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Addendum to ARL-TR-8974, *A Survey of Methods for Estimating Pulse Width and Pulse Repetition Interval*

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Addendum to ARL-TR-8974, *A Survey of Methods for Estimating Pulse Width and Pulse Repetition Interval*

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14. ABSTRACT

In ARL-TR-8974, we presented several classes of algorithms designed to estimate pulse repetition intervals. At the time, however, we failed to mention some interesting statistical approaches based on minimization of the mean squared error. The problem formulation in these approaches leads to the incorporation of target tracking techniques based on the Kalman filter. Thus, these approaches attack the problem from a different perspective than those previously considered. This “new” perspective warrants coverage within the framework of the earlier discussion.

The purpose of this addendum is to describe the origins of the “new” perspective based on linear system theory and state variable estimation.

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Contents

Preface

In ARL-TR-8974,[1] we presented several classes of algorithms designed to estimate pulse repetition intervals (PRIs). At the time, however, we failed to mention some interesting statistical approaches based on minimization of the mean squared error (MSE).[2–5] The problem formulation in these approaches leads to the incorporation of target-tracking techniques based on the Kalman filter. Thus, these approaches attack the problem from a different perspective than those previously considered. This "new" perspective warrants coverage within the framework of the earlier discussion.

The purpose of this addendum is to describe the origins of the "new" perspective based on linear system theory and state variable estimation. We begin by documenting the direct application of minimum MSE concepts and noting how this application extends naturally to the state-space model of the Kalman filter. Finally, we document the reported implementation of multihypothesis Kalman tracking algorithms for de-interleaving multiple PRIs from within a pulse stream.

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[1] Ranney K, Tom K. A survey of methods for estimating pulse width and pulse repetition interval. Adelphi (MD): CCDC Army Research Laboratory (US); 2020 June. Report No.: ARL-TR-8974.

[2] Sadler BM, Casey SD. On periodic pulse interval analysis with outliers and missing observations. IEEE Transactions on Signal Processing. 1998;46(11):2990–3002. doi: 10.1109/78.726812.

[3] Moore JB, Krishnamurthy V. Deinterleaving pulse trains using discrete-time stochastic dynamic-linear models. IEEE Transactions on Signal Processing. 1994;42(11):3092–3103, doi: 10.1109/78.330369.

[4] Liu J, Meng H, Wang X. A new pulse deinterleaving algorithm based on multiple hypothesis tracking. 2009 International Radar Conference "Surveillance for a Safer World" (RADAR 2009); 2009; Bordeaux, France. pp. 1–4.

[5] Conroy T, Moore JB. The limits of extended Kalman filtering for pulse train deinterleaving. IEEE Transactions on Signal Processing. 1998;46(12):3326–3332, doi: 10.1109/78.735307.

1. Pulse Repetition Interval Estimation Using Minimum Mean Squared Error: Linear Systems and Multi-hypothesis Tracking

Algorithm creators often design parameter estimation algorithms to minimize the mean squared error (MSE). This particular performance criterion is appealing because it provides a way of quantifying the estimator's performance in terms of a lower bound on the estimation error.[1] Hence, the pulse repetition interval (PRI) estimation problem can be cast in terms of estimating the true PRI from a collection of noisy (inaccurate) PRI measurements. If a single PRI is present within the pulse stream, the problem reduces to finding the MSE solution to an overdetermined set of equations. From this initial formulation, it is then a relatively small step to a more comprehensive formulation in terms of the state variables encountered in Kalman filtering.[2]

Sadler and Casey[3] define the fundamental problem (in the case of a single PRI) as follows:

Find the minimum mean squared error (MMSE) estimate (also the maximum likelihood estimate) of T for

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} k_2 - k_1 \\ k_3 - k_2 \\ \vdots \\ k_N - k_{N-1} \end{bmatrix} T + \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_{N-1} \end{bmatrix}, \quad \text{or} \qquad \boldsymbol{y} = \boldsymbol{X}_{diff} T + \boldsymbol{\delta}\,, \tag{1}$$

where δ_i are noise samples, and k_i are arrival time indices, which could be irregularly spaced due to missing pulses. The authors note that the solution to this equation is

$$T = \left(\boldsymbol{X}_{diff}^T \boldsymbol{R}_{\boldsymbol{\delta}}^{-1} \boldsymbol{X}_{diff}\right)^{-1} \boldsymbol{X}_{diff}^T \boldsymbol{R}_{\boldsymbol{\delta}}^{-1} \boldsymbol{y}\ , \tag{2}$$

where $\boldsymbol{R}_{\boldsymbol{\delta}}^{-1}$ is the noise covariance matrix. To deal with the problem of missing pulses, the authors develop the "modified Euclidean algorithm". The algorithm examines the greatest common divisor of the elements of $\boldsymbol{X}_{diff}$ and uses it to obtain a preliminary estimate of T. This value of T is then used together with $\boldsymbol{y}$ to obtain a new estimate of $\boldsymbol{X}_{diff}$, which is used in Eq. 2 to obtain the final estimate of T. The authors illustrate the algorithm's performance as a function of jitter for several different probabilities of dropping a pulse. That is, the probability that a pulse is dropped follows a Bernoulli(p) distribution, where p represents the probability that a pulse is dropped. The authors consider only the case where a single PRI is present; they do not attempt to extend their approach to de-interleave multiple PRIs.

Moore and Krishnamurthy[4] do address the de-interleaving problem using a stochastic discrete-time dynamic linear model. They are then able to recast the problem and implement a Kalman filter to perform the de-interleaving task. Their Kalman-filter state variables consist of vectors comprising PRIs (values of T) and arrival times (values of τ). Each potential PRI value represents a signal from a new emitter, and the dimensions of both T and τ are defined to be N, the hypothesized maximum number of emitters. T and τ are concatenated to form the state vector, which is, therefore, of dimension $2N$. Hence, de-interleaving multiple PRIs can be viewed as a tracking problem, and the different PRIs can be viewed as targets to be tracked.

Liu et al.[5] present a much more concise and straightforward translation of the PRI estimation problem into a target-tracking framework. In place of the high-dimensional state vectors defined by the previous authors, they define their states to be

$$\boldsymbol{x}(\boldsymbol{k}) = \begin{bmatrix} TOA(k) \\ PRI(k) \end{bmatrix}, \tag{3}$$

where $TOA(k)$ is the time of arrival of pulse k and PRI(k) is the pulse repetition interval of pulse k. Unlike Moore and Krishnamurthy, Liu et al. note the natural analogy between TOA (position) and PRI (velocity) common to representations of linear dynamic systems. From this, they readily identify the equations for the Kalman filter as

$$\begin{bmatrix} T\hat{O}A(k|k) \\ P\hat{R}I(k|k) \end{bmatrix} = \begin{bmatrix} T\hat{O}A(k|k-1) \\ P\hat{R}I(k|k-1) \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(TOA_0(k) - \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} T\hat{O}A(k|k-1) \\ P\hat{R}I(k|k-1) \end{bmatrix} \right) \tag{4}$$

$$\begin{bmatrix} T\hat{O}A(k+1|k) \\ P\hat{R}I(k+1|k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T\hat{O}A(k|k) \\ P\hat{R}I(k|k) \end{bmatrix} \quad , \tag{5}$$

where $TOA_0(k)$ represents the observed TOA at pulse k, $T\hat{O}A(k|k-1)$, $P\hat{R}I(k|k-1)$ represent the predicted values, and $T\hat{O}A(k|k)$ and $P\hat{R}I(k|k)$ represent the values after filtering. α and β are empirically determined constants. To initialize the algorithm, the first two estimates are replaced by the observations.

Liu et al.[5] outline in some detail the data association procedure used to identify potential target tracks (i.e., PRIs). They then present promising results obtained by applying the multi-hypothesis tracker to three simulated, interleaved pulse trains. One potential drawback, however, arises due to the initialization procedure, which requires specification of a number of candidate PRIs. The authors note that a judicious selection will enable the algorithm to behave appropriately.

Conroy and Moore[6] consider a modification of the earlier work of Moore and Krishnamurthy, analyzing an extended Kalman filter with an M-dimensional state vector. As in the earlier work, the dimensionality of the state vector corresponds to the maximum number of emitters (i.e., PRIs) hypothesized to exist. While the authors note that the algorithm is computationally efficient, they also note that it is not robust to missing pulses.

The publications documented here are intended to provide insight; they by no means represent an exhaustive survey of the topic. They do illustrate, however, another way in which researchers have approached the problem of identifying certain properties of received radar signals. These approaches must be included as an important supplement to those previously documented in ARL-TR-8974.[7]

2. References

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2. Zarchan P, Musoff H. Fundamentals of Kalman filtering: a practical approach. 2nd ed. Reston (VA): American Institute of Aeronautics and Astronautics; 2005.
3. Sadler BM, Casey SD. On periodic pulse interval analysis with outliers and missing observations. IEEE Transactions on Signal Processing. 1998;46(11):2990–3002, doi: 10.1109/78.726812.
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5. Liu J, Meng H, Wang X. A new pulse deinterleaving algorithm based on multiple hypothesis tracking. 2009 International Radar Conference "Surveillance for a Safer World" (RADAR 2009); 2009; Bordeaux, France. pp. 1–4.
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List of Symbols, Abbreviations, and Acronyms

MMSE	minimum mean squared error
MSE	mean squared error
PRI	pulse repetition interval

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