



Target Identification Problems in Inverse Scattering Theory

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Final Report**

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14. ABSTRACT This grant is concerned with three problems that are of interest to Air Force needs: 1) The development of new 'target signatures' that can be determined from measured scattering data and used for the purposes of nondestructive testing, 2) The use of electromagnetic waves to detect delamination between layers of a structure and 3) The applicability of using transmissioneigenvalues to investigate small scale effects on the mean field outside a periodic inhomogeneity.					
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Target Identification Problems in Inverse Scattering Theory
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Dr. David Colton (PI)* Dr. Peter Monk (co-PI)*
Dr. Fioralba Cakoni (co-PI)[†]

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Objectives

This grant is concerned with three problems that are of interest to Air Force needs: 1) The development of new “target signatures” that can be determined from measured scattering data and used for the purposes of nondestructive testing, 2) The use of electromagnetic waves to detect delamination between layers of a structure and 3) The applicability of using transmission eigenvalues to investigate small scale effects on the mean field outside a periodic inhomogeneity.

Accomplishments / New Findings

The main accomplishments during the period of this report are:

1. The development of new “target signatures” for nondestructive testing using measured scattering data.
2. The use of electromagnetic waves to detect the delamination of materials.
3. The use of qualitative methods to reconstruct local perturbations in a periodic layered media.
4. The use of transmission eigenvalues to detect small scale effects in bounded periodic structures.

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Status of Efforts

We have developed new methods for the nondestructive testing of both isotropic and anisotropic materials through the use of electromagnetic interrogation [1], [2], [6], [8], [15], [24]. In particular, we have identified certain “target signatures” that are characterized by novel eigenvalue problems such that these eigenvalues can be determined from measured scattering data. Changes in the structural properties of the material or the presence of flaws can then be detected by changes in the measured eigenvalues. In contrast to the use of transmission eigenvalues for this purpose, the new methods that we have introduced apply to both dielectric and absorbing materials as well as dispersive media. Nevertheless, we have continued to investigate transmission eigenvalues and in particular their numerical computation which is needed for their application to nondestructive testing [3], [7], [10], [11].

The detection of cracks or delamination between layers of a structure is an important problem in nondestructive testing of materials. In this direction, we have developed electromagnetic models to detect delamination with potential applications to detecting separation between integrated circuit components [13]. Also we have considered the reconstruction of other types of defects that exhibit a small scale parameter but do not obey the Born approximation using asymptotic methods combined with a qualitative approach and transmission eigenvalues [5], [12], [14], [20].

Periodic structures have found use in numerous applications in science and technology, including photonics, meta-materials and the theory of composites. While significant progress has been made on unbounded periodic media, little is known about scattering by bounded periodic structures due to the boundary disruption of the periodicity. In this direction, we have considered bounded periodic media and explored small scale effects on the mean field outside the periodic inhomogeneity using the theory of transmission eigenvalues [22]. In addition the recovery of a locally supported defect within a layered background that is periodic is considered in [17], where no knowledge of the background is assumed except for the fact that is periodic with known period. Also in the area of photonics, we have used methods developed for interior transmission problems to analyze a new method for approximating the Bloch variety in periodic and frequency dependent materials [26].

Finally, we have made progress on several projects related to our main research directions. We have continued the development of the time-domain linear sampling method aiming to reduce the number of receivers and transmitters in the collection of data needed to maintain good resolution for this method [4], [19]. In [25] we used a novel Bayesian approach to broaden the applicability of linearization methods to stronger scatterers. Finally, the linear sampling method is also considered to detect abnormalities inside infinite or terminated waveguides [21],[27].

Personnel Supported

- Supported visitors at Rutgers: Andreas Kirsch KIT Germany, Michael Levitin University of Reading UK, Shixu Meng University of Michigan USA.
- Supported visitors at Rutgers: Virginia Selgas (University of Oviedo).

- Supported students at Delaware: Irene de Teresa, Sam Cogar.
- Supported visitors at Delaware: Timo Lahivaara (University of Eastern Finland), Jiguang Sun (Michigan Tech.).

Interactions/Transitions

Professors Colton, Monk and Cakoni have attended numerous conferences and seminars as invited speakers both in this country and Europe and Asia.

Archival Publications (Published) During This Period

Year 2017

- [1] L. Audibert, F. Cakoni and H. Haddar, New sets of eigenvalues in inverse scattering for inhomogeneous media and their determination from scattering data, *Inverse Problems*, 33 (2017) 125011.

Abstract: We develop a general mathematical framework to determine interior eigenvalues from a knowledge of the modified far field operator associated with an unknown (anisotropic) inhomogeneity. The modified far field operator is obtained by subtracting from the measured far field operator the computed far field operator corresponding to a well-posed scattering problem depending on one (possibly complex) parameter. Injectivity of this modified far field operator is related to an appropriate eigenvalue problem whose eigenvalues can be determined from the scattering data, and thus can be used to obtain information about material properties of the unknown inhomogeneity. We discuss here two examples of such modification leading to a Steklov eigenvalue problem, and a new type of the transmission eigenvalue problem. We present some numerical examples demonstrating the viability of our method for determining the interior eigenvalues from far field data.

- [2] F. Cakoni, D. Colton and P. Monk, Qualitative methods in inverse electromagnetic scattering theory, *IEEE Antennas and Propagation Magazine* 59 (2017), 24-33.

Abstract: In recent years a new approach has been developed in the study of the inverse scattering problem for electromagnetic waves. In this approach a weak scattering assumption has been avoided nor has use been made of nonlinear optimization methods. Instead, a study is made of the analytic properties of the far field operator and the results of this study used to determine the support of the scattering object together with an estimate of the material properties of the scatterer. This paper provides an introduction to this new approach in inverse electromagnetic scattering theory which is called the qualitative approach to inverse scattering theory.

- [3] F. Cakoni and R. Kress, A boundary integral equation method for the transmission eigenvalue problem, *Applicable Analysis*, (2017) 23-38.

Abstract: We propose a new integral equation formulation to characterize and compute transmission eigenvalues for constant refractive index that play an important role in inverse scattering problems for penetrable media. Our analysis is based on only one integral equation in terms of Dirichlet-to-Neumann or Robin-to-Dirichlet operators which results in a noticeable reduction of computational costs. Further we employ the numerical algorithm for analytic non-linear eigenvalue problems for the numerical computation of transmission eigenvalues via this new integral equation.

- [4] F. Cakoni and J. Rezac, "Direct imaging of small scatterers using reduced time dependent data", *J. Comp. Physics*, 338 (2017) 371-387.

Abstract: We introduce qualitative methods for locating small objects using time dependent acoustic near field waves. These methods have reduced data collection requirements compared to typical qualitative imaging techniques. In particular, we only collect scattered field data in a small region surrounding the location from which an incident field was transmitted. The new methods are partially theoretically justified and numerical simulations demonstrate their efficacy. We show that these reduced data techniques give comparable results to methods which require full multistatic data and that these time dependent methods require less scattered field data than their time harmonic analogs.

- [5] F. Cakoni, I. Harris and S. Moskow, The imaging of small perturbations in an anisotropic media, *Computers and Mathematics with Applications*, 74 (2017) 2769-2783.

Abstract: We employ asymptotic analysis to determine information about small volume defects in a known anisotropic scattering medium from far field scattering data. The location of the defects is reconstructed via the MUSIC algorithm from the range of the multi-static response matrix derived from the asymptotic expansion of the far field pattern in the presence of small defects. Since the same data determines the transmission eigenvalues corresponding to the perturbed media, we investigate how the presence of the defects changes the transmission eigenvalues and use this information to recover the strength of the small defects. We provide convergence results on transmission eigenvalues as the size of the defects tends to zero as well as derive the first correction term in the asymptotic expansion of the simple transmission eigenvalues. Numerical examples are presented to show the viability of our imaging method.

- [6] J. Camano, C. Lackner and P. Monk, Electromagnetic Stekloff eigenvalues in inverse scattering, *Siam J. Math. Anal.*, 49 (2017) 4376-4401.

Abstract: In [F. Cakoni, D. Colton, S. Meng, and P. Monk, *SIAM J. Appl. Math.*, 76 (2016), pp. 1737-1763] it was suggested to use Stekloff eigenvalues for the Helmholtz equation to detect changes in a scatterer using remote measurements of the scattered wave. This paper investigates the use of Stekloff eigenvalues for Maxwell's equations for the same purpose. Because the Stekloff eigenvalue problem for Maxwell's equations is not a standard eigenvalue problem for a compact operator, we propose a modified Stekloff problem that restores compactness. In order to measure the modified Stekloff eigenvalues of a domain from far field measurements we perturb the usual far field equation of the linear sampling method

by using the far field pattern of an auxiliary impedance problem related to the modified Stekloff problem. We are then able to show (1) the existence of modified Stekloff eigenvalues and (2) the well-posedness of the corresponding auxiliary exterior impedance problem and (3) to provide theorems that support our claim to be able to detect modified Stekloff eigenvalues from far field measurements. Preliminary numerical results show that for some simple domains it is possible to measure a few modified Stekloff eigenvalues. (As for the Helmholtz equation, not all eigenvalues can be measured.) In addition the modified Stekloff eigenvalues are changed by perturbations of the scatterer. An open problem is to obtain a proof of the existence of modified Stekloff eigenvalues for absorbing media.

- [7] S. Cogar, D. Colton and Y. J. Leung, The inverse spectral problem for transmission eigenvalues, *Inverse Problems* 33 (2017) 055015.

Abstract: In this paper, we consider the inverse medium problem of determining the spherically stratified index of refraction $n(r)$ from given spectral data. We begin by introducing a modified transmission eigenvalue problem depending on a parameter η and an associated modified far field operator. We prove that this operator is injective with dense range provided that k is not a modified transmission eigenvalue, and we show that $n(r)$ is uniquely determined by the modified transmission eigenvalues corresponding to η whenever $0 < n(r) < \eta^2$ for $0 \leq r \leq 1$.

- [8] S. Cogar, D. Colton, S. Meng and P. Monk, Modified transmission eigenvalues in inverse scattering theory, *Inverse Problems* 33 (2017) 125002.

Abstract: We consider the scattering of an acoustic plane wave by an inhomogeneous medium of compact support. Our aim is to introduce a new class of target signatures that can be used to detect changes in the material properties of the scattering object from a knowledge of the far field pattern of the scattered field. To this end we introduce a modified far field operator depending on a parameter η and show that this operator is injective with dense range provided η is not an eigenvalue of a new problem called the modified transmission eigenvalue problem. It is explained why this class of target signatures is preferable in some ways to previously studied target signatures that are based on scattering resonances, transmission eigenvalues, or Stekloff eigenvalues.

- [9] D. Colton, Peter Monk and inverse scattering theory, *Computers and Mathematics with Applications* 74 (2017), 2640-2644.

Abstract: This is a survey of Peter monk's contributions to inverse electromagnetic scattering theory on the occasion of his 60th birthday.

- [10] A. Kleefeld and D. Colton, Interior transmission eigenvalues for anisotropic media, in *Integral Methods in Science and Engineering, Vol.1 Theoretical Techniques*, Birkhauser Springer, 2017, 139-147.

Abstract: In this paper, the numerical calculation of interior transmission eigenvalues for anisotropic media in two dimensions is considered. This is achieved by reformulating the original problem into a system of boundary integral equations. The resulting nonlinear

eigenvalue problem is solved with a recent method using complex-valued contour integrals. Numerical results show that one is also able to calculate complex-valued interior transmission eigenvalues, although the existence of those is still open.

Year 2018

- [11] F. Cakoni, O. Ivanyshyn-Yaman, R. Kress and F. Le Louer, A boundary integral equations for the transmission eigenvalue problem for Maxwell's equations, *Math. Meth. Appl. Sci.* 41, no 4, (2018) 1316-1330.

Abstract: We propose a new integral equation formulation to characterize and compute transmission eigenvalues in electromagnetic scattering, which relies on a two by two system of boundary integral equations. Our analysis is based on only one integral equation in terms of the electric-to-magnetic boundary trace operator which results in a simplification of the theory and in a considerable reduction of computational costs. We employ the numerical algorithm for analytic non-linear eigenvalue problems for the numerical computation of the transmission eigenvalues via this new integral equation.

- [12] F. Cakoni and V. Kovtunenکو, Topological optimality condition for the identification of the center of an inhomogeneity, *Inverse Problems*, 34 (2018) 035009.

Abstract: The inverse scattering problem for inhomogeneous media is considered within the topology optimization framework. Varying the complex-valued refractive index we derive a zero-order necessary optimality condition in minimizing the L^2 misfit cost functional of the far-field measurement. The topology asymptotic expansion of the optimality condition leads to an imaging operator, which is used to identify the center of the unknown inhomogeneity using few far-field measurements. Numerical tests show high precision and stability in the reconstruction using our optimality condition based imaging both in two and three dimensions.

- [13] F. Cakoni, I. De Teresa and P. Monk, "Nondestructive testing of delaminated interfaces between two materials using electromagnetic interrogation" *Inverse Problems*, 34 (2018) 065005.

Abstract: We consider the problem of detecting whether two materials that should be in contact have separated or delaminated using electromagnetic radiation. The interface damage is modeled as a thin opening between two materials of different electromagnetic properties. To derive a reconstruction algorithm that focuses on testing for the delamination at the interface between the two materials, we use the approximate asymptotic model for the forward problem. In this model, the differential equations in the small opening are replaced by approximate transmission conditions for the electromagnetic fields across the interface. We also assume that the undamaged or background state is known and it is desired to find where the delamination has opened. We adapt the linear sampling method to this configuration in order to locate the damaged part of the interface from a knowledge of the scattered field and the undamaged configuration, but without needing to know the electromagnetic properties of the opening. Numerical examples are presented to validate our algorithm.

- [14] F. Cakoni, S. Moskow and S. Rome, "Asymptotic expansions of transmission eigenvalues for small perturbation of media with generally signed contrast" *Inverse Problems and Imaging*, 12 (2018) 971-992.

Abstract: In this paper we revisit the transmission eigenvalue problem for an inhomogeneous media of compact support perturbed by small penetrable homogeneous inclusions. Assuming that the inhomogeneous background media is known and smooth, we investigate how these small volume inclusions affect the transmission eigenvalues. Our perturbation analysis requires that the contrast of the inhomogeneity is of one-sign on the boundary. Thus, our approach can handle small perturbations with positive, negative or zero (voids) contrasts. In addition to proving the convergence rate for the eigenvalues corresponding to the perturbed media as inclusions' volume goes to zero, we also provide the explicit first correction term in the asymptotic expansion for simple eigenvalues. The correction term involves computable information about the known inhomogeneity as well as the location, size and refractive index of small perturbations. Our asymptotic formula has the potential to be used to recover information about small inclusions from knowledge of the real transmission eigenvalues, which can be determined from scattering data.

- [15] S. Cogar D. Colton and P. Monk, Using eigenvalues to detect anomalies in the exterior of a cavity, *Inverse Problems* 34 (2018) 085006.

Abstract: We use modified near field operators and the generalized linear sampling method to investigate an inverse scattering problem for anisotropic media with data measured inside a cavity. The aim of this paper is to determine information on possible changes in the material properties of the surrounding medium under the assumption that the shape is known, and this will be accomplished by the introduction of a new class of eigenvalue problems for which the eigenvalues can be determined from the measured scattering data.

- [16] D. Colton and R. Kress, Looking back on inverse scattering theory, *SIAM Review* 60 (2018), 779-807.

Abstract: This is an essay on the mathematical development of inverse scattering theory for time harmonic waves over the past fifty years together with some personal memories of the author's participation in these events.

- [17] F. Cakoni H. Haddar and TP Nguyen, New interior transmission problem applied to a single Floquet-Bloch mode imaging of local perturbations in periodic media" *Inverse Problems*, 35 (2019) 015009.

Abstract: This paper considers the imaging of local perturbations of an infinite penetrable periodic layer. A cell of this periodic layer consists of several bounded inhomogeneities situated in a known homogeneous media. We use a differential linear sampling method to reconstruct the support of perturbations without using the Green's function of the periodic layer nor reconstruct the periodic background inhomogeneities. The justification of this imaging method relies on the well-posedness of a nonstandard interior transmission problem, which until now was an open problem except for the special case when the local perturbation did not intersect the background inhomogeneities. The analysis of this new

interior transmission problem is the main focus of this paper. We then complete the justification of our inversion method and present some numerical examples that confirm the theoretical behavior of the differential indicator function determining the reconstructable regions in the periodic layer.

Year 2019

- [18] F. Cakoni and S. Chanillo, "Transmission eigenvalues and the Riemann zeta function in scattering theory for automorphic forms on Fuchsian groups of type I", *Acta Mathematica Sinica, English Series*, 39 (2019) 987-1010.

Abstract: We introduce the concept of transmission eigenvalues in scattering theory for automorphic forms on fundamental domains generated by discrete groups acting on the hyperbolic upper half complex plane. In particular, we consider Fuchsian groups of type I. Transmission eigenvalues are related to those eigen-parameters for which one can send an incident wave that produces no scattering. The notion of transmission eigenvalues, or non-scattering energies, is well studied in the Euclidean geometry, where in some cases these eigenvalues appear as zeros of the scattering matrix. As opposed to scattering poles, in hyperbolic geometry such a connection between zeros of the scattering matrix and non-scattering energies is not studied, and the goal of this paper is to do just this for particular arithmetic groups. For such groups, using existing deep results from analytic number theory, we reveal that the zeros of the scattering matrix, consequently non-scattering energies, are directly expressed in terms of the zeros of the Riemann zeta function. Weyl's asymptotic laws are provided for the eigenvalues in those cases along with estimates on their location in the complex plane.

- [19] F. Cakoni H. Haddar and A. Lechleiter, On the factorization method for a far field inverse scattering problem in the time domain, *SIAM J. Math. Anal.*, 51 (2019) 854-872.

Abstract: We develop a factorization method to obtain an explicit characterization of a (possibly nonconvex) Dirichlet scattering object from measurements of time-dependent causal scattered waves in the far field regime. In particular, we prove that far fields of solutions to the wave equation due to particularly modified incident waves characterize the obstacle by a range criterion involving the square root of the time derivative of the corresponding far field operator. Our analysis makes essential use of a coercivity property of the solution of the Dirichlet initial boundary value problem for the wave equation in the Laplace domain. This forces us to consider this particular modification of the far field operator. The latter in fact can be chosen arbitrarily close to the true far field operator given in terms of physical measurements.

- [20] F. Cakoni M. Bonnet, "Analysis of topological derivative as a tool for qualitative identification" *Inverse Problems*, 35 (2019) 1044007.

Abstract: The concept of topological derivative has proved effective as a qualitative inversion tool for a wave-based identification of finite-sized objects. Our paper develop the analysis of justification of topological sensitivity for the case of anisotropic scatterers and background

with near field data. Topological derivative-based imaging functional is analyzed using a suitable factorization of the near fields, which became achievable thanks to a new volume integral formulation. Our results include justification of sign heuristics for the topological derivative in the isotropic case with jump in the main operator and for some cases of anisotropic media, as well as verifying its decaying property in the isotropic case with near field spherical measurements configuration situated far enough from the probing region.

- [21] F. Cakoni, L. Borcea and S. Meng, "A direct approach to imaging in a waveguide with perturbed geometry" *J. Comp. Physics*, 392, 556-577 (2019).

Abstract: We introduce a direct, linear sampling approach to imaging in an acoustic waveguide with sound hard walls. The waveguide terminates at one end and has unknown geometry due to compactly supported wall deformations. The goal of imaging is to determine these deformations and to identify localized scatterers in the waveguide, using a remote array of sensors that emits time-harmonic probing waves and records the echoes. We present a theoretical analysis of the imaging approach and illustrate its performance with numerical simulations.

- [22] F. Cakoni, B. Guzina, S. Moskow and T. Pangburn, Scattering by a bounded highly oscillating periodic medium and the effect of boundary correctors, *SIAM J. Appl. Math.*, 79 (2019) 1448-1474.

Abstract: We study the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficient in the lower-order term of the Helmholtz equation. The squared index of refraction is assumed to be a periodic function of the fast variable, specified over the unit cell with characteristic size ϵ . We obtain improved convergence results that assume lower regularity than previous estimates (which also allow for periodicity in the second-order operator), and we describe the asymptotic behavior of boundary correctors for general domains at all orders. In particular we show that, in contrast to Dirichlet problems, the $O(\epsilon)$ boundary corrector is nontrivial and can be observed in the far field. We further demonstrate the latter far field effect is larger than that of the "bulk" corrector—the so-called periodic drift, which is found to emerge only at $O(\epsilon^2)$. We illustrate the analysis by examples in one and two spatial dimensions.

- [23] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer Verlag Series in Applied Mathematics, Vol. 93, 4th Edition, 2019.

Abstract: This is the fourth edition of a book that first appeared in 1992.

- [24] S. Cogar, D. Colton and P. Monk, Eigenvalue problems in inverse electromagnetic scattering theory, in *Maxwell's Equations: Analysis and Numerics*, De Gruyter, 2019, 145-165.

Abstract: This article compares three different eigenvalue problems for which the eigenvalues can be determined from the scattering data and each of which can be potentially used as target signatures for the purpose of nondestructive testing.

- [25] J.P. Kaipio, T. Huttunen, T. Luostari, T. Lahivaara, and P.B. Monk, A Bayesian approach to improving the Born approximation for inverse scattering with high-contrast materials, *Inverse Problems*, 35 (2019) 084001.

Abstract: Time harmonic inverse scattering using accurate forward models is often computationally expensive. On the other hand, the use of computationally efficient solvers, such as the Born approximation, may fail if the targets do not satisfy the assumptions of the simplified model. In the Bayesian framework for inverse problems, one can construct a statistical model for the errors that are induced when approximate solvers are used, and hence increase the domain of applicability of the approximate model. In this paper, we investigate the error structure that is induced by the Born approximation and show that the Bayesian approximation error approach can be used to partially recover from these errors. In particular, we study the model problem of reconstruction of the index of refraction of a penetrable medium from measurements of the far field pattern of the scattered wave.

- [26] C. Lackner, S. Meng and P. Monk, Determination of electromagnetic Bloch variety in a medium with frequency-dependent coefficients, *J. Comp. Appl. Math.*, 359 (2019), 359-373.

Abstract: We provide a functional framework and a numerical algorithm to compute the Bloch variety for Maxwell's equations when the electric permittivity is frequency dependent. We incorporate the idea of a mixed formulation for Maxwell's equations to obtain a quadratic eigenvalue for the wave-vector in terms of the frequency. We reformulate this problem as a larger linear eigenvalue problem and prove that this results in the need to compute eigenvalues of a compact operator. Using finite elements, we provide preliminary numerical examples of the scheme for both frequency independent and frequency dependent permittivity.

- [27] P. Monk, V. Selgas and F. Yang, Near-field linear sampling method for an inverse problem in an electromagnetic waveguide, *Inverse Problems*, 35 (2019) 065001.

Abstract: We consider the problem of determining the shape and location of an unknown penetrable object in a perfectly conducting electromagnetic waveguide. The inverse problem is posed in the frequency domain and uses multistatic data in the near field. In particular, we assume that we are given measurements of the electric scattered field due to point sources on a cross-section of the waveguide and measured on the same cross-section, which is away from the scatterer but not in the far field.

The problem is solved by using the linear sampling method (LSM) and we also discuss the generalized LSM. We start by giving a brief discussion of the direct problem and its associated interior transmission problem. Then, we adapt and analyze the LSM to deal with the inverse problem. This extends the work on the LSM for perfectly conducting scatterers in a waveguide by one of us (Yang) to the detection of penetrable objects. We provide several useful results concerning reciprocity and the density of fields due to single layer potentials. We also prove the standard results for the LSM in the waveguide context. Finally we give numerical results to show the performance of the method for simple shapes.