Developing a Hybrid Stochastic and Deterministic Alpha, Beta, Gamma Filter with SNR for Sensorless Control Using Propagation of Uncertainties with a Two Phase Stepper Motor as an Example

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***Abstract*— Sensorless control of systems requires precise measurements for feedback in recursive algorithms however measurement noise and process noise injects error into the signal where the error can cascade and cause the system to become uncontrollable. Sensorless control using the Kalman filter requires significant computation time that can degrade the processor performance and instigate timing issues with the processor while the alpha, beta, gamma filter requires less computational resources; requires tuning; and less accurate. In this paper, one reformulates the alpha, beta, and gamma gains to develop an alpha, beta and gamma filter with signal to noise ratio using propagation of uncertainties in order to conserve computation time by tuning the gain and increase the performance and accuracy of the alpha, beta, gamma filter through the stochastic process of the Kalman filter. In addition, the system is modeled with two plants for the purposes of tuning the filter to compensate for the environmental effects, noise, other perturbations such as the squared errors and others.**

Keywords—Alpha, Beta, Gamma Filter, stepper motor.

I. Introduction

Sensorless control in an application requires controlling a system without any direct measurements of the system being controlled. The issues involved with sensorless control is the imperfect measurement process and noise that introduces error between the target set point and the actual set point in controlling systems. Sensorless control of open loop systems involves taking measurements such as in the case for a stepper motor, voltage and current measurements to indirectly close the loop between the target set point and actual set point for improved tracking and control of the system being observed. One example of sensorless control involves the measurement of the stepper motor position in order to track phase, angular velocity and angular acceleration of the stepper motor using a 3-state observer from a tracking algorithm such as the alpha, beta, and gamma filter. Additionally, for the example, one improves the rotor environment for position control by filtering the phase currents of a two-phase stepper motor. In this paper, one reformulates the alpha, beta, and gamma gains using SNR ratios in order to take advantage of the deterministic formula of the alpha, beta, and gamma filter and the recursive, stochastic model of the Kalman filter; incorporate propagation of uncertainties; and cascade the alpha, beta, and gamma gains into an additional plant using z-scores to better model the environment and enable a tuning capability. The reformulated alpha, beta, gamma filter, a reduced order observer, is versatile in that the filter can be tuned as required while deterministically calculating the beta, and gamma gains to save the computation overhead of continuously recalculating gains for systems that have performance limitations due to timing constraints and processing constraints. Finally, the alpha, beta, and gamma filter can be utilized as a stochastic filter by taking measurement statistics for all three gains and improving the accuracy by increasing the sample size.

II. The Alpha, Beta and GammaFilter

The alpha, beta, gamma, α, β, γ, tracking filter is a state observer that uses a deterministic equation to track and follow a target using input data from instruments for direct or indirect measurements while a Kalman filter uses a repeated, incremental measurement model referred to as the innovation and a state observer to statistically track and follow a target. The Kalman filter has an computation overhead of an order of magnitude of 1 and approximately 20 percent increase in performance when compared to the alpha, beta filter according to one study. [1] Depending on the design constraints such as computation time and performance, one may designate the use of the alpha, beta, gamma filter for their target tracking algorithm.

Generally, α, β, γ are fixed constants that are heuristically adjusted to estimate the actual measurement and noise in order for a correction to the error introduced in the residual between the measurement and prediction. The measurement residual otherwise known as the estimation error is sent to the microprocessor tracking function and then an alpha, beta, gamma filter utilizes the measurement residual for tracking the target by forecasting the next position of the target relative to the present measurement. The α, β, γ filter utilizes the power of the microprocessor to computationally track a target by discretizing a continuous time signal in finite time constants otherwise known as a filter cycle over the tracking interval using the position, velocity and acceleration filter functions to estimate the state vectors for the current filter cycle by employing alpha gain for position; the beta and gamma gains for velocity and acceleration estimates. The position, velocity and acceleration measurement of the target is updated with position, velocity and acceleration information from the partial observer where the three states are given as follows:

$$\hat{\theta}_k = [\theta_k \, \dot{\theta}_k \ddot{\theta}_k \,]^T$$

The predictor equations are based on the partial physical model of the target that is being tracked using ideal equations with no noise in order to allow the alpha, beta, gamma filter to make corrections to the measurement between the measured and predicted output. The three state prediction equations are given as follows:

$$\hat{\theta}_P = [\theta_P \; \dot{\theta}_P \; \ddot{\theta}_P]^{\mathrm{T}}$$

The predictor equations provides a baseline for comparison for the optimality of the measurement residual when compared to the actual states of sensors where the signal is combined with measurement noise and process noise. The three state equations are tracked each cycle using the fixed gain constants that are experimentally adjusted to remove the noise from the measurement, sensor output.

Static alpha, beta, and gamma gains are beneficial in tracking systems that have limited processor capability or functionally demanding multi-sensor or multiple instrumented environments. In considering the gain values, one must consider the tradeoff between a fast response time and effective noise reduction where large alpha, beta and gamma gains and sizeable time constants improve response time but increases noise while small alpha, beta, and gamma gains and less significant time constants reduces noise but diminishes the response time in tracking the target where response time would be a design constraint in a dynamic system.[1] The alpha, beta, gamma gain constants are given as follows:

$$B_K = [\; \alpha \; \beta \; \gamma \;]$$

The current state of the model given by the time update equations using the gains are as follows:

$$\theta_K = \theta_{K-1} + \dot{\theta}_{k-1} \cdot \mathrm{T} + \ddot{\theta}_{k-1} \cdot \frac{T^2}{2} + \alpha_{\,k-1} \cdot (\, \theta_k - \theta_P \,)$$

$$\dot{\theta}_k = \dot{\theta}_{k-1} \cdot \mathrm{T} + \ddot{\theta}_{k-1} \cdot \frac{T^2}{2} + \beta_{\,k-1} \cdot (\, \dot{\theta}_{k-1} - \dot{\theta}_P \,)$$

$$\ddot{\theta}_k = \ddot{\theta}_{k-1} \cdot \frac{T^2}{2} + \gamma_{\,k-1} \cdot (\ddot{\theta}_{k-1} - \ddot{\theta}_P \,)$$

One can formulate the filter using the Kalman gains for an alpha, beta, gamma filter by utilizing the process variance, σ_w, and the noise measurements, σ_v, to calculate the α, β and γ gains by multiplying by a tracking index, Γ. Using the standard equations and calculating the alpha, α, position gain, directly using the ratio between the process variance and measurement noise, one can solve the remaining simultaneous equations for beta, β, velocity gain, and gamma, γ , acceleration gain as follows[1]:

$$\delta = \frac{\sigma_w}{\sigma_v} \cdot \Gamma^2$$

$$\alpha = -\delta^2 + \frac{\sqrt{\delta^4 + 16 \cdot \delta^2}}{8}$$

$$\beta = 2(2 - \alpha) - 4\sqrt{1 - \alpha}$$

$$\gamma = \frac{\beta^2}{2\alpha}$$

Stochastic alpha, beta and gamma gains are preferable in signal environments that are in a state of constant flux due to measurement noise and outside disturbances that can be smoothed while static alpha, beta, and gamma gains are preferable in signal environments in which the measurement noise, and outside disturbances are constant.

III. Alpha-Beta Filter using Z-Score and Signal to Noise Ratio (SNR)

One can use signal to noise ratio in the alpha, beta and gamma filter to determine the strength of the signal compared to unwanted noise. The target measurement is updated with position, velocity, and acceleration information with an error that is introduced in the position, velocity and acceleration measurement residuals. For each finite time constant, the data set for the discretized continuous time signal can be smoothed through averaging to obtain a single revised measurement for the time update equation in order to develop a stochastic alpha, betta and gamma filter similar to the Kalman filter.

$$\mu_{\theta,k} = \frac{1}{i}\sum_{k=1}^{i}((k-1) \cdot \mu_\theta + \theta_k)$$

$$\sigma_\theta = \sqrt{\frac{1 \; \sum_{k=1}^{i}(\,\theta_k - \mu_\theta\,)^2}{i}}$$

$$\mu_{\dot{\theta}k} = \frac{1}{i}\sum_{k=1}^{i}((k-1) \cdot \mu_{\dot{\theta}} + \dot{\theta}_k)$$

$$\sigma_{\dot{\theta}} = \sqrt{\frac{1 \; \sum_{k=1}^{i}(\,\dot{\theta}_k - \mu_{\dot{\theta}}\,)^2}{i}}$$

$$\mu_{\ddot{\theta}k} = \frac{1}{i}\sum_{k=1}^{i}((k-1) \cdot \mu_{\ddot{\theta}} + \ddot{\theta}_k)$$

$$\sigma_{\ddot{\theta}} = \sqrt{\frac{1 \; \sum_{k=1}^{i}(\,\ddot{\theta}_k - \mu_{\ddot{\theta}}\,)^2}{i}}$$

Since the measurements are combined with noise (i.e., quantization noise, round off errors, etc.) with other interferences, Gaussian white noise or process noise corrupts the measurements, and the latency, between measurement and microprocessor, introduces additional error into the measurements due to timing errors. The latency error and other noise can be removed from the measurement by smoothing the noise data set over a filter cycle and subtracting the smoothed noise measurement from the discretized continuous time signal for each finite time constant updated equation. The noise in the measurement, $\Delta\theta_k$, can be expressed as the difference between the measurement, θ_M, and the predicted value, θ_P. The smooth noise measurements, $\mu_{\theta kn}$, $\mu_{\dot{\theta}n,k}$, $\mu_{\ddot{\theta}n,k}$, and the standard deviation for the noise measurements, $\sigma_{\theta n}$, $\sigma_{\dot{\theta}n}$, and $\sigma_{\ddot{\theta}n}$ are given as follows:

$$\Delta\theta_k = \theta_M - \theta_P$$

$$\mu_{\theta kn} = \frac{1}{i}\sum_{k=1}^{i}((k-1) \cdot \mu_{\theta n} + \Delta\theta_k)$$

$$\sigma_{\theta n} = \sqrt{\frac{1 \sum_{k=1}^{i}(\theta_{kn} - \mu_{\theta})^2}{i}}$$

$$\Delta\dot{\theta}_k = \dot{\theta_M} - \dot{\theta}_p$$

$$\mu_{\dot{\theta}n,k} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\cdot\mu_{\dot{\theta n}} + \Delta\dot{\theta}_k)$$

$$\sigma_{\dot{\theta n}} = \sqrt{\frac{1 \sum_{k=1}^{i}(\dot{\theta}_{kn} - \mu_{\dot{\theta n}})^2}{i}}$$

$$\Delta\ddot{\theta}_k = \ddot{\theta_M} - \ddot{\theta}_p$$

$$\mu_{\ddot{\theta}n,k} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\cdot\mu_{\ddot{\theta n}} + \Delta\ddot{\theta}_k)$$

$$\sigma_{\ddot{\theta n}} = \sqrt{\frac{1 \sum_{k=1}^{i}(\ddot{\theta}_{kn} - \mu_{\ddot{\theta n}})^2}{i}}$$

In order to complete the smoothing process for the final signal measurement, one needs to remove the smoothed noise measurement from the smoothed signal measurement which will remove the unwanted disturbances and the latent effects, computation time and other outside perturbations, from the smoothed signal measurement.

$$\mu_{\theta,k} = \mu_{\theta,k} - \mu_{\theta kn}$$

$$\mu_{\dot{\theta}k} = \mu_{\dot{\theta}k} - \mu_{\dot{\theta}n,k}$$

$$\mu_{\ddot{\theta}k} = \mu_{\ddot{\theta}k} - \mu_{\ddot{\theta}n,k}$$

In electronics, signal to noise ratio, SNR, is represented as a ratio of signal power to noise power where the noise power consists of johnson noise, shot noise, thermal noise, and disturbances from interference effects.

$$\text{SNR} = \frac{S}{N} = \frac{P_{signal}}{P_{noise}}$$

Noise power reduces the effective signal power and disturbs proper electronic functionality and can create propogation errors or cascade errors in instruments and sensor measurements which degrades the tracking function over time when the magnitude of the error gradually increases to the point of losing the target. Ideally, one would like a high signal to noise ratio for optimal conditions for the tracking function, however, that is not always possible as previously discussed in contrast to a low signal to noise ratio which complicates tracking a target since the noise perturbs the signal with error. Thus, there are many different sources for uncertainties introduced into the residuals from the instruments, therefore, the different process and measurement uncertanties can be combined into the signal to noise ratio using the propagation of uncertainties provided the variables are linear and independent. Obviously, one can linearize uncertainties that are nonlinear and include in the signal to noise ratio equation for combination of uncertainties. For each filter cycle, the signal to noise ratio is expressed as signal over noise using the principals of propagation of uncertainties for fractional errors (i.e., inverse), therefore, SNR_k, is given as follows:

$$SNR_k = \left(\frac{\mu_\theta}{\sigma_\theta}\right) + \left(\frac{\mu_{\dot{\theta}}}{\sigma_{\dot{\theta}}}\right) + \left(\frac{\mu_{\ddot{\theta}}}{\sigma_{\ddot{\theta}}}\right) + \cdot + \cdot$$

Using the discretized finite measurements for a fixed time interval, τ, one can smooth the signal measurement for a data set, μ, where μ represents the smoothed signal and $\delta\theta$ represents the spread of the measurement error and the deviation from the mean for position, $\delta\theta$, velocity, $\delta\dot{\theta}$, and acceleration, $\delta\ddot{\theta}$, and the total error, $\delta\theta_{Total}$ in the alpha, beta, gamma filter. Generally, signals with smaller range of variances are more stable than signals with large variances and are also, more accurate. The position error, average angular velocity error and average angular acceleration error, and total error is given as follows:

$$\delta\theta = (\theta_k - \mu_\theta)$$

$$\delta\dot{\theta} = \frac{(\theta_k - \mu_\theta)}{\tau} = (\dot{\theta}_k - \mu_{\dot{\theta}})$$

$$\delta\ddot{\theta} = \frac{(\dot{\theta}_k - \mu_{\dot{\theta}})}{\tau} = (\ddot{\theta}_k - \mu_{\ddot{\theta}})$$

$$\delta\theta_{Total} = (\theta_k - \mu_\theta) + (\dot{\theta}_k - \mu_{\dot{\theta}}) + (\ddot{\theta}_k - \mu_{\ddot{\theta}})$$

One can make comparisons between different measurement residuals using the z-score, $\theta_{Z-SCORE}$, by taking the ratio between the spread, $\delta\theta_k$, which is the difference between the actual instrument measurement output and the smoothed instrument measurement output, to the standard deviation for the data set of the instrument measurement of a filter cycle. For instrument sensor errors, the z-score is designated as the distance of the measurement from the mean in a Gaussian distribution, thus, providing a reference for the instrumentation measurement error due to disturbances from the signal which is given as follows:

$$\hat{\theta}_{Z-SCORE} = \frac{\delta\theta_k}{\sigma_\theta}$$

$$\hat{\dot{\theta}}_{Z-SCORE} = \frac{\delta\dot{\theta}_k}{\sigma_{\dot{\theta}}}$$

$$\hat{\ddot{\theta}}_{Z-SCORE} = \frac{\delta\ddot{\theta}_k}{\sigma_{\ddot{\theta}}}$$

The alpha, beta, gamma gains are composed of signal and error, therefore, one can express the gains as the ratio between the smooth signal, μ_θ, and the sum of the smoothed signal and error, the spread between the data point and mean, outputs from the sensors, as follows :

$$\alpha = \frac{\mu_\theta}{\mu_\theta + \sigma_\theta}$$

$$\beta = \frac{\mu_{\dot{\theta}}}{\mu_{\dot{\theta}} + \sigma_{\dot{\theta}}}$$

$$\gamma = \frac{\mu_{\ddot{\theta}}}{\mu_{\ddot{\theta}} + \sigma_{\ddot{\theta}}}$$

Without full knowledge of the system model and a detailed noise model, using the process of combined uncertainties, one can calibrate the mean and equivalently incorporate vectors into the combined uncertainty to isolate the vector component responsible for the error for the specified gain in terms of signal to noise ratio provided that the signal to noise ratio can be measured. Otherwise a close approximation can be made for the vector component using the signal to noise ratio equation since the principal components are represented in the signal to noise ratio equations which would still allow a suitable gain calculation for the suboptimal model. In order to improve accuracy, one can utilize propagation of uncertainties and isolate the component for the specific gain as follows:

$$\alpha = \frac{\mu_\theta \cdot \frac{SNR_{Range}{}'}{SNR_k}}{\mu_\theta \cdot \frac{SNR_{Range}{}'}{SNR_k} + \sigma_\theta \cdot \frac{SNR_k}{SNR_{Range}{}'}}$$

$$\beta = \frac{\mu_{\dot\theta} \cdot \frac{SNR_{Velolcity}{}'}{SNR_k}}{\mu_{\dot\theta} \cdot \frac{SNR_{Velolcity}{}'}{SNR_k} + \sigma_{\dot\theta} \cdot \frac{SNR_k}{SNR_{Velolcity}{}'}}$$

$$\gamma = \frac{\mu_{\ddot\theta} \cdot \frac{SNR_{Acceleration}{}'}{SNR}}{\mu_{\ddot\theta} \cdot \frac{SNR_{Acceleration}{}'}{SNR_k} + \sigma_{\ddot\theta} \cdot \frac{SNR_k}{SNR_{Acceleration}{}'}}$$

Where, the instantaneous signal to noise measurement for the specific vector can be found as follows:

$$SNR_{Range}{}' = \left(\frac{\mu_\theta}{\delta\theta}\right)$$

$$SNR_{Velolcity}{}' = \left(\frac{\mu_{\dot\theta}}{\delta\dot\theta}\right)$$

$$SNR_{Acceleration}{}' = \left(\frac{\mu_{\ddot\theta}}{\delta\ddot\theta}\right)$$

$$SNR'_k = \left(\frac{\mu_\theta}{\delta\theta}\right) + \left(\frac{\mu_{\dot\theta}}{\delta\dot\theta}\right) + \left(\frac{\mu_{\ddot\theta}}{\delta\ddot\theta}\right) + \cdot + \cdot$$

And, the smoothed signal to noise ratio measurement for the specific vector can be found as follows:

$$SNR_{Range} = \left(\frac{\mu_\theta}{\sigma_\theta}\right)$$

$$SNR_{Velolcity} = \left(\frac{\mu_{\dot\theta}}{\sigma_{\dot\theta}}\right)$$

$$SNR_{Acceleration} = \left(\frac{\mu_{\ddot\theta}}{\sigma_{\ddot\theta}}\right)$$

$$SNR_k = \left(\frac{\mu_\theta}{\sigma_\theta}\right) + \left(\frac{\mu_{\dot\theta}}{\sigma_{\dot\theta}}\right) + \left(\frac{\mu_{\ddot\theta}}{\sigma_{\ddot\theta}}\right) + \cdot + \cdot$$

The gain shall then be further refined with an additional plant to further account for environmental effects by closing the loop between the signal and the error. One can develop a gain using the z-score since measurement error can be represented as the distance of the measurement from the smoothed signal. A high z-score would indicate a large error in the measurement residual, therefore, would require a large correction while a small z-score would indicate a small error and require a smaller correction which we can incorporate into the gain. Using z-scores, the vector component responsible for the error for the specified gain can be isolated.

$$\alpha' = \frac{\alpha \cdot \hat\theta_{Z-SCORE}{}^{-1}}{\alpha \cdot \hat\theta_{Z-SCORE}{}^{-1} + \hat\theta_{Z-SCORE} \cdot \sigma_\theta}$$

$$\beta' = \frac{\beta \cdot \hat{\dot\theta}_{Z-SCORE}{}^{-1}}{\beta \cdot \hat{\dot\theta}_{Z-SCORE}{}^{-1} + \hat{\dot\theta}_{Z-SCORE} \cdot \sigma_{\dot\theta}}$$

$$\gamma' = \frac{\gamma \cdot \hat{\ddot\theta}_{Z-SCORE}{}^{-1}}{\gamma \cdot \hat{\ddot\theta}_{Z-SCORE}{}^{-1} + \hat{\ddot\theta}_{Z-SCORE} \cdot \sigma_{\ddot\theta}}$$

Again, in order to be consistent, using signal to noise ratio, one can utilize propagation of errors for the process of combined uncertainties and isolate the component for the specific gain with the following equivalent expressions:

$$\alpha' = \frac{\alpha \cdot \frac{SNR_{Range}{}'}{SNR_k}}{\alpha \cdot \frac{SNR_{Range}{}'}{SNR_k} + \frac{SNR_k}{SNR_{Range}{}'} \cdot \sigma_\theta}$$

$$\beta' = \frac{\beta \cdot \frac{SNR_{Velolcity}{}'}{SNR}}{\beta \cdot \frac{SNR_{Velolcity}{}'}{SNR} + \cdot \frac{SNR}{SNR_{Velolcity}{}'} \cdot \sigma_{\dot\theta}}$$

$$\gamma' = \frac{\gamma \cdot \frac{SNR_{Acceleration}{}'}{SNR_k}}{\gamma \cdot \frac{SNR_{Acceleration}{}'}{SNR_k} + \frac{SNR_k}{SNR_{Acceleration}{}'} \cdot \sigma_{\ddot\theta}}$$

Or, when full knowledge of the noise is available and either an accurate system model or a signal to noise ratio can be measured, one can determine the error using vector analysis as follows:

$$\alpha' = \frac{\alpha \cdot \frac{SNR_{Range}}{SNR'_k}}{\alpha \cdot \frac{SNR_{Range}}{SNR'_k} + \frac{SNR'_k}{SNR_{Range}{}'} \cdot \delta\theta_{Total}}$$

$$\beta' = \frac{\beta \cdot \frac{SNR_{Velolcity}}{SNR_k}}{\beta \cdot \frac{SNR_{Velolcity}}{SNR'_k} + \frac{SNR'_k}{SNR_{'Velolcity'}} \cdot \delta\dot\theta_{Total}}$$

$$\gamma' = \frac{\gamma \cdot \frac{SNR_{Acceleration}}{SNR'_k}}{\gamma \cdot \frac{SNR_{Acceleration}}{SNR'_k} + \frac{SNR'_k}{SNR_{Acceleration}{}'} \cdot \delta\ddot\theta_{Total}}$$

Now that the gains for the alpha, beta, gamma filter has been reformulated as gains based on the z-score, one can perform the correction to the state observers succeeding each filter cycle and remove the error from the time update equations provided that the target tracking is linear over time and the tracking model errors, the raw data, are Gaussian. In the process, in order to correct the estimation error, the signal quantity can be retained through the removal of the process variances and noise variances which is the spread between the measurement and predicted output, from the time update equations for the state observers.

$$\theta_k = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \alpha' \cdot (\theta_K - \theta_P)$$

$$\theta_k = \theta_k - \Delta\theta_k$$

$$\dot\theta_k = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \beta' \cdot (\dot\theta_K - \dot\theta_P)$$

$$\dot\theta_k = \dot\theta_k - \Delta\dot\theta_k$$

$$\ddot{\theta}_k = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \gamma' \cdot (\ddot{\theta}_K - \ddot{\theta}_P)$$

$$\ddot{\theta}_k = \ddot{\theta}_k - \Delta\ddot{\theta}_k$$

One can save computational overhead by applying the deterministic gains to the alpha, beta, gamma filter and benefit from the constant tuning of the alpha gain through stochastic measurements and calculating the gains for beta and gamma using the previously discussed deterministic equations. In the process, the computational overhead has been reduced by approximately two-thirds since beta and gamma gains are deterministically determined every filter cycle and alpha is tuned at some fixed time interval according to designated time instances. In other words, one is only required to compute the associated statistical measurements of one variable out of the three variables each filter cycle while the other two variables are deterministically calculated using a single stochastic gain variable. The Kalman filter can use both deterministic and stochastic gains in a similar manner. An example for a hybrid stochastic filter type gain combined with deterministic gains for an alpha, beta, gamma filter follows:

$$\alpha = \frac{\mu_\theta \cdot \frac{SNR_{Range}'}{SNR_k}}{\mu_\theta \cdot \frac{SNR_{Range}'}{SNR_k} + \sigma_\theta \cdot \frac{SNR_k}{SNR_{Range}'}}$$

Or,

$$\alpha' = \frac{\alpha \cdot \hat{\theta}_{Z-SCORE}^{\;-1}}{\alpha \cdot \hat{\theta}_{Z-SCORE}^{\;-1} + \hat{\theta}_{Z-SCORE} \cdot \sigma_\theta}$$

$$\beta = 2(2 - \alpha') - 4\sqrt{1 - \alpha'}$$

$$\gamma = \frac{\beta^2}{2\alpha'}$$

After the gain is smoothed, one can utilize the smoothed gain for tuning and perform additional iterations with the subsequent gain calculations using z-scores for a gain, α', in order to preserve computation time of the gain calculations; therefore, the gains can be tuned according to the environmental effects on the signal to preserve processing power and improve performance. As an example, in an environment with a high signal to noise ratio in which the data from the instrument output is constant, the gains can be tuned one time while in noisy environments, the signal, gains for both plants, can be tuned every filter cycle. The new alpha, beta, and gamma gain calculations are not as computationally intensive as the previous given Kalman gains which utilizes a computation with a power of four in the gain equation while powers greater than two are generally computationally intensive when fractions are involved. Computer calculations of powers of three or greater in a given equation can have a performance impact through stalled executions and increased frequencies of disruptions to processes which can result in a lost track and many other problems for most embedded systems. Finally, cubic roots in alpha, beta, and gamma equations can stall a processor and degrade performance as well.[11]

IV. Presenting the Dynamics of the Physical Model in the Hybrid Alpha-Beta-Gamma Filter with SNR using the Stepper Motor as an Example

The stepper motor operates on the principal of exciting the stator windings, which are wrapped around permanent magnets, causes the rotor, a permanent magnet with opposite poles to the stator, to rotate at precise angles. The stepper motor rotor rotates each time a pulse is applied to an H-Bridge circuit which causes the stator windings to be excited to create an electromagnet with a magnetic north and south pole. Pulses generated for each phase is given as follows [5]:

$$P_j = \sin(\varphi_j \cdot n + \varphi_0)$$

This in turn causes the permanent magnetic motor to rotate by alignining the rotor teeth with the electromagnetic fields. The number of steps per revolution of the rotor is as follows [4]:

$$S = 2 \cdot n \cdot m$$

Where n = number of rotor pairs and m = number of phases
The step angle is given as the following [5]:

$$\theta = \frac{2\pi}{nm}$$

The motor torque is given as the following with a sinusoidal magnetic field contribution from the air gap [5]:

$$\tau_m = k_m I_j(t) \sin(\varphi_j + N\theta(t))$$

Where:
k_m = motor constant
φ_j = location of the coil in the stator
I_j(t) = current in coil as a function of time
θ(t) = actual rotor position

$$V(t) = v_{emf} + RI(t) + L\frac{dI}{dt}$$

The electromagnetic field generated by each phase of the stepper motor coil is as follows [4]:

$$v_{emf} = k_m(t)\left(\sin(\varphi + N\theta(t)\right) \cdot \omega(t)$$

Typical methods to monitor the rotational velocity would be to use the optical encoder for closed loop feedback to obtain the average rotational velocity and average rotational acceleration and then using a filter to improve the estimate. Another technique for indirect measurement of the actual rotational velocity can also be monitored using the phase voltage equations by isolating the rotational velocity from the equations below. The phase voltages, V_a and V_b, are applied to the stator windings in which a current flows that experiences an ohmic loss, $R_a I_a$, and an inductance, L, and back emf, V_b, with a rotor pitch, p. [4]

$$V_a = L\frac{dI}{dt} + R_a I_a + \omega(t) p \psi_m \sin(p\theta)$$

$$V_b = L\frac{dI}{dt} + R_a I_a + \omega(t) p \psi_m \cos(p\theta)$$

One can obtain the rotational acceleration of the stepper motor as follows [8]:

$$\frac{d\omega}{dt} = \frac{-k_m \sin(N\theta) + k_m \cos(N\theta) - B\omega - Tl}{J}$$

The motor's torque is given as the moment from the shaft, J, with a damping coefficient of B.

$$\tau_m = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt}$$

The torque applied to the rotor by the stator magnetic field is as follows [4]:

$$T_a = i_a p \varphi_m \sin(p\theta)$$

$$T_b = i_b p\varphi_m \cos(p\theta)$$
$$T_{total} = T_a - T_b$$

One can integrate the physical model which must be linear into the alpha, beta, gamma filter which in this case would be a stepper motor and model the stepper motor coefficients using discrete time intervals in the A-matrix for each filter cycle with the predictor equations, X_P, and the sensor outputs, X_K.

$$A = \begin{matrix} 1 & dt & dt^2 \\ 0 & dt & dt^2 \\ 0 & 0 & dt^2 \end{matrix}$$
$$X_K = [\ \theta\ \dot{\theta}\ \ddot{\theta}\ ]_K^T$$
$$X_P = [\ \theta\ \dot{\theta}\ \ddot{\theta}\ ]_P^T$$

On a stepper motor, the position, angular velocity, and angular acceleration can be measured by a partial physical model of the stepper motor where the rotor position is non-ideal and varies due to mechanical and the electrical disturbances according to the winding currents and deviates from the actual angular position to give rise to phase angle mismatch error between the actual phase angle and the measured phase angle. While the rotor position can be corrected, for this example, one will only be concerned with the phase currents, particularly, the phase angle mismatch error for the phase currents.

$$\Delta\theta = \theta \mp \Delta\theta$$

In addition to the phase angle mismatch error, the actual amplitude and the measured amplitude of the phase current can be mismatched principally due to electrical disturbances to give rise to amplitude mismatch error which will be required to be corrected. The amplitude mismatch error and phase mismatch error can be attributed to cross modulation, quantization errors, round-off errors, harmonics, etc. For this example, one shall only be concerned with the phase angle mismatch error. The initial state equations for the stepper motor are given as follows:

$$\theta_r = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \alpha \cdot (\theta_K - \theta_P)$$
$$\dot{\theta}_r = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \beta \cdot (\dot{\theta}_K - \dot{\theta}_P)$$
$$\ddot{\theta}_r = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \gamma \cdot (\ddot{\theta}_K - \ddot{\theta}_P)$$

The initial predictor equations for the stepper motor are given as follows:

$$\theta_p = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1)$$
$$\dot{\theta}_p = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1)$$
$$\ddot{\theta}_p = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1)$$

The stepper motor phase A and phase B current are given as follows [6]:

$$\omega_0 = \frac{d\theta}{dt}$$
$$I_A = I_{a0} \cdot \cos(\omega_0 t + \theta)$$
$$I_B = I_{b0} \cdot \cos(\omega_0 t + \theta + \frac{\pi}{2})$$

In order for the partial observer to be linear, the stepper motor phase currents must be linearized [8].

$$\dot{I}_{A,B} = (\frac{V_{A,B}}{R} - I_{A,B} + K_M \cdot \theta_J \sin(\theta_J \cdot k \cdot dt)) \cdot \frac{1}{L}$$
$$I_{A,B} = I_{A,B} + \dot{I}_{A,B}$$

The data for the measurement residuals, position, velocity and acceleration for the time update equations can be indirectly determined from the current measurement in the stepper motor windings while smoothing the data. The phase for stepper motor phase current, $\varphi_{AM,BM}$, is given as follows:

$$\varphi_{AM} = cos^{-1}(\frac{I_A}{I_{a0}})$$
$$\varphi_{BM} = cos^{-1}(\frac{I_B}{I_{b0}})$$

Direct measurements are generally more accurate and produces less error than indirect measurements, however, if direct measurements are not available, one can indirectly calculate the remaining data points. The average rotational velocity and average rotational acceleration for the phase current can be directly calculated as follows:

$$\dot{\varphi}_{s,AM,BM} = \frac{(\varphi_{s,AM,BM} - \varphi_{s-1,AM,BM})}{dt}$$
$$\ddot{\varphi}_{s,AM,BM} = \frac{(\dot{\varphi}_{s,AM,BM} - \dot{\varphi}_{s-1,AM,BM})}{dt}$$

The smoothed measurement for the position, angular velocity and angular acceleration and the smoothed noise measurement for position, angular velocity and angular acceleration can then be calculated.

$$\mu_{\varphi_{A,B}} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\mu_{\varphi_{A,B}} + \varphi_{AM_k,BM_k})$$
$$\mu_{\dot{\varphi}_{A,B}} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\mu_{\dot{\varphi}_{A,B}} + \dot{\varphi}_{AM_k,BM_k})$$
$$\mu_{\ddot{\varphi}_{A,B}} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\mu_{\ddot{\varphi}_{A,B}} + \ddot{\varphi}_{AM_k,BM_k})$$
$$\mu_{\varphi_{An,Bn}} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\mu_{\varphi_{An,Bn}} + \varphi_{An_k,Bn_k})$$
$$\mu_{\dot{\varphi}_{An,Bn}} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\mu_{\dot{\varphi}_{A,B}} + \dot{\varphi}_{An_k,Bn_k})$$
$$\mu_{\ddot{\varphi}_{An,Bn}} = \frac{1}{i}\sum_{k=1}^{i}((k-1)\mu_{\ddot{\varphi}_{An,Bn}} + \ddot{\varphi}_{An_k,Bn_k})$$

The mean noise measurement can then be removed from the mean measurement.

$$\mu_{\varphi_{A,B}} = \mu_{\varphi_{A,B}} - \mu_{\varphi_{An,Bn}}$$
$$\mu_{\dot{\varphi}_{A,B}} = \mu_{\dot{\varphi}_{A,B}} - \mu_{\dot{\varphi}_{An,Bn}}$$
$$\mu_{\ddot{\varphi}_{A,B}} = \mu_{\ddot{\varphi}_{A,B}} - \mu_{\ddot{\varphi}_{An,Bn}}$$

After making the calculations for smoothing the signal measurement and smoothing the data for a measurement output, the calculation for the signal to noise ratio can be determined.

$$SNR_\theta = \frac{\mu_\theta}{\sigma_\theta} + \frac{\mu_{\dot{\theta}}}{\sigma_{\dot{\theta}}} + \frac{\mu_{\ddot{\theta}}}{\sigma_{\ddot{\theta}}} + \frac{\mu_I}{\sigma_I} + \cdot + \cdot$$

Next, for this stepper motor example, one can calculate the z-scores.

$$\theta_{Z-SCORE} = \frac{\delta\theta_k}{\sigma_\theta}$$
$$\dot{\theta}_{Z-SCORE} = \frac{\delta\dot{\theta}_k}{\sigma_{\dot{\theta}}}$$
$$\ddot{\theta}_{Z-SCORE} = \frac{\delta\ddot{\theta}_k}{\sigma_{\ddot{\theta}}}$$
$$I_{a,b\,Z-SCORE} = \frac{\delta I_{a,b,k}}{\sigma_{a,b}}$$

$$\delta\theta_{Total} = (\theta_k - \mu_\theta) + (\dot{\theta}_k - \mu_{\dot{\theta}}) + (\ddot{\theta}_k - \mu_{\ddot{\theta}}) + (I_a - \mu_{I_a}) + (I_b - \mu_{I_b})$$

And then, one can calculate the gains for the stepper motor.

$$\alpha_\theta, \beta_{\dot{\theta}}, \gamma_{\ddot{\theta}}$$

And finally, the smoothed gain can be converted to a gain based on z-scores or signal to noise ratio for the tuned gains.

$$\alpha_{\theta}', \beta_{\dot{\theta}}', \gamma_{\ddot{\theta}}'$$

Finally, the current phase, angular velocity, and angular acceleration equations can be updated with a correction to the residual.

$$\theta_k = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \alpha \cdot (\theta_K - \theta_P)$$

$$\theta_k = \theta_k - \Delta\theta_k$$

$$\dot{\theta}_k = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \beta \cdot (\dot{\theta}_K - \dot{\theta}_P)$$

$$\dot{\theta}_k = \dot{\theta}_k - \Delta\dot{\theta}$$

$$\ddot{\theta}_k = A(1,1) \cdot X(1) + A(1,2) \cdot X(1) + A(1,3) \cdot X(1) + \gamma \cdot (\ddot{\theta}_K - \ddot{\theta}_P)$$

$$\ddot{\theta}_k = \ddot{\theta}_k - \Delta\ddot{\theta}$$

V. Results for Applying the Hybrid AlphA-Beta-Gamma Filter with SNR to the Stepper Motor as an Example

The hybrid extended alpha, beta, gamma filter has previously been discussed and then applied to the stepper motor using a MATLAB simulation. In this example taken from [8], here are the following specifications for the stepper motor (mks): J = $2.00\text{x}10^{-6}\,\frac{m}{kg^3}$, B = $1.1\text{x}10^{-3}$, L = 1H, R = 2.5Ω, N=100 turns, K_m=0.03, $I_{a,init}$=2.5A, $I_{b,init}$=2.5A, V_a = 5.0V, V_b = 5.0V. The step angle of the motor is 1.8° for 200 steps per revolution. The stepper motor is a two-phase stepper motor with a magnetic rotor. Now that the motor specifications have been established, one can discuss the non-idealities of the stepper motor that were assumed.

In simulating the phase current of the stepper motor, one must consider the phase current noise due to the environment, therefore, for this simulation, the noise was assigned a value of 0.1 due to the assumption of the 10% tolerance for the nominal resistance value of the stepper motor phase wire resistance. For velocity, one assume an error proportional to the wire resistance of 0.1 and for acceleration, the assumption was for an error proportional to the wire resistance of 0.005. In electronics, noise can be caused by RFI, EMI, cross talk from neighboring conductors and shot noise due to random fluctuations in electrical current.[6] One can simulate the alpha, beta and gamma gains to a hybrid alpha, beta and gamma gains using statistical measurements. The simulation of the hybrid extended alpha, beta, gamma motor was simulated by smoothing the gain every filter cycle for an interval of 5 discrete time constants of 1.5 us over a time interval of 3ms with a sample size of 5 samples. The phase A, B currents were sinusoidal as expected which can be seen in figure 1.

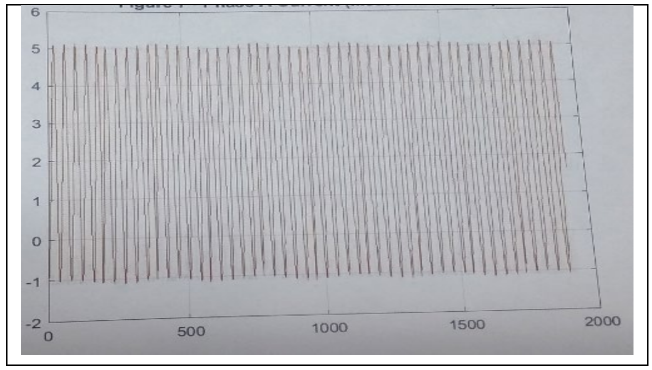

Figure 1: Stepper Motor Phase Current in Matlab Simulation

Now that the specifications of the stepper motor were discussed, one can discuss the results of the Matlab simulation.

One simulated the alpha, beta, and gamma filters using both the stochastic gains and the hybrid, deterministic and stochastic gains in order to make a comparison. Firstly, one can discuss the results for the alpha, beta, and gamma filter using the hybrid gains where statistical measurements were used for the alpha gain while the beta and gamma gains were determined using the deterministic equations. The position (phase) error was a maximum of $2\ x\ 10^{-4}$ degrees which can be seen in figure 2.

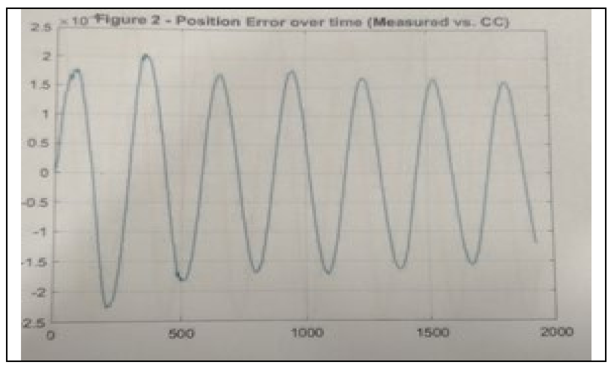

Figure 2: Stepper Motor Position Error for Phase Currents in Matlab Simulation

The angular velocity error was a maximum of $\pm 8\ x\ 10^{-6}$ degrees/second which can be seen in figure 3 and the angular acceleration gain was a maximum of $\pm 4\ x\ 10^{-7}$ degrees/seconds^2 which can be seen in figure 4.

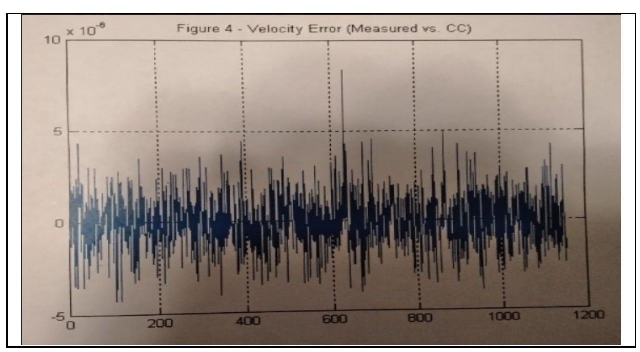

Figure 3: Stepper Motor Velocity Error for Phase Currents in Matlab Simulation

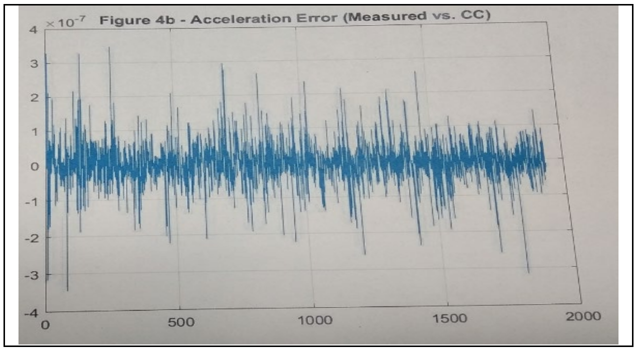

Figure 4: Stepper Motor Acceleration Error for Phase Currents in Matlab Simulation

Stepper motors are known to have high harmonic noise content due to the square wave current driven by pulse-width modulation of an H-bridge circuit and mechanical vibration. The phase A, B current error was approximately ± 3.5 x 10^{-4}A maximum in figure 5. One may observe the error cascading in an interval which can be attributed to the deterministic gain calculation.

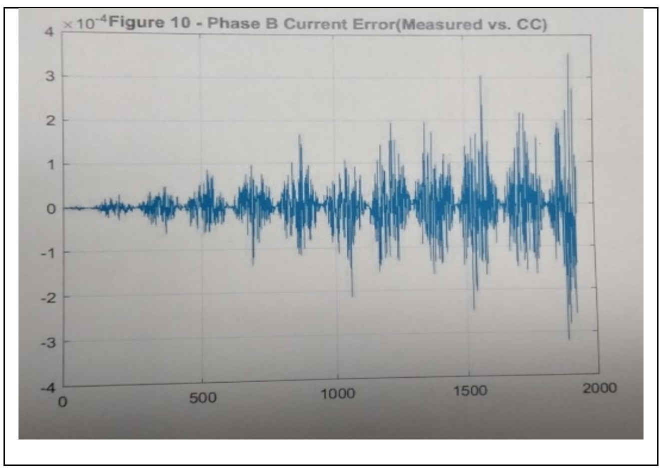

Figure 5: Stepper Motor Phase Current Error in Matlab Simulation

Finally, in figure 6, one can see the position of the stepper motor over a number of filter cycles.

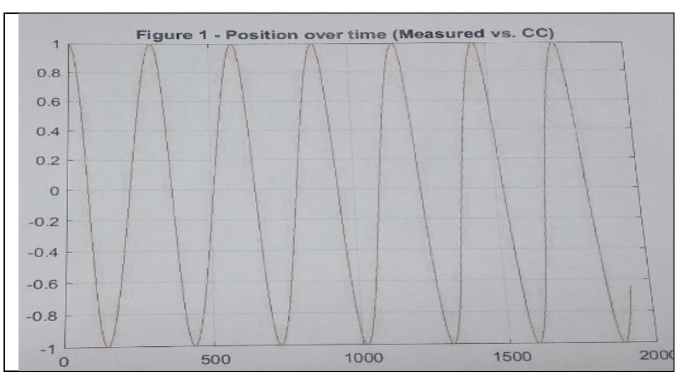

Figure 6: Stepper Motor Phase Current Error in Matlab Simulation

One can further compare the alpha, beta and gamma gains for a hybrid alpha, beta and gamma gains using statistical gains. The position error in figure 7 is $\pm 1.5x10^{-6}$ degrees which is more accurate than the position error of $2\ x\ 10^{-4}$ degrees for the alpha, beta, and gamma gains using hybrid gains.

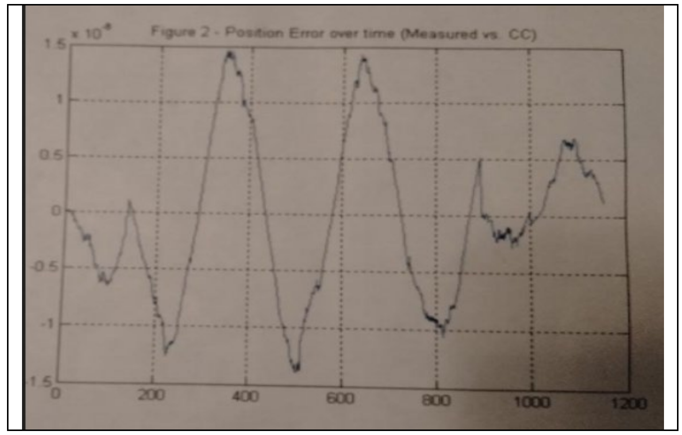

Figure 7: Stepper Motor Position Error in Matlab Simulation

The angular velocity was $\pm 2x10^{-5}$ degrees/second which has a higher accuracy when compared to the hybrid alpha, beta, and gamma gains in figure 3 which can be seen in figure 8.

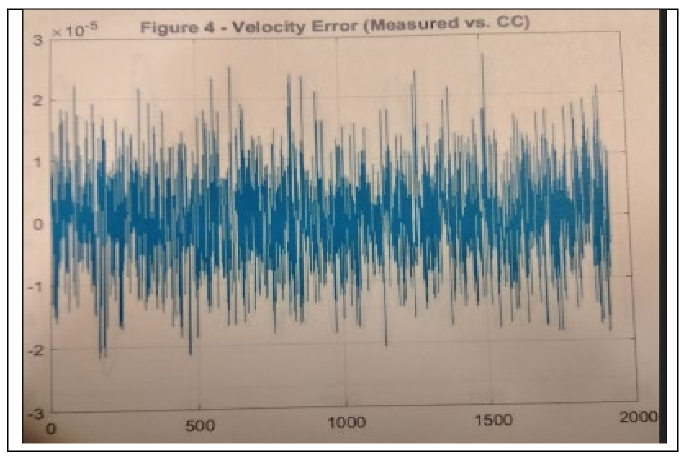

Figure 8: Stepper Motor Phase Current Velocity Error in Matlab Simulation

The angular acceleration was significantly less than $\pm 2x10^{-8}$ degrees/seconds^2 which is more accurate when compared to the angular acceleration of the hybrid gains which can be seen in figure 9.

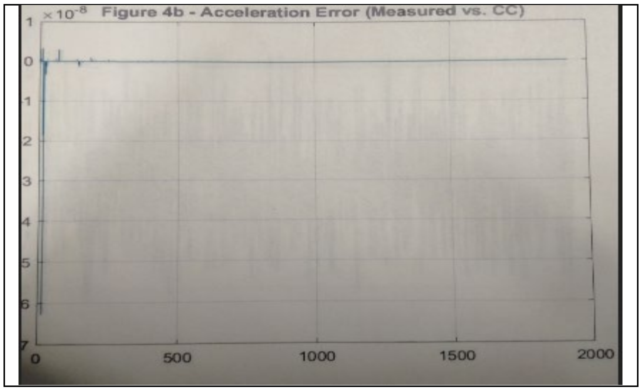

Figure 9: Stepper Motor Phase Current Acceleration Error in Matlab Simulation

The phase current error was $\pm 2x10^{-4}$ A as shown in figure 10 which is comparable to the hybrid alpha, beta, and gamma gains.

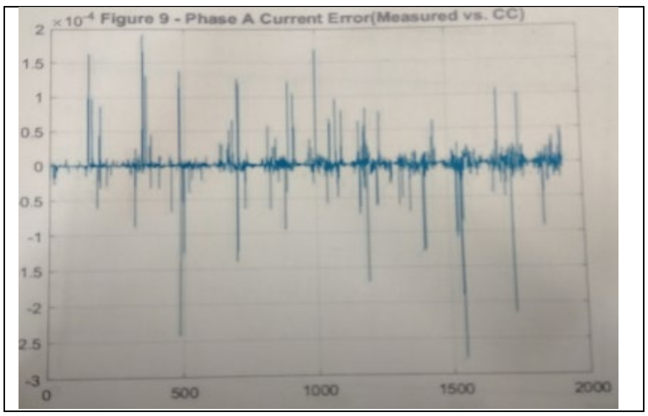

Figure 10: Stepper Motor Phase Current Error in Matlab Simulation

The accuracy between the three stochastic gains of the alpha, beta and gamma filter and the hybrid, stochastic and deterministic, gains of the alpha, beta, and gamma filter were nearly comparable in accuracy, however, the stochastic gains provided a little more accuracy than the hybrid gains. In addition, the error cascades in an interval when using the deterministic equations but still remains controllable. The accuracy can be improved by increasing the number of samples in the measurement statistics.

The tracking of phase current statistics can be useful in improving the phase A and B currents of the stepper motor. For this example, one can reduce the electrical disturbances in the phase current by adjusting the current phase modulation via the H-Bridge circuit, thus, reducing the magnetic disturbances on the rotor for improved positioning since the phase currents gives rise to the electromagnetic torque applied to the rotor of the stepper motor. Electronic suppression techniques of spurious signals in conjunction with standard noise suppression techniques with analog components can be employed in the stepper motor to quiet down the phase currents to reduce position errors. Electronic switching techniques to improve efficiency such as synchronous switching; switches used for zero current switching or zero voltage switching, can also be employed by using the tracking algorithm. Thus, through tracking of the stepper motor phase currents, more accurate phase currents can be provided for more accurate positioning of the rotor. In addition, one can perform filtering on the rotor position by measuring the phase current wave forms and adjusting the rotor velocity and acceleration. Alternatively, one can improve the rotor position by using the same techniques to reduce the error of the rotor position as previously stated by using an optical encoder on the shaft to provide feedback. The simulation has demonstrated the reliability and accuracy of using the reformulated gains on the alpha, beta, and gamma filter on stepper motors using the phase current adjustments as an example.

VI. Conclusion

In this paper one discussed the design of a hybrid stochastic and deterministic alpha, beta, gamma filter. The Kalman filter requires an optimal model of the system being tracked which requires extensive research to account for the system dynamics, perturbations and noises that can affect the system in order to obtain the accurate tracking while the alpha, beta, gamma filter only requires a sub-optimal model of the system being tracked and, therefore, only requires a partial physical model of the system which requires less research. In addition, since the Kalman filter requires a full physical model with the noise carefully placed in the model, the control algorithm requires more processing power to execute the additional lines of code which impacts timing and performance. Additionally, the alpha, beta, gamma filter discussed in this paper can be utilized in an optimal model for the system being tracked when all the observables and noise estimates are available, therefore, the algorithm can be used as a deterministic or stochastic tracking filter.

Both, the Kalman filter and alpha, beta and gamma filter can utilize deterministic and stochastic gain calculations. The Kalman filter requires a formal set up of the equations while the alpha, beta, and gamma filter can be set up and refined as required for the system being modeled. For the alpha, beta, gamma filter, the signal to noise ratio is commonly measured and a readily available measurement, therefore, one can vectorally obtain the desired measurement of the signal being observed without full knowledge of the system dynamics and perturbations involved. The alpha, beta, and gamma filter can be more easily modeled by scaling the mean using signal to noise ratios and using the process of propagation of uncertainties in one line by summing the fractional errors (i.e., inverse) in the system and extracting the desired error. The alpha, beta, gamma filter using propagation of uncertainties was equivalently expressed using both, z-scores and signal to noise ratio which is a readily available measurement. As a final point, with full knowledge of the system dynamics and system noises, one can utilize the alpha, beta and gamma filter as a stochastic filter to improve accuracy by instantaneously quantifying the error and using the smoothed signal measurement similar to the Kalman filter.

Finally, the reformulated alpha, beta, and gamma filter using propagation of uncertainties for fractional errors has a second stage to the gain allowing the gains to be tunable to environmental effects. On the other hand, the Kalman filter is not tunable to different environmental effects which makes the filter less adaptive to noisy environments which can degrade the performance of the Kalman filter in noisy environments due to the duration of time to perform the necessary statistical calculations. The second stage to the alpha, beta, and gamma filter in this paper allows the signal to noise ratio to be compensated on the fly which makes the filter more adaptive to noisy environments. Or, if more accuracy is desired, one can recalculate both gain stages with more iterations according to environmental checks.

The versatility of the reformulated hybrid alpha, beta, and gamma filter allows one to use the hybrid alpha, beta, gamma as a tuning filter with deterministic gains or as a stochastic filter similar to the Kalman filter. In addition, the reformulated alpha, beta, gamma filter is more versatile under a variety of design constraints such as computation time, accuracy, noise, etc. Also, the reformulated alpha, beta, gamma filter has been successfully demonstrated using MATLAB on a two phase stepper motor. The hybrid gains performed optimally in terms of execution speed and accuracy when compared to the filter with stochastic gains. However, the accuracy of the stochastic filter improved when the sample size was sufficiently increased. Also, the optimal model for the alpha, beta, and gamma filter had better accuracy compared to the suboptimal models. By reformulating the stochastic Kalman filter with the alpha, beta and gamma filter, the filter provides another versatile and optimal solution for tracking filter designs.

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