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Stochastic Surveillance and Distributed Coordination

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14. ABSTRACT

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For the setting of faults or randomly-appearing intruders, we proposed quickest detection algorithms and computed and optimized the so-called hitting time of a single and of multiple Markov chains. For example, we analyzed the meeting time between a pair of pursuer and evader performing random walks on digraphs. We obtained the closed-form expression for the expected meeting time and setup and studied the minimization problem for the expected capture time for a pursuer/evader pair.

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1 Summary

This project focused on robotic surveillance in complex environments via autonomous vehicles. The chief aim was to design fast and unpredictable motion strategies for surveillance agents. The technical approach focused on Markov chain modeling and optimization methods.

For the setting of faults or randomly-appearing intruders, we proposed quickest detection algorithms and computed and optimized the so-called hitting time of a single and of multiple Markov chains. For example, we analyzed the meeting time between a pair of pursuer and evader performing random walks on digraphs. We obtained the closed-form expression for the expected meeting time and setup and studied the minimization problem for the expected capture time for a pursuer/evader pair.

On the topic of unpredictable strategies, we proposed two notions of entropy for robotic motion. We first studied the problem of maximizing the entropy rate generated by a random walk. We showed the equivalence to a semi-definite program for reversible chains. We then introduced a novel concept of unpredictability based on the average entropy of the return time variables at the environment locations. We formally studied this optimization problem and validated the performance of projected gradient algorithms for this problem. We validated our algorithms on basic and random graphs and on a publicly-available dataset describing crime statistics in San Francisco.

We distributed Matlab and Julia implementations of our proposed algorithms in an open-source ”RoboSurv” library available on github. The grant also provided partial support for work by the PI on a network systems book and a few related topics, including synchronization in pulse-coupled oscillators, graph-theoretic small gain theorems for positive and monotone systems, capture strategies for 3D reach-avoid games, and collective cell migration.

2 Excerpt from proposal introduction and report organization

It is convenient to report here an excerpt from the Introduction section of the proposal narrative.

We propose the design of efficient surveillance, information gathering, and coordination strategies for robotic networks in dynamic environments and their application to DoD scenarios. We argue that sensor scheduling, motion planning and coordination algorithms for surveillance and anomaly detection is a crucial objective. The key technological challenge is how to search an area in a persistent manner, with minimal average time to detection, with unpredictable trajectories and with optimally-partitioned workload among multiple assets. The key scientific subject is the study of optimization criteria, convexity properties, relaxations and coordination strategies defined via the theory of Markov chains and random walks. Our technical approach is based on a combination of tools from the study of Markov chains, convex optimization, dynamical systems, distributed algorithms, robotic coordination, and network systems. Our research effort on stochastic surveillance is articulated in the following tasks:

(T1) characterizing optimal stationary distributions, i.e., deciding where to focus the search efforts,

(T2) computing optimal reversible Markov chains via convex optimization,

(T3) computing optimal non-reversible Markov chains on lifted spaces,

(T4) computing optimal Markov chains with entropy-rate constrains, i.e., designing unpredictable fast searchers,

(T5) designing and characterizing search strategies for randomly-moving evaders, and

(T6) defining, analyzing and optimizing an appropriate notion of group Kemeny constant (i.e., a mean first passage time for multiple walkers), thereby designing multivehicle surveillance policies.

3 Detailed review of activities by year

During the first performance period June 2015 - May 2016, we made solid contributions on the main thrust of this proposal, that is, Markov chain modeling and optimization methods with application to robotic surveillance and coordination. Specifically, we made progress on three of the tasks outlined in the proposal narrative: tasks (T1), (T2) and (T6) by publishing two journal articles and one conference article on these.

During the second performance period June 2016 - May 2017, we made solid contributions on the stochastic surveillance main thrust of this proposal. Specifically, submitted work on tasks (T4) “computing optimal Markov chains with entropy-rate constrains” and (T5) “designing and characterizing search strategies for randomly-moving evaders”. We also made preliminary progress (but not yet submitted) on (T3) “computing optimal non-reversible Markov chains on lifted spaces”. During Year 2, we also made progress on two other related tasks (that were only partly supported by this grant): the publication of a textbook entitled “Lectures on Network Systems” and work on collective cell migration in systems biology.

During the third performance period June 2017 - May 2018, we continued to work on stochastic surveillance as the main thrust of this project. Specifically, we submitted a main new paper on the general topic of “computing optimal Markov chains with entropy-rate constrains” — this new contribution provides a new formulation with practical and theoretical advantages over previous methods. We made progress also on a network systems textbook, an MS and a PhD theses, and other documents, as documented below.

Specifically, we completed our work on a new optimization problem for Markov chains based on the concept of “return time entropy.” The corresponding paper was revised and finally accepted by IEEE TAC – the corresponding conference paper was presented at CDC. As main new effort, we started working on strategic intruders and formulated the robotic surveillance problem as a Stackelberg game, where the surveillance agent knows the best response of the intruder and optimizes her strategy accordingly. We design a catalog of relevant problems, completed a preliminary theoretical analysis, and are now performing numerical experiments comparing various solvers (specifically, a solver based on sequential quadratic programming and a solver based on branch and bound methods). Finally, we made progress also on a network systems textbook (revision 1.2), on a journal submission on network small gain theorems and on a journal submission on distributed optimization of support vector machines.

During the final fifth performance period June 2019 - December 2019 (7 months), we continued to work on stochastic surveillance as the main thrust of this project. In particular, we continued our investigation on the Stackelberg game formulation of the robotic surveillance problem where the intruder is assumed to be omniscient. In the case of unit travel times, we looked at different prototypical graph topologies and tried to perform mathematically rigorous analysis on the optimal strategies for the surveillance agent. On a separate note, we analyzed the meeting time between a pair of pursuer and evader performing random walks on digraphs. We obtained the closed-form expression for the expected meeting time and setup and studied the minimization problem for the expected capture time for a pursuer/evader pair. Finally, we developed and published a software package, called *RoboSurv*, with both MATLAB and Julia implementations, that feature the computation and optimization of various metrics of Markov chains.

In the following sections, we discuss our scientific progress and accomplishments in each of these areas.

4 Markov chain modeling and optimization methods with application to robotic surveillance and coordination

4.1 Work completed during Year 1 (15jun15-31may16)

During the first performance period we made good progress and submitted/published work on tasks (T1), (T2), and (T6). We also made preliminary progress (but not yet submitted) on (T3) and (T5). In the next few paragraphs we review our publications to-date.

In pre-proposal preparation work in reference [1], we provide analysis and optimization results for the *mean first passage time*, also known as the *Kemeny constant*, of a Markov chain. First, we generalize the notion of Kemeny constant to environments with heterogeneous travel and service times, denote this generalization as the *weighted Kemeny constant*, and we characterize its properties. Second, for reversible Markov chains, we show that the minimization of the Kemeny constant and its weighted counterpart can be formulated as convex optimization problems and, moreover, as semidefinite programs. Third, we apply these results to the design of stochastic surveillance strategies for quickest detection of anomalies in network environments. We numerically illustrate the proposed design: compared with other well-known Markov chains, the performance of our Kemeny-based strategies are always better and in many cases substantially so.

In reference [2] on task (T1) and (T2), in collaboration with former PhD student Pushkarini Agharkar (now graduated), we study the problem of quickest detection of anomalies in an environment under large uncertainties in sensor measurements. The robotic roadmap corresponding to the environment can be represented as a graph with an arbitrary topology. We analyze the Ensemble CUSUM Algorithm for this surveillance problem. We quantify the delay in detection of anomalies using the Ensemble CUSUM Algorithm and also frame an optimization problem to minimize this detection delay. We then provide an upper bound on the optimal detection delay and frame a convex optimization problem to minimize this upper bound. We also propose an efficient policy which achieves this upper bound and which can be computed by solving a semidefinite program. We illustrate the efficacy of the Ensemble CUSUM Algorithm using numerical simulations. We observe that the efficient policy outperforms policies based on other well-known Markov chains. This trend is more noticeable for higher levels of uncertainties and noise in sensor measurements.

In references [3] and [4] on task (T6), in collaboration with PhD alumn Rush Patel and visiting student Andrea Carron, we provide analysis results for the weighted hitting time and the pairwise weighted hitting time of a Markov chain. Specifically, we provide a novel method for calculating the hitting time for multiple random walkers, which we denote as the *group hitting time*. We also provide closed form solution for calculating the hitting time between specified nodes for both the single and multiple random walker cases. Our results allow for the multiple random walks to be different and, moreover, for the random walks to operate on different subgraphs. The results are also applicable to robotic roadmaps with heterogeneous travel times. Finally, using sequential quadratic programming, we show the combination of transition matrices that generate the minimal group hitting time for various graph topologies are often different. These optimization results are directly applicable to the design of motion planning strategies for multiple searchers in a robotic roadmap.

4.2 Work completed during Year 2 (1jun16-31may17)

During the second performance period we made progress and submitted work on tasks (T4) and (T5). We also made preliminary progress (but not yet submitted) on (T3). In the next few paragraphs we review the submitted work during this period.

In reference [5] on (T4), we provide an algorithm to obtain the maximum entropy-rate Markov chain or the *maxentropic chain*. Finding the maximum entropy Markov chain subject to graph and stationary distribution constraints is known to be a convex program. By analyzing the solutions to the dual of the convex program, we establish that maximum entropy Markov chains have a unique structure which is codified by appropriate nonlinear maps, defining an appropriate maxentropc vector. The maxentropic vector is obtained by solving a system of quadratic equations. By analyzing this system of equations, we show that when graphs have self-loops that this map is a global diffeomorphism and provide an iteration that converges to the solution for the maxentropic vector for any prescribed stationary distribution.

In incomplete work on (T5), we rigorously analyze the *meeting time* between pursuers and evaders performing random walks on digraphs. There exists several bounds on the expected meeting time between random walkers on graphs in the literature, however, closed-form expressions are limited in scope. By utilizing the notion that multiple random walks on a common graph can be understood as a single random walk on the Kronecker product graph, we are able to provide the first analytic expression for the meeting time in terms of the transition matrices of the random walkers when modeled by either discrete-time Markov chains or continuous-time Markov chains. We further extend the results to the case of multiple pursuers and multiple evaders performing independent random walks. We present various sufficient conditions for pairs (or tuples) of transition matrices that satisfy certain conditions on the absorbing classes for which finite meeting times are guaranteed to exist.

4.3 Work completed during Year 3 (1jun17-31may18)

In reference [6] on (T4), we study the novel problem of maximizing the return time entropy of a Markov chain, subject to a graph topology with travel times and stationary distribution. The return time entropy is the weighted average, over all graph nodes, of the entropy of the first return times of the Markov chain; this objective function is a function series that does not admit in general a closed form.

The paper [8] (and the earlier conference version [7]) features theoretical and computational contributions. First, we obtain a discrete-time delayed linear system for the return time probability distribution and establish its convergence properties. We show that the objective function is continuous over a compact set and therefore admits a global maximum; a unique globally-optimal solution is known only for complete graphs with unitary travel times. We then establish upper and lower bounds between the return time entropy and the well-known entropy rate of the Markov chain. To compute the optimal Markov chain numerically, we establish the asymptotic equality between entropy, conditional entropy and truncated entropy, and propose an iteration to compute the gradient of the truncated entropy. Finally, we apply these results to the robotic surveillance problem. Our numerical results show that, for a model of rational intruder over prototypical graph topologies and test cases, the maximum return time entropy chain performs better than several existing Markov chains, including Kemeny constant minimizing chain from (0) and the entropy rate maximizing chain from (4).

In the Masters Thesis of Mr. Sean Wang, we obtain promising preliminary results on the design of high-entropy motion planning strategies with a prescribed refresh frequency at each environment location. By prescribing a refresh frequency at a location we are requiring the surveillance agent to revisit the location within a given maximum time duration. Mr. Swan Wang obtained his BS in Mechanical Engineering and is now a PhD student at CMU.

4.4 Work completed during Year 4 (1jun18-31may19)

With regards to work on the return time entropy of Markov chains, during this performance year, (1) we presented a conference paper [7] at the IEEE CDC 2018 and (2) we complete the revision of the journal paper [8], which is to appear in the IEEE TAC.

In ongoing work we consider strategic intruders and formulate the robotic surveillance problem as a Stackelberg game, where the surveillance agent knows the best response of the intruder and optimizes her strategy accordingly. Based on different assumptions of the intruder's knowledge about the surveillance strategy and the environment, we propose a catalog of problem formulations for the surveillance agent. One formulation that provides the worst-case performance guarantees is the scenario when the intruder is omniscient, i.e., the intruder knows the exact surveillance strategy and the real time location of the surveillance agent. In this case, we first discover and analyze the dominance in the intruder's strategy and thus reduce the strategy space for the intruder; under the assumption that the travel times are unitary, we develop and partly prove various conjectures about the optimal surveillance strategy over simple graph topologies. Since the objective functions (capture probabilities) of the problems in the catalog all involve the hitting time probabilities (which are polynomial functions of the transition matrix [8]), searching for global optimal solutions for these nonlinear nonconvex optimization problems is generally intractable. For the case of omniscient intruder, we experiment with two numerical solvers, i.e., a solver based on sequentially quadratic programming (SQP) and a solver based on branch and bound algorithms. We perform our numerical experiments over different graph topologies, including line, ring, grid, complete and Erdős-Rényi. We found that the SQP solver solves the problem in a relatively short amount of time with acceptable solution quality, but without performance guarantees. Instead the BnB solver takes extremely long time to solve the problem, but provides rigorous performance guarantees. We are currently working on designing numerical algorithms that solve the problem efficiently (fast) and reliably (with performance guarantees) by exploiting the structure of the objective functions as well as symmetries in the graph topology.

4.5 Work completed during Year 5 (1jun19-31dec19)

We continue our investigation on the Stackelberg game formulation of the robotic surveillance problem where the intruder is assumed to be omniscient. This problem formulation provides worst-case performance guarantees measured by the capture probability. In the case when the travel times are unitary, we look at different prototypical graph topologies such as star graphs and line graphs and develop preliminary results. In particular, we rigorously show the optimal solution in the star graph case and prove a few properties of the optimal solution in the line case.

The paper [9] analyzes the *meeting time* between a pair of pursuer and evader performing random walks on digraphs. The existing bounds on the meeting time usually work only for certain classes of walks and cannot be used to formulate optimization problems and design robotic strategies. First, by analyzing multiple random walks on a common graph as a single random walk on the Kronecker product graph, we provide the first closed-form expression for the expected meeting time in terms of the transition matrices of the moving agents. This novel expression leads to necessary and sufficient conditions for the meeting time to be finite and to insightful graph-theoretic interpretations. Second, based on the closed-form expression, we setup and study the minimization problem for the expected capture time for a pursuer/evader pair. We report theoretical and numerical results on basic case studies to show the effectiveness of the design.

In reference [10], we have developed and published a software package, called *RoboSurv* with both MATLAB and Julia implementations, that feature the computation and optimization of various metrics of Markov chains. We include the most relevant metrics related to robotic surveillance such as the Kemeny constant and the return time entropy, as well as a few generic quantities that are of general interest such as the entropy rate and the mixing time. The packages integrate different computation and optimization functions and they are lightweight, extensible and easy to use. The implementations in MATLAB and Julia also allow us to compare the efficiency of these two languages in the computation and optimization of different objects.

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5 Textbook on “Lectures on Network Systems” [Work completed during Years 1, 2. Official publication and revision 1.1 during Year 3. Revision 1.2 during Year 4. Revision 1.3 during Year 5.]

The textbook [11] is a long-term project by the PI and has been supported only partly on this AFOSR grant. During Year 1, the PI worked on developing these lecture notes and version 0.85 was posted for free download on the PI’s website at the end of May 2016. During Year 2, the book reached version .95. During Year 3, and specifically during the period March-May 2018, the book reached version 1, the book is now widely available online and through a self-publish print-on-demand Amazon CreateSpace for the low price of $24. At version 1, the book is 300 pages, 200 pages in exercises and solutions, 250 illustrations. During Year 4, the PI prepared and published revision 1.2 (corrected several typos and inconsistencies, redrawn a few figures, added a few new exercises). During Year 5, the PI prepared and published revision 1.4 (new chapter on diffusively-coupled linear systems, polished Section 6.4, polished Section 7.1.1, rewritten Section 7.4, rewritten Section 5.1) with 316 pages.

The textbook, related teaching slides, all figures, and other related documentation, are freely available online at

`http://motion.me.ucsb.edu/book-lns`

A solution manual for the 164 questions is available upon request. The PDF of the book (and of its earlier versions) has been download 4700 times since June 1, 2016. The textbook has already been adopted by approximately 35 instructors in 16 countries around the world.

The textbook is intended primarily for graduate students interested in network systems, distributed algorithms, and cooperative control. The objective is to answer basic questions such as: What are fundamental dynamical models of interconnnected systems? What are the essential dynamical properties of these models and how are they related to network properties? What are basic estimation, control, and optimization problems for these dynamical models?

The book is organized in three parts: Linear Systems, Topics in Averaging Systems, and Nonlinear Systems. The Linear Systems part, together with the Topics in Averaging Systems, includes

(i) several key motivating examples systems drawn from social, sensor, and compartmental networks, as well as additional ones from robotics,

(ii) basic concepts and results in matrix and graph theory, with an emphasis on Perron–Frobenius theory, algebraic graph theory and linear dynamical systems,

(iii) averaging systems in discrete and continuous time, described by static, time-varying and random matrices, and

(iv) positive and compartmental systems, described by Metzler matrices, with examples from ecology, epidemiology and chemical kinetics.

The Nonlinear Systems part includes

(v) formation control and coordination problems for relative sensing networks,

(vi) networks of phase oscillator systems with an emphasis on the Kuramoto model and models of power networks, and

(vii) population dynamic models, describing mutualism, competition and cooperation in multi-species systems.

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6 Synchronization in pulse-coupled oscillator networks [Work completed during Year 1]

Due to their rich behaviors, pulse-coupled oscillator (PCO) networks find application in numerous natural and man-made engineering systems. PCO network-based algorithms have been used for clock synchronization for wireless transceivers, in cellular mobile radio, robotics, wireless sensor networks, scheduling and management. Prior research has focused on systems with excitatory, inhibitory, and mixed excitatory/inhibitory coupling, as well as on systems with and without delays in pulse transmission.

This article [12] focuses on PCO networks with delayed excitatory/inhibitory coupling. We consider a simple phase transition rule and show that the resulting PCO network is a linear time-varying control system with the delays as input disturbances. We define the synchronization error as the length of the arc containing the oscillators' phases. We show that the synchronization error converges exponentially fast to a final value proportional to the maximum transmission delay, under the following sufficient conditions: (i) the coupling strength is sufficiently small, (ii) the network has a globally reachable node, and (iii) the delays are sufficiently small. A corollary to this result is that, when all the delays are zero, the network synchronizes exactly and exponentially fast. We also estimate the rate of convergence, final synchronization error, and basin of attraction of the final state, and analyze special cases where synchronization occurs even in the presence of delays. We then extend the analysis to PCO networks with delayed inhibitory coupling, and identify sufficient conditions for synchronization that are less conservative than those in existing literature.

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7 Connecting individual to collective cell migration [Work completed during Year 2]

Collective cell migration plays a pivotal role in the formation of organs, tissue regeneration, wound healing and many disease processes, including cancer. Despite the considerable existing knowledge on the molecular control of cell movements, it is unclear how the different observed modes of collective migration, especially for small groups of cells, emerge from the known behaviors of individual cells. In [13] we derive a physical description of collective cellular movements from first principles, while accounting for known phenomenological cell behaviors, such as contact inhibition of locomotion and force-induced cell repolarization. We show that this theoretical description successfully describes the motion of groups of cells of arbitrary numbers, connecting single cell behaviors and parameters (e.g., adhesion and traction forces) to the collective migration of small groups of cells and the expansion of large cell colonies. Specifically, using a common framework, we explain how cells characterized by contact inhibition of locomotion can display coherent collective behavior when in groups, even in the absence of biochemical signaling. We find an optimal group size leading to maximal group persistence and show that cell proliferation prevents the buildup of intercellular forces within cell colonies, enabling their expansion.

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8 Graph-theoretic small gain theorems for network flow systems [Work completed during Year 4]

Much attention in recent years has been focused on multi-agent systems, but the majority of efforts has been devoted to averaging dynamics and consensus behavior with simple dynamical modoels. Much less attention has been drawn to dynamical flow systems (for commodities such as power, water, gas) as they appear in societal infrastructures. Notable exceptions are a collection of recent papers motivated by applications to traffic and biological systems as well as the long-standing interest in positive systems. Despite these remarkable recent works, many open questions remain.

The paper [14] focuses on a key foundational question for linear monotone systems, i.e., positive systems modeled by Metzler matrices, and on its application to the study of nonlinear monotone systems: what are graph-theoretical conditions for the Hurwitzness of a Metzler matrix? While a graph theoretical treatment is available for a subclass of Meztler matrices known as "compartmental matrices," a general treatment is lacking. This is in stark contrast with the comprehensive understanding of the graph theoretical conditions guaranteeing convergence to consensus for row-stochastic matrices in averaging systems. Graph-theoretic conditions are particularly useful because they allow us to analyze stability based on the structural properties of the interconnection network given the existence of perturbations or uncertainties on the parameters.

In other words, the paper studies the graph-theoretic conditions for stability of positive monotone systems. Using concepts from the input-to-state stability and network small-gain theory, we first establish necessary and sufficient conditions for the stability of linear positive systems described by Metzler matrices. Specifically, we define and compute two forms of input-to-state stability gains for Metzler systems, namely max-interconnection gains and sum-interconnection gains. Then, based on the max-interconnection gains, we show that the cyclic small-gain theorem becomes necessary and sufficient for the stability of Metzler systems; based on the sum-interconnection gains, we obtain novel graph-theoretic conditions for the stability of Metzler systems. All these conditions highlight the role of cycles in the interconnection graph and unveil how the structural properties of the graph affect stability. Finally, we extend our results to the nonlinear monotone system and obtain similar sufficient conditions for global asymptotic stability.

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9 Distributed stochastic subgradient descent for SVMs [Work completed during Year 4]

The Pegasos algorithm is an efficient centralized method to compute Support Vector Machines (SVM). Motivated by practical applications to distributed storage and computation, the paper [15] proposes a distributed stochastic subgradient algorithm to compute SVM. We consider both dynamic network topology with static nodes and networks with dynamic nodes which means the nodes may quit or rejoin the network at some moment. To the best of our knowledge, this paper is the first to show the convergence rate of primal stochastic subgradient descent algorithm for distributed SVM computation with diminishing step sizes over a network with dynamic topology. Compared to traditional methods, our algorithm has advantages in efficiency, fault-tolerance, and security. We provide a rigorous theoretical analysis of our algorithm. First, we establish the convergence rate in expectation for a network with static nodes but dynamic topology. Second, we analyze the feasibility of our algorithm and provide a consensus result for a network with dynamic nodes.

This work was supported only in part by the AFOSR grant.

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10 Capture strategies for 3D reach-avoid games [Work completed during Year 5]

In this work we study a 3D multiplayer reach-avoid game with a goal region and a play region. Multiple pursuers defend the goal region by consecutively capturing multiple evaders in the play region. The players have different moving speeds and the pursuers have different capture radii. First, we introduce an evasion space (ES) method characterized by a potential function to construct a guaranteed pursuer winning strategy. Then, based on this strategy, we develop conditions to determine whether a pursuit team can guard the goal region against one evader. It is shown that in 3D, if a pursuit team is able to defend the goal region against an evader, then at most three pursuers in the team are necessarily needed. We also compute the value function of the Hamilton-Jacobi-Isaacs (HJI) equation via a convex program and obtain optimal strategies for the players. To capture a maximum number of evaders, we formulate a maximum bipartite matching problem with conflict graph (MBMC). We show that the MBMC is NP-hard and design a polynomial-time constant-factor approximation algorithm to solve it. Finally, we propose a receding horizon strategy for the pursuit team where in each horizon an MBMC is solved and the pursuers adopt the optimal pursuit strategy. We also extend our results to the case of a bounded convex play region where the evaders escape through an exit. Two numerical examples are provided to demonstrate the obtained results.

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