



Non-equilibrium dynamics and many body localization in ultracold atoms

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REGENTS OF THE UNIVERSITY OF COLORADO

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14. ABSTRACT AFOSR support enabled the completion of twenty five distinct projects, of which twenty are published and five more are under review. Signature achievements included the demonstration that many body localization can occur in long range interacting systems (publications 1-3 below), the discovery of a new class of phenomena in quantum dynamics (publications 7-11 below), including both localization in translation invariant systems, which is robust to noise, and also (an infinite number of) new subdiffusive hydrodynamic universality classes, and finally the application of this new theoretical understanding to experiments on ultracold atoms. All goals of the original proposal were met.					
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1) Summary of accomplishments made during this grant period

AFOSR support enabled the completion of twenty five distinct projects, of which twenty are published and five more are under review. Signature achievements included the demonstration that many body localization can occur in long range interacting systems (publications 1-3 below), the discovery of a new class of phenomena in quantum dynamics (publications 7-11 below), including both localization in translation invariant systems, which is robust to noise, and also (an infinite number of) new subdiffusive hydrodynamic universality classes, and finally the application of this new theoretical understanding to experiments on ultracold atoms. All goals of the original proposal were met or exceeded. A detailed description of what was accomplished is provided at the end of this report.

2) Archival publications

Published articles

1. **Many body localization with long range interactions** *Rahul M. Nandkishore and S.L. Sondhi, Phys. Rev. X* **7**, 041021 (2017) [Selected as a research highlight by *Nature Physics* **13**, 1037 (2017)]
2. **Localization of extended quantum objects** *Michael Pretko and Rahul M. Nandkishore, Phys. Rev. B* **98**, 134301 (2018)
3. **Symmetry breaking and localization in a random Schwinger model with commensuration** *A.A. Akhtar, Rahul M. Nandkishore and S.L. Sondhi, Phys. Rev. B* **98**, 115109 (2018)
4. **AC conductivity crossover in localized superconductors** *A.T. Schmitz, Michael Pretko and Rahul M. Nandkishore, Phys. Rev. B* **98**, 144203 (2018)
5. **Quantum dynamics of disordered spin chains with power-law interactions.** *A. Safavi-Naini, M.L. Wall, O.L. Acevedo, A.M. Rey and R.M. Nandkishore, Phys. Rev. A* **99**, 033610 (2019) [Editor’s suggestions].
6. **Emergent phases of fractonic matter** *Abhinav Prem, Michael Pretko and Rahul M. Nandkishore, Phys. Rev. B* **97**, 085116 (2018)
7. **Localization in fractonic random circuits.** *Shriya Pai, Michael Pretko and Rahul M. Nandkishore Phys. Rev. X*, **9**, 021003 (2019) *Erratum: Phys. Rev. X*, **9**, 021003 (2019)
8. **Localization from Hilbert space shattering: From theory to physical realizations.** *Vedika Khemani, Michael Hermele and Rahul Nandkishore, Phys. Rev. B* **101**, 174204 (2020) [Editors’ suggestion]

9. **Anomalous subdiffusion from subsystem symmetries.** Jason Iaconis, Sagar Vijay and Rahul Nandkishore, *Phys. Rev. B* **100**, 214301 (2019)
10. **Fracton hydrodynamics.** Andrey Gromov, Andrew Lucas, and Rahul M. Nandkishore. *Phys. Rev. Research* **2**, 033124 (2020)
11. **Quantum entropic self-localization with ultracold fermions.** M. Mamaev, I. Kimchi, M. A. Perlin, R. M. Nandkishore, and A. M. Rey. *Phys. Rev. Lett.* **123**, 130402 (2019)
12. **Dynamical scar states in driven fracton systems.** Shriya Pai and Michael Pretko, *Phys. Rev. Lett.* **123**, 136401 (2019)
13. **Fractons from confinement in one dimension** Shriya Pai and Michael Pretko, *Phys. Rev. Research* **2**, 013094 (2020)
14. **Entanglement Spectra of Stabilizer Codes: A Window into Gapped Quantum Phases of Matter.** Albert T. Schmitz, Sheng-Jie Huang, and Abhinav Prem. *Phys. Rev. B* **99**, 205109 (2019).
15. **Gauge Structures: From Stabilizer Codes to Continuum Models,** Albert T. Schmitz, *Annals of Physics* **410** (2019): 167927
16. **Symmetric Tensor Gauge Theories on Curved Spaces.** Kevin Slagle, Abhinav Prem and Michael Pretko. *Annals of Physics* **410** (2019) 167910
17. **Fracton phases of matter.** Michael Pretko, Xie Chen and Yizhi You, *Int. J. of Modern Physics A*, Vol **35**, No. 6, 2030003 (2020).
18. **Remnants of Anderson localization in pre-thermalization induced by white noise** S. Lorenzo, T. Apollaro, G.M. Palma, R. Nandkishore, A. Silva and J. Marino, *Phys. Rev. B* **98**, 054302 (2019)
19. **Exploring many-body localisation in open quantum systems via Wegner-Wilson flows.** Shane P. Kelly, Rahul Nandkishore and Jamir Marino, *Nucl. Phys. B* **951**, 114886 (2020)
20. **Many body localization proximity effects in platforms of coupled spins and bosons** J. Marino and R.M. Nandkishore, *Phys. Rev. B* **97**, 054201 (2018)

Articles submitted and under review, currently available in preprint form on the arXiv

1. **Thermalization and its absence within Krylov subspaces of a constrained Hamiltonian.** Sanjay Moudgalya, Abhinav Prem, Rahul Nandkishore, Nicolas Regnault, B. Andrei Bernevig. arXiv: 1910.14048
2. **Fractons from polarons and hole-doped antiferromagnets: Microscopic realizations.** John Sous and Michael Pretko, arXiv: 1904.08424
3. **Distilling Fractons from Layered Subsystem-Symmetry Protected Phases.** A. T. Schmitz. ArXiv: 1910.04765.
4. **Thermal stability of dynamical phase transitions in higher dimensional stabilizer codes.** A.T. Schmitz, ArXiv 2002.11733
5. **Disorder controlled relaxation in a 3D Hubbard model quantum simulator.** W. Morong, S.R. Muleady, I. Kimchi, W. Xu, R.M. Nandkishore, A.M. Rey, B. DeMarco. arXiv: 2001.07341

Change in research objectives: None

Change in AFOSR program manager: None

Extensions granted or milestones slipped: None

Inventions or patent disclosures: None

Detailed summary of accomplishments

We now provide a detailed summary of what was accomplished. These projects can be grouped into three distinct themes (following the original grant proposal).

A. Non-equilibrium dynamics and many body localization with long range interactions

Recent developments in the understanding of many body quantum dynamics and many body localization (MBL) have focused on systems with interactions that are short range in real space. However, many 'real' systems (of charges, dipoles, etc) exhibit interactions that are long ranged in real space. The quantum dynamics of long range interacting many body quantum systems is thus an important open frontier for research, and was a key theme of my grant proposal.

Over the course of the grant period we made major progress on this front, thanks to AFOSR support. This progress came in the form of five completed and published projects that are described below.

- (i) **Many body localization with long range interactions** *Rahul M. Nandkishore and S.L. Sondhi, Phys. Rev. X 7, 041021 (2017)*

Many-body localization (MBL) has emerged as a powerful paradigm for understanding non-equilibrium quantum dynamics. Folklore based on perturbative arguments holds that MBL arises only in systems with short-range interactions. In this work, supported by AFOSR, we advance nonperturbative arguments indicating that MBL *can* arise in systems with long-range (Coulomb) interactions, through a mechanism we dub "order enabled localization." In particular, we show using bosonization that MBL can arise in one-dimensional systems with $\sim r$ interactions, a problem that exhibits charge confinement. We also argue that (through the Anderson-Higgs mechanism) MBL can arise in two-dimensional systems with $\log(r)$ interactions, and speculate that our arguments may even extend to three-dimensional systems with $1/r$ interactions. Our arguments opened the door to investigation of MBL physics in a wide array of long-range interacting systems where such physics was previously believed not to arise.

This work was published in *Physical Review X* [*Phys. Rev. X 7, 041021 (2017)*], and was selected as a research highlight by *Nature Physics* [*Nature Physics 13, 1037 (2017)*]

- (ii) **Localization of extended quantum objects** *Michael Pretko and Rahul M. Nandkishore, Phys. Rev. B 98, 134301 (2018)*

A quantum system of particles can exist in a localized phase, exhibiting ergodicity breaking and maintaining forever a local memory of its initial conditions. In this work, supported by AFOSR, we generalized this concept to a system of extended objects, such as strings and membranes, arguing that such a system can also exhibit localization in the presence of sufficiently strong disorder (randomness) in the Hamiltonian. This work extends the notion of localization (and localization protected order) to a host of settings where such ideas previously did not apply.

Importantly, this work also allows us to demonstrate localization in *three* dimensional systems with $1/r$ (Coulomb) interactions – a result with obvious experimental relevance. We predict the existence of a new ‘localized superconductor’ phase in which localization of quantum flux lines could stabilize superconductivity at energy densities where a normal state would arise in thermal equilibrium. This could have significant technological applications.

This work is published. It introduced the notion of ‘MBL superconductors.’

- (iii) **Symmetry breaking and localization in a random Schwinger model with commensuration** A.A. Akhtar, Rahul M. Nandkishore and S.L. Sondhi, *Phys. Rev. B* **98**, 115109 (2018)

In this work we subject the scenario of many body localization with long-range interactions to stringent numerical tests using both exact diagonalization and density matrix renormalization group techniques. Specifically, we focus on the one dimensional realization of these ideas (‘Schwinger model’) – which can be accessed experimentally using trapped ions [see e.g. *Nature* **534**, 516 (2016)]. We numerically investigate the interplay of confinement, lattice commensuration, and disorder, in the form of a random chemical potential. A careful examination of the structure of low lying excited states reveals disorder induced localization, consistent with the analytic expectations from the PI’s earlier grant funded work *Phys. Rev. X* **7**, 041021 (2017).

This work is published.

- (iv) **AC conductivity crossover in localized superconductors** A.T. Schmitz, Michael Pretko and Rahul M. Nandkishore, *Phys. Rev. B* **98**, 144203 (2018)

In (i) above we introduced the notion of a ‘localized superconductor’ phase for three dimensional systems with a $1/r$ interaction. In this work we discuss how such a phase may be experimentally characterized using optical conductivity measurements. In localized phases the low-frequency AC conductivity typically vanishes as ω^ϕ . The exponent $\phi = 2$ for Anderson insulators, whereas for many body localized insulators ϕ is a continuously varying exponent $1 \leq \phi \leq 2$. In this work, we show that in localized superconductors, the exponent ϕ can be markedly different from the characteristic value for localized insulators. This difference occurs due to singularities in the low-energy density of states, permitted by the effective particle-hole symmetry around the Fermi level. In particular, in certain symmetry classes at zero temperature, we obtain $\phi > 2$. We further identify an interesting temperature dependent crossover in the scaling form of the AC conductivity, which could be useful for the experimental characterization of localized superconductors.

This work is published.

- (v) **Quantum dynamics of disordered spin chains with power-law interactions.** A. Safavi-Naini, M.L. Wall, O.L. Acevedo, A.M. Rey and R.M. Nandkishore, *Phys. Rev. A* **99**, 033610 (2019) [*Editor’s suggestions*].

This work extends the PI’s investigations of MBL in long range interacting systems beyond the

previously investigated cases of Coulomb interactions. In this work, we numerically investigated the dynamics of spin chains with power law long range interactions of tunable range. Specifically, we used extensive numerical simulations based on matrix product state methods to study the quantum dynamics of spin chains with strong on-site disorder and power-law decaying ($1/r^\alpha$) interactions. We focused on two spin-1=2 Hamiltonians featuring power-law interactions: Heisenberg and XY, and characterized their corresponding long-time dynamics using three distinct diagnostics relevant to AMO experiments: decay of a staggered magnetization pattern $l(t)$, growth of entanglement entropy $S(t)$, and growth of quantum Fisher information $FQ(t)$: For sufficiently rapidly decaying interactions $\alpha > \alpha_c$ we found a many-body localized phase, in which $l(t)$ saturates to a non-zero value, entanglement entropy grows as $S(t) \sim t^{1/\alpha}$, and Fisher information grows logarithmically. Importantly, entanglement entropy and Fisher information do not scale the same way (unlike short range interacting models). The critical power α_c is smaller for the XY model than for the Heisenberg model.

This work is published.

B: Connections between quantum information theory and many body dynamics

A second theme of my grant proposal was exploring the connection between quantum information theory and quantum dynamics. On this front we benefited from a stroke of luck: the discovery of ‘fracton’ models, which sit at the intersection between quantum dynamics and quantum information. AFOSR support enabled us to explore the possible phases of matter that may arise in such models. This work was conducted together with Abhinav Prem and Albert Schmitz (two graduate students partially supported by this grant, both of whom graduated during the grant period) and was reported in the publications listed below.

- (vi) **Localization in fractonic random circuits.** Shriya Pai, Michael Pretko and Rahul M. Nandkishore *Phys. Rev. X*, **9**, 021003 (2019). *Erratum: Phys. Rev. X*, **9**, 021003 (2019)

In this work we introduced a model of quantum dynamics subject to *constraints*, motivated directly by the study of fracton systems. We numerically discovered that this model can exhibit *localization* even in the presence of temporal noise, and in the absence of spatial disorder, unlike any other model of localization that we are aware of. It thus appears to constitute a qualitatively new route to localization.

This work is published.

- (vii) **Localization from Hilbert space shattering: From theory to physical realizations.** Vedika Khemani, Michael Hermele and Rahul Nandkishore, *Phys. Rev. B* **101**, 174204 (2020) [*Editors’ suggestion*]

This work continues the study of the fractonic random circuit model introduced by the PI in (vi) above, and provides a complete and rigorous analytic understanding of the new type of localization arising therein. Specifically, the localization in this model is rigorously shown as arising from a constraint induced ‘shattering’ of Hilbert space into a huge number of dynamically distinct subsectors. This ‘shattering’ phenomenon is shown

to be robust. Illuminating connections are drawn to several other areas of considerable current interest, in particular the study of ‘quantum many body scars.’ We also explain how the phenomenon may be extended to systems in arbitrary spatial dimensions, and how it may be realized in near term ultracold atom experiments.

This work is published.

(viii) **Thermalization and its absence within Krylov subspaces of a constrained Hamiltonian.**

Sanjay Moudgalya, Abhinav Prem, Rahul Nandkishore, Nicolas Regnault, B. Andrei Bernevig. arXiv: 1910.14048

This work continues the study of constrained models of the form discussed in publications (vi) and (vii) above. While the previous works focused on the ‘localized’ (i.e. small) dynamical subspaces of the quantum dynamics, this work explored the ‘large’ subspaces of the quantum dynamics, and whether the dynamics is thermal within these ‘large’ subspaces. Dynamics *within* a particular dynamical subspace is shown to be thermal for some subspaces, but non-thermal for others, with the non-thermality arising due to an unusual form of integrability, which becomes apparent only when the dynamics is projected on a particular dynamical subspace.

This work is under review and is currently available on the arXiv.

(ix) **Anomalous subdiffusion from subsystem symmetries.** Jason Iaconis, Sagar Vijay and Rahul Nandkishore, *Phys. Rev. B* **100**, 214301 (2019)

In this work, we introduced a new method for simulating quantum dynamics, which allows us to efficiently simulate to long times ‘almost chaotic’ quantum dynamics. This new method is based on a combination of cellular automaton ideas and Monte Carlo methods. We apply this new method to simulate the quantum dynamics of systems with ‘fracton’ like constraints in two and three space dimensions and numerically discover subdiffusion. We analytically explain the origin of the subdiffusion in the particular cases. We also argue that the class of dynamics simulable using our new technique is much closer to ‘generic’ chaotic quantum dynamics than the kinds of dynamics that were simulable previously.

This work is published

(x) **Fracton hydrodynamics.** Andrey Gromov, Andrew Lucas, and Rahul M. Nandkishore. *Phys. Rev. Research* **2**, 033124 (2020)

In this work we examine the behavior of constrained quantum dynamics (along the lines studied in (vi)-(viii) above), in the case when the ‘non-thermal’ regions of Hilbert space discussed above

are measure zero in the thermodynamic limit. In this case, generic initial conditions do thermalize, but we argue that they do so in *subdiffusive* manner. We introduce new classes of hydrodynamic theories to describe the thermalization of such fracton matter. Each of these theories exhibits subdiffusive thermalization, and constitutes a new universality class of hydrodynamic behavior. There are infinitely many such classes, each with distinct subdiffusive exponents, all of which are captured by our formalism. Our framework naturally explains recent results on dynamics with constrained quantum circuits, as well as recent experiments with ultracold atoms in tilted optical lattices. We identify crisp experimental signatures of these novel hydrodynamics, and explain how they may be realized in near term ultracold atom experiments.

This work is published.

- (xi) **Quantum entropic self-localization with ultracold fermions.** M. Mamaev, I. Kimchi, M. A. Perlin, R. M. Nandkishore, and A. M. Rey. *Phys. Rev. Lett.* **123**, 130402 (2019)

This work examines a model of constrained quantum dynamics motivated by experiments on atomic clocks, and reveals it to exhibit an unusual kind of few-body localization. This self-localization is revealed to be of ‘quantum entropic’ origin, as revealed by a mapping to a non-interacting Anderson impurity model. Using techniques introduced by the PI in (vi) above, the results are extended also to many-body systems, and a connection is made to the dynamics of ‘Hilbert space fracture’ discussed in (vi) – (viii) above.

This work is published.

- (xii) **Dynamical scar states in driven fracton systems.** Shriya Pai and Michael Pretko, *Phys. Rev. Lett.* **123**, 136401 (2019)

This work examines fractonic circuit models of the type introduced by the PI in (vi) above, and makes a connection between their behavior and quantum scars. It does not involve the PI as an author but is grant relevant and was carried out by an AFOSR funded student and postdoc. This work is published.

- (xiii) **Fractons from confinement in one dimension** Shriya Pai and Michael Pretko, *Phys. Rev. Research* **2**, 013094 (2020)

This work explains how ‘fracton’ constraints, whose consequences are explored in (vi)-(xii) above, may arise in one dimensional confining quantum systems, such as U(1) gauge theory and the quantum Ising model. This work does not involve the PI as an author, but is grant relevant and was carried out by a postdoc funded in part by AFOSR. This work is published.

- (xiv) **Entanglement Spectra of Stabilizer Codes: A Window into Gapped Quantum Phases of Matter.** Albert T. Schmitz, Sheng-Jie Huang, and Abhinav Prem. *Phys. Rev. B* **99**, 205109 (2019).

The entanglement spectrum (ES) provides a barometer of quantum entanglement and encodes physical information beyond that contained in the entanglement entropy. This paper explores the ES of stabilizer codes, which furnish exactly solvable models for a plethora of gapped quantum phases of matter. In particular, it studies models harboring fracton order, and compares the resulting ES with that of both conventional topological order and of (strong) subsystem symmetry protected topological (SSPT) states. This sheds further light on the interplay between geometric and topological effects in fracton phases of matter. It further shows that a version of the edge-entanglement correspondence, established earlier for gapped two-dimensional topological phases, also holds for gapped three-dimensional fracton models.

This is work that does not involve the PI as an author, but is grant relevant and was led by the PI's AFOSR funded student. It is published.

- (xv) **Gauge Structures: From Stabilizer Codes to Continuum Models**, Albert T. Schmitz, *Annals of Physics* **410** (2019): 167927

This paper develops a new formal perspective capable of describing fracton phases of matter. It illuminates the general structure of fracton phases, as well as how they connect to continuum theories. This is work that does not involve the PI as an author, but is grant relevant and was led by the PI's AFOSR funded student. This work is published.

- (xvi) **Symmetric Tensor Gauge Theories on Curved Spaces**. Kevin Slagle, Abhinav Prem and Michael Pretko. *Annals of Physics* **410** (2019) 167910

Fractons and other subdimensional particles are an exotic class of emergent quasi-particle excitations with severely restricted mobility. A wide class of models featuring these quasi-particles have a natural description in the language of symmetric tensor gauge theories. This work investigates the fate of symmetric tensor gauge theories in the presence of spatial curvature. It shows that weak curvature can induce small (exponentially suppressed) violations on the mobility restrictions of charges, leaving a sense of asymptotic fractonic/sub-dimensional behavior on generic manifolds. Nevertheless, certain symmetric tensor gauge theories maintain sharp mobility restrictions and gauge invariance on certain special curved spaces, such as Einstein manifolds or spaces of constant curvature.

This is work that does not involve the PI as an author, but is grant relevant and involves the PI's AFOSR funded student (Prem), who has since graduated. This work is published.

- (xvii) **Emergent phases of fractonic matter** Abhinav Prem, Michael Pretko and Rahul M. Nandkishore, *Phys. Rev. B* **97**, 085116 (2018)

Fractons are emergent particles which are immobile in isolation, but which can move together in dipolar pairs or other small clusters. These exotic excitations naturally occur in certain quantum phases of matter described by tensor gauge theories. Previous research has focused on the properties of small numbers of fractons and their interactions, effectively mapping out the “standard model” of fractons. In the present work, however, we consider systems with a finite density of either fractons or their dipolar bound states, with a focus on the U(1) fracton models. We study some of the phases in which emergent fractonic matter can exist, thereby initiating the study of the “condensed matter” of fractons. We begin by considering a system with a finite density of fractons, which we show can exhibit microemulsion physics, in which fractons form small-scale clusters emulsed in a phase dominated by long-range repulsion. We then move on to study systems with a finite density of mobile dipoles, which have phases analogous to many conventional condensed matter phases. We focus on two major examples: Fermi liquids and quantum Hall phases. A finite density of fermionic dipoles will form a Fermi surface and enter a Fermi liquid phase. Interestingly, this dipolar Fermi liquid exhibits a finite-temperature phase transition, corresponding to an unbinding transition of fractons. Finally, we study chiral two-dimensional phases corresponding to dipoles in “quantum Hall” states of their emergent magnetic field. We study numerous aspects of these generalized quantum Hall systems, such as their edge theories and ground state degeneracies.

This work is published.

(xviii) Fractons from polarons and hole-doped antiferromagnets: Microscopic realizations.

John Sous and Michael Pretko, arXiv: 1904.08424

This work shows that boson-affected hopping models can provide a natural realization of fractons, either approximately or exactly, depending on the details of the system. It first considers a generic one dimensional boson-affected hopping model, in which single particles move only at sixth order in perturbation theory, while motion of bound states occurs at second order, allowing for a broad parameter regime exhibiting approximate fracton phenomenology. It then studies a special type of boson-affected hopping models with mutual hard-core repulsion between particles and bosons, accessible in hole-doped mixed-dimensional Ising antiferromagnets, in which the hole motion is one dimensional in an otherwise two-dimensional antiferromagnetic background. This system, which is within the current reach of ultracold-atom experiments, exhibits perfect fracton behavior to all orders in perturbation theory. Diagnostic signatures of fractonic behavior in these systems are discussed. This work presents boson-affected hopping systems as a natural platform for studying important aspects of fracton physics, such as restricted thermalization.

This is work that does not involve the PI as an author, but is grant relevant and involves the PI’s AFOSR funded postdoc (Pretko). It is under review and currently available on the arXiv.

(xix) Distilling Fractons from Layered Subsystem-Symmetry Protected Phases. A. T. Schmitz.

ArXiv: 1910.04765.

This paper explains how paradigmatic 'fracton models' including the Haah cubic code may be obtained via a 'layer construction' from lower dimensional phases. Fracton phases were the inspiration for the explorations of quantum dynamics in (vi) – (xvii) above. This work does not involve the PI as an author but is grant relevant and was carried out by the PI's AFOSR funded student.

This work is publicly available as a preprint on the arXiv. The student (and sole author) has graduated and moved to an industry job at Intel Corp. so it is likely that this will remain a preprint.

(xx) Thermal stability of dynamical phase transitions in higher dimensional stabilizer codes. A.T. Schmitz, ArXiv 2002.11733

This paper explains how dynamical phase transitions (i.e. singularities in the Loschmidt echo) can be obtained in dimensions higher than one, and at non-zero temperature. The paper does not involve the PI as an author but is grant relevant and was carried out by the PI's AFOSR funded student. The manuscript is publicly available as a preprint on the ArXiv. The author has since graduated and moved to an industry job (at Intel corp). so it seems likely this will remain a preprint.

(xxi) Fracton phases of matter. Michael Pretko, Xie Chen and Yizhi You, *Int. J. of Modern Physics A*, Vol **35**, No. 6, 2030003 (2020). Invited review on fractons written by the PI's AFOSR supported postdoc, Dr. Michael Pretko.

C: Decoherence of localized systems

The final theme of my grant proposal was studying the decoherence of localized systems coupled to heat baths and/or subject to noise. We also made progress on this theme in the past year, in joint work with Dr. Jamir Marino, a postdoc who was paid partially through this grant, who has since moved on to a faculty position at the University of Mainz. This took the form of three completed projects described below.

(xxii) Remnants of Anderson localization in pre-thermalization induced by white noise S. Lorenzo, T. Apollaro, G.M. Palma, R. Nandkishore, A. Silva and J. Marino, *Phys. Rev. B* **98**, 054302 (2019)

We study the non-equilibrium evolution of a one-dimensional quantum Ising chain with spatially disordered, time-dependent, transverse fields characterized by white noise correlation dynamics. We establish pre-thermalization in this model, showing that the quench dynamics of the on-site transverse magnetization first approaches a metastable state unaffected by noise fluctuations, and then relaxes exponentially fast towards an infinite temperature state as a result of the noise. We also consider energy transport in the model, starting from an inhomogeneous state with two domain walls which separate regions characterized by spins with opposite transverse magnetization. We observe at intermediate time scales a phenomenology akin to Anderson localization:

energy remains localized within the two domain walls, until the Markovian noise destroys coherence and accordingly disorder induced localization, allowing the system to relax towards the late stages of its non-equilibrium dynamics. We benchmark our results with the simpler case of a noisy quantum Ising chain without disorder, and we find that the pre-thermal plateau is a generic property of weakly noisy spin chains, while the phenomenon of pre-thermal Anderson localisation is a specific feature arising from the competition of noise and disorder in the real-time transport properties of the system.

This work is published.

(xxiii) Many body localization proximity effects in platforms of coupled spins and bosons J. Marino and R.M. Nandkishore, *Phys. Rev. B* **97**, 054201 (2018)

In this work we discuss the onset of many body localization in a one-dimensional system composed of a XXZ quantum spin chain and a Bose-Hubbard model linearly coupled together. We consider two complementary setups depending whether spatial disorder is initially imprinted on spins or on bosons; in both cases, we explore the conditions for the disordered portion of the system to localize by proximity of the other clean half. Assuming that the dynamics of one of the two parts develops on shorter time scales than the other, we can adiabatically eliminate the fast degrees of freedom, and derive an effective Hamiltonian for the system's remainder using projection operator techniques. Performing a locator expansion on the strength of the many-body interaction term or on the hopping amplitude of the effective Hamiltonian thus derived, we present results on the stability of the many body localized phases induced by proximity effect. We also briefly comment on the feasibility of the proposed model through modern quantum optics architectures, with the long-term perspective to realize experimentally, in composite open systems, Anderson or many-body localization proximity effects.

This work is published.

(xxiv) Exploring many-body localisation in open quantum systems via Wegner-Wilson flows. Shane P. Kelly, Rahul Nandkishore and Jamir Marino, *Nucl. Phys. B* **951**, 114886 (2020)

We apply a Flow Equation method to study the problem of a many-body localised system of spinless fermions, coupled via density-density interactions to a second clean chain of fermions. In particular, we focus on the conditions for the onset of a many-body localised phase in the clean sector of our model by proximity to the dirty one. We find that a many-body localisation proximity effect in the clean component is established when the density of dirty fermions exceeds a threshold value, in a way reminiscent of recent experiments on many body localised systems coupled to a bath. Tuning the control parameters of the model we establish thresholds for the induction of a many-body localised phase in the clean sector, using a joint set of emergent integrals of motion for the clean and dirty components as ansatz for the solution of the flow equations. Furthermore, by engineering the geometry of the inter-chain couplings, we

show that the dynamics of the model can be described, on intermediate time scales, by a Hamiltonian with a novel set of emergent integrals of motion.

This work is published.

- (xxv) **Disorder controlled relaxation in a 3D Hubbard model quantum simulator.** W. Morong, S.R. Muleady, I. Kimchi, W. Xu, R.M. Nandkishore, A.M. Rey, B. DeMarco. arXiv: 2001.07341

Understanding the collective behavior of strongly correlated electrons in materials remains a central problem in many-particle quantum physics. A minimal description of these systems is provided by the disordered Fermi-Hubbard model (DFHM), which incorporates the interplay of motion in a disordered lattice with local inter-particle interactions. Despite its minimal elements, many dynamical properties of the DFHM are not well understood, owing to the complexity of systems combining out-of-equilibrium behavior, interactions, and disorder in higher spatial dimensions. Here, we study the relaxation dynamics of doubly occupied lattice sites in the three-dimensional (3D) DFHM using interaction-quench measurements on a quantum simulator composed of fermionic atoms confined in an optical lattice. In addition to observing the widely studied effect of disorder inhibiting relaxation, we find that the cooperation between strong interactions and disorder also leads to the emergence of a dynamical regime characterized by *disorder-enhanced* relaxation. To support these results, we develop an approximate numerical method and a phenomenological model that each capture the essential physics of the decay dynamics. Our results provide a theoretical framework for a previously inaccessible regime of the DFHM and demonstrate the ability of quantum simulators to enable understanding of complex many-body systems through minimal models.

This work is under review and currently available on the arXiv.

Student theses

Two PhD theses were completed partially with support from this grant. The first was by Dr. Abhinav Prem, now a postdoctoral fellow at the Princeton Center for Theoretical Science. The title of this thesis is *Aspects of Topology in Quantum Phases of Matter: A Journey through Lands both Flat and Not*. The AFOSR supported work included therein is published, as publication 6 in the list at the beginning of this report. The full thesis can be obtained online from the University of Colorado library.

The second PhD thesis was by Dr. Albert Schmitz, now a research scientist at Intel Corp. This thesis was entitled *Linear Gauge Structures: Theory and Uses*. The AFOSR supported portions of this thesis were published as publications 14 and 15, and also preprints 3 and 4 in the list at the beginning of this report. The full thesis can be obtained online from the University of Colorado library.