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DECAY OF LARGE UNDERWATER BUBBLE OSCILLATIONS

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Decay of Large Underwater Bubble Oscillations

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Director

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Decay of Large Underwater Bubble Oscillations

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Executive Summary: This report presents a fluid dynamic theory to explain the rapid damping observed in the oscillation of large underwater bubbles. The results have application to explosive and implosive sources currently used as underwater acoustic probes.

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Decay of Large Underwater Bubble Oscillations

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Abstract: Pressure - time series from breathing mode oscillation of large (centimeter scale or larger) underwater bubbles reveal much higher decay rates than can be explained using viscous, thermal, or radiative mechanisms which apply to microbubbles. We show that if one assumes energy transfer to shape oscillations (surface capillary waves) of large amplitude in subharmonic resonance with the breathing mode [Longuet - Higgins, JASA **91**, 1414, 1992], then the shape oscillations can drive fluid motions outside the bubble capable of exciting turbulent instabilities. Application of an appropriate eddy viscosity from mixing length theory to the viscous decay mechanism appears to offer a credible explanation for the observed large decay rates. We give an analysis to show that energy is transferred from the breathing mode to surface capillaries fast enough to make the proposed decay mechanism viable.

Keywords: bubble pulse ○ underwater implosion ○ underwater explosion ○ eddy viscosity

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Introduction

Ocean acoustic research is sometimes carried out using explosions or implosions, both of which result in a large (a few to several cm radius) bubble which pulsates at a frequency governed by depth and bubble parameters. After the large oscillating bubble is created, the pressure time series from its linear oscillation (excluding the shock wave pulse in the case of explosives) has a shape resembling a strongly damped sinusoid. In fact, the damping tends to be about one factor of ϵ per oscillation cycle regardless whether the source is an imploding lighbulb or nearly a kilogram of TNT. We will review some of the data leading to this conclusion and then seek to determine within an idealized model how the energy loss rate of a large bubble can be so great and so nearly independent of bubble size and depth.

The implosion of glass spheres[1, 2] or lightbulbs[3, 4] has been investigated with increasing interest as an acoustic source for probing the underwater environment in part because of regulatory restrictions and dangers inherent in the use of explosives in shallow water environments. In addition, small implosive sources may be used in the vicinity of receiving arrays with minimal concern about equipment damage. Lightbulbs provide an inexpensive and safe implosion device when combined with a breaking mechanism to trigger the implosion. A useful discussion of several lightbulb implosion experiments has recently been given by Heard, *et al*[3]. While useful relations exist[3, 4] for predicting the peak level and breathing frequency, no current theory adequately predicts the oscillation decay rate for large bubbles.

Figure 1 gives examples of lightbulb implosion pressure time series taken by Saclantcen [4] at depths of 36 and 91m. Although pressure time series from implosion experiments are subject to variability due to the nonsphericity of the source and chaotic interaction among water, gas, and glass fragments, signatures of the type shown in Figure 1 are quite typical of such implosions at depths down to approximately 200m. In fact Figure 2 of ref [3] displays a lightbulb implosion pressure time series for a depth of 18m which is remarkably similar in shape to Figures (1a) and (1c). One notable feature of the time series in Figure 1 is the short decay time of the acoustic pressure, which is on the order of the bubble oscillation period.

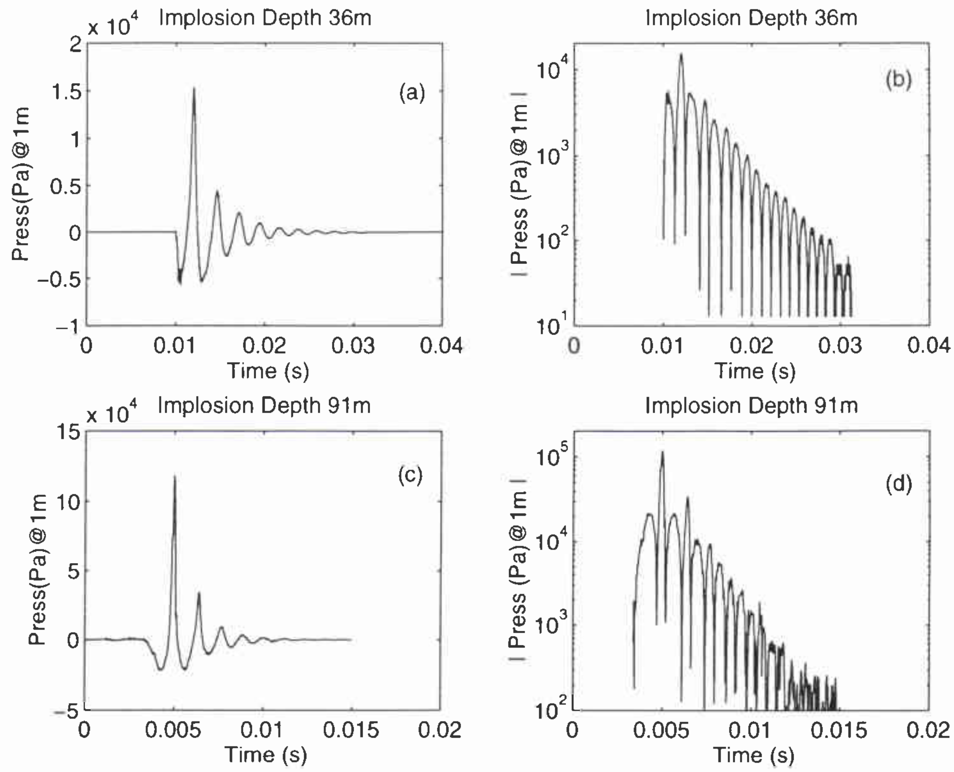

Figure 1. Pressure time series[4] at 1m radius for lightbulb implosions at depths 36 and 91m. Left panels use linear scale, while log scale for absolute value of the pressure is shown in right panels to demonstrate linear decay.

At depths greater than about 200m, lightbulbs implode spontaneously because of the hydrostatic pressure. For implosions at depths greater than about 200m, glass spheres can be constructed with enough strength to withstand the pressure until the breaking mechanism is triggered. At depths much greater than 200m, a different type of time series is found. At depths ranging between approximately 2500 and 3500m, Orr and Schoenberg[1] recorded implosion pressure time series consisting of one large spike with hardly any oscillatory tendancy. As we will show below, a decrease in number of oscillations with implosion depth is consistent with acoustic radiation as being the dominant decay mechanism at great depth.

The literature applicable to the decay of bubble oscillations[5]-[9] is primarily oriented toward small (sub- centimeter) bubbles. The gas inside A 100-Watt lightbulb[3] is approximately 150cm^3, and would correspond to a sphere of radius 3.3cm at atmospheric pressure. Theoretical work for the implosion and oscillation of bubbles of this approximate size is scarce. Perhaps the reason for this is that such large bubbles in the ocean are rare, rise quickly to the surface, and are subject to breaking up.

Although the current work emerged as an attempt to understand the oscillation of implosion bubbles, we came to realize that our results also apply to some features of bubble oscillations produced by underwater explosions (*e.g.* SUS charges). Chapman[10] has given a thorough and useful review of the properties of bubble pulses following SUS charge detonations. Results of that review are incorporated into our comparisons between theory and data below.

After reviewing the dominant decay mechanisms (thermal, viscous, and radiative) normally invoked for small bubble oscillations (and which are inadequate to explain the damping of centimeter - scale bubble oscillations at depths less than about 200m) we will propose a simple model which appears consistent with the observed decay of oscillations in lightbulb - sized bubbles. The model invokes turbulence in the fluid just outside the bubble excited by shape oscillations[9] which are in subharmonic resonance with the bubble's breathing mode. The model yields a predicted ratio of decay time to oscillation period which is independent of bubble radius and depth, in approximate agreement with data for depths less than about 200m.

2

Bubble Oscillation Damping Constants

For spherical bubbles linear oscillations involving only radial motion are of the form $\exp(-\beta_{TH} - \beta_R - \beta_V - i\omega_b)t$, where the breathing frequency of the bubble is

$$\omega_b = a^{-1}\sqrt{\frac{3\gamma p}{\rho}}, \tag{1}$$

a is the equilibrium bubble radius, γ is the ratio of specific heats of the gas (1.4 for air), p is the ambient pressure and ρ is the fluid density. We are concerned with bubbles of sufficient size to be regarded as adiabatic, so that $a \propto p^{-1/3\gamma}$. The damping constants are as follows[5, 9].

Thermal damping ($\omega_b/2\pi < 10\text{kHz}$):

$$\frac{\beta_{TH}}{\omega_b} = \frac{3(\gamma-1)}{2a}\left(\frac{D}{2\omega_b}\right)^{1/2} \tag{2}$$

Radiative damping:

$$\frac{\beta_R}{\omega_b} = \frac{1}{2c}\left(\frac{3\gamma p}{\rho}\right)^{1/2} \tag{3}$$

Viscous damping:

$$\beta_V = \frac{2\nu}{a^2} \tag{4}$$

Constants introduced in (2) - (4) are the thermal diffusivity of air, $D \simeq 0.2$ cm^2/s, the water sound speed $c = 1500$m/s, and the kinematic viscosity of water $\nu \simeq .01$ cm^2/s. The damping rates per oscillation period due to thermal, viscous, and radiative effects for a lightbulb - sized bubble $a_0 = 3$cm of gas at pressure 1 atmosphere imploded at depth are shown in Figure 2. The shape oscillation damping rate is discussed below. Also plotted in Figure 2 are data for bubbles created by lightbulb implosions and bubble pulses (excluding the initial shock wave) from 820g TNT SUS charges[10]. Remarkably, the decay per oscillation of the bubble created by 820g TNT SUS charges is within a factor of two of that for lightbulb implosion bubbles, suggesting that some sort of scale independent mechanism may be at work.

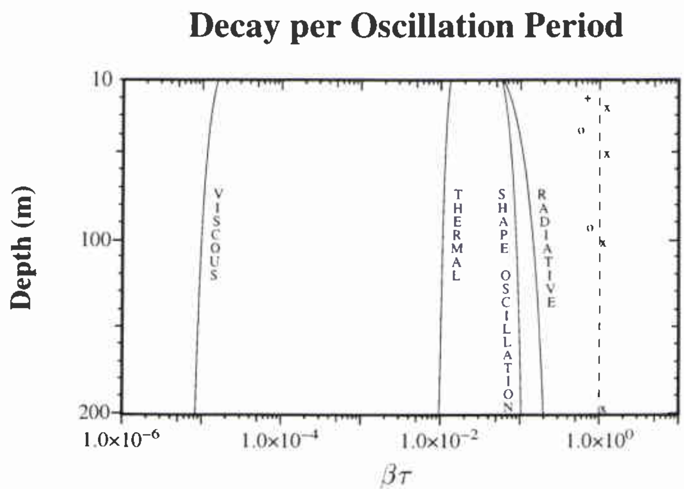

Figure 2. Curves: Decay exponent $\beta\tau$ for various mechanisms as a function of depth in the oscillation of a bubble of radius 3cm at pressure 1atm taken to depth. τ is the bubble oscillation period $\tau = 2\pi/\omega_b$. Dashed line: Decay per oscillation from present model with $\kappa = 0.5$ in eq. (14). Data points: + from ref [3], Fig. 2; o from Fig. 1.; x from ratio of second to third bubble pulse peak pressure for SUS charges [10].

It is clear from Figure 2 that the sum of the theoretical decay rates for a nominal 3cm radius bubble taken adiabatically to depth is far too small to explain the observed damping rates, of order one e-fold per oscillation period. Thus we are motivated to consider other mechanisms that would explain the observed decay rate. We first considered nonlinearity in a purely radially oscillating bubble. The numerical integration carried out by Prosperetti and Lezzi[11] retained dominant nonlinearities for a bubble collapse from a radius 4 times its pressure equilibrium radius. At the end of the first oscillation period, the bubble had lost approximately 74 percent of its initial energy. For a lightbulb of internal pressure 0.8 atm[3], this simulation would imply an implosion depth of approximately 2700m. At such a depth the linear decay $\exp(-\beta_R\tau)$ in amplitude due to radiation is approximately 0.5. Near equilibrium, a submerged bubble's internal energy increases approximately quadratically with the difference between bubble radius and its equilibrium value. Thus the 74 percent energy loss appears mostly linear in nature for a scaled implosion depth of 2700m. Since radiative and nonlinear losses decrease with decreasing implosion depth, the mechanism proposed by [11] will not account for the observed energy loss for shallow (less than 200m depth) implosions.

A second possible rapid energy loss mechanism by large bubbles is that of shape oscillations in subharmonic resonance with the breathing mode[9]. Longuet - Higgins had considered this mechanism to explain large damping rates observed in sub- centimeter scale bubbles. Shape oscillations may be larger in amplitude than the breathing mode oscillations because they do not change the bubble's volume. The normal modes of shape oscillations are such that the radial displacements of the bubble surface are spherical surface harmonics[9], and are thus proportional to $P_n(\cos\theta)$, $P_n(x)$ being the Legendre polynomial of order n, and θ is the polar angle with respect to some arbitrary direction. The surface harmonics consist of standing capillary waves on a sphere with frequency[9]

$$\sigma_n^2 = (n-1)(n+1)(n+2)\frac{T}{\rho a^3} \tag{5}$$

where $T = 73$ dyne/cm is the surface tension of water. The subharmonic resonance condition invoked by Longuet - Higgins is

$$2\sigma_n = \omega_b \tag{6}$$

Substituting equations (1) and (5) into (6) yields the resonance condition

$$(n-1)(n+1)(n+2) = a\frac{3\gamma p}{4T} \tag{7}$$

We can take the large n limit in (7) as justified by substitution of minimal parameters $a = 1$cm, $p = 1$atm, yielding a value of 1.437×10^4 for the right hand side, and $n \simeq 24$. Lightbulb implosions will always lead to larger values of a and p, so the resonance condition becomes

$$n^3 \simeq a\frac{3\gamma p}{4T} \tag{8}$$

Even though n is constrained to integer values, the resonance is a very broad one due to the rapid decay of oscillations. This means that it is likely for large n that several harmonics near the value given in (8) will be excited.

According to Longuet - Higgins[9], the dominant loss mechanism for shape oscillations is viscosity, resulting in a decay constant

$$\beta_{nV} = (n+2)(2n+1)\nu/a^2$$
$$\simeq \frac{2\nu n^2}{a^2} \tag{9}$$

The decay per oscillation period due to shape oscillations is shown with the other mechanisms in Figure 1, and is again too small to explain the damping of order one e-fold per oscillation at these depths for centimeter scale bubbles.

3

A Model Invoking Eddy Viscosity

We hypothesize that fluid instabilities similar to those involved in turbulence may be generated by the shape oscillations of large bubbles, with a result of high energy decay rates. One of the most basic ways to model turbulent- like effects in a fluid is to use an eddy viscosity ν_e from mixing length theory:

$$\nu_e \propto < \tilde{v}\lambda > \tag{10}$$

where $\tilde{v}$ is the velocity of a parcel of fluid relative to the average flow velocity in its vicinity, λ is the mixing length or characteristic distance the parcel travels before desolving into the background fluid, and the brackets $<>$ indicate an ensemble average.

If large shape oscillations are occuring (with large meaning that maximum radial displacement is a non negligible fraction of the oscillation wavelength), it is reasonable to take λ proportional to the shape oscillation wavelength, and $\tilde{v}$ proportional to the capillary wave speed. Thus

$$\lambda \propto \frac{a}{n} \tag{11}$$

and

$$\tilde{v} \propto \omega_b \lambda \tag{12}$$

The proportionality constants in (10-12) are of order unity. Thus we hypothesize that

$$\nu_e = \kappa \omega_b \frac{a^2}{n^2} \tag{13}$$

where κ is a dimensionless constant of order unity.

If we adapt the decay constant (9) to include ν_e rather than the molecular value ν, we find

$$\begin{aligned}\beta_{ne} &\simeq \frac{2\nu_e n^2}{a^2} \\ &= 2\kappa\omega_b \end{aligned} \tag{14}$$

where β_{ne} stands for the eddy transport decay rate of surface harmonic n. The decay rate per oscillation period for this hypothesized mechanism is

$$2\pi\beta_{ne}/\omega_b = 2\kappa = const. \tag{15}$$

The above mechanism with $\kappa \simeq 0.5$ then would lead to approximately one e-fold per oscillation, independent of bubble radius and depth (as long as n is large, which will apply for centimeter scale bubbles) in agreement with the time series of Figure 1. As shown in Figure 2, the radiative contribution to the decay increases as the square root of depth, so that at some large depth radiative damping would dominate and further enhance the total decay rate. This seems to be consistent with the results of Orr and Schoenberg[1] indicating very strong damping and almost no oscillation tendency at implosion depths of approximately 2500m and greater.

If fluid instabilities are indeed operative in damping the oscillation of large bubbles, one should consider whether the traditional criterion for turbulence is satisfied; namely that the Reynolds number for the fluid motion be large compared to some value determined by the geometry of the system. In a system with characteristic length and velocity scales L and v, the Reynolds number is vL/ν, where ν is the molecular kinematic viscosity. For shape oscillations of mode n, the characteristic length scale is taken to be the average capillary wavelength, $L = 2\pi a/n$, and the velocity scale is taken to be $v = \omega_b L$. The Reynolds number is thus

$$R_e = \frac{\omega_b}{\nu}\left(\frac{2\pi a}{n}\right)^2 \tag{16}$$

The dependence upon depth of R_e for shape oscillations of the nominal 3cm bubble taken to depth is shown in Figure 3. R_e is of order 10^4, certainly large enough to indicate fluid instability. Its depth dependence may be shown from (16) to be proportional to $p^{-1/6-1/9\gamma}$, where $1/6 + 1/9\gamma = 0.246$. We have shown in the dotted line of Figure 3 that a fit of the form $p^{-1/4}$ is quite accurate for the depth dependence of R_e.

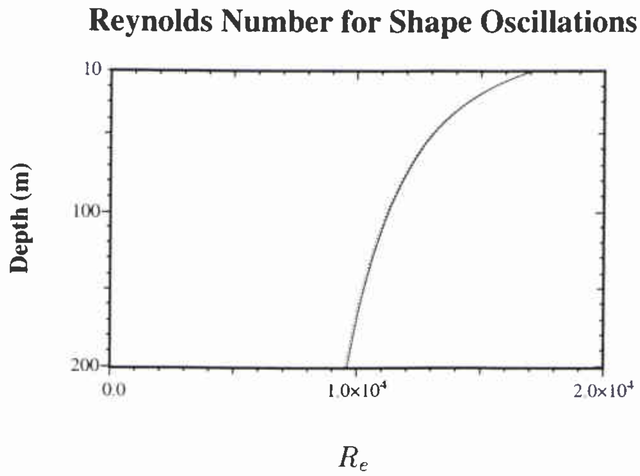

Figure 3. Reynolds number (eq. (16)) based on capillary wavelength and phase speed of shape oscillations of a 3cm bubble taken to depth. The dotted line is a fit to $p^{-1/4}$.

The large values of R_e for depths less than about 200m and its gradual decrease with depth support our hypothesis that at small to moderate depths turbulence may be active in damping oscillations of large bubbles, while at great depth radiative damping becomes dominant.

4

Energy Transfer: Rayleigh Taylor Instability

In order for the proposed mechanism to explain the rapid damping of the breathing mode, there must be a mechanism for transferring energy to shape oscillations on a time scale shorter than the oscillation period. We will present some theoretical arguments to the effect that energy is indeed transferred rapidly to capillary waves during the outward acceleration phases of bubble oscillation. The mechanism is the Rayleigh Taylor instability, familiar as the operative instability mechanism when a heavy fluid overlays a lighter one. In such a configuration an initially plane interface between the fluids developes ripples, and then fingers of the two fluids migrate into each other as the heavier fluid attempts to sink and the lighter one attempts to rise. In the case of an oscillating bubble, gravity is replaced by the outward acceleration of the bubble wall during half the oscillation period. Although this acceleration is time dependent, we will obtain order of magnitude estimates for instability growth time using a constant root mean squared (rms) value for the acceleration.

For capillary wavelengths much smaller than the bubble radius, we may use results for a plane interface between air and water with x along the interface and y normal to it. For disturbances of the form $\exp(ikx - k|y| + \sigma t)$ the instability growth rate including surface tension but neglecting the density of air when added to or subtracted from that of water is[12]

$$\Gamma^2 = Ak - \frac{T}{\rho}k^3 \tag{17}$$

where A is the acceleration of the interface taken to be positive in the direction of the water side of the interface. One should notice that during a contraction (negative A), (17) leads to imaginary growth rate (propagating rather than growing capillary waves). Thus the alternation of the sign of A leads to altenrating periods of capillary growth and propagation, but not to capillary decay.

The growth rate of the most unstable wave is from (17)

$$\Gamma_{max}^2 = \frac{2}{3}A^{3/2}\sqrt{\frac{\rho}{3T}} \tag{18}$$

For A we take the value $a_0(2\sqrt{2})^{-1}(1-(p_0/p)^{1/3\gamma})\omega_b^2$. This is a rough estimate of the rms acceleration during an imploded lightbulb's first expansion phase. Substituting lightbulb parameters cited above, we use eqs. (1) and (18) to estimate the number of instability growth times $\Gamma_{max}\tau/2$ during the first outward acceleration half cycle, where as above $\tau = 2\pi/\omega_b$. The result is shown as a function of depth in Figure 4.

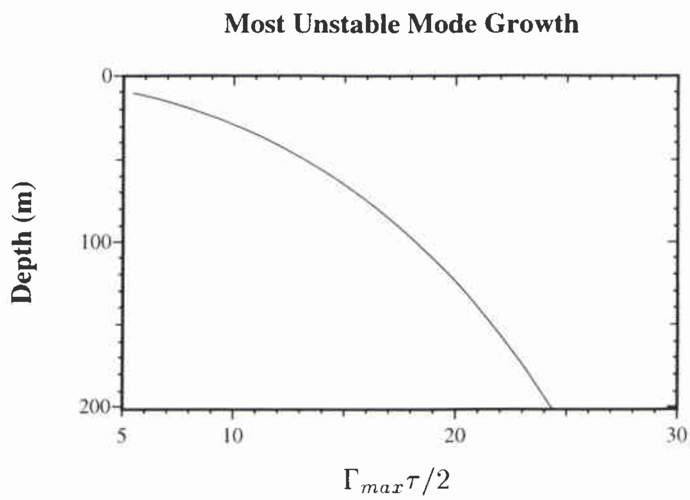

Figure 4. The number of growth times for the most unstable Rayleigh - Taylor mode during the first outward acceleration half cycle following a lightbulb implosion.

Figure 4 indicates that in the first outward acceleration half cycle following a lightbulb implosion there is ample time for fingering instabilities to develope. Their kinetic energy is parasitic to the breathing mode, so that energy transfer to shape oscillations appears vindicated by the above estimates.

5
Conclusion

We have shown that a possible candidate for the decay mechanism of large underwater bubble oscillations is fluid turbulence outside the bubble forced by shape oscillations in subharmonic resonance with the bubble's breathing mode. Namely, we have shown that the decay times for shape oscillation conform to the observations at modest depths (200m or less for lightbulb implosions) when eddy viscosity (13) is used in (9), with one free parameter $\kappa = 0.5$. In order to make a more rigorous argument for this mechanism, one needs to examine in detail the subharmonic resonance as discussed by Longuet- Higgins[9] as a means of transferring energy quickly from the breathing mode to the shape oscillations. We have made a step in this direction by showing in the analysis leading to Figure 4 that the Rayleigh Taylor fingering instability has ample time to generate parasitic capillary waves during the first outward acceleration following a lightbulb implosion.

6
Acknowledgements

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Keywords

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