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A multi-physics approach to validation of failure models in extreme thermoacoustic environments

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Summary / Abstract

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Chapter 1 Introduction

1.1 Programme Overview

This is a final report on a 46-month programme conducted at the University of Liverpool (UoL) in collaboration with Professor John Lambros at the University of Illinois at Urbana-Champaign (UIUC) and supported by the US Air Force. The long-term goal of this collaborative work is the understanding and modelling of the coupled thermoacoustic fatigue failure of aircraft structures and developing a validation framework for the complex high-end numerical models, which would lead to the eventual success of a “digital twin” concept.

Currently there is only limited structural level validation quality data available for cases of extreme thermoacoustic loading. Validation quality data has different, and often more stringent, requirements than experimental data associated with the investigation of physical phenomena on a laboratory scale. For example, as the number of actual experiments may be limited, there is a need to capture as much information as possible from a single experimental configuration, implying that many different experimental techniques could be needed simultaneously. There is also a need for three-dimensional (3D) information of stress and strain over large areas of the structure, and data acquisition over many length and time scales may be required. The AFRL/RQ has several unique facilities that could be used for the validation of simulations being developed by the collaborative efforts of the AFRL in-house Structural Sciences Center (SSC) and the present research team. These facilities, the Combined Environment Facility (CEAC), and its smaller counterpart, the Sub-Element facility (SEF), are capable of producing sustained acoustic loading on a structural panel while maintaining variable thermal loading of up to 1,650°C (3,000°F) by means of quartz lamp heating. These are ideal devices for the performance of such thermoacoustic experiments. However, at the start of this programme there was nothing formulated, either in terms of techniques or methodology or even understanding, that can meet the stringent experimental validation requirements of SSC. The reason was the existence of multiple knowledge gaps in all three areas: experimental techniques for thermoacoustic fatigue assessment, in-depth understanding for appropriate thermoacoustic failure model development, and finally methodologies for multi-scale multi physics validation. The work carried at UoL out as part of this 46-month project has addressed key aspects of all of the three above-mentioned areas.

1.2 Report structure

The core of this report (Chapters 2-8) is the PhD thesis submitted by Ana Catarina dos Santos Silva in September 2019 and successfully defended in November 2019. Her work was focused on investigating the effects of coupled thermal and acoustic loading on aerospace grade metallic panels. Acquired

experiment data was used in the development and validation of computational models which aimed to predict the structural response of the panels when subjected to a range of temperature distributions. A self-contained Chapter 9 describes the work carried out by Elias Lopez Alba when he visited UoL from University of Jaén, Spain in summer 2017. He performed experiments to investigate the phenomenon of mode shifting and jumping that occurred in rectangular plate subject to asymmetrical heating beyond the point at which thermal buckling appears. The last two self-contained chapters of this report are contributed by Khurram Amjad who worked on this project as a post-doctoral researcher between October 2017- December 2017 and December 2018 - September 2019. Chapter 9 describes the investigation on the use of thermoelastic stress analysis technique for simultaneously measuring full-field stresses and determining mode shapes in conditions investigated by Silva and Alba. A novel approach for quantitative validation of volumetric datasets is described in Chapter 10. This validation approach, which was partly developed in support of the work on a separate AFOSR grant (FA9550-17-1-0272), is applicable to volumetric data sets with any combination of spatial and temporal variation along three orthogonal dimensions of the volume. A list of publications originating from the work performed under this grant is provided in Appendix G provided at the end of this report. The remainder of this chapter contains the sections from the introduction chapter of Silva's PhD thesis.

1.3 Motivation

When the speed of a travelling aircraft exceeds the speed of sound, environmental conditions lead to extreme loading scenarios that significantly shorten the lifecycle of materials and structures. In supersonic and hypersonic flight, aircraft structures reach high temperatures due to aerodynamic heating¹, which are coupled with a broad spectrum of acoustic and vibratory loads generated by fluctuating air pressures and high velocity gradients that characterise the aircraft's boundary layer (illustrated in Figure 1.1). The aerodynamic heating is not uniform across the surfaces of the aircraft, being particularly significant at its leading edges and creating severe temperature gradients across the structure. Thermal and acoustic loads from the engine add to the complexity of this highly transient environment, making it often difficult to identify which load or combination of loads is responsible for component failure². Thin-gauge components, such as the aircraft skin, are especially affected by high thermal stresses which may cause a reduction or loss of structural stability as a result of the development of in-plane compressive stresses³. Therefore, despite not being well understood, the effect of high temperatures and vibro-acoustic loads heavily influence both the analysis and design of supersonic and hypersonic aircrafts⁴.

The difficulty in obtaining measured data relevant to the study of aerospace structures under hypersonic conditions makes it crucial to implement ways to predict material and structural responses throughout a component's lifecycle. This can be achieved using simulation methods that are continuously being revised and improved. Ideally, future developments will lead to predictive response models capable of

combining measured structural information with mission history information, making it possible to reliably predict future structural performance, given the known starting state of the structure⁵. In order to suitably validate these models, experimental data needs to be accrued in situ, from flight tests, and in controlled environments. Past work on the subject was largely limited to point-wise acquisition and analysis of data but more recent developments have used optical techniques to acquire full-field displacement measurements of plates under thermal and thermo-vibratory loads. These investigations aimed to develop the methodologies for the acquisition of high-quality experimental data and validation of predictive models but provided little information on the response of structures under various thermal and thermo-mechanical loading scenarios, which the present work tries to address.

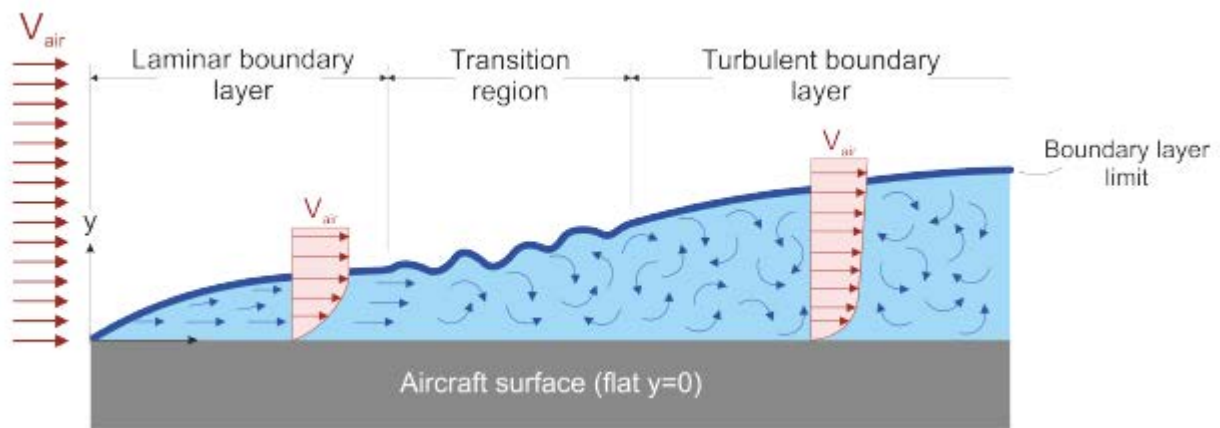


Figure 1.1 Development of a boundary layer over a flat plate. Adapted from ⁶.

1.4 Background

The quest for hypersonic flight has led researchers in the study of structures subjected to thermal and thermo-vibratory loading. The acquisition of experimental data has been historically difficult due to the complexity and cost associated with the process. This leads to a limited availability of quality data to be used in the design and development of structures, which often limits the optimisation of components, both aerodynamically and cost-effectively. This creates a clear need for laboratory-acquired data that represents realistic loading conditions and can be used in the validation of predictive models. In the past, the majority of studies in this area were limited to point-wise measurements due to the use of strain gauges and thermocouples in the acquisition of data, which narrowed the volume of measurements used in the validation of predictions. Whilst this is acceptable in the monitoring of simple loading conditions, these methods are insufficient in the characterisation of complex loading cases, such as ones representative of hypersonic flight, that have highly non-linear structural responses. The advent of recent optical techniques have made it possible to acquire full-field displacement and temperature data that thoroughly describe the conditions of the loading and the behaviour of the structures. Recent work by Beberniss et al.⁷ used digital image correlation (DIC) in the measurement of the vibratory response of a steel panel subjected to shock impingement using a progressive wave tube (PWT). Jin et

al.⁸ have also used DIC in the monitoring of deformation of a thermally-stressed composite panel. Abotula et al.⁹, Berke et al.¹⁰ and Sebastian¹¹ used different heating methods to impart high temperatures to small Hastelloy X plates which were also subjected to either shock-wave loading or vibro-acoustic excitation. The work cited above has developed a means to gather experimental data for the validation of predictive models of aerospace components; it yielded significant findings on the dynamic behaviour of thermally stressed plates, prompting further developments on the topic. As large strains associated with resonant modes and large displacements contribute to the fatigue of components and can severely shorten their lifecycle, it is critical for the robust design and maintenance of an aircraft to develop computational models which are capable of providing reliable predictions of structural response. Therefore, the present project builds upon the work previously undertaken by focusing on the characterisation of structural responses associated with thermal and thermo-vibratory loading and the development of finite element (FE) models that can adequately describe them.

1.5 Aims and Objectives

The aim of this PhD research project was to investigate the effects of thermal and thermo-vibratory loading on aerospace-grade metallic panels in laboratory conditions. Data acquired from experiments was used in the development and validation of computational models which aimed to predict the structural response of the panels when subjected to a range of temperature distributions. As the driving force behind this work is the future merger of mission history data with structural information acquired in-situ, all investigations were kept at a component level (macroscale). To achieve the aim of this project, the following objectives have been set:

1. To experimentally study the effect of temperature on the dynamic response of aerospace panels, particularly the influence of non-uniform temperature distributions.
2. To develop and validate a computational solid mechanics model with temperature-dependent material properties, capable of predicting the behaviour of components when subjected to and combined thermo-vibratory loads;
3. To perform experimental thermal and thermo-vibratory loading of aerospace-grade metal panels in order to acquire full-field, high-quality displacement and temperature data to be used in the development and validation of predictive models;

Chapter 2 Literature Review

The structural behaviour of thin-walled structures has been a problem tackled by engineers since the early days of aviation and the wood-steel-fabric biplane era¹². The vast majority of this early work focused on the effect of mechanical loads on plates and shells used in the construction of aircraft. After World War II, however, the pursuit of supersonic flight brought to light the effect of elevated temperatures induced by aerodynamic heating on the material selection and structural design practices¹². At transonic speeds, air compression and increased friction between the aircraft and the air flow were identified to be the main mechanisms responsible for this local increase in temperatures¹. In 1992, Thornton¹³ published a journal paper surveying the advancements in the analysis of thermal structures from the first considerations for supersonic flight to the developments of hypersonic aircraft. Thornton listed the ways in which elevated temperatures are detrimental to structural behaviour: 1) the most evident effect is the decrease in Young's modulus with temperature, which decreases the ability of the structure to withstand loads and reduces allowable stresses; 2) time-dependent material behaviour such as creep can become a determining factor in material and structural design; 3) thermal stresses are introduced due to existing structural constraints and non-uniform material expansion/contraction. These stresses can deformation, alter structural dynamic behaviour, affect component fatigue life and uniquely affect the stability of components.

Several research programs on the effect of combined thermo-mechanical loading in aerospace structures have been sponsored by the United States (U.S.) government. The importance of research on, what came to be known as the "thermal barrier", was first recognised in 1944 with the development of the transonic aircraft, the Bell X-1, which reached Mach 1.94 in a 1957 research mission. Whilst data acquired during such missions showed aircraft skin temperatures to be below 100 °C, the rapid increase in speed above the sonic threshold demonstrated the need to consider aerodynamic heating. The first program aimed at the development of a hypersonic aircraft ran from 1954 to 1968 and resulted in the design and manufacture of three rocket-powered aircraft - the X-15. These were capable of flying at altitudes of 10,000 feet (approximately 3,000 m) or higher and achieved speeds of Mach 5. Under these conditions, data gathered on a 1965 mission showed the temperatures at the leading edges of the wings to reach over 700 °C (1325 °F)¹⁴. The development of the successor of the X-15 started in 1982, after a long period dedicated to more fundamental research. The U.S. government terminated the funding for this project in 1994 and concept models of the X-30 National Aerospace Plane (NASP) were never developed into a full-scale aircraft, which was intended to fly at speeds of Mach 25¹⁵. The program yielded significant findings on high-temperature fatigue of materials and structures as the 1\3 concept demonstrator was studied using a high-temperature wind tunnel. Results suggested real in-flight airframe temperatures to exceed 1650 °C¹⁶. The U.S. then faced a lack of a cohesive program of

hypersonic technology development as a series of research efforts followed the cancellation of the X-30 project in an attempt to keep research momentum throughout the 1990s. One such program resulted in the Hyper-X or X-43A aircraft and used NASP technology to expedite its readiness level towards the demonstration of hypersonic air-breathing propulsion in flight¹⁷. Three expendable X-43A vehicles were built and tested in the early to mid-2000s. The first two test flights fell short of the accomplishments of the third, which set the speed record for a jet aircraft at Mach 9.6. In 2006, McClinton gave a lecture on the significance of data gathered during these test flights and showed measured surface temperatures to have reached approximately 1090°C (2000°F)¹⁸. In 2003, the U.S. Air Force Research Laboratory began the design and development of a hypersonic aircraft powered by a jet engine: the X-51A Waverider. A total of four aircraft were built, none of them being designed to be recovered after the test flight (akin to the X-43A). The X-51A maiden flight took place in 2010 followed by two unsuccessful attempts. In 2013, the final X-51A aircraft travelled more than 230 nautical miles in just over six minutes, reaching a peak speed on Mach 5.1¹⁹. Lane published information on the design criteria for the X-51A²⁰, which included considerations on the temperature range experienced by the vehicle's skin and exhaust nozzle - from approximately 815 °C (1500 °F) to 1920 °C (3500 °F).

Whilst the data gathered in modern hypersonic research flights is not widely available to the public, investigations into the effect of thermal and thermovibratory loads on aerospace structures have continued. The vast majority of this work has been carried out using computational mechanics simulations or analytical models due to the cost-effective nature of these investigations. Nevertheless, results have provided important insights into the predicted aerothermal and acoustic loads to be encountered by a vehicle travelling on a typical trajectory. One such example is the extensive analytic work by Blevins et al.^{2,21} which predicted aircraft panel temperatures to reach values of 1140-1790 °C, depending on the airflow and ascent trajectory. Aeroacoustic loads due to boundary layer turbulence were found to be between 130 and 145 dB, whilst engine noise was estimated to be as high as 170 dB. A conference paper by Beberniss et al.⁷ noted high-cycle or sonic fatigue to occur in thin, lightly-damped aircraft structures with vibrational modes below 500 Hz. Recent developments in optical techniques have enabled the full-field acquisition of measurements, prompting further advances in the subject, which are addressed later in this review.

As the aim of the work presented in this thesis includes both experimental and computational studies on the effect of thermal loads on the dynamic behaviour of thin-walled panels, a brief review of the current literature on experimental and numerical advancements is presented. The first part discusses the contribution of past analytical and predictive work largely derived from studies on thermal buckling, which is defined as a highly non-linear event in which transverse pressure and, or in-plane compressive loads lead to a loss of structural stability and the change in the stable configuration of the component²². The second part focuses on the findings and experimental advances in the modal analysis of panels under thermo-vibratory loads; including the determination of the resonant frequencies of the structure

and the out-of-plane displacement maps that represent a pattern of vibration at those frequencies (mode shapes). The third part of this review examines the current state-of-the-art in the experimental thermal loading of panels and the effects of temperature on the deformation of these structures. Finally, the fourth part gives an appraisal of the current techniques used in the analysis of full-field data, acquired using optical techniques.

2.1 Analytical and computational studies on thermally and thermo-mechanically loaded plates

Plate theory has been an extensively studied field of engineering, both analytically and computationally. Existing work on analytical plate theory was compiled and collated by Leissa²³ and published in 1969 by NASA. In his monograph, Leissa presented a comprehensive set of results based on linear plate theory for the resonant frequencies and corresponding mode shapes associated with the free vibration of plates. This compendium includes different plate shapes, aspect ratios and boundary conditions. Major contributors to the theory presented in Leissa's monograph were Timoshenko and Woinowsky-Kriege²⁴. These studies gave little to no consideration to the effect of temperature on the structural behaviour of the analysed plates.

Early work by Lurie²⁵ and Bailey²⁶ found a clear relationship between resonant frequencies and loads applied to the plate (mechanical and thermal loads, respectively). Following a linear approach in his analytical study, Bailey's results suggested the first resonant frequency to decrease with temperature until the stressed state of a cantilever is such that the structure buckles²⁷. As such, ΔT_{cr} defines the state of thermal stresses at which a component buckles and the resonant frequency corresponding to the first free vibration mode becomes zero, leading to the "disappearance" of the mode. In his studies, Bailey^{26, 27} also noted that the resonant frequencies of certain modes are capable of moving across the frequency spectrum during the thermal loading of a plate, enabling resonant modes to change position in the frequency spectrum. Jones et al.²⁸ built upon work done by Pal²⁹⁻³¹ and Berger³² and obtained more accurate analytical solutions for the static and dynamic behaviour of plates at elevated temperatures. A recent study by Jeyaraj et al.³³ on flat plates confirmed a decrease in frequency for each resonant mode when exposed to a uniform increase in temperature and under various boundary conditions: fully clamped, simply supported and free edges. The "disappearance" of the first mode as the temperature approached the critical buckling value was also witnessed. The authors performed detailed simulation work on isotropic, rectangular plates using a combination of FE analysis and boundary element methods (BEM) to predict the resonant frequencies and mode shapes of a flat plate at different stages of uniform thermal loading, assuming temperature-independent material properties.

In 1973, replicating the linear analytical approach used by Bailey²⁶ and Simons and Leissa³⁴, Mead³⁵ focused on the analytical and computational study of ideal Kirchhoff plates, rectangular in shape and

with free edges. These were subjected to in-plane thermal stresses resulting from non-uniform temperature distributions that were doubly symmetrical about the centrelines of the plates. Results showed that the temperature at which the plate buckled was strongly dependent on the temperature distribution applied to the plate. Mead built upon the fact the resonant frequencies of beams and plates are known to vary depending on the loading conditions to which they are subjected, concluding in his studies that compressive thermal stresses induced by high temperatures reduce the local stiffness while tensile stresses at cold areas of the plate increase it. For simplicity, Mead assumed a constant Young's modulus and a linear expansion coefficient. Mead verified that buckling of the centre of a plate is associated with compressive stresses in the same region (Figure 2.1). These stresses stem from a spatially-distributed thermal load imparted by highest temperatures at the centre of the plate. Conversely, edge-buckling occurs when the centre of the plate is subjected to tensile stresses associated with higher temperatures at the edges. Moreover, Mead found edge-buckling to occur at lower absolute temperatures, as the free edges are less constrained by plate tension than the central region.

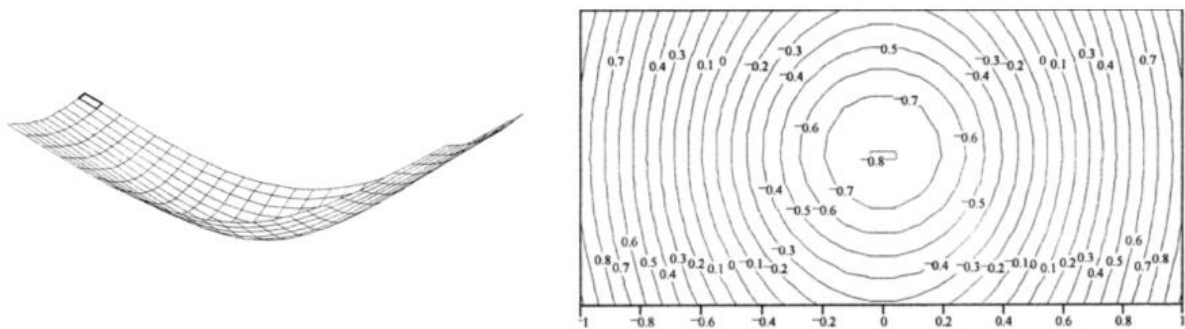


Figure 2.1 Non-dimensional surface and contour plot of plate buckling at a non-dimensional temperature of 209.25 °C. From Mead³⁵.

Mead's analysis suggested that, with an increasing temperature in the centre of the plate relative to its edges, there was an increase in the resonant frequencies for the doubly symmetrical and antisymmetrical modes of a rectangular plate. His results also showed particular mode shapes to switch frequencies in a phenomenon called mode shifting. Chen and Virgin³⁶ further investigated this phenomenon by combining an analytical approach with Finite Element (FE) analysis in the study of thermally post-buckled, aluminium plates. These authors assumed material properties to be temperature-independent when developing their model in ANSYS. Shell elements were programmed in the model and simply-supported boundary conditions were simulated. Their predictions showed a change in the resonant frequencies with temperature which demonstrated the dynamic instability of the thermally buckled structure. Mode shifting was found to be absent in the post-buckled regime if the plate underwent a second buckling event.

In recent years, more powerful computing facilities made the development of increasingly complex mechanics models possible. One such example is the inclusion of temperature-dependent material

properties in FE-based simulations that can describe thermal softening under elevated temperatures which has been adopted by several researchers. Ko³⁷ predicted the post-buckled shapes of rectangular plates subject to dome-shaped temperature distributions and found the critical temperature at which buckling occurred; Ibrahim et al.³⁸ studied the non-linear random response of functionally graded (FG) panels under combined thermal and acoustic loads using a non-linear finite element model; while Talha et al.³⁹ investigated the random vibration response of FG plates and analysed the effect of uncertainty in material properties on the frequency response. The majority of the relevant models using temperature-dependent material has, in fact, focused on investigations of the vibratory response of FG structures by modelling the thermal properties of their constituent materials⁴⁰⁻⁴³. However, none of these studies investigated the effect of temperature on resonant frequencies and mode shapes of isotropic plates, for which Berke et al.¹⁰ developed an FE-model based on small strains and a linear elastic response with no thermal expansion or conduction included. The dependence of the Young's modulus on temperature was introduced by Berke et al. by assigning specific values of modulus to nodes based on a discrete temperature distribution, which was calculated from measured strain data and the material properties published by the manufacturer. These authors also used full-field experimental techniques to acquire measurements of the shape of the plate, which was a much more realistic approach to the geometric model than idealised plates used in the other investigations discussed here. This has a very significant influence in the resonant frequencies and mode shapes of the structure as shown by Murphy et al.^{44, 45} who tested a fully-clamped plate subjected to a uniform thermal load and compared experimental measurements with theoretical results. By modelling an initial central deflection of the plate⁴⁴, their results did not show the "disappearance" of resonant frequencies with an increase in temperature, as predicted by Bailey²⁷ and Jeyaraj et al.³³ In fact, imperfections were proven to increase the structural stiffness of the stressed plate. This was in line with Lurie's work²⁵ which shows that a small deflection specified as initial imperfection causes an increase in stiffness as the structure undergoes further gradual deflection with increasing load.

Cui and Hu⁴⁶ developed a temperature-dependent theoretical model to predict the effect of uniform heating on an ideal isotropic plate with a complex boundary condition: stick-slip-stop. This meant the in-plane expansion of the plate was initially limited by normal or frictional forces, followed by a subsequent slip and a final stop. Their results showed the resonant frequencies of the plate to be highly dependent on the conditions that constrain the plate initially, before slipping occurs. A high level of initial constraints was shown to lead to a decrease in the first three resonant frequencies of the plate when subjected to a uniform temperature rise, particularly in the case of the first resonant mode. However, when these constraining forces allowed the plate to expand more freely, the initial decrease in resonant frequencies was followed by an increase before diminishing once again.

2.1.1 Conclusions

Extensive analytical and computational research has been carried out on the static and dynamic behaviour of plates subjected to various loads and boundary conditions. Important conclusions have been drawn from these studies, namely the strong influence of geometric non-linearities and boundary conditions on the dynamic behaviour of thermally stressed plates; non-uniform temperature distributions across components were shown to yield non-linear responses to thermal and thermo-mechanical loading. However, it is important to highlight that very few studies have used temperature-dependent material models in the study of resonant frequencies and mode shapes of isotropic plates. The most interesting examinations carried out to date have been performed by Cui and Hu⁴⁶ and Berke et al.¹⁰, despite some identifiable limitations. Cui and Hu restricted their study to the analysis of through-thickness gradients of an ideal plate, opting for a uniform in-plane temperature distribution. Berke et al. compared their simulation results to experimental measurements and some differences were observed between the predicted and experimentally determined resonant frequencies. These discrepancies were possibly due the overly simplistic material model which did not include thermal expansion or conduction, and therefore did not suitably replicate the thermal loading process. The assumption of constant material characteristics throughout the thermal loading process has repercussions on the geometric stiffness of the component and affects the predicted vibratory response. Notably, most of the reviewed predictive studies did not include any attempt at validation by comparison with measurements.

2.2 Experimental studies on vibro-acoustic loading of thermally stressed plates

A limited amount of experimental research has been performed on the modal response of thermally stressed plates. The majority of existing work has been performed using the Thermal Acoustic Fatigue Apparatus (TAFA) at the NASA Langley Research Center. Ng and Clevenson⁴⁷ performed experiments on 304.8 x 381 x 1.6 mm (12 x 15 x 0.063 in.) aluminium plates using the TAFA in order to study the random motion of buckled plates when exposed to thermoacoustic loading. For their work, Ng and Clevenson exposed the specimens to noise levels up to 160 dB using pressurised air supplied by two electro-pneumatic modulators. Heating of the plates was achieved using twelve 2.5 kW quartz lamps. Instrumentation on the aluminium plate consisted of thermocouples and strain gauges located on both the front and back of the plate while the noise level was measured using microphones. The plate was fully clamped at its edges using steel brackets for the mounting and experiments were conducted with and without insulation between the plate and mounting frame. Non-uniform temperatures were achieved for both the uninsulated (121.1°C at the plate's centre and 43.3°C at the edges) and insulated (121.1°C centre temperature and approximately 87°C at the edges) experiments. Results showed the increase or decrease in resonant frequencies of the plate when compared to room-temperature conditions to depend on the magnitude of displacements induced by the thermal load; however, no

analysis of mode shapes was conducted. Whilst the experimental results agreed with the FEA predictions for the fully-clamped uninsulated plate, the same was not achieved for the insulated plate due to the non-linear properties of the insulating material which modified the plate's boundary conditions. The uncertainty introduced by thermal insulation in the boundary conditions of the specimen was also highlighted by Leatherwood et al.⁴⁸ when using an equivalent experimental setup in the thermo-acoustic loading of flat and blade-stiffened panels to failure. The recurrent failure of instrumentation used in tests up to approximately 650°C (1200°C) clearly illustrated the complexity of high-temperature experiments using contact methods for data acquisition.

In 1991, Snyder and Kehoe⁴⁹ reported on experiments using a suspended aluminium plate subjected to uniform, non-uniform and transient thermal loads. The plate was instrumented with 18 accelerometers and 30 thermocouples and heated in an oven using an array of quartz lamps placed on one side of the structure. A calibrated hammer was used to provide impact excitation to the plate. Their results showed an increase in temperature, uniform or otherwise, to yield a reduction in resonant frequency for the first four modes of the plate. Despite representing a considerable contribution to the field, this study was restricted to small in-plane temperature gradients with maximum local temperatures just over 200°C, similar to Ng and Clevenson⁴⁷.

Istenes et al.⁵⁰ studied the effect of thermal load on the dynamic behaviour of four fully-clamped panels made from a graphite polymeric composite with different laminate layups. Strain was measured using fourteen strain gauges mounted in symmetric pairs on both sides of the panels and in seven different locations. A thermocouple was mounted at the centre of each panel to monitor their temperature and an infrared imaging radiometer confirmed the temperature distribution across the specimens to be near uniform. Clamping of the four edges of the panels was achieved using two steel support frames which provided the clamping force needed when bolted together. The interface between the steel clamping frame and the test specimen was insulated to minimise the thermal dissipation via conduction between the specimen and the rig. An electrodynamic shaker was used to mechanically load the specimens whilst heating was provided by two 100W lamp banks with 8 lamps each, located on one side of the panels. Each specimen underwent broadband loading at room temperature followed by the narrowband excitation of the first two vibration modes. The same procedure was applied in experiments at 121.1°C, 148.9°C and 176.7°C. Their results suggested high in-plane strains introduced by the thermal load to act as a stabiliser to the motion of the plates when vibrating about their buckled position.

In two complimentary studies, Murphy et al.^{44, 45} studied the resonant frequencies of an AISI 321 stainless steel plate subjected to thermo-acoustic loading. Experiments were conducted at the TAFA (see above) where a plate of dimensions 381 x 304.8 x 3.175 mm was fully clamped onto a side wall of the test area and thus subjected to a grazing acoustic load. Placed opposite to the specimen and across the TAFA chamber, a set of ten quartz lamps was used to provide a radiant thermal load. The clamp was

continuously cooled using a built-in water channel in order to avoid a change in the boundary conditions of the specimen due to thermal expansion of the frame; an insulating blanket was sandwiched between the plate and the clamp for the same purpose. The temperature was monitored using several thermocouples distributed across the width and length of the plate. The output of these thermocouples was used to provide feedback to a control system which adjusted the energy distribution of the banks of lamps so that a near-uniform temperature distribution was achieved across the plate. Displacement and strain data was measured using strain gauges and a scanning laser vibrometer, with the latter being used to create a map of the velocity magnitude and phase across the specimen. The dynamic behaviour of the plate was recorded at room temperature and the process repeated at several different intervals of the thermal loading. The authors compared the experimental results with analytical predictions and highlighted the importance of initial imperfections on analytical results, as discussed in section 2.1. They also recognised there were some inevitable in-plane displacements between the clamping frame and test plate due to the ceramic coating used for insulation purposes. This meant the fully-clamped boundary conditions in the FE model were not reproduced experimentally.

Further experiments on fully-clamped aluminium plates were performed by Geng et al.⁵¹ and their results showed a moderate increase in uniform temperature (approximately 20°C) to decrease the resonant frequencies of the plate. The first five resonant modes of the plate were identified using an impact hammer test at room temperature before increasing the thermal load and repeating the process. A modal exciter was used to dynamically load the plate at resonant frequencies whilst a set of accelerometers measured the vibratory response. The authors attributed the decrease in resonant frequencies to the thermal softening of the material and compressive stresses accumulated in the structure due to the limited thermal expansion allowed in fully-clamped conditions. Complimentary simulations performed by the authors correlated well with their experimental findings; however, this computational work also suggested initial deflections of the plate to greatly affect its resonant frequencies and could even counteract the effect of thermal softening.

Recent investigations departed from the use of contact instruments for the acquisition of experimental data and opted for full-field optical methods. Beberniss et al.⁷ used stereoscopic image correlation to capture the dynamic response of a thin panel subjected to shock impingement. Whilst no thermal load was considered initially, the authors conducted a follow-up investigation⁴ in which the panel was exposed to a heated airflow, reaching non-uniform temperatures of 80°C. These studies helped the authors in the development and planning of future experiments as preliminary results showed the panel to present highly non-linear behaviour under combined loading. Similarly, Berke et al.¹⁰ showed the change of resonant frequencies of a 120x80x1 mm Hastelloy X plate under thermal load to vary from mode to mode. The authors conducted experiments using an induction coil to heat the specimen and a shaker to provide vibratory excitation. A non-uniform in-plane temperature distribution was achieved and estimated using high-temperature strains measurements divided by the material's temperature-

dependent coefficient of thermal expansion; magnitudes were determined to have reached approximately 600°C. Mode shapes were acquired using high-speed stereoscopic DIC. Results showed most of the resonant frequencies of the small plate to decrease with thermal load but that was not the case for every mode.

2.2.1 Conclusions

The work described above has greatly contributed to the acquisition of experimental data pertaining to the resonant frequencies and mode shapes of plates and panels under combined thermo-vibratory loading. The vast majority of these investigations used point-based techniques for the acquisition of data, particularly temperature data, which limited the scope and depth of the studies. Particular attention was given to the effect of insulating material in imparting elasticity to fully-clamped boundary conditions, which hinders the development of predictive models that suitably replicate experiments. Similar to the findings of analytical studies, experimental results showed geometric imperfections in the plates to influence the increase or decrease of its resonant frequencies with thermal load. Most published work was performed using fully-clamped plates under uniform temperatures and showed evidence of the thermal softening of the material leading to a decrease in resonant frequencies. However, results from Berke et al.¹⁰ using a free-edge plate subjected to non-uniform heating suggest the frequency of some resonant modes to increase at elevated temperatures. A single temperature distribution was used in this investigation, which does not provide a comprehensive study on the resonant frequencies and mode shapes of the plate when exposed to a change in in-plane temperature. This constitutes a particular point of interest as the predictions by Jeyaraj et al.³³ suggest a change in mode shapes with thermal load when one or more edges are free.

2.3 Experimental advances on the influence of thermal load on the deformation of panel structures

Experimental work on thermally stressed components has been limited, partly due to the complexity behind the necessary experimental setup. The acquisition and analysis of experimental data have also been restricted by available techniques, which target the gathering of point-wise data and neglect full-field phenomena.

Thornton et al.⁵² studied the elastic and inelastic thermal buckling behaviour of an unreinforced Hastelloy X plate under non-uniform temperature distributions using a single quartz bulb. Temperatures at the surface of the 381 x 254 x 3.175 mm (15 x 10 x 0.125 inch) plate were monitored using a set of 29 thermocouples mounted along the centrelines of the plate and out-of-plane displacements were measured using 15 linear variable differential transformers (LVDTs) placed in an equivalent pattern. The longitudinal edges of the plate were inserted in plastic tubes with negligible bending stiffness and

cooled by flowing water. The plate was mounted horizontally and simply supported at its corners at only four points as shown in Figure 2.2.

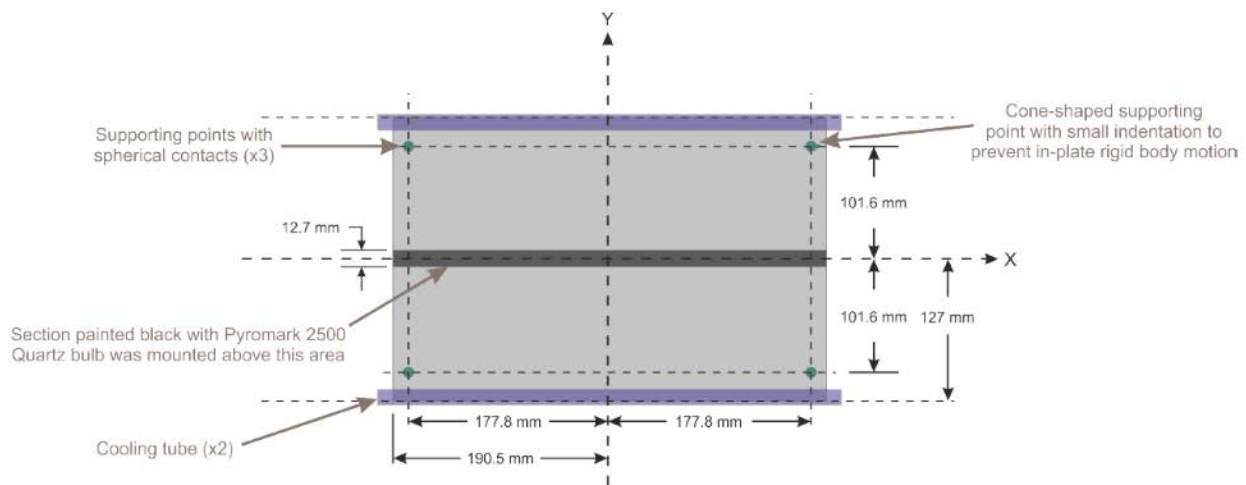


Figure 2.2 Experimental setup used by Thornton et al. in the thermal loading of a Hastelloy X plate. Adapted from Thornton et al.⁵².

The specimen was subjected to five experiments at increasing temperatures and the power output of the lamp was tailored to each test using a power controller. All heating was turned off after the maximum temperature recorded by the thermocouples was reached. The results showed elastic buckling to occur at a maximum local temperature of 190.6°C (375°F) after approximately 300 seconds. Inelastic buckling occurred by repeating the test at a higher heating rate so the maximum local temperature of the plate reached 537.8°C (1000°F) in 90s. The authors concluded thermal buckling was still possible in non-clamped plates as out-of-plane bending was strongly related to compressive membrane stresses caused by in-plane temperature gradients. This reaffirmed the findings of Heldenfels and Roberts⁵³, when performing equivalent experiments using an aluminium specimen, who observed thermal stresses to induce out-of-plane deformation of the plate in a combined bending-buckling phenomenon associated with the initial imperfections of the structure.

Advances in optical techniques and the development of low-cost CCD cameras allowed experimentalists to depart from the exclusive acquisition of point-wise data and allowed for full-field displacements and temperature measurements. Smith et al.⁵⁴ used stereoscopic DIC to measure the surface strain during cyclic heat loading of pre-stressed aluminium specimens. The applied mechanical pre-stress and thermal cycles were designed to recreate the conditions of the production of aluminium sheets used in the automotive industry. The specimens were placed in a split-box furnace and subjected to a heat cycle ranging from 130 to 190°C. DIC cameras were positioned outside the furnace, behind a 101.6 x 127 mm optical quartz window, and the temperature was monitored using a single K-type thermocouple. Displacements and strain data were used as the basis for the development of an FE model to simulate the effects of the thermal cycles.

Jin et al.⁸ focused on the use of stereo DIC in the investigation of displacements and strains associated with the thermal buckling of circular composite plates exposed to a uniformly distributed thermal load. The specimens were made of glass/epoxy fabric pre-preg and simply supported by a titanium ring. The specimen was heated in a chamber from 30°C to 120°C at two different load rates: 2°C/min and 7°C/min. Images were acquired throughout the thermal loading at 5° increments, resulting in a total of 19 image pairs for analysis. Experimental results were compared to predictions from an FE model developed in ABAQUS using temperature-independent material properties and including a generic factor that accounted for geometric imperfections of the plate. This mirrored the model of Lee et al.⁵⁵ who studied the effect of initial geometric imperfections on the critical buckling temperature of laminated composite panels and found a clear relationship between the two: the critical buckling temperature increased as the imperfection scaling factor decreased. Experimental results from Jin et al.⁸ showed the buckling temperature to be strongly dependent on the rate at which the plate was loaded. In fact, the authors noted that buckling occurred when applying a 7°C/min thermal load rate, but not when using a 2°C/min heating. The buckling temperature was experimentally determined by analysing the temperature-displacement relation at the centre of the plate and the DIC displacement map corresponding to the buckling shape. A very similar methodology was used by Yuan et al.⁵⁶ to experimentally determine the buckling temperature and buckled shape of a sandwich panel with a stainless steel truss core. The square specimen was heated in a furnace where it was horizontally clamped using a cast iron frame; a stereoscopic DIC system was used to monitor the displacements of the panel during heating. The cameras were positioned outside the furnace and a view to the specimen was provided through a double layer quartz glass window. Temperature data was acquired using thermocouples fitted to the centre and two opposite corners of the specimen. Results from numerical and theoretical models were found to over-predict the buckling temperature of the panel due to the lack of defects and imperfections, which were not included in the models. DIC results also showed the buckled shape to strongly depend on these imperfections as some local yielding was witnessed and not predicted analytically.

Pan et al.⁵⁷ also focused on composite structures when studying the thermally-induced out-of-plane deflection in honeycomb sandwich panels used for thermal protection in hypersonic flight. Their thick, multi-material sandwich panels were placed in the vertical plane standing on an edge without any fixation and heated using infrared radiation on one side. A stereoscopic DIC system was used to acquire displacement data and thermocouples were attached to the front and back surfaces of the panel. The authors concluded that the maximum deflections of the panel were dependent on the through-thickness temperature gradient at each point of the loading. Their results showed maximum deflections were achieved at the same time as this gradient was at its highest value.

Recently, there have been some attempts at the combined acquisition of full-field displacement and temperature, namely in the study of mechanical energy and heat sources developed locally during

tensile load⁵⁸, in the investigation of the yield behaviour of semi-crystalline polymers⁵⁹ and in biomedical applications for the diagnosis and follow-up of diabetic foot disease⁶⁰. Orteu et al.³³ have published a journal article describing the use of a single CCD camera for the acquisition of both displacements and temperatures by performing a radiometric and geometric calibration of the device. The main limitation of this technique is the amount of preparatory work needed before experiments are conducted and the temperatures it can measure (values above 300°C).

2.3.1 Conclusions

It is possible to conclude from the above literature review that past work has focused on the experimental determination of the buckling temperature of simply-supported and fully-clamped plates. Several findings from these studies are highlighted below:

- Compressive stresses due to high in-plane temperature gradients were found to be sufficient to thermally buckle thin, unclamped plates;
- Maximum out-of-plane deflections of a thick, composite panel were shown to depend on the temperature gradients across its thickness;
- Several authors noted that structural imperfections strongly influence the out-of-plane deflections of thermally-stressed plates; and
- A limited number of investigations in the field have used full-field displacement acquisition techniques and no examples of full-field temperature measurements have been found.

2.4 Analysis and comparison of full-field datasets

Studies presented in ^{8, 61, 62} aimed to establish a degree of correlation between displacement and temperature. However, the relationship between experimentally acquired datasets has been limited to point-based comparisons, either due to practical limitations of the experimental methodology (single point readings provided by strain gauges and thermocouples), and/or the difficulty in comparing full-field datasets which do not share the same scale or physical units. This may lead to an under-sampling of pertinent findings and, therefore, oversimplification of the physical phenomena. However, the increase in the volume of data acquired when using current optical techniques that provide full-field measurements poses a new challenge: to distil the significant information and extract relevant conclusions. Recent developments on the field of FE model validation have proposed a quantitative methodology for the validation and updating of computational solid mechanics models using experimental full-field displacement or strain data⁶³⁻⁶⁵. At its core, this work involves the comparison of two datasets - FE predictions and experimental data. On a point-by-point basis, these are data-rich analyses with extensive data maps that require substantial processing. The authors proposed a data-fitting method based on orthogonal decomposition that allows for the reduction of comparable two-dimensional data from thousands of data points to a set of terms. The 2D monomials used as kernel

functions in this polynomial fit are usually known as geometric moment descriptors (GMD)⁶⁶ or, simply, shape descriptors (SD)⁶⁷. The choice of polynomial depends on the data to be represented and several different studies have been conducted on this topic. The basis of this work prompted the development of standard directives for the validation of computational models which were published by the Comité Européen de Normalisation (CEN)⁶⁸. Wang and Mottershead⁶⁷ have provided a thorough breakdown on the advantages and disadvantages of the multiple polynomials used in orthogonal decomposition for continuous and discrete data.

Teague⁶⁹ was the first to use continuous Zernike and Legendre polynomials to form shape descriptors used in the decomposition of data. The fundamental difference between the two was the domain in which they were defined as Zernike descriptors are orthogonal within a unit circle (therefore, invariant to rotation of coordinates), while the Legendre polynomials are defined on a rectangular coordinate system. Wang et al.⁷⁰ applied orthogonal decomposition using Zernike descriptors as a tool for the recognition of vibration mode shapes from full-field DIC data. Patki and Patterson⁷¹ proposed a hybrid Fourier-Zernike descriptor to overcome several shortcomings found when using the two descriptors separately. Whilst the Fourier descriptor did not yield a significant data compression, Zernike presented limitations in the description of images with discontinuities from geometric features such as holes and cut-outs. Wang et al.⁶³ attempted to resolve this issue by modifying the Zernike descriptor using a Gram-Schmidt orthonormalisation of the polynomials over a non-circular domain. The modified descriptor was then used to compare experimental and computational data pertaining to the measured strain map for a uniaxially-loaded rectangular plate with a circular hole. However, this method was bespoke to a specific geometry and would result in a costly process when applied to multiple geometric discontinuities. One such example is the work proposed by Wang et al.⁷² and Burguete et al.⁶⁵ in the analysis of the dynamic response of geometrically-complex structures under mechanical excitation. The authors used an adaptive geometric moment descriptor (AGMD) determined by projecting the transient responses of a car bonnet onto a two-dimensional orthonormal space obtained by mapping the 3D surface of the structure onto a planar domain. Nevertheless, the discretisation of Zernike/Legendre polynomials into shape descriptors inevitably introduces numerical errors in the decomposition process. According to Wang et al.⁶³, the use of discrete Chebyshev (or Tchebichef), Krawtchouk or Hahn polynomials⁴ has been proposed to reduce these errors in the evaluation of digital images⁷³⁻⁷⁵.

Mukundan et al.⁷³ introduced discrete Chebyshev polynomials as descriptors in image analysis as a method to extract global features from a dataset, i.e. no specific emphasis is given to a particular portion or region of the image. Local features, such as edges and other high-contrast areas, were found to be better represented by Krawtchouk descriptors⁷⁴. Both Chebyshev and Krawtchouk descriptors are particular cases of Hahn descriptors⁷⁵, depending on the parametric definition of the more generalised polynomial. Non-uniform lattices may also be described by other discrete polynomials, such as the Racah and the dual Hahn polynomials⁶⁷. Chebyshev descriptors are of special interest in the analysis of

full-field optical data, as they are defined in a Cartesian coordinate space that better relates to the regular grid of a pixel array. Wang et al.⁷⁶ showed the full-field vibration mode shapes of a rectangular plate with 10^3 - 10^4 data points to be effectively represented using 10^0 - 10^1 shape descriptors, which depicts an efficient compression of full-field data. This is because the 2D shapes described by the discrete Chebyshev polynomials coincide with the mode shapes of an ideal free-free rectangular plate. The authors used Chebyshev polynomials to decompose and compare experimentally-acquired mode shapes using DIC and FE predictions. Sebastian et al.⁷⁷ used a similar method in the comparison of experimental and computational strain maps of a composite panel under uniaxial compression. A new procedure for the use of image decomposition as a tool in the validation of FE models was developed and proposed by Sebastian et al.⁶⁴ in 2013. In this work, the comparison of full-field displacement and strain maps from computational models and experiments was shown to be performed effectively using shape descriptors. The authors deemed validation to be achieved when the coordinate pairs of shape descriptors from the decomposition of experimental and predictive data lie within a scatter band defined by the measurement uncertainty. By following this procedure and the protocol established by the CEN guide⁶⁸, Berke et al.¹⁰ used Chebyshev shape descriptors to validate predictions from their computational model against mode shape data acquired using DIC. A recent investigation into the use of Principal Component Analysis (PCA) for the description of full-field experimental results showed this technique to be a powerful tool in the compression of data (K. Dvurecenska, personal communication, November 13, 2018)⁷⁸. This is achieved by a rotation of the axes of the original variable coordinate system to new orthogonal axes, also known as principal axes, such that the new axes coincide with directions of maximum variation of the original observations⁷⁹. However, the study showed the resulting vectors from the analysis to be difficult to interpret due to the lack of consistent shape descriptors to be used as reference. Extracting a physical meaning behind the compressed data without that reference was found impractical.

2.4.1 Conclusions

Orthogonal decomposition has been shown to be a powerful tool to distil the essentials of a data field. A number of authors found the use of shape descriptors as a basis for fitting of two-dimensional data to assist in the identification of gradients and patterns across datasets, regardless of the scale and physical domain in which they occur. The current state-of-the-art evidences a wide range of applications in which orthogonal decomposition has been used; from validation of predictive models to the identification of mode shapes acquired experimentally. The choice of descriptor used in data decomposition is based on the type of data to be analysed, shape of the original datasets and whether the polynomial is to represent global or local features.

2.5 Knowledge Gaps

From the conclusions sections in the above review, it was possible to identify the following gaps in past research:

1. **From section 2.1, knowledge gap number one:** There has been a lack of integration of structural and temperature data in the developed analytical and numerical models, which has led to unrealistic predictions. This highlights the need for the development of a finite element model capable of predicting resonant frequencies and mode shapes for a plate under non-uniform thermal loading using temperature-dependent material properties and a realistic geometric representation of the initial curvature of the structure.
2. **From section 2.2, knowledge gap number two:** No examples of concurrent full-field acquisition of displacement and temperature have been found when studying the modal response of panel structures. Whilst Istenes et al⁵⁰ used a radiometer to confirm uniform temperature distributions across their specimens, there has been no experimental modal analysis at high temperatures with full-field thermal mapping. This is likely the reason behind the deficiency in information on the effect of different non-uniform temperature distributions of the resonant behaviour of panels. The present project aimed to perform such a study using a commercially-available thermal camera.
3. **From section 2.3, knowledge gap number three:** Thin plates have been shown to buckle under thermal load due to high in-plane temperature gradients. Experiments have been conducted on isotropic, simply-supported plates and functionally-graded panels using various boundary conditions and temperature ranges. However, there is a lack of experimental data describing the behaviour of geometrically-reinforced isotropic plates under thermal load. The literature includes some examples of full-field displacement measurements but none coupled with full-field temperature monitoring. Similarly, no studies have been found on the effect of temperature distribution on thermally loaded panels. An attempt to bridge these gaps is presented in this thesis.
4. **From sections 2.3 and 2.4, knowledge gap number four:** The current state of the art shows that correlations between temperature and displacement measurements have been established in a point-wise domain. The present thesis proposes the use of orthogonal decomposition to analyse full-field results and contribute towards an efficient use of experimental data in the validation of predictive models. The literature shows this technique is yet to be used in the comparison of data fields with different physical dimensions (i.e. temperature and displacement) and it is adopted here as a global analysis method applied to multi-physics data.

Chapter 3 Numerical and experimental methods

This chapter summarises the methods and techniques used in the acquisition and processing of data presented in this thesis. The first part addresses the numerical methods and overall assumptions used in the development of a temperature-dependent computational mechanics model. The second and final part focuses on the experimental methods used to thermally load aerospace-grade metal panels and the techniques used to measure full-field displacements and temperatures across the specimens.

3.1 Numerical methods

Finite element (FE) modelling has become established as the universally-accepted analysis method in structural design⁸⁰. It can be used as a predictive tool capable of solving a wide variety of mechanical, structural and non-structural problems, including static and dynamic loading, heat transfer and fluid mechanics. In a general sense, a solution to the FE problem can be achieved in three general steps:

Step 1 - Pre-processing or "defining the problem": construction of a discrete system of individual elements that represent the mass and stiffness of a continuous structure, known as a mesh. This is known as a piecewise approximation of a continuous domain. Each element defines a discrete area (in shell geometries) or volume (for solid geometries) and is defined by a mathematical description, regardless of the overall geometric complexity of the structure. The boundaries of each element are defined by a unique polynomial curve or surface that connects the boundary node points between elements and are used to establish a relation between individual elements and the displacements of the nodes, in a process called shape function interpolation⁸⁰. In the pre-processing of an FE analysis, nodes, elements and boundaries relevant to the structure are defined, along with material properties which will affect the mathematical description of the elements. Loading conditions are also programmed into the analysis through the application of a distributed load or pressure, whilst boundary conditions specify how the structure is constrained. The analyst must also select the type of analysis based on the physical behaviour of the model. This determines the mathematical model to be used in the solving of the analysis and the governing equations of the model; it may include, for instance, defining whether the analysis is linear/non-linear or if it represents a mechanical/thermal/combined loading scenario. The output of the pre-processor is a set of definitions and instructions that are fed to the solver.

Step 2 – Solving the problem: The FE solver processes the code from the pre-processor using specific algorithms and provides an output that corresponds to the solutions of the analysis. For the present work, the chosen solver was LS-DYNA (Livermore Software Technology Corporation, California, USA) as it is particularly powerful when solving dynamic problems. When processing the instructions from the pre-processor, the FE solver converts the governing mathematical model into element models by fitting a set of trial functions into the original governing equations and minimising the error between the two.

For each element, the solver determines three descriptors: 1) element shape functions, 2) element stiffness matrices (geometric and material stiffness), and 3) element load vectors. The solver then assembles the mathematical description of every element (either one or a set of partial differential equations) in a large system of equations that models the entire problem and enforces the continuity of displacements between element boundaries. This is achieved through a transformation of the element models from local coordinates systems to a global coordinate system.

When assuming a linear relation between load and displacement, the solver assembles the global stiffness (K) and force (F) vectors and, for a static analysis, solves the equation:

$$KU = F \quad (3.1)$$

where U is the unknown displacement vector.

However, when the global stiffness matrix changes throughout the analysis, the displacement-load relation becomes non-linear and the same assumption cannot be made. This may occur due to non-linearities in the displacement-strain relation (geometric non-linearities, e.g. large displacements) or strain-stress relation (material non-linearities, e.g. plastic deformation). In such cases, the solution to the above equation becomes:

$$KU(U) = F \quad (3.2)$$

which requires the recalculation of the global stiffness matrix throughout the loading, as it becomes dependent on the global state of each element. Current FE software provides the analyst with a choice on which method to use in the calculation of the displacement vector, U . Given the iterative process required in the recalculation of the global stiffness matrix, however, non-linear analyses are typically more time consuming.

Step 3 – Post-processing:

The post-processor decodes the output from the solver and gives the analyst several tools to view the results, such as: deflection plots, stress/strain contour diagrams, nodal displacements graphs, matrices or tables, etc. HyperWorks (Altair Engineering, Michigan, USA) was used for the pre-processing of analyses and post-processing of results due to its meshing capabilities and easy interface with LS-DYNA.

The specification of algorithms to be used by the solver in the processing of analyses was selected based on the requirements of the study. A structural-thermal solution procedure in LS-DYNA was required when defining temperature-dependent properties in the model. Heat transfer analysis was coupled with the mechanical solver to calculate the effect of temperature on the structure.

An eigenvalue analysis was used in the investigation of the modal response of the plates to compute the solution of the equation of motion for the resonant frequencies (eigenvalues) and corresponding

vibration mode shapes (eigenshapes). This method assumes negligible damping and no applied loading, reducing the equation of motion to the eigenequation:

$$[K] - \lambda[M]\{\phi\} = 0 \quad (3.3)$$

where ³³ is the stiffness matrix of the component, ^{70, 81, 82} corresponds to its mass matrix, λ is the resonant frequency and $\{\phi\}$ is the corresponding mode shape. Alternatively, a modal frequency response could have been performed by applying a mechanical excitation to a thermally loaded plate and computing its response. However, this is a time-consuming method that requires a separate computational analysis to be performed for each resonant frequency and temperature distribution. The eigenvalue method expedited the acquisition of results by enabling the extraction of resonant frequencies and mode shapes at different increments of the thermal loading using a single computational analysis. This allowed for a quasi-continuous monitoring of the modal response of the structure from room temperature to a high-temperature steady state. The eigensystem solutions were extracted using the linear iterative Lanczos extraction method, which is standard in most FE packages⁸³; no damping was modelled.

A double precision executable LS-DYNA solver was used in order to improve accuracy of results as the single precision executable yielded inconsistent results due to truncation errors in the calculations associated with the fine mesh applied to the model⁸⁴.

All analyses were performed using an implicit solver. In implicit analysis, displacements calculated at each time step are not a function of time so that velocities and accelerations are zero, and mass and damping factors can be neglected. The time dependency of the solution is not an important factor in modal analysis, as resonant frequencies and shapes are extracted by solving a static eigenvalue problem. By using an implicit solver, solutions have been calculated using mathematical iterations to enforce equilibrium at each time step until a final result is reached. As this is an iterative process which requires the inversion the stiffness matrix, it is a computationally expensive process. However, this is often offset by the fact it does not require small time steps unlike explicit methods which define displacements as a function of time.

3.2 Experimental methods

3.2.1 Infrared heating

Infrared lamps were used to heat the specimens up to maximum temperatures of approximately 700°C due to their inexpensive and replaceable nature and the ability to easily control their power output to create different temperature distributions across the panels. In the context of this work, these advantages greatly outweigh the typical higher energy losses associated with the use of radiant heating in comparison with induction methods which rely on electric currents induced internally in the material

to be heated⁸⁵. In induction heating, the flow of AC current through a conductive coil (generally copper) generates an alternating magnetic field, which induces eddy currents in the specimen. The specimen behaves like a resistance through which the eddy currents flow generating an increase in temperature due to resistive (or Joule) heating. Thus, the method strongly relies on the electric conductivity of the material to be heated, which limits its use when studying non-ferrous specimens.

In the work here presented, halogen quartz lamps with a power output of 1kW each and a colour temperature of 3210 K were used for heating the specimens (QIR 240 1000 V2D, Ushio, Steinhöring, Germany). The lamps were mounted in uniformly spaced arrays of six with a reflective back-plate so to maximise the energy provided to the test specimens. The centre-to-centre distance between lamps was approximately 3.5 cm. The power output of the lamps was regulated by a controller using TRIAC semiconductors. The specimens and lamps were not enclosed in a furnace so that there were substantial heat losses from the apparatus, but the optical access was excellent.

3.2.2 Full-field infrared thermography

The use of optical techniques in the monitoring and measuring of temperatures was expedited in the early 1900s after the photon effect based on photoconductivity was discovered by W. Smith in 1873⁸⁶. Two different types of detectors were then developed: photon detectors, and thermal detectors or bolometers. Photon detectors rely on the principle of photoconductivity as photons of specific wavelengths are absorbed by the sensor and create free electron-hole pairs, which can be detected as electrical current. However, the output signal is very small and easily overshadowed by internally-generated thermal noise in the device, which results in the need for it to be cryogenically cooled. Active cooling is not required in bolometers, which produce an electrical response based on a change in temperature of the sensor. The need for thermal equilibrium to be reached within the bolometer detector after the absorption of radiation (thermalisation) caused them to have a lower sensitivity and their response time to be considerably slower than photon detectors. However, manufacturing advancements in the 1970s made it possible to produce smaller, thinner pixel-based thermal detectors which narrowed the differences between the two technologies⁸⁶.

In the high-temperature work presented in this thesis, both types of infrared detectors were used. Initially, a commercially-available photon detector (SC7650E, FLIR, North Billerica, MA, USA) was used to monitor the temperature distribution on the surface of the specimen. The camera has a focal plane array of 640 x 512 pixels with a maximum recording rate of 100 frames per second at this resolution and a typical sensitivity of 20 mK at wavelength from 3 μm to 5 μm . Despite the high sensitivity of the camera, the system was not calibrated for temperatures above the 150°C, which restricted the maximum temperatures detected before the sensor saturated. A reflective neutral density (ND) filter with a usable range between 2 and 16 μm (NDIR20B, Thorlabs Inc., New Jersey, USA) was fitted to the

camera to prevent saturation and a manual calibration was performed with the aid of a laser-guided infrared thermometer (Fluke 62, Fluke

UK Ltd, Norfolk, UK). This calibration method for the camera was prone to error due to the complicated process involved; and so, in subsequent experiments a micro-bolometer with an accuracy of ± 2 °C or ± 2 % (whichever is greater) was used. The commercially-available uncooled micro-bolometer (TIM 400, MICROEPSILON UK, Birkenhead, UK) was a more economic option in the monitoring of temperatures than the photon detector. Its $25\text{ }\mu\text{m} \times 25\text{ }\mu\text{m}$ focal plane array sensor has a 382×288 pixel resolution and a maximum frame rate of 80Hz. The sensor is sensitive to a spectral range of $7.5\text{ }\mu\text{m}$ to $13\text{ }\mu\text{m}$ and a thermal sensitivity of 0.1K with the chosen telephoto lens ($13^\circ \times 10^\circ$ FOV; $F = 1.0$). The device is factory-calibrated and includes an automated "ag" system that is periodically positioned in front of the sensor as a constant temperature reference to reduce the variations in the response of each pixel to the infrared energy. Using the control software of the micro-bolometer (TIM Connect, MICRO-EPSILON UK, Birkenhead, UK), this non-uniformity correction was programmed to take place in-between data acquisition frames to avoid the introduction of erroneous readings if performed when recording data. The range of temperatures studied bridged two available calibration ranges: 0-250 °C and 150-900 °C; hence, a compromise was made to use a camera calibrated in the 150-900 °C range with an option selected that extended the calibration down to room temperature. This resulted in an increase in the amount of noise for room temperature readings, which decreased the system's accuracy to 5-7 K (worst case scenario), according to the manufacturer. Since the work presented here is primarily focused on phenomena at temperatures beyond 150 °C, the higher uncertainty of measurements at lower temperatures was accepted.

3.2.3 Digital image correlation

Several full-field techniques can be used for the measurement of displacements and strain at high temperatures. In a broad sense, the methodology spectrum can be divided into two major groups: 1) interference methods, such as holography or moire interferometry, which make use of coherent light and, therefore, depend on its wave properties, and 2) other methods that use non-coherent light and include photoelasticity, digital image correlation (DIC), fringe projection and grid methods. Sebastian¹¹ has provided a detailed breakdown of viable full-field measurement techniques and their inherent strengths and weaknesses. In the present work, stereoscopic-DIC was used in the acquisition of displacements and specimen shape data because it is capable of gathering full-field information at a large range of temperatures, specimen sizes, time scales and loading rates. It is a robust technique with a relatively simple setup and low sensitivity to unintentional vibration and rigid body movements during testing; both these factors contribute towards it being an attractive method to be used in industrial settings. Despite requiring very simple specimen preparation using readily available temperature-resistant materials, this technique provides a rich data set by being capable of measuring in-plane and

out-of-plane displacements, as well as object shape. DIC uses digital images of an object or structure at difference stages of deformation and employs tracking techniques to measure surface displacement and build full-field 2-D and 3-D displacement fields and estimated strain maps. 2-D DIC is the simplest example of this method, as a single camera is used in the acquisition of images and only in-plane deformation can be measured. Stereoscopic-DIC (or 3D-DIC) uses two cameras to acquire image pairs at each deformation stage, which introduces the ability to trace out-of-plane displacements and provide data pertaining to the curvature of the object through point triangulation. It is also possible to perform stereoscopic-DIC with a single camera using additional optical devices like diffraction grating, a bi-prism or a set of planar mirrors, which are used to record pseudo images of the imaged object⁸⁷.

Experimentalists have also successfully used systems of multiple cameras (2+) to acquire groups of images per deformation stage, which was shown to be an effective way to enlarge the measurement field without losing effective resolution in the area of interest⁸⁸⁻⁹¹.

In DIC, each digital image is an effective intensity array which is divided into overlapping, square subsets, called facets. The size of these subsets is based on the spatial frequency of the pattern being imaged to ensure each facet is unique and the DIC system cannot mistakenly identify it as a different facet. For this purpose, Sutton et al.⁹² recommended that an optimised facet should contain at least nine uniformly distributed speckles. Prior to loading, an image of the object or structure is acquired and defined as the reference image. Subsequent images are acquired at different stages of loading and are termed deformed images. Image correlation is performed by comparing the facets from the reference image to the facets from each of the deformed images using a correlation algorithm based on image matching. A point within a facet of the reference image, $P(x, y)$ can be mapped to a point in a facet of each of the deformed images, $P^*(\tilde{x}, \tilde{y})$ using the displacement components u and v in Equations 3.4.

$$\begin{aligned}\tilde{x} &= x + u(x, y) \\ \tilde{y} &= y + v(x, y)\end{aligned}\tag{3.4}$$

A schematic of the image correlation process applied to 2D-DIC is shown in Figure 3.1, where point P_1 in the reference image is matched to point P_1^* in the deformed image. The shape of the deformed facet is modelled using a high order shape function that transforms pixel coordinates in the reference facet into coordinates in the image after deformation⁹². In the work here presented, the DIC software used a second-order function (shape function) to locate and map each subset to its distorted shape⁹³, as defined by the following equations:

$$\begin{aligned}\tilde{x} &= x_0 + u_0 + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial^2 u}{\partial x \partial y}\Delta x \Delta y \\ \tilde{y} &= y_0 + v_0 + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y + \frac{\partial^2 v}{\partial x \partial y}\Delta x \Delta y\end{aligned}\tag{3.5}$$

where (x_0, y_0) and $(\tilde{x}, \tilde{y})$ represent the coordinates of the centre of the facet in the reference and deformed image, respectively. The in-plane displacements are defined by u_0 and v_0 , whilst the remaining parameters in the shape function define surface slopes and distortion. The terms in Equation 3.5 are determined using a sum of squares deviation to minimise the differences between the reference and deformed facets.

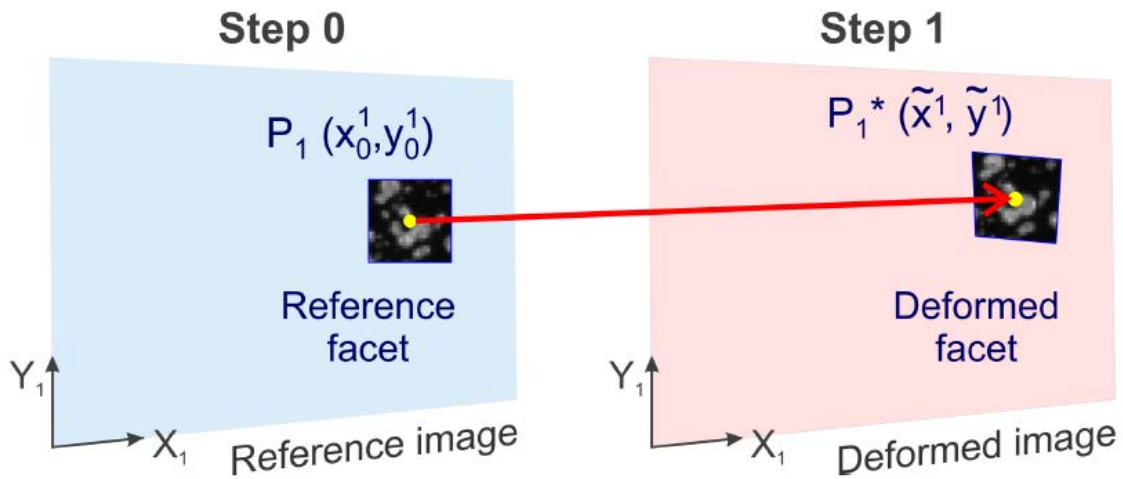


Figure 3.1 Schematic representation of the DIC process in which an undeformed facet in the reference image is mapped onto a deformed facet in an image acquired post-loading.

The deformation of the object or structure introduce non-integer displacements of the corresponding positions in the reference image⁹⁴, which requires the discrete, pixel-based intensity values of the deformed images to be interpolated and converted into a continuous functional representation of the original images. This allows for displacements to be determined to a sub-pixel accuracy. Different interpolation methods may be used in this process. Figure 3.2 a) shows an example of a 16 x 16 pixel greyscale image which was then plotted in Figure 3.2 b) to illustrate the discrete intensity values at each pixel that range from 0 (black) to 255 (white). A bi-linear interpolation of the original pixel map is shown in Figure 3.2 c) and represents a computationally efficient option but at a cost: a large bias, particularly when processing noisy images⁹⁵. In the work presented in this thesis, a bi-cubic spline was used to acquire sub-pixel accuracy of displacements, which is exemplified in Figure 3.2 d). Whilst computationally more expensive, investigations by Bornert et al.⁹⁵ and Sutton et al.⁹² shows the bi-cubic spline to yield the lowest bias error and the lowest random error when compared to other typically used methods such as bilinear and bi-cubic polynomial interpolation. According to Bornert et al.⁹⁵,

interpolation using an optimised bi-cubic spline filter can result in a bias of approximately 1.1×10^{-3} pixels for noiseless images and 13×10^{-3} pixels for images with added noise.

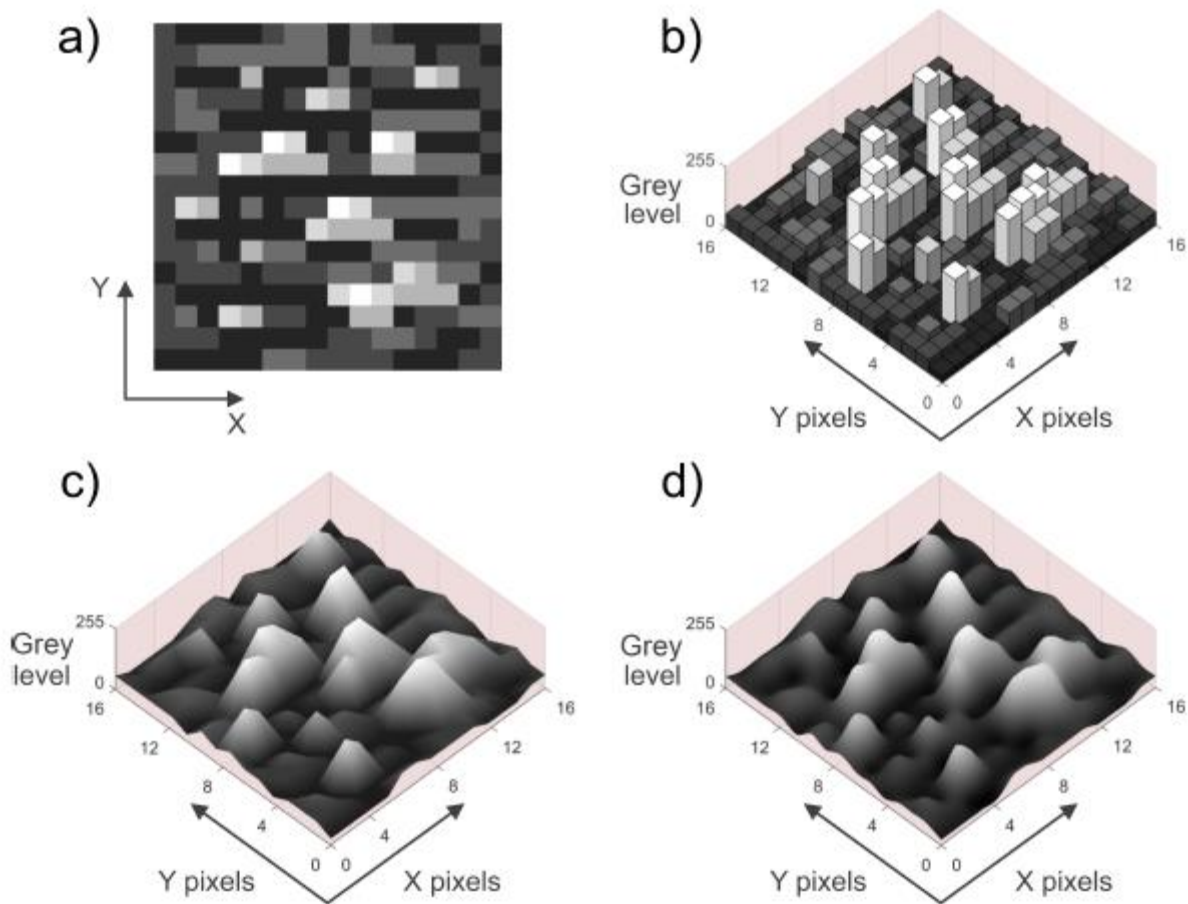


Figure 3.2 Discrete grey level representation of intensity values for a 16 x 16 pixel array. Image credit. The b) Three-dimensional, c) bi-linear and d) bi-cubic spline representations of the same pixel array are included.

2D-DIC requires the calibration of the system so that a total of eleven independent parameter are determined: six extrinsic parameters that describe the orientation and position of the camera, and five intrinsic parameters which define the imaging parameters for the camera⁹³. This is achieved by acquiring several images of a known target that undergoes arbitrary motion. In the calibration process, the DIC software effectively estimates the transfer function of the DIC system by recognising a pattern in the acquired images of the calibration target (fiducial points) and analysing it against known characteristics of that pattern. In a stereoscopic rig, both cameras acquire images of the object or structure at the same stage of deformation and, as such, they must share a common coordinate system. An equivalent calibration process to the one described for 2D-DIC is applied to determine the intrinsic parameters for each camera and the relative positions and orientations of the cameras with respect to each other⁹³. When calculating the deformations experienced by an object or structure under load, these parameters are used to calculate the 3D position of each surface point and provide both in-plane and out-of-plane displacement data. According to Sutton⁹⁴, the typical accuracy of these measurements is in the order of

$\pm 1/100$ pixels and for in-plane displacement components and $\pm Z/50000$ for out-of-plane displacement components, where Z is the distance from the cameras to the object or structure.

Another advantage of stereoscopic DIC compared to its 2D counterpart is its ability to provide information about the shape of the imaged object or structure. The DIC software selects the subsets in the reference image, acquired using the master camera, and performs image correlation to locate the matching positions in the image acquired using the second camera. The intrinsic and extrinsic parameters calculated during the calibration of the DIC system are then used to determine the optimal three-dimensional position of the common object point for each matching pair of subsets, which results in a dense set of three-dimensional points that represent the shape of the object. Figure 3.3 shows a schematic of the stereoscopic DIC process when tracking a point P , which represents the centre of the highlighted facet.

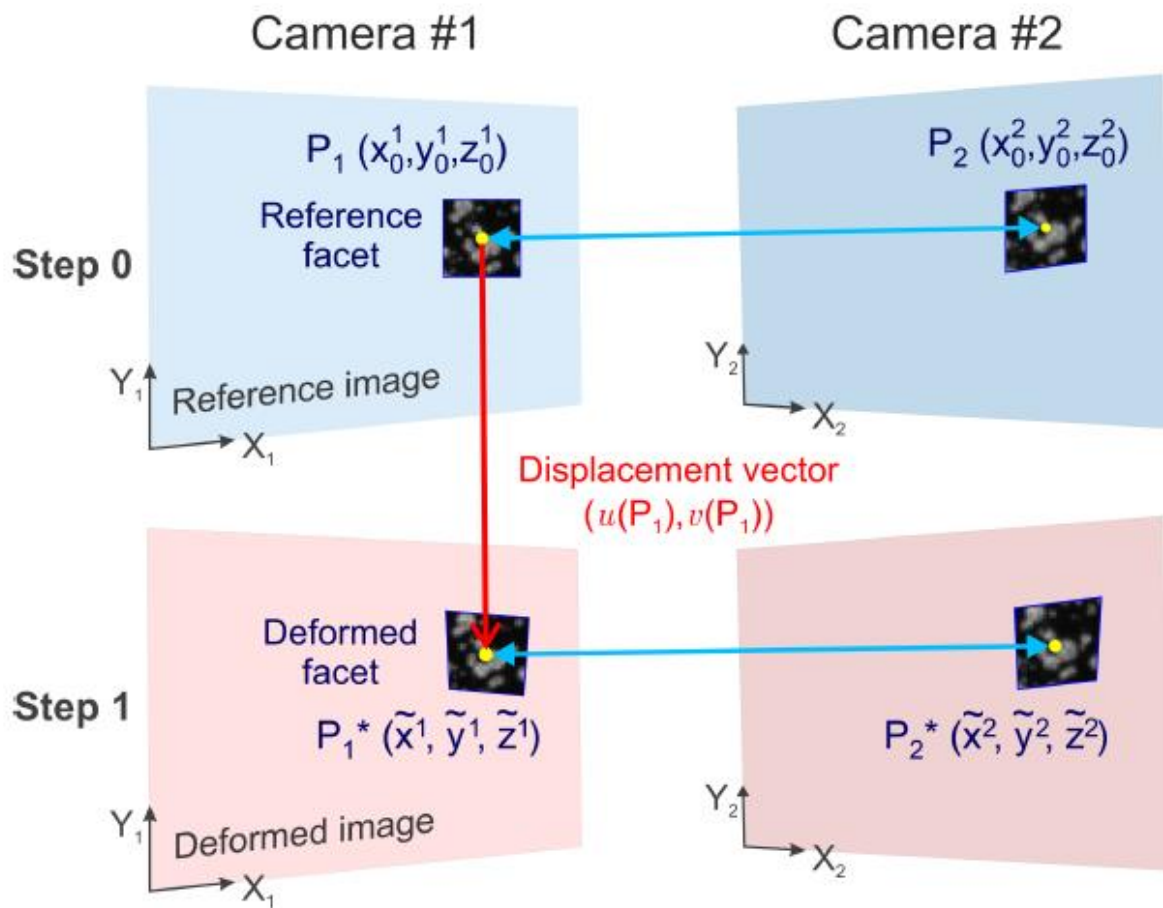


Figure 3.3 Schematic representation of the typical stereoscopic-DIC process.

In the work presented in this thesis, a commercially-available stereoscopic DIC system was used (Q-400 system, Dantec Dynamics GmbH, Ulm, Germany), which consisted of two 1624x1234 pixel CCD Firewire cameras (2MP Stingray F-201b, Allied Vision Technologies GmbH, Stradtroda, Germany) _fitted with a set of 12 mm lenses (Cinegon, 1.4/12, Schneider). The maximum frame rate of the cameras was 14 frames per second at full resolution.

The surfaces of specimens are prepared for DIC measurements by applying a high-contrast random pattern to the regions of interest; this is fundamental in the characterisation of facet position and displacement tracking. Whilst DIC has been performed using many types of object-based patterns, such as lines, grids, dots, random arrays and features, the most commonly used method is the use of random speckle patterns achieved using high-contrast paints. In this work, specimens were prepared for high temperature measurements using DIC by spraying them with a uniform layer of commercially-available refractory black paint (VHT Flameproof, Cleveland, Ohio, USA). A white version of the same paint was used to create a speckle pattern. The more usual reverse colour scheme was considered but disregarded because the black background used here enables a larger transfer of radiation and results in higher specimen temperatures. The scale of the region of interest determined the process used for the creation of the speckle. A large speckle size requires the use of a large subset that can approximately accommodate the nine, uniformly-distributed speckles recommended by Sutton⁹²; however, this compromises spatial resolution. A speckle size that is too small increases the noise level in the results and the risk of aliasing. Multiple optimal speckle sizes have been recommended in literature but the most generally accepted convention is approximately 3-5 pixels in diameter^{92, 96}. The misting or spraying of the base colour with a contrasting paint is typically a good method for the generation of small, dense speckle patterns. In the present work, however, a suitable speckle size could not be achieved using spray nozzles as the full-field view of the specimen rendered the resulting speckle to be greatly below the conventionally accepted size. Promising results were achieved using a short bristle brush, which was slightly distressed, and lightly dipped in paint and flicked onto the specimen; however, there was a directionality to the speckle due to the momentum of the paint droplets. This directionality is undesirable as oriented features limit the determination of motion vector components which are orthogonal to the object or structure⁹². Qualitatively, the most even speckle distribution was achieved by using a 1 inch (2.54 mm) wide brush dipped in paint and held horizontally, approximately 20 cm above a horizontal specimen. By firmly tapping it, the resulting speckle was achieved by force of gravity and lateral deviations to the droplet trajectories were kept to a minimum. Fiji by ImageJ^{97, 98} was used to analyse a high-quality image of the speckled specimens acquired using one of the CCD cameras: a threshold equal to the mean grey level across the plate was defined and the particle analysis tool available in the software was used to calculate the average speckle size. This was found to be around 14 pixels², which corresponds to an average speckle diameter of approximately 4 pixels when assuming a perfectly circular shape. Images of the resultant speckle pattern can be found in Chapters 5 and 6, where the experimental results of this work are documented.

Compared to its room temperature counterpart, there are several added challenges when performing DIC at high temperatures. The most common problem is the increased in black body radiation. The most common problem is the increase in black body radiation. Whilst, in the past, traditional DIC methods without any special optical setup have been used successfully up to approximately 600°C⁹⁹, radiation

from the test specimen becomes visible to the naked eye when its temperature goes beyond 1000K (727°C). However, most CCD camera sensors are sensitive to radiation emitted by bodies below this temperature, which interferes with the imaging process by altering the contrast between surface features and often saturates the camera sensors. Several researchers have shown that undesired radiation from high-temperature experiments can be eliminated by the use of narrow band lighting and, or optical bandpass filters^{10, 11, 100-103}. Experimental displacement data presented in this thesis was acquired using stroboscopic illumination of the specimen provided by a pulsed-laser with a wavelength corresponding to the peak quantum efficiency of the CCD sensors in the DIC cameras. This consisted of an Nd:YAG laser (Nano L200-10, Litron, Rugby, England) that emitted a 4-nanosecond pulse of green light (532 nm) which was expanded after passing through a spatial filter to produce a speckle-free beam of approximately 1 m in diameter. The laser was placed directly in front of a specimen, providing it with uniform illumination and granting equal exposure for the images captured by both cameras. Optical bandpass filters with a centre wavelength of 532 nm and 4nm bandwidth were fitted to the standard DIC cameras in order to block any light outside of the wavelength of the laser. Simultaneous triggering of the laser pulse and DIC image acquisition was achieved by connecting both systems to a timing box.

The deterioration or oxidation of the paint at high temperatures is another challenge to overcome, as it tends to volatilise with the increase in temperature. Researchers have applied very sophisticated methodologies to solve this problem at temperatures above 1000°C, from the airbrushing of alumina and zirconia paints¹⁰¹, to the use of plasma spray to speckle tungsten powder onto carbon composites⁶². Commercially-available, temperature-resistant paint has been shown to withstand temperatures of about 600°C without any significant degradation in the accuracy of the measurements⁹⁹. However, Sebastian¹¹ experienced some problems with the flaking and discolouration of temperature-resistant paint at equivalent temperatures. So to prevent this from happening in the experiments described here, P120 sandpaper was used to lightly score the surface of the specimens before the black VHT Flameproof base paint was applied; this was done in accordance to the directions of the paint manufacturer and no flaking of the paint was witnessed. No visible discolouration was witnessed at the temperature range studied in this thesis (from room temperature to just below 700°C), which was confirmed by Berke et al.¹⁰³ who found this particular paint to discolour at temperatures on the order of 750°C.

The final issue identified by researchers when performing high temperature DIC was the role of a heat haze in the distortion of results. This is typically induced by the transient variations of the refractive index of the heated air¹⁰⁴ when the air temperature between the cameras and the specimen being imaged is not homogeneous. Unfortunately, the influence of the heat haze in DIC measurements is not yet well-understood as it heavily depends on the experimental setup used in each study¹⁰⁵. However, Pan et al.¹⁰⁴ have showed the influence of the heat haze to be negligible when positioning the specimen vertical and perpendicular to the optical axis of the imaging system. In the experiments presented here, that configuration was adopted. The distance between the cameras and the specimen was also kept to a

necessary safe minimum, as large distances between the specimen and cameras promote the formation of a heat haze. To facilitate the mixing of the air around the experimental setup, tests were performed in a ventilated room with an extractor fan, as recommended by Lyons et al.¹⁰⁶.

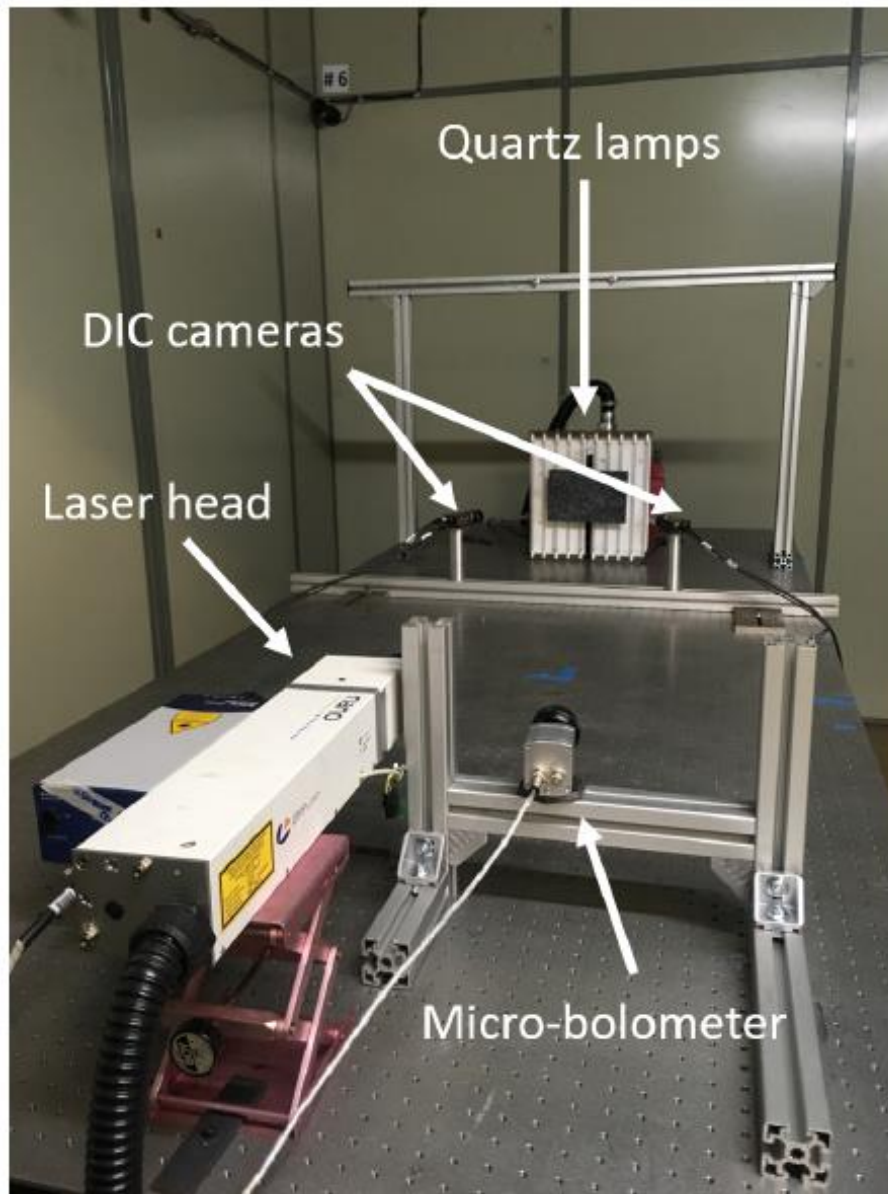


Figure 3.4 Photograph of the setup used in the experiments performed as part of this project.

In each experimental chapter of this thesis, a detailed schematic of the setup is presented, however, Figure 3.4 shows an overview of the rig used in the experimental work performed as a part of this project. Note the DIC-ready specimen included in the photograph and mounted in front of the quartz lamps.

Chapter 4 Development of a temperature-dependent material model using finite element analysis

4.1 Introduction

One objective of this project focused on the development of a Finite Element (FE) model, with temperature-dependent material properties, to be used as a tool capable of accurately predicting resonant frequencies and mode shapes of a thin plate subjected to elevated temperatures and at different stages of the thermal loading.

In this development chapter, an in-depth description of the model is provided and each assumption is explained. Results are presented and compared to experimental and predictive data published in 2015 by Berke et al.¹⁰ as a way to build on previous modelling work undertaken by the research group. This previous model assumed small strain linear elastic response with no thermal expansion or conduction modelled. As explained in section 2.1, Berke et al. introduced a temperature-dependent Young's modulus by assigning specific values to the nodes affected by a discrete temperature distribution divided into 6 levels. This temperature distribution was computed from stereo-DIC strain data and material data provided by the manufacturer¹⁰⁷. The analysis was defined as a static event with no dynamic increase in temperature having been programmed into the model. Their frequency predictions yielded no significant changes in resonant frequency with temperature, which was not confirmed experimentally. There were several possible causes for this disparity of results: 1) the use of an overly simplistic material model; 2) not capturing thermal stresses accurately when performing a static analysis using FE; 3) poor depiction of the temperature changes during the oscillation period. Whilst the latter can only be experimentally tested, a systematic study was undertaken to identify which parameter or number of parameters were responsible for the shortcomings of this past model. Results were also compared to predictions from Bailey^{26, 27}, Jeyaraj et al.³³ and Murphy et al.⁴⁴ on thermally stressed plates in an attempt to establish a framework for further developments that are presented in Chapter 5 and involved the integration of experimentally-acquired geometric data (out-of-plane curvature of the plate) and temperature data in the model. Hence, it was critical to develop a robust, reliable model that could not only be used to recreate the conditions of future experimental work, but could help in the design the experimental setup, techniques and protocols.

The structure of this chapter deviates from that of the subsequent results chapters because it describes an iterative development process of a computational model; thus, the results of the multiple separate studies are presented before an overall discussion and consequent conclusions are drawn in the context

of this project. This chapter is based on the following conference paper, presented at the British Society for Strain Measurement 12th International Conference on Advances in Experimental Mechanics:

A. C. Santos Silva, C. M. Sebastian, E. A. Patterson, "Acoustic response of thermally stressed plates using a temperature dependent finite element material model", BSSM 12th International Conference on Advances in Experimental Mechanics, 2017.

4.2 Definition of a temperature-dependent material model applied to a static loading analysis

An initial attempt at a predictive model with temperature-dependent material properties aimed to computationally recreate the laboratory conditions described by Berke et al.¹⁰. The resulting model was then used to study the first hypothesis proposed by these authors to explain the discrepancies between their experimental results and FE predictions: an overly simplistic material model. Consequently, the focus was not on the computation of thermal stresses but on the development of the material model itself and its application to a plate under constant load (static analysis). The specimen studied was a 120 x 80 x 1.016 mm Hastelloy X (Haynes International, Kokomo, IN USA) plate modelled in Hyperworks (Altair Engineering, Michigan, USA) using 1 x 1 mm quadrilateral shell elements (9801 nodes in total, akin to Berke et al.'s FE model). These elements were applicable due to the specimen being assumed to be a Kirchhoff-Love plate (thin plate) which can be fully described in two-dimensional form using its mid-surface plane. The recommended shell formulation for implicit analyses in LS-DYNA was selected (type 16 - fully integrated with 4 in-plane and 5 through-thickness integration points)¹⁰⁸, as it uses a "local element coordinate system that rotates with the material to account for rigid body motion"¹⁰⁹ and avoid element locking phenomena¹. The plate was modelled in the x – y plane with out-of-plane displacements in the z-direction. A single central node (x,y) = (0, 0) was completely restrained (zero degrees of freedom). It is important to stress that, at this point in the study and unlike the work from Berke et al., this geometry was assumed to be an ideally flat plate and did not account for geometric imperfections.

The values of temperature-dependent material properties of a Hastelloy X plate were provided by the manufacturer¹⁰⁷ and integrated into an isotropic thermal material model available in LS-DYNA (MAT ELASTIC PLASTIC THERMAL) by defining property curves that described how the Young's modulus, mean coefficient of thermal expansion, thermal conductivity and heat capacity changed with temperature. These curves have been plotted and are available in Appendix A. The Poisson's ratio of the material was assumed to be constant as values provided by the manufacturer showed this parameter not to be significantly affected by temperature. The material was assumed to have an elastic response and a constant mass density at 8220kg/m³. Despite the fact that metal plates are often manufactured using a rolling process that induces a directionality to the grain orientation, the material was assumed isotropic

for simplicity and because that level of material data can be difficult to acquire for existing aircraft structures.

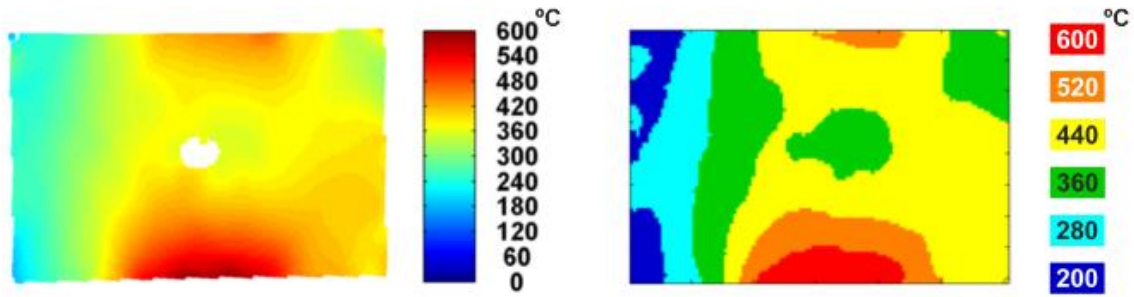


Figure 4.1 Experimental temperature distribution estimated by Berke et al.¹⁰ (left) and the recreated map using the developed MATLAB script (right).

The analysis was defined assuming a linear relation between load and displacement and the stiffness matrix was assumed constant throughout the study. A uniform room temperature (22°) was applied to the plate and the first nine eigenvalues and eigenshapes were extracted. The process was repeated at higher temperatures using an equivalent temperature distribution to Berke et al. A MATLAB script was written to replicate the discrete temperature distribution used by the authors and assign the correct temperature values to the FE nodes shown in Figure 4.1. The original image on the left of Figure 4.1 was converted to greyscale and its intensity values divided into six discrete levels. Then, each of these levels was assigned a temperature value that reproduced the distribution used by Berke et al. in their simulation work. Any discontinuities in the data map in Figure 4.1 (left) were interpolated using MATLAB's default nearest neighbour function.

The combination of static loading, a flat geometry and a fine mesh prevented hourglassing from occurring and so no correction was necessary. Studies on the impact of gravity loading on the modal response plate were undertaken and no effect was witnessed. The developed static model with temperature-dependent material properties was then used to perform two simple investigations that aimed to improve predictions. The first investigation was conducted to understand how the FE predictions would be affected by the simplification of the material model due to the lack of mechanical and, or thermal performance data. A first degree polynomial was used to interpolate the temperature-dependent Young's modulus, mean coefficient of thermal expansion and thermal conductivity; the resulting curves are shown in Figure B.1 of Appendix B. Heat capacity and Poisson's ratio remained unchanged. Predictions using the original and simplified material models (F_s) were compared to the experimentally-acquired frequencies published by Berke et al. (F_E) by calculating the relative error of the FE results as:

$$Relative\ error = \frac{|F_E - F_s|}{F_E} \times 100\% \quad (4.1)$$

Uniform plate

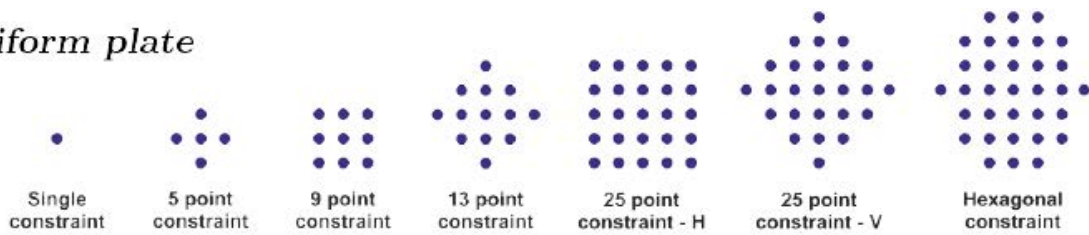


Figure 4.2 Constrained nodes studied for the uniform plate. All permutations were centred on the plate's central node: $(x, y) = (0, 0)$.

Plate with 5mm hole

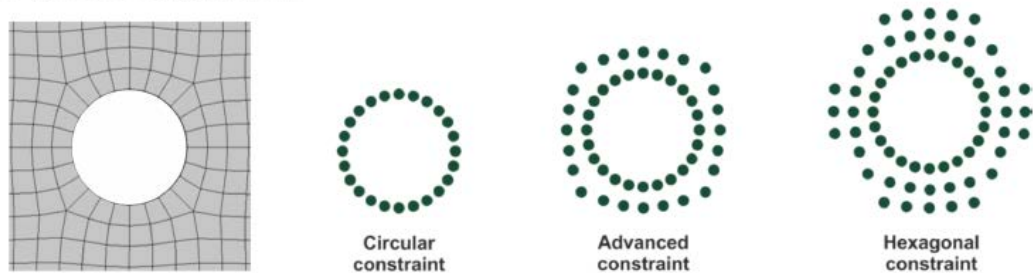


Figure 4.3 Constrained nodes studied for the plate with a hole. All circular constraints were centred on the plate's central node: $(x, y) = (0, 0)$.

Results from this study are also presented in Appendix B. Room temperature frequency predictions did not present any significant change with the simplification of the material model. High temperature predictions were seen to be only marginally more sensitive to this simplification, as their mean relative error increased by 0.1% when using the fitted material model. Regardless, the difference in relative error against experimental data for all resonant frequencies predictions using the original and simplified models was below 0.6%, which demonstrated a good agreement of results. Hence, the model was found to be robust to variations in the temperature-dependent material properties that might occur when a limited amount of material data is available to the analyst. The original material data provided by the manufacturer¹⁰⁷ was used in all analyses on Hastelloy X components presented in this thesis.

The second investigation aimed to identify which boundary conditions best replicate Berke et al.'s experimental setup and various permutations of fully-constrained nodes were analysed. The developed static model previously described in this chapter was used and a spatially uniform room temperature distribution was applied. Predictions using different patterns of nodal constraints were compared to the experimental results from Berke et al. All of the patterns were centred on the central node of the plate: $(x, y) = (0, 0)$. Figure 4.2 depicts the different types of nodal constraints applied to the flat plate. The hexagonal constraint aimed to replicate the nut attaching the plate to the stinger in the experimental setup.

The geometry used in the static FE analysis was then substituted by a flat plate with a 5 mm circular hole modelled at its centre. This aimed to better replicate the plate used in the DIC experiments performed by Berke et al. which included a hole through which a stinger was attached. Figure 4.3 illustrates the different fully constrained nodal patterns on the plate with a hole.

Comparing the resonant frequency results of each iteration against experimental data, the minimum relative error computed was found for the single, central constraint type, which was then used in the subsequent studies. Evidence of this study is available in Appendix C.

4.2.1 Results

Figure 4.4 shows three data sets pertaining to the first nine mode shapes of the 120 x 80 x 1.016 mm Hastelloy X plate at room temperature; no hole was modelled in the plate which was fully constrained about its single central node. Whilst Figure 4.4 a) and b) show the experimental and predicted results published by Berke et al., respectively, Figure 4.4 c) includes predictions from the developed static model with temperature-dependent material properties.

Qualitatively, Figure 4.4 shows that both the current static model and Berke et al.'s model depict the shapes of the plate's first nine natural modes accurately at room temperature. As experimental displacements heavily depend on the amplitude of the excitation signal, the analysis was focused around the overall shape of each mode (relative displacements) instead of the absolute magnitude of displacements. Similarly, high temperature results relative to the temperature distribution in Figure 4.1 are shown in Figure 4.5.

Resulting resonant frequencies were compared with the DIC experimental data tabulated by Berke et al. in ¹⁰, which was taken as reference, and relative errors calculated. This comparison was made at a uniform temperature of 22°C and at higher temperatures distributed according to Figure 4.1. The average relative error of both analyses can be found in Table 4.1. A more comprehensible comparison of resonant frequencies results is available in Figure 4.6.

Table 4.1 Mean relative error of Berke et al.'s¹⁰ FE model and the current static model against experimental data.

	Room temperature	High temperature
Berke et al. FE model	22.1%	19.8%
Static model	10.9%	12.8%

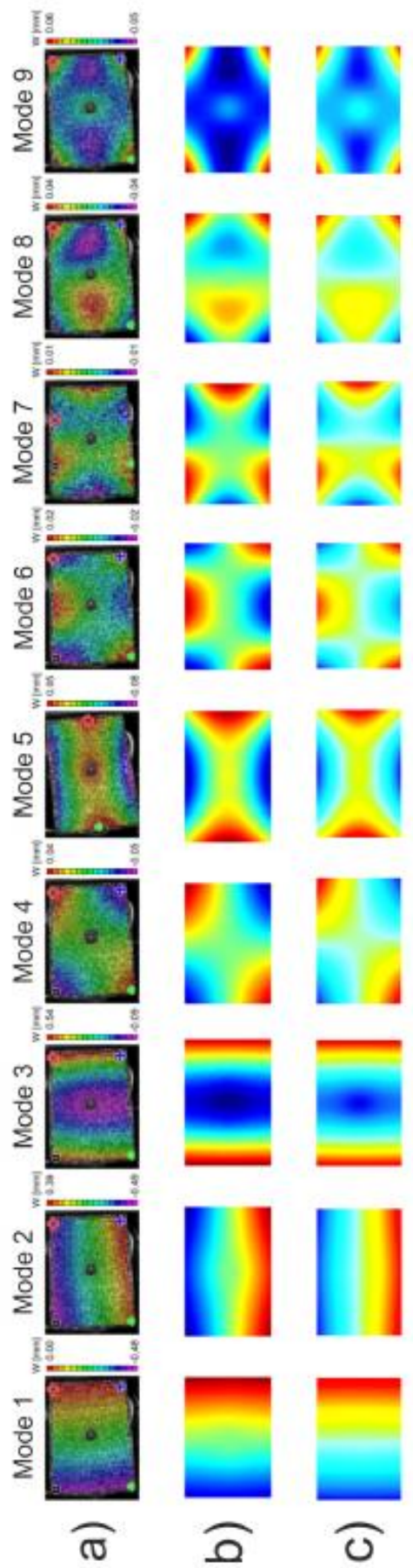


Figure 4.4 Mode shapes results of a 120 x 80 x 1.016 mm Hastelloy X plate at room temperature, which include a) experimental results adapted from Berke et al.¹⁰, b) predictions from the model developed by Berke et al.¹⁰, and c) predictions from the current temperature-dependent FE model with static loading.

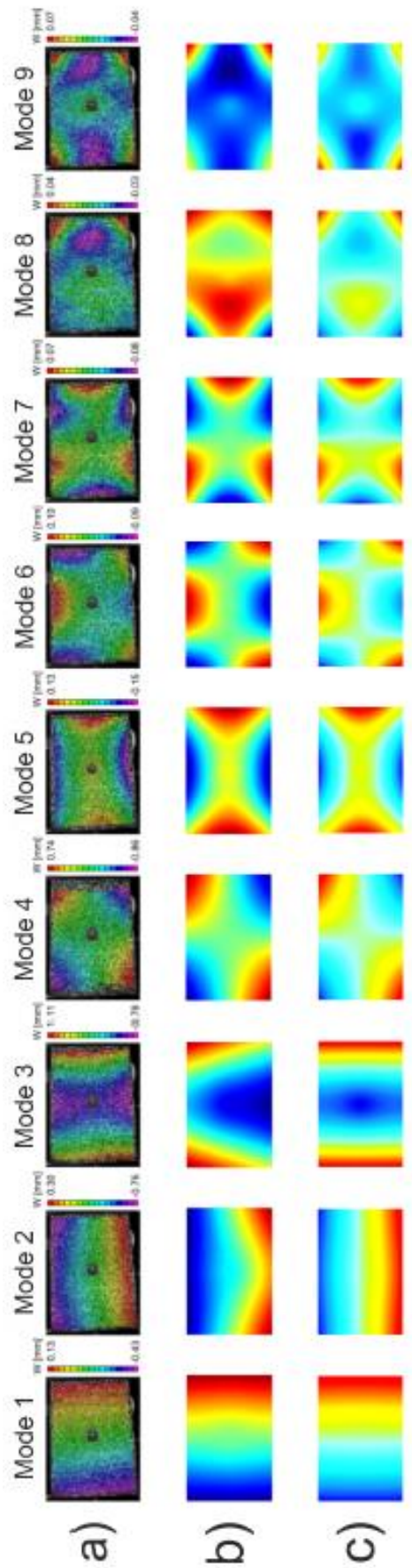


Figure 4.5 Mode shapes results of a 120 x 80 x 1.016 mm Hastelloy X plate at the temperature distribution in Figure 4.1, which include a) experimental results adapted from Berke et al. , b) predictions from the model developed by Berke et al.¹⁰, and c) predictions from the current temperature-dependent FE model with static loading.

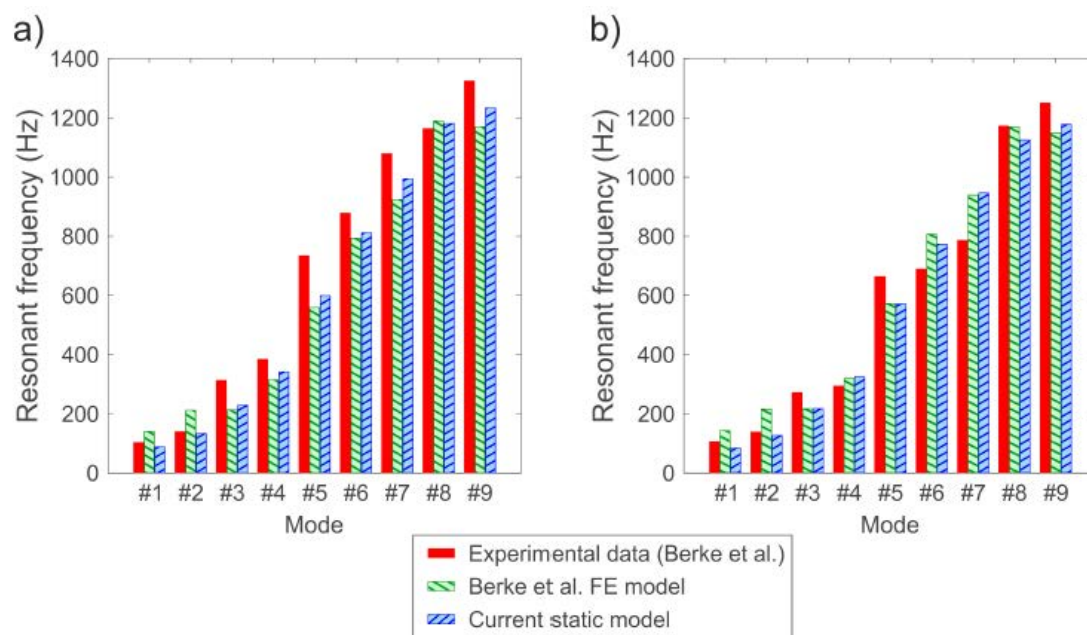


Figure 4.6 Resonant frequency results from Berke et al. ¹⁰ and predictions using the current static model at a) room temperature and b) high temperature.

4.3 Numerical methods for the effective representation of the thermal load

4.3.1 Establishing a benchmark

Investigations then focused on the role of thermal stresses in the predictions of results by using transient loading of the plate. Past work by Jeyaraj et al.³³ on the modal behaviour of thermally stressed plates was used for an initial comparison of results and establishing a performance benchmark. Their study began by determining the critical buckling temperature (T_{cr}) of an ideal at plate subjected to uniform thermal load followed by a modal analysis at different stages of the loading up to the buckling temperature. To build confidence in the ability to perform a buckling analysis using LS-DYNA, the present work started by setting up a mechanical buckling example using a rectangular plate with an aspect ratio of 1.6, with two length-wise fully clamped edges and two width-wise simply supported edges. The FE results were compared to theoretical data, published by Young and Budynas, and reproduced in Roark's compendium¹¹⁰, and validated in terms of critical load and buckling shape.

Studies progressed into thermal buckling by defining a transient change in temperature applied to a plate equivalent to the one described by Jeyaraj et al., i.e. a $0.5 \times 0.4 \times 0.01 \text{ m}^3$ rectangular steel plate with a density $\rho = 7850 \text{ kg/m}^3$ a constant Young's modulus $E = 2.1 \times 10^{11} \text{ N/m}^2$ and Poisson's ratio $\nu = 0.3$, using a $0.02 \times 0.02 \text{ m}^2$ quadrilateral mesh. Temperature was linearly increased from zero degrees Celsius to 175°C in a non-representative period of a second; this was due to the fact that Jeyaraj et al. did not provide any information pertaining to the heating rates used in their studies. By specifying the command `CONTROL_IMPLICIT_BUCKLE` in the pre-processor, it was requested of the solver to perform

an eigenvalue buckling analysis. This is a linear analysis method which predicts the theoretical buckling load of an elastic structure. Unfortunately, it often provides non-conservative estimates due to discretisation errors associated with the meshing of structures and the unavoidable material and structural nonlinearities in real-world components. However, this is the most commonly used method by engineers as it offers a quick and easy estimation of the temperatures at which buckling occurs and the corresponding buckled shape of the component, which suited the purpose of the present study. In an eigenvalue buckling problem, the FE solver reduces the equilibrium equation of the component to an eigenproblem as:

$$(K_m + \lambda K_g \{\varphi\}) = 0 \quad (4.2)$$

where K_m is the stiffness matrix of the base structure, K_g is the additional geometric stiffness due to the stresses caused by the loading, λ is the buckling load factor (BLF) and $\{\varphi\}$ is the buckled shape associated with the BLF. In LS-Dyna, the BLF is multiplied by the temperature load applied (T_{cr}) to obtain the magnitude of the critical buckling temperature, T_{cr} . LS-DYNA performs this analysis at the end of an implicit simulation, after a thermal load has been applied to the plate and thermal stresses have developed.

As the heating rate information was lacking in the literature, there was the need to investigate the robustness of the model when calculating the buckling temperature. To do so, the effect of the baseline temperature on predictions was studied by setting up two separate analyses using the fully-clamped boundary conditions applied to the steel plate. In both analyses the load was applied during a single second and two uniform temperature distributions were defined, the baseline temperature and the end temperature. The baseline temperature is the starting temperature of the plate, which was increased with each timestep (0.1seconds), thus creating a thermal gradient during the test. The end temperature of the thermal gradient was set to 100°C in both analysis. However, the baseline temperature was defined as 0°C in one of the models, and 30°C in the other. The results showed a difference of 0.2°C for the predicted buckling temperature, T_{cr} , which was considered negligible.

Jeyaraj et al. did not detail the type of assumptions with regards to the linearity of the solver when predicting the plate's response to thermal loading. However, as the temperature increases and the plate deforms, the component undergoes a highly non-linear process during the buckling phenomenon due to large deflections. It was, therefore, logical to define a non-linear analysis. The five different boundary conditions studied are shown in Figure 4.7 and were specified according to Ko's³⁷ definitions.

After determining T_{cr} for each boundary condition, the resonant frequencies of the plate were extracted at different intervals of the thermal loading using the eigenvalue method described in Chapter 3. The temperature was raised uniformly through the plate and in four equal increments from zero degrees Celsius to 99% of its critical buckling temperature and eigenvalues obtained at each increment. The

extraction of eigenvalues at the critical temperature was avoided because the non-linearities associated with the large deflections at the moment of buckling can lead to false solutions to the eigen-problems, which are intrinsically linear in nature. The studies were repeated with and without hourglass correction and no significant difference in results was found.

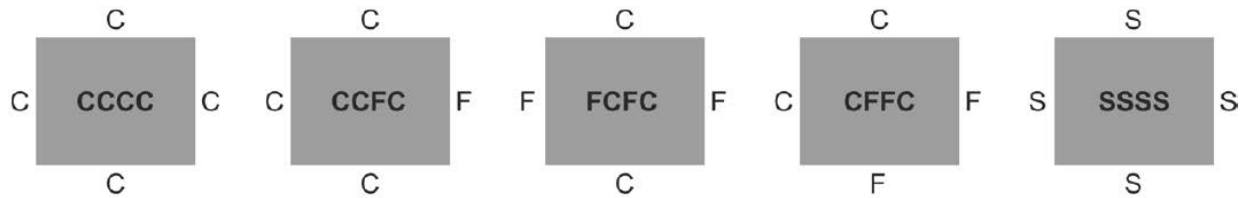


Figure 4.7 Boundary conditions as defined by Jeyaraj et al.³³: C – fully clamped; F - free; S - simply supported.

4.3.2 Knowledge transfer to the current problem

An equivalent model to the benchmark case was developed for the $120 \times 80 \times 1.016\text{mm}^3$ Hastelloy X plate used by Berke et al.¹⁰, assuming it to be perfectly flat. Both the geometry and temperature-dependent material model from section 4.2 were used in this analysis. The first step included determining the buckling temperature of the plate when exposed to a temperature distribution equivalent to the one used by Berke et al. in their model and Figure 4.1; this was achieved using the CONTROL_IMPLICIT_BUCKLE command in LS-DYNA and linearly increasing the temperature for 300 seconds, from a uniform 22°C to the temperature distribution in Figure 4.1.

Then, in a separate analysis, the resonant frequencies and modes shapes of the plate throughout the loading were investigated by performing an eigenvalue modal analysis. To do so, a linear transient load ranging from room temperature (22°C) to T_{cr} was defined by applying different heating rates to the six areas of the plate identified by Berke et al. until T_{cr} was reached after 300 seconds. Then, the moments of the loading at which eigenvalues were extracted had to be defined. As such, the temperature of the plate was increased in eight equal increments from room temperature to 80% of the overall temperature increase ($T_{cr} - 22^\circ\text{C}$) and the first 10 eigenvalues extracted at each increment. Four equal increments were then specified between 80% of ($T_{cr} - 22^\circ\text{C}$) and the critical buckling temperature. Smaller increments would have been possible but would hardly be reproducible in experimental work; larger increments also allowed for an easier identification of mode shapes between consecutive eigenvalue extraction points. According to the available literature on the subject (see Chapter 2), the immediate post-buckling period is of particular interest as past analytical and computational work have predicted this regime to include the "disappearance" of the first non-rigid body mode and the possible switching of frequencies between particular mode shapes (mode shifting). Hence, the analysis was extended past T_{cr} to 10% of T_{cr} . By analysing the first 10 eigenvalues, it was possible to assess if the model had predicted any higher-level mode shifting.

4.3.3 Results

The relative difference between results in Table 4.2 is below 1% which represents a sound agreement between the results. Appendix D includes the results acquired when studying the change in modal natural frequencies at different values of the thermal loading for the various boundary conditions. These compare outputs from the current model and the data from the literature. The relative difference between the datasets can also be found in Appendix D. The results from the transient model are in good agreement with literature (below 3% relative difference) except for the first mode at $0.99 \times T_{cr}$ temperature, which is likely tied to the highly non-linear nature of the buckling phenomenon, as previously discussed. This hypothesis was supported when repeating the analysis using a linear solver as the results showed the predictions for the resonant frequency of the first mode to be the most affected by this change. Data pertaining to this investigation is available in Appendix E.

Table 4.2 presents the critical buckling temperature data from Jeyaraj et al.³³ and the results from the benchmark model.

Boundary condition	Jeyaraj et al. [33]	Current transient model	Relative difference (%)
CCCC	139	139.2	0.0
CCFC	154	154.7	0.5
FCFC	153	153.5	0.3
CFFC	348	349.0	0.3
SSSS	51	51.4	0.8

After establishing this benchmark, an equivalent method was used to predict the results experimentally acquired by Berke et al.¹⁰. The temperatures at which buckling occurs and the corresponding buckled shape of the at Hastelloy X plate when exposed to the temperature distribution applied by Berke et al. were determined and are shown in Figure 4.8. The eigenvalue analysis yields the predicted buckled shape assumed by the structure when it buckles out-of-plane in a particular mode (Figure 4.8 c); however, numerical values are merely relative and do not represent the actual magnitude of displacements¹¹¹. This is because the buckling shape determined by the eigenvalue analysis is defined by an eigenvector which is, by definition, a normalised vector that can change by a scalar factor when a linear transformation is applied. Hence, the shape in Figure 4.8 c) has been normalised and is, therefore, dimensionless.

Figure 4.9 shows the change in the resonant frequencies of the plate at different instances of the loading when increasing the temperature from 22°C to a non-uniform temperature distribution with magnitudes 10% higher than the estimated values at buckling, shown in Figure 4.8 b). The transient FE model predicted a decrease in the resonant frequency for every one of the first ten modes of the plate, with the most evident change being the "disappearance" of modes 3 and 4 just before T_{cr} . An exception to this trend are modes 1 and 2, which represent rigid body modes and seem to be insignificantly

affected by the thermal load, apart from slight fluctuations near the buckling instant. The results in Figure 4.9 also evidenced mode shifting between modes 9 and 10, which seems to occur at about 97% of T_{cr} .

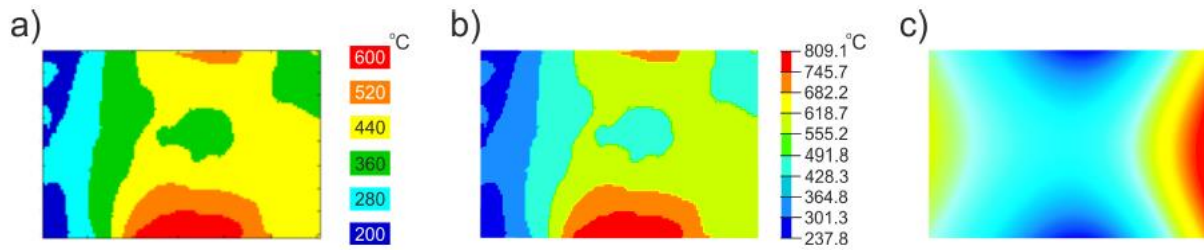


Figure 4.8 Temperature distribution equivalent to Berke et al. ¹⁰, computed using a custom MATLAB script, b) Temperature map at buckling point (T_{cr}) and c) the corresponding buckled shape, which has been normalised between 1 (red) and -1 (dark blue).

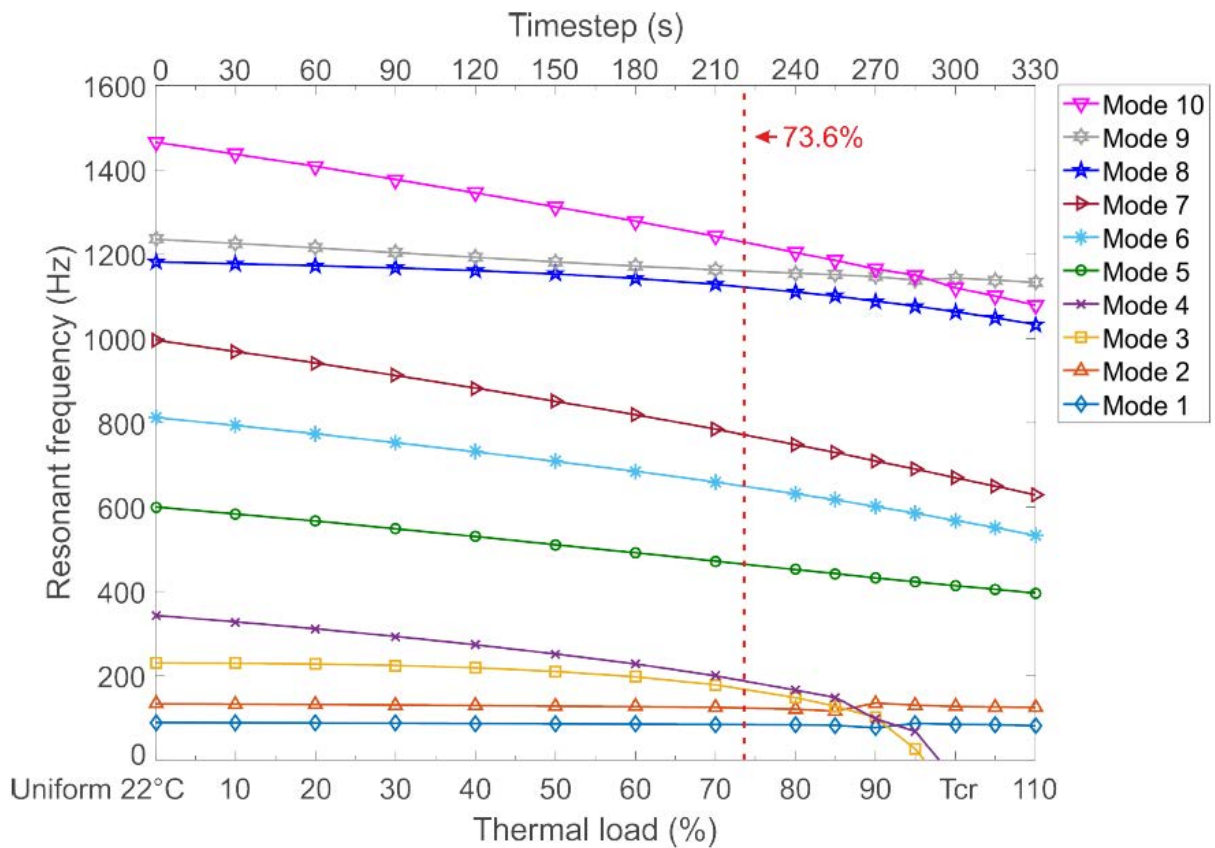


Figure 4.9 Predicted resonant frequencies with thermal load using the transient model in which a flat plate was heated up to 110% of T_{cr} , which is shown in Figure 4.8 b).

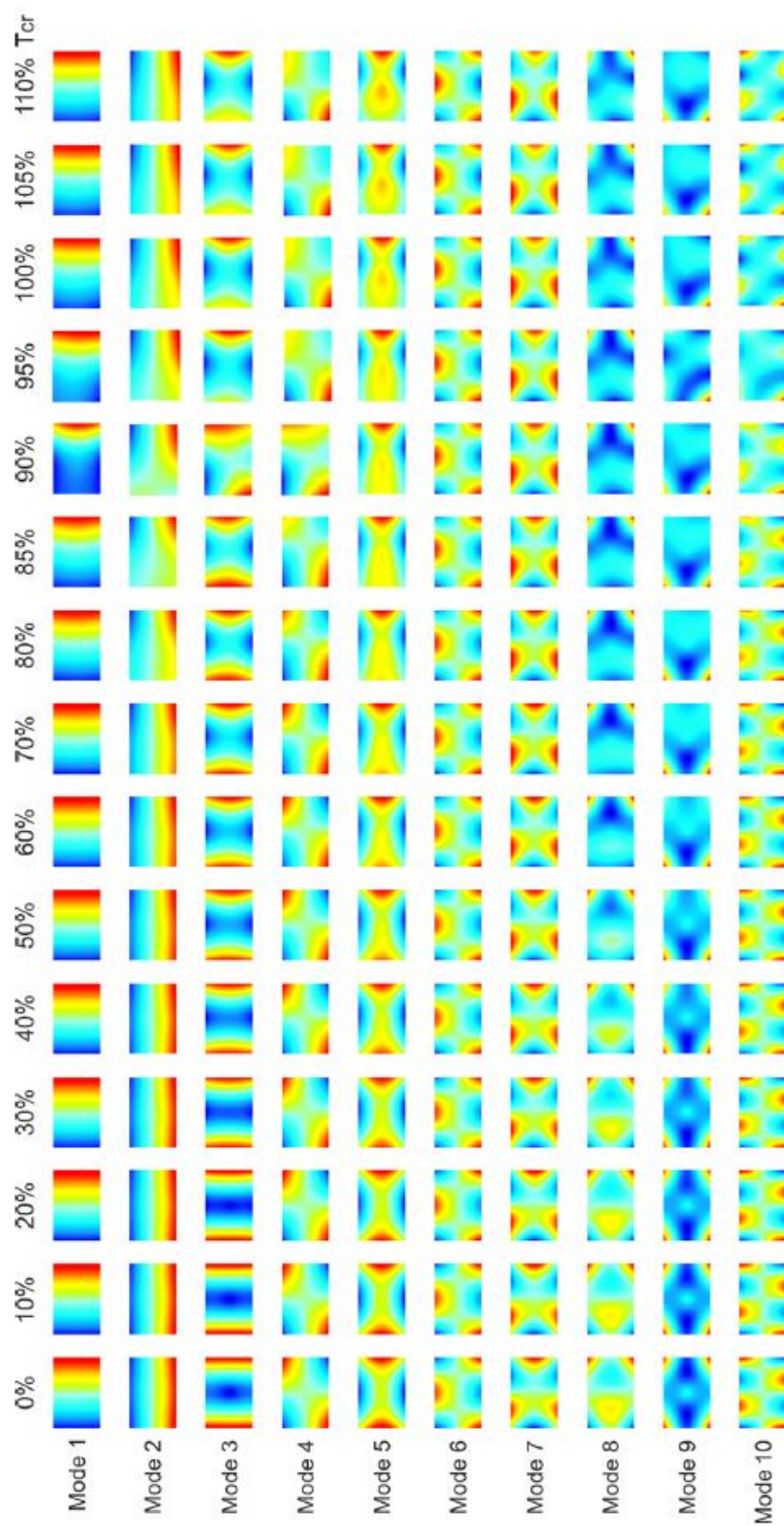


Figure 4.10 Predicted mode shapes at different thermal loads using the transient model in which a flat plate was heated from uniform room temperature to 110% of T_{cr} , which is shown in Figure 4.8 b).

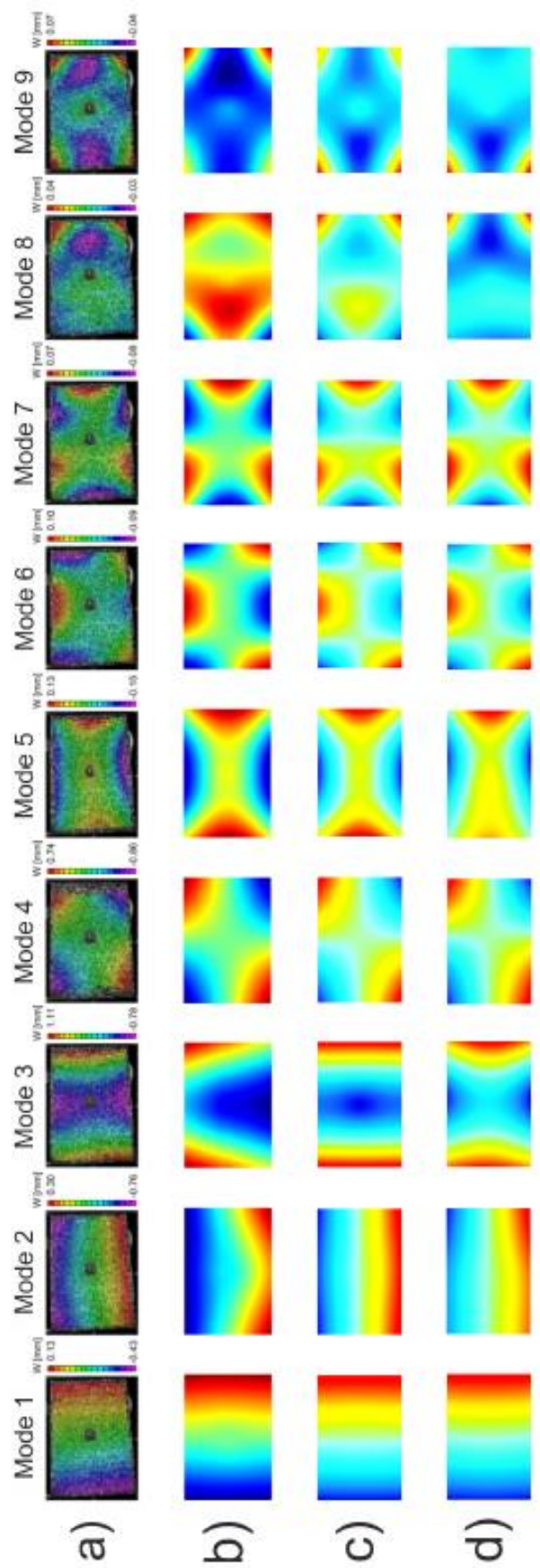


Figure 4.11 Mode shapes at high temperatures using the distribution in Figure 4.1: a) and b) show the experimental and FE results from Berke et al.¹⁰, respectively; c) shows predictions using the static model and d) the transient model.

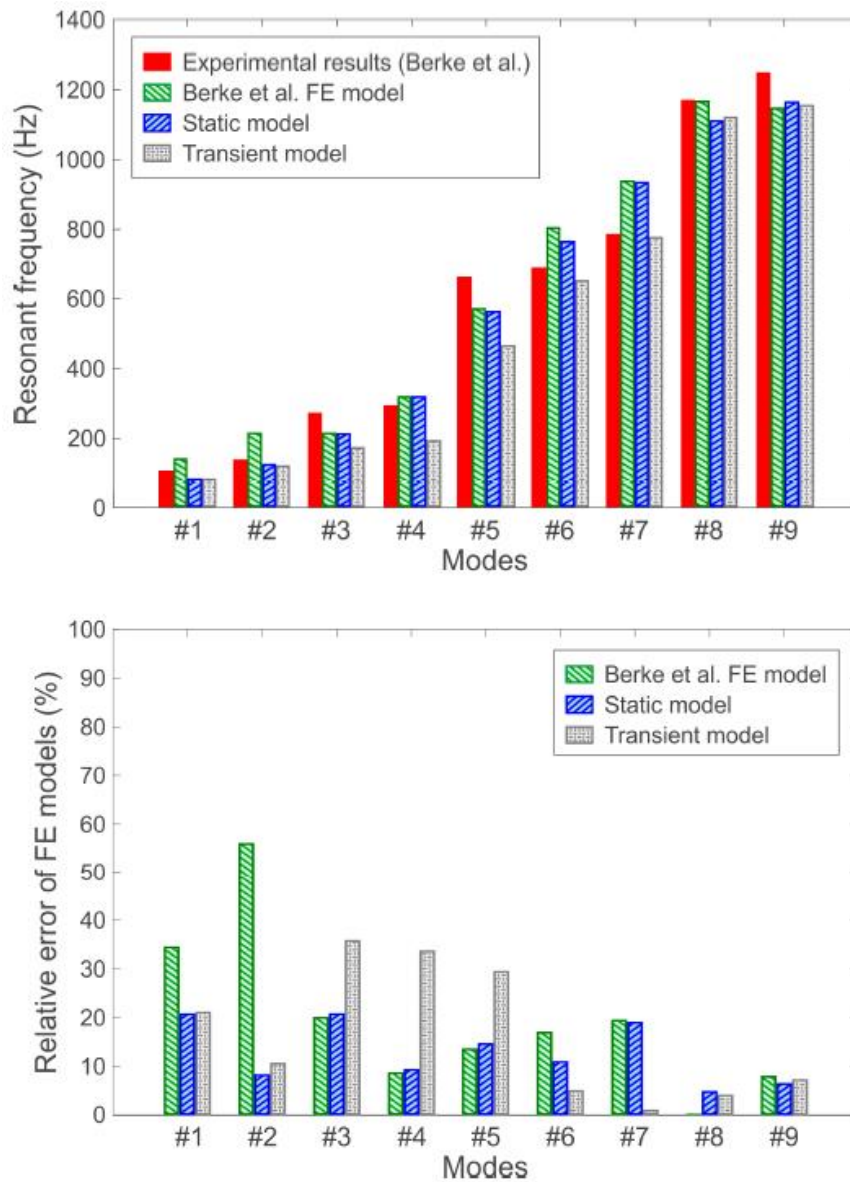


Figure 4.12 High-temperature, resonant frequency predictions from multiple models (top) and the relative error of each one against experimental data from Berke et al. ¹⁰ (bottom).

The corresponding mode shapes predicted by the transient model can be found in Figure 4.10. The results show the shapes of the majority of the modes to significantly change with the temperature increase. Despite the results in Figure 4.9 showing the "disappearance" of modes 3 and 4 near the occurrence of buckling, the FE eigenvalue analysis still recognises these modes as having negative frequencies and corresponding mode shapes, which have no physical meaning. Purely for continuity purposes in this thesis, the post-buckling modal shapes of modes 3 and 4 were included in Figure 4.10 but do not represent realistic modes of vibration under those conditions.

A particular point of interest occurs at approximately 73% of T_{cr} , when the temperature reaches the distribution calculated by Berke et al. and a direct comparison to past experimental results can be made. Figure 4.11 shows a qualitative improvement in the high-temperature mode shapes predicted using the transient model when compared to the previous static model and the FE analysis conducted by Berke et al., particularly for mode shapes numbers 3, 5 and 8.

Figure 4.12 shows the resonant frequency results for each model at the elevated temperatures in Figure 4.1 (top) and the relative error against the experimental results from Berke et al. (bottom). The transient model was found to have the highest relative error for modes 3, 4 and 5 (first three non-rigid body modes) and the most accurate predictions for modes 6, 7 and 8.

4.4 Analysis of non-ideal plates by inclusion of experimental shape measurements

Predictions by Murphy et al.^{44, 45} have showed the effect of thermal load on the resonant frequencies of an ideally at plate to be markedly different when geometric imperfections of the structure are included in the model. There are several inevitable factors that can prevent the use of an ideal scenario as a representation of a realistic component or condition, including: 1) material imperfections from inclusions and discontinuities to directionality of grain orientation; 2) geometric imperfections relative to the intended shape of the component; 3) inability to maintain or reproduce boundary conditions effectively throughout the analysis. A micro-scale investigation of material defects would be challenging in existing aircraft structures. Similarly, analysts have found the accurate modelling of boundary conditions in components to be difficult^{45, 47, 48}; for instance, the inclusion of insulation between a thermally loaded plate and its clamp was seen to impart an elasticity to the boundary condition that could not be easily replicated in a computational model⁴⁷. In the present work, the DIC software Istra 4D was used to estimate the contour map of the initial shape of a small Hastelloy X plate, similar in dimensions to the one tested by Berke et al.¹⁰. The software correlates an image pair acquired at room temperature and calculates the three-dimensional positions of each facet in the image. A flat plane is then fit onto the data using the method of least squares and the shape of the plate is presented as a distance relative to that plane. This provided a map of macroscale geometric imperfections in the structure that was used in the model. A custom MATLAB script converted specimen shape data exported from Istra 4D into x, y and z nodal values of a 1 x 1 mm mesh, which defined 9600 quadrilateral shell elements and is shown in Figure 4.13. This geometry was then imported into the pre-processor and an equivalent model to the one described in section 4.3.2 was defined. The plate was fully-constrained at its single central node.

This marked the introduction of macroscale geometric imperfections to the original shape of the plate in the FE model and it was found to generate warping of the structure when subjected to thermal loads. Thus, the inclusion of hourglass control in the analysis was required, as warped geometries are prone to degrade the FE solution when using shell elements. By applying hourglass correction type 8, LS-DYNA activates warping stiffness and correctly solves the coupled torsion-bending problem associated with the loading (the "Twisted Beam" problem)¹¹². No control factors for the warping stiffness were available and the LS-DYNA default settings for hourglass correction type 8 were used.

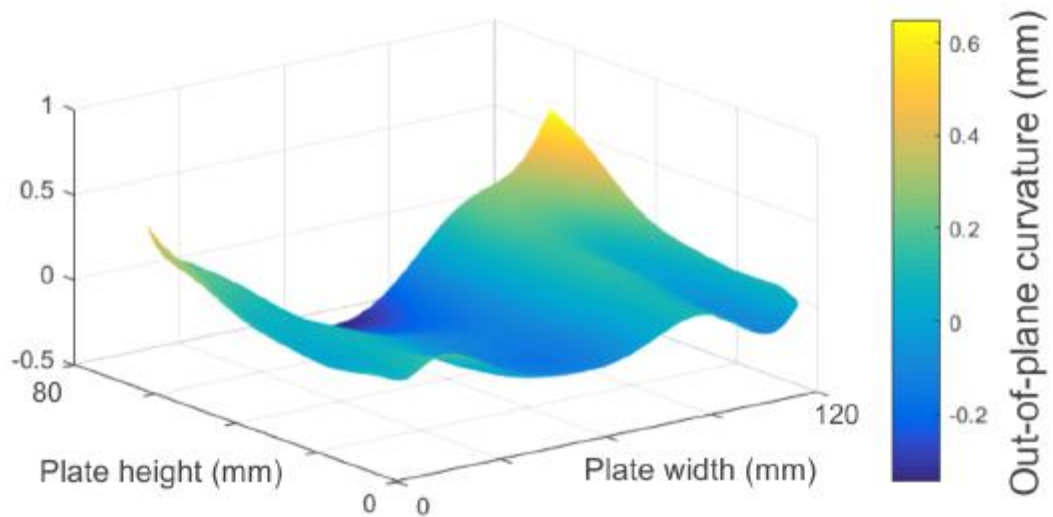


Figure 4.13 Macroscale, measured plate geometry used in the transient thermal loading from a uniform room temperature to the temperature distribution in Figure 4.8 b).

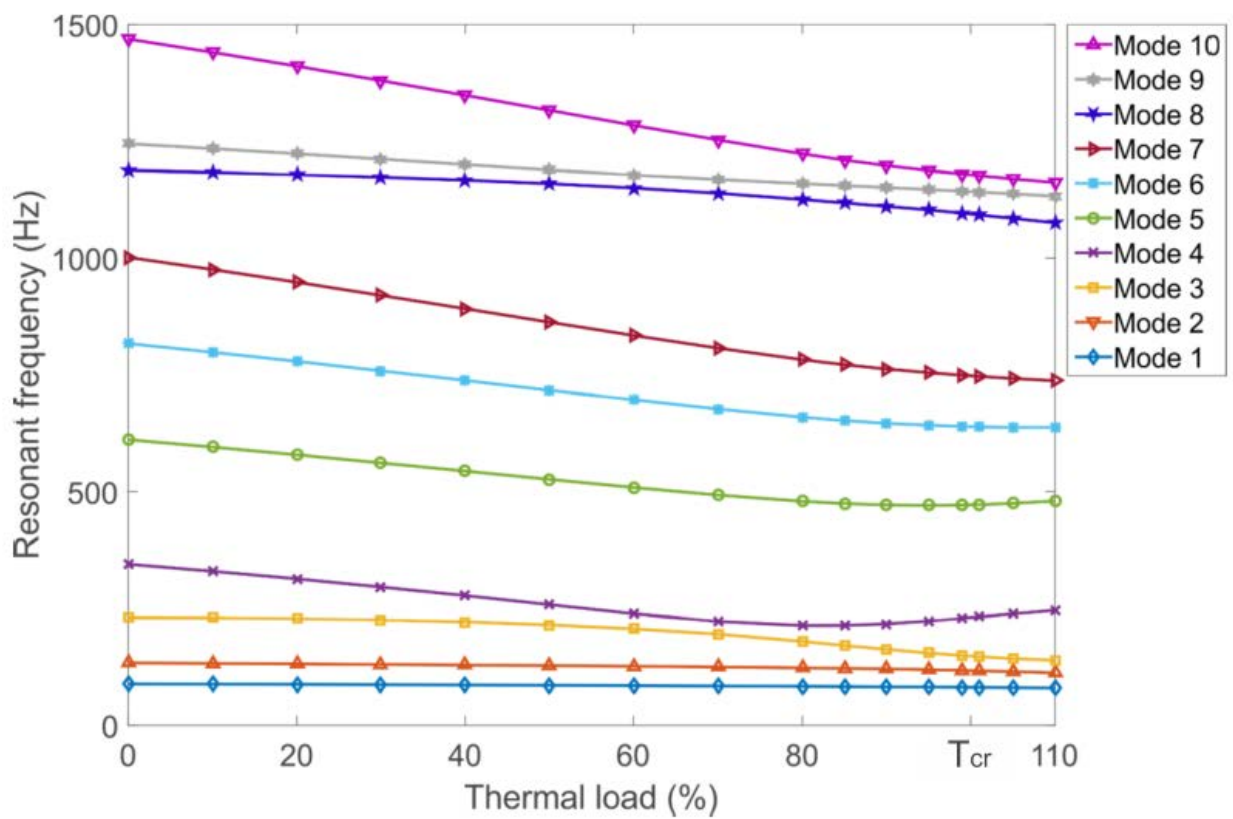


Figure 4.14 Predicted resonant frequencies with thermal load using the transient model in which an imperfect plate was heated up to T_{cr} , shown in Figure 4.8 b).

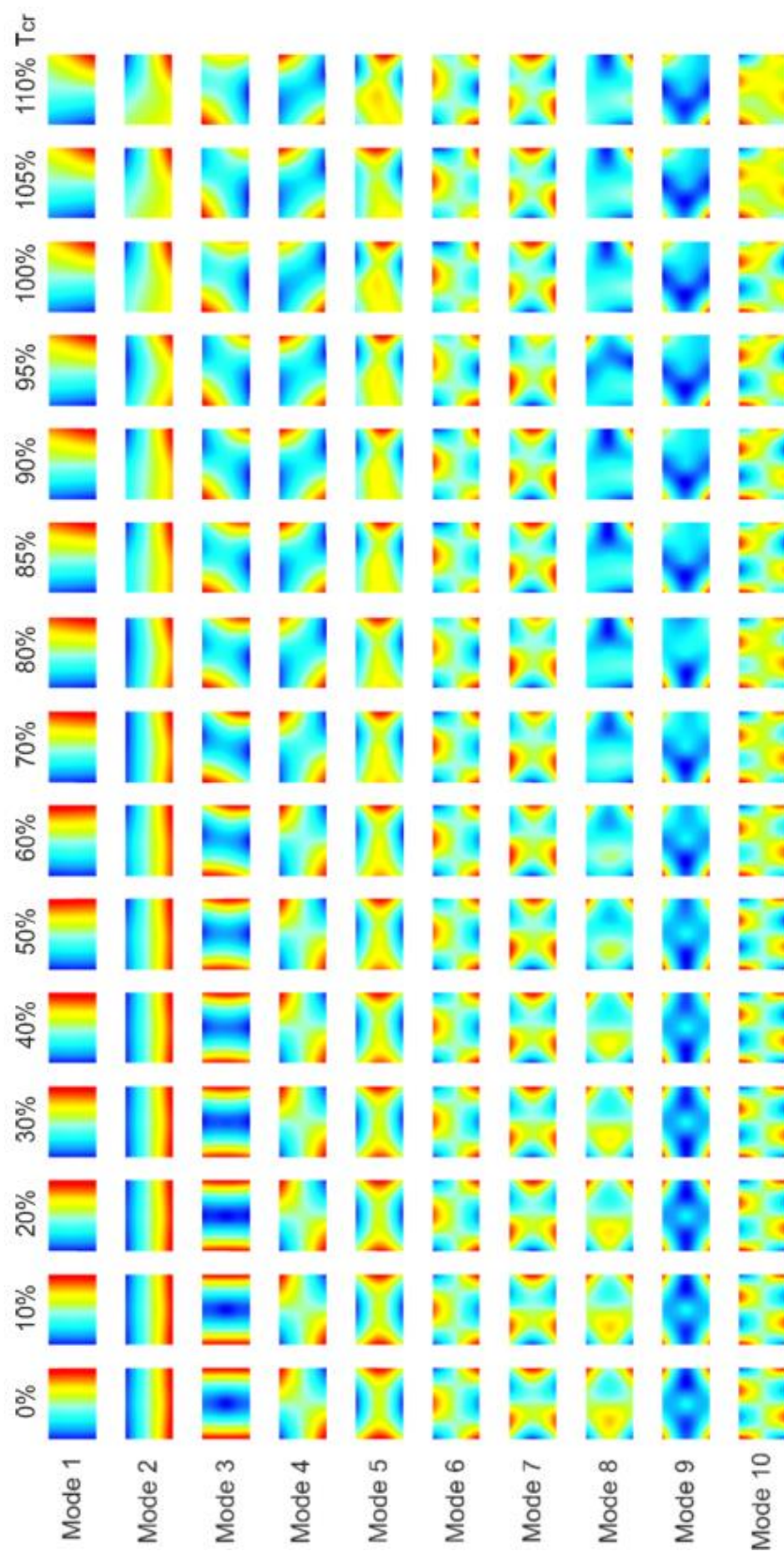


Figure 4.15 Predicted mode shapes with thermal load using the transient model in which a measured plate geometry was heated up to T_{cr} , shown in Figure 4.8 b).

4.4.1 Results

Figure 4.14 presents the resonant frequency results from the computational modal analysis of a 120 x 80 x 1.016 mm Hastelloy X plate with a measured geometry, using a temperature dependent material model and a transient thermal load. These predictions show the resonant frequencies of the plate to decrease with the increasing temperature. However, at approximately 80% of T_{cr} , the rate at which they did was seen to change significantly and, in the case of modes 4 and 5, their resonant frequencies began to steadily increase until 110% T_{cr} was reached. This agrees with predictions by Murphy et al.^{44, 45} as the model with measured geometry yielded results which contradicted studies using at plates and did not show the "disappearance" of modes at the buckling temperature. Interestingly, the mode shifting predicted when loading the flat plate was also absent in the analysis using a measured geometry. However, similarly to the results for the flat plate, the resonant frequencies of rigid body modes (modes 1 and 2) were shown to remain minimally affected. The predicted mode shapes corresponding to the frequencies plotted in Figure 4.14 can be found in Figure 4.15 and further support that the increasing thermal load affects the deformation patterns at which the plate resonates. This was evident for most investigated modes, apart from mode numbers 6 and 7, which present little to no change in mode shape.

4.5 Discussion

Table 4.3 provides a summary of the accuracy of the resonant frequency predictions made by the developed models with temperature-dependent material properties against the results published by Berke et al.¹⁰. For each one of the nine resonant frequencies acquired experimentally by Berke et al., the relative error between predictions (F_s) and measurements (F_E) was calculated as explained in section 4.2, and the mean relative error associated with a given model calculated.

Table 4.3 Mean relative error of the studied FE models against experimental data acquired by Berke et al.¹⁰.

FE model	Mean relative error (%)	
	Room temperature	High temperature
Berke et al.'s model	22.1	19.8
Static model	10.9	12.8
Transient model	10.9	16.6

Expectedly, no change was identified between the results from the static model and the transient model at room temperature. However, and more interestingly, Berke et al.'s model was found to provide the least accurate predictions for the room temperature resonant frequencies of the plate, despite including an added level of detail with a realistic initial shape of the specimen. This suggests the model developed by Berke et al. suffered from a seemingly inappropriate definition of the initial conditions of the

specimen, either in the characterisation of boundary conditions, or in the specification of the analysis and/or element type.

Table 4.3 also shows that, at elevated temperatures, both developed models represent an improvement upon Berke et al.'s results. However, the transient model was less accurate at predicting the resonant frequencies of the plate than the static model, which can be largely attributed to the relative errors associated with the FE results for modes 3, 4 and 5, which were the highest in magnitude compared to the other modes, as shown in Figure 4.12. To ascertain whether this difference was due to the use of a non-linear solver in the transient analysis or the inclusion of thermal stresses through the application of a temperature gradient, a non-linear static analysis was performed. This yielded near-identical results to the linear static model with a 13.0% mean relative error against Berke et al.'s DIC data at high temperatures and a maximum difference of 1 Hz between the predictions of the same mode by the two models. Hence, it was concluded that the inclusion of transient loading was responsible for the difference in the high-temperature results presented in Table 4.3, as static models neglect the stressed state of the structure at the time of eigenvalue extraction.

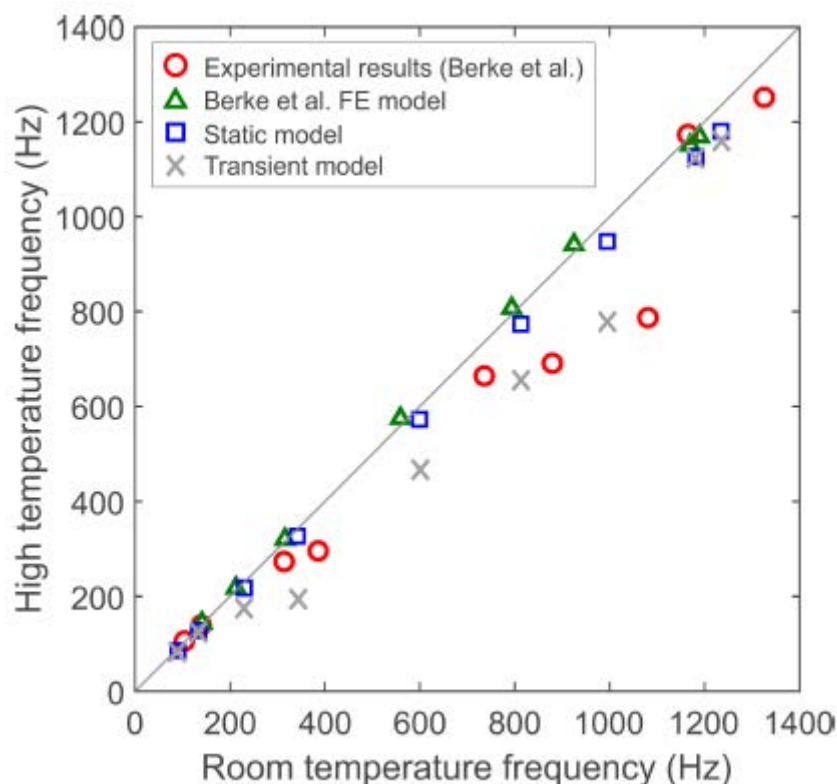


Figure 4.16 High-temperature resonant frequencies as a function of room-temperature results using the developed models and the results from Berke et al. ¹⁰.

Figure 4.16 provides a plot of high-temperature resonant frequencies as a function of room-temperature results for all the FE models and the experimental data from Berke et al. The latter shows the plate's resonant frequencies to be affected non-linearly by the thermal load. This was not seen in the results of Berke et al.'s model, as they predicted no significant changes in frequency with

temperature. The developed static model yielded predictions that showed an approximately linear decrease of 5% in resonant frequencies with temperature for all modes. The transient model was better at predicting the non-linear changes in frequency with elevated temperatures, despite the increase in mean relative error when compared to the static model. A more in-depth comparison between the accuracy of the static and the transient models could have been performed by knowing the uncertainty in the experimental frequency data; however, such information was not provided in the article by Berke et al.

Results for mode shapes from the transient model when loading the plate from room temperature to just above the critical buckling temperature (T_{cr}) are presented in Figure 4.10. Curiously, model predictions at 50-60% T_{cr} are in qualitatively better agreement with Berke et al.'s experimentally-acquired mode shapes than the results at 70% of T_{cr} , which approximately corresponds to the temperatures estimated by the authors in their work. This is particularly clear for mode numbers 2, 3 and 9 shown in Figure 4.10. However, temperatures at 50-60% T_{cr} are lower than the temperatures computed by Berke et al. at the start of the experiment and assumed constant throughout; hence, they do not correspond to the temperature distribution in Figure 4.1. Whilst this suggests conditions may have varied during their experiments, it is impossible to confirm this because the authors did not continuously monitor temperature. Another possibility for the discrepancy between predictions and experimental results, which has not been explored in the present work, is the over-discretisation of the temperature distribution. The 80°C step change in temperature between the discrete levels in Figure 4.1 equates to an approximate 2% difference in the modulus of elasticity of adjacent FE elements of the Hastelloy X plate. This may cause the model to estimate the development of unrealistic thermal stresses across the specimen, which would have otherwise been avoided by appropriately representing the component's in-plane temperature gradient.

The resonant frequency results from the transient model using a flat plate presented in Figure 4.9 show the frequencies of modes 3 and 4 to decrease to 0 Hz and "disappear" between 95 and 100% of T_{cr} . This confirms predictions by Bailey²⁷ and Jeyaraj et al.³³ of the "disappearance" of the first non-rigid body mode when the thermal load applied to a flat plate approaches the critical buckling temperature (T_{cr}). According to Lurie's²⁵ analytical work on simply supported plates, this may be because the first vibration mode (non-rigid body mode) and the thermally buckled shape are identical for flat plates. The fact modes 3 and 4 present very similar resonant frequencies at 90% of T_{cr} , shortly before "disappearing", may be an indication of bifurcation buckling in which there are two possible stable buckling configurations of the stressed plate for the same magnitude of load applied.

However, by repeating the non-linear transient analysis using a measured geometry, results in Figure 4.15 confirmed findings by Murphy et al.^{44, 45} and showed that the "disappearance" of resonant modes near T_{cr} may not occur for a plate with a measured curvature. This is likely because the measured

geometry is an inherently unstable structure due to its macroscale geometric non-linearities and deflects gradually with the increase in temperature. On the other hand, a flat plate is a stable structure which remains flat until T_{cr} is reached and the plate buckles into a shape equal to the vibration mode that requires the lowest excitation energy and excludes modes that describe only the translation or rotation of the structure (rigid body motion).

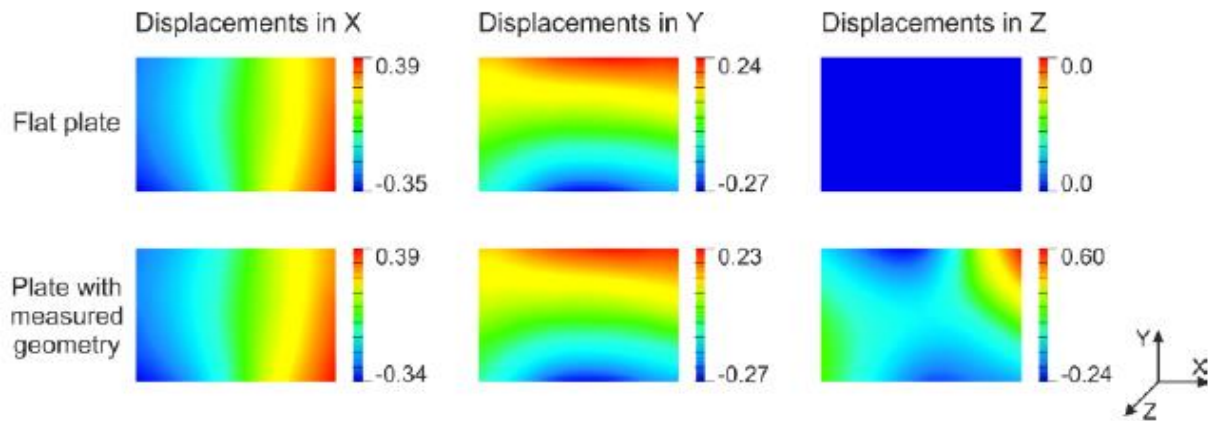


Figure 4.17 Predictions of in-plane and out-of-plane deformation of an ideally flat plate (top) and plate with measured geometry (bottom) using the transient model to load the structures up to Berke et al.'s¹⁰ temperature distribution in Figure 4.8.

A further investigation of the results obtained using the transient model using a flat plate has revealed the model to predict in-plane deformation caused by the increase in temperature; the unconstrained edges allowed the material to freely expand in the X and Y directions, as shown in Figure 4.17 (top). However, the model did not predict the out-of-plane deformation of the plate, which was experimentally verified by Berke et al. as an increase of its curvature induced by the thermal load (approximately 5.3 times larger than at room temperature). This implies that, whilst the transient model using a flat plate accounted for the changes in material stiffness at high temperatures, the changes in geometric stiffness were not appropriately represented when computing the resonant frequencies and mode shapes. Conversely, when using a plate with the measured geometry in Figure 4.13, the transient model was able to predict a temperature-induced change in curvature of the plate, as shown in Figure 4.17 (bottom). The inclusion of a measured geometry acquired at room temperature introduced macroscale geometric nonlinearities to the component and added local constraints to the thermal expansion of the plate that provided the conditions to initiate the out-of-plane deformation of the structure. It is worth noting, however, these predictions were small when compared to the curvature measured by Berke et al. at high temperatures (over 2.5 times smaller). Once again, these findings agree with those of Murphy et al.⁴⁴ who concluded that a flat plate remains flat as the applied temperature increases until it suddenly buckles at a critical temperature (T_{cr}); however, a plate with a curvature experiences gradual deflection with the increase in temperature until a new equilibrium state is reached. The discrepancy between the experimental deformation data at high temperature and the predicted deformed shape of the plate when using the transient model is likely due to the initial plate

shape used in the model, which did not directly correspond to the curvature of Berke et al.'s specimen at room temperature and was purely representative of a plate with a curvature. This hypothesis is supported by Thornton's⁵² work, in which the initial shape of thin plates was found to strongly influence the way the structure deforms out-of-plane. Due to the undeniable difference in the plate shape at both room and high temperature between the developed model and Berke et al.'s experiments, no conclusive comparison between the quality of predictions using a plate with a measured geometry and Berke et al.'s experimental results was prudent. However, results from the current transient model using a measured geometry, even if inaccurate, were encouraging: Berke et al.'s high-temperature distribution corresponds to 64% of T_{cr} in Figure 4.14 and yielded a mean relative error against their experimental data of 10.6% and 13.8% for room temperature and high temperature resonant frequencies, respectively. This represents an improvement on their FE predictions.

4.6 Conclusions

In this Chapter, a number of FE analyses were developed using a temperature-dependent material model and their results compared to experimental measurements published by Berke et al.¹⁰. This exercise aimed to develop a computational model with temperature-dependent material properties that could accurately predict the resonant frequencies and mode shapes of a thin plate subjected to elevated temperatures.

The lack of direct access to Berke et al.'s raw experimental data was unfortunate, as it created a few limitations to the breath of the work undertaken. If available, data describing the initial shape of the specimen at room temperature could have been used in the definition of the measured geometry of the model in section 4.4. Most significantly, however, access to the mode shape data could have been used to quantify the quality of mode shape predictions. Whilst Berke et al. have provided images of each experimentally-acquired mode shape, which could be quantitatively compared to predictions using orthogonal decomposition (see section 2.4), these images included coloured markers on the areas of largest incursion of the mode shapes. This would interfere with the decomposition process and their removal would require the extrapolation of large areas of the displacement maps, negatively impacting on the representation of the actual mode shapes. Regardless, important conclusions were drawn from the study and used as best practices in the development of the FE model presented in Chapter 5 which is validated against newly-acquired experimental data pertaining to the resonant frequencies and mode shapes of a rectangular plate with free edges. These conclusion were:

- Results showed that the transient loading of the plate is integral to the representation of the non-linear effect of temperature on the resonant frequencies of the structure, which was experimentally observed. This requires the appropriate monitoring of the temperature across the specimen when acquiring experimental data.

- The inclusion of a measured geometry of the plate was found to be critical when calculating the out-of-plane deformation of the structure induced by the thermal load. As the resonant modes of a plate depend on both its material and geometric stiffness, the curvature of the component must be appropriately depicted so as to improve predictions.

The inclusion of temperature-dependent material properties in the LS-DYNA model was found to be a simple and robust way to acquire high-temperature predictions of resonant frequencies and mode shapes of a plate with unconstrained edges. Results from the developed static and transient models were a significant improvement on predictions from Berke et al., both at room temperature and high temperature. However, it was not possible to conclusively establish the reason behind this improvement due to the lack of experimental raw data to be used in the definition of the model and in the analysis of results; namely usable raw data depicting the initial geometry of the plate, temperature distribution and mode shapes.

Chapter 5 High temperature modal analysis of a non-uniformly heated rectangular plate

5.1 Introduction

A limited amount of experimental research has been performed on the modal response of thermally stressed plates; which results in a gap in knowledge pertaining to the effect of temperature on their resonant frequencies and the out-of-plane displacement maps representative of a pattern of vibration at those frequencies (mode shapes). Resonance is defined as a large-amplitude vibration caused by a small periodic stimulus having the same, or nearly the same, period as the system's natural vibration¹¹³. Consequently, if a low amplitude energy source causes a structure or machine to oscillate at its natural frequency, the large amplitudes of this periodic motion impart elevated strains that often lead to the fatigue of components or even their catastrophic failure. An improved understanding of the issue is particularly important in the design and monitoring of structures in demanding environments such as those associated with hypersonic flight, where fluctuating air pressure and velocity gradients promote vibro-acoustic excitation whilst inducing aerodynamic heating. Similarly, fusion energy reactors also experience thermo-mechanical loading as panels which operate at high temperatures are affected by turbulence promoted by plasma circulation in the divertor.

The present chapter aims to address the first and second knowledge gaps identified in section 2.5 by pursuing three objectives: 1) to concurrently acquire high-quality, full-field displacement and temperature data whilst exciting a thermally-loaded thin plate to its natural frequencies; 2) to study the effect of different non-linear temperature distributions on the mode shapes and resonant frequencies of the structure; 3) to develop a finite element model capable of predicting the resonant frequencies and associated mode shapes of the plate using different temperature distributions applied to the structure.

This chapter is based on the following paper, written published in the Journal of Sound and Vibration:

A. C. Santos Silva, C. M. Sebastian, J. Lambros, E. A. Patterson, "High temperature modal analysis of a non-uniformly heated rectangular plate: Experiments and simulations", Journal of Sound and Vibration (2019) 443, pp.397-410.

5.2 Test specimen

Experiments were conducted on a 219 x 146 x 1 mm plate, made from Hastelloy-X (sheet form) [American Special Metals Inc., Miami, FL, USA], which is a nickel-chromium-iron-molybdenum super

alloy, which offers a high resistance to corrosion and is widely used in high temperature industrial applications. This is a refractory material that exhibits good ductility after prolonged exposure to high temperatures (650, 760 and 870°C for 16,000 hours), which makes it ideal for applications such as combustion zone components in gas turbines, industrial furnaces and structures used in chemical processing and nuclear engineering. Due to its good performance under elevated temperatures, Hastelloy X has been widely used in aircraft components such as jet engine combustion chambers, cabin heaters, exhaust and afterburner components. More details on the mechanical properties of Hastelloy X are presented later in this report.

A 5 mm hole was drilled in the centre of the Hastelloy X plate for the insertion of a stinger (300 mm M4 stainless steel threaded rod) to attach the specimen to a mechanical shaker via a bolted connection. No other constraints or supports were used to restrain the plate, as the work documented in the existing literature emphasised the increased difficulty in replicating the boundary conditions of a clamped plate in computational mechanics models. Also, predictions by Jeyaraj et al.³³ suggest the mode shapes of a rectangular plate change with thermal load when at least one edge is free, which was considered an interesting challenge to predict with the developed computational model. The option of an off-centre mounting hole was considered but preliminary room-temperature results showed this configuration required added support to be provided to the plate due to the asymmetric distribution of its weight on either side of the bolted connection. Similarly, room-temperature tests were also performed using larger plates, but the added weight was found to cause the stinger to bend, which would have influenced the experimental results. As both alternative geometries of the specimen required additional supports that would further increase the complexity of the boundary conditions to be replicated computationally, they were excluded from subsequent investigations.

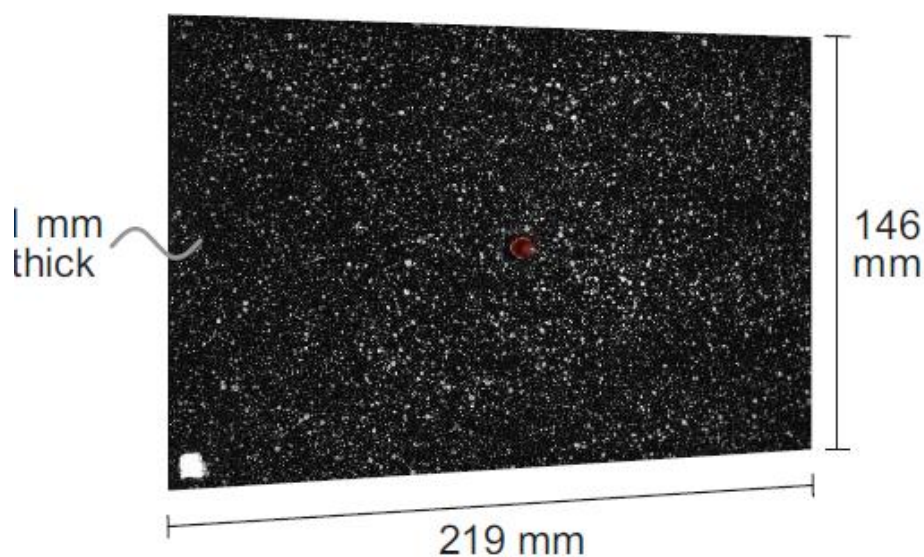


Figure 5.1 Photograph of the plate showing the painted speckle pattern used for digital image correlation (DIC). Image from the left camera of the stereo-vision system shown.

The plate was prepared for high temperature DIC by applying a high-contrast speckle pattern using the methods described in section 3.2.3. An image of the plate and resulting speckle pattern is shown in Figure 5.1. All experiments were conducted with the longitudinal axis of the plate in the horizontal orientation.

5.3 Experimental methods

The experimental work presented in this thesis was performed using a test rig originally built by Sebastian¹¹ and then further developed to include temperature monitoring and the remote control of equipment for safer operation. The test rig was relocated from a laboratory space occupied by the analyst to a laser cabin that allowed experiments to be conducted remotely from the outside of the enclosure (Figure 3.4).

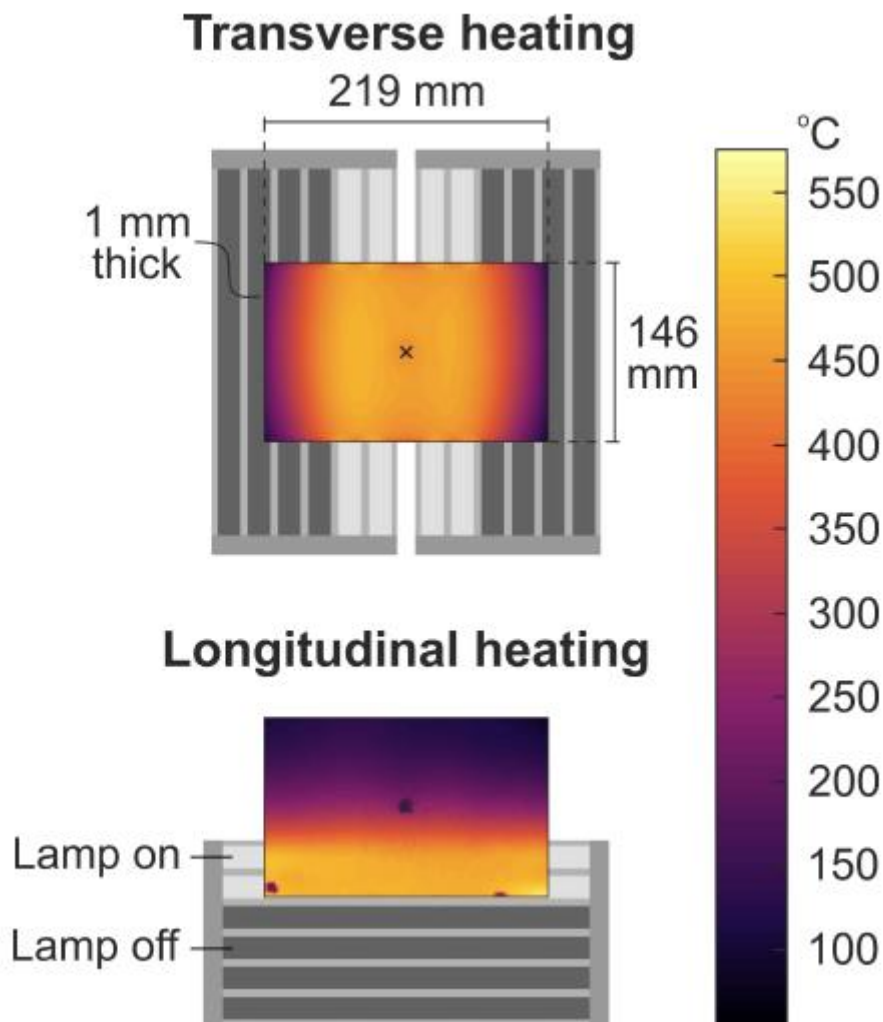


Figure 5.2 Configuration of lamps in grey illustrating transverse heating with four lamps in light grey (top) and longitudinal heating with two lamps in light grey (bottom) together with the resultant measured temperature distributions for the plate shown as overlays. Note that the anomalies in the temperature distributions correspond to zones of increased reflectivity where the vibrometer was focused.

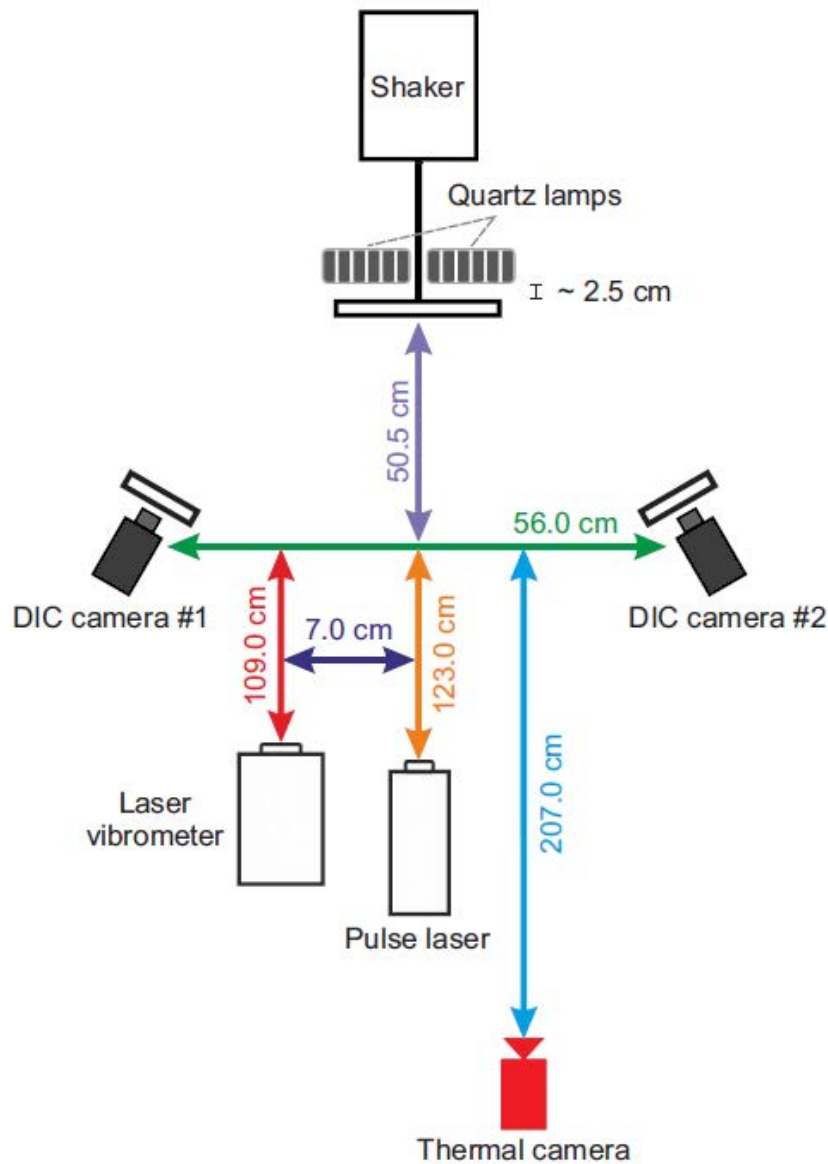


Figure 5.3 Schematic diagram of test setup used in the thermo-vibratory

The plate was thermally loaded using two different lamp setups, shown schematically in Figure 5.2, to create different temperature distributions on the specimen: 1) transverse heating using four vertically-oriented lamps centred around the plate's bolted constraint; and 2) longitudinal heating using two horizontally-oriented lamps aligned with the bottom edge of the plate. For reference purposes, a uniform room temperature (25°C) distribution was also studied.

Two types of vibratory loading were applied at each temperature distribution: broadband loading and single-frequency sinusoidal loading. These experiments were conducted after 130 seconds of heating, at which point the plate had reached a steady state temperature distribution. Figure 5.3 shows a schematic of the test setup used in both the broadband and modal tests.

5.3.1 Broadband loading

Mechanical excitation of the specimen was achieved by a commercially-available shaker (V100, DataPhysics, San Jose, CA, USA) and 1kW power amplifier system (DSA1-1K, DataPhysics) which was capable of imparting a random force of 533 N (RMS) and a maximum cyclic force of 1000 N. A random signal between 0 and 800 Hz from a vibration controller (ABACUS, DataPhysics) was used to drive the shaker. The frequency range is typical of those experienced by thin, lightly-damped structures in aircraft that are susceptible to high-cycle or sonic fatigue⁷.

In order to determine the resonant frequencies of the specimen, a Fast Fourier Transform (FFT) was computed while the plate was subjected to broadband excitation. This provided the response's transfer function in which the peaks correspond to the resonant frequencies. The input, or reference signal for the response function, was supplied by an accelerometer attached to the shaker's membrane; this location was preferable to the plate itself where the accelerometer would have influenced the behaviour of the plate and the measured signal was more representative of the input to the plate than the output from the vibration controller. The output signal used in the calculations of the system's transfer function was measured using a commercially-available laser Doppler vibrometer (OFV-503, Polytec GmbH, Waldbronn, Germany) focused on a corner of the plate. The vibrometer uses the Doppler effect from a low-powered laser beam to compute the point-wise velocity at the surface of the specimen; an integration card is then used to calculate displacement. Once again, the use of a noncontact method for the measurement of plate's response aims to minimise the influence of instrumentation on the behaviour of the specimen. Optimisation of the vibrometer's signal strength was achieved by aiming the laser at a reflective section on the surface of the plate. At room temperature, adequate surface reflectivity was achieved using a piece of retroreflective tape, whilst for high temperature tests a section of approximately 7.5 x 7.5 mm on the bottom left corner of the plate was polished with wet P800 sanding paper. The input and output signals were supplied to signal analyser software (SignalCalc, Data Physics) which computed the transfer function associated with the reference signal and the output signal. The form of some mode shapes made it so the bottom left corner had near zero displacement when at resonance, which made it impossible to identify their resonant frequencies; such cases required the laser vibrometer to be aimed at the middle left edge of the plate during the broadband loading.

Six repetitions were performed to determine the mean resonant frequencies of the first eleven modes. This was then repeated for each of the three temperature distributions analysed: room temperature, transverse heating, and longitudinal heating. A detailed diagram of the experimental setup used in broadband tests is shown in Figure 5.4. The plate was kept at high-temperature steady state for no longer than 10 minutes to avoid material aging, which occurs when heating Hastelloy X to temperatures of approximately 760°C for around 3 hours¹¹⁴. Similarly, temperatures were kept below the recrystallisation or critical temperature of the material (1163 - 1191°C) to avoid annealing or solution treating¹¹⁴. After each experiment, the specimen was allowed to air cool to room temperature.

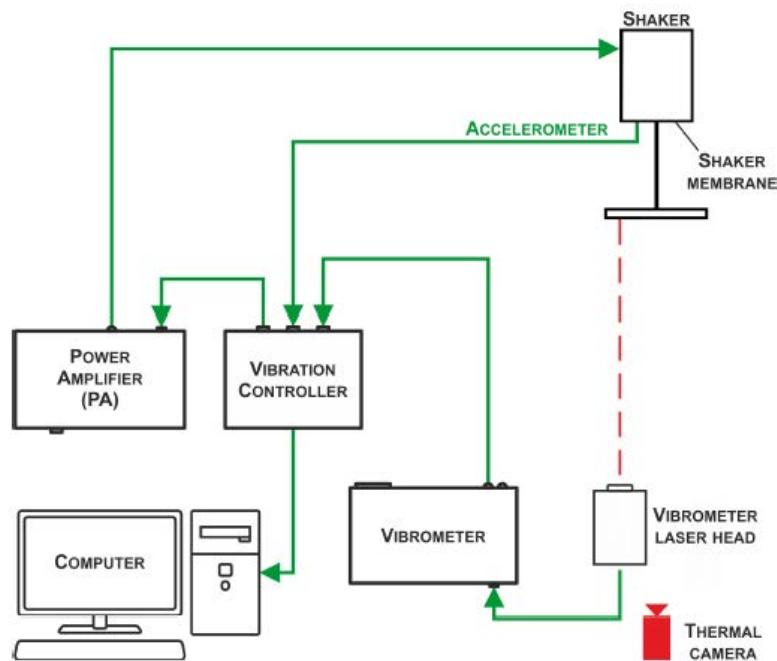


Figure 5.4 Schematic of the experimental setup used in broadband loading.

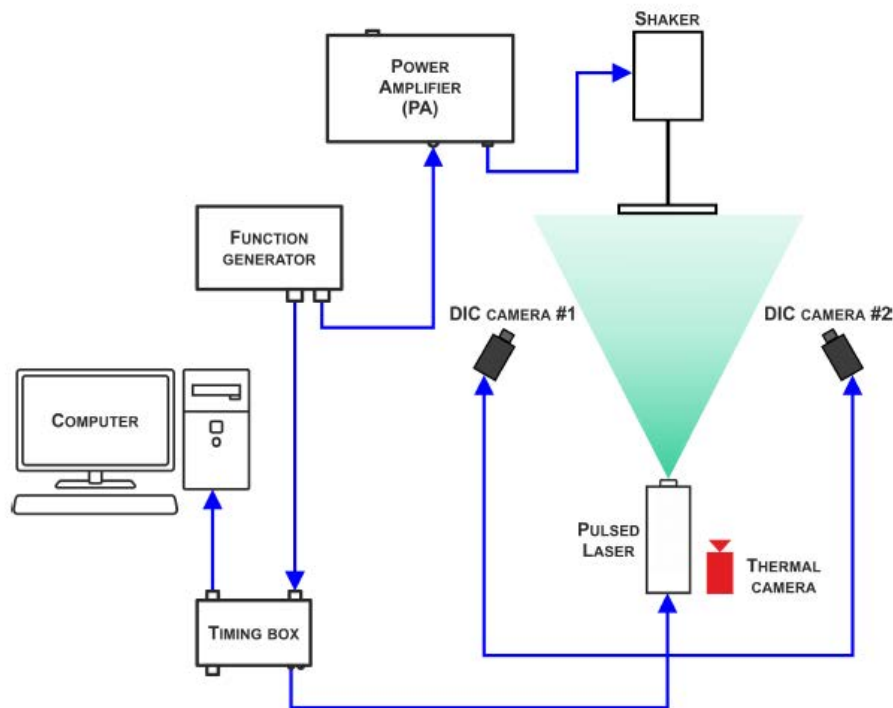


Figure 5.5 Schematic of the experimental setup used in single-frequency sinusoidal

5.3.2 Single-frequency sinusoidal loading

A function generator was used to control the V100 DataPhysics shaker and to create a sine waveform to excite the specimen at each of its resonant frequencies, which had been determined using broadband excitation. The amplitude of excitation was specified to approximately match the values recorded by the Doppler vibrometer during broadband loading in order to satisfy small amplitude assumptions required for linear modal analyses. The full-field out-of-plane displacement of the plate was measured using the

stereoscopic DIC system (described in detail in Chapter 3) fitted with a set of 12 mm lenses adjusted to an f-stop of 2.8. This setup resulted in an average spatial resolution of 0.18 mm/pixel. Image capture and displacement calculation was performed using Istra 4D software supplied with the DIC system. Image correlation was performed with facets of 25 x 25 pixels with a centre-to-centre spacing of 21 pixels. The experimental setup used in single-frequency sinusoidal loading is detailed in Figure 5.5.

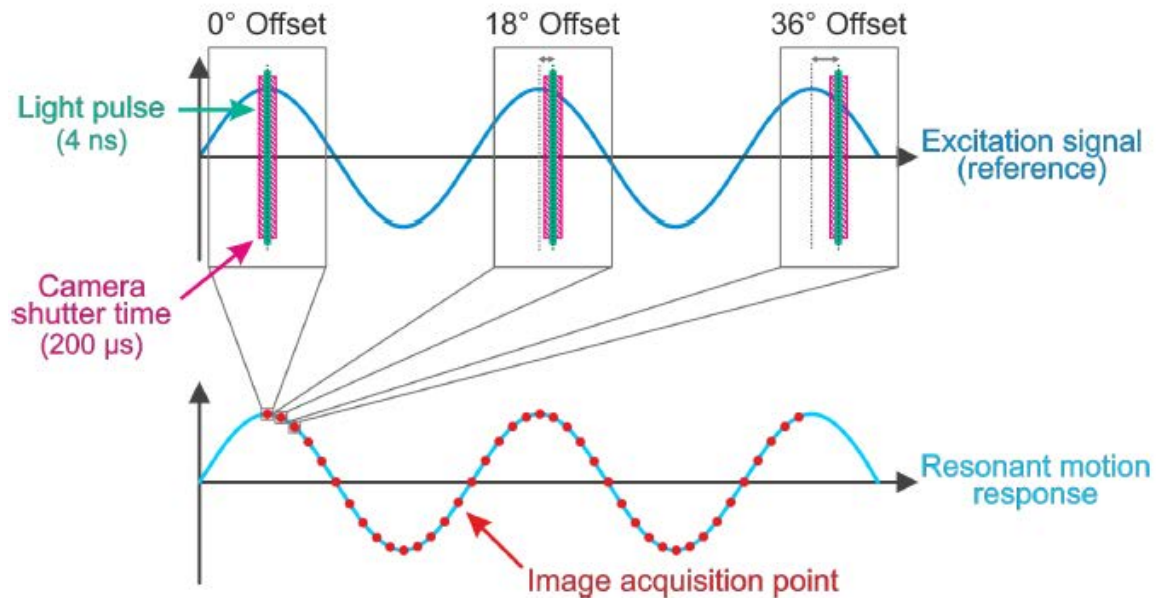


Figure 5.6 Phase stepping acquisition process.

Stroboscopic illumination provided by the 4ns-pulse laser was used to "freeze" the motion of the plate and eliminate motion blurring in the acquired image pairs.

Simultaneous triggering of the laser pulse and DIC image acquisition was achieved by routing the signal from the function generator to a timing box connected to both the laser and CCD cameras. The dynamic motion of the plate was captured by phase-stepping the image acquisition relative to the excitation signal and 40 images were captured at $\pi/10$ increments¹¹⁵, which corresponds to two vibration cycles. This process is shown in schematic form in Figure 5.6 for illustrative purposes.

An alternative approach to the image capture of a vibrating component and the "freezing" of its motion would be the use of very bright, constant illumination coupled with a very short shutter time^{116, 117}. When capturing very fast motion (including high-frequency excitation), however, its use may be limited by the minimum shutter time for the DIC cameras used in the experiments (20 μ s for the Stingray cameras used in this work), which is typically in the order of microseconds and much longer than the 4ns pulse provided by the laser. High-speed cameras are often used in the acquisition of DIC images of specimens under dynamic loading because they have very short acquisition times (1 μ s) and their frame rates can approach 1 million frames per seconds (fps)¹¹. For many years, there were no available high-speed camera options to operate in the UV spectrum, which limited their use in high-temperature experiments where glowing of the specimen occurs. A few camera models now satisfy that requirement

and are currently available in the market¹¹⁸⁻¹²⁰. However, they are large in size and represent a high level of capital investment, which limits their use when studying structures in industrial settings where weight and ease of use are paramount. Moreover, the increase in frame rate when using high-speed cameras often requires a compromise on image resolution, which can sometimes go as low as 64 x 12 pixels at maximum frame rate. This is a clear disadvantage for digital image correlation as a reduction in the size of the image array will lead to a decrease in spatial resolution¹²¹. Sebastian¹¹ has shown that the developed Pulsed Laser Digital Image Correlation (PL-DIC) system using standard CCD cameras is a more affordable, more compact option that "performed as well or better than a high speed system".

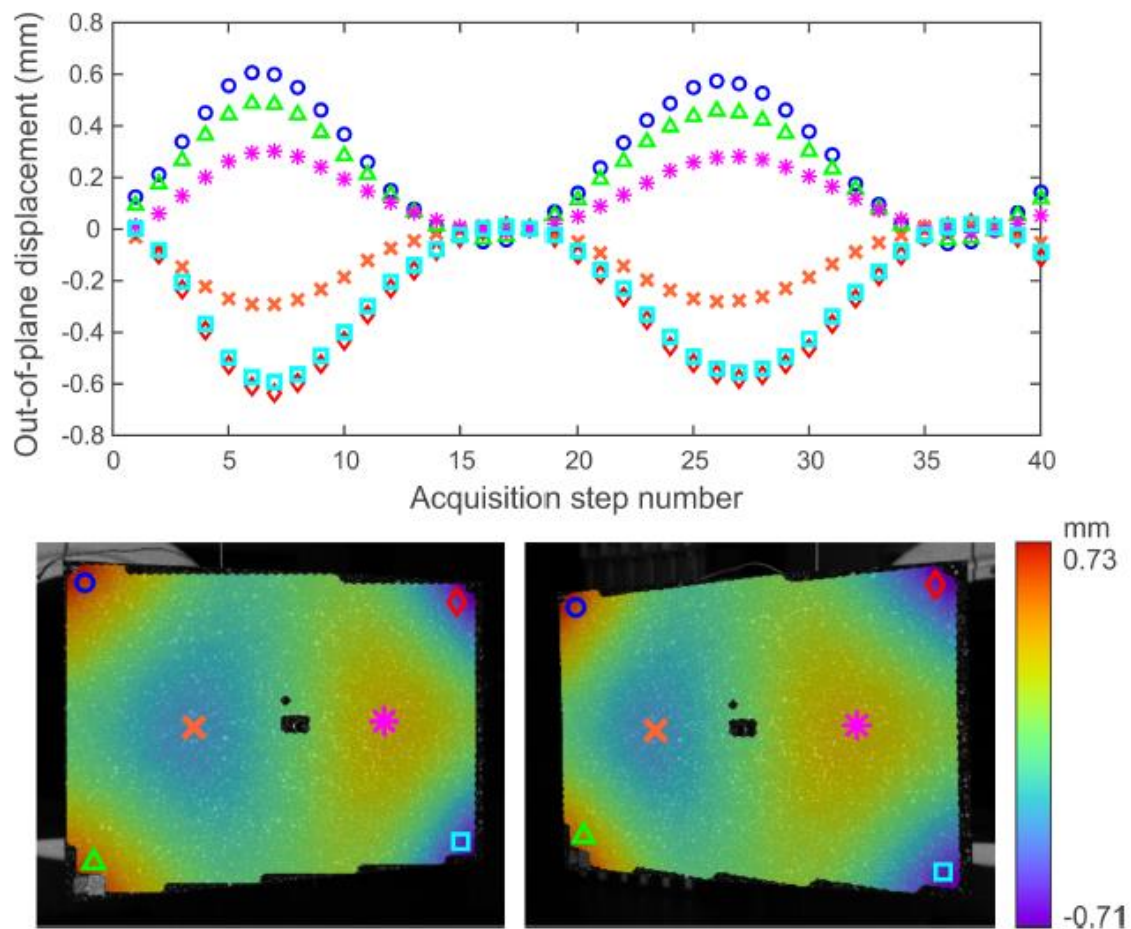


Figure 5.7 Typical out-of-plane displacement readings throughout two vibration cycles of a resonant mode (standing wave). The locations of each of the six plotted coloured markers are shown in the DIC images provided.

In the work presented here, the phase-stepping approach yielded the deformed shape through the vibration cycle and allowed it to be confirmed that the measured shape was a standing (or stationary) wave corresponding to a mode shape, i.e. all parts of the structure oscillated in time but the profile of peak displacements was fixed in space. An example of the typical displacement readings of a measured shape over the vibration period is shown in Figure 5.7. The identified mode shapes corresponding to the maximum excursion are shown in Figure 5.8.

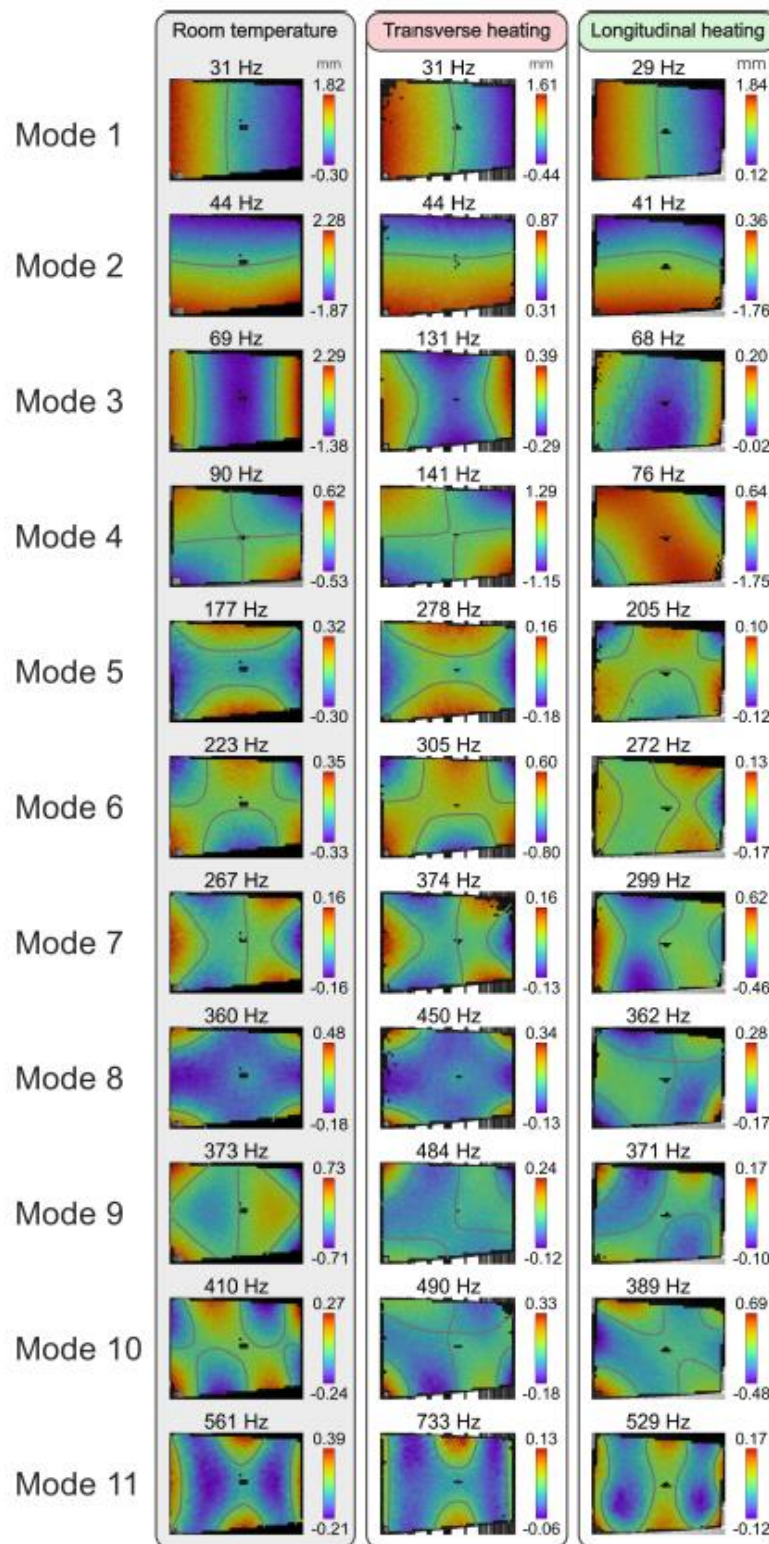


Figure 5.8 Measured (DIC) displacement maps for the test plate subject to the three temperature regimes: room temperature (left), transverse heating of the centre of the plate (middle) and longitudinal heating on one edge (right). All displacements are in mm.

As previously explained in section 3.2.2, two types of thermal camera were used in the high temperature experiments. Initially, a neutral density filter was fitted to the FLIR photon detector, which was used to monitor the temperature distribution on the surface of the specimen. Manual calibration of the system was performed using a laser-guided, infrared thermometer (Fluke 62, Fluke UK Ltd, Norfolk,

UK). The specimen occupied an area of 278 x 184 pixels in this camera's field of view (FOV), which was used to acquire the temperature distribution during transverse heating, as shown in Figure 5.3 (top). The error-prone calibration method necessary when using the FLIR camera was impractical; and so, in subsequent experiments the factory-calibrated micro-bolometer was used. Typical measurements from this bolometer are shown in Figure 5.3 (bottom). A K-type thermocouple was used to compare point-wise temperature measurements to those acquired with the micro-bolometer in order to estimate the influence of temperature on the emissivity of the painted plate. Using the micro-bolometer software, the emissivity setting was varied until the imaged temperature matched the thermocouple temperature. The results showed an increase in emissivity of 0.08 between room temperature (0.89) and approximately 600°C (0.97). This difference in emissivity is relatively small and, thus, the discrepancy in the temperature readings was accepted and the full-field temperature maps measured at the elevated temperatures were corrected assuming a uniform emissivity of 0.97. The change in the measured temperatures shown in Figure 5.3 due to temperature-dependent emissivity would correspond to a maximum increase of 0.09% in the Young's modulus of Hastelloy X¹⁰⁷. This is a very small change when compared to the approximate 15% decrease in the Young's modulus the material actually experiences between room temperature and 600°C (from 205 GPa to approximately 175 GPa).

The specimen was allowed to cool between each excitation experiment; consequently for each experiment: 1) the initial deformed shape of the plate at room temperature was acquired using DIC; 2) the temperature distribution across the plate's surface was monitored and recorded using a thermal camera; and 3) the FE model was updated to include the measured initial shape of the plate and the achieved temperature distribution. The resultant measured and predicted modal shapes are shown in Figure 5.8 and Figure 5.9 respectively. The nodal lines in these figures separate the regions of the plate vibrating with opposite phase.

5.4 Finite element modelling

A finite element model based on the work described in Chapter 4 was developed using HyperWorks for the pre- and post-processing and LS-Dyna as the solver. Temperature-dependent material property values provided by the manufacturer¹⁰⁷ and available in Appendix A were incorporated into LS-DYNA's isotropic thermal material model (MAT_ELASTIC_PLASTIC_THERMAL). A small strain elastic response was assumed in all of the simulations. To account for the initial geometric shape of the plate by integrating experimental data in the model, the geometry of the FE mesh was defined using the shape of the specimen at room temperature, computed from the DIC data using Istra 4D and shown in Figure 5.10.

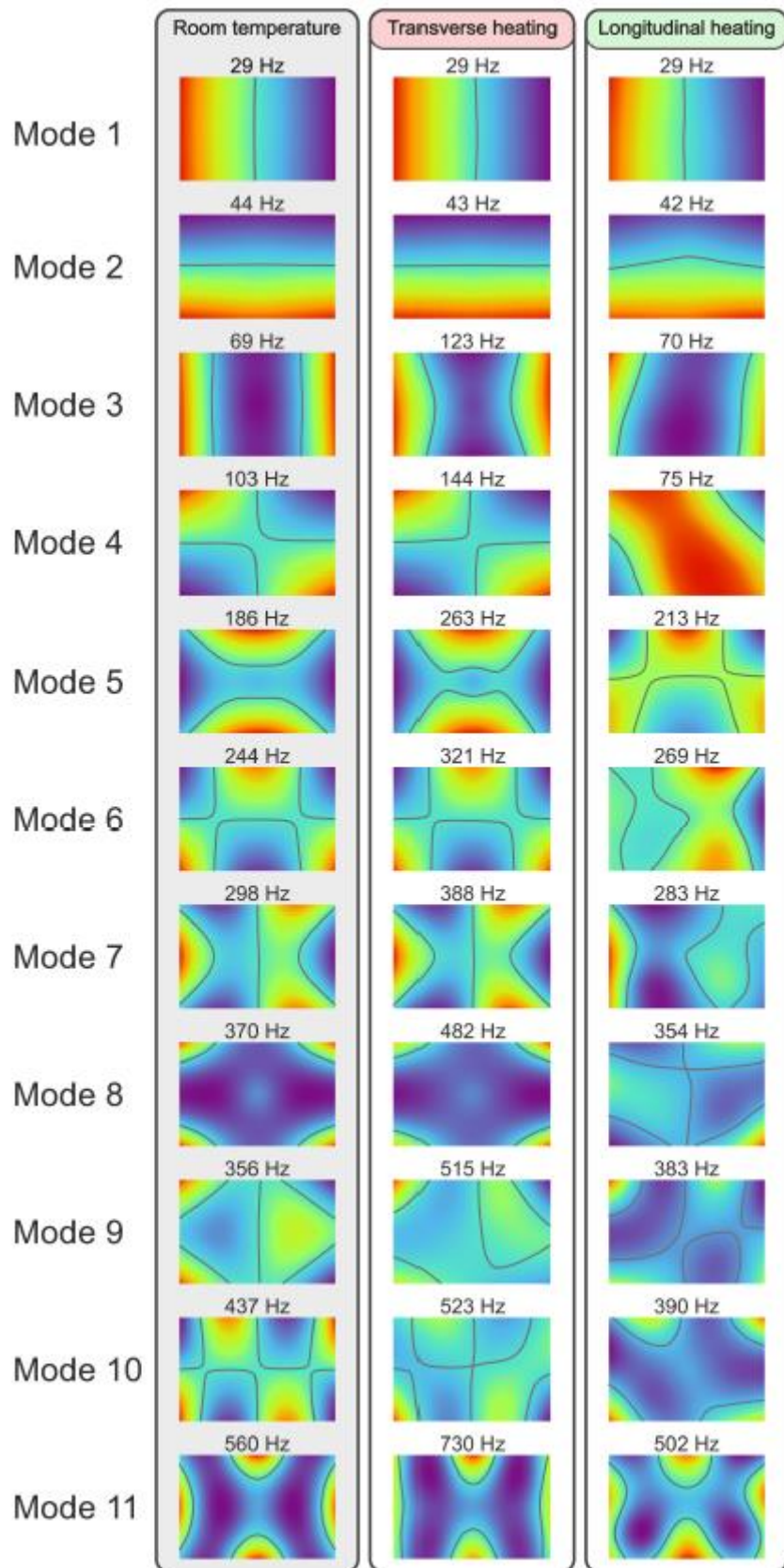


Figure 5.9 Predicted (FE) mode shapes for the test plate subject to the three temperature regimes: room temperature (left), transverse heating of the centre of the plate (middle) and longitudinal heating on one edge (right).

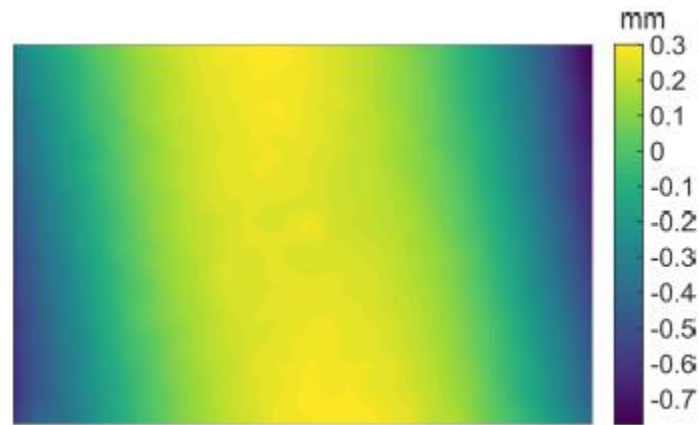


Figure 5.10 Finite element shape computed using DIC contour measurements of the plate at room temperature.

To do so, and as explained in section 3.2.3, Istra determines the 3D positions of each facet in the image, which results in a dense set of three-dimensional points that represent the shape of the plate. The software then fits a plane on to the shape data using the method of least squares and the out-of-plane shape of the plate is presented as the distance relative to that plane, as depicted in Figure 5.11.

Any missing values in the raw DIC data were interpolated using the cubic convolution method, which was chosen to avoid distortion and mosaic effects that can be introduced by other methods¹²². The geometry was discretised into 2 x 2 mm shell elements with hourglass control to avoid spurious modes of deformation; a non-linear solver was used to account for large deflections induced by the load. A custom MATLAB script assigned a temperature value to each node of the mesh based on the measured temperature distributions. It was assumed that through-thickness temperature gradients were negligible and no damping was modelled.



Figure 5.11 Schematic of the best fit plane method used by Istra 4D to estimate the shape of the plate.

Similarly to the work presented in Chapter 4, the boundary conditions in this model were a simplification of those in the experiments because the hole for the stinger and the bolted connection were not represented and instead a single node at the centre point of the plate was fully constrained. The spatial distribution of temperature was increased proportionally assuming a linearly increasing thermal load with time (over a time period of 130 seconds) applied to each node to reflect the experimental protocol, starting at room temperature and ending with the corresponding measured steady state temperature. An implicit eigenvalue analysis was used to extract the first eleven resonant frequencies and corresponding mode shapes, shown in Figure 5.9, at both room temperature and after heating for 130 seconds.

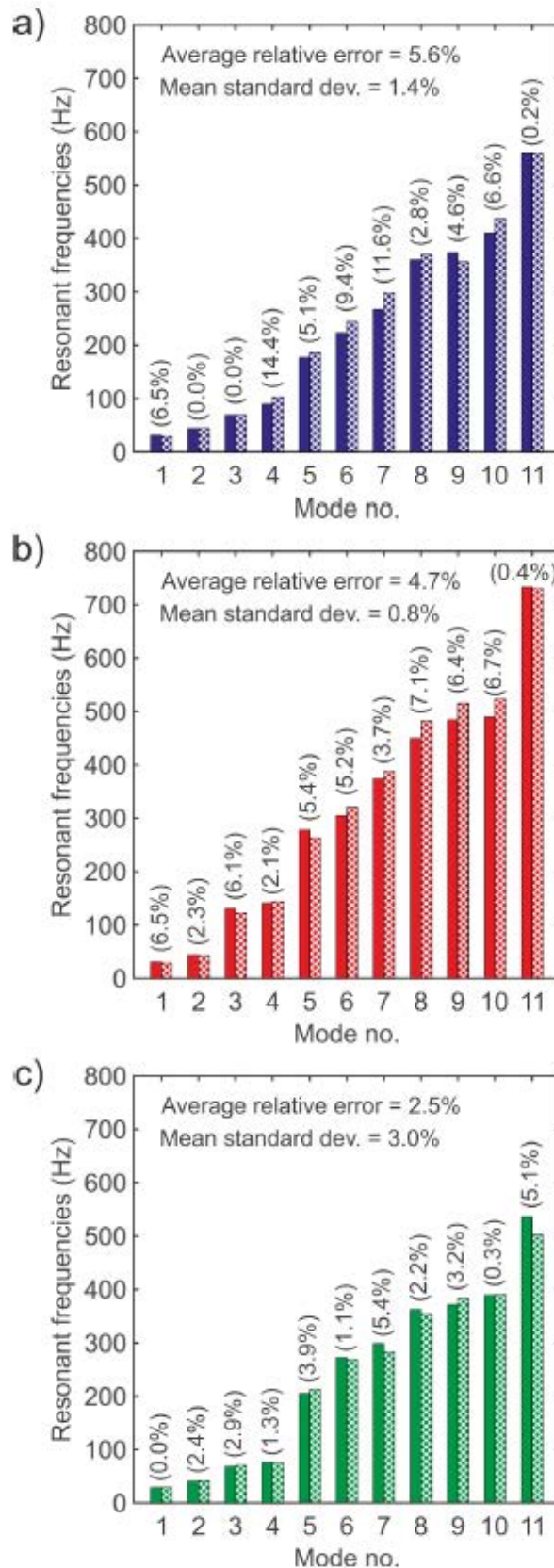


Figure 5.12 Measured (solid bars) and predicted (shaded bars) resonant frequencies for a) uniform room temperature; b) transverse heating of the centre of the plate; c) longitudinal heating on one edge. Relative errors of predictions against measurement data are shown in parenthesis.

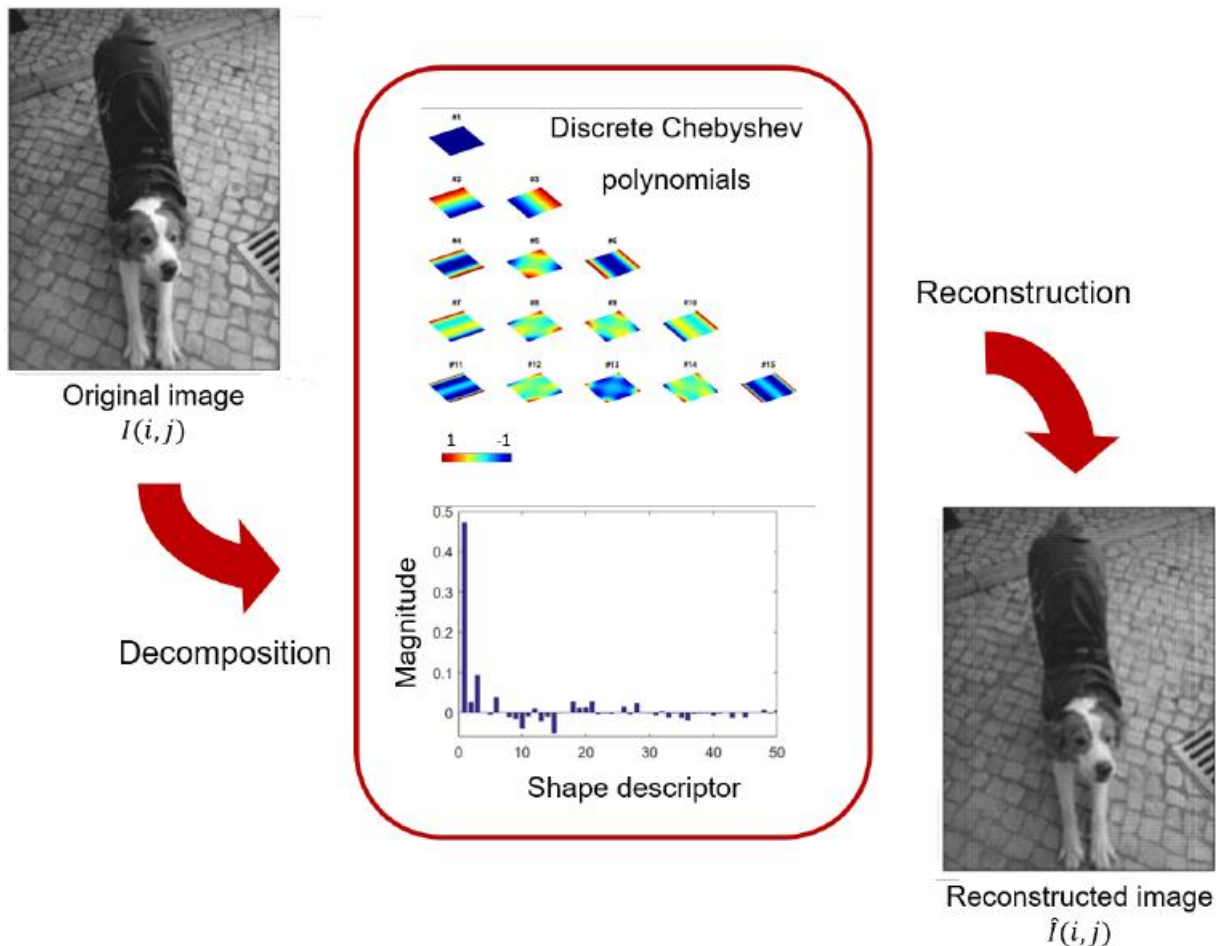


Figure 5.13 Illustration of the decomposition process using Chebyshev polynomials as the orthogonal basis for the definition of descriptors of $I(i, j)$. The reconstruction process is also represented and the resulting image, $\hat{I}(i, j)$, is shown.

The predicted and measured values of resonant frequency are compared in Figure 5.12. The predicted and measured modal shapes were compared using proper orthogonal image decomposition, following the method recommended by CEN⁶⁸ and employed by Berke et al¹⁰. This method has a simple but powerful premise based on polynomial fitting: 2D data fields with a large number of data points can be fully represented by a comparatively small set of coefficients (shape descriptors) which describe an overall function (feature vector). This allows for a substantial reduction in the dimension of the data to be analysed and facilitates the comparison of data sets of different magnitudes or units. As seen in section 2.4 of the literature review, the choice of polynomial basis used in orthogonal decomposition depends on the data to be represented. In this work, the data describing a mode shape were treated as an image, $I(i, j)$, and decomposed using discrete Chebyshev polynomials to generate a set of shape descriptors because this polynomial base provides an efficient dimensional reduction of the mode shape data of a rectangular plate, as shown by Wang et al.⁷⁶ (i.e. mode shapes with 10^3 - 10^4 data points can be fully described using only 10^0 - 10^1 shape descriptors). This is due to the fact that the mode shapes of an ideal free-free rectangular plate coincide with the shapes represented by the discrete Chebyshev polynomials. An example is shown in Figure 5.13, in which the original image was decomposed using

1,000 terms in the Chebyshev polynomials. For illustrative purposes, only the first fifteen Chebyshev polynomial terms and the first 50 associated descriptors are presented.

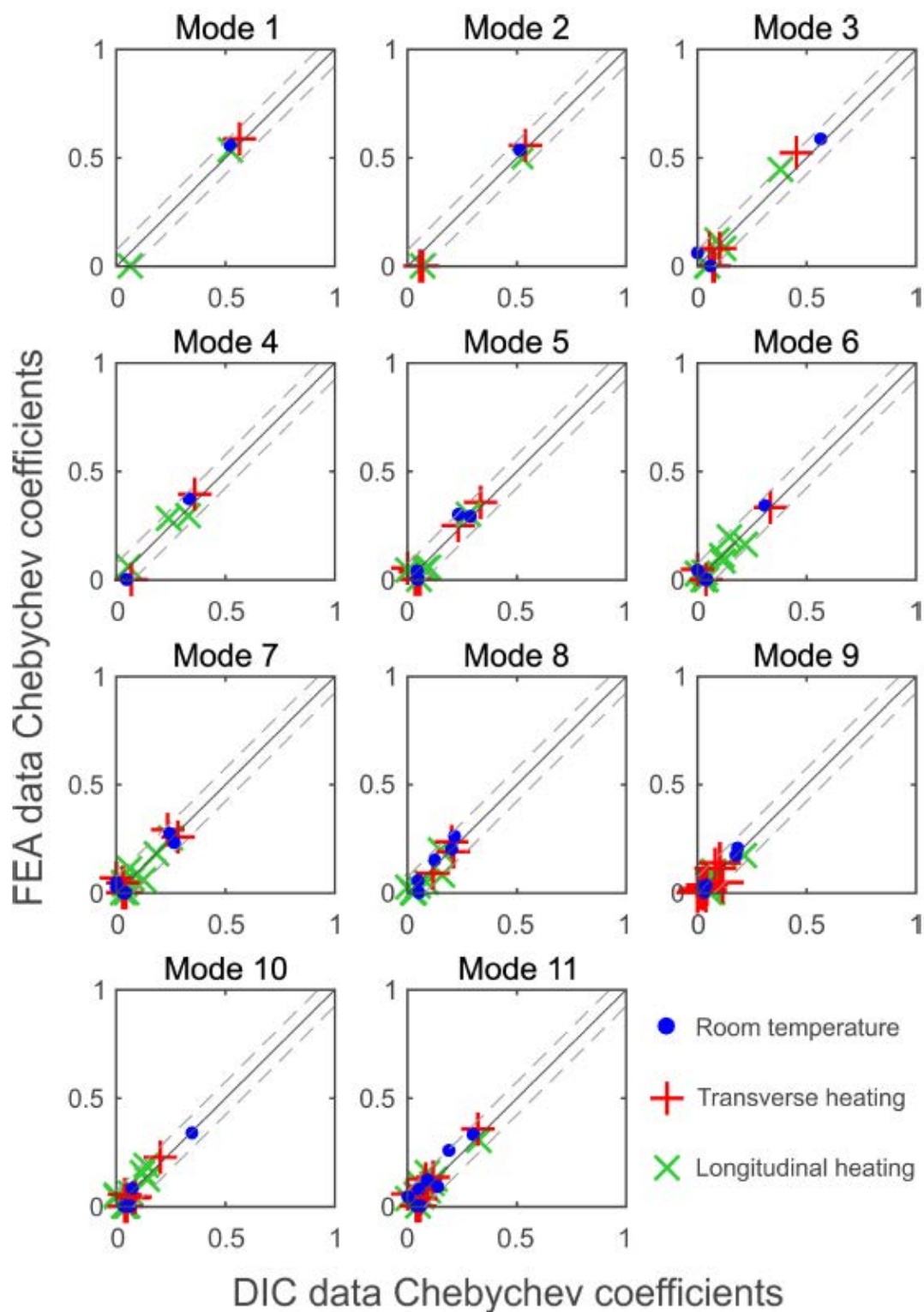


Figure 5.14 Comparison of Chebyshev coefficients from the orthogonal decomposition of measurements (horizontal axis) and predictions (vertical axis) data. Fifty Chebyshev kernels were used in the decomposition of the measurement and prediction data. The first kernel was excluded from the plots as it describes rigid out-of-plane translation only and is unrelated to the deformation of the plate.

As represented in Figure 5.13, the shape descriptors and associated Chebyshev polynomials can then be used to calculate a reconstruction of the data, $\hat{I}(i, j)$, using an inverse transform. The average residual from the reconstruction (u_{resid}) can be calculated as:

$$u_{resid} = \sqrt{\frac{1}{N} \sum_{i,j}^N (\hat{I}(i, j) - I(i, j))^2} \quad (5.1)$$

In this work, the predicted and measured mode shapes were decomposed using 50 Chebyshev polynomial terms; u_{resid} was calculated and found to vary from 0.005 to 0.054 for the room temperature results, from 0.014 to 0.097 for the transverse heating results and from 0.017 to 0.083 for the longitudinal heating results.

The CEN guide recommends that no clusters of residuals between the original and reconstructed data should be greater than 3 times the average square residual of the data fit; where a cluster is a group of adjacent pixels comprising 0.3% of the total of number of pixels in the dataset⁶⁸. Pixels corresponding to missing data in the original dataset were excluded from this criterion.

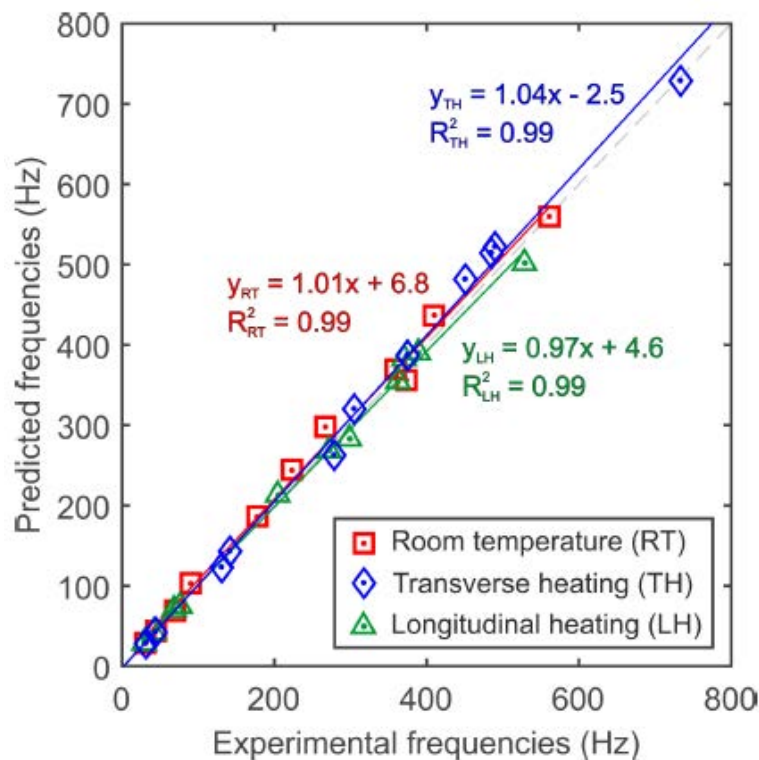


Figure 5.15 Predicted resonant frequencies using a temperature-dependent material model plotted against experimental results.

Euclid¹²³, a specially written MATLAB program by Christian and Patterson, was used to decompose the measured and predicted modal shapes and the resultant coefficients are shown in Figure 5.14. Missing measurement data, for instance around the nut attachment area, were interpolated by the program

using the default nearest neighbour interpolant function, which allowed for the fast decomposition of the large datasets.

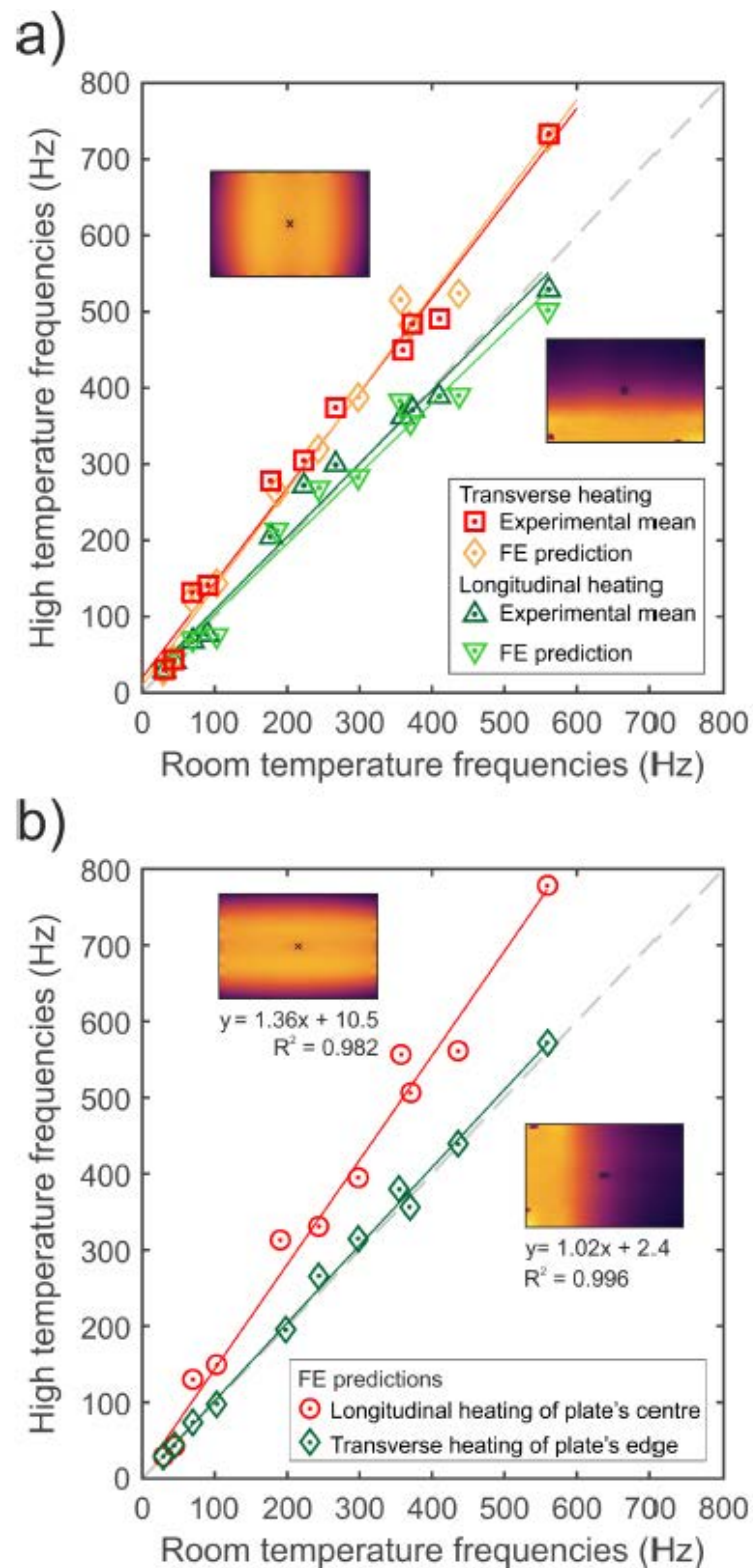


Figure 5.16 High temperature resonant frequencies plotted against room temperature results for: a) transverse heating along the centre of the plate and longitudinal heating along a single horizontal edge (experiments and simulations); b) longitudinal heating of the centre of the plate and transverse heating of a single vertical edge (simulations only). Insets for the corresponding temperature map for each data set are presented and their temperature colour bar as shown in Figure 5.2.

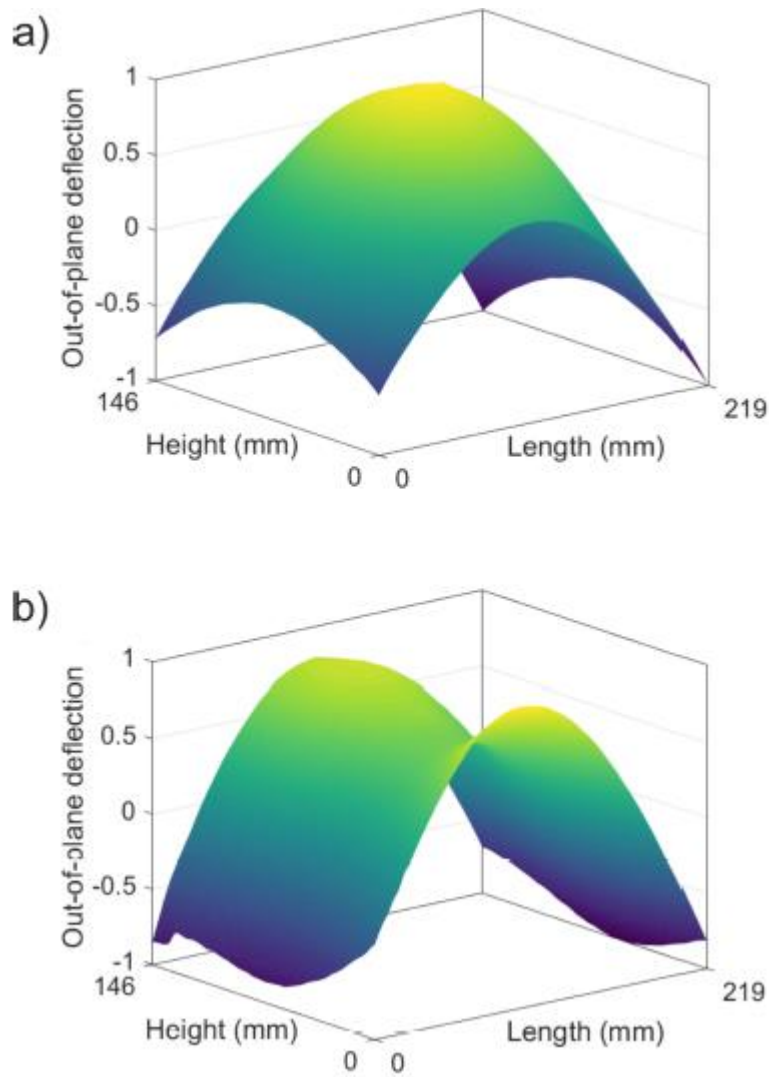


Figure 5.17 Measured deformed shape of the plate in the absence of mechanical excitation but following the transverse heating along the centre of the plate (top) and of a single longitudinal edge (bottom). The datasets have been normalised between -1 (dark purple) and 1 (yellow) because the energy inputs in the two cases are different and hence the absolute deformations are not directly comparable. Missing DIC data due to the bolted constraint has been interpolated using the cubic convolution method.

5.5 Results and discussion

There is an excellent agreement between the predicted and measured resonant frequencies at room temperature and at high temperatures, which are plotted against one another in Figure 5.15. The measured resonant frequencies are presented as the mean values of the six repetitions performed experimentally and their standard deviations were calculated and are presented in Appendix F. The maximum standard deviations of the measured resonant frequencies are 5.5 Hz for room temperature (corresponding to mode 10), 2.1 Hz for transverse heating (mode 10) and 14.3 Hz for longitudinal heating (mode 11). These standard deviations are all smaller than the difference in frequency between modal frequencies which implies that they are not significant and there is a clear identification of the individual modes.

Room temperature resonant frequencies were plotted against the corresponding high temperature data in Figure 5.16 a) so as to identify the influence of temperature on the resonant frequency response. For longitudinal heating, there is no significant change in resonant frequencies because all of the data points corresponding to both the measurements and predictions lie along the line of equality. A linear regression between the frequencies acquired experimentally at room temperature and during longitudinal heating supports this observation as it has a gradient of 0.96 with an R^2 of 0.98 (gradient of 0.92 and R^2 of 0.97 for FE predictions). However, when transverse heating is applied, the resonant frequency of each mode increased by approximately 24% based on a linear regression with an R^2 value of 0.99; while the FEA predicted a 28% increase with an R^2 value of 0.99 for equivalent conditions. The difference in behaviour is likely a result of the difference in the deformed shapes that occur due to the heating alone, which are shown in Figure 5.17. At steady state the through-thickness temperature difference was below the uncertainty range of the micro-bolometer; however, the high in-plane temperature gradients caused localised differential expansions of the plate leading to non-uniform in-plane compressive and tensile stresses that caused bending of the initially deformed plate. The transverse heating generated a doubly-curved "dome" shape whereas the longitudinal heating created a largely singly-curved shape.

These results appear to support the mechanism postulated by Mead³⁵, based on his simulation results, that non-uniform heating induces thermal stresses which provide a higher structural resistance to deformation. However, this might not always occur because the material would be expected to soften at high temperatures, i.e., the material stiffness will reduce with temperature. This will result in a competition between the changes in stiffness caused by thermal softening and by structural or geometric stiffening. For the conditions in the reported experiments, the changes in geometric stiffness appear to dominate the shape changes with higher frequency, shorter wavelength excitation required when the plate stiffness increases. The double curvature generated by the transverse heating would be expected to provide an enhanced structural or geometric resistance to the deformation required to generate all mode shapes and hence to exhibit higher resonance frequencies at higher temperatures as seen in Figure 5.16 a); while the single curvature associated with the longitudinal heating would enhance the resistance to some modes and reduce it to others. The absence of a significant change in resonant frequency with either transverse or longitudinal heating for rigid body modes (modes 1 and 2) provides some additional support for this hypothesis.

In Figure 5.12, the relative error of each frequency prediction (F_s) was calculated against experimental measurements (F_e) as explained in section 4.2. For each resonant frequency acquired experimentally during the six repetitions (available in Appendix F), the standard deviation of the experimental readings was calculated as a function of their mean value:

$$\text{Standard deviation (\%)} = \frac{\text{Standard deviation (Hz)}}{\text{Mean (Hz)}} \times 100\% \quad (5.2)$$

Figure 5.12 shows that the FE results for resonant frequencies at room temperature had the highest average relative error against experimental data at approximately 5.6%, which is larger than the mean standard deviation for the experimental readings (1.4%). These discrepancies might be a result of the difference in boundary conditions between experiment and simulation. In particular, the bolted connection that experimentally constrains the plate prevents a displacement gradient forming whereas the single node constraint used in the FE model does not. The magnitude of the material's Young's modulus is at its highest at room temperature, therefore providing an increased resistance to deformation in response to the excitation force provided. Hence, at higher temperatures, the influence of the structural reinforcement of the constraint is less significant and less energy is needed to deflect the material around the bolted connector. Thus, because the FE model includes a simplified constraint which allows a displacement gradient to form, frequency and shape results at room temperature are more affected by the difference between simulation and experimental boundary conditions. This is consistent with the lower average relative errors against experimental data for the frequency predictions at high temperatures, i.e., 4.7% in transverse heating (maximum of 7.1% for mode 8) and 2.5% in longitudinal heating (maximum of 5.4% for mode 7). Figure 5.8 (middle) and Figure 5.8 (right) show DIC out-of-plane displacement maps for the temperature distributions shown in Figure 5.2. By comparing the mode shapes presented in these figures with the room temperature results in Figure 5.8 (left), it is possible to identify some changes in modal shape with thermal load. A mode shifting phenomenon is also present when thermally loading the plate, as mode 6 at room temperature becomes mode 5 following longitudinal heating. This means the natural order of occurrence of the modes in the frequency spectrum changes as the temperature increases and the plate changes shape. This behaviour is similar to the computational results obtained by Chen and Virgin^{36, 124} who found veering or switching of modes occurred when there was a change in the static configuration of a plate caused by a spatially non-uniform temperature. Further work was conducted by Lopez-Alba et al.¹²⁵ and experimentally confirmed this phenomenon by heating a simple rectangular plate 2. The authors used the same experimental rig described in this thesis to asymmetrically heat the plate whilst providing broadband excitation using the shaker. The laser vibrometer was used to record a time-frequency spectrogram during the heating sequence. The spectrogram guided the investigation of mode shapes and the micro-bolometer was used to monitor temperature, which allowed a correlation between mode shapes, frequency, time and temperature to be established. Their results confirmed Chen and Virgin's^{36, 124} predictions as mode veering and shifting was seen to be associated with thermally-induced buckling promoted by the asymmetric heating.

Qualitatively, predicted mode shapes in Figure 5.9 agree with experimentally measured displacement maps in Figure 5.8 but a quantitative comparison was needed for the rigorous validation of the model. Traditionally, this quantitative analysis was made using the Modal Assurance Criterion (MAC)¹²⁶, which is a statistical indicator of compliance between mode shapes. The MAC uses a least squares form of linear regression analysis to compare two characteristic vectors of the mode shapes (eigenvectors): one computed from experimental readings and another one resulting from analytic predictions¹²⁷. The experimentally-acquired vector is determined using the transfer function between the excitation signal and the structural response to excitation, which is measured using point-based techniques. Vectors acquired using a predictive model present the structure's response as an eigenvalue problem to which solutions represent the resonant frequency (eigenvalue) and displacement map of the resulting mode shape (eigenvector). The experimental and predictive eigenvectors are then used to calculate the normalised scalar product between the two. Results can range from zero to one, with one representing the highest level of agreement between datasets and zero indicating no agreement. Mottershead and his co-workers^{70, 81, 82} have used both the MAC and the orthogonal decomposition approach for the comparison of measured and predicted mode shapes and concluded that the MAC is inadequate for revealing subtle differences in the data sets. This is particularly significant when analysing high order mode shapes with increasing geometric complexity. Hence, orthogonal decomposition was used here to quantify the agreement between the predicted and measured mode shapes following the process used by Berke et al.¹⁰ and recommended in the CEN guide for validation of computational mechanics models⁶⁸. The CEN guide suggests that the predictions from a model can be considered to exhibit acceptable agreement with measurements when, following decomposition, the coefficients representing the predicted data field are plotted as functions of the corresponding coefficients representing the measured data and fall within:

$$S_P = S_M \pm 2u(S_E) \quad (5.3)$$

where S_M and S_P are the shape descriptors representing the measured and predicted data, respectively; and $u(S_E)$ is the experimental uncertainty⁶⁸. The minimum measurement uncertainty (u_{meas}) was taken as 1.9% of the data range based on an evaluation of the same system by Sebastian et al.¹¹⁵ and a methodology described by Hack et al.¹²⁸. The CEN guide recommends combining the minimum measurement uncertainty and the reconstruction error to obtain the experimental uncertainty. In this case, that would result in different values for the acceptability limit for each dataset defined by Equation 5.3, which makes interpretation more difficult; hence, instead the reconstruction error was replaced by the minimum measurement uncertainty to give a constant value for the acceptability limits in Figure 5.14. When this analysis was performed for each of the 11 resonant mode shapes at each of the three thermal conditions, i.e., room temperature, longitudinal heating and transverse heating, it was found

that all of the predicted shapes were acceptable. Whilst 50 kernels were used in the decomposition of data, the presentation of results in Figure 5.14 was simplified by employing a methodology similar to one described by Lampeas et al.¹²⁹ in which only the coefficients above or equal to 10% of the largest coefficient are used in the representation of data whilst maintaining a Pearson's correlation coefficient greater than 0.95 between the original data fields and those reconstructed from the remaining coefficients.

After the reliability of the finite element model had been demonstrated, it was used to explore the influence of the applied temperature profile on the behaviour of the plate. The transverse heating in Figure 5.2 (top) involved transversely heating the centre of the plate whilst the longitudinal heating shown in Figure 5.2 (bottom) was applied to only a single longitudinal edge. The FE model was modified to study the effect of heating (a) the centre of the plate longitudinally and b) heating a single transverse edge. The corresponding deformed shapes due to these heating regimes were obtained by rotating the shape data in Figure 5.2 through $\pi/2$ and transforming them to conform to the FE mesh. The resultant predicted resonant frequencies are presented in Figure 5.16 (b) and show, as in Figure 5.16 a), that heating a single edge produces little change in resonant frequencies; but, longitudinally heating the plate's centre causes an upward shift in the resonant frequencies, except for the rigid body modes which remain unchanged.

In this study, the plate was heated into the post-buckling regime and different spatial distributions of applied thermal load were used to induce different buckled shapes, as shown in Figure 5.17. Inevitably, the plate will have contained some residual stresses induced by the processes used to manufacture it and these resulted it being non-planar at room temperature, as shown in Figure 5.10. However, this initial curvature at room temperature was small compared to the post-buckled shapes, which were approximately 4.8 and 7.8 times larger for longitudinal and transverse heating shapes, respectively; and, hence it is reasonable to conclude that thermal buckling dominates the shape changes that occurred with heating, rather than the residual stresses due to manufacturing.

5.6 Conclusions

In this Chapter, the first concurrent acquisition of full-field temperature and full-field out-of-plane displacement data has been presented for the resonant response of a plate subjected to combined thermal (up to approximately 550°C) and vibratory loads (up to 800 Hz). The study of the first eleven resonant modes of the structure expanded upon the number of modes investigated in the published literature, allowing the measurement of modal shapes with an increasing level of geometric complexity. The inclusion of quasi-continuous, full-field experimental measurements of temperature and shape in a temperature dependent material model was shown to improve the reliability of predictions when compared to more simplistic models. For a centrally-constrained plate with free edges, the results

showed resonant frequencies to change with temperature and to be strongly dependent on the spatial temperature distribution across the structure. For spatially non-uniform temperature distributions, the plate changed shape and sought to reach a stable state, which influenced the amount of energy required to excite each mode shape. Experimentally, transverse heating yielded an increase in resonant frequency with temperature, whilst longitudinal heating did not generate a significant change. FE predictions using the same temperature distributions rotated by $\pi/2$ suggested that these results are a consequence of the difference in curvature of the plate at high-temperatures in a steady state. The influence of thermal load on the resonant response of the plate at high temperature has been numerically and, for the first time, experimentally demonstrated and the results correlated well with Mead's³⁵ analytical predictions. Consequently, it is critical for the reliability of FE predictions of behaviour at high temperatures that thermal loading is effectively represented because it is responsible for both changes in material properties and changes in the structural properties, i.e., changes in geometric stiffness.

Chapter 6 Dynamic response of a thermally stressed plate

with reinforced edges

6.1 Introduction

The results in Chapter 5 agree with the predictions by Jeyaraj et al.³³ and highlight the effect of the distribution of temperature on the deformation of a plate with free edges. These boundary conditions are commonly used in modal testing since it is easier to replicate them in computational models used in the prediction of behaviour. Free edges are typical of lightweight constructions; in aircraft design, circumferential frames attached to the fuselage skin have what it is commonly referred to as "mouse-holes", which are openings in the frame that allow the stringers to pass through it¹³⁰. Due to these holes, the frame has free edges at the stringer positions. However, the aircraft's skin presents more complex boundary conditions, which are defined by the stringers and ribs that constrain the skin but also experience deformation themselves.

The third knowledge gap, which was identified in the literature review, indicated that a limited amount of experimental analysis has been performed on thermally-loaded plates using full-field measurement methods. This review also highlighted the focus of past investigations on functionally-graded plates, with few studies on isotropic plates being featured, apart from Thornton et al.⁵² who acquired point-wise displacement and temperature data when thermally loading a simply-supported, Hastelloy X plate.

More recent work, using computational mechanics, has included the development of models of functionally-graded plates resting on elastic foundations and subject to thermo-mechanical loading^{131, 132}. Adineh et al.¹³² applied a sudden uniform mechanical load to a simply-supported aluminium plate and their results showed the centre of the plate to spring-back after reaching a maximum out-of-plane displacement. This reflected earlier results for functionally-graded plates subjected to mechanical¹³³ and thermo-mechanical^{134, 135} loading. As a combined result of through-thickness thermal gradients and the different elastic properties of the constituent materials of these structures, Vel and Batra¹³⁶ predicted a similar out-of-plane dynamic response to a rapidly increasing thermal load. The aim of the work presented in this chapter was to gather high-quality temperature and displacement data throughout the thermal loading of a thin plate with reinforced edges. A relationship between the out-of-plane deflection and temperature was established both at a local and a global level.

This chapter is based on a paper entitled "Dynamic response of a thermally stressed plate with reinforced edges", written for publication by the journal *Experimental Mechanics*.

6.2 Test specimen

The study was conducted using a 250x150 mm Hastelloy X (American Special Metals Inc., Miami, FL, USA)¹⁰⁷ plate with reinforced edges as shown in Figure 6.1 and vertically suspended using two stainless steel wires looped through two small holes in the plate's upper corners. The wires allowed the plate to hang from an aluminium portal frame bolted to an optical table. The specimen was manufactured by machining a 4.5 mm thick plate down to 1 mm thickness in the centre, while leaving a 10 mm wide frame of thickness 4.5 mm around its edges. The intention of the design was to represent the reinforcement in aircraft skins produced by stringers and ribs, which do not fully clamp the thin-gauge structure but constrain it by imparting a higher resistance to deformation. Several small holes were drilled in the reinforced edges so that multiple mounting options were available, but only one option was used in the results reported here.

The plate was prepared for high-temperature DIC by spraying a uniform layer of commercially-available black paint (VHT Flameproof, Cleveland, Ohio, USA) onto the front and back surfaces. A white variation of the same paint was used to create a random speckle pattern on the plate. The reinforced edges of the plate were painted black but not speckled, as the focus of this study was the thin plate.

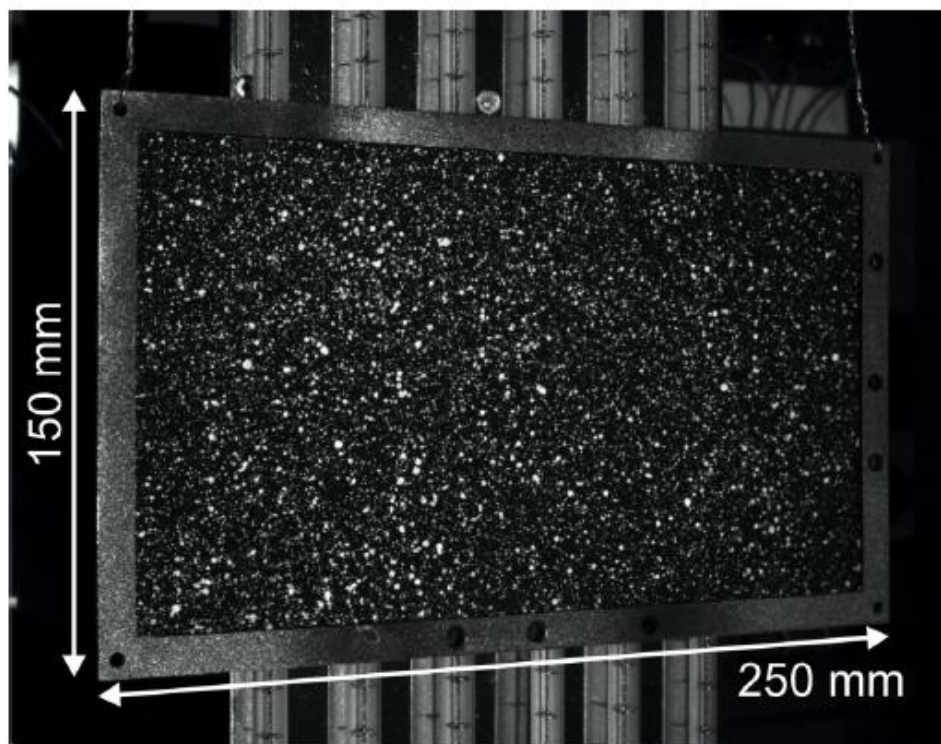


Figure 6.1 Photograph of the reinforced plate showing the speckle pattern and lamp setup.

6.3 Experimental methods

The specimen was thermally loaded using an array of halogen quartz lamps (QIR 240 1000 V2D, Ushio, Steinhöring, Germany), detailed in section 3.2.1. The lamps were uniformly spaced in arrays of six as

shown in Figures 6.1 and 6.2 and their power output was regulated by a controller using the TRIAC semiconductors.

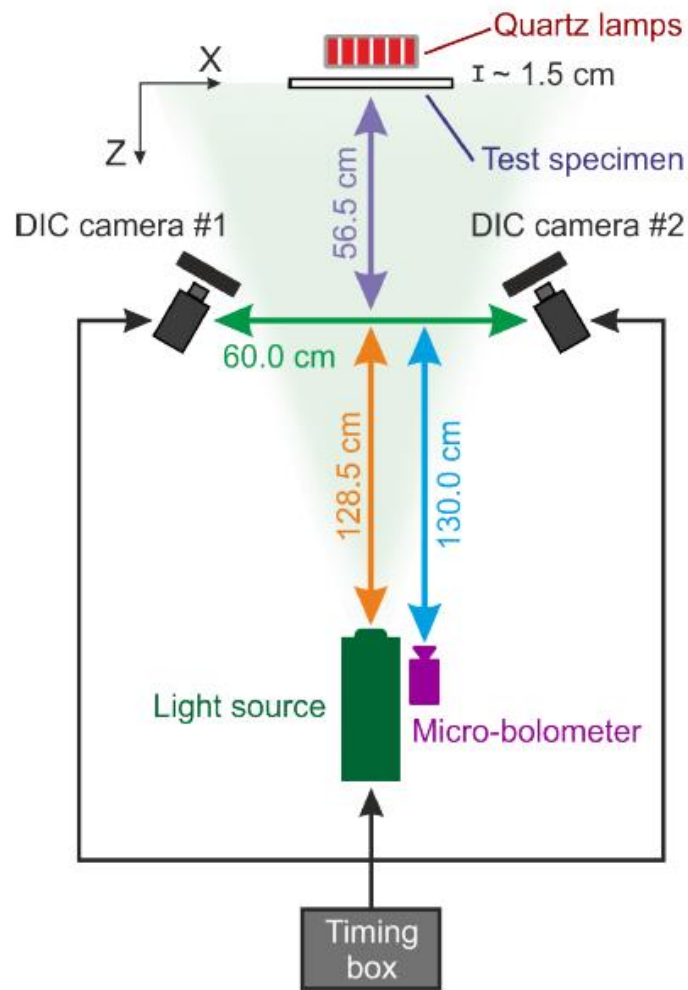


Figure 6.2 Schematic of the test set-up used in the data acquisition of the thermal loading of the reinforced plate.

The full-field temperature distribution in the plate was monitored using the micro-bolometer (TIM 400, MICRO-EPSILON UK, Birkenhead, UK). A schematic of the experimental setup is shown in Figure 6.2 which resulted in a spatial resolution of 1.1 mm/pixel. The effect of temperature on the emissivity of the painted plate was analysed using a K-type thermocouple and comparing pointwise temperature measurements to those acquired with the micro-bolometer. Thermocouple measurements at different temperatures were plotted against local readings at the corresponding location from the micro-bolometer and a second-degree polynomial was fit to the data ($R^2 = 0.99$) and used to correct the full-field temperature maps. The corrected temperature maps were used in the analysis of the data presented in this chapter; however, the raw, uncorrected data was also analysed and results showed the change of emissivity with temperature to have no impact on the conclusions of this investigation. At room temperatures, the difference in emissivity of the white and black paint used in the DIC speckle yielded differences in apparent temperature that were below the resolution of the camera; so that, the speckle pattern was not visible in the infrared images from the micro-bolometer. The scale of the

temperature gradients when thermally loading the plate was much larger than the difference in readings imparted by the different paints.

The Pulsed Laser Digital Image Correlation (PL-DIC) system described in section 3.2.3 was used to acquire and process displacement fields of the thin section of the plate during thermal loading. Image processing and calculation of data fields of out-of-plane displacement was performed using the Istra 4D software supplied with the system, employing square facets of 25 pixels (5.75 mm) with a 4 pixel (0.92 mm) overlap that corresponded to the pitch of the displacement data.

6.3.1 Experiment preparation

Prior to any experiments, the plate was thermally loaded for 10 minutes using six lamps at full power, symmetrically positioned about the plate's centre. This procedure was followed to relieve some of the residual stresses introduced by the manufacturing of the specimen and to create an initial curvature in the plate, as shown in Figure 6.3, which better represents real components that are rarely at. As seen by Thornton et al.⁵², who called it the plate's "initial warpage", this initial curvature determined the direction of deformation in the following experiments.

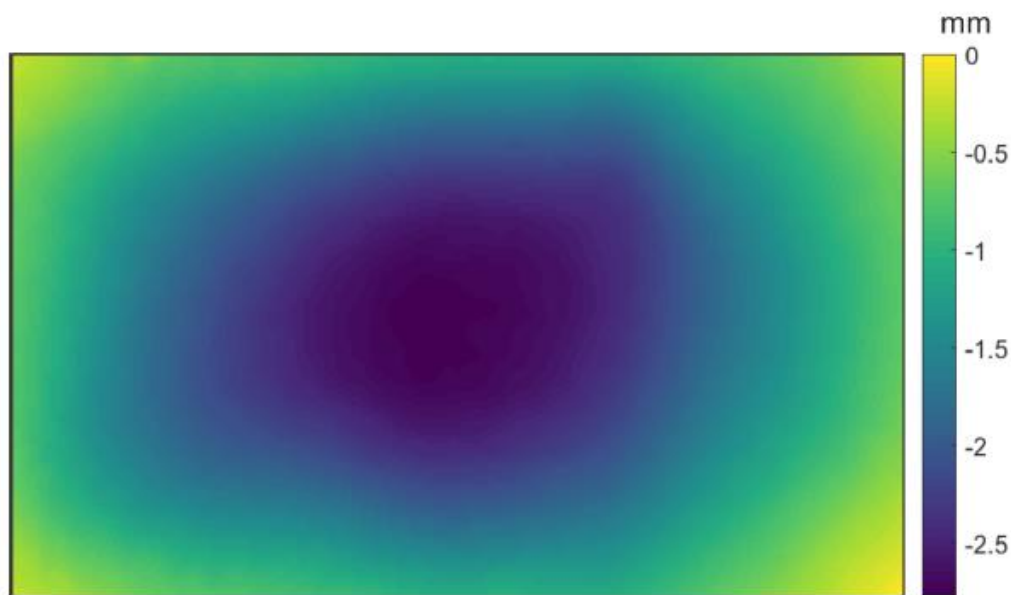


Figure 6.3 Initial shape of the thin plate at room temperature calculated from DIC measurements using Istra 4D following an initial thermal cycle corresponding to heating with six lamps at full power for ten minutes from room temperature and then allowing the plate to cool in air to room temperature. The shape is shown as z-direction displacements from the x-y plane.

Two types of experiments were conducted, in which the energy supplied to the plate was incremented: in the first set of experiments, different numbers of lamps were used in the thermal loading of the specimen whilst the second set involved heating the plate with a constant number of lamps whilst controlling their power output. These heating regimes resulted in non-uniform temperature distributions and near-uniform temperature distributions, respectively, which are shown in Figure 6.4.

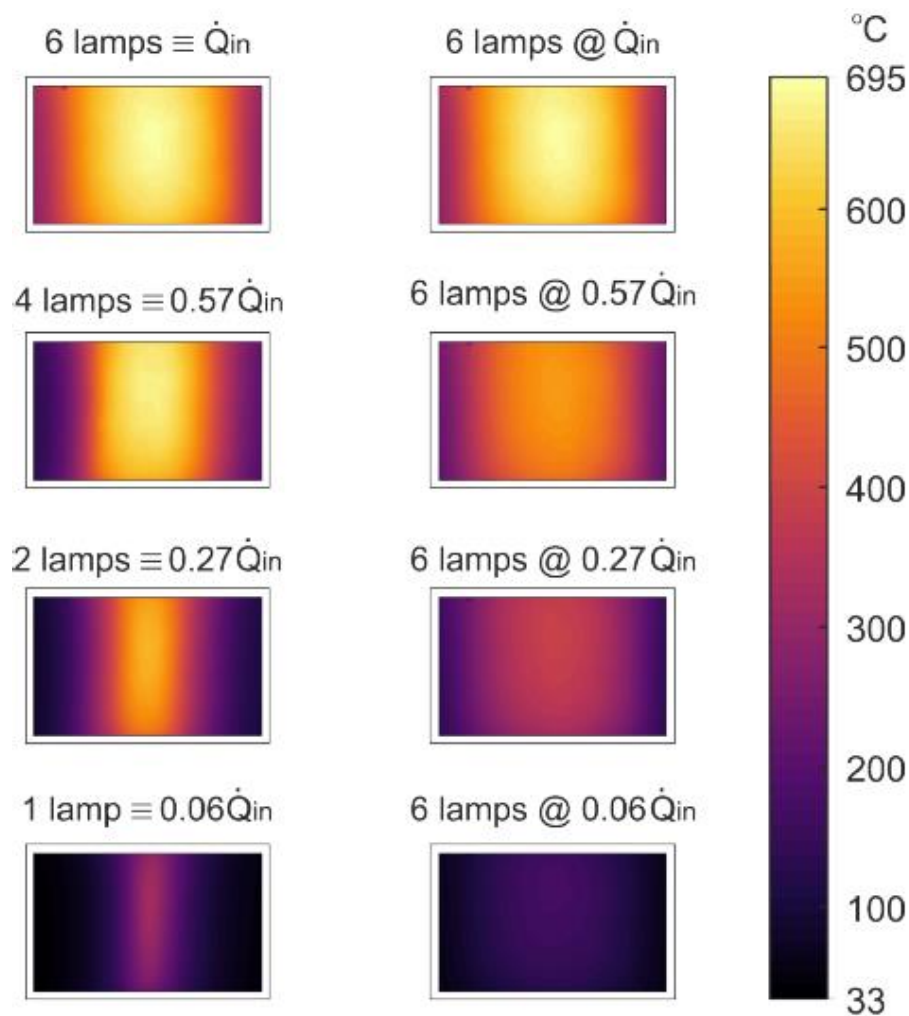


Figure 6.4 Steady state temperature measurements for the thin section of the reinforced plate subjected to non-uniform heating using a variable number of lamps at full power (left) and nominally uniform heating using six lamps (right); with constant rates of energy supplied on each row but decreasing from top to bottom.

Experiments were limited to 10 minutes duration to avoid accumulating changes due to material aging, which occurs when heating to temperatures around 760°C for approximately 3 hours¹¹⁴. Similarly, all experimental temperatures were kept below the recrystallisation or critical temperature of the material ($1163 - 1191^{\circ}\text{C}$) to avoid annealing or solution treating¹¹⁴. After each experiment, the specimen was allowed to air cool to room temperature.

To estimate the level of the noise floor for the measurements, a series of 30 images were acquired with the panel in a static configuration at room temperature and then processed using Istra 4D. The mean value of the out-of-plane displacement component was then calculated and the noise floor was found to be approximately $16.6\mu\text{m}$.

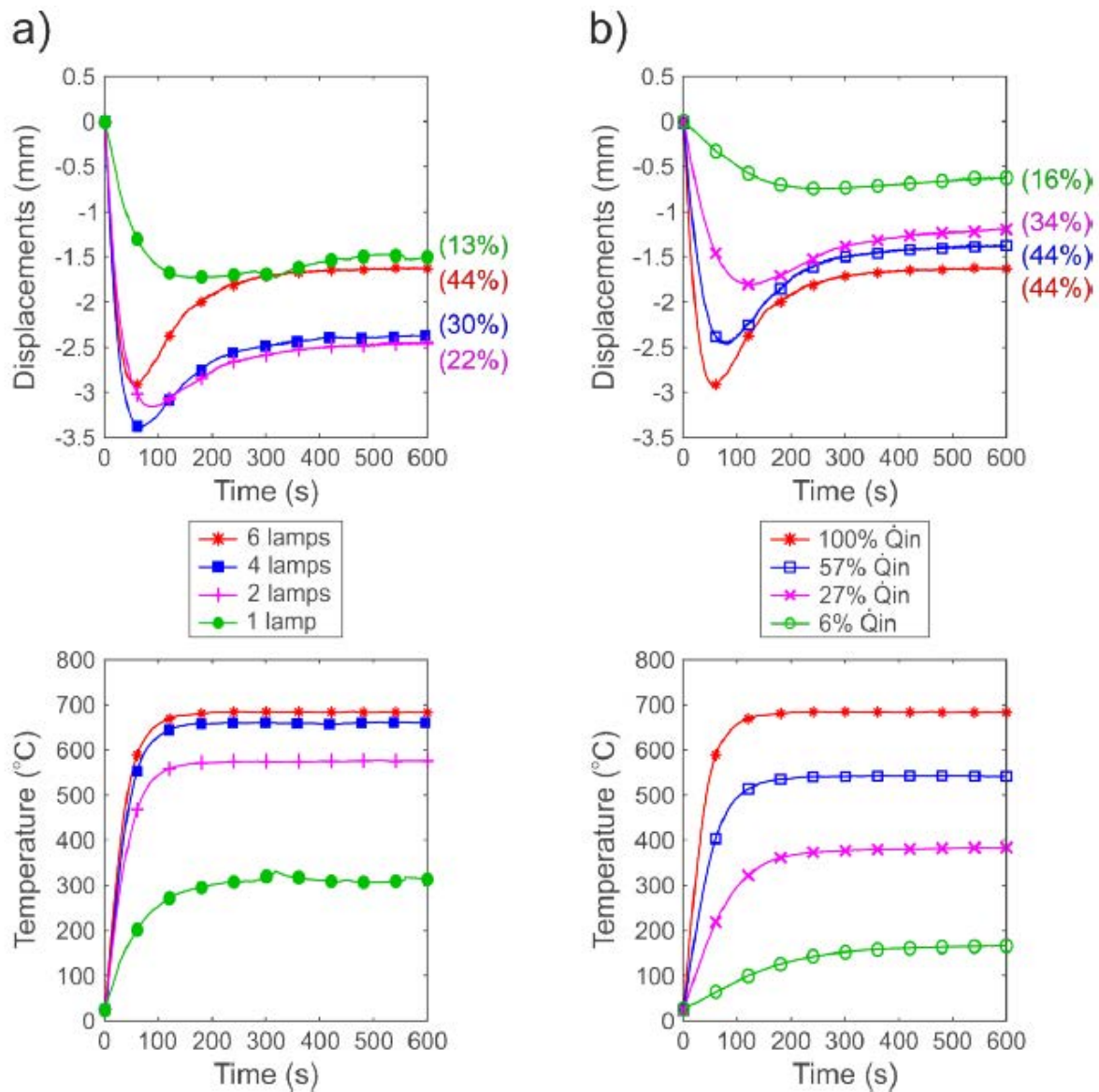


Figure 6.5 Point-wise displacement at the centre of the plate (top) and corresponding temperature measurements (bottom) as a) a function of the number of lamps (non-uniform heating) and b) the power output of six lamps (nominally uniform heating). For illustrative purposes, symbols were only included with the plotted lines at 60 seconds intervals. The level of central spring-back is shown in brackets in the displacement plots and the displacements are relative to the initial shape of the plate shown in Figure 6.3 (convex towards the lamps) and exclude rigid body motion.

6.3.2 Experiments on non-uniform temperature distributions

The suspended reinforced plate was thermally stressed using successively six, four, two and one quartz lamps, with their axes parallel to the short side of the plate. The lamps were uniformly distributed and positioned symmetrically about the plate's transverse and longitudinal axes. DIC and temperature data acquisition began at room temperature after which the lamps were switched on at full power and after approximately 200 seconds the plate reached a stable temperature distribution. The acquisition frequencies were 1 Hz and 4 Hz for the temperature data and DIC images, respectively.

In order to facilitate comparison with results from previous investigations^{3, 35, 36, 47}, the Istra software was used to remove rigid body motions (RBM) prior to selecting the data for the out-of-plane deflection at

the centre of the plate and plotting it as a function of time together the corresponding point-wise temperature results for each thermal loading scenario in Figure 6.5a).

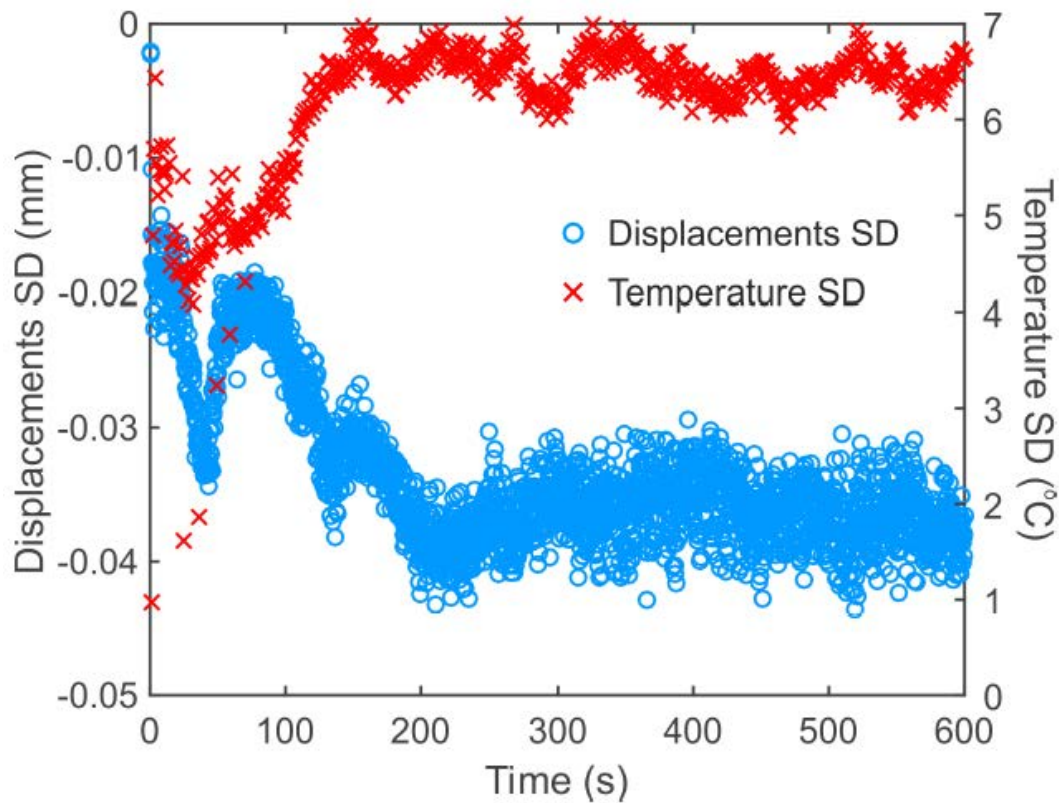


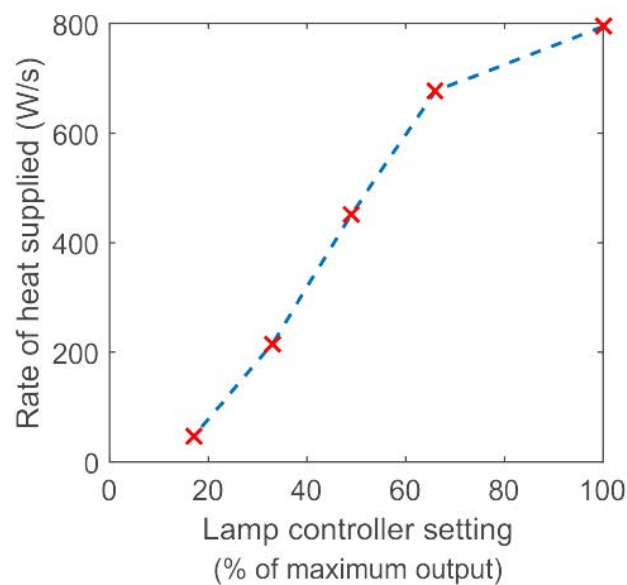
Figure 6.6 Standard deviation (SD) of displacements with rigid body removed (left axis and blue markers) and temperature (right axis and red markers) measurements at the centre of the plate based on six repetitions of heating the plate to a steady state using two lamps at full power.

To estimate the random error associated with these measurements, the same acquisition method for full-field temperature and DIC data was used when transversely loading the plate six times using 2 lamps at full power. The standard deviation of displacements (RBM removed) and temperatures at the central data point (facet) of the datasets was calculated and is shown in Figure 6.6. In addition, for each repeat, the location of the facet where the maximum displacement occurred was identified and this location varied by no more than a vector pitch (4.83 mm) in both the transverse and longitudinal directions.

6.3.3 Experiments on uniform temperature distributions

The use of a different numbers of lamps, in the previous section, produced a series of non-uniform spatial distributions of temperature along the longitudinal axis of the plate; and, the energy supplied to the plate (Q_{in}) was also changed by the number of lamps used. Hence, to better understand the observed behaviour, experiments were conducted using the maximum number of lamps to achieve an approximately uniform temperature distribution; but, adjusting the energy supply to the lamps so that Q_{in} corresponded to values in the previous set of experiments. Six lamps were used, and the energy output of each lamp was defined as a function of the rate of energy supplied to the plate at steady state, $\dot{Q}_{in}$, when using six, four, two and one lamps. It was assumed, for steady state conditions, that the

heat transfer from the plate to the environment was equal to heat supplied, $\dot{Q}_{in}$. This heat transfer was evaluated using the average temperature of the plate calculated using a surface integral fitted to the steady-state, full-field temperature measurements using the nearest neighbour method in MATLAB. The conduction heat losses through the hanging wire were found to be negligible and so, the sum of the steady-state convection and radiation heat transfer rates were equated to the unknown rate of heat supplied, $\dot{Q}_{in}$. The reinforced edges were excluded from these calculations. Based on this analysis, a calibration graph of rate of heat supplied, $\dot{Q}_{in}$ against controller settings was obtained using the temperature data collected with six lamps and used to identify controller settings for the six lamps that yielded an approximate rate of heat supplied, $\dot{Q}_{in}$ comparable to four, two and one lamps at full power. This graph is available in Figure 6.7.



Non-uniform heating	Lamp controller setting	Uniform heating
6 lamps @ $\dot{Q}_{in}$	100%	6 lamps @ $\dot{Q}_{in}$
4 lamps $\equiv 57\% \dot{Q}_{in}$	49%	6 lamps @ 57% $\dot{Q}_{in}$
2 lamps $\equiv 27\% \dot{Q}_{in}$	33%	6 lamps @ 27% $\dot{Q}_{in}$
1 lamp $\equiv 6\% \dot{Q}_{in}$	17%	6 lamps @ 6% $\dot{Q}_{in}$

Figure 6.7 Calibration graph of rate of heat supplied to the plate against controller settings (top). The settings used to yield approximately equivalent rates of heat supplied using four, two and one lamps at full power are also shown (bottom).

These settings were deployed using the same experimental methodology and data analysis as described in section 6.3.2. The resultant centre out-of-plane deflection and temperature data are plotted in Figure 6.5.

Table 6.1 DIC and temperature data: mean reconstruction errors, their standard deviation and number of retained shape descriptors after filtering.

Dataset	Mean reconstruction error (200 shape descriptors)		Number of retained shape descriptors		Mean reconstruction error after filtering	
	DIC	Temperature	DIC	Temperature	DIC	Temperature
6 lamps @ $\dot{Q}_{in}$	0.3% $\pm$ 0.1	1.8 °C $\pm$ 0.2	13	14	1.7% $\pm$ 0.5	4.6 °C $\pm$ 0.4
6 lamps @ 57% $\dot{Q}_{in}$	0.4% $\pm$ 0.1	1.8 °C $\pm$ 0.3	10	10	2.5% $\pm$ 0.6	5.0 °C $\pm$ 0.4
6 lamps @ 27% $\dot{Q}_{in}$	0.5% $\pm$ 0.1	1.1 °C $\pm$ 0.2	9	8	2.6% $\pm$ 0.6	5.7 °C $\pm$ 0.5
6 lamps @ 6% $\dot{Q}_{in}$	1.1% $\pm$ 0.4	0.6 °C $\pm$ 0.0	18	4	1.7% $\pm$ 0.4	4.9 °C $\pm$ 1.1
4 lamps $\equiv$ 57% $\dot{Q}_{in}$	0.3% $\pm$ 0.1	1.8 °C $\pm$ 0.2	12	22	2.5% $\pm$ 0.5	4.8 °C $\pm$ 0.5
2 lamps $\equiv$ 27% $\dot{Q}_{in}$	0.5% $\pm$ 0.1	1.3 °C $\pm$ 0.1	13	16	3.0% $\pm$ 0.4	5.9 °C $\pm$ 0.3
1 lamps $\equiv$ 6% $\dot{Q}_{in}$	1.2% $\pm$ 0.2	1.3 °C $\pm$ 0.1	27	16	3.3% $\pm$ 0.4	6.2 °C $\pm$ 0.5

Table 6.2 Threshold values for the filtering of shape descriptors from DIC and temperature data.

Data set	DIC data (mm)	Temperature data (°C)
6 lamps @ $\dot{Q}_{in}$	3.44×10^{-2}	2.24
6 lamps @ 57% $\dot{Q}_{in}$	4.14×10^{-2}	4.74
6 lamps @ 27% $\dot{Q}_{in}$	3.89×10^{-2}	4.06
6 lamps @ 6% $\dot{Q}_{in}$	4.24×10^{-3}	6.40
4 lamps $\equiv$ 57% $\dot{Q}_{in}$	4.96×10^{-2}	2.47
2 lamps $\equiv$ 27% $\dot{Q}_{in}$	3.50×10^{-2}	3.12
1 lamps $\equiv$ 6% $\dot{Q}_{in}$	9.19×10^{-3}	2.85

6.3.4 Full-field analysis of displacement and temperature data

The point-wise analysis, the results from which are shown in Figure 6.5, described the behaviour of the centre of the plate and was useful for comparing the response to different loading conditions and to previous studies. However, the digital image correlation provided displacement fields over the whole surface of the plate which can be quantitatively compared using orthogonal decomposition following the methodologies proposed by Patterson and his co-workers^{63, 64, 71}. As previously explained in this report, this decomposition method involves the data field being treated as an image and a set of orthogonal polynomials being fitted to the intensity values in the image. The coefficients of the polynomials, or shape descriptors, and can be collated in a feature vector from which the original data can be reconstructed^{65, 68, 70, 76, 137}. Similarly to the data decomposed in Chapter 5, the DIC and temperature data acquired experimentally are discrete data fields, defined in a rectangular coordinate system. Thus, and because the choice of polynomial is critical to the feature vector being a reliable representation of the original data, Chebyshev polynomials were used in the decomposition process, as proposed by Mukundan et al.⁷³ and previously adopted in other work^{10, 68, 115}. The protocol recommended by the CEN guideline for decomposition of displacement fields⁶⁸ was implemented using Euclid¹²³, which is freely available. Each data field was decomposed using Chebyshev polynomials following a similar methodology to the one developed by Lampeas et al.¹²⁹. This included decomposing the data with large number of polynomials terms but only those shape descriptors with values greater than a selected threshold were retained in the feature vector. In this work, 200 terms were used in the

decomposition process to ensure the reconstruction error was significantly smaller than the uncertainties of the measurements. The threshold was then selected to ensure that the root mean squared residual between each original data field and the reconstruction from its feature vector was less than or equal to the measurement uncertainty for every data field acquired during loading from 20 to 600 seconds. Hence, to achieve a suitable filtering of the shape descriptors, different thresholds were required for the displacement and temperature datasets acquired in each loading scenario. The initial 20 seconds of heating were excluded from the criterion for the threshold because some of the displacement measurements were below the noise floor level of the measurement system during this time period.

The uncertainty in temperature measurements was assumed to be 7°C, based on the manufacturer's calibration; whilst uncertainty in the displacement measurements were based on earlier work using a similar set-up by López -Alba et al.¹²⁵ and Sebastian¹¹ who found the errors associated with the PL-DIC system to be less than 4% of the amplitude of the displacements in dynamic events. This filtering of the original descriptors resulted in feature vectors that contained between 9 and 27 shape descriptors for the displacement fields and between 4 and 22 shape descriptors for the temperature fields. The number of retained shape descriptors after filtering is presented in Table 6.1, as well as the mean and standard deviation of the reconstruction errors for the decomposition of DIC and temperature data maps throughout the 20-600 seconds of loading. The threshold values used in the filtering of displacement and temperature data are presented in Table 6.2.

6.4 Results and discussion

The temperature and corresponding fields of z-direction displacement from the initial shape of the plate measured, in each experiment, in the steady-state after 10 minutes are presented in Figure 6.4 and Figure 6.10, respectively. In all cases, the plate is suspended from wires at its top corners and hence the displacement at these corners is approximately zero; however, the z-displacement in the bottom corners has positive values varying from close-to-zero to just above 1.5 mm as the plate rotates about its attachment points to counteract the buckling of its centre in the negative z-direction, i.e. to keep its centroid in the same x-y plane as the attachments. As would be expected, for a constant number of lamps supplying energy, the out-of-plane displacement field increases in magnitude with the rate of energy supplied, $\dot{Q}_{in}$ to the plate but without a significant change in shape (right column in Figure 6.10). However, the response was different when the number of lamps was varied (left column in Figure 6.10) because both the magnitude and shape of the displacement field changed with heat supplied.

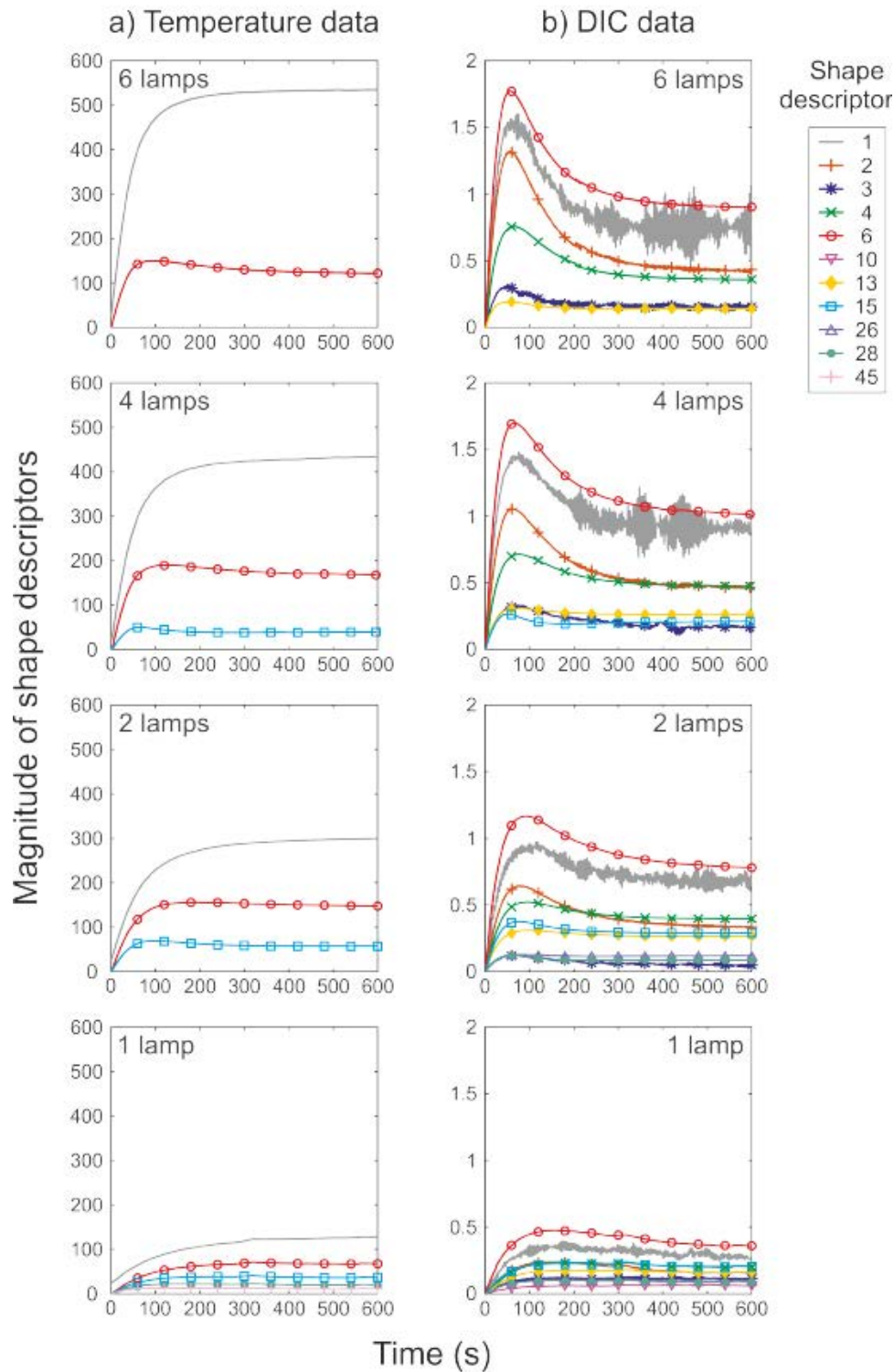


Figure 6.8 Magnitude of Chebyshev shape descriptors for a) temperature and b) displacement fields (including rigid body motion and relative to the initial shape in Figure 6.4) during thermal loading using non-uniform heating. For illustrative purposes, only the descriptors with values greater than 10% of the maximum-valued shape descriptor are presented. Absolute values were used and symbols were only included with the plotted lines at 60 seconds intervals.

The measured displacement fields are unlikely to contain significant errors because it has been previously shown that the errors associated with the PLDIC system are less than 4% for dynamic events¹²⁵. Room temperature measurements of the curvature of the plate were made before and after each thermal loading and the difference was found to be below the noise floor of the setup (16.6 μm). Therefore, it was concluded that the shape of the plate had not significantly changed as a consequence

of each loading scenario. The plate appeared to follow the same deformation path under heating and cooling conditions.

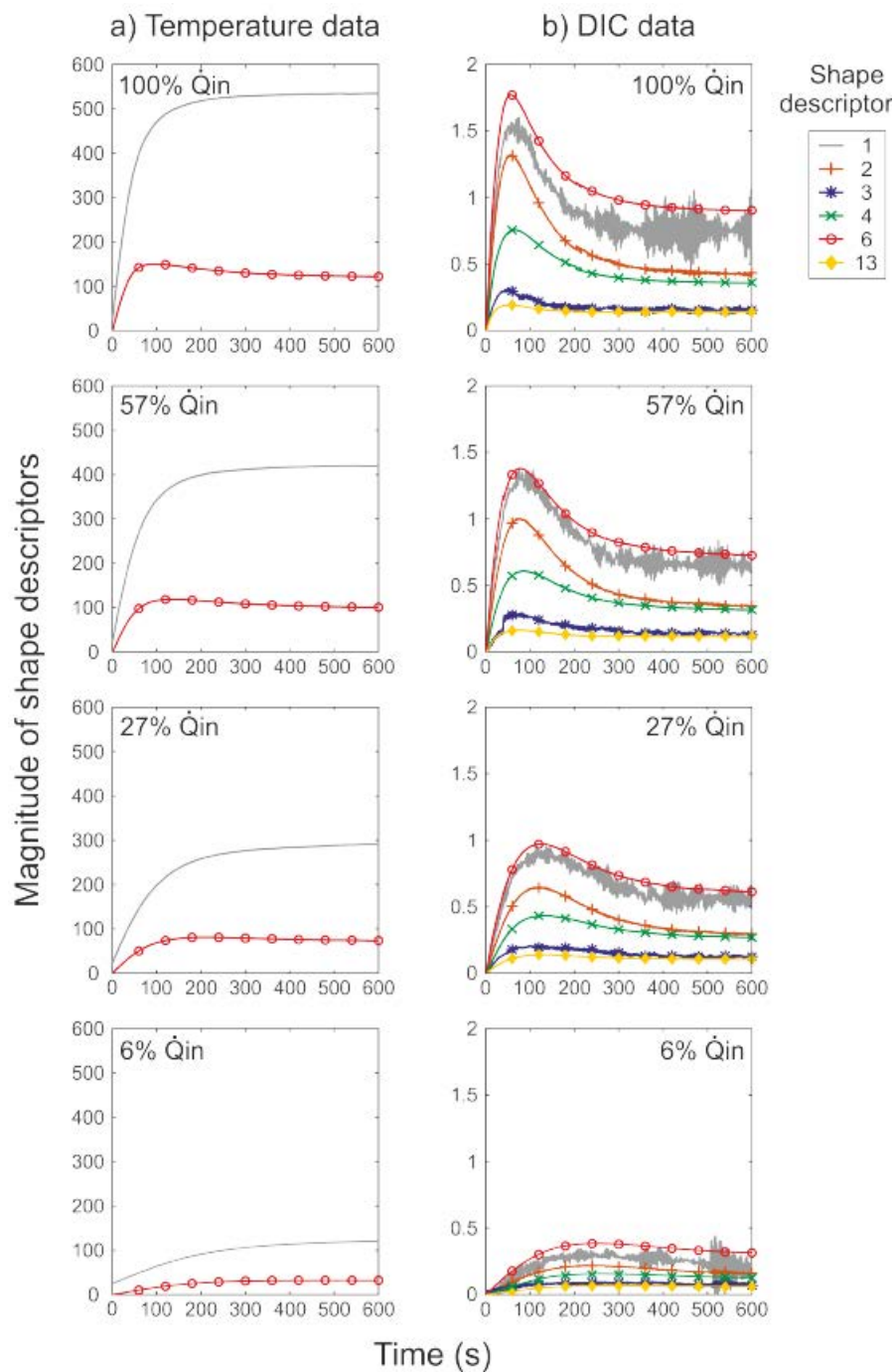


Figure 6.9 Magnitude of Chebyshev shape descriptors for a) temperature and b) displacement maps (including rigid body motion and relative to the initial shape in Figure 6.4) during thermal loading using nominally uniform heating with six lamps. For illustrative purposes, only the descriptors with values greater than 10% of the maximum-valued shape descriptor are presented. Absolute values were used and symbols were only included with the plotted lines at 60 seconds intervals.

The point-wise analysis of the out-of-plane displacement at the centre facet of the plate is presented in the top row of Figure 6.5 as a function of time and shows a dynamic response to the thermal loading. When the temperature reaches a constant value, the rate of plate deformation reverses and the plate relaxes to a steady-state position with a spring-back of the order 40% for heating with six lamps at full power. This behaviour is present for all lamp and power configurations in Figure 6.4, except for the

lowest level of heat supplied to the plate, i.e. one lamp at full power and six lamps at 6% of ($\dot{Q}_{in}$). In these two low energy cases, the rate of temperature increase with time was substantially lower than in the other cases and asymptotically approached a constant value with the result that the plate relaxes gently to a deformation of 13% to 16% of the maximum deflection. For the experiment conducted with a single lamp at full power, there appeared to be have been some fluctuation of lamp power output, which is evident in the temperature and DIC measurements as small perturbations. The error estimates in Figure 6.6 show the standard deviation of both displacement and temperature readings at the centre of the plate to be below $45\text{ }\mu\text{m}$ and 7°C , respectively, which supports the conclusion that the differences between the experiments, in terms of the displacement and temperature measurements in Figure 6.5, are significant. The temperature data in Figure 6.5 (bottom) was plotted as a function of the displacement measurements in Figure 6.5 (top) and the results are shown in Figure 6.11.

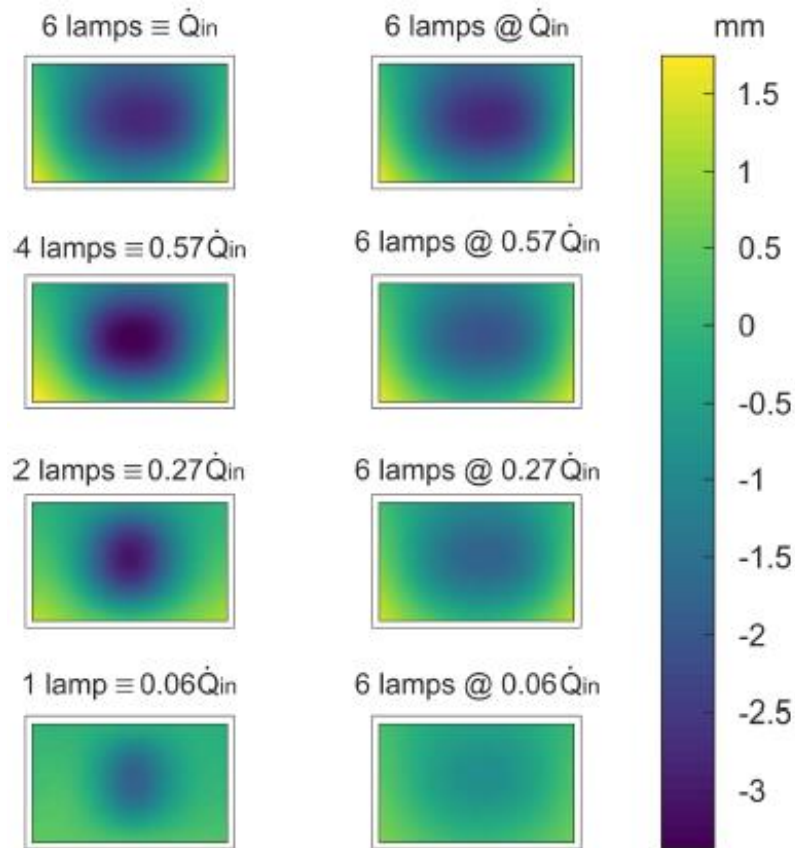


Figure 6.10 Steady-state measured out-of-plane displacements (including rigid body motion and relative to the initial shape in Figure 6.3) for the thin section of the reinforced plate subject to the thermal loads characterised by the temperature measurements shown in Figure 6.4.

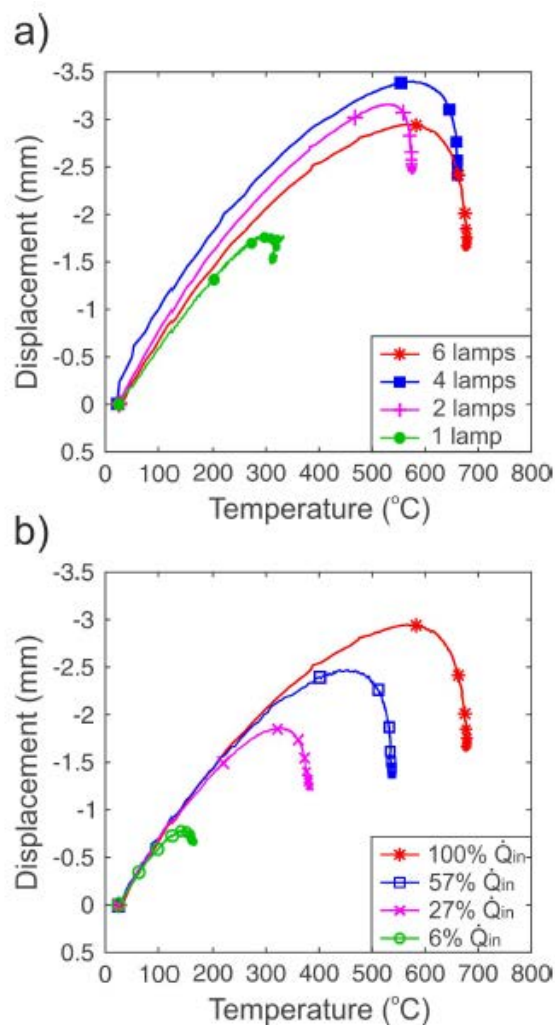


Figure 6.11 Point-wise displacement values at the centre of the plate as a function of the corresponding temperature when heating as a function of (a) the number of lamps and (b) the power output of six lamps. For illustrative purposes, symbols were only included with the plotted lines at 60 seconds intervals of loading. The displacements are relative to the initial shape of the plate shown in Figure 6.3 (convex towards the lamps) and exclude rigid body motion.

Table 6.3 Measured rate of temperature change and displacement at the centre of the plate for each condition in the first 10 seconds of heating. Rigid body motion has been excluded in the displacement measurements.

Nominal temperature distribution	Heating condition	Initial rate of temperature change (°C/s)	Initial displacement rate ($\mu\text{m/s}$)	Initial displacement/temperature gradient ($\mu\text{m}/^\circ\text{C}$)
Non-uniform	6 lamps	13.6 ± 5.1	117.6 ± 12.7	8.6 ± 2.5
	4 lamps	10.7 ± 5.1	127.7 ± 12.7	11.5 ± 2.5
	2 lamps	9.7 ± 5.1	97.9 ± 12.7	10.1 ± 2.5
	1 lamps	4.0 ± 5.1	25.6 ± 12.7	6.4 ± 2.5
	Mean	9.5 ± 5.1	92.2 ± 12.7	9.2 ± 2.5
Uniform	100% $\dot{Q}_{in}$	13.6 ± 5.1	117.6 ± 12.7	8.6 ± 2.5
	57% $\dot{Q}_{in}$	7.7 ± 5.1	72.0 ± 12.7	9.4 ± 2.5
	27% $\dot{Q}_{in}$	3.5 ± 5.1	32.2 ± 12.7	9.1 ± 2.5
	6% $\dot{Q}_{in}$	0.4 ± 5.1	5.3 ± 12.7	10.8 ± 2.5
	Mean	6.3 ± 5.1	56.8 ± 12.7	9.5 ± 2.5

For both sets of experiments, the gradients of these graphs for the first ten seconds are within 1.5 standard deviations of the mean of each set of experiments, see Table 6.3; i.e., varying the power or the number of lamps does not have a significant effect on the initial rate of change in the central

displacement with temperature. Figure 6.5a) and Figure 6.11a) show that the largest central displacement occurred when using four lamps at full power, with the next largest being for two lamps at full power. Hence, the central deflection of the reinforced plate under thermal load was not proportional to the rate of energy supplied when the spatial distribution of the energy supplied was also changed.

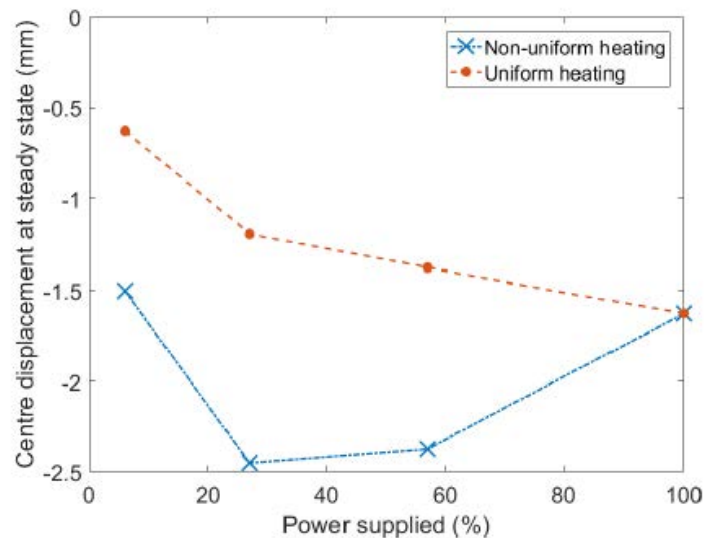


Figure 6.12 Point-wise displacement of the central facet of the plate as a function of power supplied. The measurements presented were acquired at steady state and rigid body motion has been removed.

A comparison of the results, in Figure 6.11a) and Figure 6.11b), suggests that the larger central deflections are associated with the thermal loading of a narrower region of the plate and, therefore, a steeper gradient in the spatial distribution of temperature in the longitudinal direction. This agrees with early analytical work by Mansfield¹³⁸ and computational predictions by Liu et al.¹³¹ that suggested the buckling temperature of a composite plate decreased with an increasing in-plane temperature gradient. A likely explanation of this behaviour is that the maximum temperature in the four-lamp case is approximately the same as the six-lamp case (see Figure 6.5 bottom left panel); so, the local reduction in modulus and local thermal expansion will be approximately the same in the centre. However, the temperature towards the edge of the plate is much lower in the four-lamp case (see Figure 6.4 top left two panels) so that the local reduction in modulus will be smaller leading to a higher level of constraint and this renders a local out-of-plane deflection at the centre more likely in the four-lamp case, as seen in the measurements (see Figure 6.11).

In experiments using nominally uniform heating, however, the location and number of the heat sources was not changed, hence the same area of the plate was heated in every case, causing the shape of the temperature fields shown in Figure 6.4 to remain similar and represented by shape descriptors # 1 and # 6, which were found to be the most significant descriptors in Figure 6.9a). In this scenario, the central displacement is proportional to the power supplied, except at the lowest level of power supplied, as shown in Figure 6.12.

These results for the out-of-plane deflection at the centre of an isotropic plate with reinforced edges are similar those observed by Pan et al.⁵⁷ for the dynamic behaviour of a thick honeycomb sandwich panel subject to thermal loads. Pan et al. noted the central displacement of the heated panel to be strongly dependent on the temperature difference between the front and back surfaces of the structure.

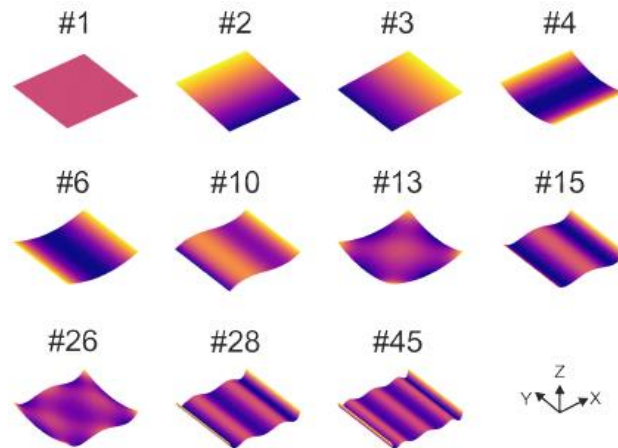


Figure 6.13 Shape represented by the Chebyshev shape descriptors used to describe the distributions of temperature and displacement shown in Figure 6.4 and Figure 6.10, respectively.

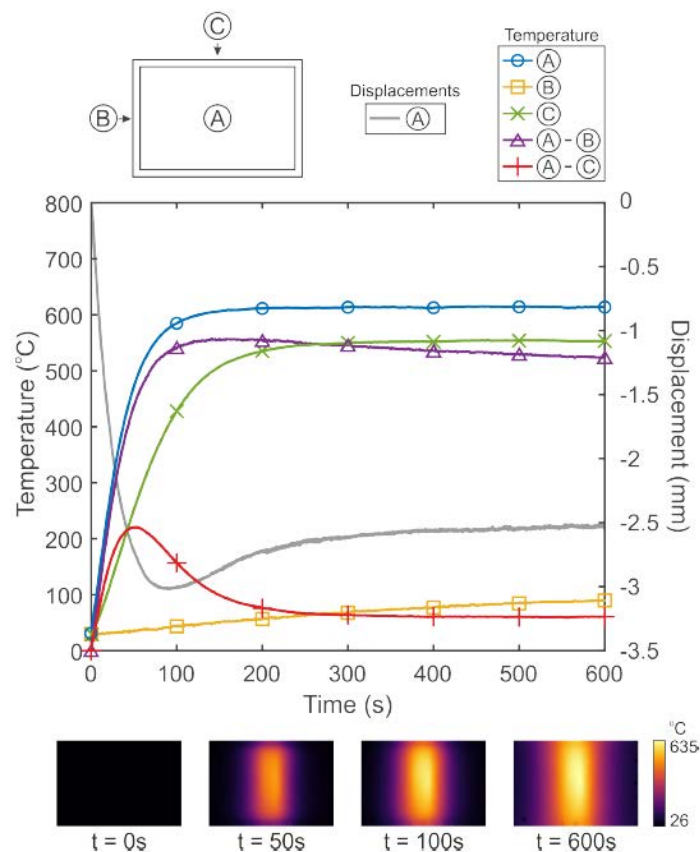


Figure 6.14 Point-wise temperature measurements at the centre and edges of the plate (top) when heating the structure with 2 lamps which results in the temperature distributions shown (bottom) that include the reinforced edges. The temperature differences between the point locations shown in the top diagram are also shown. The central displacement of the plate with rigid body motion removed is provided for comparative purposes and symbols are shown for illustrative purposes only.

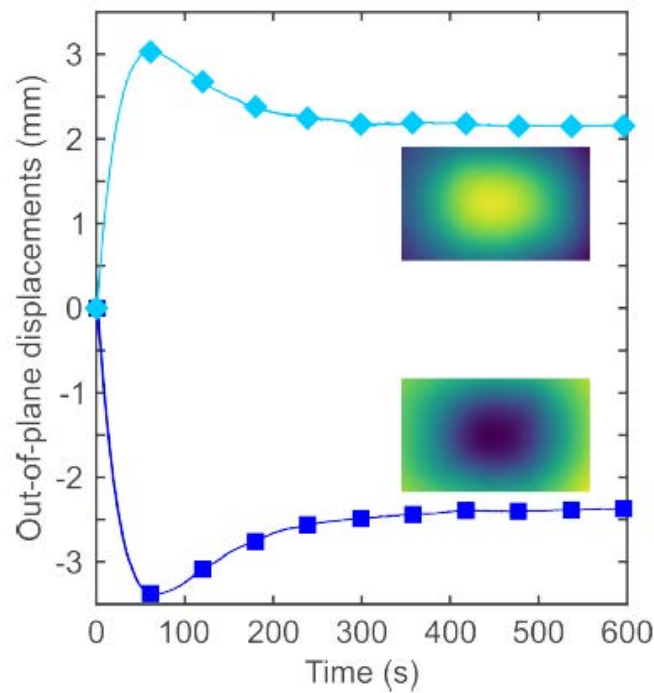


Figure 6.15 Centre displacement, with rigid body motion removed, of the plate in its original orientation (square markers) and rotated by π about its transverse axis (diamond markers) together with the shape of the thin plate at steady state as insets.

In the present study, it was not possible to measure this temperature difference because the heating lamps obstructed optical access to one surface of the plate. However, the micro-bolometer was used to measure temperatures on the top edge of the plate during one experiment using 4 lamps at full power. The results showed the difference between the temperatures at the front and back edges of the reinforced edge was less than 5% of the temperatures measured on the front of the plate. This suggests that additional mechanisms, to those postulated by Pan et al., are contributing to the deformation observed in this study. Namely, that the maximum temperature at the centre of the plate, shown in all of the spatial distributions of temperature in Figure 6.4, will lead to a larger thermal expansion in the centre than at the edges, which in turn, will cause the plate to buckle out-of-plane to accommodate this local expansion.

The shapes represented by the terms of the Chebyshev polynomials corresponding to the shape descriptors used in Figure 6.8 and Figure 6.9 are plotted in Figure 6.13. It can be seen that shape descriptor # 1 corresponds to a piston-term describing the average height of the shape, i.e., the average magnitude of the displacement or temperature distribution. The terms corresponding to shape descriptors # 2 and # 3 describe rigid body rotations about the x and y-axes respectively, and the higher order terms represent increasingly complicated shapes. Hence, as shown in Figure 6.9, shape descriptors # 1 and # 6, in decreasing magnitude, are the major contributors to the distribution of temperature in the plate heated with six lamps (Figure 6.4, right column) for all rates of energy supplied. However, a significant contribution of shape descriptor # 15 is needed to describe the temperature distribution generated when fewer lamps are used and a narrower section across the plate received most of the heat. The same shape descriptors appear as significant contributors to representing the corresponding

shape of out-of-plane displacement fields induced by the heating. However, shape descriptors # 2, # 4 and # 13 also make considerable contributions. Shape descriptor # 2 represents a rotation about the x- or longitudinal axis which implies a shift in the centroid of the plate causing it to rotate about the line through the holes to which the supporting wires are attached. Shape descriptor # 4 is indicative of bending about the x-axis and implies that the plate buckles as a consequence of in-plane compressive stresses in the y- or transverse direction; as well as buckling due to in-plane stress in the longitudinal direction indicated by shape descriptor # 6. The bi-directional buckling also leads to a small contribution from shape descriptor # 13 which corresponds to a dome-shape.

The results from the orthogonal decomposition of the displacement fields confirmed the dynamic behaviour of the plate identified in the point-wise analysis. The shape descriptors describing the displacement fields due to both non-uniform and nominally uniform temperature loading in Figure 6.8b) and Figure 6.9b) show that Chebyshev shape descriptors # 1, # 2, # 4 and # 6 behave dynamically before the plate reaches a steady state, i.e. their magnitudes reach a maximum before relaxing to a lower steady state value. The latter two represent half-waves along the longitudinal and transverse axes of the plate, respectively. Although the magnitude of shape descriptor # 1, describing the average magnitude of the temperature distribution, tends asymptotically towards a steady state value, the shape descriptors describing the shape of the temperature distribution, i.e. # 6 and # 15, exhibit the same form of curve with a maximum before relaxing to a lower steady state value. Probably, this is because the thin plate heats up more quickly than the reinforced-edge due to the difference in thermal capacities resulting from the difference in thickness; hence, when the plate initially buckles, the reinforced-edge is at a lower temperature and continues to expand due to its rising temperature which relaxes the constraint on the thin plate causing the magnitude of the out-of-plane displacement in the thin plate to be reduced.

Some evidence for this is provided in Figure 6.14 where point-wise temperature values at the centre and edges of the plate when using two lamps have been plotted as a function of time. The centre of plate was always hottest and transverse edge was always the coolest; however, the temperature differences between the plate's centre and the longitudinal and transverse edges are not constant and exhibit maxima that occur shortly before and after, respectively, the maximum out-of-plane displacement. These differences drive the differential thermal expansion and hence the in-plane membrane stress; thus, when the temperature differences are greater the buckling would be expected to be greater and to decrease as the temperature differences reduce to the steady state. This mechanism will operate regardless of whether the initial shape of the plate is convex or concave towards the heat source, as shown in Figure 6.15.

To the knowledge of the author of this thesis, the present work represents the first time that orthogonal decomposition has been applied to measurements of displacement and temperature fields acquired

concurrently during the thermal loading of an edge-reinforced plate. The resultant shape descriptors support the conclusions from the point-wise data and show that the in-plane temperature distribution had a significant impact on the magnitude and shape of its deformation, with localised heating causing larger deflections to occur. This is particularly relevant knowledge in cases, such as hypersonic flight, where the thermal loading is cyclic and non-uniform, which contributes to component fatigue and, eventually, ultimate failure of the structure.

6.5 Conclusions

Thermal loading of an edge-reinforced thin plate has been conducted using multiple configurations of heating lamps and power outputs that achieved maximum temperatures of around 700°C. Full-field displacement and temperature data were concurrently acquired while thermal loading of the plate for 10 minutes during which the temperature rose from room temperature to an approximately steady state temperature in about 200 seconds. The results showed that the reinforced plate behaved as a dynamic system in response to this thermal loading with a post-buckled shape that was dome-shaped. It was observed that the plate deformation was not proportional to the amount of energy supplied to the plate but dependent on the in-plane temperature distribution because heating of narrow sections across the plate appear to induce substantial compressive membrane stresses that caused out-of-plane buckling. This correlates well with predictions from Mansfield¹³⁸ and Liu et al.¹³¹.

Full-field data analysis using orthogonal decomposition confirmed the dynamic behaviour of the plate and suggested it to be at least partially associated to the change in temperature difference between the thin plate and its reinforced edges during the loading. The decrease in this temperature difference reduced the differential thermal expansion and hence relaxed the constraint imposed by the reinforced edges on the thin plate, which sprung back until it reached a stable state

Chapter 7 Discussion

The aim of this project was to investigate the effects of thermal and thermomechanical loading of aerospace-grade metal panels in laboratory conditions and to contribute towards the development of reliable computational mechanics models capable of predicting the structural response in such scenarios, which are typical of hypersonic flight. All investigations were done at a macro-scale level as the project's industrial pull is the ultimate integration of high-quality experimental data acquired in-situ and mission history data to aid in the prediction of future component performance. The objectives of this project were:

1. To experimentally study the effect of temperature on the dynamic response of aerospace panels, particularly the influence of non-uniform temperature distributions;
2. To develop and validate a computational solid mechanics model with temperature-dependent material properties, capable of predicting the behaviour of components when subjected to combined thermo-vibratory loads;
3. To perform experimental thermal and thermo-vibratory loading of aerospace-grade metal panels in order to acquire full-field, high-quality displacement and temperature data to be used in the development and validation of predictive models.

This discussion chapter relates these objectives with the project findings presented in Chapters 4-6 and the knowledge gaps identified in Chapter 2 and summarised below:

1. Knowledge gap number one: Lack of a successful integration of structural and temperature data in the developed analytical and numerical models. Very few past studies have used temperature-dependent material models in the study of the resonant response of isotropic plates.
2. Knowledge gap number two: No previous examples of concurrent full-field acquisition of displacement and temperature when studying the modal response of panel structures.
3. Knowledge gap number three: No studies had been found on the effect of temperature distribution on thermally loaded panels and, in particular, reinforced panels.
4. Knowledge gap number four: Correlations between temperature and displacement measurements have only been established in a point-wise domain.

An initial attempt to complete the second objective of this project and to bridge the first knowledge gap is presented in Chapter 4, where the development of an FE model with temperature-dependent material properties is detailed and results are focused on the resonant response of a thin plate, centrally constrained, and with free edges. The predicted resonant frequencies were quantitatively compared to previously published experimental data¹⁰. The same data was qualitatively compared with the

associated mode shapes. This work was a systematic study of the best practices in the development of a robust predictive model, which were later adopted in the model described in Chapter 5. The work yielded two major findings. First, an effective representation of the thermal load applied to the specimen is critical in the acquisition of reliable predictions of resonant frequencies and associated mode shapes. When the plate is subjected to a non-uniform increase in temperature, there is a localised expansion of the material, which promotes component deformation and, consequently, a change in its geometric stiffness. Computational models calculate the structural response based on both the geometric and material stiffness matrices of the plate, which requires the temperature-induced change in both these matrices to be adequately represented. Figure 4.16 best illustrates the fact that a transient model representing the change in thermal load was capable of predicting the non-linear changes in resonant frequencies of the structure when heated; which was not possible to when using a static model. Secondly, and closely related to the previous finding, the inclusion of the initial geometric shape in the plate is fundamental when predicting how the plate deforms under thermal load. In Figure 4.17, computational results using a flat plate showed the component to deform in-plane when subjected to a non-uniform temperature increase; however, the model was incapable of predicting out-of-plane deformation of the plate. This was resolved by including an initial, nominal curvature of the plate, which provided the local geometric stiffness needed for the initiation of the out-of-plane deformation of the structure. This supports observations made previously by Murphy et al.⁴⁴ who investigated the dynamic response of thermally loaded plates using a uniform temperature distribution and found initially curved structures to be inherently unstable, gradually deflecting with an increase in temperature. Contrarily, at plates are stable structures which remain at until a critical load is reached.

The model described in Chapter 5 included temperature-dependent material properties and was developed according to the findings described above. It included full-field measurements of specimen shape (Figure 5.10) and temperature (Figure 5.2), which were acquired experimentally and used in the definition of the plate's geometry and a linearly increasing thermal load, respectively. This integration of structural and temperature data in the model bridged the first knowledge gap. By quantitatively validating the results of this model against high-quality experimental data, the second objective of this project was completed. As shown in Figure 5.12, resonant frequency predictions were compared to experimental results by calculating their relative error against experimental data. However, the comparison of mode shape predictions and experimentally-acquired displacement maps has been a more challenging process for researchers as Mottershead and his co-workers^{70, 81, 82} have shown the method traditionally used for this purpose, the Modal Assurance Criterion (MAC), to be inadequate for revealing subtle differences in the data sets, particularly in high-order mode shapes. These authors also showed that orthogonal decomposition was able to overcome this problem.

Hence, the quantitative comparison of predicted and experimentally-acquired mode shapes was performed using orthogonal decomposition and guided by the method proposed by the Comité

Européen de Normalisation (CEN)⁶⁸ on the validation of computational solid mechanics. Mode shape data was decomposed using discrete Chebyshev polynomials and the resulting coefficients from experiments and simulations were compared and presented in Figure 5.14. An excellent agreement between predictions and experimental results for both frequencies and mode shapes was demonstrated and represented a significant improvement on past efforts by Berke et al.¹⁰, whose predictions relied on a static analysis performed using a simplistic material model and a discretised temperature distribution across the specimen. Unlike the model developed in Chapter 5 of this thesis, their model failed to predict any significant change in resonant frequencies with an increasing temperature and, thus, did not correspond to experimental observations.

Knowledge gap number two highlighted the lack of experimental, full-field displacement and temperature data when thermo-mechanically loading a structure. The acquisition of such relevant data on aircraft structures has been a difficult and expensive process, as documented in Chapter 2, and it has included the development and instrumentation of full-scale test aircraft. This has created a data vacuum that limits the knowledge behind the thermo-mechanical behaviour of structures in extreme environments, restricts the optimisation of their design and leads to overly conservative maintenance plans. Hence, there is the need for high-quality data acquired in laboratory conditions that can be used to validate predictive models. Whilst very few past investigations have focused on the whole-field measurement of displacements^{7, 10}, none included the acquisition of temperature fields. The third objective of this project aimed to address this gap and it was achieved with the work presented in Chapter 5. Resonant frequencies of a thin Hastelloy X plate were acquired using an established method in experimental modal analysis that involved the broadband excitation of the structure using a mechanical shaker. Excitation frequencies were kept between 0 and 800 Hz, which characterises the range at which thin, lightly-damped aircraft structures experience high-cycle fatigue. The plate's response was monitored using a laser Doppler vibrometer, which allowed for the calculation of the system's response function. The mode shapes associated to each resonant frequency were captured using a Pulsed Laser Digital Image Correlation (PL-DIC) system initially developed by Sebastian¹¹ and performing a single frequency sinusoidal excitation of the plate. The PL-DIC system permitted the motion of the plate to be "frozen" by synchronising the acquisition of DIC images with a 4 ns pulse of bright, green light. Optical bandpass filters with a wavelength corresponding to the laser light (532 nm) were fitted to the DIC cameras, which prevented the radiation from the heating lamps from saturating the camera sensors. This resulted in an experimental setup with excellent optical access to the specimen. Whilst this is a relatively novel method in the acquisition of mode shapes, studies by Sebastian¹¹ have shown the accuracy of the PL-DIC system to be equal to or better than alternative methods using high-speed cameras because the increase in frame rate sacrifices image resolution, which greatly affects the accuracy of DIC measurements.

The acquisition of temperature data was initially achieved using a photon detector which was not factory-calibrated for high temperatures. Measurements were obtained by fitting the detector with a neutral density filter and performing a manual calibration with the help of a hand-held thermometer. However, this was found to be an error-prone method and a difficult process for the researcher because the infrared lamps used to heat the specimens required the use of safety goggles which also eliminated the low-power laser beam from the laser-guided thermometer. As such, the photon detector was abandoned and replaced by a more affordable, commercially-available micro-bolometer. The complete remote operation of both the DIC system and thermal camera was achieved by setting up a high-temperature test chamber, pictured in Figure 3.4, with fitted CCTV cameras that assures the safety of the operator and allows for the monitoring of experiments as they are performed. These full-field displacement and temperature data acquisition systems were then used to collect high-quality experimental data, which was the base for consequent investigations.

The combined findings of the work presented in Chapters 5 and 6 provide a joint contribution to the completion of the first objective of this project by studying the effect of the distribution of temperature on the structural behaviour of thin plates. This is a significant contribution to the field of hypersonic structures as past investigations have mostly targeted uniform temperature distributions, which do not reflect real-world scenarios. Chapter 5 focused on the thermomechanical loading of a thin plate with free edges and the concurrent acquisition of full-field displacement and temperature data, bridging knowledge gap number two. Experiments were repeated using different heating lamp configurations which allowed for the effect of temperature distribution across the structure on the resonant frequencies and mode shapes of the plate to be studied, therefore addressing the third knowledge gap. Chapter 6 also contributed towards the third identified knowledge gap by focusing on the acquisition of full-field displacement and temperature data of a reinforced plate heated using a number of different lamp configurations and power outputs. The fourth knowledge gap has been addressed in Chapter 6 by using orthogonal decomposition to attempt to distil significant information from this full-field displacement and temperature data and extract relevant conclusions.

In Chapter 5, the resonant frequencies of a centrally-constrained plate with free edges were acquired experimentally at room temperature and compared to results obtained with two distinct high-temperature distributions: one acquired during the transverse heating of the plate's centre and a second one acquired when longitudinally heating the plate's bottom edge, as shown in Figure 5.2. The two heating scenarios yielded a spatial variation in the distribution of temperature, which was achieved at steady state. The change in temperature distribution was shown to affect the shape of the plate at high temperature as its free edges allowed the plate to freely deform and adopt different geometric configurations, dependent on the thermal stresses generated by the thermal load (Figure 5.17). Compared to room-temperature results, the double curvature induced by the transverse heating of the

plate in Figure 5.17 (top) was shown to lead to an increase in resonant frequencies. This is evident in Figure 5.16 and likely due to a high geometric stiffness - typical of domed structures - which appears to dominate over the material softening at high temperatures. These results support simulation work published by Mead³⁵, and show that the non-uniform heating of a thin plate with free edges can induce thermal stresses that change the geometric stiffness of the structure, competing with the decrease in material stiffness caused by thermal softening and promoting a structural resistance to deformation. When heated longitudinally along its bottom edge (Figure 5.17 (bottom)), the plate adopted a saddle shape and, compared to its room-temperature counterparts, the resonant frequencies were shown to increase for some modes and decrease for others (Figure 5.16). Such behaviour is expected, as the single curvature of the plate is likely to provide varying degrees of structural resistance to the deformation patterns, contrarily to the double curvature which forms what Berke et al.¹⁰ termed a "stabilising ring" for all modes of vibration. The behaviour of simply-supported rectangular plates heated into the post-buckling regime has been investigated previously using point measurements by Murphy et al.⁴⁴ and using analytical and numerical modelling by Chen and Virgin^{33, 36} and by Jeyaraj et al.³³. However, this study represents a significant advance by developing and demonstrating the technology to acquire high-quality full-field measurements and by making quantitative comparisons with predictions from a computational model.

The findings from Chapter 5, made it apparent that the temperature-induced deformation of the structure strongly influenced its dynamic behaviour. So to better understand the influence of the distribution of temperature in the deformation of aerospace panel structures, a geometry more realistic of aircraft skin structures was developed and studied in Chapter 6. This consisted of a specimen designed to emulate the stringers and ribs used as reinforcements for aircraft skins. A 4.5 mm Hastelloy X plate was machined down to 1 mm while leaving a 10 mm wide frame of the original thickness. The specimen was conditioned prior to experiments to impart an initial curvature to the structure, as real aerospace components are rarely at. No mechanical excitation was provided to the specimen. Different numbers of quartz lamps at full power (six, four, two and one lamps) were used to thermally load the central transverse axis of the panel, which promoted a variation of the spatial distribution of temperature and the energy supplied to the plate, Q_{in} . For each of the loading scenarios, the full-field temperature measurements at steady state were used to calculate an equivalent rate of heat supplied, $\dot{Q}_{in}$, using a set number of six lamps. A controller was then used to adjust the power output of the six lamps and load the plate with an approximate rate of heat supplied comparable to four, two and one lamp at full power. Displacement results were acquired using the PL-DIC system and showed that the deformation of the plate was not directly proportional to the amount of energy supplied but dependent on the in-plane temperature distribution, as shown in Figure 6.5 and Figure 6.10. This is likely due to the induction of compressive membrane stresses when narrowly heating the plate, which confirms predictions made by Mansfield¹³⁸ and Liu et al.¹³¹. It was observed that the plate behaved dynamically

with an increase in temperature, both locally (Figure 6.5) and globally (Figure 6.8 and Figure 6.9). This behaviour was investigated for the first time using a full-field analysis method - orthogonal decomposition, or image decomposition. A freely available orthogonal decomposition software was used to decompose the full-field data into shape descriptors associated to Chebyshev polynomials. The analysis of the most significant shape descriptors of displacement and temperature data acquired throughout the several experiments revealed the shape of both displacement fields and temperature distribution maps to behave dynamically. Further inspection of the temperature difference between the centre of the plate and its reinforced edges throughout the loading was presented in Figure 6.14 and suggested this to drive a differential in thermal expansion between the two. The centre of the plate was shown to reach steady state earlier than its edges due to the higher heat capacity of the thicker frame relative to the thin plate. Hence, as the local temperature at the plate's centre stabilises, the reinforced edges continue to heat up and expand. This mechanism appeared to be responsible for the relaxation of thermally-induced stresses which causes the plate to spring back. The findings in Chapter 6 are important in designing the skin of structures subject to high heat fluxes as they support that the distribution of the energy flux across the components strongly influences their behaviour. However, other parameters including plate thickness, variation of mechanical properties of the material with temperature and residual stresses introduced during manufacturing, are all likely to influence the behaviour. Nevertheless, the results in Chapter 6 will be relevant in cases where the thermal loading is cyclic and the distribution of temperature non-uniform. One such example is, once again, hypersonic flight in which aircraft components are exposed to the described conditions that can lead to premature failure of the structures due to fatigue.

Chapter 8 Conclusions

The pursuit of hypersonic flight requires an understanding of the effect of thermal and thermo-mechanical loads on the behaviour of aircraft structures and, in particular, thin-gauge panels which are susceptible to a loss of stability due to in-plane compressive stresses. To date, however, the number of test flights resulting from hypersonic technology development programs have been few and the acquisition of relevant data is difficult. Whilst computational mechanics models can be used to predict the structural response of components and optimise aircraft design, these need to be suitably validated using high-quality experimental data acquired in controlled environments. A review of the available literature conducted for this project showed little research on the dynamic behaviour of thermally-stressed panels using full-field techniques in the acquisition of displacement and temperature data. Notably, limited investigations have been found on the effect of non-uniform temperature distributions across aircraft panels.

In this research project, full-field experimental data has been acquired on the dynamic behaviour of simple and reinforced plates which have been heated non-uniformly up to approximately 700°C. Moreover, the best practices in the development of a temperature-dependent computational model have been investigated, contributing significantly to the knowledge. The work undertaken during this project has yielded two conference papers, two published journal papers^{125, 139} and a third paper has been accepted for publication. The major contributions are as follows:

The influence of the distribution of temperature on the dynamic behaviour of a thermally-loaded thin panel has been demonstrated experimentally, for the first time, using the concurrent acquisition of full-field temperature and displacement data. Experiments on both free-edge and edge-reinforced plates have shown the distribution of temperature to greatly affect how the plates deform and, in the case of the free-edge plate, how it responds to mechanical excitation. These results confirm experimentally, and for the first time, predictions published in literature^{33, 35, 36, 131, 138} and represent a significant advance in the development of the technology used to acquire full-field, high-quality displacement and temperature measurements. The findings are relevant in the design and monitoring of panels operating in high-temperature environments, such as exhaust-wash structures associated with embedded engines in aircraft, reusable space vehicles, sustained hypersonic flight vehicles and breeder blankets in fusion reactors.

For a centrally-constrained plate with free edges, investigations showed the energy required to excite the structure to change with temperature and to be strongly dependent on its spatial distribution across the plate. The transverse heating of the plate yielded an increase in resonant frequency with temperature, whilst longitudinal heating did not generate a significant change. This was found to be a consequence of the difference in curvature of the plate at high temperatures. The work yielded a new

and useful dataset for a plate under combined thermal and mechanical loading by using full-field experimental methods in the acquisition of high-quality displacement and temperature data and by making quantitative comparisons with predictions from a computational model. The results provide more detailed information on the behaviour reported in previously published work and highlight the requirements for a reliable computational model. This contribution bridges the second and third knowledge gaps identified in Chapter 2.

Heating an edge-reinforced thin plate up to temperatures of around 700°C showed the thin plate to behave as a dynamic system in response to the thermal loading. For the first time, full-field temperature and displacement maps acquired throughout the thermal loading have been analysed using orthogonal decomposition and the full-field data suggest the plate's dynamic behaviour to be at least partially associated with the evolution of temperature difference between the thin plate and its reinforced edges during the loading. Experiments were conducted using multiple configurations of heating lamps and power outputs and results showed that the deformation of the thin plate was not proportional to the amount of energy provided to the structure but depend more so on the in-plane spatial distribution of temperature. This contribution addresses the third and fourth knowledge gaps identified in Chapter 2.

The development of a computational model which included temperature-dependent material properties and experimental readings of temperature and plate geometry. The model was validated against experimental data and shown to be capable of reliably predicting the resonant frequencies and mode shapes of a thermally stressed plate. This was a three-fold exercise that involved: 1) determining the best practices on the development of a finite element model using temperature-dependent material properties, which emphasised the need to appropriately represent both the shape of the plate and the transient thermal load applied to it, 2) achieving a successful integration of experimental data in the development of the predictive model, and 3) validating the predictions against resonant frequencies and mode shapes acquired experimentally. Results showed the inclusion of full-field experimental measurements of temperature and shape in a temperature-dependent material model to improve the reliability of predictions when compared to more simplistic models. This contribution bridges the first knowledge gap identified in Chapter 2.

Chapter 9 Experimental study of mode shifting in an asymmetrically heated rectangular plate^r

9.1 Abstract

Modal shifting and jumping in thermally buckled plates has been investigated previously using computational models; however, relatively few experimental explorations have been reported. In this study, the modal shifts and jumps in a simple rectangular plate or panel were investigated using digital image correlation to obtain detailed mode shapes. One face of the planar panel, which was 219x146 mm, was speckled and viewed with a stereoscopic digital image correlation system using pulsed-laser illumination and with a laser vibrometer at a single point. The other face was heated using a set of 1 kW quartz lamps to produce a non-uniform temperature distribution which progressively increased from room temperature to around 760K. An infra-red camera recorded the time-varying temperature distribution on the panel while it was excited with an electrodynamic shaker. A waterfall plot, or time-frequency spectrogram, showing natural frequencies as a function of time was generated, which highlighted the shifting response of the panel with temperature. The heating sequence was repeated and the panel excited at the natural frequencies identified in the waterfall plot, which permitted the corresponding mode shapes to be measured. These data showed that mode shifting and jumping was present with asymmetric heating, but not with spatially uniform heating, and was associated with thermally-induced buckling.

9.2 Introduction

It has been recognised for some time that panels or plates subject to spatially non-uniform temperature distributions undergo buckling as the temperature distribution varies with time. In plates that are also subject to vibration, the buckling causes mode shifting²³ and mode jumping¹⁴⁰. Mode shifting is the exchange of vibration modes when the plate is heated into the post-buckled state; while mode jumping is sudden transitions from one buckling mode to another. This type of behaviour could potentially occur in the panels that form the skin of aircraft designed to travel at hypersonic speeds or near the engine exhausts in more conventional aircraft as well as in fusion reactors. The presence of this type of

^r This chapter describes work performed by Dr Elias Lopez-Alba during 2017 and has been published as Lopez-Alba E, Sebastian CM, Santos Silva AC & Patterson EA, Experimental study of mode shifting in an asymmetrically heated rectangular plate, *J. Sound & Vibration*, 439:241-250, 2019.

behaviour could have a significant impact on the structural integrity and, hence, the service life of these structures.

It is well-known that the natural frequencies of vibrating plates or panels vary with in-plane stress such that compressive stresses reduce frequencies and tensile stress increase them¹⁴¹. When in-plane stresses are caused by constraint due to thermal expansion then increases in temperature may cause natural frequencies to reduce and, or increase. The behaviour is complicated by the presence of boundary conditions that cause buckling of the plate; and spatially non-uniform temperature distributions can induce such conditions, without any external constraint, as a consequence of local areas of tension and compression³⁵. Bailey²⁷ investigated this type of behaviour in double-wedge square-cantilever plates with thin edges to induce local constraint, while Kaldas and Dickinson¹⁴² induced in-plane stresses by introducing a weld along the centre line of a rectangular plate; in both cases, the temperature range observed was not large, but they confirmed the linear relationship between the natural frequency and compressive load for some modes but not for others. When narrowband acoustic loading is combined with thermal loading, clamped rectangular plates have been shown to exhibit aperiodic snap-through phenomena that are characterised by large non-linear amplitudes⁴⁴, while pre- and post-buckling regimes with small amplitudes exhibited linear responses. There are many studies reported in the literature of analytical and numerical modelling of post-buckling behaviour and snap-through phenomena, including more recently by Chen and Virgin^{36, 124} who investigated mode jumping and shifting in simply-supported rectangular plates heated deeply into the post-buckled state; while Ribeiro¹⁴³ studied plates with fixed boundaries. More recently, using the finite element method, Jeyaraj¹⁴⁴ investigated the behaviour of vibrating isotropic plates subject to non-uniform thermal loading and Han et al.¹⁴⁵ studied the free vibration and buckling of foam-filled corrugated sandwich plates under thermal loading. However, there is very little experimental evidence to support these modelling studies¹⁴⁶, primarily because of the difficulty associated with acquiring modal shapes at elevated temperatures. Murphy et al.⁴⁵ used a central strain gauge to confirm the behaviour of rectangular steel plates of dimensions 300x375x1.5mm subject to sinusoidal and broadband excitation with modest temperatures. Jeon et al.¹⁴⁷ investigated the free vibration of rectangular plates (100x100x2mm) plate subject to rapid thermal loading with halogen lamps using a laser scanning vibrometer with thermocouples to monitor the temperature; and Jin et al.¹⁴⁸ used digital image correlation to characterise thermal buckling of a compound disc consisting of a 50mm aluminium disc surrounded by a titanium ring which was heated to 160°C, without mechanical loading or excitation. Recently, time-frequency spectrograms or waterfall plots have been measured to characterise the temporal variation in modal behaviour of structures during heating using a nominally uniform temperature distribution¹⁴⁹. More generally, Helfrick et al.¹⁵⁰ and Reu et al.¹⁵¹ reviewed the advantages and disadvantages of using DIC for full-field vibration measurements, while Beberniss and his co-workers^{152, 153} have used high-speed digital image correlation to capture the nonlinear response of

structures included computing the FRF using data from selected locations. Scanning laser vibrometers also enable high-resolution displacement fields to be measured but the sequential measurements result in very long test periods particularly when the region of interest is large¹⁵⁴. The aim of this study was to generate detailed measurement data for a vibrating plate subject to asymmetric heating that could be used to support and validate computational modelling. To this end, a time-frequency spectrogram has been obtained for a simple rectangular plate subject to a spatially non-uniform temperature distribution as it is heated from room temperature into the post-buckling state and, through the use of a pulsed laser system for stereoscopic digital image correlation (DIC)¹¹⁵, the corresponding mode shapes have been measured.

9.3 Methodology

A rectangular plate 219mm x 146mm was cut from a 1mm thick plate of Hastelloy X (American Special Metals Inc., Miami FL, USA) and a 5mm diameter hole was drilled at its centre. The physical properties of the plate were not measured but typical values can be found in Table 1¹⁵⁵. The plate was spray-painted with a thin coat of matt black paint (VHT Flameproof, Cleveland OH, USA) on the front and back surfaces to aid heat transfer and to provide a base for a DIC speckle pattern. The speckle pattern was sprayed onto one face of the plate using white paint (VHT Flameproof, Cleveland OH, USA). The plate was mounted on a shaker (V100, Data Physics, San Jose CA, USA) using a M4 stainless steel rod, which was secured through the hole using a nut. The stinger was the only external attachment to the plate so that it can be considered a ‘free-free-free-free’ plate. The speckled surface faced away from the shaker and a set of halogen quartz lamps (QIR 240 1000 V2D, Ushio, Steinhöring, Germany) were placed between the shaker and the plate, on either side of the stinger, as shown in Figure 9.1 and Figure 9.2. The lamps were arranged in sets of five in specially designed mounts with stainless steel backing sheets that reflected their output towards the plate and shielded the shaker. Each lamp had an output of 1kW and a colour temperature of 3210K. The lamps were located 1.5cm from the surface of the plate and were spaced at 3.5cm (centre-to-centre) with lamps adjacent to the stinger being 5cm from it. A typical heat map for one lamp is shown in Figure 9.3.

The behaviour of the plate was monitored using a laser vibrometer (OFV-503, Polytec GmbH, Waldbronn, Germany) which was aimed at a small area in one corner of the plate where the paint had been removed and the metal surface polished using wet P800 paper in order to provide a strong reflection of the signal. An uncooled micro-bolometer (TIM 400, Micro-Epsilon UK, Birkenhead, UK) was used to monitor the temperature distribution on the specked surface of the plate. A pulsed-laser stereoscopic digital image correlation (DIC) system was used to record the shape of the plate. This system consisted of an Nd: YAG laser (Nano L200-10, Litron, Rugby, UK) that emitted a 4 nanosecond pulse of light at 532nm, which passed through a specially designed set of optics that expanded the laser beam and removed the speckle using an optical Fourier filter. The images for digital correlation were

acquired by a pair of 1624x1234 pixel CCD cameras (2MP Stingray F-201b, Allied Vision Technologies GmbH, Stradtroda, Germany) to which were fitted optical narrowband filters centred on 532nm, so that data was only acquired during the laser pulse and the cameras were not saturated by the heating lamps. The cameras were controlled by a trigger box and software (Q-400 system, Dantec Dynamics GmbH, Ulm, Germany) and had a frame rate of 10 frames per second. The images were processed using the Istra 4D software supplied with the DIC system and employing square facets of 25 pixels with a centre-to-centre spacing of 21 pixels.

Table 9.1 Typical physical properties of Hastelloy X [from¹⁵⁵]

	Temperature, °C	Metric Units
Density	22	8.22 g/cm ³
Melting range	1260 - 1355	
Dynamic modulus of elasticity (heat treated at 1177°C, rapid cooled)	Room	205 GPa
	93	203 GPa
	204	197 GPa
	316	192 GPa
	427	184 GPa
	538	178 GPa
	649	170 GPa
	760	161 GPa
	871	153 GPa
	982	141 GPa
Mean coefficient of thermal expansion	26 - 93	13.9 10 ⁻⁶ m/m-°C
	26 - 538	15.1 10 ⁻⁶ m/m-°C
	26 - 649	15.5 10 ⁻⁶ m/m-°C
	26 - 732	15.8 10 ⁻⁶ m/m-°C
	26 - 816	16.0 10 ⁻⁶ m/m-°C
	26 - 899	16.4 10 ⁻⁶ m/m-°C
	26 - 982	16.6 10 ⁻⁶ m/m-°C
Poisson's ratio	22	0.32

The experiment was conducted in two stages using the set up shown in Figure 9.1 and Figure 9.2. First, the vibrometer was used to establish a time-frequency spectrogram for the chosen heating sequence, as shown in Figure 9.3 and Figure 9.4. In the second stage, time-frequency spectrogram was employed as a

map and the modal shapes of the plate were determined, for particular thermal conditions and the corresponding natural frequencies, using the digital image correlation system.

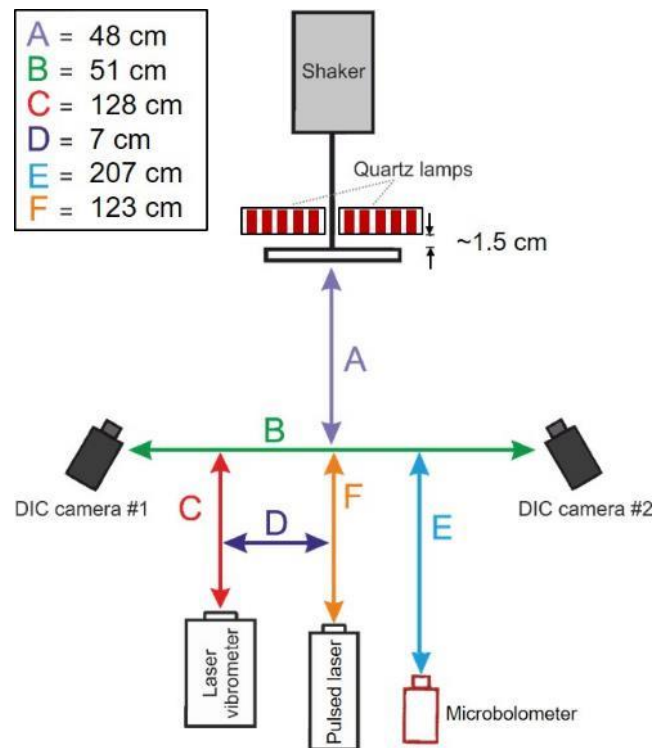


Figure 9.1 Diagram showing the experimental arrangement.

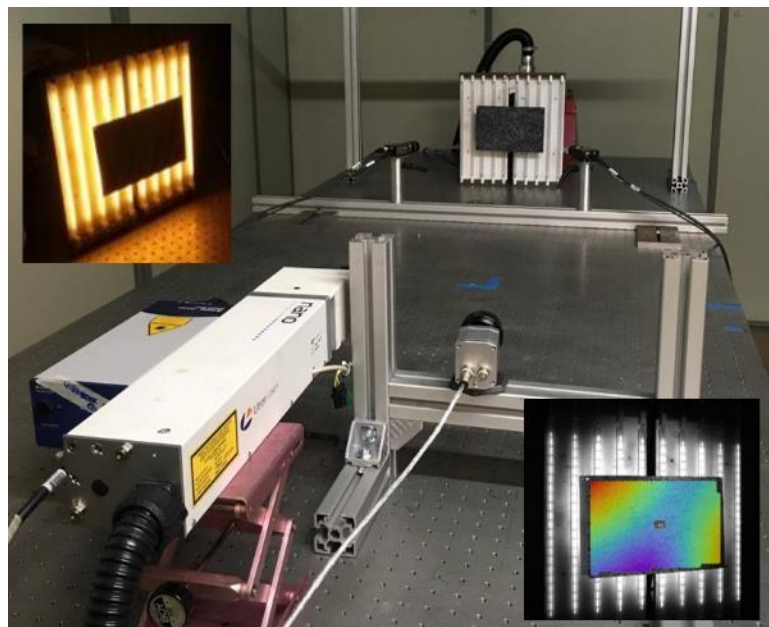


Figure 9.2 Photograph of the experimental arrangement showing the plate in the background in front of two sets of lamps with the shaker behind them and the vibrometer, pulsed-laser and micro-bolometer (from left to right) in the foreground; and inset, with the lamps switched on: the image obtained from DIC camera with the narrowband filters removed (top left), and the out-of-plane data from DIC overlaid on a DIC camera image obtained using the pulse-laser illumination (bottom right).

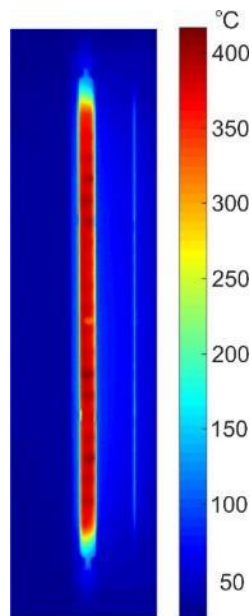


Figure 9.3 Heat map for a typical quartz lamp at steady state under full power.

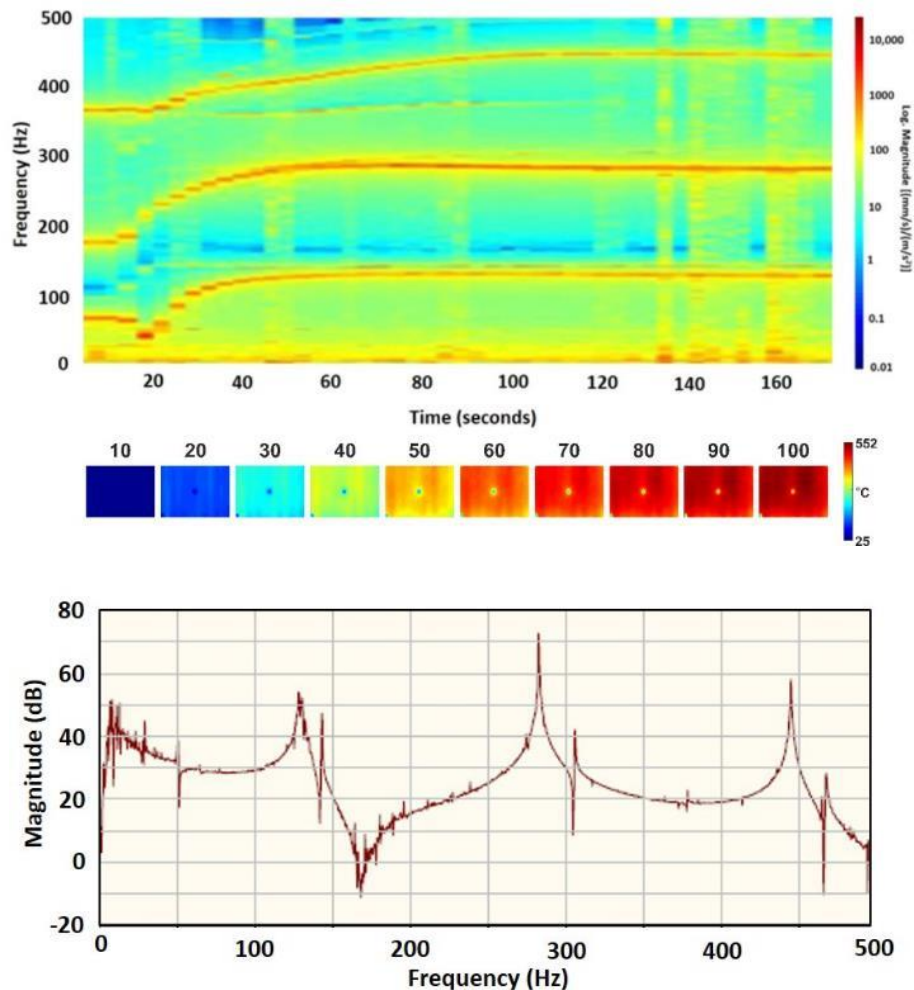


Figure 9.4 Time-frequency spectrogram (top) for the plate subject to uniform heating from room temperature by two sets of lamps that were switched on after 10 seconds, as illustrated by the temperature distributions (middle) measured by the microbolometer. Each vertical slice through the spectrogram represents the transfer response function for that instant in time, as shown in the bottom graph.

In the first stage of the experiment, a broadband random excitation between 0 and 500 Hz was applied to the plate using vibration controller (ABACUS, Data Physics, San Jose CA, USA) and a 1kW power amplifier system (DSA1-1K, Data Physics, San Jose CA, USA). An accelerometer, attached to the output face of the actuator of the shaker, provided the input signal while the output signal was taken from the vibrometer. The two signals were processed, using the signal analyser provided with the shaker system (SignalCalc, Data Physics, San Jose CA, USA), to obtain the transfer response function for the plate. This measurement was started at room temperature and then, after a nominal 10 seconds, the lamps were switched on, at full power, to heat the plate while the analysis was performed continuously. The results are shown in Figure 9.4, in the form of the time-frequency spectrogram, for the uniform heating of the plate using two sets of lamps arranged on each side of the stinger. The contours of high magnitude values of the transfer function show the change in natural frequencies as the temperature of the plate increases; there is no evidence of mode shifting or jumping because the uniform temperature distribution does not generate any of the constraints that are required to induce this type of behaviour. However, the time-frequency spectrogram for the asymmetric heating shown in Figure 9.5 exhibits much more complex behaviour with evidence of both mode shifting and jumping taking place, which were investigated in more detail in the second stage of the experiment. The asymmetric heating was produced using only four of the five lamps in the set on the left side of the plate, which produced the time-varying temperature distributions shown in Figure 9.5. The second stage of the experiment was conducted with only the asymmetric heating.

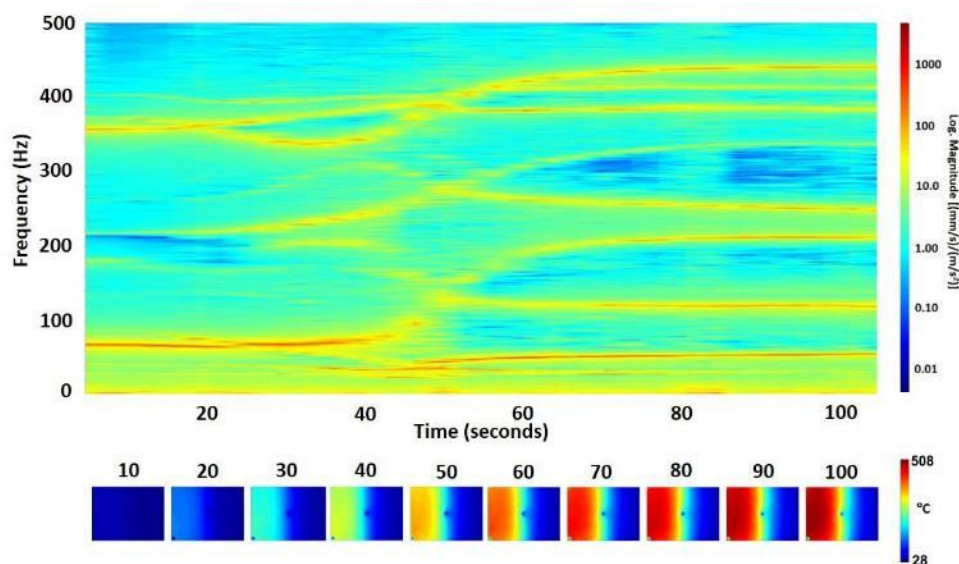


Figure 9.5 Time-frequency spectrogram for the plate subject to heating from room temperature by the asymmetric arrangement of four 1kW lamps switched on at 10 seconds (top), together with the resultant temperature distributions obtained from the microbolometer (bottom).

The goal of the second stage of each experiment was to track the modes through the corresponding spectrogram from the first stage. Consequently, the heating of the plate was repeated but a single frequency excitation was applied to the plate using the shaker controlled by a function generator and

data recorded using the digital image correlation system. The single frequencies were selected using the time-frequency spectrogram to identify the natural frequency at a specific time in the heating sequence. In general, DIC data could only be acquired for one natural frequency during each heating sequence and, hence, the heating sequence had to be repeated many times to allow the modal shapes to be measured along the contour lines in the spectrogram.

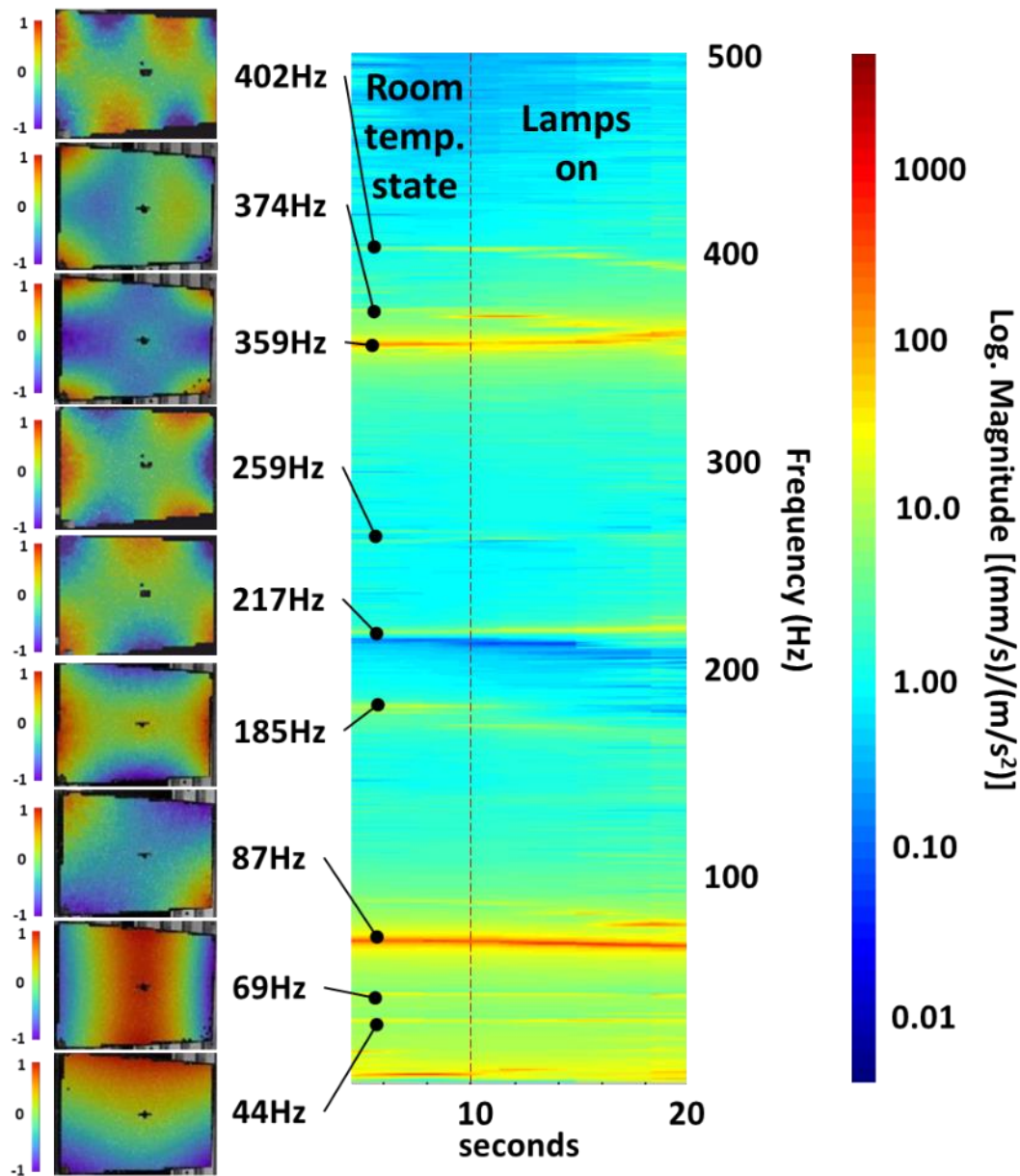


Figure 9.6 Normalised modal shapes (left) measured using digital image correlation at room temperature when the plate was excited at the single frequencies identified from the time-frequency spectrogram (right) prior to switching on the lamps at nominally ten seconds (as shown by vertical dashed line).

9.4 Results and Discussion

A large number of deflection shapes were measured in the second stage of the experiment at single frequencies corresponding to the natural frequencies identified in the time-frequency spectrogram in Figure 9.5; and hence, it is appropriate to refer to them as mode shapes, as opposed to operating

deflection shape which would contain components of forced vibration⁵². The results obtained from digital image correlation at room temperature are shown in Figure 9.6 together with the initial twenty seconds of the spectrogram for the asymmetric heating shown in Figure 9.5. The room temperature portion of the spectrograms were always identical.

The modal shapes were measured using stereoscopic digital image correlation and then used to track the mode shifting and jumping that occurred during heating of the plate. For instance, Figure 9.7 shows that in the range 300 to 500 Hz there are three modes that shift during asymmetric heating of the plate. The higher frequency mode, which at room temperature is found at 402 Hz, initially reduces in frequency with heating and then rises to a steady-state of 413Hz during plate heating. However, the two lower frequency modes exhibit large excursions from their room temperature states and shift position twice before reaching a steady state after about 60 seconds of heating. It is noticeable that the form of the modal shapes changes after the first shift at 19 seconds with some loss of symmetry in the modal shape that occurs at 359 Hz at room temperature and a phase reversal of the shape that occurs at 374Hz at room temperature. There is further distortion of the shapes after the second shift that occurs at 49 seconds.

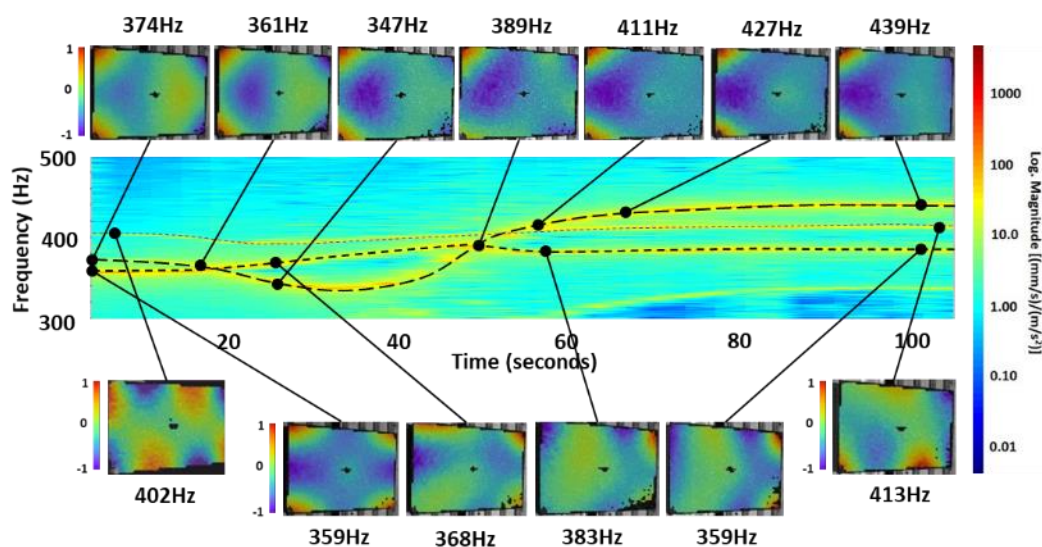


Figure 9.7 Time-frequency spectrogram (middle) between 300 and 500 Hz for the asymmetric heating shown in Figure 9.5 with two modes shown that exhibit mode shifting (long and medium dashed lines) together with the corresponding normalised mode shapes obtained from digital image correlation, together with a third mode whose frequency is almost constant with heating (short dashes). The colour bar refers to the time-frequency spectrogram.

In the interval between 200 Hz and 350 Hz (see Figure 9.8), two modes are present at room temperature that almost merge after approximately 50 seconds before separating again without any shifting or jumping.

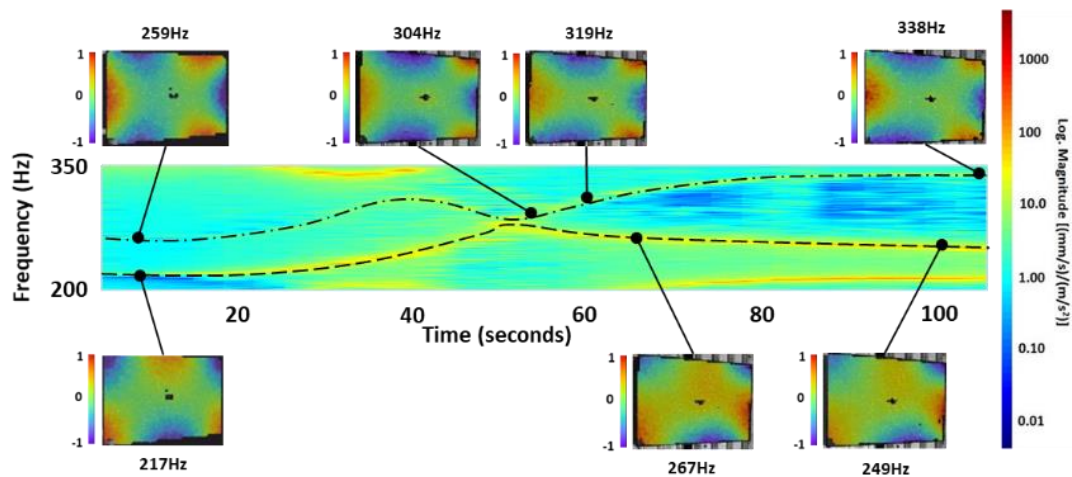


Figure 9.8 Time-frequency spectrogram (middle) between 200 and 350 Hz for the asymmetric heating shown in Figure 9.5 with two modes at room temperature that almost merge after approximately 50 seconds before separating again, together with their corresponding mode shapes obtained from digital image correlation. The colour bar on the right refers to the time-frequency spectrogram and shapes are normalised.

Below 200Hz (shown in Figure 9.9) the dominant first bending mode at 69Hz disappears around 40 seconds into the heating sequence. The less prominent diagonal bending mode is present at 87Hz at room temperature and shifts down in frequency to 57Hz under the steady state heating conditions at 100 seconds. However, a second diagonal bending mode appears at 60 seconds into the heat sequence at 120 Hz. Unfortunately, because of constraints on resources, it was not possible to track the modes below 50Hz or to obtain more detail on the mode jumping and shifting between 50 and 200 Hz.

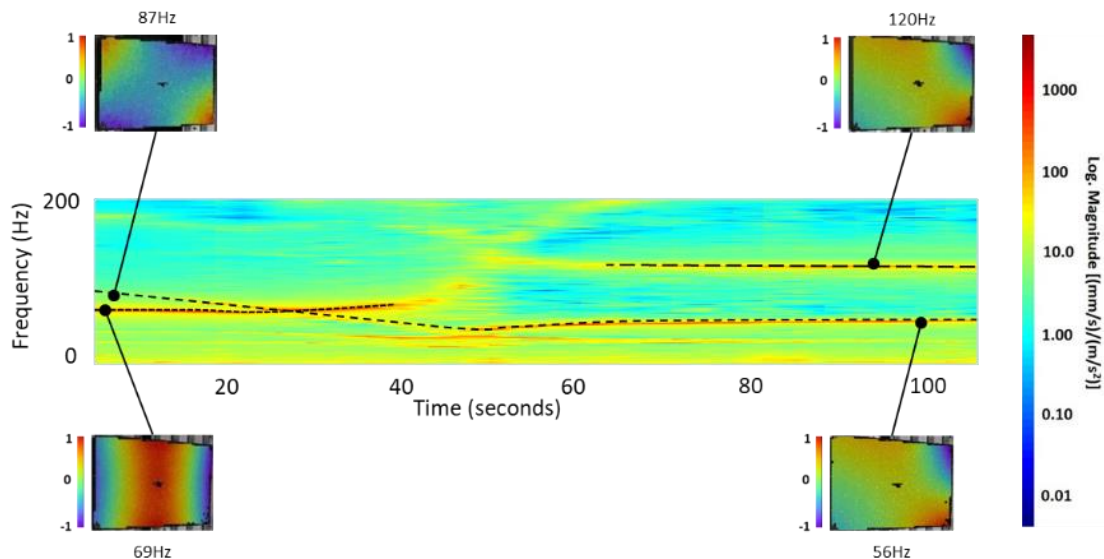


Figure 9.9 Time-frequency spectrogram (middle) below 200 Hz for the asymmetric heating shown in Figure 9.5 showing the first bending mode at 69Hz (short dashes) disappearing about 40 seconds into the heating sequence and an additional diagonal bending mode (long dashes) appearing about 60 seconds into the sequence at 120Hz; a diagonal bending model at 87Hz at room temperature shifts frequency to 56 Hz during the heating sequence (medium dashes). The colour bar on the right refers to the time-frequency spectrogram and the shapes are normalised.

It can be seen from the time-frequency spectrum in Figure 9.5, as well as the extracts in Figure 9.7 to Figure 9.9, that the plate undergoes a major change in behaviour between 40 and 60 seconds into the asymmetric heating sequence, with mode shifting and jumping over a wide range of frequencies. When

the plate was subject to the same heating sequence in the absence of the mechanical excitation, then it exhibited buckling at the same stage of the sequence, as shown by the plot of out-of-plane displacement as function of time in Figure 9.10. The plate takes up a new stable shape after about 70 seconds, i.e. following a buckling process in which it transitions from one equilibrium shape to another. Hence, it would appear that the mode shifting and jumping is associated with a change in the static configuration of the plate caused by the spatially non-uniform temperature distribution as predicted by Chen & Virgin^{36, 124}. This behaviour is similar to the elastic buckling observed by Thornton et al⁵², when they heated a rectangular plate with a single quartz bulb to a maximum temperature of about 190°C and measured the temperature distribution with 29 thermocouples and displacement with 15 linear variable differential transducers.

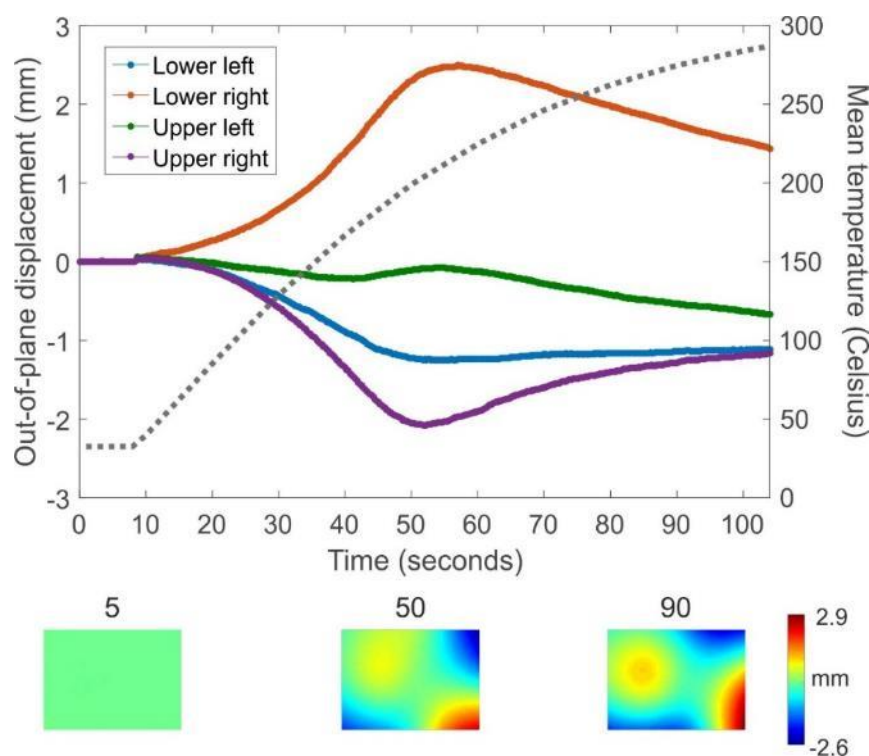


Figure 9.10 Out-of-plane displacements at the corners of the asymmetrically heated plate as a function of time during heating sequence together with the mean temperature of the plate (dashed lines) with displacement maps shown below; the displacement measurements were made using digital image correlation in the absence of any mechanical excitation and show the thermal buckling of the plate occurring between 40 and 60 seconds.

It has been shown previously that the relative measurement uncertainty for the pulsed-laser digital image correlation system is less than 4% when used to measure modal shapes for frequencies from 120 Hz to 2000 Hz. So, it is unlikely there are significant errors in the shape measurements; however, it was difficult to control the synchronisation of the heating process and the capture of images for the DIC and, hence, there could be some errors associated with a lack of synchronisation. These errors could be responsible for the apparent phase reversal of some of the modal shapes.

The data presented illustrate the potential for experimental validation of numerical and analytical models of mode shifting and jumping. A relatively small selection of data has been presented in the

interests of space; however, very extensive data can be acquired relatively easily although the experiments are time-consuming. Sebastian et al¹¹⁵ have discussed the use of image decomposition to reduce the dimensionality of modal data to allow statistical comparisons with detailed predictions from models; and the modal shape data from a time-frequency spectrogram could be compressed in a similar way to allow equivalent quantitative comparisons for the purpose of model validation and updating.

9.5 Conclusions

Physical tests have been conducted on a simple rectangular plate subject to broadband excitation between 0 and 500 Hz while being heated to around 760K using multiple quartz lamps. The arrangement of the lamps could be easily modified to achieve non-uniform heating both spatially and temporally, while recording the surface temperature distribution of the plate with a micro-bolometer and its surface shape using a pulse-laser digital image correlation system. The investigation of modal shapes has been guided by a time-frequency spectrogram recorded using a laser vibrometer during the heating sequence, which allowed the correlations between modal shapes, frequency and time or temperature to be made. The results showed that mode shifting and jumping was present when asymmetric heating was applied to the plate but was not present with spatially uniform heating, which concurs with earlier modelling studies; and it is associated with the thermally-induced buckling.

Chapter 10 Study of a rectangular plate under acoustic loading

using thermoelastic stress analysis

10.1 Abstract

In the past, limited number of studies have used thermoelastic stress analysis (TSA) technique to analyse the vibratory response of structures. There has been a lack of clarity in the literature on the interpretation of the TSA data to obtain mode shapes and stresses from components subject to vibrational loading. Hence, the aim of this study was to establish the interpretation of TSA data to accurately determine mode shapes and quantitative stresses. A rectangular plate was cyclically loaded at four different resonant frequencies and the response was measured using both TSA and pulsed-laser digital image correlation (PL-DIC). It was hypothesised that X-image of the TSA data is indicative of mode shape whereas the R-image provides a quantitative measure of the first stress invariant. The hypothesis was reinforced by comparing the X- and phase-images of the TSA data with the mode shape and the constructed phase maps acquired from PL-DIC, respectively. The results demonstrate the potential capability of TSA in accurately determining the mode shapes and quantitative stress maps of a panel under thermo-acoustic loading.

10.2 Introduction

TSA is a well-established, non-contact technique used for obtaining full-field stress data from the surface of a cyclically loaded component. This technique utilises an infrared detector to measure minute temperature fluctuations associated with the time-varying elastic deformation in a solid. A volume of material when subjected to elastic compressive stress experiences a slight increase in temperature, whereas, applied tensile stress results in a small decrease in temperature. The theoretical basis for this phenomenon, known as the thermoelastic effect, was first proposed by Lord Kelvin in the 1850s. A detailed description of the mathematical theory underpinning TSA has been presented in a review by Pitarresi and Patterson¹⁵⁶. The thermodynamic relation that relates the temperature change (ΔT) to the change in strains for an elastic solid is as follows:

$$\Delta T = \frac{T}{\rho c_\epsilon} \sum \frac{\partial \sigma_{ij}}{\partial T} \Delta \epsilon_{ij} + \frac{\delta q}{c_\epsilon} \quad (10.1)$$

where T , ρ , c_ϵ , σ_{ij} , ϵ_{ij} and δq are the absolute temperature, density, specific heat capacity at constant strain, stress tensor, strain tensor and heat transfer per unit mass, respectively. TSA is typically

performed at a sufficiently high frequency to ensure pseudo-adiabatic conditions, thereby allowing the second term in Eq. 1 to be ignored:

$$\Delta T = \frac{T}{\rho c_\epsilon} \sum \frac{\partial \sigma_{ij}}{\partial T} \Delta \epsilon_{ij} \quad (10.2)$$

For an isotropic material under plane stress condition, Eq. 10.2 can be simplified to:

$$\Delta T = -\frac{\alpha T}{\rho c_\sigma} \Delta(\sigma_{11} + \sigma_{22}) \quad (10.3)$$

where α and c_σ are the coefficients of linear thermal expansion and specific heat capacity at constant stress, respectively. ΔT is measured using an infrared detector in terms of the voltage output (S); therefore, the working form of Eq. 10.3 for practical TSA is:

$$AS = \Delta(\sigma_{11} + \sigma_{22}) \quad (10.4)$$

S is defined as the output signal which corresponds to the thermoelastic effect and is proportional to the change in sum of principal stresses or the first stress invariant. A is the calibration factor and is considered to be a function of both the material properties of the component being investigated and the detector parameters. The calibration factor is typically determined experimentally by measuring the response from the surface of a calibration specimen with a known stress state. The surface of the specimen is usually painted with a uniform coating of matt-black paint to enhance the surface emissivity, and hence, the photon flux¹⁵⁶. It also helps minimising the reflections of infrared radiation from the component's surface which are not associated with the thermoelastic response¹⁵⁶.

A signal processing unit called a 'lock-in analyser' is used to extract the thermoelastic signal (S), which is embedded in an inherently noisy signal output from the infrared detector¹⁵⁷. To extract S, the lock-in analyser requires a reference signal at a loading frequency, which is usually obtained from the function generator of a loading frame. The lock-in analyser can therefore be simply considered as a band pass filter that discards all frequency components in a raw detector signal other than the component, representing the thermoelastic signal, whose frequency is same as the loading frequency of the object¹⁵⁸. The thermoelastic signal (S) is presented in the form of vector, whose magnitude represents the thermoelastic response and whose orientation represents its phase difference with respect to the reference signal. The phase shift can result from many factors such as low cyclic frequencies, high stress gradients, plastic deformation or even variation in paint thickness.

Very few studies have analysed the vibratory response of the structures using TSA. Phan et al.¹⁵⁹ employed TSA to determine the mode shapes of a cantilever beam and a rectangular plate with clamped

edges at excitation frequencies below 300 Hz. Later, Backman and Greene¹⁶⁰ explored the efficacy of TSA in obtaining the mode shapes of a gas turbine blade over a frequency range of 68 Hz to 3.4 kHz. There has been a lack of clarity in these studies about the interpretation of TSA data in obtaining both the mode shape and the quantitative stress information. As mentioned earlier, TSA signal is in the form of a vector where the X component of the signal is in phase with the loading signal and Y component is 90° out-of-phase with respect to the loading signal. For a specimen subject to uniaxial cyclic loading, assuming adiabatic conditions, the X component of the TSA signal is typically in phase with the loading signal resulting in negligible Y component. In this scenario, X component represents the first stress invariant. In case of a rectangular plate subject to cyclic loading at one of its resonant frequencies, it is entirely possible for different regions in the plate to be under tension and compression simultaneously. This causes a 180° shift in the TSA response acquired from regions experiencing tension and compression at the same time. In this scenario, the R component of the TSA signal, which is a vector sum of X and Y components, should represent the stresses in the plate. In the earlier studies, the X component was instead attributed to the stresses in the material. Hence, the focus of this study is to establish an interpretation of the TSA data to allow the deduction of both the mode shape and the surface stress map from a rectangular plate subject to cyclic loading at a resonant frequency.

10.3 Experiment methods

A 210×148 by 1mm thick Hastelloy X plate was attached to the shaker using a stainless steel stinger. A broadband test, described in Section 5.3, was performed to determine the frequency response function (FRF) and consequently the resonant frequencies of the plate over a frequency span of 1000 Hz. The FRF of the plate is shown in Figure 10.1. The plate was then excited at four different resonant frequencies i.e. 365, 374, 603 and 880 Hz and TSA data were obtained. To capture the raw infrared response from the plate's surface, an infrared camera (SC7650, FLIR) was employed and the thermoelastic signal was extracted using the DeltaTherm acquisition system. The camera is capable of acquiring images using the full sensor of 640×512 pixels at a maximum frame rate of 250 Hz. For these experiments, the camera was operated at a frame rate of 327 Hz with a reduced active sensor window of 320×256 pixels. The camera was mounted with a 13mm lens and placed at a distance of approximately 400mm from the plate. The images were acquired using the exposure time of 600 μs and integrated over a period of 2 to 5 minutes, depending on the excitation frequency, to obtain the thermoelastic response. It has been demonstrated by Backman and Greene¹⁶⁰ that it is entirely possible to acquire a thermoelastic response, with the help of the lock-in analyser, at excitation frequencies exceeding the frame rate of the infrared camera; however, it is important to ensure that the camera frame rate is not the same or too close to the excitation frequency and that the exposure time should not exceed one-third of the oscillation period. Pulsed-laser digital image correlation method, described in Section 3.2, was also used to acquire full-field out-of-plane displacements of the plate at the above-mentioned four resonant frequencies.

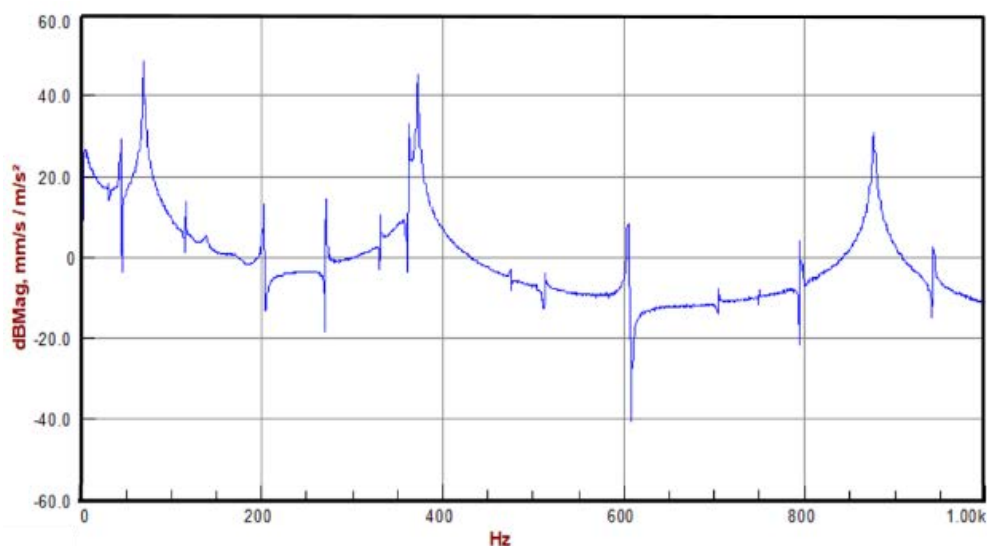


Figure 10.1 Frequency response function for the 210x148mm plate.

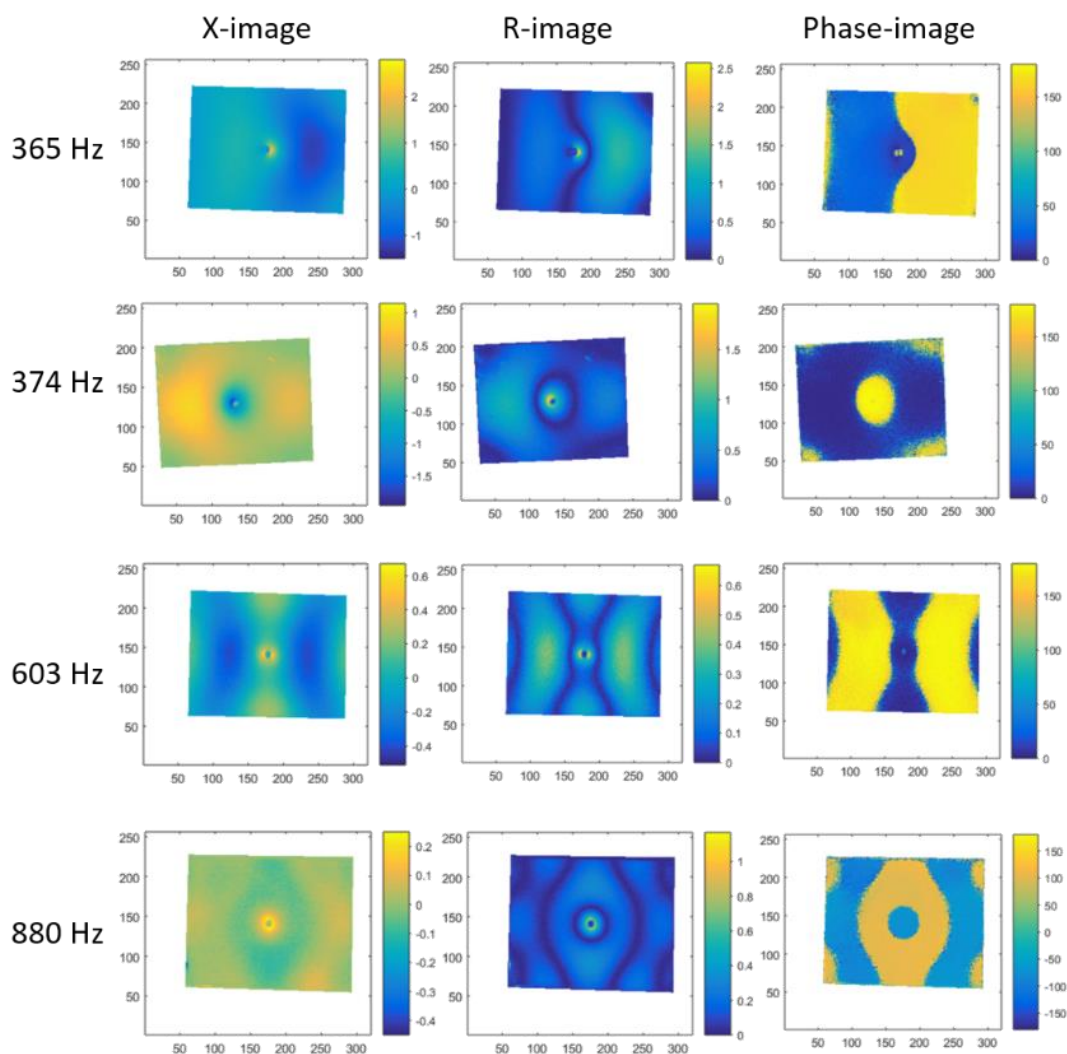


Figure 10.2. X-(left), R-(middle) and phase-images (right) of the TSA response acquired at different resonant frequencies of the plate. X- and R-images are not calibrated and are defined in terms of detector units. The angles in phase images are defined in degrees ($^{\circ}$).

10.4 Results and Discussion

As briefly described in an earlier section, TSA image is presented as a vector where the X component of the image (referred to as X-image) is in-phase with the reference signal and the Y component (Y-image) is 90° out-of-phase with respect to the reference signal. R-image is defined as a vector sum of the X and Y images. There are various sources which could induce phase shifts in the TSA image. It is important to understand and characterise the causes of these phase shifts in order to obtain quantitative stress information from the TSA image. The phase shifts can be characterised as global i.e. uniform over the field of view or localised¹⁶¹. Global phase shifts are often a function of the loading frequency and the source of reference signal used¹⁶¹. The thermoelastic response resulting from elastic deformation, under adiabatic conditions, is considered to be in-phase with the loading frequency. Therefore, if a global phase difference is present in the acquired TSA image, it could be offset to zero to maximise the magnitude of the X-image, which would correspond to the first stress invariant, or simply the R-image can be taken as the map representing the first stress invariant in a case where the phase is not shifted to zero. Localised phase shifts can be caused by non-adiabatic conditions, non-linear mechanics such as plastic deformation and/or variations in the paint coating over the component's surface¹⁶¹. For a structure subjected to excitation forces at a resonant frequency, all the regions oscillate sinusoidally with the same (resonant) frequency and with a fixed phase relation. In other words, at a given instant during the oscillation period, different regions within the same structure could potentially be in tension and compression, which will induce localised phase shifts in the TSA image. It is, therefore, postulated that these localised variations in the phase difference over the component's surface provide information about the mode shape, whereas, the R-image represents the map of first stress invariant, provided the deformation can be assumed as elastic, under adiabatic conditions and the paint coating is uniform. It is pertinent to mention here that the earlier studies^{159, 160} on the vibratory response of structures at resonant frequencies have instead associated the X-image to the first stress invariant.

The X-, R- and phase-images of the plate at four different resonant frequencies are shown in Figure 10.2. Localised regions in the plate undergoing compression and tension simultaneously can be clearly identified from the phase-image because of the shift in the phase of the TSA signal with respect to the loading frequency. This phase shift also results in a sign change for the X-component of the TSA signal (see left plots in Figure 10.2). Hence, the X-image is indicative of the mode shape of the plate. The nodal locations can be identified as narrow strips between the positive and negative phase regions. It is expected that at these nodal locations, the magnitude of the R-component, which is proportional to the first stress invariant, will be close to zero; this can be observed from the R-images in Figure 10.2.

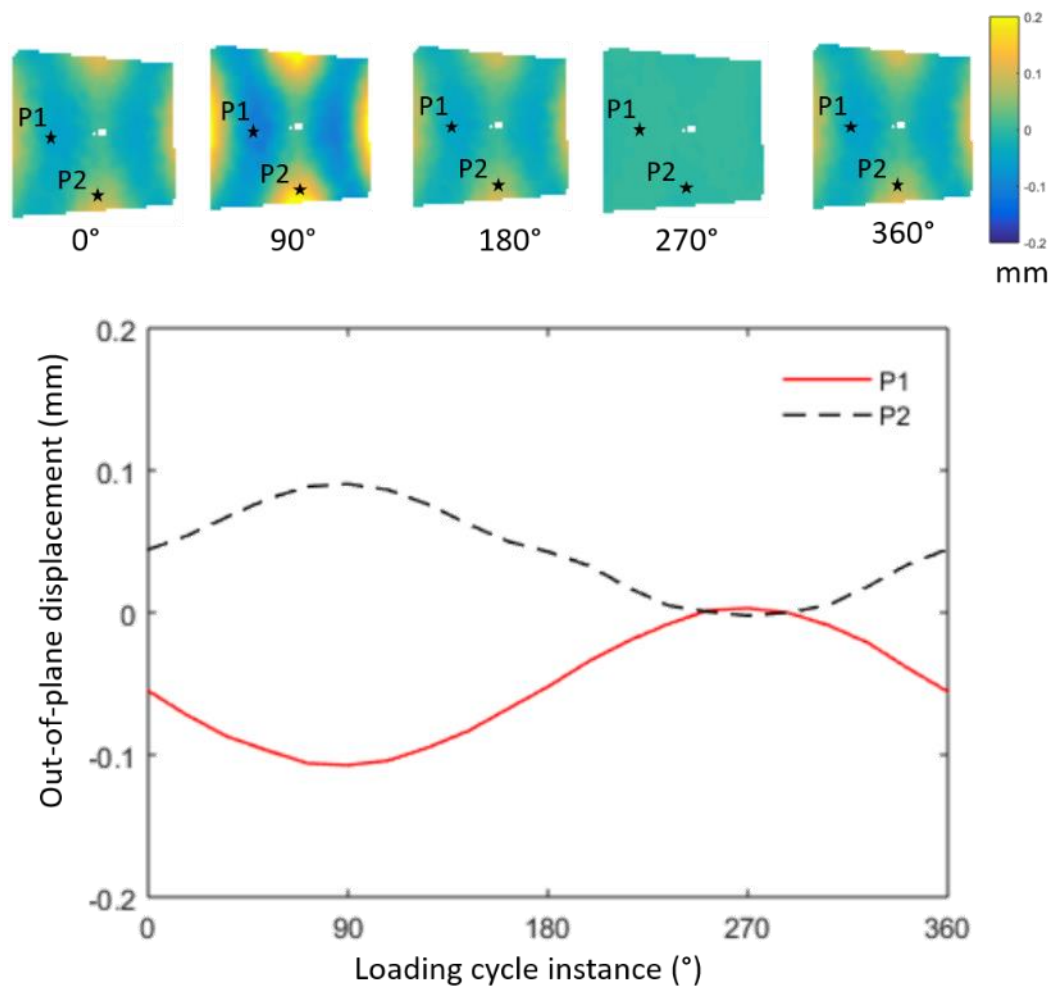


Figure 10.3 Top plot shows the out-of-plane displacements at 90° intervals during a loading cycle of the plate at a resonant frequency of 603 Hz. Bottom graph shows the variation in the magnitude of out-of-plane displacement at two points (P1 and P2), annotated on the displacement maps in the top plot, during one complete loading cycle.

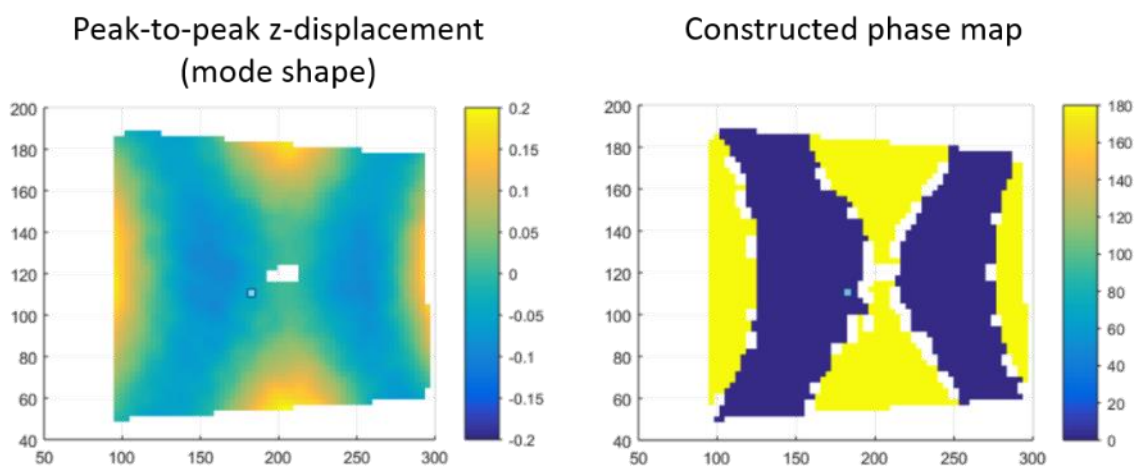


Figure 10.4 Plot of peak-to-peak out-of-plane displacement representing the mode shape (left) and the constructed phase map (right) of the plate for the resonant frequency of 603 Hz.

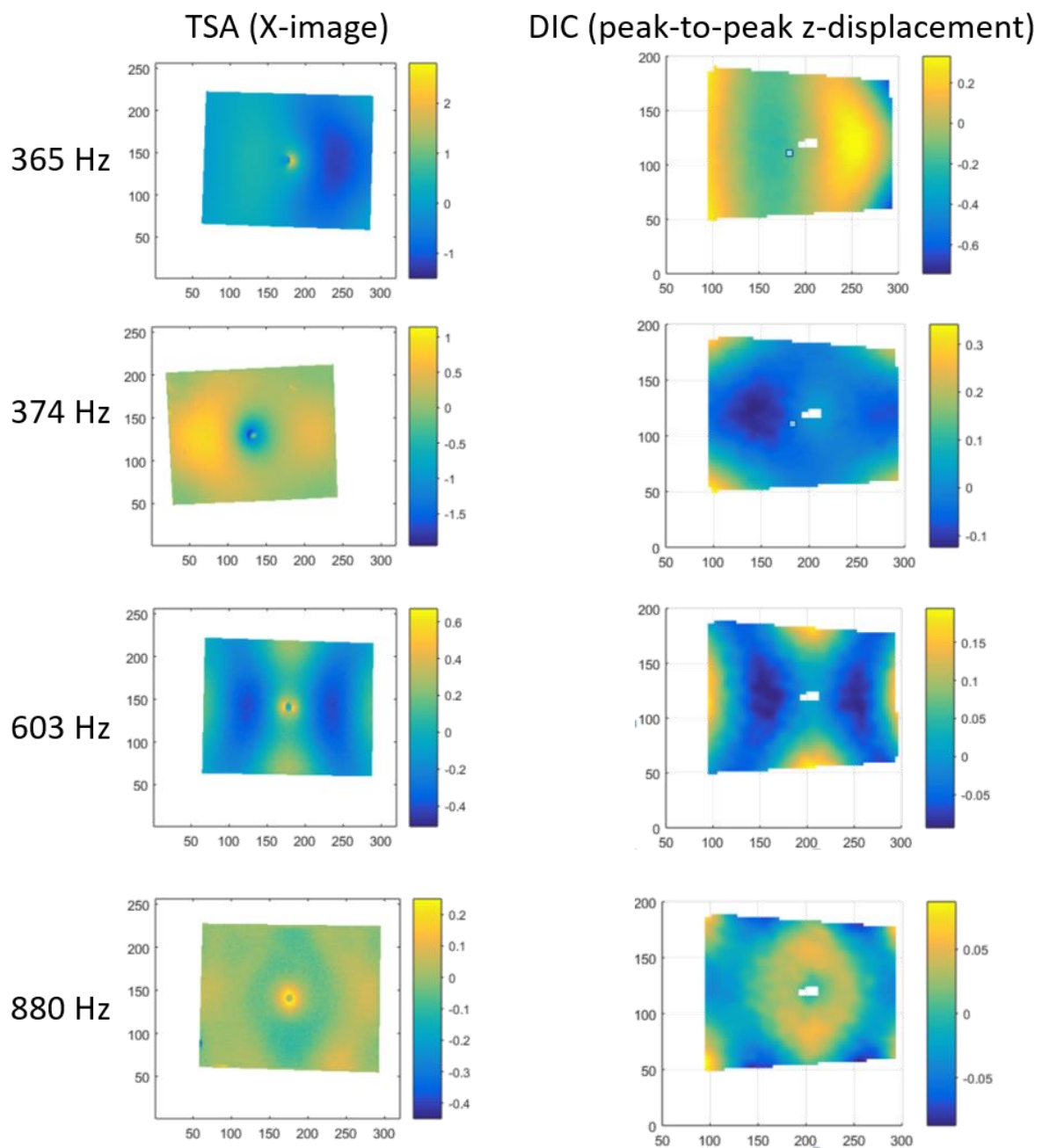


Figure 10.5 Comparison between the X-image from TSA data indicative of mode shape (left) and peak-to-peak out-of-plane displacement maps from PL-DIC representing the mode shape (right) at four different resonant frequencies of the plate.

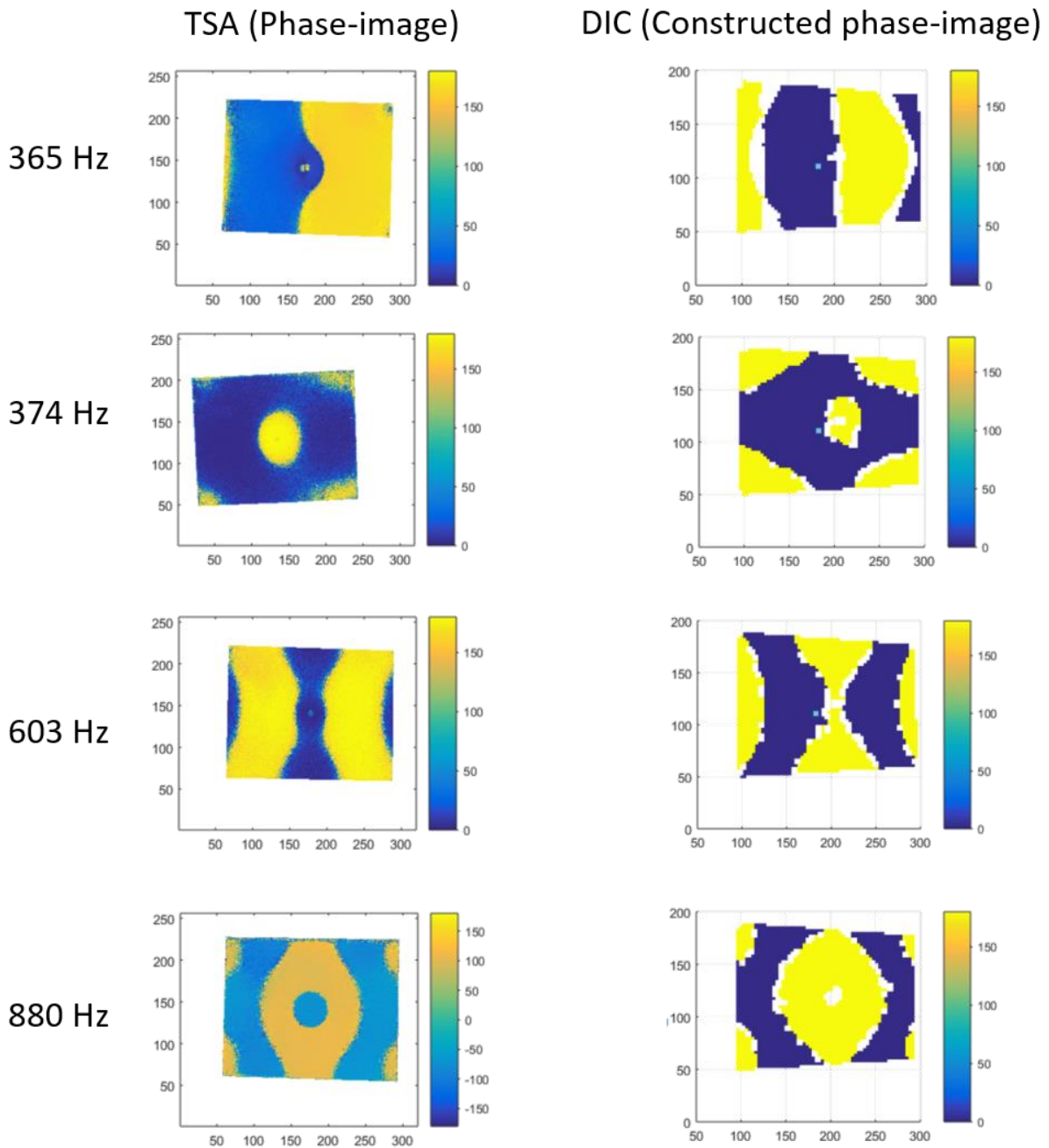


Figure 10.6 Comparison between the phase-map from TSA data (left) and the constructed phase map from PL-DIC (right) at four different resonant frequencies of the plate.

The top plot in Figure 10.3 shows the out-of-plane displacement maps acquired from PL-DIC during one loading cycle of the plate at a resonant frequency of 603 Hz. The bottom graph shows the temporal variation in the out-of-plane displacement at two different locations on the plate for one complete loading cycle. The two sinusoidal curves have a fixed phase shift of 180° . To construct the phase map from the displacement maps acquired using PL-DIC, a point on the plate i.e. P1 was chosen as a reference and the phase shift for the temporal variations in the out-of-plane displacements at all other location on the plate was evaluated with respect to the reference point. The right-hand plot in Figure 10.4 shows the phase map constructed from the PL-DIC data at the resonant frequency of 603 Hz. The mode shape was acquired by identifying the maps with the maximum and minimum out-of-plane

displacements at the reference point and subtracting the two images (see left-hand plot in Figure 10.4). The X-maps from the TSA data indicating the mode shape are compared with the mode shape acquired from PL-DIC in Figure 10.5. To further reinforce the hypothesis regarding the interpretation of TSA data for mode shape and quantitative stresses, the phase maps from TSA data are compared with the constructed phase maps from PL-DIC in Figure 10.6. An attempt was made at determining the map of strain invariant from the in-plane displacements measured using PL-DIC for direct comparison with the R-image of the TSA data, which represents the first stress invariant. However, the in-plane strains induced from the vibrational loading were less than $100\mu\epsilon$ in magnitude. Hence, it was not possible to extract the strain invariant pattern from the typical level of noise expected in the DIC strain data. Nonetheless, results in Figure 10.5 and Figure 10.6 provide sufficient evidence to suggest that R-image of TSA can be calibrated to give a quantitative measure of the first stress invariant and X-image of the TSA data can be attributed to the mode shape.

10.5 Conclusions

This study was carried out to establish the interpretation of thermoelastic stress analysis (TSA) data to deduce both the mode shape and the surface stresses from a rectangular plate subject to acoustic loading. TSA data and PL-DIC data were acquired from a surface of a rectangular plate subject to cyclic loading at four different resonant frequencies. The mode shapes and phase maps determined from both the techniques were compared to suggest that TSA can be used for simultaneous acquisition of mode shape and stresses under thermo-acoustic loading.

Chapter 11 Quantitative comparison of volumetric datasets*

11.1 Abstract

An orthogonal decomposition algorithm has been developed for the compression of volumetric datasets. The algorithm provides a powerful tool for computationally-efficient analysis and straightforward quantitative comparison of information-rich volumetric datasets by representing them as feature vectors. Data compression capability of the algorithm was established by decomposing a three-dimensional array of data on a vibratory response of an aerospace panel. The data array was constructed from time-varying, two-dimensional out-of-plane surface displacements of the aerospace panel, subjected to excitation at one of its resonant frequencies, which were measured using pulsed-laser digital image correlation technique. The constructed array was successfully represented as a feature vector, with representation error as low as the minimum measurement uncertainty in the measured data, giving a compression ratio of 155 : 1. The decomposition algorithm was also employed to successfully perform quantitative validation of the predicted data from a finite element model simulating the vibratory response of the aerospace panel.

11.2 Introduction

The emergence of relatively low-cost digital sensors has revolutionised experimental measurements by allowing information-rich data fields to be acquired in real-time, in a wide range of environments and with adjustable spatial and temporal resolutions. The examination of papers on experimental mechanics, published between 2000 and 2015, has revealed that the popularity and use of digital image correlation (DIC), which is the most robust full-field technique for measuring surface deformations in components, has far surpassed the once widely-used point-measurement method of resistance strain gauge¹⁶². The relative ease of acquiring information-rich data from full-field measurement techniques has allowed researchers to make significant advances in the areas of non-destructive evaluation (NDE) data fusion¹⁶³ and finite element model updating⁶³ over the last two decades. A major computational challenge which the researchers have long faced was the difficulty in comparing large sets of data containing at least 10^3 values that often do not share the same orientation, scale, coordinate system or data pitch.

* This chapter reports work performed by Dr Khurram Amjad and will be submitted shortly as Amjad K, Christian WJR, Dvurecenska K, Sebastian CM, Mollenhauer D & Patterson EA, Compression and quantitative comparison of volumetric datasets using orthogonal decomposition, *Int. Journal of Solids & Structures*.

The work carried out at the University of Liverpool, spanning almost over a decade, has overcome this difficulty for two-dimensional (2D) data fields by treating them as images and decomposing them using approaches developed for applications such as iris recognition and target acquisition. Essentially, a set of orthogonal polynomials are used to describe the image and the coefficients of the polynomials are collated in a column vector. This column vector, which is referred to as the feature vector, provides a unique and accurate representation of the original data but using, typically, less than hundred coefficients instead $> 10^3$ data values. These developments have led to new approaches for updating⁶³ and quantitative validation^{64, 164} of computational mechanics models using displacement and/or strain fields in structures. The ‘distance’ between feature vectors describing the strain fields has been used to develop innovative approaches for determining the onset of damage⁷¹ and predicting the residual strength of impacted composite laminates¹⁶⁵. More recently, orthogonal decomposition was employed to uniquely describe local defects in a continuous fibre-reinforced ceramic matrix composite specimen, which allowed significant compression of the original data size and straightforward comparisons of local defects between specimens for rigorous quality control of manufactured components¹⁶⁶. This paper extends the orthogonal decomposition approach, which was previously developed for 2D data fields, to three dimensions. This innovative step allows the volumetric data sets, typically obtained from techniques such as X-ray computed tomography and automated serial-sectioning, to be orthogonally decomposed and uniquely described as feature vectors resulting in significant data compression on the order of at least 100 : 1. The algorithm is equally applicable to 2D data fields which vary in the temporal domain and, for the purpose of orthogonal decomposition, can be treated as a volumetric data.

11.3 Algorithm for orthogonal decomposition of volumetric data

Three-dimensional (3D) kernel functions are required to decompose a 3D grid of data. These kernels are formed from the one-dimensional discrete Chebyshev polynomials, which are defined using the recursive formula⁷³:

$$t_m(x) = \frac{(2m-1)t_1(x)t_{m-1}(x) - (m-1)\left(1 - \frac{(m-1)^2}{M^2}\right)t_{m-2}(x)}{m}, \quad (11.1)$$

$$m = 2, 3, \dots, M-1$$

$$t_0(x) = 1 \quad (11.2)$$

$$t_1(x) = \frac{2x+1-M}{M} \quad (11.3)$$

where m , is the order of the polynomial and M , the number of sampling points. These discrete polynomials can be combined to obtain a three-dimensional orthogonal kernel, of dimensions $M \times N \times O$, using:

$$T_{m,n,o}(x,y,z) = t_m(x)t_n(y)t_o(z) \quad (11.4)$$

where m , n and o are the order of the one-dimensional polynomials. When combined, the order of the three-dimensional kernel is calculated as:

$$\omega_{m,n,o} = m + n + o \quad (11.5)$$

To use the orthogonal kernels for decomposition they must first be normalized by dividing each kernel by its associated norm, and given as:

$$\mathcal{P}_{m,n,o} = \rho_m \rho_n \rho_o \quad (11.6)$$

$$\rho_m = \frac{M \left(1 - \frac{1}{M^2}\right) \left(1 - \frac{2^2}{M^2}\right) \cdots \left(1 - \frac{m^2}{M^2}\right)}{2m+1} \quad (11.7)$$

The data can then be decomposed into coefficients, T_{mno} , using:

$$T_{m,n,o} = \sum_{x,y,z=0}^{M,N,O} I(x,y,z) \frac{1}{\sqrt{MNO}} \frac{T_{m,n,o}(x,y,z)}{\sqrt{\mathcal{P}_{m,n,o}}} \quad (11.8)$$

The reconstruction of the image is calculated as:

$$\hat{I}(x,y,z) = \sum_{m,n,o=0}^{M,N,O} T_{m,n,o} \sqrt{MNO} \frac{T_{m,n,o}(x,y,z)}{\sqrt{\mathcal{P}_{m,n,o}}} \quad (11.9)$$

The coefficients are arranged as a three-dimensional matrix, these can be permuted using the ordering system described by Bateman¹⁶⁷ which has been extended here to three dimensions. Using this system, the coefficient $T_{m,n,o}$ comes before $T_{p,q,r}$ in the feature vector if either of the following conditions are true:

$$\omega_{m,n,o} < \omega_{p,q,r} \quad (11.10)$$

$$(\omega_{m,n,o} = \omega_{p,q,r}) \wedge (m + nN + oNO < p + qN + rNO) \quad (11.11)$$

where $\wedge$, is the mathematical notation for “logical and”. This results in a feature vector f , ordered as follows:

$$\mathbf{f} = \begin{Bmatrix} T_{0,0,0} \\ T_{1,0,0} \\ T_{0,1,0} \\ T_{0,0,1} \\ T_{2,0,0} \\ T_{1,1,0} \\ \vdots \\ T_{M-1,N-1,O-1} \end{Bmatrix} \quad (11.12)$$

The number of coefficients in the feature vector $\mathbf{f}$, when using all the kernels up to a maximum order of ω_{max} is calculated as:

$$\Omega = \frac{1}{6} \omega_{max}^3 + \omega_{max}^2 + \frac{11}{6} \omega_{max} + 1 \quad (11.13)$$

This equation can be inverted using the cubic equation, resulting in two imaginary roots and a single real root equal to the order of the polynomials required to populate a feature vector of any arbitrary length.

11.3.1 Representation error

The representation error is the difference between the original data volume and its reconstruction.

When reconstructing real-valued data, a common technique for quantifying the representation error is to use the root mean squared error, calculated as:

$$u_{rms} = \sqrt{\frac{1}{MNO} \sum_{x,y,z=0}^{M,N,O} (I(x,y,z) - \hat{I}(x,y,z))^2} \quad (11.14)$$

This measure can also be used for assessing the reconstruction of binary volumes of data. However, in this situation the mean absolute error is more effective as it calculates the proportion of incorrect voxels in the reconstruction. The mean absolute error is given as:

$$u_{mae} = \frac{1}{MNO} \sum_{x,y,z=0}^{M,N,O} |I(x,y,z) - \hat{I}_{bin}(x,y,z)| \quad (11.15)$$

The feature vectors can be filtered by truncating them at a particular length or setting all coefficients that have an absolute value less than a threshold to zero. When filtering a feature vector, it is necessary to calculate the representation error to assess whether additional filtering is appropriate to further reduce the number of elements in the vector while satisfying the requirements for the quality of the representation. This requires repetitive calculations of equations (11.9) and (11.14) greatly increasing the computation time. As the kernels used to represent the data volume are orthogonal the representation error can be calculated without actually reconstructing the data using Parseval's theorem as¹⁶⁸:

$$u_{rms} = \sqrt{\frac{1}{MNO} \sum_{x,y,z=0}^{M,N,O} I(x,y,z)^2 - \sum_i \tilde{f}_i^2} \quad (11.16)$$

where $\tilde{f}$ denotes the filtered feature vector. Using this equation, it is possible to decompose a volume into a feature vector containing a high number of coefficients and then rapidly determine the minimum number of coefficients required to just achieve an arbitrary representation error.

11.4 Exemplar volumetric datasets

A prototype panel, representing an 800 mm × 400 mm rectangular section from an aircraft wing flap, was milled from a single aluminium block. A schematic of the panel is shown in Figure 11.1. The surface of the panel was sprayed with a random black and white speckle pattern and suspended using string from a rigid frame, which was affixed to an optical table. The panel was excited at its third resonant frequency of 59 Hz using an electromagnetic shaker (V100, DataPhysics, CA). During excitation, images of the painted surface were captured using a stereoscopic pulsed-laser DIC system¹¹⁵, which was designed to acquire full-field periodic displacements of the panel by phase shifting the image acquisition with respect to the excitation signal. A total of 41 pairs of images were acquired using an incremental phase shift of 9° to cover a complete (360°) loading cycle of the panel. The image correlation was performed using a commercial DIC software, Istra (Dantec Dynamics, Germany). The out-of-plane displacement maps from Istra were stacked in the z-direction at fixed intervals of 9° to construct a 3D array comprising of 1.18×10^5 data points, shown in Figure 11.1.

A FE model was created using 170,000 first-order hex elements using a commercial FE package (Altair, Optistruct, USA). An eigenvalue analysis was first performed to identify the resonant frequencies of the panel, followed by modal frequency response analysis to acquire full-field out-of-plane displacements at its third resonant frequency of 59 Hz. Further information about the experimental setup and FE model can be found in the paper by Sebastian et al¹¹⁵. The simulation data from modal frequency response analysis was available in the form of 21 grey scale images, representing the contour maps of out-of-plane displacement of the panel at uniform phase intervals of 18°. The images were imported into Matlab and were subsequently cropped after identifying a rectangular region, corresponding to the region of interest from which the measured displacement data was acquired, using the Matlab image registration function. A 3D array comprising of 7.96×10^6 data points was constructed from the predicted data by stacking rectangular segments from each of the 21 grey scale images in the z-direction at regular phase intervals of 18°. The constructed volumes for both the measured and predicted data are shown in Figure 11.1. Normalisation was performed on both datasets to transform them such that their values ranged between -1 and 1. This was to enable comparisons between the predicted and experimental data.

11.5 Compression of volumetric data

The data compression capabilities of the proposed orthogonal decomposition algorithm were assessed by decomposing the measured 3D array described in the previous section. When an array of acquired data, for example experimental measurements or simulation predictions, is orthogonally decomposed into a feature vector, it is important to determine whether the feature vector provides an acceptable representation of the original data. According to the recommendations in the CEN CWA-16799 guide⁶⁸, a feature vector is considered to be an acceptable representation of the measured data array if the representation error, u_{rms} does not exceed the minimum measurement uncertainty, u_{cal} of the measurement system. The representation error can be evaluated by calculating the root-mean-square of the difference between the original and the reconstructed array from equation (11.14) or directly using the feature vector from equation (11.16). To perform orthogonal decomposition based on the CEN guide recommendations, it was essential to first establish u_{cal} for the measured data arrays. For the measured aerospace panel dataset, u_{cal} was previously evaluated to be 4 μm or 1% of the measured data range based on the DIC calibration procedure proposed by Sebastian and Patterson¹⁶⁹. The plot in Figure 11.2 shows the variation of representation error, defined as a ratio of u_{cal} , with increasing number of coefficients in the feature vector for the measured data array.

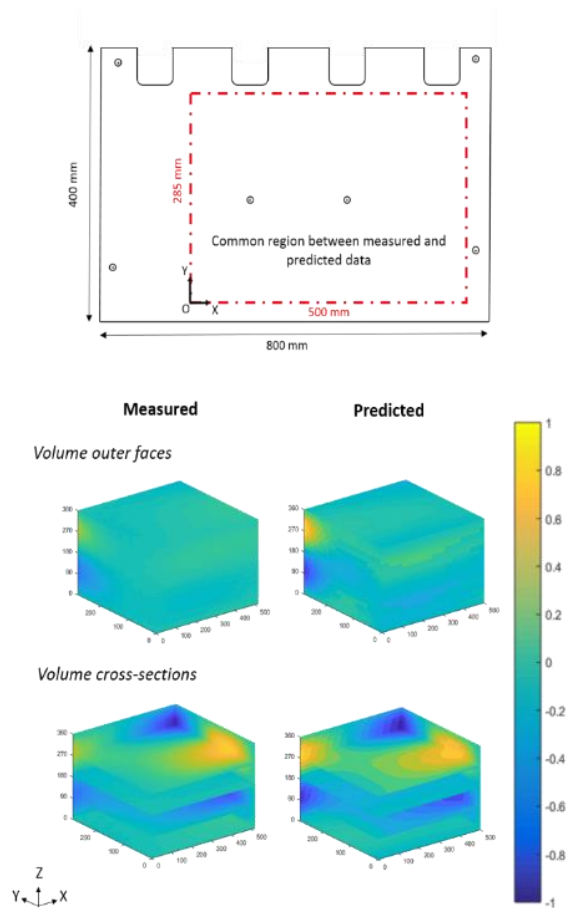


Figure 11.1 Schematic of the aerospace panel (top) and the volumetric arrays (bottom) constructed from measured (left) and predicted (right) data over the common region of interest.

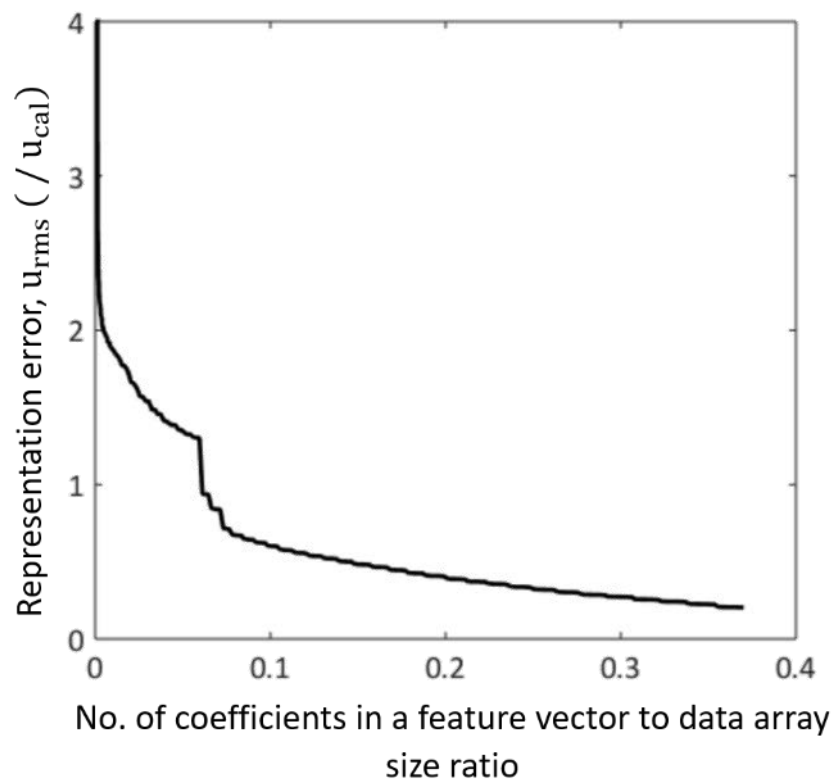


Figure 11.2 The plot of representation error, defined as a ratio of the minimum measurement uncertainty, against the ratio of number of coefficients in the unfiltered feature vector to data array size for the measured data array shown in Figure 11.1.

For 2D data fields, it has been a common practice^{129, 165} to perform decomposition using a large number of kernels such that u_{rms} is significantly lower than u_{cal} . An arbitrary threshold level is then defined to filter out those coefficients in the unfiltered feature vector whose absolute magnitude is lower than the threshold. This results in the retention of only a small number of significant coefficients in the filtered feature vector, resulting in higher data compression ratios. To meet the CEN guide recommendation, the magnitude of the threshold needs to be determined such that u_{rms} for the filtered feature vector does not exceed u_{cal} . An alternate approach to filtering is proposed here to identify the smallest set of significant coefficients from the unfiltered feature vector which results in u_{rms} being less than u_{cal} . In this approach, coefficients are selected one by one from an unfiltered feature vector based on their absolute magnitudes, such that the coefficients with the highest absolute magnitude are selected first. After each selection, u_{rms} is evaluated using equation (11.16) and compared with u_{cal} . The selection process is stopped when u_{rms} becomes less than u_{cal} .

The unfiltered feature vectors with u_{rms} ranging from $0.2 u_{rms}$ to $1 u_{rms}$ were filtered using the proposed filtering approach for the measured data array. The plot of u_{rms} for the unfiltered feature vectors against the number of coefficients in a feature vector prior to and after the filtration is shown in Figure 11.3. It can be observed that the plot for filtered feature vectors converged to a constant number of significant coefficients. It is difficult to establish this convergence with the threshold based filtration approach^{129, 165} primarily because the threshold level at which $u_{rms} \approx u_{cal}$ is a function of the unfiltered feature vector length. The compression ratio for the measured data array, evaluated based on the converged set of significant coefficients in the filtered feature vector, was found out to be 155 : 1. The array reconstructed from the filtered feature vector is compared with the original array in Figure 11.4.

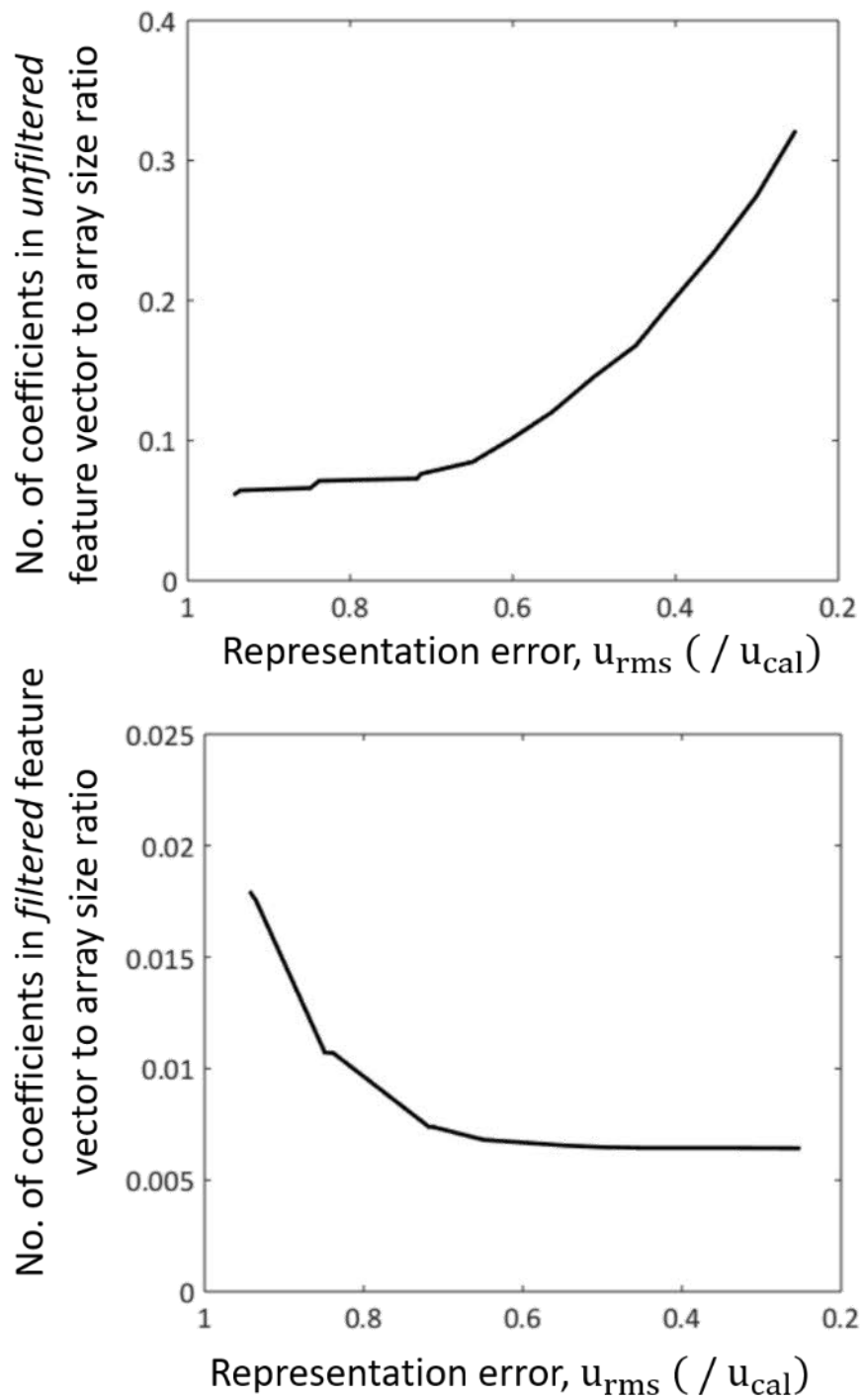


Figure 11.3 Plot of the ratio of number of coefficients in the unfiltered feature vector to data array size (top) and the number of retained coefficients after filtering to data array size (bottom) against the representation error for the unfiltered feature vectors for the measured data array shown in Figure 11.1.

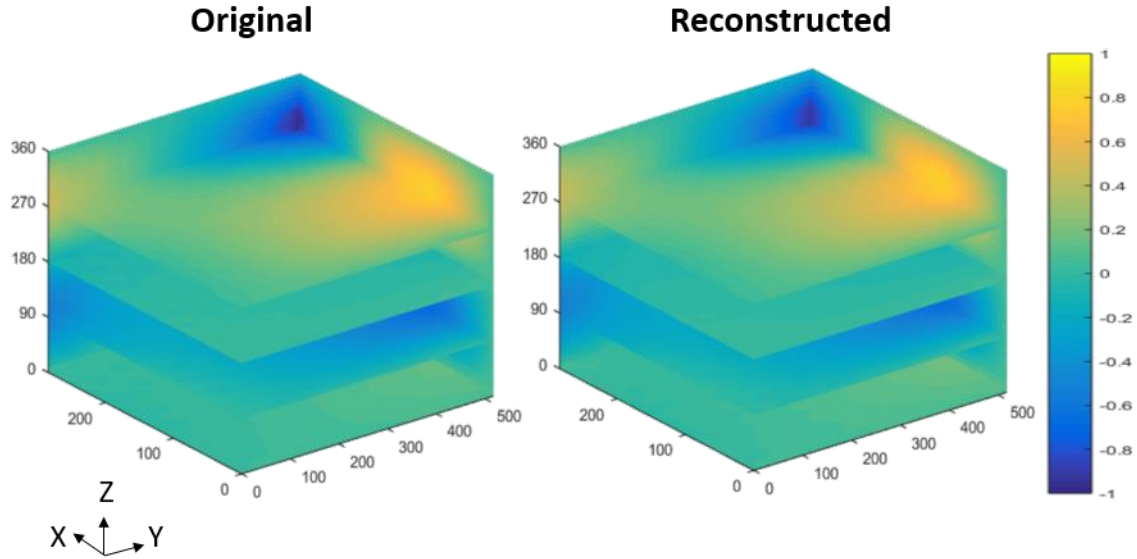


Figure 11.4 Measured (left) and reconstructed (right) data array from the filtered featured vector.

11.6 Comparison of predicted and measured volumetric data

The applicability of methodologies^{68, 164}, which were developed for quantitative comparison of 2D data fields utilising orthogonal decomposition, are analysed in this section for the 3D data arrays. The CEN CWA-16799 guide⁶⁸ has outlined a method for making a comparison between the measured and predicted 2D data fields for the purpose of validation of computational solid mechanics models. In this method, the measured and predicted data fields are first represented as feature vectors by performing orthogonal decomposition. The coefficients of the two feature vectors are then plotted against one another for a simple graphical comparison. The CEN guide recommends that the computational model is considered acceptable if all of the pairs of coefficients in the two feature vectors fall within the uncertainty zone defined by:

$$S_{P_i} = S_{M_i} \pm 2u_{exp}, i = 1, 2, \dots, n \quad (11.17)$$

where S_{P_i} and S_{M_i} are the i th coefficients in the feature vectors representing the predicted and the measured data fields, respectively, and n is the total number of coefficients in the feature vector. u_{exp} is the total uncertainty which can be determined by:

$$u_{exp} = \sqrt{u_{cal}^2 + u_{rms}^2} \quad (11.18)$$

where u_{cal} is the minimum uncertainty in the measured data field and u_{rms} is the representation error in the reconstructed data field.

To illustrate this method for volumetric datasets, the coefficients in the filtered feature vectors representing the measured and predicted displacement arrays are plotted against one another in Figure

11.5. It can be observed from the plot in Figure 11.5 that some of the points are outside the uncertainty zone, defined by the two dashed lines, which, according to the CEN guide criterion, makes the predictions of the computational model unacceptable. This approach does not provide any information about the degree to which the prediction results represent the measured data or in this case, how bad is the computational model. To fill this gap, a probabilistic validation approach¹⁶⁴ has been recently developed which evaluates a validation metric, VM representing the probability that the prediction results belong to the same population as the measured data. Four steps are involved in determining VM , which are briefly described here. In the first step, the normalized relative error, e_i for each pair of coefficients in the feature vectors representing the measured and predicted data are calculated using:

$$e_i = \left| \frac{S_{p_i} - S_{M_i}}{\max_{m \in S_M} |S_{M_m}|} \right| \quad (11.19)$$

The weight, w_i for each of the normalised error terms are then determined using:

$$w_i = \frac{e_i}{\sum_{i=1}^n e_i} \times 100 \quad (11.20)$$

In the third step, an error threshold, e_t is defined by dividing the expanded total uncertainty, $2u_{exp}$ by the coefficient in the feature vector for the measured data with the maximum absolute magnitude, i.e.

$$e_t = \frac{2u_{exp}}{\max_{m \in S_M} |S_{M_m}|} \times 100 \quad (11.21)$$

In the last step, VM is determined by summing the weights for all of the normalised errors terms that are found to be below the defined error threshold:

$$VM = \sum_{i=1}^n w_i \quad \text{for } e_i < e_t \quad (11.22)$$

From experience with historical 2D data, acquired from full-field techniques such as DIC and TSA, it has been established that 2D displacement and strain fields can be decomposed into feature vectors with low representation errors using, typically, less than hundred Chebyshev kernels. The feature vectors representing 3D data arrays can have more than a thousand coefficients, which is consistent with the increased number of grid points in 3D array with addition of the third dimension. In order to correctly apply the above-described probabilistic validation approach to 3D arrays, it is important to first analyse the sensitivity of VM to the number of coefficients in the feature vector pair of measured and predicted data. The top plot in Figure 11.6 shows the sensitivity of VM to the total number of coefficients in the feature vectors representing the measured and predicted data. The number of normalised error terms (e_i) that are found to be below the error threshold (e_t) increases with the inclusion of more coefficients in the unfiltered feature vector. This causes the accumulative weight of the error terms below the error threshold to increase as well, which is defined as the validation metric, VM according to equation

(11.22). Hence, in order to acquire unbiased values for VM , it is imperative to filter out those coefficients in the feature vector pairs representing the measured and predicted arrays, whose associated kernels do not make a significant contribution in defining the inherent distribution in the original array.

It was established in the previous section that with the proposed approach for feature vector filtering, a smallest set of significant coefficients can be identified for an arbitrary representation error. With this set of significant coefficients, an unbiased, converged value for VM can be obtained. The top plot in Figure 11.7 shows the variation in the number of retained coefficients after filtering against the total number of coefficients in the unfiltered feature vector for the measured 3D array. The bottom plot in the same figure shows the effect of filtering on VM . It can be observed from the trends in Figure 11.7 that VM converges to a constant value as the retained coefficients in the filtered feature vector converge to a constant set of significant coefficients. The converged values of VM , are not dependent on the feature vector length and hence provide an unbiased quantitative measure of the confidence associated with the agreement between the predicted and measured results. The converged value of VM was found out to be 51.7%.

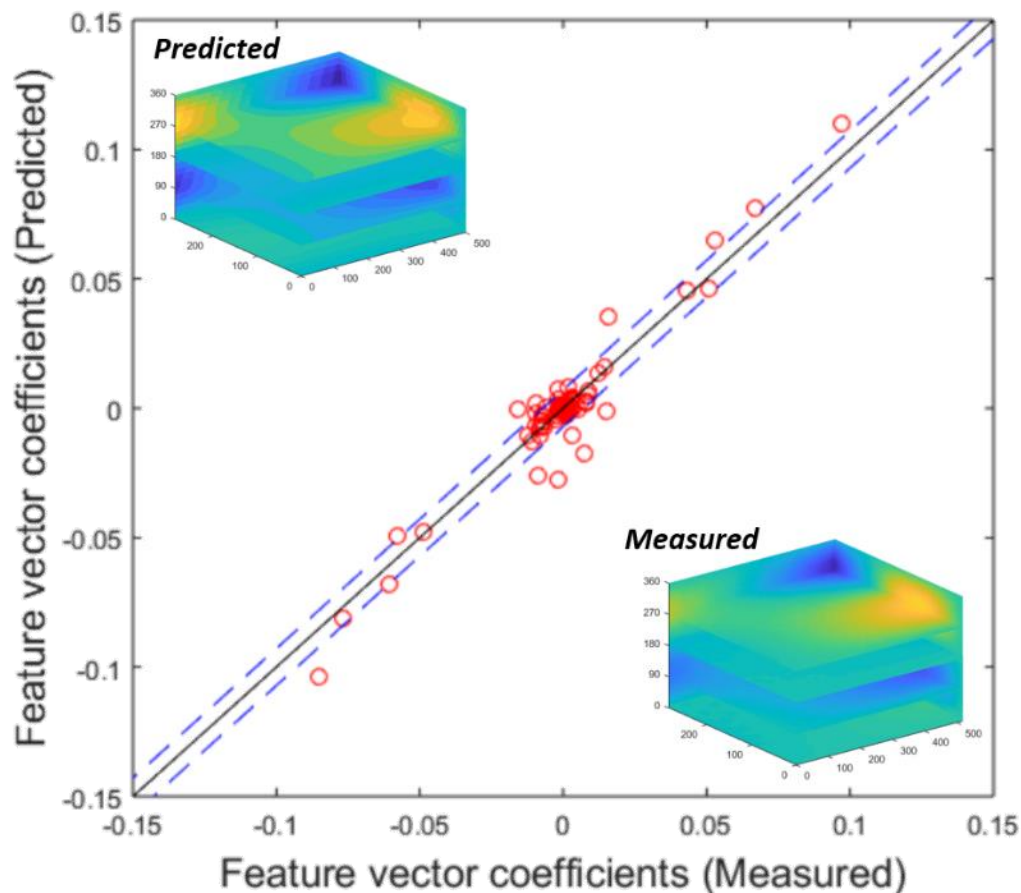


Figure 11.5 The graph of coefficients of the filtered feature vectors representing the measured and predicted displacement arrays, shown in Figure 11.1, plotted against one another. The blue-dashed lines represent the total expanded uncertainty, $2u_{exp}$.

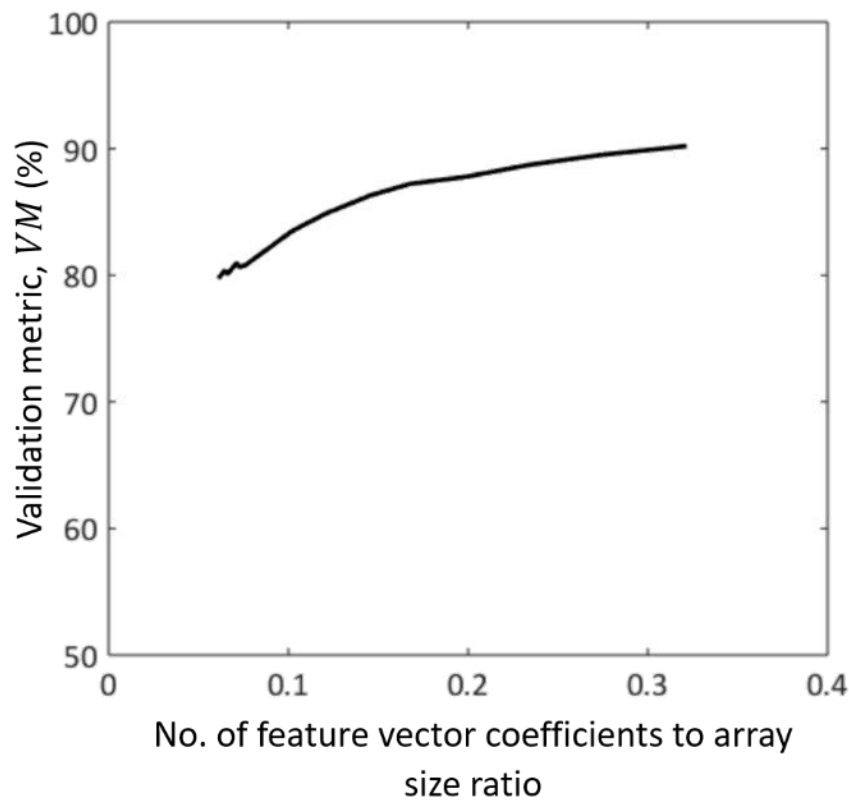


Figure 11.6 The plot of validation metric against the number of coefficients in the unfiltered feature vector to data array size ratio for the pair of measured and predicted data arrays shown in Figure 11.1.

11.7 Discussion

Quantitative comparison of large datasets has been a challenging task for the researchers primarily because datasets from different sources, which needs to be compared, often do not share the same grid size, data pitch or coordinate system. A rudimentary approach for data comparison, which is still predominant in industry and academia, involves identifying critical locations in the measured datasets and qualitatively establishing the agreement between the predicted and measured data points at these critical locations¹²⁹. In this paper, a 3D orthogonal decomposition algorithm based on discrete Chebyshev polynomials has been proposed. This algorithm is capable of decomposing volumetric datasets into feature vectors, thereby giving data compression ratios of at least 100 : 1. Furthermore, it allows quantitative comparisons of volumetric datasets in a straightforward manner within the feature vector space, irrespective of whether they share the same grid size, data pitch or coordinate system.

The plot in Figure 11.2 reveals that the representation error (u_{rms}) decreases in an exponential manner with the inclusion of more coefficients in the feature vector. It can be observed in Figure 11.2 that the plot exhibits a jagged profile. This jaggedness is caused by the inclusion of coefficients whose associated kernel makes a significant contribution to representing the inherent distribution in the original data array, thereby causing a significant drop in u_{rms} when those coefficients are included. The filtering approach proposed in this study identifies the smallest set of significant coefficients that results in an

arbitrary u_{rms} , maximising the compression ratios. In this study, filtering was performed so that u_{rms} was just less than the minimum measurement uncertainty (u_{cal}) of the measurement system, as per the recommendations of the CEN CWA-16799 guide⁶⁸, which states that u_{rms} should not exceed u_{cal} . The reduction in number of coefficients in the feature vector after removal of the redundant coefficients by implementing the filtering approach has been illustrated using the plots in Figure 11.3.

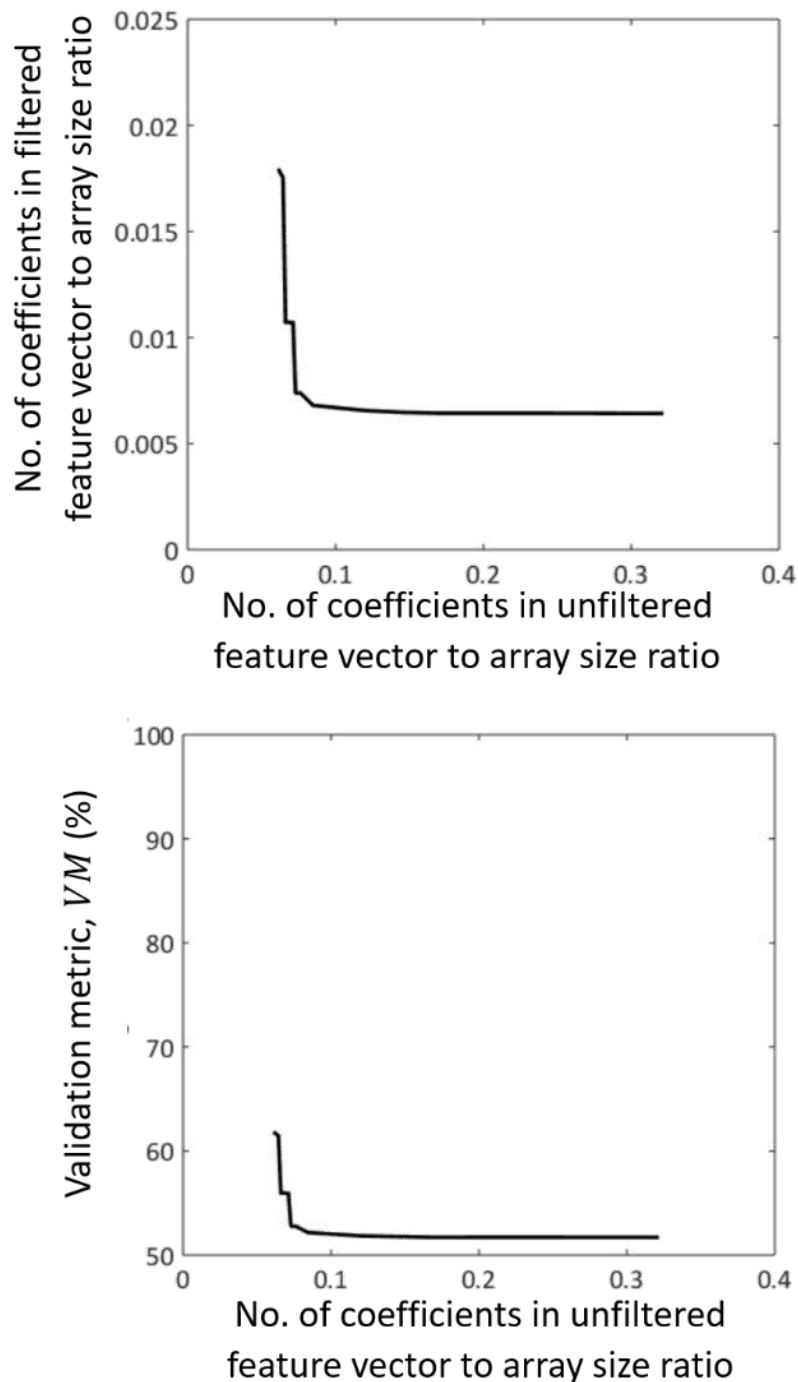


Figure 11.7 Plot of the ratio of retained coefficients after filtering (top) and the validation metric acquired from filtered feature vectors (bottom) against total number of coefficients in the unfiltered feature vector to data array size ratio.

As mentioned earlier, one of the key applications of orthogonal decomposition lies in making meaningful comparisons of the datasets. This has led to a recently-developed approach for quantitative

validation of computational solid mechanics models¹⁶⁴. In this approach, the measured and predicted data arrays are first represented by a pair of feature vectors obtained using orthogonal decomposition. The cumulative weight of the normalised differences between the individual coefficients of the two feature vectors, which are found to be below an error threshold based on the total measurement uncertainty, is then computed to obtain a validation metric, VM . The magnitude of VM represents the probability that the prediction results belong to the same population as the measured data; and hence, provide a quantitative measure of the quality of the prediction data. In this study, the applicability of this validation approach to 3D data arrays was assessed. VM was found to be sensitive to the number of coefficients in the feature vector, which can be seen in the plot in Figure 11.6. It was demonstrated in Figure 11.7 that a stable and unbiased value of VM can be acquired by employing filtering of the feature vectors.

11.8 Conclusions

An orthogonal decomposition algorithm based on one-dimensional discrete Chebyshev polynomials was developed which can decompose volumetric datasets, such as those acquired from X-ray computed tomography or automated serial sectioning, representing them as feature vectors. The algorithm is equally applicable to 2D data fields which vary in the temporal domain and, for the purpose of orthogonal decomposition, can be treated as a volumetric data. The data compression capabilities of this algorithm was assessed by decomposing a volumetric data array, representing the full-field out-of-plane displacements of a panel subjected to excitation at one of its resonant frequencies, which was acquired using pulsed-laser digital image correlation. The feature vector acquired using orthogonal decomposition accurately represented the original distribution in the measured array, with representation error lower than the estimated minimum measurement uncertainty of the measurement system, giving a compression ratio of at least 155 : 1. The proposed algorithm provides a powerful tool for making quantitative comparisons of information-rich volumetric datasets, which do not share a common grid size data pitch or coordinate system, by representing them as feature vectors. The decomposition algorithm was successfully employed to perform quantitative validation of the 3D predicted data from a finite element model simulating the vibratory response of the aerospace panel.

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Appendix A Temperature-dependent material properties

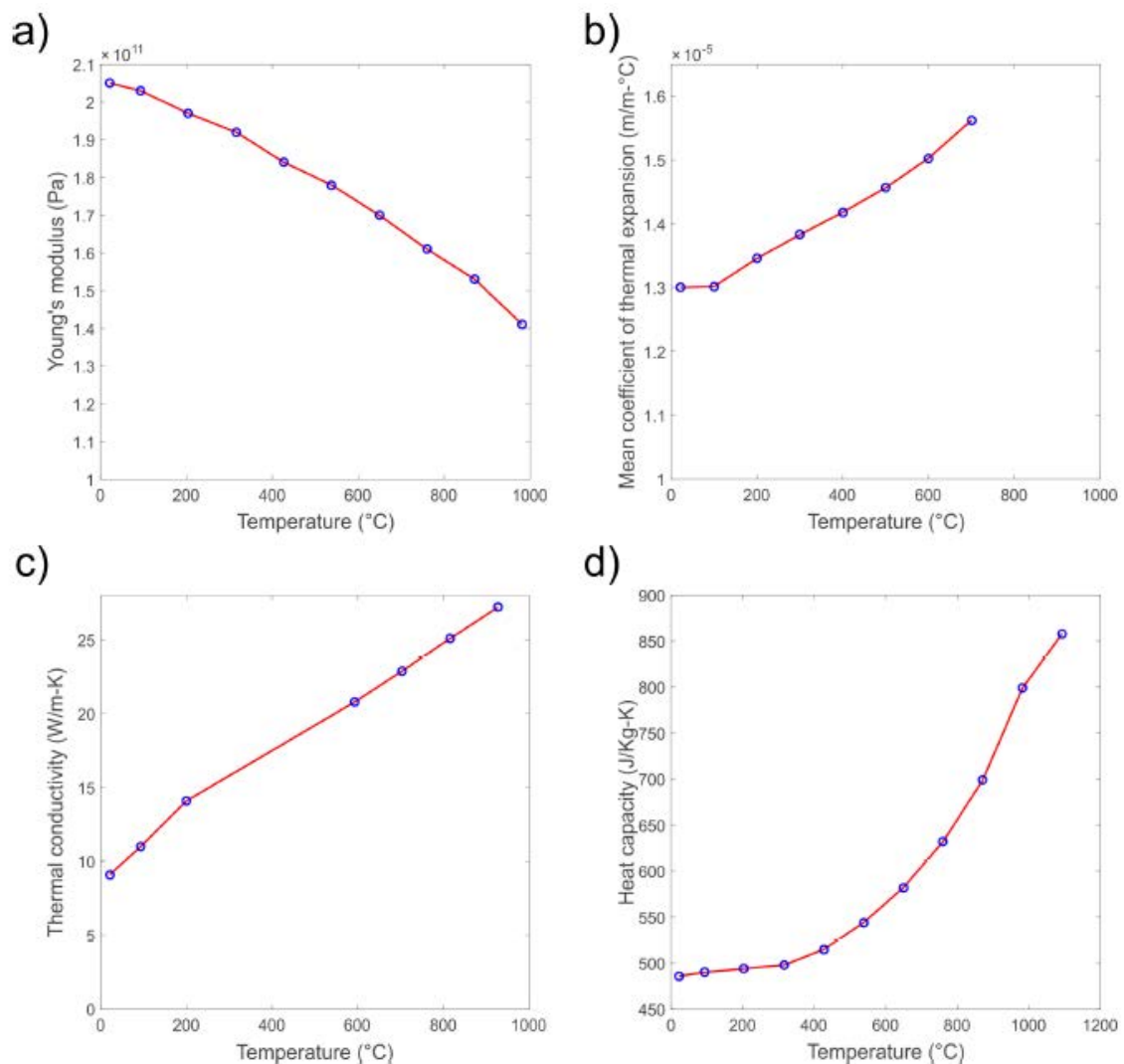


Figure A.1 Temperature-dependent material properties, as provided by the manufacturer¹⁰⁷: a) Young's modulus, b) mean coefficient of thermal expansion, c) thermal conductivity and d) heat capacity.

Appendix B Original and simplified material models

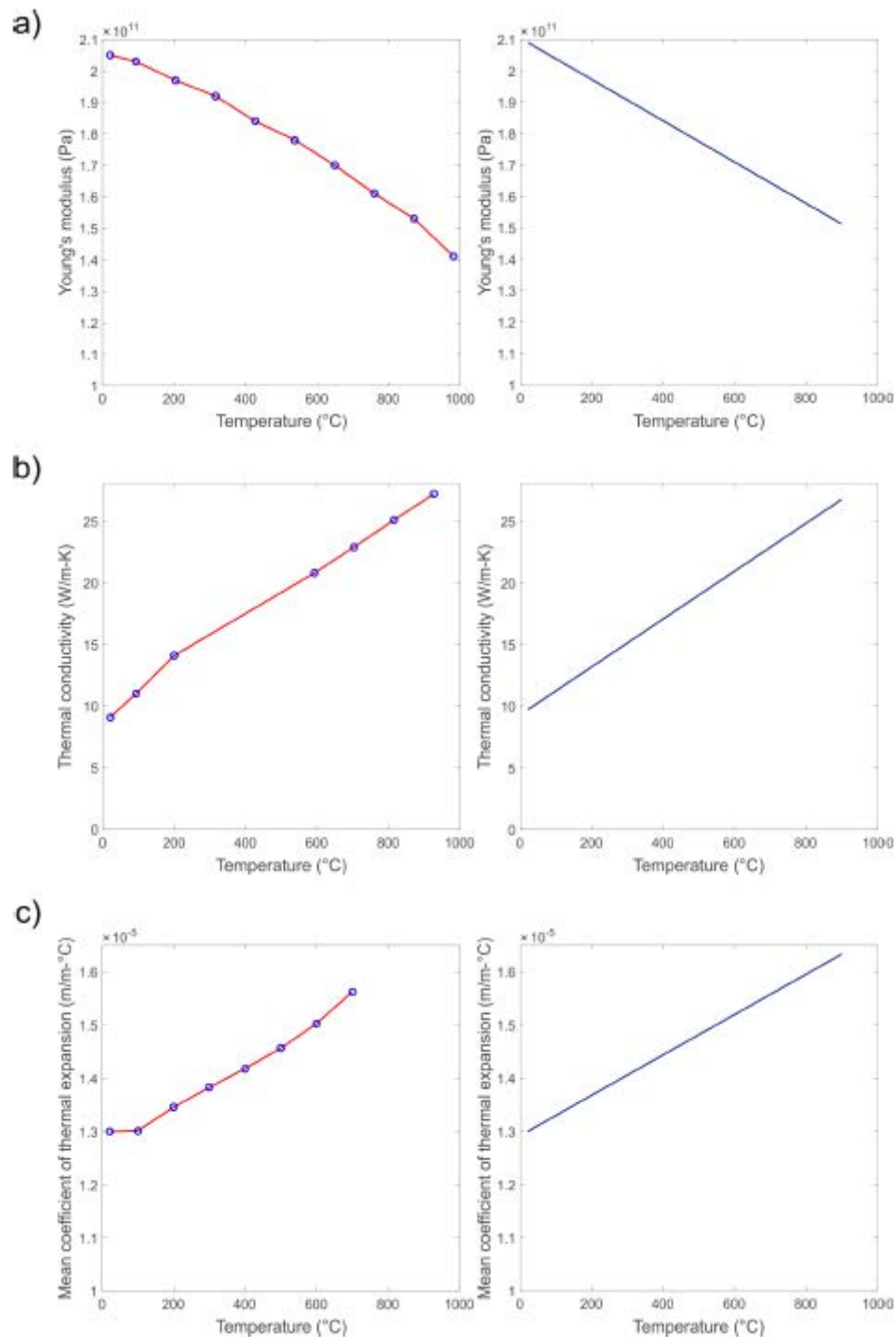


Figure B.1 Temperature-dependent a) Young's modulus, b) thermal conductivity and c) mean coefficient of thermal expansion as provided by the manufacturer¹⁰⁷ (left) and fitted using a first degree polynomial (right).

Table B.1 High temperature predictions of an FE model using simplified temperature-dependent material properties presented in B.1 (right) and the original material properties provided by the manufacturer¹⁰. The analysis was performed by statically loading a flat plate.

	Experimental measurement [10] (Hz)	Simplified material model (Hz)	Relative error	Original material model (Hz)	Relative error
Mode 1	104	89	14.1%	89	14.1%
Mode 2	140	134	4.0%	134	4.0%
Mode 3	314	230	26.7%	230	26.7%
Mode 4	385	343	10.9%	343	10.9%
Mode 5	735	600	18.4%	600	18.4%
Mode 6	879	812	7.6%	812	7.6%
Mode 7	1080	996	7.8%	996	7.8%
Mode 8	1164	1182	1.5%	1182	1.5%
Mode 9	1325	1235	6.8%	1234	6.8%
		Mean	10.9%	Mean	10.9%

Table B.2 High temperature predictions of an FE model using simplified temperature-dependent material properties presented in B.1 (right) and the original material properties provided by the manufacturer¹⁰. The analysis was performed by statically loading a flat plate.

	Experimental measurement [10] (Hz)	Simplified material model (Hz)	Relative error	Original material model (Hz)	Relative error
Mode 1	107	85	20.7%	85	20.3%
Mode 2	139	128	8.2%	128	7.7%
Mode 3	273	217	20.5%	218	20.0%
Mode 4	295	325	10.0%	326	10.6%
Mode 5	665	569	14.4%	572	14.0%
Mode 6	690	770	11.6%	774	12.2%
Mode 7	787	943	19.8%	947	20.4%
Mode 8	1173	1121	4.5%	1126	4.0%
Mode 9	1251	1174	6.1%	1179	5.7%
		Mean	12.9%	Mean	12.8%

Appendix C Relative error of predictions using different constraints applied to a uniform at plate and a flat plate with a hole

Constraint type	Relative error (%)						
	Single node	5 nodes	9 nodes	13 nodes	25 nodes (H)	25 nodes (V)	Hexagonal
Mode 1	14.1%	29.2%	39.9%	48.2%	61.3%	62.0%	65.3%
Mode 2	4.0%	44.7%	56.7%	65.9%	80.6%	81.3%	88.7%
Mode 3	26.7%	26.4%	26.1%	25.8%	25.0%	24.9%	24.7%
Mode 4	10.9%	10.9%	10.8%	10.7%	10.2%	10.4%	10.0%
Mode 5	18.4%	17.9%	17.6%	17.0%	16.1%	15.8%	15.1%
Mode 6	7.6%	4.0%	2.7%	1.6%	0.2%	0.4%	1.5%
Mode 7	7.8%	7.1%	6.9%	6.8%	6.6%	6.6%	6.5%
Mode 8	1.5%	6.3%	7.3%	9.4%	12.0%	12.1%	12.8%
Mode 9	6.8%	6.2%	5.1%	5.3%	4.2%	4.1%	3.7%
Mean	10.9%	17.0%	19.2%	21.2%	24.0%	24.2%	25.4%

Table C.1 Relative error of room-temperature predictions using a uniform at plate constrained according to the patterns in Figure 4.2.

Table C.2 Relative error of room-temperature predictions using a flat plate with a hole constrained according to the patterns in Figure 4.3.

Constraint type	Relative error (%)		
	Single node	5 nodes	Hexagonal
Mode 1	62.6%	77.1%	83.0%
Mode 2	82.2%	99.0%	107.5%
Mode 3	24.9%	23.6%	22.9%
Mode 4	10.2%	9.2%	8.9%
Mode 5	15.8%	13.9%	12.4%
Mode 6	0.5%	3.1%	4.7%
Mode 7	6.6%	6.3%	6.3%
Mode 8	12.3%	15.5%	16.9%
Mode 9	4.0%	2.5%	1.6%
Mean	24.3%	27.8%	29.3%

Appendix D Verification of resonant frequency predictions

using a temperature-dependent material model

against literature

Table D.1 Resonant frequencies for CCCC boundary conditions. The relative difference was calculated with respect to predictions published by Jeyaraj et al.³³.

	$T_{uniform}$ (°C)	Transient model (Hz)	Jayaraj et al. (Hz)	Relative difference (%)
Mode 1	$0xT_{cr}$	461.3	465	0.8
	$0.25xT_{cr}$	401.2	405	0.9
	$0.5xT_{cr}$	329.1	333	1.2
	$0.75xT_{cr}$	233.9	237	1.3
	$0.99xT_{cr}$	47.1	47	0.2
Mode 2	$0xT_{cr}$	806.3	818	1.4
	$0.25xT_{cr}$	737.5	747	1.3
	$0.5xT_{cr}$	660.9	670	1.4
	$0.75xT_{cr}$	573.4	582	1.5
	$0.99xT_{cr}$	473.5	481	1.6
Mode 3	$0xT_{cr}$	1052.4	1066	1.3
	$0.25xT_{cr}$	984.9	996	1.1
	$0.5xT_{cr}$	912.0	922	1.1
	$0.75xT_{cr}$	832.4	842	1.1
	$0.99xT_{cr}$	747.4	756	1.1
Mode 4	$0xT_{cr}$	1358.5	1386	2.0
	$0.25xT_{cr}$	1287.2	1312	1.9
	$0.5xT_{cr}$	1211.4	1238	2.2
	$0.75xT_{cr}$	1130.1	1151	1.8
	$0.99xT_{cr}$	1045.9	1068	2.1
Mode 5	$0xT_{cr}$	1365.4	1390	1.8
	$0.25xT_{cr}$	1293.2	1316	1.7
	$0.5xT_{cr}$	1216.6	1240	1.9
	$0.75xT_{cr}$	1134.6	1153	1.6
	$0.99xT_{cr}$	1049.7	1070	1.9

Table D.2 Resonant frequencies for CCFC boundary conditions. The relative difference was calculated with respect to predictions published by Jeyaraj et al. ³³.

	$T_{uniform}$ (°C)	Transient model (Hz)	Jayaraj et al. (Hz)	Relative difference (%)
Mode 1	$0xT_{\sigma}$	360.0	363	0.8
	$0.25xT_{\sigma}$	316.7	320	1.0
	$0.5xT_{\sigma}$	263.4	266	1.0
	$0.75xT_{\sigma}$	190.4	193	1.3
	$0.99xT_{\sigma}$	39.1	39	0.3
Mode 2	$0xT_{\sigma}$	506.7	515	1.6
	$0.25xT_{\sigma}$	460.5	469	1.8
	$0.5xT_{\sigma}$	408.9	417	1.9
	$0.75xT_{\sigma}$	349.7	358	2.3
	$0.99xT_{\sigma}$	281.9	288	2.1
Mode 3	$0xT_{\sigma}$	847.5	864	1.9
	$0.25xT_{\sigma}$	801.9	819	2.1
	$0.5xT_{\sigma}$	753.1	771	2.3
	$0.75xT_{\sigma}$	700.5	716	2.2
	$0.99xT_{\sigma}$	645.5	661	2.3
Mode 4	$0xT_{\sigma}$	963.9	974	1.0
	$0.25xT_{\sigma}$	907.2	917	1.1
	$0.5xT_{\sigma}$	844.9	853	1.0
	$0.75xT_{\sigma}$	775.2	782	0.9
	$0.99xT_{\sigma}$	699.3	704	0.7
Mode 5	$0xT_{\sigma}$	1121.3	1142	1.8
	$0.25xT_{\sigma}$	1062.3	1082	1.8
	$0.5xT_{\sigma}$	1000.0	1019	1.9
	$0.75xT_{\sigma}$	933.6	952	1.9
	$0.99xT_{\sigma}$	865.3	882	1.9

Table D.3 Resonant frequencies for FCFC boundary conditions. The relative difference was calculated with respect to predictions published by Jeyaraj et al. ³³.

	$T_{uniform}$ (°C)	Transient model (Hz)	Jayaraj et al. (Hz)	Relative difference (%)
Mode 1	$0xT_{\sigma}$	344.2	347	0.8
	$0.25xT_{\sigma}$	300.5	303	0.8
	$0.5xT_{\sigma}$	247.5	249	0.6
	$0.75xT_{\sigma}$	176.7	178	0.7
	$0.99xT_{\sigma}$	35.7	37	3.5
Mode 2	$0xT_{\sigma}$	385.6	391	1.4
	$0.25xT_{\sigma}$	348.9	354	1.4
	$0.5xT_{\sigma}$	307.1	312	1.6
	$0.75xT_{\sigma}$	257.8	262	1.6
	$0.99xT_{\sigma}$	197.8	203	2.6
Mode 3	$0xT_{\sigma}$	547.7	560	2.2
	$0.25xT_{\sigma}$	515.7	528	2.3
	$0.5xT_{\sigma}$	481.6	494	2.5
	$0.75xT_{\sigma}$	445.0	457	2.6
	$0.99xT_{\sigma}$	406.8	419	2.9
Mode 4	$0xT_{\sigma}$	885.3	909	2.6
	$0.25xT_{\sigma}$	859.0	883	2.7
	$0.5xT_{\sigma}$	828.0	835	0.8
	$0.75xT_{\sigma}$	760.9	767	0.8
	$0.99xT_{\sigma}$	689.1	694	0.7
Mode 5	$0xT_{\sigma}$	945.6	955	1.0
	$0.25xT_{\sigma}$	889.2	898	1.0
	$0.5xT_{\sigma}$	831.7	856	2.8
	$0.75xT_{\sigma}$	803.4	828	3.0
	$0.99xT_{\sigma}$	775.2	794	2.4

Table D.4 Resonant frequencies for CFFC boundary conditions. The relative difference was calculated with respect to predictions published by Jeyaraj et al.³³.

	$T_{uniform}$ (°C)	Transient model (Hz)	Jayaraj et al. (Hz)	Relative difference (%)
Mode 1	$0xT_{cr}$	87.7	88	0.4
	$0.25xT_{cr}$	82.5	83	0.6
	$0.5xT_{cr}$	75.1	76	1.2
	$0.75xT_{cr}$	62.1	63	1.5
	$0.99xT_{cr}$	14.6	17	14.2
Mode 2	$0xT_{cr}$	267.3	271	1.4
	$0.25xT_{cr}$	253.0	257	1.5
	$0.5xT_{cr}$	233.7	237	1.4
	$0.75xT_{cr}$	205.7	209	1.6
	$0.99xT_{cr}$	163.8	167	1.9
Mode 3	$0xT_{cr}$	371.7	376	1.1
	$0.25xT_{cr}$	359.7	365	1.5
	$0.5xT_{cr}$	344.4	349	1.3
	$0.75xT_{cr}$	324.3	330	1.7
	$0.99xT_{cr}$	297.8	303	1.7
Mode 4	$0xT_{cr}$	591.9	605	2.2
	$0.25xT_{cr}$	566.8	580	2.3
	$0.5xT_{cr}$	538.9	552	2.4
	$0.75xT_{cr}$	507.9	520	2.3
	$0.99xT_{cr}$	475.0	487	2.5
Mode 5	$0xT_{cr}$	651.2	662	1.6
	$0.25xT_{cr}$	628.5	638	1.5
	$0.5xT_{cr}$	600.7	611	1.7
	$0.75xT_{cr}$	566.1	577	1.9
	$0.99xT_{cr}$	524.4	535	2.0

Table D.5 Resonant frequencies for SSSS boundary conditions. The relative difference was calculated with respect to predictions published by Jeyaraj et al.³³.

	$T_{uniform}$ (°C)	Transient model (Hz)	Jayaraj et al. (Hz)	Relative difference (%)
Mode 1	$0xT_{cr}$	251.1	253	0.7
	$0.25xT_{cr}$	217.5	219	0.7
	$0.5xT_{cr}$	177.6	179	0.8
	$0.75xT_{cr}$	125.6	128	1.9
	$0.99xT_{cr}$	25.1	33	23.9
Mode 2	$0xT_{cr}$	543.4	548	0.8
	$0.25xT_{cr}$	511.1	516	1.0
	$0.5xT_{cr}$	476.6	481	0.9
	$0.75xT_{cr}$	439.4	444	1.0
	$0.99xT_{cr}$	400.5	405	1.1
Mode 3	$0xT_{cr}$	708.3	714	0.8
	$0.25xT_{cr}$	676.3	682	0.8
	$0.5xT_{cr}$	642.7	648	0.8
	$0.75xT_{cr}$	607.3	613	0.9
	$0.99xT_{cr}$	571.2	577	1.0
Mode 4	$0xT_{cr}$	994.3	1006	1.2
	$0.25xT_{cr}$	962.5	974	1.2
	$0.5xT_{cr}$	929.7	941	1.2
	$0.75xT_{cr}$	895.7	906	1.1
	$0.99xT_{cr}$	861.7	875	1.5
Mode 5	$0xT_{cr}$	1028.6	1039	1.0
	$0.25xT_{cr}$	996.9	1007	1.0
	$0.5xT_{cr}$	964.1	975	1.1
	$0.75xT_{cr}$	930.3	941	1.1
	$0.99xT_{cr}$	896.5	907	1.2

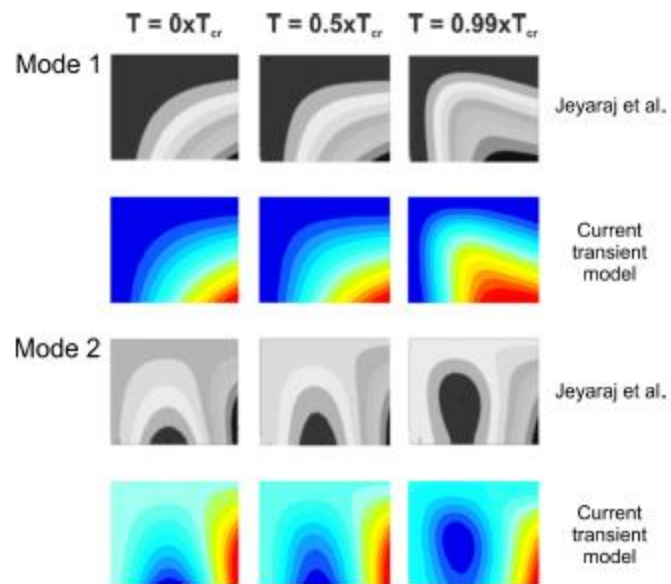


Figure D.1 CFFC first and second mode shapes predictions from Jeyaraj et al. ³³ and using the transient, temperature-dependent material model.

Appendix E Resonant frequency predictions using linear and non-linear solvers to calculate the effect of a transient thermal load up to buckling

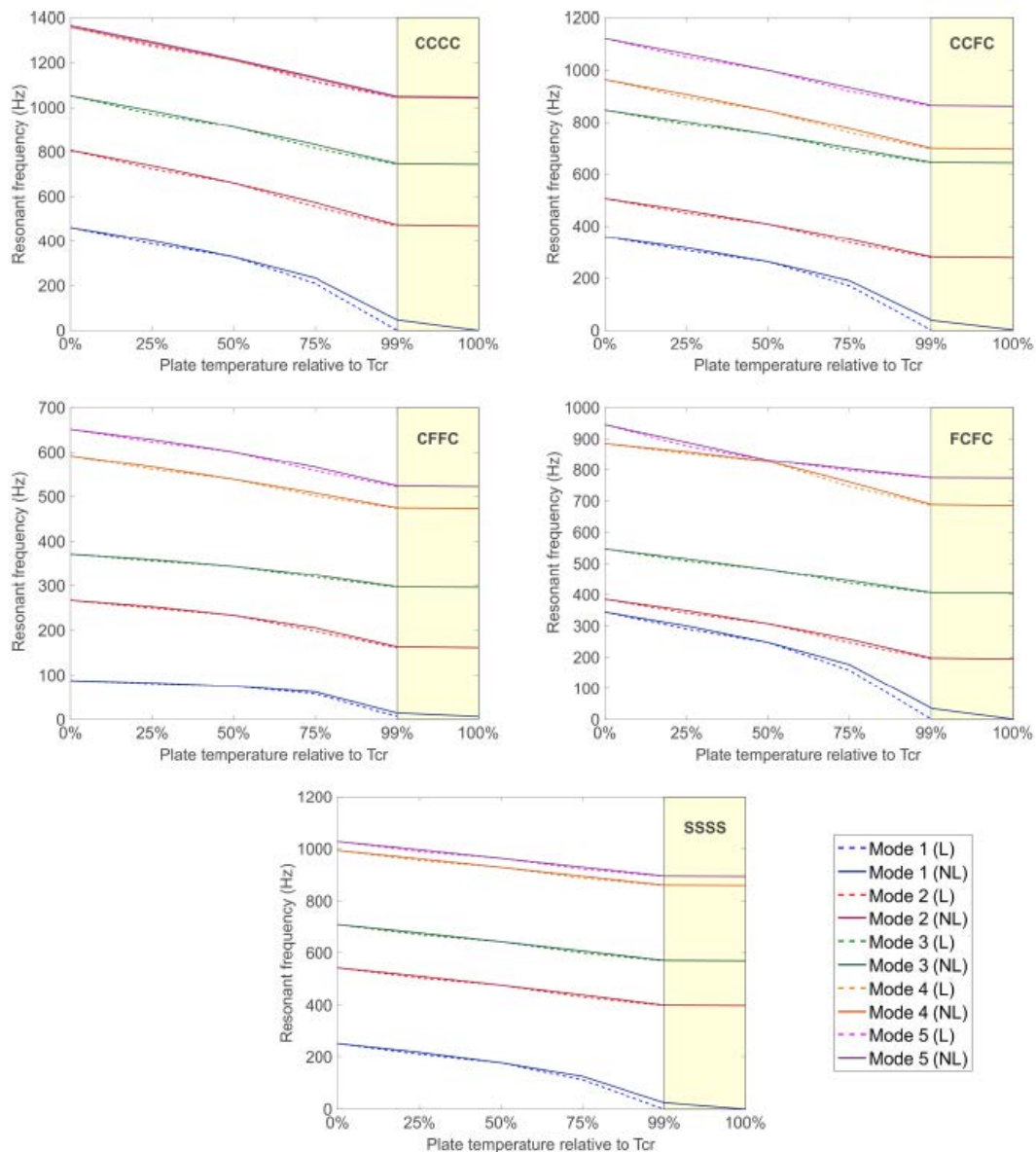


Figure E.1 Predictions from the transient model using a linear (L) and non-linear (NL) solvers. The shaded area in yellow was emphasised as the scale of the x-axis has been changed to better show the difference between the results near the buckling point (T_{cr}).

Appendix F Experimentally-acquired resonant frequency

results of a thin plate

Experimental readings											
Frequencies (Hz)											FE predictions
Mode 1	30.8	31.5	30.8	30	30.5	30.5	31	0.5	1.5	Mode 1	29
Mode 2	44.6	44.3	45	45.3	43.5	43.5	44	0.7	1.8	Mode 2	44
Mode 3	69	69.8	69.4	68.5	68.5	68.5	69	0.5	1.3	Mode 3	69
Mode 4	91.1	91	90.4	87.5	88.5	88.5	90	1.4	3.6	Mode 4	103
Mode 5	174	174	174	174	182.5	182.5	177	4.0	8.5	Mode 5	186
Mode 6	228	229	228	217	218	218	223	5.4	12.0	Mode 6	244
Mode 7	273	272	273	262	262	262	267	5.3	11.0	Mode 7	298
Mode 8	364	365	363	355	355.5	355	360	4.5	10.0	Mode 8	370
Mode 9	372	375	372	372	372.5	372.5	373	1.1	3.0	Mode 9	356
Mode 10	416	415	416	405	405	404	410	5.5	12.0	Mode 10	437
Mode 11	561	561	561	560.5	560.5	560.5	561	0.3	0.5	Mode 11	560

Table F.1 Resonant frequency results acquired at room temperature and corresponding predictions using a finite element (FE) model with temperature-dependent material properties

Experimental readings											
Frequencies (Hz)											FE predictions
Mode 1	30.5	30	30.5	30	30.5	32	30	Mean	Stand. Dev.	Range	Frequencies (Hz)
Mode 1	30.5	30	30.5	30	30.5	32	30	31	0.7	2.0	Mode 1
Mode 2	43	43.5	43	44	44	44	44	44	0.4	1.0	Mode 2
Mode 3	130	131	131	131	130	130	131	131	0.5	1.0	Mode 3
Mode 4	141	141	141	141	142	141	141	141	0.4	1.0	Mode 4
Mode 5	278	278	278	278	278	278	278	278	0.0	0.0	Mode 5
Mode 6	306	305	305	304	304	304	304	305	0.7	2.0	Mode 6
Mode 7	375	375	375	374	374	374	373	375	0.5	1.0	Mode 7
Mode 8	452	452	451	449	449	448	448	451	1.4	3.0	Mode 8
Mode 9	484	484	483	485	485	483	483	484	0.7	2.0	Mode 9
Mode 10	492	488	487	488	491	492	492	489	1.9	5.0	Mode 10
Mode 11	734	732	734	732	732	733	731	733	0.9	2.0	Mode 11

Table F.2 Resonant frequency results acquired when transversely heating the plate and corresponding predictions using a finite element (FE) model with temperature-dependent material properties

Experimental readings											FE predictions	
Frequencies (Hz)											Frequencies (Hz)	
Mode 1	31	29	26.5	29	30	29.5	29	1.5	29	4.5	Mode 1	29
Mode 2	42	40	42	40	41	40.5	41	0.9	41	2	Mode 2	42
Mode 3	70	67	60	69	69.5	71	67	3.7	67	10	Mode 3	70
Mode 4	75	72	82	76	75	76	76	3.3	76	10	Mode 4	75
Mode 5	199	214	202	189	205.5	221	202	8.2	202	25	Mode 5	213
Mode 6	269.5	269	293	271	271.5	257	275	9.1	275	24	Mode 6	269
Mode 7	297	297	310	297	299	292.5	300	5.1	300	13	Mode 7	283
Mode 8	360.5	360	372	361	359.5	361.5	363	4.7	363	12.5	Mode 8	354
Mode 9	369	369	382	370	369.5	365.5	372	5.1	372	13	Mode 9	383
Mode 10	386	386	400	386	388	385	389	5.5	389	14	Mode 10	390
Mode 11	513.5	540	505	541	541	531.5	528	15.6	528	36	Mode 11	502

Table F.3 Resonant frequency results acquired when longitudinally heating the plate and corresponding predictions using a finite element (FE) model with temperature-dependent material properties.

Appendix G List of Journal Publications

- Berke, R.B., Sebastian, C.M., Chona, R., Patterson, E.A. and Lambros, J., 2016. **High temperature vibratory response of Hastelloy-X: stereo-DIC measurements and image decomposition analysis.** *Experimental Mechanics*, 56(2), pp.231-243.
- Sebastian, C.M., López-Alba, E. and Patterson, E.A., 2017. **A comparison methodology for measured and predicted displacement fields in modal analysis.** *Journal of Sound and Vibration*, 400, pp.354-368.
- Patterson, E. A., Sebastian, C.M., Christian, W.J.R. and & Lopez-Alba, E., 2018. **The use of CCD cameras for characterizing the mean deflected shape of an aerospace panel during broadband excitation.** *Journal of Strain Analysis for Engineering Design*, 54(1), 13–23.
- Patterson, E.A. and Whelan, M.P., 2019. **On the validation of variable fidelity multi-physics simulations.** *Journal of Sound and Vibration*, 448, pp.247-258.
- Lopez-Alba, E., Sebastian, C.M., Silva, A.S. and Patterson, E.A., 2019. **Experimental study of mode shifting in an asymmetrically heated rectangular plate.** *Journal of Sound and Vibration*, 439, pp.241-250.
- Silva, A.S., Sebastian, C.M., Lambros, J. and Patterson, E.A., 2019. **High temperature modal analysis of a non-uniformly heated rectangular plate: Experiments and simulations.** *Journal of Sound and Vibration*, 443, pp.397-410.
- Silva, A.S., Lambros, J., Garner, D.M. and Patterson, E.A., 2019. **Dynamic Response of a Thermally Stressed Plate with Reinforced Edges.** *Experimental Mechanics*, <https://doi.org/10.1007/s11340-019-00536-w>.
- Amjad, K., Christian, W.J.R., Dvurecenska, K., Sebastian, C.M., Mollenhauer, D. and Patterson, E.A., (2019) **Compression and quantitative comparison of volumetric datasets using orthogonal decomposition.** In preparation for submission to *International Journal of Solids and Structures*.