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An overview of methods to convert Cone Index to Bevameter Parameters

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Journal of Terramechanics

Cover letter

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Title of Manuscript: An overview of methods to convert Cone Index to Bevameter Parameters

Editor-in-Chief

Dear: Editor

I am submitting a journal article for your review on conversion of soil strength from cone index to bevameter. This is a basic overview of existing equations I found through exhaustive research. I also included data that was collected by the US Army in an attempt to describe the variability in the relationships. I took some liberties in conversion and comparison. Dr. Wong has requested an opportunity to review the manuscript as he is interested in the Bevameter constructs. Thank you for giving me the opportunity to submit the manuscript and the use of your invaluable time to review the works.

Respectively,

George L. Mason

Highlights:

- Equations exist for the conversion of Cone Index to select Bevameter parameters.
- The Cone index correlation relates to the bearing capacity Bevameter parameters Kc, K_{φ}, and n
- Correlations do not exist between cone index and surface shear parameters c, ϕ
- The equations are supported by limited validation in select regional areas of sand and clay

An overview of methods to convert Cone Index to Bevameter Parameters

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Abstract

This paper reviews experimental methods for the conversion of cone index measurements to bevameter parameters in support of vehicle soil/tire/track interactions for two general soil types, sand and lean clay. The accurate prediction of traction, motion resistance, and sinkage of tire/tracks off-road requires estimates of soil strength. Equipment used in the measurement of soil strength to support predictions of off-road mobility include the bevameter and the cone penetrometer. The portability of the cone penetrometer and rapid estimates of spatial/temporal variability in all terrain conditions make it an invaluable tool. The bevameter, a less portable tool, is used for the mechanical analysis of soils. The bevameter measures parameters defining soil strength in terms of cohesive modulus of soil deformation (k_c), frictional modulus of soil deformation (k_φ), exponent of soil sinkage (n), cohesion (c), angle of internal friction (φ), and the plate pressure at 1 inch (2.54 cm) of penetration (K) (Bekker, 1969). The field of terramechanics would greatly benefit from having the ability to convert cone penetrometer data

in areas where bevameter parameters are difficult to collect. That ability to convert from cone index to bevameter parameters could be used for the large sets of existing cone index data to support determination of traction and motion resistance. This paper examines those methods for converting cone index to bevameter plate penetration parameters k_c, k_φ, and n.

1. Introduction

The proper evaluation of the strength properties of a given terrain is essential to predicting off-road vehicle mobility (e.g., net traction, gross traction, and motion resistance). Strength properties of soils are measured by several different techniques, two of which include the cone penetrometer and the bevameter.

The cone penetrometer was developed to determine a bearing capacity in terms of an average cone index (CI) [lb/in^2 , Pa] based in part on a rate of penetration 1.2 in/sec (30 mm/s) and a critical depth related to the load exerted by the tire and/or track, typically 6 inches (15.24 cm). The bevameter measures normal and shear stresses of soil to derive a number of soil parameters (k_c, k_φ, n, c, φ, and K) providing a solution for the Bekker vehicle/tire interaction equation (Wong, 1989).

The behavior of soils varies greatly under a wide variety of conditions including composition, moisture content, density, temperature, and many other factors (Laughery, Gerhart, Muench, 2000). For example, a three percent change in moisture content can greatly change soil strengths, as shown in Figure 1. Figure 1 was taken from (Bekker, 1960) and appended to include remold cone index (RCI) for a sandy clay (SC) as suggested by the sandy loam classification and pictures by Bekker, furthermore utilizing equations correlating moisture to RCI as defined in (Mason, 2001).

			
Moisture Content (%)	22%	20%	19%
Angle of Internal Friction (ϕ)	36	38	36
Cohesion (c)	0.25	0.53	0.6
Frictional Modulus of Soil Deformation (k_{φ})	2.2	7	9
Cohesive Modulus of Soil Deformation (k_c)	2.5	16	20
Exponent of Soil Sinkage (n)	0.18	0.17	0.16
Remold Cone Index (RCI)	30	40	46

Figure 1. Consistency changes and the moisture content of a sandy loam (Bekker 1960).

Several sources tend to agree that finding bevameter parameters from cone index values to be inconceivable or impossible (Janosi, 1959; WES, 1964). M. G. Bekker agreed with this conclusion in his *Introduction to Terrain-Vehicle Systems, 1969*; however, he included a footnote stating:

> [the reverse process is inconceivable] if only a single index value is available. For the minimum of three 'cone indices' measured at various depths the k_c, k_{φ}, and n values may be theoretically calculated by means of an iterative computerized procedure using Janosi's Equation 1-6, as shown recently by J. Eilers of Land Locomotion Laboratory. Experimental verification, however, is still lacking. (Bekker, 1969)

Bekker's footnote suggested the possibility of converting cone index values to bevameter parameters utilizing differential pressure measured by the cone with depth. An equation developed by Zoltan Janosi, discussed later, provides a relationship between the bevameter parameters of the plate penetration part and cone index.

2. Background

Test methods and instruments were developed over the past several decades in attempt to measure soil properties to predict soil/tire/track interaction. Janosi states, "the bevameter is the standard method for scientific and engineering exploration of soils and off-road vehicle design" (Janosi, 1959). The bevameter measures multiple soil values supporting numerical and analytical simulations of cohesive and internal frictional forces to predict tire/track soil interaction. The cone penetrometer measures one value at a set rate of penetration for a given unified soil classification, supporting vehicle trafficability on a "go" or "no go" basis to support the prediction of gross and net traction. The U.S. Army elected to use the cone penetrometer due to its simplicity and ability to quickly collect spatial information (Janosi 1959).

Both the bevameter and cone index methods use a load (P) to displace the soil at a depth (z) at a constant rate. The measurements from the bevameter plate load test and cone penetration tests are both used to determine sinkage of the tire, supporting computations of motion resistance and traction. The bevameter uses two or more rectangular or circular plates of width/diameter (b) depending the characteristics of the soil and tire (Nguyen Van et. Al., 2007). The standard mobility cone penetrometer consists of a 0.5 or 0.2 inch (12.7 or 5.08 mm) diameter cone depending on soil type and vehicle loading. Both the bevameter and cone have a manual and automated system supporting data collection in the field or laboratory.

3. Bevameter

M. G. Bekker originally proposed a system of seven soil values (k_c, k_ϕ, n, c, ϕ, k_1, and k_2) to describe the behavior of soils under vehicles (WES, 1964). In order to predict those properties, the bevameter was developed. The bevameter is a device that replicates the sinkage of the vehicle as it travels over a deformable surface while there is a change in surface traction of the tire/track. In other words, it tries to replicate loading conditions a vehicle exerts on the soil (Bekker, 1969). The bevameter has two separate tests, the plate penetration and shear ring, which quantify soil parameters to support a solution of the tractive effort (Figure 2). The shear test supports the prediction of surface traction while the plate penetration tests supports the prediction as a result of bearing capacity. Wong suggested, “the bevameter technique provides the closest simulation of vehicle loading conditions among the various measuring techniques presently in use,” however, the bevameter is not widely available and not as portable as the cone penetrometer (Wong, 1989).

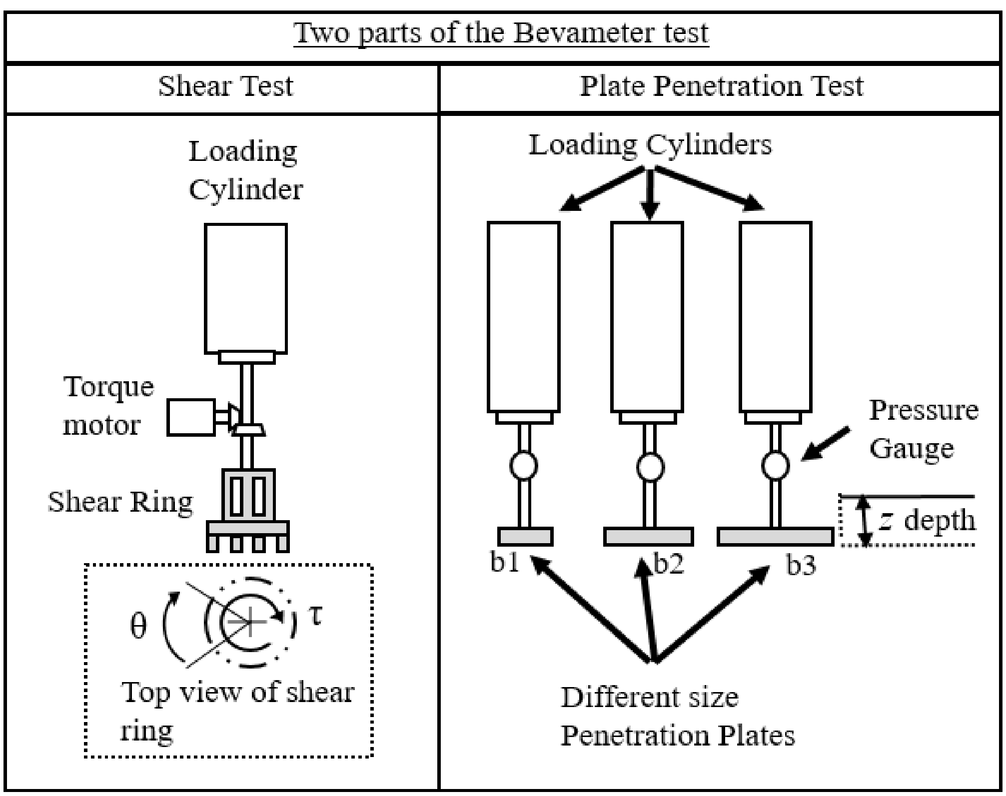


Figure 2. Schematic view of a bevameter type instrument (Redrawn from Bekker 1969)

3.1 Plate Penetration Test

The bevameter parameters k_c, k_φ, and n are determined by a minimum of two different size penetration plate tests. Plates of different sizes (b1, b2, b3) are used to define the pressure-sinkage relationship. Rectangular or circular plate sizes range from ⅜ of an inch to three inches (9.52 to 76.2 mm) in width or diameter respectively. Normal loads are replicated with the set of penetration plate tests (Bekker, 1969).

The exponent of soil sinkage, n [dimensionless], is the tangent of the slope angle from the sinkage vs. pressure relationship. Typical test results are illustrated in Figure 3 from tests conducted in Vicksburg, MS jointly by the Tank Automotive Command (TACOM) and Waterways Experiment Station (WES) (WES, 1964).

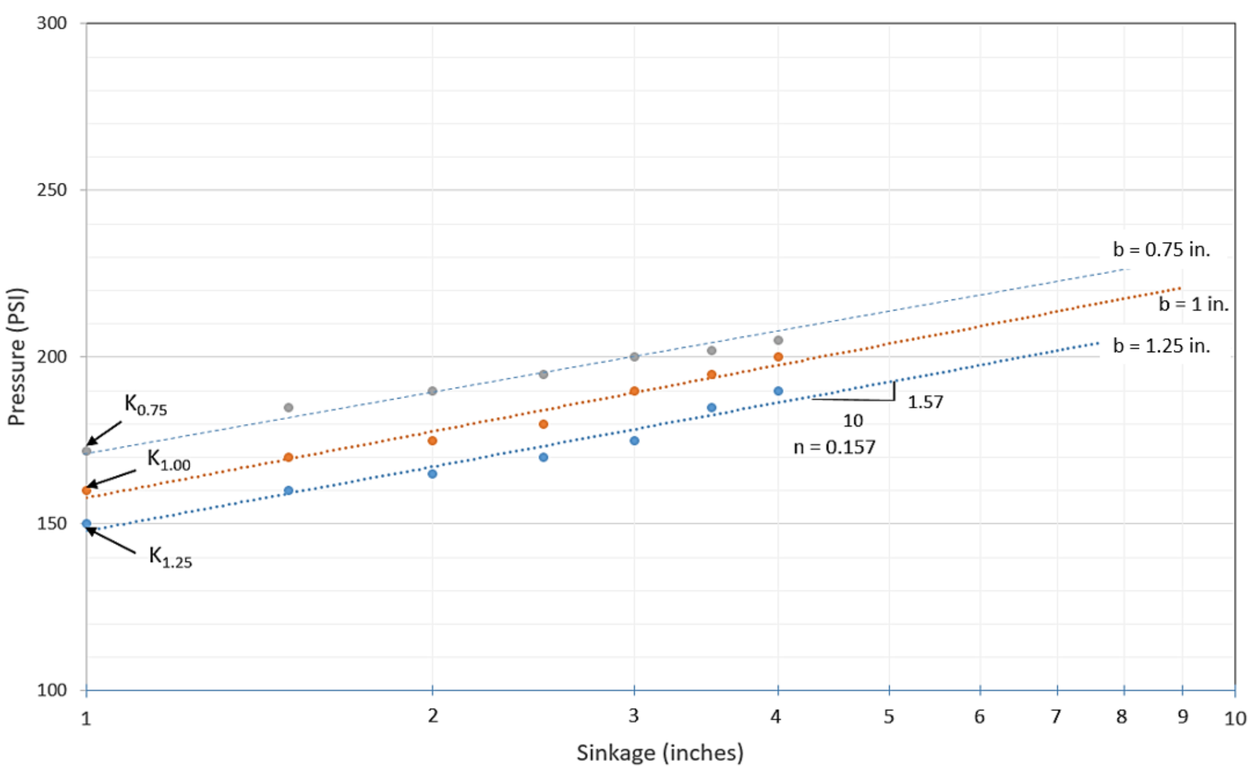


Figure 3. Determination of n (Redrawn from WES 1964)

It was considered in the (WES, 1964) report that n could be considered a constant for each soil type. The n value varied between 0.11 and 0.20 for both the lean and heavy clays tested in the fluvial plain of Vicksburg, Mississippi and sandy loam ranged between 0.2 and 0.15 as illustrated in Table 1. Dry sand tested in the laboratory had n values of 1.1. Sandy loam in Michigan and Maryland areas ranged between 0.3 and 0.8. The WES study of 1964 suggested that n did not change significantly with changes in moisture content or density. This seemed consistent from the sets of values provided in (Bekker, 1969).

Bekker defines the K value as the measured plate pressure at 1 inch (2.54 cm) of penetration. K has different values for each size plate, and is considered to be a composite of k_c, k_φ, and b (width of plates) as shown in the algebraic example in Figure 4 (WES, 1964). This is better explained in Figure 3, where the subscript of the K along the y-axis is the width of each plate and the values of each K is where the three lines intercept the y-axis. In this example, two of those intercepts are 150 and 160 psi, correlating with the two K values in the algebraic solution in Figure 4. The cohesive modulus of soil deformation as defined by Bekker, k_c [lb/in$^{(n+1)}$, kN/m^{n+1}], and the frictional modulus of soil deformation, k_φ [lb/in$^{(n+2)}$, kN/m^{n+2}], can be found graphically or algebraically by a solution of simultaneous equations. An algebraic example of solving for the values is shown in Figure 4.

$$K_{1.25} = \frac{k_c}{b} + k_\phi \qquad K_{1.00} = \frac{k_c}{b} + k_\phi$$

$$150 = \frac{k_c}{1.25} + k_\phi \quad 160 = \frac{k_c}{1.00} + k_\phi$$

$$187.5 = k_c + 1.25 k_\phi$$

$$-\ 160 = k_c + 1.00\, k_\phi$$

$$27.5 = 0.25\, k_\phi$$

$$k_\phi = 110$$

$$k_c = 50$$

Figure 4. Algebraic example of k_c and k_φ

It is generally understood that for clays, the size of the plate compared to the size of the footing does not change the results; however for sands, test plates smaller than the footing tend to overestimate the sinkage/load relationship and give conservative factors of safety. There is reason to assume that for wheels operating on sand the bevameter plate load tests may operate in the same manner.

The unit conversion from English to metric units for k_c and k_ϕ is complex.. The units for k_ϕ and k_c are measured in units of force (F) per unit length (L) and force (F) per unit length (L) squared respectively. The units for k_ϕ and k_c are in terms of n defined by the slope of the line derived during testing (WES, 1964). Equation 1 presents the conversion of k_ϕ and k_c from English units to metric.

$$Given{:}\; KL^n = \left(\frac{k_c}{L} + k_\varphi\right) L^n \qquad (1)$$

$$\therefore k_c = \frac{Force}{Lenght^{n+1}} \quad , \quad k_\varphi = \frac{Force}{Lenght^{n+2}}$$

3.2 Shear Test

The shearing ring of the bevameter finds the surface shear strength parameters of c, ϕ, and the slip coefficient K (or k_1 and k_2). An annular shear ring or plate is placed on the soil with an applied normal load and rotated at a constant rate. The shearing ring finds the shear stress vs. shear displacement relationship. A shear ring can have an outside diameter of 13.5 inches (34.29 cm) and an inside diameter of 10.5 inches (26.67 cm). The torque applied to the shear ring can be around 2000 lb-in (225.96 N-m). "The test is performed at a range of normal loads to determine the Coulomb shear strength parameters (Equation 2) corresponding to the soil/metal or soil/rubber shear" (Shoop, 1993). Similar to the plate load tests, values of c and φ are found graphically and algebraically by the solution of simultaneous equations using the following equation (Bekker, 1960).

$$\boldsymbol{\tau} = c + p \tan\varphi \tag{2}$$

Where:

$\boldsymbol{\tau}$ is shear strength, lb/in^2 (Pa)

c is cohesion, lb/in^2 (Pa)

p is normal load, lb/in^2 (Pa)

φ is the angle of internal friction, degrees

The shear strength vs. soil deformation graph in Figure 5 presents two different soil relationships, a brittle soil, which is typically granular soil (e.g., sand) and a plastic soil such as (e.g., clay). For brittle soils that display a decay in the stress-deformation graph after a peak shear strength, the slip coefficients k_1 and k_2 can be determined either algebraically or graphically.

This K-parameter can be found by using Equation 3.

$$\tau = (c + p \times \tan\varphi)\,(1-e^{-j / K}) \tag{3}$$

Where:

τ is shear strength, lb/in^2 (Pa)

c is cohesion, lb/in^2 (Pa)

p is normal load, lb/in^2 (Pa)

φ is the angle of internal friction, degrees

j is deformation, in (m)

K is the slip coefficient for plastic soils

The K parameter is the distance between the y-axis and the point of intersection of two straight lines tangent to both the sloped and horizontal portions of the shear strength vs. deformation curve as shown in Figure 4. (Bekker, 1969). Soils that do not display a rapid decay or "hump" and represent an ideal plastic material can be defined by a single K parameter (Figure 5) (Bekker, 1969). It is important to note that this K parameter is different from the K value used to find k_c and k_ϕ in the plate penetration part of the bevameter.

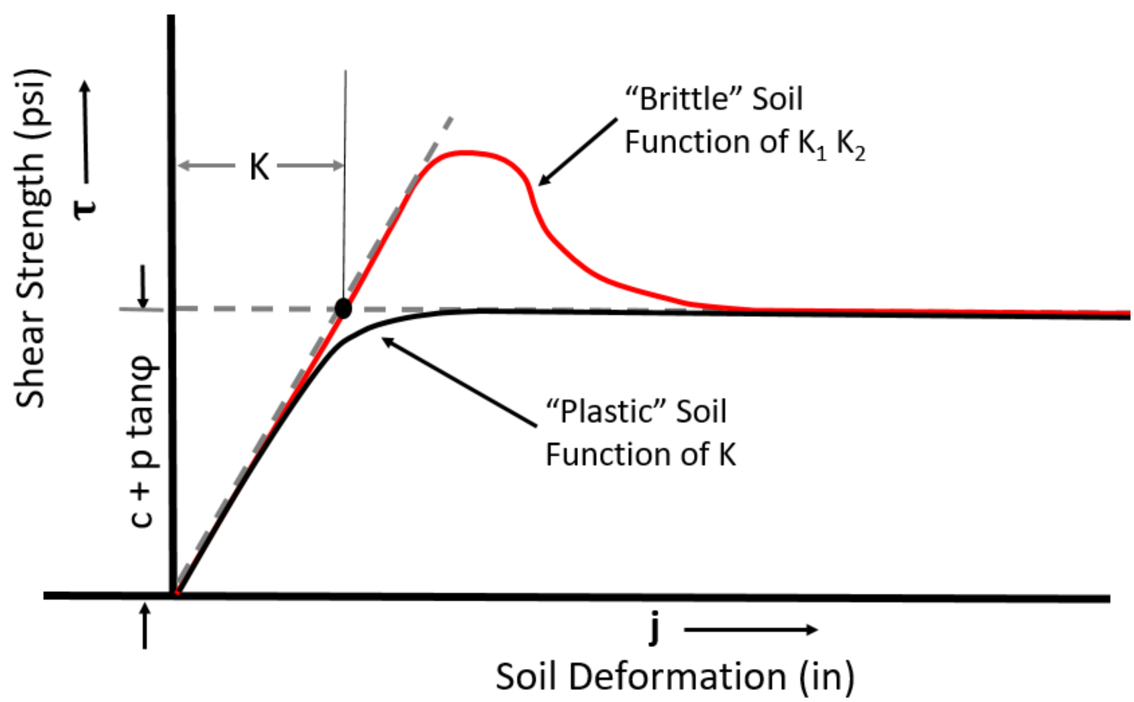
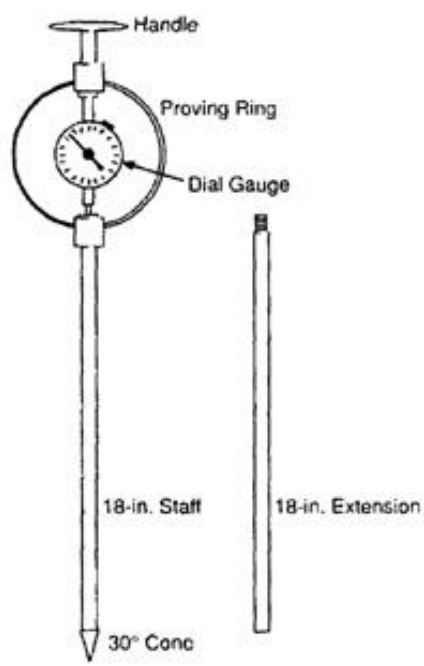


Figure 5. Definition of Slip Coefficient K for plastic and brittle soils (Bekker, 1969)

4. Cone Penetrometer

The Waterways Experiment Station (WES) of the U.S. Army Corps of Engineers developed the cone penetrometer during the Second World War. The cone penetrometer, as shown in Figure 6, is a device typically consisting of a ⅝ inch (1.58 cm) diameter rod, a 30-degree or 60-degree right circular cone having base area of 0.5 in.^{2} (323 mm^2) or 0.2 in.^{2} (130 mm^2) respectively and 1.5 inches (3.81 cm) tall. The recommended rate of penetration is about 1.2 inch/s (30 mm/s) (Wong, 1989). The cone index is the force required to sink the cone divided by the cone's base area. The reading from the dial on the penetrometer is twice the value of the force required to penetrate the soil (Janosi, 1959). It was developed for the purpose of providing military intelligence and reconnaissance with a simple, portable field device for predicting the soil trafficability on a "go" and "no go" basis. In other words, it is used to determine if the soil is capable of supporting certain vehicles without the vehicle becoming immobilized. The cone penetrometer is lightweight, portable, and reliable for determining basic trafficability (Mason, 2015; Wong, 1989).



Unclassified.

Figure 6. A cone penetrometer (Shoop, 1993)

5. Theory

Equation 4 is used in the foundation of Bekker's bevameter tests in which a plate is considered to penetrate a soil to depth *z* [in.] under pressure *p* [psi]. The empirical curve *p(z)* can be fitted with the following equation, where *k* is the modulus of inelastic deformation and n is the exponent of sinkage (Bekker, 1969).

$$P \approx k\, z^{n} \tag{4}$$

The K value in Equation 4 was very sensitive to the form of the test plate, which is unacceptable for making generalizations of soil parameters. Equation 4 was later developed into the following Equation 5, in which k_c and k_φ are cohesive and frictional moduli of deformation, and (b) is the diameter of smaller loading plate (Bekker, 1960).

$$P = [\left(\frac{k_c}{b}\right) + k_\varphi]z^{n} \tag{5}$$

Thus, soil is defined in a load penetration test by three parameters, k_c , k_φ, and n, which define empirically the vertical stress-strain relationship (Bekker, 1969).

In the relating of this equation to the cone penetrometer, the pressure (P) applied by a cone or plate is the total force (W) divided by the area of the plate or cone. The cone is shown in Figure 7 as a series of plates with varying diameter (dD_x). In order to find the total force (W), it is necessary to consider the equilibrium of the elemental truncated cone indicated by the cross-hatching. The vertical active force (dW) equals the sum of the pressure acting against the mantle of the cone element (Equation 6):

$$dW = p_x dA \tag{6}$$

Where (dA) is the area of the annulus shown at the bottom of Figure 7. The increment (dA) can be expressed as Equation 7.

$$dA = \pi[D_x - dD_x]\frac{dD_x}{2}$$

$$\therefore dW = \frac{\pi}{2} p_x D_x dD_x \qquad (7)$$

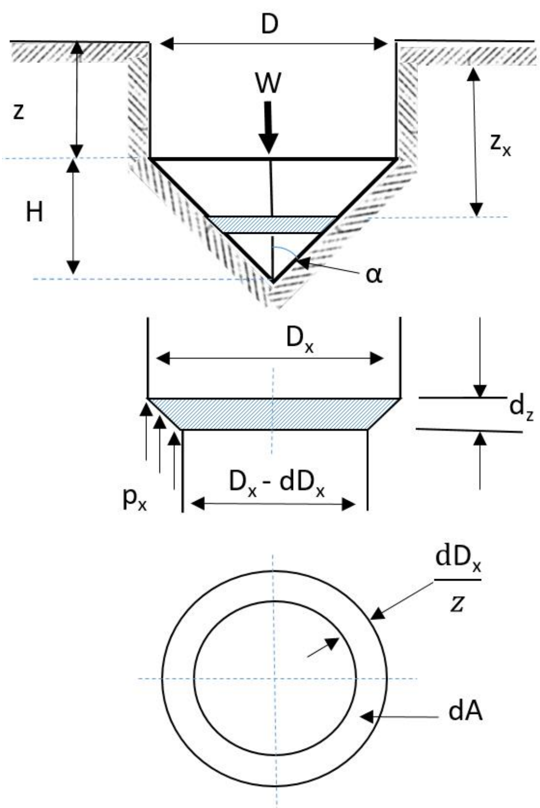


Figure 7. Notation and Equilibrium of Forces (Redrawn from Janosi 1959)

Where:

α is the vertical angle of the cone, 15 degrees

D is the diameter of the cone, 0.8 inches

D_x is the diameter of the top of the truncated cone

dD_x is the change in the diameter of the truncated cone with respect to d_z

dA is the area of the annulus

d_z is the height of the truncated cone

H is the height of the cone, 1.5 inches

p_x is the soil resistance force

z_x is the height from the base of the cone to the height of a given diameter

z is the depth of the top of the cone to the surface

Since $\frac{(dD_x)^2}{2} \approx 0$

Substituting in Equation 7 we have Equation 8:

$$dW = (\frac{k_c}{D_x} + k_\varphi) z_x^n \frac{\pi}{2} D_x \frac{dD_x}{2} \tag{8}$$

From Fig. 7

$$\frac{D}{D_x} = \frac{H}{z + H - z_x}$$

Solving for D_x:

$$D_x = \frac{D}{H}(z + H - z_x)$$

And differentiating D_x with respect to z_x

$$\frac{dD_x}{dz_x} = -\frac{D}{H}$$

Giving:

$$dW = \frac{\pi}{2}[k_c + k_\varphi \frac{D}{H}(z + H - z_x)] z_x^n (-\frac{D}{H}) dz_x \tag{9}$$

The total force, W, is found by integrating Equation 9 between the limits (z) and (z+h):

$$W = -\frac{\pi D}{2H} \int_z^{z+H} [k_c + k_\varphi \frac{D}{H}(z + H - z_x)] z_x^n (-\frac{D}{H}) dz_x$$

Giving:

$$W = -\frac{\pi D}{2H} \{ \frac{k_c}{n+1} [(z + H)^{n+1} - z^{n+1}] + \frac{k_\varphi D}{H} [\frac{(z+H)^{n+2}}{(n+1)(n+2)} + \frac{z^{n+2}}{n+2} - \frac{(z+1)z^{n+1}}{n+1}]\} \tag{10}$$

From Fig. 7: $\frac{D}{2H} = tan\alpha$ or $\frac{D}{H} = 2tan\alpha$

The minus sign in Equation 10 can be neglected since it only means that W and the reaction force act in opposite directions so that we are given Equation 11:

$$W = \pi tan\alpha\{\frac{k_c}{n+1}[(z+H)^{n+1} - z^{n+1}] + 2k_\varphi tan\alpha[\frac{(z+H)^{n+2}}{(n+1)(n+2)} + \frac{z^{n+2}}{n+1} - \frac{(z+H)z^{n+1}}{n+1} \quad (11)$$

As previously mentioned, the dial of the cone penetrometer indicates a cone index reading that is twice as large as the force, (W), required for penetration of the soil layer. If we denote the cone index by C_{WES} then

$$C_{WES} = 2W$$

The dimensions of the standard cone are:

H = height of the cone, 1.5 inches (38.1 mm)

$\alpha = \frac{30°}{2} = 15°$

$2\ \pi\ \tan\alpha = 1.625$

$2\ \tan\alpha = 0.5175$

Giving Janosi's CI Equation:

$$C_{WES} = 1.625\{\frac{k_c}{n+1}[(z+1.5)^{n+1} - z^{n+1}] + 0.517k_\varphi[\frac{(z+1.5)^{n+2}}{(n+1)(n+2)} + \frac{z^{n+2}}{n+1} - \frac{(z+1.5)z^{n+1}}{n+1}]\} \quad (12)$$

The cone index, as defined by Janosi's equation, is predicted as a function of the k_c, k_φ, n, and the sinkage in inches, z (Janosi, 1959). Bevameter and cone penetrometer tests were run concurrently under identical soil conditions. Numerous experiments were performed to check the validity of the above equation. According to Janosi, the test results indicate that the equation is accurate (Janosi, 1959).

6. Discussion

A computerized tool, like Microsoft Excel's Solver, was used to run iterative procedures to solve for k_c, k_φ, and n. The iterative solution was used to convert from cone index to bevameter values, varying in method depending on soil type. The method of predicting the Bekker surface cohesion "c" and angle of internal friction "φ", measured from the bevameter shear ring, were not investigated in this project. However, additional research published by Rohani and Baladi (1981) and Perkins, Meier, and Farr (1992) derived the mechanical properties of the soil in terms of c and φ from cone index.

6.1 Comparison of Cone Index to Bekker Parameters

As previously discussed, most references on this topic state that predicting k_c, k_φ, and n, from CI is impossible. Using Bekker's suggestion in *Introduction to Terrain-Vehicle Systems,* an iterative procedure using Janosi's CI equation, comparable soil values, equations from reports using similar soils, and field-measured cone index equations were used to predict values for k_c, k_φ, and n. As previously mentioned, c and φ values were not found directly from CI values in this project.

Various sets of data from several sources were used to give common values for CI, k_c, k_φ, and n. Data sets with k_c, k_φ, and n values with corresponding CI values were of particular interest. The following tables were used as reference.

Table 1. Sets of Values for Soils and Moisture Contents (Bekker 1969)

Moisture Content %	k_φ [lb/in$^{(n+2)}$]	k_c [lb/in$^{(n+1)}$]	n	c (lb/in^{2})	φ (degrees)	Type/Location
13	7	5	0.80	1	29	Sandy Loam, Michigan (Strong, Buchele)
11	6	11	0.90	0.7	20	
23	20	5	0.70	1.6	25	
23	27	15	0.40	1.4	35	
21	38	14	0.40	2.5	22	
32	1.2	0.7	0.50	0.75	11	Sandy Loam, Maryland (Hanamoto)
31	1.2	1.5	0.40	0.8	15	
30	0.1	7.5	0.40	0.9	23	
29	2.7	1.6	0.60	2	26	
	3	2.2	0.60	2.1	26	
26	6.8	5.3	0.30	2	22	
0	3.9	0.1	1.10	0.15	28	Dry Sand (Land Locomotion Lab)
45	13	14	0.30	0.8	17	Clayey Soil (Thailand)
47	8	24	0.60	1.1	14	
38	16	12	0.50	0.6	13	
185	3	3	1.00	0.5	11	
55	5	7	1.00	0	11	
55	14	7	0.70	0.3	10	
43	4	22	0.90	0.26	10	
25	140	45	0.13	10	34	Heavy clay (WES)
30	65	25	0.12	7	22	
35	30	14	0.13	5	14	
40	10	7	0.11	3	6	
22	120	45	0.20	10	20	Lean clay (WES)
24	80	30	0.17	7	18	
26	45	20	0.17	5	15	
28	30	10	0.16	4	12	
30	20	8	0.16	2.5	11	
32	10	5	0.15	2	11	
22	3	7	0.20	0.2	38	Sand Loam (LLL)

Table 2. Selected Excerpt of Bevameter Sinkage from WES 1964 and WES 1964a

Bevameter Sinkage								
Lean Clay WES 1964					Yuma Sand WES 1964a			
Average Cone Index of 0 to 6 in. [psi]	n	k_c [lb/in$^{(n+1)}$]	k_φ [lb/in$^{(n+2)}$]		Average Cone Index of 0 to 6 in. [psi]	n	k_c [lb/in$^{(n+1)}$]	k_φ [lb/in$^{(n+2)}$]
207	0.365	75	133		34	0.91	1.4	9.7
216	0.365	75	133		38	0.95	-0.6	9
210	0.365	75	133		36	0.88	0.8	10.3
211	0.365	75	133		37	0.97	0.8	9.1
218	0.365	75	133		42	0.86	2.8	8.6
201	0.365	75	133		15	0.79	0.6	7.2
158	0.193	15.7	69.5		46	0.76	0.3	15.8
152	0.193	15.7	69.5		40	0.89	0.6	11.3
158	0.193	15.7	69.5		61	0.78	3.1	17.9
124	0.218	21.4	57.3		60	0.62	1.4	19.7
128	0.218	21.4	57.3		44	0.94	0.3	10.2
120	0.218	21.4	57.3		25	0.89	1.3	6.9
99	0.211	26.6	31.1		23	1.02	-0.1	6.3
99	0.211	26.6	31.1		24	0.81	1.7	7.8
100	0.211	26.6	31.1		48	0.62	4.1	13.2
87	0.185	9.5	32.8		42	0.66	1.3	13.9
87	0.185	9.5	32.8		40	0.64	2.5	13
84	0.185	9.5	32.8		50	0.62	-3.5	19.2
62	0.146	0	44.2		43	0.7	0.1	14.8
60	0.146	0	44.2		35	0.66	1.9	12.6
69	0.146	0	44.2		23	0.89	0	7.6

Rula and Nuttall generated a set of predicted and measured cone indices based on a "rearranged form of Janosi's equations". The equations listed for the prediction of cone index include Equations 13-16 (Rula Nuttall, 1971).

$$CI = \frac{1}{K\lambda_1}\left(k_\varphi + \frac{K\lambda_2}{d} k_c\right) \quad (13)$$

Where:

$$K\lambda_1 = \frac{(n+1)(n+2)}{2H^n} * \frac{(\lambda-1)^{n+2}}{\lambda^{n+2}-(n+2)\lambda+(n+1)} \tag{14}$$

$$K\lambda_2 = \frac{(n+2)}{2} * \frac{(\lambda-1)(\lambda^{n+1}-1)}{\lambda^{n+2}-(n+2)\lambda+(n+1)} \tag{15}$$

$$\lambda = \frac{H}{z} = 1 \tag{16}$$

Where:

H = cone length, from tip to base, 1.5 in. (38.1 mm)

z = cone sinkage measured at the cone base, in.

d = diameter, 0.8 in. (20.3 mm)

This method was compared to Janosi's equation (Equation 12) to test the similarity of solutions given by two methods. It was found that when using the Rula and Nuttall method to predict CI, the equations produced an answer that was double the actual CI values. In other words, we were only able to replicate their data when using their exact equations and dividing the final answer by two. This new CI equation was able to produce similar CI values to the existing CI equation that was used for the purpose of this paper; however, the k_c, k_φ, and n values that were generated were not comparable. It was decided after the comparison that the cone index equation from Janosi's 1959 paper was the desired method and worked better with other data sets.

Two different soil types were investigated, sand and lean clay. The prediction of k_c, k_φ, and n for sand was more complex than that of clay. The methods for each soil type are explained in the following sections.

6.1.1 Sand

For the prediction of k_c, k_φ, and n for sand, three methods were used. All three of the methods used an iterative procedure to solve for one or more unknown variables using

Janosi's CI equation (Equation 12). Janosi's CI equation contains four variables: z, k_c, k_φ, and n. The depth of sinkage z was held at a constant of 6 inches (15.24) for all three methods. For the first method, n was held as a constant set equal to one and k_c was set equal to zero, assumed from typical values for dry sand. The k_φ value was then solved using an iterative solution. The second method held the n value constant at one, and used a linear relationship between k_c and cone index (Equation 17) from the *Measuring Soil Properties in Vehicle Mobility Research* (WES, 1964a) report for the k_c value.

$$k_c = 0.15 * CI - 3.20 \tag{17}$$

As in the first method, the k_ϕ value was solved for using an iterative solution. The first and second method do not absolutely need an iterative solution to find the only missing variable k_ϕ. The third method was truly an iterative procedure. All three of Bekker's parameters, k_c, k_φ, and n, were variables. The (WES, 1964a) report had k_c, k_φ, and n values with corresponding cone index values. This full range of data was also used to find an acceptable range for each of the three Bekker parameters to vary within. The maximum and minimum value for each parameter were expanded by five percent to give an ideal range. Equation 12 was set equal to field measured CI values, and the three Bekker parameters were varied iteratively within the bounds of the ideal range and the CI equation.

All three methods were compared with the original set of data from WES 1964a report to see how closely they matched the field measured values. Table 3 shows the standard deviation between the measured and predicted for each method.

Table 3. Standard Deviation of three methods in sand

Standard Deviation Values: Measured vs. Predicted

	k_c	k_φ	n
Method 1	2.748	4.085	0.161
Method 2	2.770	4.250	0.161
Method 3	2.483	3.194	0.173

Method 3 had the least deviation for k_c and k_ϕ, but a slightly larger value for n compared to the other two methods.

6.1.2 Clay

In order to generate k_c, k_φ, and n values for clay soil types, linear soil relationships and Janosi's equation (Equation 12) were used. According to *Strength-Moisture-Density Relations of Fine-Grain Soils in Vehicle Mobility Research* (WES 1964), heavy and lean clay's k_c and k_φ values share linear relationships with cone index. The following equations (Equation 18 and 19) are the linear relationships, as shown in Figures 8 and 9 (WES, 1964).

$$k_c = 0.19 * CI \quad (18)$$

$$k_\varphi = 0.48 * CI \quad (19)$$

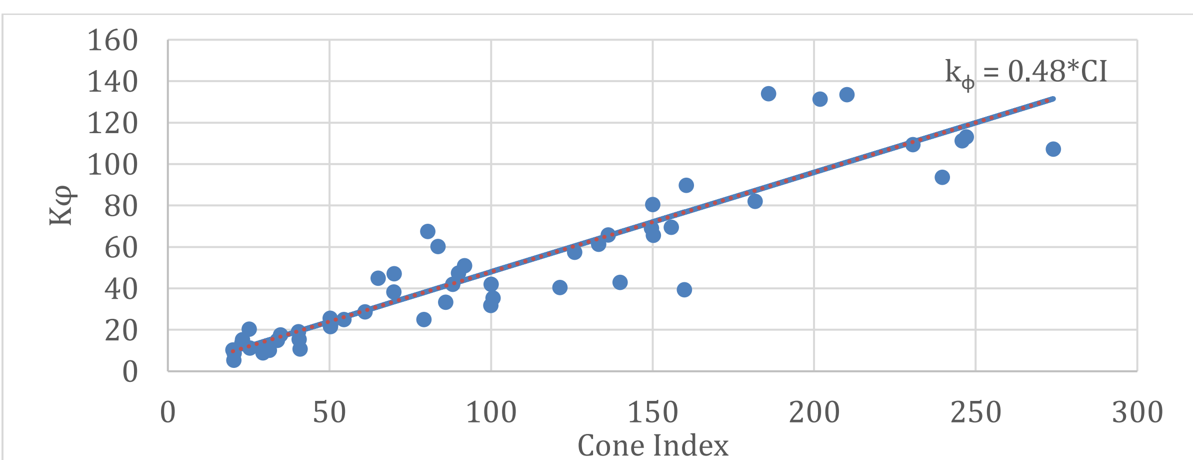


*Figure 8. k_φ vs. Cone Index (*Redrawn from *WES 1964)*

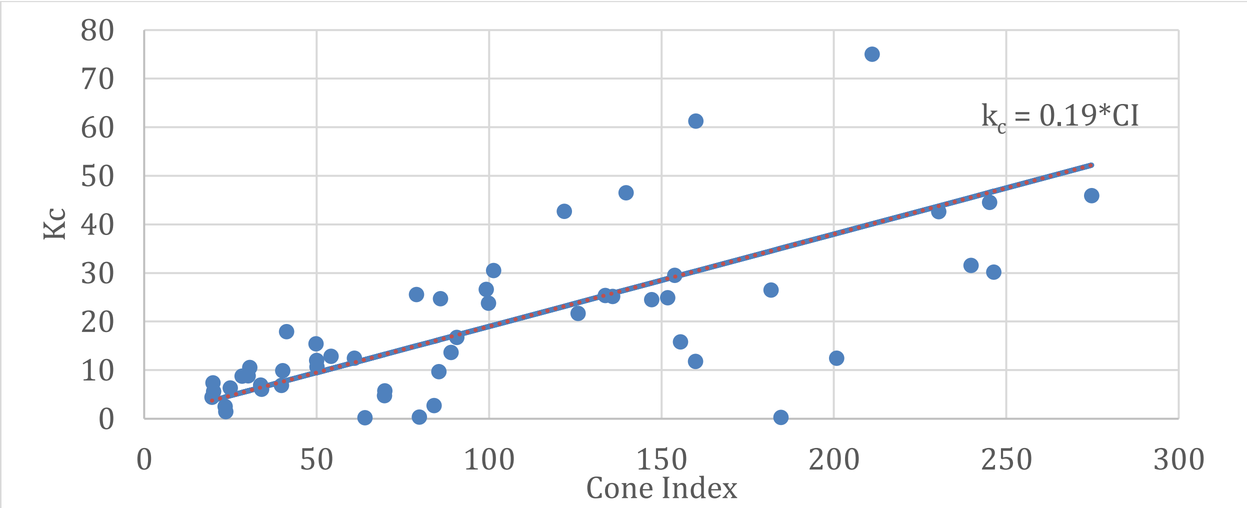


Figure 9. k_c *vs. Cone Index (*Redrawn from *WES 1964)*

These equations allowed for the generation of k_c and k_φ values that were used in Janosi's CI equation to solve for the unknown n value. Since n is an exponent in Janosi's cone index equation, the slightest change in the hundredth decimal place had an effect on the final predicted result. The process of finding only one missing variable did not need an iterative procedure, like methods one and two in sand. The linear relationships between CI, k_c and k_φ reduced the amount of uncertainties in the prediction in clayey soils

7. Summary

The results that were obtained from this experimental process are not proven and have considerable room for error. This process relies on assumptions made regarding soil values from published reports containing values for similar soils. Those typical values were measured by different teams working over a span of several decades with no standard technique. However, some of the data did exhibit a consistent change with moisture and showed a believable quantitative relationship between soil values (Bekker, 1969). For future use, these methods could be applied to other locations, assuming that those soils were similar enough to the Vicksburg heavy/lean clays, and the Yuma Sands.

The assumptions made were reasonable based on the soil type of this project, but would not have been feasible if there was an absence of pre-existing soils data to compare with the soil type of the given project. To create bounds for an iterative solution to work within, average values for cone index and corresponding bevameter parameters are needed.

The bevameter parameters from the WES 1964a report, when plugged into the CI equation, did not produce a CI value similar to the measured CI value. Some data sets produced CI values exactly double what they actually were supposed to be, while others had no consistent deviation from the original measured value. This brings the validity of Janosi's equation, or the data sets into question. In respect to the computerized iterative process, there is also considerable room for error. The output values of the iterative solver that was used in this project were only one set of many possible combinations of values. The variety of possible combinations were due to the complexity of Janosi's equation and the nature of the iterative process.

8. Conclusion/Recommendations

The process of predicting Bekker parameters from known cone index values has been considered impossible by many sources. It was suggested that the process of predicting the three unknown bevameter variables in Janosi's cone index equation may be theoretically worked by means of an iterative procedure if additional information is provided such as soil type. This effort used previously found relationships and known test values to predict bevameter parameters from cone index data using direct and indirect methods in an iterative process. The validity of this process and its results relies on the accuracy of previously determined soils data, relationships, and the iterative process with Janosi's cone index equation. It is recommended for future tests that a cone penetrometer and a bevameter be used in conjunction to verify the predicted values when compared against the actual field-measured values. This experiment only used a small scope of data in testing the experimental methods due to the small amount of bevameter data with corresponding CI data. In these circumstances, the results cannot be confirmed and are inconclusive. This process should be further explored to find if a more accurate and reliable method of predicting Bekker's k_c, k_φ, and n parameters from cone index values is possible.

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10. References

Ahlvin, R.B. (1992) *NATO Reference Mobility Model Edition II, NRMM II* Users Guide

Bekker, M. G. (1960). *Off-the-road locomotion; research and development in terramechanics*. Ann Arbor: University of Michigan Press.

Bekker, M. G. (1969). *Introduction to Terrain-Vehicle Systems*. University of Michigan Press

Janosi, Zoltan. (1959) *Prediction of "WES Cone Index" by Means of a Stress-Strain Function of Soils*.

Laughery, S., Gerhart, G., & Muench, P. (2000). *Evaluating Vehicle Mobility Using Bekker's Equations*. Warren, MI: US Army TARDEC 48397-5000.

Mason, G., Ahlvin R., and Green, J., (2001) “Short-Term Operational Forecasts of Trafficability” TR-01-22 Engineering Research and Development Center

Mason, G., Priddy, J. D., Stevens, T. M., Towne, B. W., and Osorio, J. E., (2015) “Test Procedures for the Determination of One-Pass Vehicle Cone Index (VCI)” Conference: Proceedings of the 13^{th} ISTVS European Conference, Rome Italy

Nguyen Van, Nang., Matsuo, Takaaki., Koumoto, Tatsuya., and Inaba, Shigeki (2008) Experimental Device for Measuring Sandy Soil Sinkage Parameters” Bulletin Fac. Agriculture, Saga University. No. 93

Perkins, W. E., Meier, R. W., & Farr, J. V. (1992, May). *Strength Property Estimation for Dry, Cohesionless Soils Using the Military Cone Penetrometer* [PDF]. Washington D.C.: US Army Corps of Engineers.

Rohani, B., & Baladi, G. Y. (1981). *Correlation of mobility cone index with fundamental engineering properties of soil*: Final report. Washington, D.C.: GPO.

Rula, A. A., & Nuttall, C. J. (1971). *An analysis of ground mobility models (ANAMOB)*. Vicksburg, MS: Waterways Experiment Station.

Shoop, S. A. (1993). *Terrain characterization for trafficability*. Hanover, NH: U.S. Army Corps of Engineers Cold Regions Research & Engineering Laboratory.

Waterways Experiment Station (1964). *Strength-Moisture-Density Relations of Fine-Grain Soils in Vehicle Mobility Research* Waterways Experiment Station TR 3-639, Vicksburg, Miss.

Waterways Experiment Station (1964a). *Measuring Soil Properties in Vehicle Mobility Research, Strength-Density Relations of an Air-Dry Sand;* Waterways Experiment Station TR 3-652, Report 1, Vicksburg, Miss.

Wong, J. Y. (1989). *Terramechanics and off-road vehicles*. Amsterdam: Elsevier.