AFRL-AFOSR-VA-TR-2020-0051

Electromagnetic Beam Propagation Through Dispersive Material

Natalie Cartwright
RESEARCH FOUNDATION OF STATE UNIVERSITY OF NEW YORK THE

12/18/2019
Final Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTB1
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
08-06-2020

2. REPORT TYPE
Final Performance

3. DATES COVERED *(From - To)*
15 Mar 2016 to 14 Sep 2019

4. TITLE AND SUBTITLE
Electromagnetic Beam Propagation Through Dispersive Material

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-16-1-0241

5c. PROGRAM ELEMENT NUMBER
61102F

6. AUTHOR(S)
Natalie Cartwright

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
RESEARCH FOUNDATION OF STATE UNIVERSITY OF NEW YORK THE
75 S MANHEIM BLVD
NEW PALTZ, NY 125612447 US

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AF Office of Scientific Research
875 N. Randolph St. Room 3112
Arlington, VA 22203

10. SPONSOR/MONITOR'S ACRONYM(S)
AFRL/AFOSR RTB1

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-AFOSR-VA-TR-2020-0051

12. DISTRIBUTION/AVAILABILITY STATEMENT
A DISTRIBUTION UNLIMITED: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The objective of AFOSR grant 'Electromagnetic Beam Propagation Through Dispersive Material" (FA9550-16-1-0241) was to analyze the propagation of both pulsed waves and pulsed beam fields through dispersive material using asymptotic techniques. Asymptotic methods provide valuable approximations based upon the full, causal electromagnetic response of the dispersive material over all frequencies. In this reporting period, we have continued to analyze waveforms for synthetic
aperture radar through dispersive material and have continued to investigate asymptotic methods for multiple integrals with complex phase.

15. SUBJECT TERMS
dispersive media, asymptotic expansions

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	UU		NACHMAN, ARJE
					19b. TELEPHONE NUMBER *(Include area code)* 703-696-8427

FINAL REPORT

Electromagnetic Beam Propagation Through Dispersive Material

Dr. Natalie A. Cartwright
State University of New York at New Paltz

The objective of AFOSR grant " Electromagnetic Beam Propagation Through Dispersive Material" (FA9550-16-1-0241) was to analyze the propagation of both pulsed waves and pulsed beam fields through dispersive material using asymptotic techniques. Asymptotic methods provide valuable approximations based upon the full, causal electromagnetic response of the dispersive material over all frequencies. In this reporting period, we have continued to analyze waveforms for synthetic aperture radar through dispersive material and we have continued to investigate asymptotic methods for multiple integrals with complex phase, as described in the following summary.

SAR Waveforms. When an electromagnetic pulse travels through a dispersive material each frequency of the transmitted pulse changes in both amplitude and phase, and each frequency at its own rate. As a consequence, broadband pulses propagating in dispersive material experience significant amplitude distortion and changes in pulse velocity. Asymptotic analysis of the exact integral representation of the propagated field, which utilizes the full causal dispersion relation of the dispersive material, provides a complete far-field description of the propagated pulse. In 2005, Oughstun [1] proposed a *Brillouin pulse* as a pulse that experiences the least attenuation due to material absorption. It has been hypothesized that the slow decay rate of the Brillouin pulse can be used to advantage in radar detection and imaging applications [1, 2, **?**].

The question of whether or not the Brillouin pulse is the ideal waveform for synthetic aperture radar imaging through dispersive material was partially addressed by Varslot, Morales, and Cheney [3, 4] in a pair of papers appearing in 2010 and 2011. These authors used a filtered back-projection algorithm to derive an optimal filter and its associated optimal waveform. The optimization is based upon minimizing the mean square error of the L^2-norm between the ideal image and the reconstructed image. The optimal waveform is numerically derived for each noise level by solving a fixed point equation for the absolute value of the optimal pulse spectrum and then using a minimum-phase algorithm to obtain the optimal waveform. The authors concluded that the optimal minimum-phase waveforms have a "transmit spectrum that is concentrated around the frequencies which are conducive to the generation of precursors" [4], but no conclusive statement could be made.

We define a *scattering precursor* based on the asymptotic expansion of the propagated electric field component of the impulse response due to scattering by an isotropic point source in a frequency-dependent dispersive (and lossy) material. This scattering precursor is given by

$$E^{sc}(s,t) = \int_{\mathcal{C}} \frac{e^{-i2\omega(t-n(\omega)|\mathbf{r}_{s,\mathbf{y}}|/c_0)}}{16\pi^3|\mathbf{r}_{s,\mathbf{y}}|^2}\,\omega^2 P(\omega)d\omega\tilde{T}(\mathbf{y})d\mathbf{y} \tag{1}$$

$$\propto \int_{\mathcal{C}} \exp\left[\frac{2|\mathbf{r}_{s,\mathbf{y}}|}{c_0}\phi(\omega,\theta)\right]\omega^2 P(\omega)d\omega, \tag{2}$$

where the complex phase function ϕ is defined as

$$\phi(\omega,\theta) = i\omega\left[n\left(\omega\right)-\theta\right] = i\omega\left[\sqrt{\epsilon(\omega)}\right], \tag{3}$$

and $\theta = c_0 t/2|\mathbf{r}_{s,\mathbf{y}}|$ is a dimensionless space-time parameter. Here, $P(\omega)$ is the spectrum of the transmitted pulse, $\tilde{T}$ is a modified target that accounts for both the target reflectivity and non-planar surface area, $\mathbf{y} = (y_1, y_2)$ is a two dimensional vector position, and $\mathcal{C}$ is a Bromwich contour in the upper half plane. The complex-valued index of refraction of the dispersive material is $n(\omega) = \sqrt{\epsilon(\omega)}$, the flight path $\gamma(s)$ is parameterized by the slow time s, and $\mathbf{r}_{s,\mathbf{y}} = |\mathbf{\Psi}(\mathbf{y})-\gamma(s)|$ is the distance between the antenna and the target. Here, it is assumed that the targets are stationary and are comprised of linear materials, the antenna is an isotropic point source, the same antenna is used to transmit and receive, the antenna's position remains fixed from transmit to receive, scattering occurs at a known surface $\mathbf{\Psi}(x_1,x_2) = [x_1, x_1, \psi(x_1,x_2)]^T$, and the target dispersion in known [3, 4].

Our comparisons of the propagation characteristics and reconstruction images of the scattering precursor to the optimal waveform of Varslot et al. and a rectangular modulated sinusoid [5] reveals that, as expected, higher frequency components that remain in the propagated pulse improve the resolution of the image. However, as all high frequency components are attenuated with propagation through the material, all signals become dominated by low frequencies. Thus, we return to the age-old trade off between minimizing attenuation to increase the signal to noise ratio (preference for low frequencies) and resolution (preference for high frequencies).

Beam Propagation. For electromagnetic *beams* propagating in *three dimensions*, analytic approximations have been almost entirely restricted to complex amplitudes of monochromatic scalar wave fields propagating through lossless material; presumably due to the lack of a straightforward manner in which to deal with multiple integrals with complex phase. In a linear, homogeneous, isotropic, source-free material, each component of the scalar wave field may be represented in terms of its angular spectrum representation

$$E(\mathbf{r},t) = \frac{1}{4\pi^3}\mathrm{Re}\int_{ia-\infty}^{ia+\infty} d\omega \iint_{-\infty}^{\infty} dk_x dk_y \tilde{E}(k_x,k_y,0,\omega)e^{i\gamma(\omega)z}e^{i(\mathbf{k_T}\cdot\mathbf{r_T}-\omega t)}, \quad (4)$$

where $\tilde{E}(k_x,k_y,0,\omega)$ is the spatiotemporal spectrum of the electric field component on the plane $z=0$, the wave vector is

$$\mathbf{k}(\omega) = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + \gamma(\omega)\hat{\mathbf{z}} = \mathbf{k_T} + \gamma(\omega)\hat{\mathbf{z}}, \quad (5)$$

with position vector $\mathbf{r} = \mathbf{r_T} + z\hat{\mathbf{z}}$, square of the complex wave number $k^2(\omega) = \mathbf{k}(\omega)\cdot\mathbf{k}(\omega)$, and

$$\gamma(\omega) = \left[k^2(\omega) - k_T^2\right]^{1/2} \quad (6)$$

is, in general, a complex-valued function of angular frequency. The angular spectrum representation Eq. (4) has the general form

$$I(\lambda) = \int_{\mathcal{C}} g(\mathbf{z})e^{-\lambda\phi(\mathbf{z})}d\mathbf{z}, \quad (7)$$

where g is the amplitude function, ϕ is the complex phase function, λ is a large parameter, and $\mathcal{C}$ represents a d-chain in $\mathbb{C}^d$.

In the general case, in which the electromagnetic beam propagates through a dispersive (and hence, lossy) material, $k(\omega)$ is not strictly real-valued and separation of the integration region into homogeneous and evanescent regions is not possible. It is our opinion that a multidimensional analog of the saddle point method would be useful in the analysis of both the monochromatic wave $E(\mathbf{r},\omega)$ and the pulsed beam field $E(\mathbf{r},t)$. The theory of asymptotic analysis of multiple integrals with complex phase is by no means complete and has not been used extensively in applications, see [6] for an overview. For the case in which the critical points are isolated and nondegenerate and the phase function has a strict minimum at each critical point, the following theorem due to Pemantle and Wilson [6, 7] is applicable:

Suppose that the real part of ϕ is strictly positive except at the origin and that its Hessian matrix H is nonsingular there. Let g be any analytic function not vanishing at the origin and define $I(\lambda)$ by (7). Then

$$I(\lambda) \sim \sum_{l\geq 0} c_l\lambda^{-d/2} \quad (8)$$

where

$$c_0 = g(\mathbf{0})\frac{(2\pi)^{-d/2}}{\sqrt{\det H}} \quad (9)$$

and the choice of sign is defined by taking the product of the principal square roots of the eigenvalues of H.

This resulting asymptotic form is akin to those obtained for a purely imaginary phase function (via the method of stationary) or purely real phase function (via the method of Laplace). However, the increased difficulty in dealing with a complex phase function comes from the transformation to canonical form (in this case, a quadratic) and finding the appropriate d-chain in which the real part of the phase obtains a minimum and the imaginary part remains constant. We have not made noteworthy progress in this area and note that research in this area remains as was.

References

[1] K. E. Oughstun, "Dynamical evolution of the Brillouin precursor in Rocard-Powles-Debye model dielectrics," *IEEE Trans. Antennas Propag.*, vol. 53, no. 5, pp. 1582–1590, 2005.

[2] "Method, technique, and system for detecting Brillouin precursors at microwave frequencies for enhanced performance in various applications," U. S. patent US 8570207 B1.

[3] T. Varslot, J. H. Morales, and M. Cheney, "Synthetic-apertures radar imaging through dispersive media," *Inverse Problems*, vol. 26, no. 2, 2010.

[4] ——, "Waveform design for synthetic-aperture radar imaging through dispersive media," *SIAM J. Appl. Math.*, vol. 71, no. 5, pp. 1780–1800, 2011.

[5] N. Cartwright and K. Muller, "Precursors for SAR," forthcoming.

[6] R. Pemantle and M. C. Wilson, "Asymptotic expansions of oscillatory integrals with complex phase," *Contemp. Math.*, vol. 520, pp. 221–239, 2010.

[7] ——, *Analytic Combinatorics in Several Variables.* New York: Cambridge Univ. Press, 2013.