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Exact algorithms for the (A)symmetric Traveling Salesman Problem with order dependent constraints and objective function

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# FINAL PERFORMANCE REPORT

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Exact algorithms for the (A)symmetric Traveling Salesman Problem with order dependent constraints and objective function

Dr. Valentina Cacchiani

Period of performance: 15 Dec 2016 - 14 Dec 2019 (Three years)

## Table of Contents

In this report, we present the research we performed during the three years and the achieved results. In Section 1, we give a summary of the main contributions. Section 2 describes the studied problems, while Section 3 reports the proposed solving methods. The results we obtained are reported in Section 4. Finally, we conclude the report in Section 5.

## 1 Summary

During the three years of research, we worked on the following problems:

- Traveling Salesman Problem with Pickups, Deliveries, and Draft Limits (TSP-DL);
- Traveling Salesman Problem with Time-dependent Service Times (TSP-TS);
- Pollution Traveling Salesman Problem (PTSP);
- Route Optimization with Unmanned Aerial Base Stations Aided Networks (RO-UAB);
- Multi-trip Pickup and Delivery Problem with Time Windows and Synchronization (MT-PDTWS);
- Sum Coloring Problem (SCP);
- $k$ -Vertex Cut Problem ( $k$ -VCP);
- Knapsack Problem with Penalised Fractional Items (KP-frac);
- Chance Constrained Problems (CCP);

The first four problems are extensions of Traveling Salesman Problem (TSP), while the fifth deals with a variant in which more vehicles (rather than only one) are involved. Problems six and seven are generalizations of Graph Coloring and Partitioning, while the eighth problem is a generalization of the Knapsack Problems. For these problems, we propose Mixed Integer Linear Programming (MILP) models, exact, approximate, heuristic and metaheuristic algorithms. Finally, we considered Chance Constrained Problems in general, from a methodological viewpoint.

The following works were published in International Journals:

- E. Malaguti, S. Martello, A. Santini. The traveling salesman problem with pickups, deliveries, and draft limits. *Omega*, 74:5058, 2018.
- E. Malaguti, M. Monaci, P. Paronuzzi, U. Pferschy. Integer Optimization with Penalized Fractional Values: The Knapsack Case. *European Journal of Operational Research*, 273:874-888, 2019.
- A. Bettinelli, V. Cacchiani, T. G. Crainic, D. Vigo. A Branch-and-Cut-and-Price Algorithm for the Multi-trip Separate Pickup and Delivery Problem with Time Windows at Customers and Facilities. *European Journal of Operational Research*, 279(3):824-839, 2019.
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- F. Furini, I. Ljubic, E. Malaguti, P. Paronuzzi. On integer and bilevel formulations for the k-vertex cut problem. *Mathematical Programming Computation*, in press <https://doi.org/10.1007/s12532-019-00167-1>
- A. Lodi, E. Malaguti, G. Nannicini, D. Thomopulos. Nonlinear chance-constrained problems with applications to hydro scheduling. *Mathematical Programming*, in press <https://doi.org/10.1007/s10107-019-01447-3>

The work “The traveling salesman problem with pickups, deliveries, and draft limits”, *Omega*, 74:5058, 2018 (E. Malaguti, S. Martello, A. Santini) was awarded the best Omega paper award in year 2018.

The following works were published in Proceedings of International Conferences:

- V. Cacchiani, C. Contreras-Bolton, J.W. Escobar, L.M. Escobar-Falcon, R. Linfati, P. Toth. An Iterated Local Search Algorithm for the Pollution Traveling Salesman Problem. In: Daniele P., Scrimali L. (eds) *New Trends in Emerging Complex Real Life Problems*. AIRO Springer Series, vol 1. Springer, Cham, pp. 83-91, 2018.
- S. Mignardi, C. Buratti, V. Cacchiani, R. Verdone. Path Optimization for Unmanned Aerial Base Stations with Limited Radio Resources. *IEEE 29th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)-Workshop WS-02 on “UAV Communications for 5G and Beyond”*, 2018.
- F. Furini, E. Malaguti, S. Martin, I.C. Ternier. ILP models and column generation for the minimum sum coloring problem. *Proceedings of the 8th International Network Optimization Conference (INOC20017)*, ENDM 64, pp. 215224, 2018.

In addition, our results were presented in the following conference talks:

- C. Contreras-Bolton, V. Cacchiani, P. Toth. Models and algorithms for the traveling salesman problem with time-dependent service times. AIRO 2016 Conference, Trieste (Italy), September 2016
- P. Toth, V. Cacchiani, C. Contreras-Bolton. Relaxations and Algorithms for the Traveling Salesman Problem with Time-Dependent Service Times. IEOM 2017 Conference, Rabat (Morocco), April 2017
- P. Toth, V. Cacchiani, C. Contreras-Bolton. Exact and Heuristic Algorithms for the Traveling Salesman Problem with Time-Dependent Service Times. NOW 2017 Conference, Viterbo (Italy), June 2017
- V. Cacchiani, C. Contreras-Bolton, P. Toth. Exact Algorithms for the Traveling Salesman Problem with Time-dependent Service Times. VeRoLog 2017 Conference, Amsterdam (The Netherlands), July 2017
- C. Contreras-Bolton, V. Cacchiani, P. Toth. Exact and heuristic algorithms for the Traveling Salesman Problem with time-dependent service times, ODS 2017 Conference, Sorrento (Italy), September 2017
- P. Toth, V. Cacchiani, C. Contreras-Bolton. Models and Algorithms for the Traveling Salesman Problem with Time-dependent Service Times, INFORMS Annual Meeting, Houston (Texas), October 2017.
- V. Cacchiani, C. Contreras-Bolton, P. Toth. Algorithms for the Traveling Salesman Problem with Time-Dependent Service Times. Odysseus 2018, Cagliari (Italy), June 2018.
- E. Malaguti, M. Monaci, P. Paronuzzi, U. Pferschy. Integer optimization with penalized fractional values: the knapsack case. MIP2018, Greenville, US, June 2018.
- E. Malaguti, M. Monaci, P. Paronuzzi, U. Pferschy. The fractional knapsack problem with penalties. ISMP2018, Bordeaux, France, July 2018.
- V. Cacchiani, C. Contreras-Bolton, L. M. Escobar-Falcon, P. Toth. Models and metaheuristics for the Pollution Traveling Salesman Problem. IEOM 2019, Bangkok, Thailand, March 2019.
- V. Cacchiani, C. Contreras-Bolton, L. M. Escobar-Falcon, P. Toth. Algorithms for the Pollution Traveling Salesman Problem. VeRoLog 2019, Seville, Spain, June 2019.

## 2 Introduction

The (A)symmetric Traveling Salesman Problem with order dependent constraints and objective function deals with the problem of determining a minimum cost Hamiltonian circuit where i) the objective function or ii) the constraints depend on the visit order of the vertices. Problems in this class include real-life features arising in distribution logistics and general freight transportation. They fall in the category of NP-hard problems, being generalizations of the well-known TSP. In this section, we briefly describe the studied problems.

The TSP-DL is a generalization of TSP with pickup and delivery, that stems from applications in maritime logistics, in which each node represents a port and has a known draft limit. Each

customer has a demand, characterized by a weight, and pickups and deliveries are performed by a single ship of given weight capacity. The ship is able to visit a port only if the amount of cargo it carries is compatible with the draft limit of the port.

The TSP-TS is a variant of TSP in which the customer service duration depends on the time at which the service starts at the customer, and therefore cannot be simply included as a constant in the travel time. The goal of TSP-TS is to minimize the total tour duration, which includes the total travel times and the total service times.

The PTSP is a generalization of the TSP, which aims at finding a Hamiltonian tour that minimizes a function of fuel consumption (dependent on distance travelled, vehicle speed and load) and driver costs.

The RO-UAB is a variant of the Orienteering Problem ([7]) that deals with the problem, arising in future moving networks, of determining an optimal route for a drone, with the goal of serving the maximum number of users that cannot be served by static Terrestrial Base Stations ([3]). The users have activation and expiration times that define a time-window when they can be served, and the drone can fly at different speeds and has a maximum battery capacity. In addition, we consider a limited number of radio resources to be assigned to the users along the route.

The MT-PDTWS arises in the context of two-tiered City Logistics Systems ([1]), where the first tier refers to the transportation between the city distribution centers, in the outskirts of the city, and intermediate facilities, while the second tier refers to the transportation of goods between the intermediate facilities and the (pickup and delivery) customers. In MT-PDTWS, synchronization of vehicle arrivals and operating time windows of facilities and customers is an important element. The MT-PDTWS calls for determining minimum cost multi-trip routes, for a given fleet of vehicles, to service pickup and delivery customers, while taking into account vehicle capacity, time windows, and synchronization.

The SCP generalizes the Vertex Coloring Problem where adjacent vertices must receive different colors, and the cost of used colors has to be minimized. The latter is given by the index of the color times the number of vertices receiving the color. The problem has applications in scheduling and telecommunications.

The  $k$ -VCP asks for determining the minimum cardinality subset of nodes whose removal disconnects a given graph into at least  $k$  components. The problem falls within the family of critical node detection problems, and has applications in network analysis and protection.

The KP-frac is a variant of the Knapsack Problem in which items can be split at the expense of a penalty depending on the fractional quantity. We study it as a prototype problem for integer optimization problems where variables can potentially take fractional values, but this occurrence is penalized in the objective function.

Finally, CCPs are optimization problems with probabilistic constraints, where the solution to the problem must optimize the objective function subject to being inside the feasible region for a set of realizations of a random variable that occurs with probability at least a given value.

### 3 Methods, Assumptions and Procedures

For the described problems, we propose MILP models, and exact, heuristic and metaheuristic algorithms.

In particular, for the TSP-DL we present an Integer Linear Programming (ILP) formulation and show how classical valid inequalities from the literature can be adapted to the considered problem.

We introduce heuristic procedures and a branch-and-cut exact algorithm.

For TSP-TS, we propose an exponential-size formulation incorporating subtour elimination constraints. This formulation is used within an ad hoc developed branch-and-cut algorithm to solve the problem to optimality, where the subtour elimination constraints are separated by using the separation procedure proposed in [4]. An improved lower bound on the total service time is also proposed. Furthermore, we propose a genetic algorithm to quickly derive heuristic solutions, whose values are also used as cut-off in the branch-and-cut method.

PTSP is formulated on a directed graph, and subtour elimination constraints are incorporated in the model also in this case. The objective function consists of the following components to be minimized: the fuel consumption, that takes into account the energy required by speed variations and that used to carry the curb weight and the load on the vehicle, and the driver wage, which depends on the tour duration and thus on speed and service times. An iterated local search algorithm is proposed for solving the PTSP, and is compared with a Cut-and-Branch algorithm we developed.

RO-UAB is modelled by using a time-space graph. Each node corresponds to a position of the drone in time and space, and can serve a subset of the users. An ILP arc flow model is proposed, which takes into account the limited available resources. We propose a Rolling Horizon heuristic algorithm, where the planning horizon is divided into time windows and the model is iteratively solved for each single time window.

MT-PDTWS is a generalization of the Vehicle Routing Problem (VRP). We propose a formulation with exponentially many route variables. We develop a branch-and-cut-and-price algorithm to solve the problem to optimality, where the pricing problem consists of solving an Elementary Shortest Path Problem with Resource Constraints, and is solved by Dynamic Programming algorithms with bi-directional search.

For the SCP we present an ILP formulation with an exponential number of variables. We introduce a heuristic procedure base on the exponential formulation.

For the  $k$ -VCP, we provide two new ILP formulations, along with families of strengthening valid inequalities. Both models involve an exponential number of constraints for which we provide poly-time separation procedures and design the respective branch-and-cut algorithms. In the second formulation, the model is derived from the perspective of a two-phase Stackelberg game in which a leader deletes the vertices in the first phase, and in the second phase a follower builds connected components in the remaining graph.

For the KP-frac, we analyze relevant properties of the problem, present alternative mathematical models, and analyze their performance from a theoretical viewpoint. In addition, we introduce a Fully Polynomial Time Approximation Scheme for the approximate solution of the general problem, and an improved dynamic programming approach that computes the optimal solution in one relevant case.

For CCPs, we present a Branch-and-Cut algorithm for a class of nonlinear chance-constrained mathematical optimization problems with a finite number of scenarios, and discuss an application in hydro scheduling.

## 4 Results and Discussion

The proposed models and methods were tested on benchmark instances from the literature or on real-world scenarios. In this section, we summarize the main achievements we obtained.

For the TSP-DL we examined, through extensive computational experiments, the impact of the various cuts and the performance of the proposed algorithms. The obtained results showed the effectiveness of the proposed methods.

The results obtained by the proposed branch-and-cut algorithm for the TSP-TS were compared with those obtained by a compact MILP model presented in [6] on instances, proposed in [6], that come from the TSPLIB library [5], showing that the proposed approach is able to reduce the computing time to solve instances to optimality. In addition, we tested both the branch-and-cut and the genetic algorithm on asymmetric instances from the TSPLIB Library and on larger size symmetric instances: the obtained results show the effectiveness of the two proposed methods.

The iterated local search and the cut-and-branch algorithms for the PTSP were tested on instances with up to 50 customers, taken from [2], and adapted to the single vehicle case. The results show that the cut-and-branch algorithm is capable of deriving the optimal solution for all the instances with up to 25 customers in very short computing times (on average about 22 seconds). As expected, instances that contain more customers are more difficult: in particular, no instance with 45 or more customers can be solved by the cut-and-branch to optimality within the time limit of two hours. The iterated local search algorithm has a very short computing time (0.50 seconds on average). By executing it 10 times for every instance and selecting the best solution found, it is able to find the best solution for 98.9% of the instances.

The arc flow model and the Rolling Horizon heuristic algorithm for RO-UAB were tested on a realistic scenario proposed in [3], with different users expiration times, duration of the Rolling Horizon window, and number of available radio resources. The computational results show that the Rolling Horizon heuristic algorithm can be effectively used in practice, since it can find good quality solutions in short computing times (about 20-30 seconds).

The branch-and-cut-and-price algorithm for MT-PDTWS was tested on instances with up to 200 customers. Our computational experiments show that the algorithm is able to solve to optimality or near-optimality most of the instances.

For the SCP, extensive computational experiments showed that the proposed approach outperforms previous algorithms from the literature.

For the  $k$ -VCP, the combination of the two ILP formulations in a hybrid Branch-and-Cut algorithm is able to largely outperform a recently published state-of-the-art algorithm from the literature.

For the KP-frac, we performed an extensive computational analysis to illustrate the performances of all the proposed approaches on a large benchmark set derived from the literature. Our experiments showed that we are able to efficiently solve medium-sized KP-frac instances, and that the hardness of the instances is closely related to the shape of the penalty function used.

Finally, a computational study using data from ten hydroplants in Greece shows that the proposed Branch-and-Cut algorithm for CCPs solves instances faster than applying a general-purpose solver for convex mixed-integer non-linear programming problems to a deterministic reformulation, and scales much better with the number of scenarios.

## 5 Conclusions

The proposed methods show their effectiveness and novelty. Our works were presented in international conferences and published in three conference proceedings and six international journals. One of the works was awarded the best Omega paper award 2018.

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