Rosen Center for Advanced Computing (RCAC) at Purdue, the Air Force Office of Scientific Research (AFOSR) grant FA9550-16-1-0209 and the AFOSR YIP (FA9550-18- 271-0292). Victor Sousa also acknowledges the support of the prestigious Lynn Fellowship administered by the interdisciplinary Computational Science and Engineering (CS&E) graduate program at Purdue University. Alexander Wagner would like to acknowledges the support of the AFOSR through grant FA9550-16-1-0456. [Victor: ask Viola about her funding](#)

Appendix A. Linear Stability Equations Solver

A.1. Linearized Navier-Stokes Equations

The governing equations for a compressible ideal gas are the continuity equation, the conservation of momentum in the three spatial directions, the conservation of energy and the equation of state. These are written below, respectively:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (\text{A } 1a)$$

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla P + \nabla \cdot [\lambda (\nabla \cdot \mathbf{u}) \mathbf{I}] + \nabla \cdot [\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)] \quad (\text{A } 1b)$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T \right] = \nabla \cdot (k \nabla T) + \frac{\partial P}{\partial t} + (\mathbf{u} \cdot \nabla) P + \Phi \quad (\text{A } 1c)$$

$$\Phi = \lambda (\nabla \cdot \mathbf{u})^2 + 2\mu \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \quad (\text{A } 1d)$$

$$\mu \left[\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 \right] \quad (\text{A } 1e)$$

$$P = \rho RT$$

The linearized governing equations are then obtained by assuming the existence of a steady base flow that must then be a solution of the governing equations themselves. Moreover, the following derivation assumes that the perturbations are two dimensional and that the boundary layer grows slowly spatially (parallel flow assumption). It should be noted that while curvature effects are neglected in the governing equations for the fluctuations, the base flow is derived from a steady solution with no further simplifying assumptions. The assumptions made are summarized hereafter,

$$u = \bar{U} + \tilde{u}, \quad v = \tilde{v}, \quad w = 0, \quad P = \tilde{p}, \quad (\text{A } 2)$$

$$T = \bar{T} + \tilde{T}, \quad \mu = \bar{\mu} + \tilde{\mu}, \quad \lambda = \bar{\lambda} + \tilde{\lambda}, \quad k = \bar{k} + \tilde{k}, \quad (\text{A } 3)$$

where the total quantities are related to their respective base ($\bar{\cdot}$) and perturbation ($\tilde{\cdot}$) fields. The final set of linearized governing equations for the described set of assumptions is written below dropping the overbar for the base state quantities:

$$\frac{\partial \tilde{\rho}}{\partial t} + U \frac{\partial \tilde{\rho}}{\partial x} + \rho \frac{\partial \tilde{u}}{\partial x} + \rho \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \rho}{\partial y} \tilde{v} = 0 \quad (\text{A } 4a)$$

$$\rho \left[\frac{\partial \tilde{u}}{\partial t} + U \frac{\partial \tilde{u}}{\partial x} + \tilde{v} \frac{\partial U}{\partial y} \right] = -\frac{\partial \tilde{p}}{\partial x} + \frac{\partial}{\partial x} \left\{ 2\mu \frac{\partial \tilde{u}}{\partial x} + \lambda (\nabla \cdot \tilde{\mathbf{u}}) \right\} + \quad (\text{A } 4b)$$

$$\frac{\partial}{\partial y} \left\{ \mu \left[\frac{\partial \tilde{v}}{\partial x} + \frac{\partial \tilde{u}}{\partial y} \right] + \tilde{\mu} \frac{\partial U}{\partial y} \right\}$$

$$\rho \left[\frac{\partial \tilde{v}}{\partial t} + U \frac{\partial \tilde{v}}{\partial x} \right] = -\frac{\partial \tilde{p}}{\partial y} + \frac{\partial}{\partial x} \left\{ \mu \left[\frac{\partial \tilde{v}}{\partial x} + \frac{\partial \tilde{u}}{\partial y} \right] + \tilde{\mu} \frac{\partial U}{\partial y} \right\} + \frac{\partial}{\partial y} \left\{ 2\mu \frac{\partial \tilde{v}}{\partial y} + \lambda (\nabla \cdot \tilde{\mathbf{u}}) \right\} \quad (\text{A } 4c)$$

$$\rho c_p \left[\frac{\partial \tilde{T}}{\partial t} + U \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial T}{\partial y} \right] = \frac{\partial \tilde{p}}{\partial t} + U \frac{\partial \tilde{p}}{\partial x} + \frac{\partial}{\partial x} \left[k \frac{\partial \tilde{T}}{\partial x} \right] + \frac{\partial}{\partial y} \left[k \frac{\partial \tilde{T}}{\partial y} + \tilde{k} \frac{\partial T}{\partial y} \right] + \tilde{\Phi} \quad (\text{A } 4d)$$

$$\tilde{\Phi} = 2\mu \frac{\partial U}{\partial y} \left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x} \right) + \tilde{\mu} \left(\frac{\partial U}{\partial y} \right)^2 \quad (\text{A } 4e)$$

$$\tilde{\rho} = \frac{\tilde{p}}{RT} - \frac{\rho \tilde{T}}{T} \quad (\text{A } 4f)$$

To make this system of equations simpler to solve some further steps must be taken.

First, one can relate the conductivity (k) and the second viscosity coefficient (λ) to the molecular viscosity (μ) as,

$$k = \frac{\mu c_p}{Pr}, \quad \lambda = -\frac{2}{3}\mu. \quad (\text{A } 5)$$

Following, the viscosity perturbation can be represented as a function of temperature and its perturbation,

$$\tilde{\mu} = \frac{\partial \mu}{\partial T} \tilde{T}, \quad (\text{A } 6)$$

also, one can use the fact that the base pressure is uniform through the boundary layer to remove the base density of the system of equations with the help of the equation of state,

$$\rho = \frac{\rho_e T_e}{T}. \quad (\text{A } 7)$$

Ultimately, the following ansatz is made about the perturbations,

$$\tilde{f} = \hat{f}(y)e^{\kappa x - j\omega t}, \quad (\text{A } 8)$$

which are assumed to be a wave traveling in the streamwise direction and to have a shape dependent on the wall normal coordinate. With all these assumptions the linearized Navier-Stokes reduce to the following set of equations:

$$-j\omega \left[\frac{1}{RT} \hat{p} - \frac{\rho_e T_e}{T^2} \hat{T} \right] = -U \left[\frac{1}{RT} \kappa \hat{p} - \frac{\rho_e T_e}{T^2} \kappa \hat{T} \right] - \frac{\rho_e T_e}{T} \left[\kappa \hat{u} + \frac{\partial \hat{v}}{\partial y} \right] + \frac{\rho_e T_e}{T^2} \frac{\partial T}{\partial y} \hat{v} \quad (\text{A } 9a)$$

$$\begin{aligned} \frac{\rho_e T_e}{T} \left[-j\omega \hat{u} + U \kappa \hat{u} + \hat{v} \frac{\partial U}{\partial y} \right] = & -\kappa \hat{p} + \frac{4}{3} \mu \kappa^2 \hat{u} + \frac{1}{3} \mu \kappa \frac{\partial \hat{v}}{\partial y} \\ & + \frac{\partial}{\partial y} \left\{ \frac{\partial \mu}{\partial T} \hat{T} \frac{\partial U}{\partial y} \right\} + \frac{\partial \mu}{\partial y} \left[\kappa \hat{v} + \frac{\partial \hat{u}}{\partial y} \right] \end{aligned} \quad (\text{A } 9b)$$

$$\begin{aligned} \frac{\rho_e T_e}{T} \left[-j\omega \hat{v} + U \kappa \hat{v} \right] = & -\frac{\partial \hat{p}}{\partial y} + \mu \kappa^2 \hat{v} + \frac{4}{3} \mu \frac{\partial^2 \hat{v}}{\partial y^2} + \frac{1}{3} \mu \kappa \frac{\partial \hat{u}}{\partial y} + \frac{\partial \mu}{\partial T} \frac{\partial U}{\partial y} \kappa \hat{T} \\ & + 2 \frac{\partial \mu}{\partial y} \left[\frac{2}{3} \frac{\partial \hat{v}}{\partial y} - \frac{1}{3} \kappa \hat{u} \right] \end{aligned} \quad (\text{A } 9c)$$

$$\begin{aligned} \frac{\rho_e T_e}{T} \left[-j\omega \hat{T} + U \kappa \hat{T} + \hat{v} \frac{\partial T}{\partial y} \right] = & \frac{1}{c_p} \left(-j\omega \hat{p} + U \kappa \hat{p} \right) + \frac{\mu}{Pr} \left(\kappa^2 \hat{T} + \frac{\partial^2 \hat{T}}{\partial y^2} \right) \\ & + \frac{1}{Pr} \frac{\partial}{\partial y} \left(\frac{\partial \mu}{\partial T} \hat{T} \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial y} \left[\frac{\mu}{Pr} \right] \frac{\partial \hat{T}}{\partial y} \\ & + 2 \frac{\mu}{c_p} \frac{\partial U}{\partial y} \left(\frac{\partial \hat{u}}{\partial y} + \kappa \hat{v} \right) + \frac{1}{c_p} \frac{\partial \mu}{\partial T} \hat{T} \left(\frac{\partial U}{\partial y} \right)^2 \end{aligned} \quad (\text{A } 9d)$$

A.2. Galerkin Method of Weighted Residuals

From equations A 9 one can assume a polynomial expansion for the wall normal mode shapes on as a sum of weighted basis functions $\{\psi_j\}$, where $\hat{f}$ are their weighting coefficients.

$$\hat{f}(y) \approx \sum_{i=1}^N \check{f}_i \psi_i \quad (\text{A } 10)$$

Since this is only a finite approximation of the infinite expansion which would constitute the exact solution, once we substitute this into the linearized equations a non-zero residual (R) arises. As the polynomial order (N) of the expansion increases and approaches infinity, the approximate solution tends to the exact one and the residual R vanishes. Assuming a finite N though, for each polynomial order expansion the residual can be minimized by the correct choice of the weighting coefficients. The values for $\check{f}_j$ are found via the method of weighted residuals, which restrict the residual to be orthogonal to a set of trial functions that satisfy the boundary conditions of the initial problem. In particular, the Galerkin method uses the same set of basis functions employed in the polynomial expansion as trial functions and, in the end, the orthogonality condition leads to the formulation of the following inner product:

$$\int_D \psi_i R(\check{f}_j \psi_j) d\mathbf{x} = 0 \quad (\text{A } 11)$$

which, is a linear system of equations with the weighting coefficients as unknowns.

The numerical efficiency of the method is directly influenced by the choice of the expansion basis in two ways. First, the basis can impact the computational cost of assembling the linear system, which involves the numerical integration of the inner products and, second, it affects the total matrix inversion time. Also, as the order of the polynomial expansion increases, special attention must be given to conditioning of the system, which could degrade rapidly leading to a numerically linearly dependent basis. For these reasons, ? argues in favor of using the orthogonal Jacobi polynomials. Additionally, the accuracy of this method depends on the approximation properties of the expansion functions, i.e. their capability of matching the boundary conditions of the initial problem. For the linear stability eigenvalue problem of a boundary layer, one expects the mode gradients to be concentrated near the wall and to decay as the distance from the boundary increases. Therefore, the Laguerre functions ($\hat{L}_k(y)$), a specific type of Jacobi polynomials defined on the positive half of the real number axis $[0, \infty)$ (?) was chosen.

Following the implementation of the Galerkin method using Laguerre functions developed by ?, an initial basis that satisfies the homogeneous Dirichlet boundary condition at the wall ($\check{f}(0) = 0$) and the boundary condition at infinity ($\lim_{y \rightarrow \infty} \check{f}(y) = 0$) is achieved by algebraic manipulation,

$$\hat{\phi}_k(y) = \hat{L}_k(y) - \hat{L}_{k+1}(y). \quad (\text{A } 12)$$

The resulting basis is then augmented by one Laguerre mode that is equal to 1 at the lower boundary $\{\psi_j(y)\} = \{\hat{L}_1(y), \hat{\phi}_1(y), \hat{\phi}_2(y), \dots, \hat{\phi}_{j-1}(y)\}$. The addition of this non-zero function makes it possible to lift the solution to any required value at the boundary by adjusting its weight, which could also be set to zero to yield a homogeneous Dirichlet boundary condition.

The integrals of type A 11 are evaluated using the Gauss-Radau-Laguerre quadrature, where the nodes and weights are obtained using a modified Jacobi matrix of order $N+1$, whose eigenvalues coincide with the node locations and the weights can be expressed as function of the associated normalized eigenvectors, as described by ?. If one wants to increase the accuracy of the polynomial expansion to get closer to the exact solution, modes of higher order are needed but, as the order of the Gauss-Radau-Laguerre quadrature

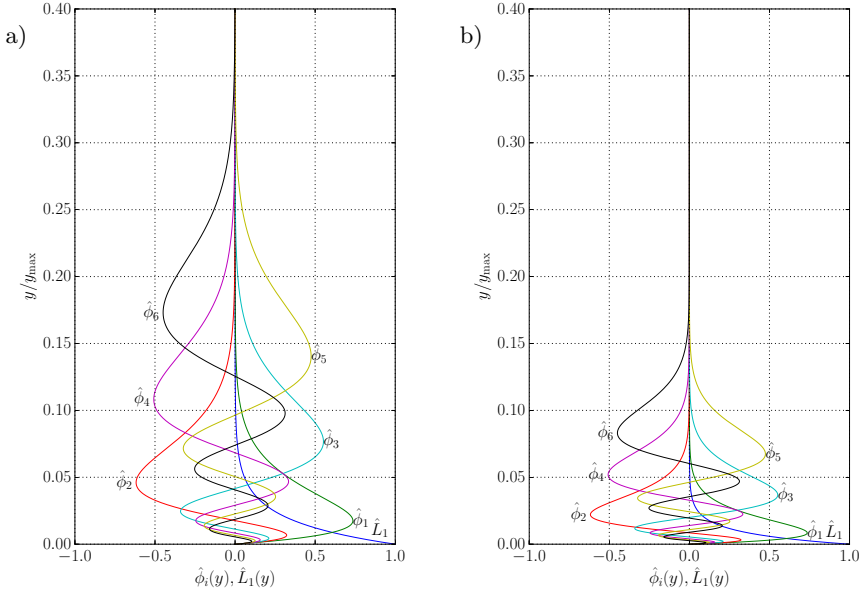


Figure 26: Mode shapes of the first 6 components of $\hat{\phi}_k(y)$ and $\hat{L}_1(y)$ for polynomial expansions of order 32 (a) and 64 (b). In order to increase the accuracy of numerical procedure, the Laguerre functions are scaled into a region of interest. In the end, an increase in the order of reconstruction not only increases the number of modes but also concentrates them in the near boundary region.

grows, the location of its nodes move away from $y = 0$ to better capture the asymptotic behavior at infinity. This results in the unwanted shift of the majority of the quadrature nodes outside the region of interest for the problem. To overcome this difficulty, a scaling of the basis functions is introduced and a lengthscale (y_{max}) where an "effective infinity" would have already been reached is used so that all the quadrature points are located below y_{max} . The scaling of the Gauss-Radau-Laguerre quadrature collocation points was shown to lead to higher numerical accuracy by ?.

If y_{max} is set to a prefixed value and the order of the polynomial expansion is increased, in addition to a higher number of modes, the scaling effect also concentrates points near the boundary, what is useful to solve the boundary layer stability problem. This effect of flattening the modes is shown in figure 26, where the first six modes of the basis $\phi_k(y)$ and $\hat{L}_1$ are plotted for two polynomial orders, $N = 32$ and $N = 64$, and the modes originated from the higher order reconstruction lay closer to the boundary in comparison to the lower order one.

Introducing the basis functions expansion ($\psi_j(y)$) in the previously derived linearized Navier-Stokes equations alongside with the discretized integration methods based on the gauss quadrature,

$$\mathbf{M}[g] = \int_D g \psi_i \psi_j d\mathbf{x}, \quad \mathbf{C}_y[g] = \int_D g \psi_i \frac{\partial \psi_j}{\partial y} d\mathbf{x}, \quad \mathbf{C}_{y^2}[g] = \int_D g \psi_i \frac{\partial^2 \psi_j}{\partial y^2} d\mathbf{x}, \quad (\text{A } 13)$$

a generalized eigenvalue problem is reached and its representation as a system of matrix equations is shown in A 14.

$$\omega \left(\mathbf{M} \left[\frac{1}{RT} \right] \cdot \check{\mathbf{p}} - \mathbf{M} \left[\frac{\rho_e T_e}{T^2} \right] \cdot \check{\mathbf{T}} \right) + \kappa \left(\mathbf{M} \left[\frac{U}{RT} \right] \cdot \check{\mathbf{p}} - \mathbf{M} \left[\frac{U \rho_e T_e}{T^2} \right] \cdot \check{\mathbf{T}} \right. \quad (\text{A } 14a)$$

$$\left. + \mathbf{M} \left[\frac{\rho_e T_e}{T} \right] \cdot \check{\mathbf{u}} \right) + \mathbf{C}_y \left[\frac{\rho_e T_e}{T} \right] \cdot \check{\mathbf{v}} - \mathbf{M} \left[\frac{\rho_e T_e}{T^2} \frac{\partial T}{\partial y} \right] \cdot \check{\mathbf{v}} = 0$$

$$\omega \left(\mathbf{M} \left[\frac{\rho_e T_e}{T} \right] \cdot \check{\mathbf{u}} \right) + \kappa \left(\mathbf{M} \left[\frac{\rho_e T_e U}{T} \right] \cdot \check{\mathbf{u}} + \mathbf{M} \cdot \check{\mathbf{p}} \right. \quad (\text{A } 14b)$$

$$\left. t - \frac{4}{3} \mathbf{M} [\mu] \cdot \kappa \check{\mathbf{u}} - \frac{1}{3} \mathbf{C}_y [\mu] \cdot \check{\mathbf{v}} - \mathbf{M} \left[\frac{\partial \mu}{\partial y} \right] \cdot \check{\mathbf{v}} \right) + \mathbf{M} \left[\frac{\rho_e T_e}{T} \frac{\partial U}{\partial y} \right] \cdot \check{\mathbf{v}}$$

$$- \mathbf{M} \left[\frac{\partial}{\partial y} \left(\frac{\partial \mu}{\partial T} \frac{\partial U}{\partial y} \right) \right] \cdot \check{\mathbf{T}} - \mathbf{C}_y \left[\frac{\partial \mu}{\partial T} \frac{\partial U}{\partial y} \right] \cdot \check{\mathbf{T}} - \mathbf{C}_y \left[\frac{\partial \mu}{\partial y} \right] \cdot \check{\mathbf{u}} = 0$$

$$\omega \left(\mathbf{M} \left[\frac{\rho_e T_e}{T} \right] \cdot \check{\mathbf{v}} \right) + \kappa \left(\mathbf{M} \left[\frac{\rho_e T_e U}{T} \right] \cdot \check{\mathbf{v}} - \frac{1}{3} \mathbf{C}_y [\mu] \cdot \check{\mathbf{u}} - \mathbf{M} [\mu] \cdot \kappa \check{\mathbf{v}} \right. \quad (\text{A } 14c)$$

$$\left. - \mathbf{M} \left[\frac{\partial \mu}{\partial T} \frac{\partial U}{\partial y} \right] \cdot \check{\mathbf{T}} + \frac{2}{3} \mathbf{M} \left[\frac{\partial \mu}{\partial y} \right] \cdot \check{\mathbf{u}} \right) + \mathbf{C}_y \cdot \check{\mathbf{p}} - \frac{4}{3} \mathbf{C}_{y^2} [\mu] \cdot \check{\mathbf{v}}$$

$$- \frac{4}{3} \mathbf{C}_y \left[\frac{\partial \mu}{\partial y} \right] \cdot \check{\mathbf{v}} = 0$$

$$\omega \left(\mathbf{M} \left[\frac{\rho_e T_e}{T} \right] \cdot \check{\mathbf{T}} - \frac{1}{c_p} \mathbf{M} \cdot \check{\mathbf{p}} \right) + \kappa \left(\mathbf{M} \left[\frac{\rho_e T_e U}{T} \right] \cdot \check{\mathbf{T}} - \frac{1}{c_p} \mathbf{M} [U] \cdot \check{\mathbf{p}} \right. \quad (\text{A } 14d)$$

$$\left. - \mathbf{M} \left[\frac{\mu}{Pr} \right] \cdot \kappa \check{\mathbf{T}} - 2 \mathbf{M} \left[\frac{\mu}{c_p} \frac{\partial U}{\partial y} \right] \cdot \check{\mathbf{v}} \right) + \mathbf{M} \left[\frac{\partial T}{\partial y} \frac{\rho_e T_e}{T} \right] \cdot \check{\mathbf{v}} - \mathbf{C}_{y^2} \left[\frac{\mu}{Pr} \right] \cdot \check{\mathbf{T}}$$

$$- \mathbf{M} \left[\frac{1}{Pr} \frac{\partial}{\partial y} \left(\frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} \right) \right] \cdot \check{\mathbf{T}} - \mathbf{C}_y \left[\frac{1}{Pr} \frac{\partial \mu}{\partial T} \frac{\partial T}{\partial y} \right] \cdot \check{\mathbf{T}} - \mathbf{C}_y \left[\frac{\partial}{\partial y} \left(\frac{\mu}{Pr} \right) \right] \cdot \check{\mathbf{T}}$$

$$- 2 \mathbf{C}_y \left[\frac{\mu}{c_p} \frac{\partial U}{\partial y} \right] \cdot \check{\mathbf{u}} - \mathbf{M} \left[\frac{1}{c_p} \frac{\partial \mu}{\partial T} \left(\frac{\partial U}{\partial y} \right)^2 \right] \cdot \check{\mathbf{T}} = 0$$

In possession of equations A 14 one can chose to solve a temporal or a spatial stability problem. To solve the time dependent problem, one should specify an imaginary wavenumber (κ) and solve for the complex frequency (ω) as described by relation A 15, where $\hat{\mathbf{u}} = (\check{\mathbf{p}}, \check{\mathbf{u}}, \check{\mathbf{v}}, \check{\mathbf{T}})$.

$$(\mathcal{K} + \omega \mathcal{M}) \hat{\mathbf{u}} = 0 \quad (\text{A } 15)$$

The equivalent spatial stability problem requires first that one linearizes the equations by solving for $\tilde{\mathbf{u}} = (\check{\mathbf{p}}, \check{\mathbf{u}}, \check{\mathbf{v}}, \check{\mathbf{T}}, \kappa \check{\mathbf{u}}, \kappa \check{\mathbf{v}}, \kappa \check{\mathbf{T}})$. After this step is done, one would specify a real frequency (ω) and solve for the complex wavenumber (κ), as shown in relation A 16.

$$(\tilde{\mathcal{K}} + \kappa \tilde{\mathcal{M}}) \tilde{\mathbf{u}} = 0 \quad (\text{A } 16)$$

M	$T_0[^\circ\text{R}]$	$\frac{T_w}{T_{\text{adb}}}$	R	α	Malik (?)	Chen <i>et al.</i> (?)	Current work
0.5	500	1.0	2000	0.1	$0.02908 + 0.002244i$	$0.02909 + 0.002244i$	$0.02911 + 0.002244i$
10	4200	0.1	2000	0.105	$0.09748 + 0.002030i$	—	$0.09743 + 0.002068i$
10	4200	1.0	1000	0.12	$0.11586 + 0.000153i$	$0.11586 + 0.000147i$	$0.11586 + 0.000138i$

Table 3: LST Validation

A.3. LST validation

The described method for solving the linear stability equations was compared against previously published results for the temporal stability of compressible boundary layers by ? and ? (see table 3), showing good agreement. For the adiabatic wall $M = 10$ boundary layer case, the current work predicts a slightly smaller temporal growth rate in comparison with the previously published results. Since the mode amplitude peak moves away from the wall in this case, it is supposed that the boundary condition at the top boundary of the domain might exert more influence in comparison to the isothermal wall cases and, therefore, the use of the Laguerre functions and its property of exponential decay should yield a more accurate outcome. Results here are normalized with respect to the lengthscale $l_e = \sqrt{\nu_e x / u_e}$, where the subscript e is used to indicate that the values are at the edge of the boundary layer. R is the Reynolds number related to l_e and it can be shown that $xRe_m = R^2$.

A.4. LST predictions for the flow of ? experiments

? performed hypersonic boundary layer transition experiments using a sharp 3° half angle cone in the BAM6QT quiet wind tunnel and the conventional H2K wind tunnel. The conditions under which the experiments were conducted are reported in table 4. Porous surfaces formed by either regular drilled micro-holes or the C/C random porous structure were employed in the experiments and the collected data showed neither a damping of the second mode nor a delay of the transition process occurred.

In ?, the authors had speculated that the reason why the experiments conducted by ? weren't able to measure any attenuation of the second mode waves with the same C/C sample from DLR is that the frequency of the second mode waves were low (around 120 kHz) at the flow conditions chosen for that study (table 4). This supposition is based on the fact that the acoustic absorption capability of the C/C surface rapidly deteriorates in the low frequency and low surface pressure region. In addition, the use of ultrasonic absorbing coating increases the wall roughness by a small amount, operating in conditions of minimal capability of absorbing acoustic energy could even lead to flow destabilization.

In order to confirm this hypothesis, it suffices to look into the experimental conditions tested under which the porous walls would be highest absorbing and, if little benefit is found under these conditions all the other ones would also be expected to lead to similar results. The flow conditions at $Re_m = 9.5 \cdot 10^6 \text{ m}^{-1}$ were chosen as an example since they had the highest free stream pressure which power spectrum measurements (?) did not show signs of nonlinear energy flowing to higher harmonics up to $x = 0.80 \text{ m}$. The experimental conditions with lower freestream pressures simply lead to lower base wall pressures and, ultimately, lower acoustic absorption capabilities (see figures 5 and 28).

At these operating conditions, a sharp cone with 3° half angle, such as the one used in the study, a Taylor-Maccoll analysis leads to surface pressure conditions of $P_w = 725 \text{ Pa}$ and, using the the homogeneous absorber theory with the parameters fitted to the

Tunnel	Re_m [1/m]	M_∞ [-]	p_0 [Pa]	T_0 [K]	p_∞ [Pa]	T_∞ [K]	Condition
H2K	$3.2 \cdot 10^6$	6.0	510	590	323	72	Noisy
H2K	$4.1 \cdot 10^6$	6.0	510	500	323	61	Noisy
BAM6QT	$2.4 \cdot 10^6$	5.8	212	422	165	55	Noisy
BAM6QT	$3.3 \cdot 10^6$	5.8	294	424	229	55	Noisy
BAM6QT	$2.3 \cdot 10^6$	6.0	225	427	143	52	Quiet
BAM6QT	$7.2 \cdot 10^6$	6.0	694	420	439	51	Quiet
BAM6QT	$9.5 \cdot 10^6$	6.0	928	424	587	52	Quiet
BAM6QT	$11.5 \cdot 10^6$	6.0	1128	425	714	52	Quiet

Table 4: Operating conditions for the experiments performed by ?

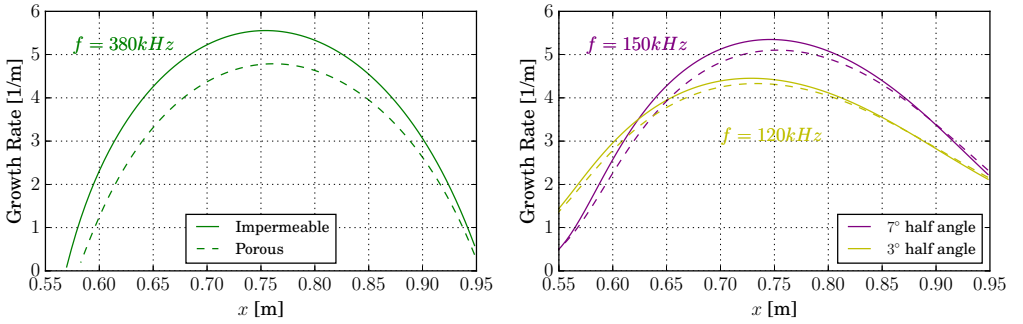


Figure 27: Results for the spatial growth rate of unstable waves inside a boundary layer over impermeable and porous walls for two different experimental conditions and two different cone geometries. On the left, the flow conditions mimic ? experiments at $Re_m = 4.0 \cdot 10^6 \text{ m}^{-1}$ and a 7° half angle cone. On the right, they are equivalent to $Re_m = 9.5 \cdot 10^6 \text{ m}^{-1}$ of ? experiments over a 3° and 7° half angle cones.

C/C response, the corresponding absorption coefficient of $\beta = 0.026$ is achieved for the frequency of 120 kHz, measured in the experiments. Comparing against ? experiments, this is much lower than the $\beta \approx 0.137$ expected for the frequency band around 380 kHz, that would be unstable at the same streamwise distance from the sharp cone tip for a 7° half angle at $Re_m = 4.0 \cdot 10^6$, for example. If the experiments by ? were performed over a 7° half angle cone, the pressure at the wall would increase to $P_w \approx 1153 \text{ Pa}$ and, in addition, the unstable frequency at a similar streamwise distance from the cone's tip would increase to $\approx 150 \text{ kHz}$. Both of these changes contribute to increasing the absorption coefficient of the C/C to $\beta \approx 0.039$, still much lower than ?.

The aforementioned LST solver code was used to predict the growth rate of the unstable frequencies for both experimental conditions using impermeable and porous surfaces as boundary conditions and the results are shown in figure 27. In the left plot, the results for the second mode growth attenuation for ? experimental conditions, what include the free stream properties and the 7 degree half angle cone, show an appreciable difference between the perturbation advection over an impermeable wall than over a porous wall. Concomitantly, the advection of a perturbation under the flow conditions of ? experiments and over both 3 and 7 degree half angled cones lead to a marginal stabilization effect

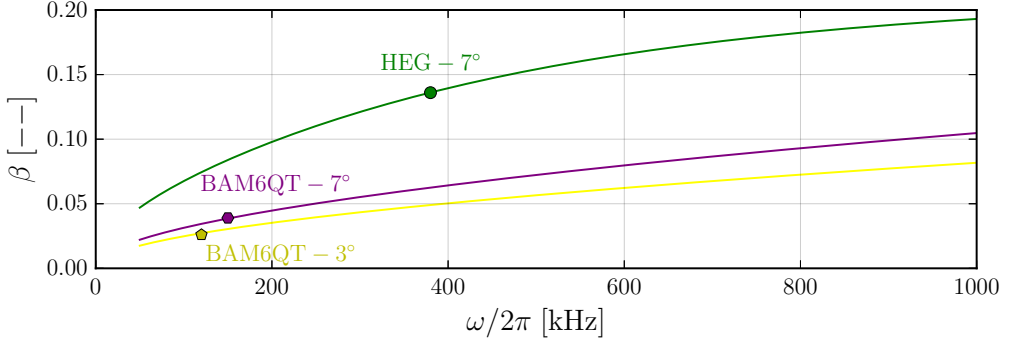


Figure 28: Absorption coefficient as a function of frequency for different wall pressure conditions corresponding to sharp cones at the experimental conditions reported by ? and by ?. For Wagner’s experiments, the 7 degree half angle cone used in the experiments was considered and the $Re_m = 4.0 \cdot 10^6 \text{ m}^{-1}$ was chosen as an example. For Willems experiments, both the 3 degree half angle cone used and a fictitious 7 degree cone were contemplated for $Re_m = 9.5 \cdot 10^6 \text{ m}^{-1}$. The symbols show what would the predicted β be for a disturbance at the aforementioned experimental conditions at $x \approx 0.75 \text{ m}$.

due to the low acoustic energy absorption coefficient of the C/C surface under such environment.

In figure 28 we collect the homogeneous absorber theory predictions for the absorption coefficient β as a function of the wall pressure for both the experimental conditions of Wagner (?), considering a 7 degree sharp cone, and Willems (?), for both 3 and 7 degree sharp cones. Results for Wagner’s experiments were already shown in figure 5 but are repeated here to make the comparison between the two cases easier. In figure 28 it is clearly shown that the low pressures and frequencies that were encountered in the BAM6QT lead to a reduced acoustic energy absorption capability in comparison with the results obtained using the HEG wind tunnel. What may be interesting for design purposes is to study the minimum absorption coefficient needed for a considerable second mode attenuation a consequent transition delay. If this minimum is, for example, $\beta = 0.1$, one could reach a fast conclusion that for wall pressures of 5 kPa unstable frequencies above 200 kHz would be absorbed while no much effect would be sense until frequencies of around 550 kHz if P_w was around 2 kPa.

The increase in amplitude and the highly nonlinear interactions that happen in the aft region of the flared cone lead to the appearance of entangled structures at the edge of the boundary layer when the magnitude of the density gradient becomes significant (figure 9). Since the optical technique of measuring second mode waves based on Schlieren relies on density gradients, a numerical Schlieren rendering is used to create a parallel with experimental results. It is interesting to highlight that these structures of fluctuations is only present in highly nonlinear regions and this is consistent with the experimental and numerical observations that the “rope waves” appear in the region preceding transition to turbulence.^{9,11,13} The simulated “rope waves” are also in agreement with the observations made by Stetson⁶ that the second mode has a wavelength of approximately two times the boundary layer thickness.

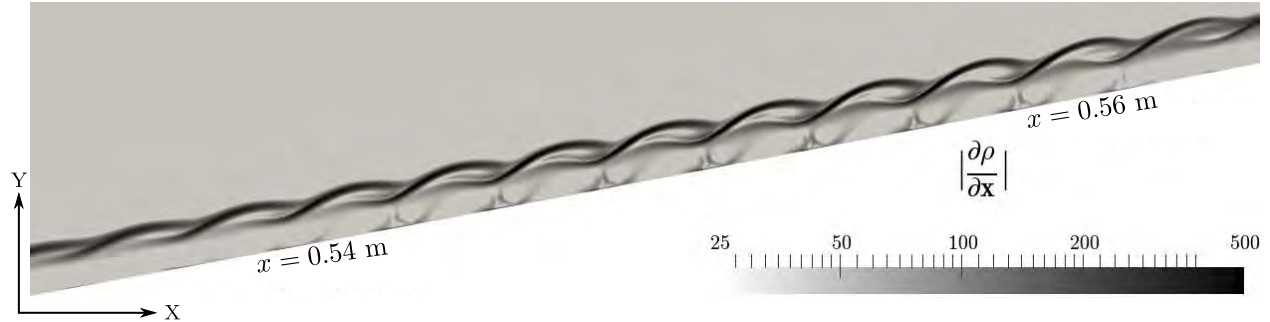


Figure 10: “Rope waves” form at boundary layer edge at the end of the computational domain in physical coordinates.

Since the physical domain is transformed into wall normal and streamwise coordinates in figure 9, the numerical Schlieren is replotted in figure 10 in physical coordinates. In this figure, the entanglement of the spatial variation of amplitude is more visible and, in addition, regions of compression and rarefaction inside the boundary layer are observed.

V. Summary

The spectral broadening of the second mode over a hypersonic flared cone was addressed in this work. A high order compact finite difference scheme was initialized from the laminar mean flow solution of a precursor simulation and was used to numerically predict the perturbation’s evolution downstream. It was observed that the flared cone designed was able to create an adverse pressure gradient that maintained the boundary layer thickness approximately constant over the entire cone. This effect concentrated the growth mechanisms of the second mode wave in a single frequency band, up to triggering nonlinear effects such as mean flow distortion and higher harmonics. The results of linear growth and nonlinear interaction was compared with a nonlinear parabolized stability equations (NPSE) solver and a good agreement was achieved. A pulse of higher amplitude was then introduced to visualize the phenomena of spectral broadening and that the so called “rope waves”, which develop only in the region of highly nonlinear interactions.

In conclusion, this work represents an intermediate step towards the understanding the relevant contributions of the nonlinearities present in a hypersonic flow and the principal energy source mechanism for second mode waves when they reach very high amplitudes. In the end, we demonstrate that we have developed the necessary tools for analyzing this in a future contribution.

Acknowledgments

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