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A major focus of the research has been on determining explicit strategies for stochastic control problems, stochastic differential games, and mean field games that included linear exponential quadratic problems, nonlinear stochastic problems, and bilinear stochastic partial differential equations. the noise has included Gauss-Volterra processes and Rosenblatt processes.

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Final Report for AFOSR Grant FA9550-17-1-0073 Studies in Stochastic Control and Differential Games

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Abstract

A major aspect of the research has been on determining explicit strategies for stochastic control problems and stochastic differential games. Generally the optimal strategies are obtained by a direct method which does not require solving Hamilton-Jacobi-Bellman partial differential equations or using a stochastic maximum principle. The problems have included linear exponential quadratic problems for control and games, nonlinear stochastic problems especially evolving in compact Lie groups or symmetric spaces, and some systems evolving in linear or bilinear stochastic partial differential equations. The noise has included Gauss-Volterra processes and Rosenblatt processes. Problems in discrete time and with input constraints have also been considered. Mean field games have been solved in a variety of formulations including jump diffusions, input constraints and model predicative control.

1 Introduction

To have an effective theory of stochastic control and stochastic differential games it is necessary to have a variety of explicitly solvable problems. Historically the control and game problems that were solved were quite restrictive

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because the solutions depended on solving Hamilton-Jacobi-Bellman equations or extending necessary conditions of stochastic maximum principles. Another typical restriction for the models has been that the noise was restricted to Brownian motions. The PIs have developed a direct method for control and game problems that allows to obtain explicit solutions without imposing restrictive conditions that have been required by other methods. Furthermore this direct method allows for noise processes that are not Gaussian or Markovian.

2 Description of the Main Results

A variety of stochastic control and stochastic differential game problems are described that were solved with the support of this AFOSR grant. Linear exponential quadratic differential game problems were solved in a simple direct manner that showed why an additional term arises in the associated Riccati equations [1]. A stochastic volatility model where the volatility was driven by a fractional Brownian motion was explicitly solved [2]. A variety of stochastic differential games were explicitly solved that evolved in some nonlinear spaces including a family of rank one compact symmetric spaces and some other spaces [3, 4, 6, 7]. A direct approach for the solution of the linear-quadratic control problem that shows why a Riccati equation appears naturally is given in [5]. A well known product-sum formula for theta functions for an arbitrary compact simple Lie group is obtained using a simple change of variables in Wiener space [8]. A partially observed multi-agent stochastic differential games with exponential quadratic payoffs is explicitly solved [9]. General Gauss-Volterra processes are used to drive linear stochastic partial differential equations to model linear-quadratic control problems that are explicitly solved in [10]. Risk-sensitive zero-sum stochastic differential games are solved in [11]. Linear-quadratic mean field games are explicitly solved in [12]. A solution of linear-quadratic mean field games with jump diffusions is solved in [13]. Some cooperative linear mean field games are solved in [14] because often cooperation occurs in games. Some linear-quadratic mean field games with multiple input constraints are solved in [15]. A description of some adaptive control methods is given in [16]. Gauss-Volterra processes are a family of Gaussian processes that includes various long range dependent Gaussian processes such as fractional Brownian motions. Some linear-quadratic stochastic differential games with Gauss-Volterra processes are solved explicitly in [17]. Some discrete time stochastic differential games are solved in [18]. A stochastic calculus for Rosenblatt processes is developed in [19]. Rosenblatt processes are a family of continuous non-Gaussian processes which have important properties for modeling physical phenomena. Some linear stochastic differential games with state dependent noise are explicitly solved in [22]. Some stochastic control problems with convexity assumptions on the state drift terms are solved directly [20]. A simple solution for indefinite linear quadratic stochastic control with random coefficients is given in [21]. Linear stochastic differential games with state dependent fractional Gaussian noise are solved in [26]. An ergodic linear quadratic control problem with Rosenblatt noise is explicitly solved in [23]. A micro-grid energy storage problem described by a mean field game is solved in [24]. A direct method is used to solve a fractional mean field game with non-quadratic costs in [25]. Linear-quadratic stochastic control with random coefficients and driven by fractional Brownian motions is solved in [26].

References

[1] T. E. Duncan, Linear exponential quadratic stochastic differential games, *IEEE Trans. Autom. Control*, 61 (9) (2016) 2550-2553.

[2] T. E. Duncan, J. Jakubowski and B. Pasik-Duncan, Stochastic volatility models with volatility driven by a fractional Brownian motion, *Commun. Info. Systems*, 15 (2015), 47-55.

[3] T. E. Duncan and B. Pasik-Duncan, Explicit strategies for some linear and nonlinear stochastic differential games, *J. Math. Engrg. Sci. Aerospace*, 7 (2016), 83-92.

[4] T. E. Duncan and B. Pasik-Duncan, Some solvable stochastic differential games in SU(3), *Commun. Info. Systems* 15 (no.4) 477-487, 2016.

[5] T. E. Duncan and B. Pasik-Duncan, A direct approach to linear-quadratic stochastic control, *Opuscula Math.* 37 (n.6), 821-827 2017.

[6] T. E. Duncan and B. Pasik-Duncan, Solvable stochastic differential games in rank one compact symmetric spaces *Intern. J. Control* (20 Jan. 2017 online) 91 (11) 2445-2450, 2018.

[7] T. E. Duncan and B. Pasik-Duncan, Some solvable stochastic differential games in SU(3) *Commun. Info. Systems* 15 (no.4) 477-487, 2016.

[8] T. E. Duncan, Theta functions and Brownian motion, *J. Theoret. Probab.* online DOI 10.1007/s10959-019-00977-y, Springer Jan. 2020.

[9] T. E. Duncan, Some partially observed multi-agent linear exponential quadratic stochastic differential games, *Evol. Eq. and Control Theory* 7 (n 6), 587-597 (2018).

[10] T. E. Duncan, B. Maslowski and B. Pasik-Duncan, Linear stochastic differential equations driven by Gauss-Volterra processes and related linear-quadratic control problems, *Appl. Math. Optim.*, 80 (2019) 369-389.

[11] J. Moon. T. E. Duncan and T. Basar, Risk-sensitive zero-sum differential games *IEEE Trans. Autom. Control* 64 (4), 1503-1518, 2019.

[12] T. E. Duncan and H. Tembine, Linear-quadratic mean-field-type games, *Games* 9 (1), Feb. 2018.

[13] J. Barreiro-Gomez, T. E. Duncan and H. Tembine, Linear-Quadratic Mean-Field-Type Games: Jump-Diffusion Process with Regime Switching, *IEEE Trans. Autom. Control* 64 (10), Oct. 2019.

[14] J. Barreiro-Gomez, T. E. Duncan and H. Tembine, Co-operative linear-quadratic mean-field-type games, *IEEE Trans. Cybernetics*, to appear.

[15] J. Barreiro-Gomez, T. E. Duncan and H. Tembine, Linear-quadratic mean-field-type games with multiple input constraints, *IEEE Control Systems Lett.*, to appear.

[16] T. E. Duncan and B. Pasik-Duncan, Stochastic adaptive control, *Encyclopedia of Systems and Control*, Springer, to appear.

[17] T. E. Duncan and B. Pasik-Duncan, Some linear-quadratic stochastic differential games driven by state dependent Gauss-Volterra processes, *IMA Stochastic Control, Computational Methods and Applications*, (eds. G. Yin and Q. Zhang) 173-180. Springer 2019,

[18] J. Barreiro-Gomez, T. E. Duncan and H. Tembine, Discrete linear-quadratic mean-field-type repeated games: Perfect, incomplete, and imperfect information, *Automatica*, to appear.

[19] P. Coupek, T. E. Duncan and B. Pasik-Duncan, A stochastic calculus for Rosenblatt processes, *Stoch. Proc. Appl.,* to appear.

[20] T. E. Duncan, B. Pasik-Duncan and L. Stettner, Bellman equations for scalar linear convex stochastic control problems, Banach Center Publications, Inst. of Math. Polish Acad. Sci., Warsaw, to appear.

[21] J. Moon and T. E. Duncan, A simple proof of indefinite linear-quadratic stochastic optimal control with random coefficients, *IEEE Trans. Autom. Control* to appear DOI 10.1109/TAC.2020.2970982.

[22] T. E. Duncan and B. Pasik-Duncan, Linear stochastic differential games with state dependent noise, *Proc. 2017 IFAC World Congress*, July 10-14, 2017, Toulouse.

[23] P. Coupek, T. E. Duncan, B. Maslowski, and B. Pasik-Duncan, An infinite time horizon linear-quadratic control problem with a Rosenblatt process, *Proc. IEEE Conf. Decision and Control*, 4973-4977, Dec. 2018, Miami.

[24] J. Barreiro-Gomez, T. E. Duncan, and H. Tembine, Linear-quadratic mean-field-type games-based stochastic model predictive control: A microgrid energy storage application, *Proc. Amer. Control Conf.*, 3234-3229, July 2019, Philadelphia.

[25] J. Barreiro-Gomez, B. Djehiche, T. E. Duncan, B. Pasik-Duncan, and H. Tembine, Fractional mean-field-type games under non-quadratic costs: A direct method, . *Proc. IEEE Conf. Decision and Control*, Dec. 2019, Nice, France.

[26] T. E. Duncan and B. Pasik-Duncan, Stochastic linear-quadratic control with stochastic coefficients and fractional Brownian motions, *Proc. IFAC Control Conference Africa*, Johannisberg, Dec. 2017