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Nonlinear Estimation Framework for Optimized Target Tracking,
Estimation, and Self-assessment

Ondrej Straka
Zapadoceska univerzita v Plzni
2732/8 Univerzitni
Plzen 3, Jizni Predmesti, 30100
CZ

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Nonlinear Estimation Framework for Optimized Target Tracking and Self-Assessment (NOTES)

Final Report

Sep 23 2016 – Sep 22 2019

Principal investigator:
Doc. Ing. Ondřej Straka, Ph.D.

Summary

The research focused on **advancing the theory of multi-dimensional non-linear, non-Gaussian state estimation methods** with the special attention on monitoring and ensuring credibility (also called consistency) of the estimates produced by the estimation algorithm. Estimate consistency is important in safety-critical applications and is often required by the user. For the purpose of monitoring consistency of estimates produced by the current estimation algorithms, two methods were designed: the *cooperative filter design* combining estimates from two filters to provide the consistency information and the *entropy-based consistency monitoring* designed specifically for the stochastic integration filter to provide the consistency information. To prevent loss of estimate consistency due to an incorrect system model, the measurement difference autocorrelation method has been proposed that is capable of estimation of system noise characteristics including time correlation of the noises and also assessment of their Gaussianity. Further, the Rao-Blackwellized point mass smoother (RBPMS) has been developed, which produces the estimates in the form of a probability density function and thus inherently ensures consistency of the estimate. The advantage of the proposed RBPMS is also in its computational efficiency, which makes the smoother feasible for medium-sized problems appearing in tracking and navigation. Last, the techniques were developed for the orbital uncertainty propagation problem, for which suitability of several measures of nonlinearity and non-Gaussianity were analyzed. The result of the analysis was used in the proposal of an adaptive method representing the uncertainty by a mixture of Gaussian terms.

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1 Introduction

The objective of the project was to **theoretically advance multi-dimensional non-linear, non-Gaussian state estimation methods**. The specific goal of the project was to augment state estimation algorithms with an inherent adaptive structure providing the state estimate supplemented with information characterizing the estimate credibility (also called consistency). Within the main objective, the research focused on four directions:

1. Efficient on-line monitoring of credibility of filter estimates;
2. Estimation of system noises characteristics;
3. Design of efficient estimation algorithm for navigation; and

4. Analysis of nonlinearity measures in orbital uncertainty propagation problem.

The first direction dealt with the problem of estimate credibility, i.e., equality of the estimate error reported by the filter and the true estimate error. The problem of inconsistent estimates usually arises when computationally efficient Gaussian filters are used for problems that are highly nonlinear, in which case the filters often report the estimate error much smaller than its true value, i.e., they are overly optimistic. The purpose of the methods proposed in the project was to monitor the estimate credibility on-line and to report overly optimistic estimates, which is critical in safety-related applications.

Besides the rough approximations of the model nonlinearities, the inconsistent estimates may be a result of the Gaussian noises which are not white. Indeed, Gaussian filters are typically designed under assumption of the white noises, but in various positioning algorithms the measurement noise may be correlated in time due to sensor bias, and thus not white. Therefore, the second research direction dealt with inconsistent estimates caused by the ignored correlation of state and measurement noises or by non-Gaussian distribution of the noises.

The third direction dealt with the point-mass smoother, which is an algorithm solving the Bayesian recursive relations. Such point-mass smoothers do not suffer from the credibility problem as it represents the state estimate by a conditional probability density function evaluated over an orthogonal and equidistant grid. Such representation is natural for navigation problems, where the state usually represents the object position and velocity. However, such representation is computationally demanding and feasible implementations of the smoother algorithms require a modification of the methods, which is in our case the so-called Rao-Blackwellization.

The fourth direction dealt with the problem of state estimate uncertainty propagation. Such problem appears when dealing with e.g., space situational awareness and can be stated as follows. Given an initial position of an object, the goal is to predict its kinematic properties at future time instants. As the measurement of the object position are scarce, the prediction must be provided for a long time interval. The initial position and velocity of the object is uncertain and thus described as a Gaussian random variable with known mean and covariance matrix. The object moves in high or low orbit and its motion is described using kinematics relations, typically ordinary differential equations. The algorithms provide a credible estimate of the object state usually adapting their structure to be efficient. The structure adaptation is governed by a measure of nonlinearity assessing the current working point. The research focused on an analysis of the nonlinearity measures and their convenience for the problem.

2 Methods, Assumptions, and Procedures

2.1 Efficient on-line monitoring of credibility of filter estimates

Within the research direction, two solutions, i.e., estimate consistency monitoring blocks, were proposed as the cooperative filter and entropy-based monitor.

The first proposed solution is based on the novel concept of the *cooperative filter design*. The cooperative filter design takes advantage of a statistical evaluation of estimates of two different Gaussian filters (e.g., the extended Kalman filter (EKF) and the unscented Kalman filter (UKF)), which are configured to perform the same estimation task. If the measures of nonlinearity or non-Gaussianity are small enough, then both filters perform almost optimally and provide consistent estimates. On the other hand, if the measures of nonlinearity or non-Gaussianity associated with the actual estimate become large, then both filters provide different (but typically inconsistent) estimates with inherently

different characteristics. Using statistical hypothesis testing, the discrepancy between the EKF and UKF estimates is detected with given confidence level. As a consequence, the proposed methodology for consistency estimate monitoring does not require specification of any threshold in terms of minimal allowed value of the measure of nonlinearity or non-Gaussianity, but the threshold is implicitly computed on the basis of probability of false alarm. Such a probability is specified by the designer on the basis of required performance.

The second proposed solution is a new algorithm called *entropy-based consistency monitoring* that was proposed for the stochastic integration filter (SIF). The SIF is a Gaussian filter, which uses the stochastic integration rule (SIR) for the computation of the state and measurement prediction moments. The rule offers, in comparison with other integration rules, beneficial properties. It provides asymptotically exact integral values and besides that it also calculates an estimate of the integral error. The second property was used to propose the entropy-based consistency monitoring.

As has been mentioned above, an optimistic estimate is an estimate, in which true error is larger than the expected estimate error corresponding to the estimate error covariance produced by the filter. This limits using the filter in safety-critical applications. The true error is assessed using the mean-square error (MSE). Then, an optimistic estimate can also be characterized by an estimate error covariance much smaller than the MSE. Hence, a consistency indicator primarily detects a too small estimate error covariance. A pessimistic estimate (true estimate error is smaller than its expectation) is usually not a problem in such cases and can be considered as being too conservative.

The proposed consistency monitoring compares the estimate error covariance produced by the filter with a lower bound of the MSE involving entropy. If the covariance is less than the lower bound, an alert is raised indicating a possibly optimistic estimate. Note that the entropy in the lower bound is calculated exploiting the ability of the SIR to calculate estimate of the integral error.

2.2 Estimation of system noises characteristics

To improve detection of inconsistent estimate caused by the ignored correlation of state and measurement noises, the measurement difference autocorrelation (MDA) method has been proposed. The method estimates the potential state and measurement noise correlation, which can be mutual or in-time. If no noises correlation is identified, then the noises are assumed to be white and thus the provided state estimate is assumed to be consistent.

The MDA method is based on a design of a measurement predictor providing estimates on the basis of past measurements only. As consequence, the predictor has a unique property that is the measurement prediction error can be shown to be a weighted sum of state and measurement noises in several successive time instants. Then, by the statistical analysis of the measurement prediction error, the statistical properties of the state and measurement noise can be found. The properties include, besides the estimated moments of the state and measurement noise, also the estimates of the mutual correlation of the noises and the time correlation. The estimates of the statistical properties are proven to converge to the true values with increasing an number of data.

Additionally, an advanced version of the MDA method has been developed to provide additionally a statistical Gaussianity assessment of the noises as knowledge of the system noise distribution and parameters is crucial for assessing the state estimate credibility. The advanced method is capable of estimating the noise covariance matrices and through the covariance matrix estimates providing information necessary for the assessment of Gaussianity of the noises.

2.3 Design of efficient estimation algorithm for navigation

The point-mass density is a natural representation of the state estimate in the form of a conditional distribution. It is capable of expressing possible multi-modalities of the estimate with high accuracy and thus naturally reducing possibility of loss of estimate consistency. Unfortunately, this advantage is paid by high computational costs of the method. We decided to focus on the smoothing problem, i.e., the estimation of the state utilizing all measurements available, even those that were acquired later. The proposed method assumes that the system can be described by a conditionally linear model. The conditional linear assumption is realistic since it is valid for many navigation problems.

The main idea of the Rao-Blackwellized point-mass smoother is to facilitate the computation of the smoothed estimate by a transformation of the Bayesian functional relations to a suitable form. Such a Bayesian form then facilitates splitting of the system state into two parts. While the conditional distribution of the first part is represented by a point-mass density (computationally demanding), the conditional distribution of the second part is represented by a Gaussian density (computationally cheap). Then, the Rao-Blackwellized point-mass smoother can be by an order of magnitude faster than the standard point-mass smoother.

2.4 Analysis of nonlinearity measures in orbital uncertainty propagation problem

Orbit uncertainty propagation is an important tracking problem appearing in space situational awareness. The uncertainty, which is initially approximately Gaussian, is transformed by the time propagation and eventually becomes non-Gaussian (see Figure 1). Gaussian representation of the uncertainty becomes gradually inaccurate, which is often addressed by splitting the Gaussian representation into a mixture of Gaussian densities, which can describe the uncertainty with arbitrary accuracy. The time instants that are suitable for the splitting can be conveniently determined by the use of measures of nonlinearity or non-Gaussianity. Within the project, several measures of nonlinearity and non-Gaussianity were analyzed with a special focus on their behavior in the object uncertainty propagation from the numerical, theoretical, and practical points of view. Measures of nonlinearity and non-Gaussianity assess the degree of the model nonlinearity around a working point. The analysis treated highly elliptical orbit, low-earth orbit (LEO) and low-earth orbit with a stochastic noise (LEO-S) scenarios. For the behavior analysis, the experiments with the Monte Carlo and unscented transformation based propagations were performed. The procedure consisted in an analysis of the time behavior of the measures for the three scenarios. Based on the analysis, two measures have been proposed to serve as nonlinearity indicators for the orbital uncertainty propagation methods based on Gaussian mixtures.

3 Results and Discussions

3.1 Efficient on-line monitoring of credibility of filter estimates

The cooperative filter design methodology was described in the FUSION 2017 conference paper **J. Duník, O. Straka, J. Ajgl, E. Blasch: “From Competitive to Cooperative Filter Design”**, In *Proceedings of the 20th International Conference on Information Fusion, Xian, China, 2017*. and details of the entropy-based consistency monitoring were published in the FUSION 2018 conference paper **O. Straka, J. Duník, “Entropy-based Consistency Monitoring for Stochastic Integration Filter”**, In *Proceedings of the 21st International Conference on Information Fusion, Cambridge, UK, 2018*.

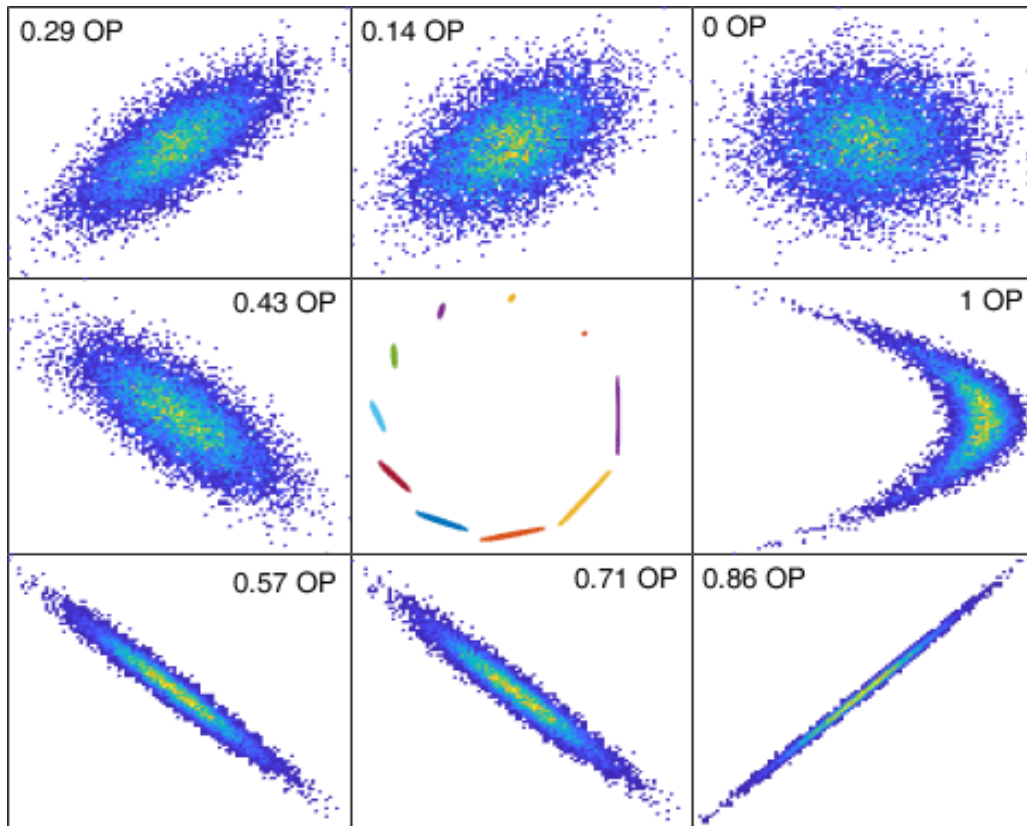


Figure 1: The transition of the Gaussian distribution to the non-Gaussian distribution for the highly elliptical orbit evaluated at several time instants within a single orbital period (OP) with the spread of points magnified ten times. The comprehensive view is in the middle inset, whereas side insets magnify some of the time instants.

The *cooperative filter design* methodology was evaluated and illustrated using a set of simulations in MATLAB. In Figure 2, performance of the proposed consistency monitoring test of the estimate provided by the EKF is illustrated. It can be seen, that between the time instants $k = 8$ and $k = 19$, the state estimate \hat{x}_k is too far from the true state x_k , which in real applications is not known, and the estimate error does not fit the interval given by the estimated covariance matrix P_k , i.e., the estimate is inconsistent. The proposed consistency monitoring test, designed with probability of false alarm $P_{FA} = 10^{-3}$, is able to detect such inconsistent behavior (without knowledge of the true state) and to raise an alert in the time interval.

The performance of the entropy-based consistency monitoring is illustrated in the three figures (Figs. 3, 4, 5), where the following quantities are depicted:

- left pane:
 - the true state (red line),
 - its SIF estimate (blue line),
- right pane:
 - estimate error (blue line),
 - 3 standard deviations (STD) (light blue region),

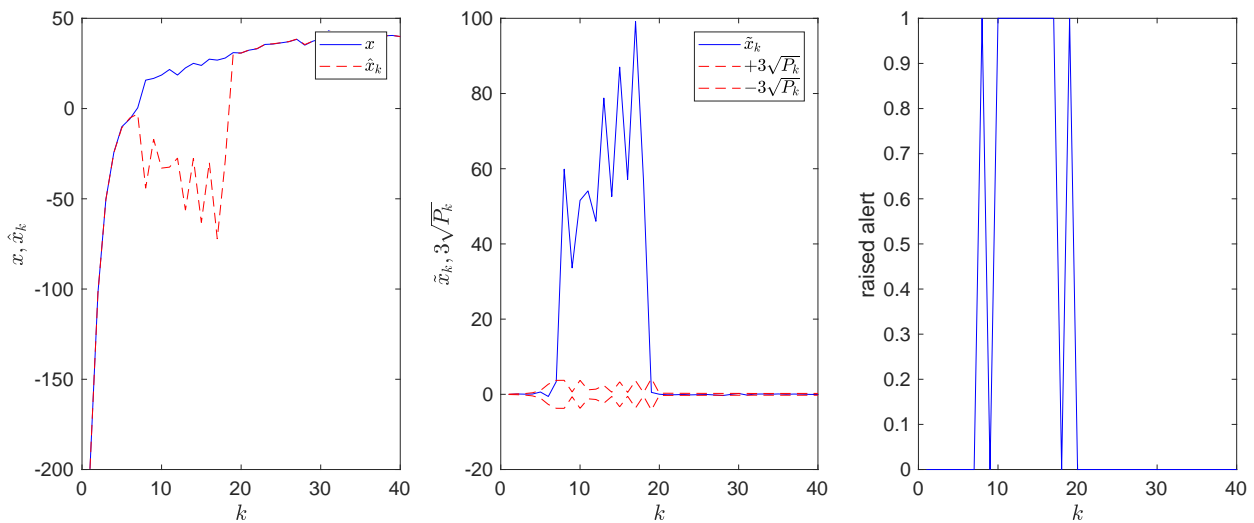


Figure 2: Illustration of proposed state estimate consistency monitoring performance.

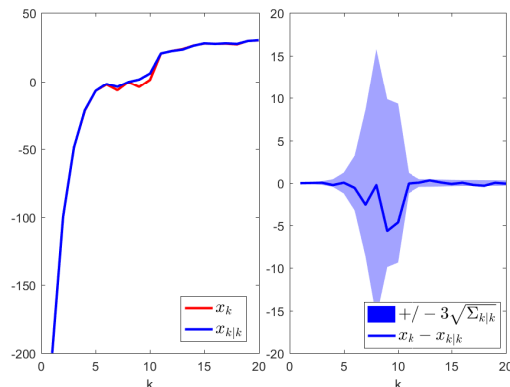


Figure 3: Example of a convergent trajectory. Left: state (blue) and its SIF estimate (red), Right: estimate error (blue), 3-STDs region (light blue), indicated optimistic estimates (black points).

- alerts raised by the consistency indicator (black points).

Note that the simulations were made for the univariate nonlinear non-stationary Gaussian model, which is often used as a benchmark problem. The results indicate that the proposed consistency monitoring tests are able to raise alerts at time instants, when the true estimate error may be larger than expected. The proposed tests will be particularly suitable as an add-on to any Gaussian filter working in safety-critical applications, such as tracking resident space objects.

3.2 Estimation of system noises characteristics

The proposed MDA method, its mathematical derivation, and numerical simulations were discussed in detail in the CDC 2018 conference paper **O. Kost, J. Duník, O. Straka, “Correlated Noise Characteristics Estimation for Linear Time-Varying Systems”, In 57th IEEE Conference on Decision and Control, Miami Beach, FL, USA, December 17-19, 2018.**

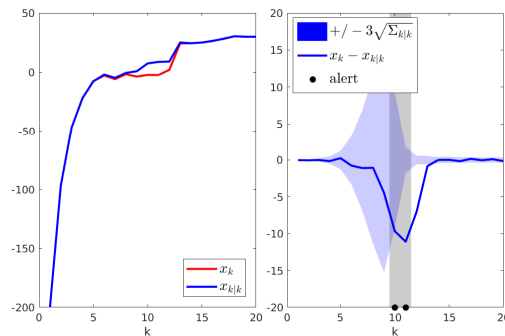


Figure 4: Example of a trajectory with temporarily optimistic estimates. Left: state (blue) and its SIF estimate (red), Right: estimate error (blue), 3-STDs region (light blue), indicated optimistic estimates (black points).

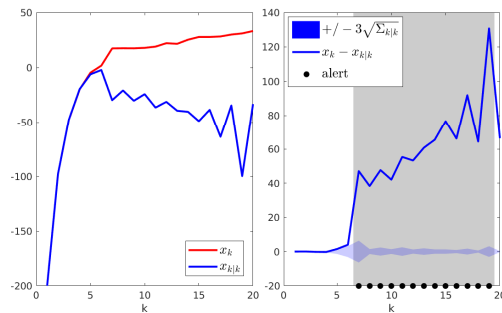


Figure 5: Example of a divergent trajectory. Left: state (blue) and its SIF estimate (red), Right: estimate error (blue), 3-STDs region (light blue), indicated optimistic estimates (black points and gray strips).

The advanced MDA method was published in a journal paper **J. Duník, O. Kost, O. Straka, E. Blasch: “Covariance Estimation and Gaussianity Assessment for State and Measurement Noise”**, which was accepted for the publication in **Journal of Guidance, Control, and Dynamics**.

The papers present a complete analytical derivation of the proposed method (and its advanced version) designed for estimation of the noise covariance matrices and noises Gaussianity assessment for linear time invariant and linear time varying models with the state noise shaping matrix. It was shown that the method provides unbiased and consistent estimates of the noise covariance matrices and generates hypothesis-testing based decisions on whether the noises are Gaussian or not.

Combination of these properties is unique in the state-of-the-art noise covariance matrix estimation methods and is important for design of many optimal navigation and tracking algorithms requiring outputs with guaranteed integrity. The performance of the method has been illustrated in an extensive simulation study using four examples. The simulations confirmed all the theoretically derived properties of the method.

Knowledge of an appropriate system state-space model will help in optimal design of many signal processing algorithms for applications such as global navigation satellite system (GNSS) based navigation and radar measurement based object tracking.

3.3 Design of efficient estimation algorithm for navigation

The results of the research concerned with the Rao-Blackwellized point-mass smoother were partially presented at the FUSION 2019 conference

J. Duník, O. Straka: “Rao-Blackwellised Point-Mass Smoothers for a Class of Conditionally Linear Dynamic Models”, In Proceedings of the 22nd International Conference on Information Fusion, Ottawa, Canada, 2019.

and the generalized version of the smoother is summarized in the paper

J. Duník, O. Straka: “Design of Rao-Blackwellized Point-Mass Smoother for Conditionally Linear and Gaussian Models”, which was accepted for publication in **IEEE Transactions on Signal Processing**.

In the papers, a novel multi-step Rao-Blackwellized point-mass smoother was proposed. Compared to the standard point-mass smoother, the Rao-Blackwellized smoother has significantly lower computational complexity as only a part of the state vector is estimated using the computationally expensive point-mass methodology, whereas the remaining part of the state vector is found using the set of efficient linear estimators. Compared to the Rao-Blackwellized particle smoother, which is popular in the literature, the proposed smoother is a deterministic algorithm and thus more suitable for safety-critical applications.

In the journal paper, the Rao-Blackwellized point-mass smoother (RBPMS) was tested using a bearings-only tracking example and its performance was compared with the performance of

- extended Kalman filter (EKF),
- extended Rauch-Tung-Striebel smoother (ERTSS),
- Rao-Blackwellized particle filter (RBPF),
- Rao-Blackwellized particle smoother (RBPS),
- Rao-Blackwellized point-mass filter (RBPMF).

The performance was assessed using the

- Mean-square error (MSE) criterion, which can be computed only when the true state is known (i.e., in simulated examples), and
- Average variance (AVar), which is computed by averaging the conditional estimate variance provided by the filters,

and the values are given in Table 1. Note that for the MSE computation, the truth knowledge must be available. From the table it is evident that the filter and smoother assuming Gaussian distribution of the state estimate (EKF and ERTSS) and the filter and smoother using a random set of particles for the estimate (RBPF and RBPS) are too optimistic, i.e. the estimate errors they report (AVar) are much smaller than the errors actually are (MSE). The filters and smoothers using the point-mass representation of the state estimate (RBPMF and RBPMS) are credible in this sense, i.e., the error they report is the same as the actual value of the error. Moreover, the proposed RBPMS achieves the best estimate quality in the computationally efficient setups. Also, it is capable to use more information from the measurements than the filter RBPMF.

Table 1: Mean square error and average variance of the filter and smoothers in the bearings-only tracking scenario.

	Gaussian		particle-based		point-mass	
	EKF	ERTSS	RBPF	RBPS	RBPMF	RBPMS
MSE	0.21	0.19	0.18	0.16	0.15	0.13
AVar	0.13	0.10	0.10	0.09	0.15	0.12

3.4 Analysis of nonlinearity measures in orbital uncertainty propagation problem

The results of the nonlinearity and non-Gaussianity measures analysis for the orbital uncertainty propagation were published in a FUSION 2019 conference paper

J. Havlík, O. Straka: “Measures of Nonlinearity and non-Gaussianity in Orbital Uncertainty Propagation” In Proceedings of the 22nd International Conference on Information Fusion, Ottawa, Canada, 2019.

The analysis revealed that from the theoretical, numerical, and implementation points of view, the measures based on the mean-square error and the divided difference are effective in measuring nonlinearity of the propagation. Figure 6 illustrates time behavior of several measures on nonlinearity (MoNL) and a measure of non-Gaussianity (MoNG) for the low Earth orbit (LEO) scenario computed using the unscented transform (UT) and Monte Carlo (MC) methods.

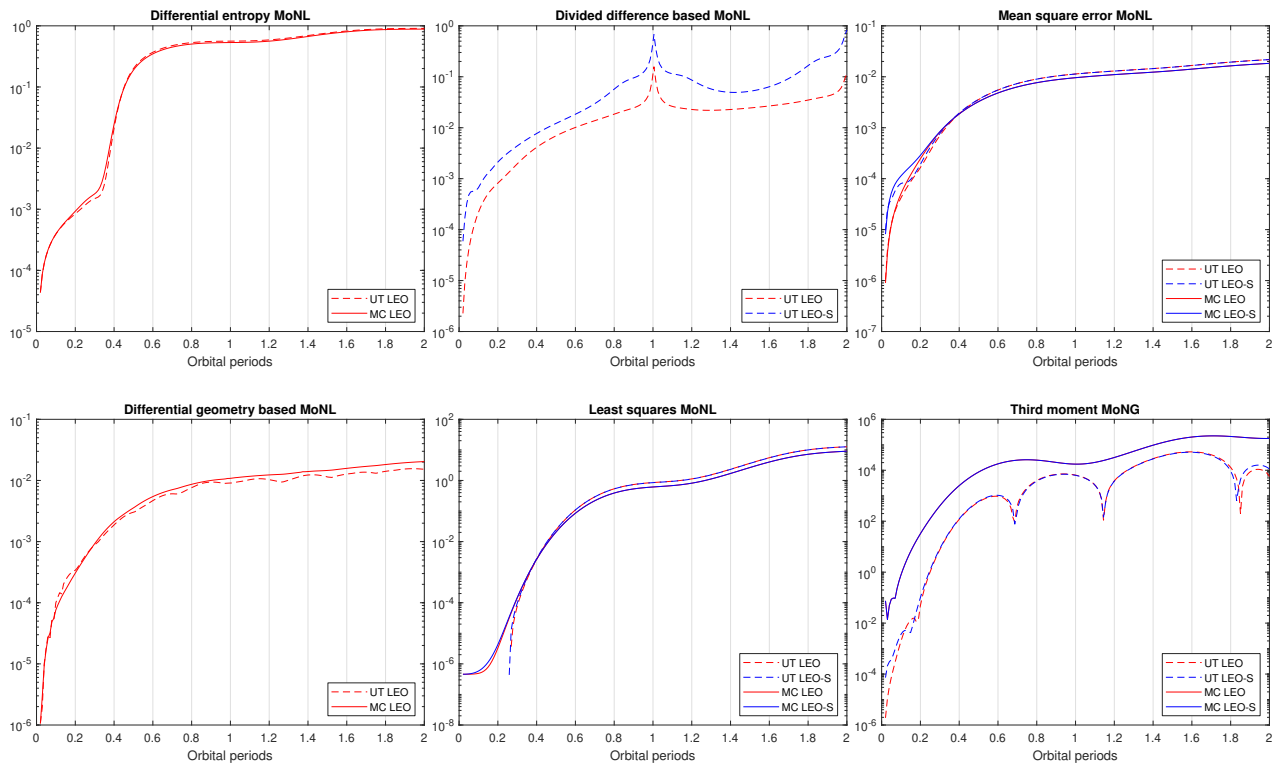


Figure 6: Evaluation of the measures for the LEO and the LEO-S

The results of the analysis will serve for the design of an efficient algorithm predicting the object position and velocity. The traditional algorithms describe the uncertainty by a Gaussian mixture

that adaptively changes the number of mixture components to represent the uncertainty efficiently. The decision whether the mixture components should be split in order to preserve the fidelity of the uncertainty description has been based on ad-hoc rules so far.

The analysis performed within the project makes it possible to select convenient measures to indicate the time instants for the splitting of a Gaussian representation of the uncertainty to a Gaussian mixture. The results of the analysis were also used to propose the efficient algorithm for object position and velocity prediction, which will be published in a conference paper "J. Krejci, O. Straka: Adaptive Gaussian Mixture Method for Uncertainty Propagation in Space Surveillance", which will be presented at the 15th European Workshop on Advanced Control and Diagnosis in November 2019, i.e. after the project deadline.

The measures of nonlinearity and non-Gaussianity were implemented within the nonlinear estimation framework NEF <https://idm.kky.zcu.cz/sw.html> which is available free for non-commercial use.

4 Conclusions

The results of the research were published using four papers presented at the FUSION 2017, 2018, and 2019 conferences and at the Conference on Decision and Control CDC 2018. The FUSION conference is a prestigious meeting of the information fusion community, and also the CDC conference is a notable conference of the automatic control community.

List of conference papers:

1. J. Duník, O. Straka, J. Ajgl, E. Blasch: "From Competitive to Cooperative Filter Design", In *Proceedings of the 20th International Conference on Information Fusion, Xian, China, 2017*.
2. O. Straka, J. Duník, "Entropy-based Consistency Monitoring for Stochastic Integration Filter", In *Proceedings of the 21st International Conference on Information Fusion, Cambridge, UK, 2018*.
3. J. Duník, O. Straka: "Rao-Blackwellised Point-Mass Smoothers for a Class of Conditionally Linear Dynamic Models", In *Proceedings of the 22nd International Conference on Information Fusion, Ottawa, Canada, 2019*.
4. J. Havlík, O. Straka: "Measures of Nonlinearity and non-Gaussianity in Orbital Uncertainty Propagation" In *Proceedings of the 22nd International Conference on Information Fusion, Ottawa, Canada, 2019*.
5. O. Kost, J. Duník, O. Straka, "Correlated Noise Characteristics Estimation for Linear Time-Varying Systems", In *57th IEEE Conference on Decision and Control, Miami Beach, FL, USA, December 17-19, 2018*.

Further, the extended and advanced versions of the proposed algorithms were and summarized in two journal papers. The journal papers have been accepted for the publication in IEEE Transactions on Signal Processing and in Journal of Guidance, Control, and Dynamics.

List of journal papers:

1. J. Duník, O. Kost, O. Straka, E. Blasch: "Covariance Estimation and Gaussianity Assessment for State and Measurement Noise", accepted for the publication in the Journal of Guidance, Control, and Dynamics.

2. J. Duňík, O. Straka: “Design of Rao-Blackwellized Point-Mass Smoother for Conditionally Linear and Gaussian Models”, accepted for the publication in IEEE Transactions on Signal Processing.

The MDA method, the measures of nonlinearity and non-Gaussianity were implemented within the MATLAB® toolbox Nonlinear Estimation Framework NEF available at <https://idm.kky.zcu.cz/sw.html> which is and available free for non-commercial use.

All the published results have contributed to the project objective, which was to theoretically advance state estimation methods for multi-dimensional non-linear and non-Gaussian systems. The primary aim of the results is to ensure consistency of the state estimate, which is of crucial importance especially when dealing with highly nonlinear or non-Gaussian models in safety-critical applications.

Impact of the project

The results of the project were presented at the 20th, 21st and 22nd International Conferences on Information Fusion (FUSION 2017, FUSION 2018, and FUSION 2019) with many attendees from DoD (NRL, AFRL, AFOSR, US Navy, ARL). Within the conferences, the project investigators co-organized special sessions on “Advanced Nonlinear Filters” and “Advanced Nonlinear State Estimation”, which involved more than 15/11 papers on nonlinear tracking with some focusing on Dynamic Data-Driven Application Systems (DDDAS). Some of the results were presented by the project investigators in the tutorials “Noise Covariance Matrices in State Space Models: Overview, Algorithms, and Comparison of Estimation Methods” (attended by an NRL member). The project research was coordinated with Dr. Blasch and discussed with Dr. Crouse (NRL) and Dr. Kaplan (ARL).

List of abbreviations

EKF	extended Kalman filter
UKF	unscented Kalman filter
SIR	stochastic integration rule
SIF	stochastic integration filter
MSE	mean-squared error
MDA	measurement difference autocorrelation
OP	orbital period
STD	standard deviation
GNSS	global navigation satellite system
ERTSS	extended Rauch-Tung-Striebel smoother
RBPF	Rao-Blackwellized particle filter
RBPS	Rao-Blackwellized particle smoother
RBPMF	Rao-Blackwellized point-mass filter
RPMS	Rao-Blackwellized point-mass smoother
AVar	average variance
MoNL	measure of nonlinearity
MoNG	measure of non-Gaussianity
LEO	low Earth orbit
LEO-S	low Earth orbit with stochastic component
UT	unscented transform
MC	Monte Carlo
NEF	nonlinear estimation framework