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RPPR Final Report

as of 12-Jun-2020

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Major Goals: The purpose of this workshop is to bring together Mathematicians, Logicians, Physicists, and Computer Scientists, to discuss the possibility of using a higher category approach to certifiably correct quantum information processing systems.

Here is a tentative list of prospective speakers/participants:

1. Dr. Emily Riehl, Assistant Professor, Department of Mathematics, Johns Hopkins University.
2. Dr. Andrew Childs, Professor, Department of Computer Science, and co-director of Joint Center for Quantum Information and Computer Science, University of Maryland.
3. Dr. Steve Zdancewic, Professor, Department of Computer and Information Science, University of Pennsylvania.
4. Dr. Louis Kauffman, Professor, Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago.
5. Dr. Michael Mislove, Pendergraft Herbert Buchanan Professor, Department of Mathematics, Tulane University.
6. Dr. Scott Aaronson, David J. Bruton Centennial Professor of Computer Science and director of the Quantum Information Center at the University of Texas at Austin.
7. Dr. David Spivak, Research Scientist, Department of Mathematics, MIT.
8. Dr. Paul Alsing, Senior Research Physicist, Information Directorate, AFRL.
9. Dr. Tristan Nguyen, Program Officer, AFSOR.
10. Dr. Spencer Breiner, Mathematician, NIST.
11. Dr. Jared Culbertson, Mathematician, AFRL.
12. Dr. Radhakrishnan Balu, Quantum Information Science, ARL.
13. Dr. Carl Miller, Mathematician, Cryptographic Technologies Group, NIST.
14. Dr. Brad Lackey, Cryptographic Designer and Mathematician, Department of Defense.
15. Dr. Jason Morton, Associate Professor of Mathematics and Statistics, Pennsylvania State University.
16. Dr. Aaron Lauda, Professor, Department of Mathematics, University of Southern California.
17. Dr. Michael Shulman, Assistant Professor, Department of Mathematics, University of San Diego.
18. Dr. Dan Freed, Professor, Department of Mathematics, University of Texas at Austin.

Some of the questions to discuss during the workshop include:

- application of quantum field theory (especially topological) to quantum information processing in the context of higher category theory.
- How do we draw inspirations from classical information processing in applying category theories?
- How do we frame theorem proving in the context of quantum computation?
- How do we connect homotopy type theory to quantum physics?

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- What is the relationship between path-integral quantization and higher categories?

We expect the Workshop to be an important event toward the exploration of synergies between higher category theory, topological quantum field theories and quantum information processing, and a milestone in the development of mathematical tools that will advance these synergies.

Accomplishments: The “Workshop on Higher Category Approach to Certifiably Correct Quantum Information Processing Systems” was held at the Sheraton College Park North Hotel, Beltsville, MD, on Monday, February 4, 2019. It attracted mathematicians, computer scientists, logicians, chemists, and physicists (both theorists and experimentalists) who discussed their research and participated in stimulating discussions on the development of mathematical tools that could advance synergies between higher category theory, topological quantum field theories, and quantum information processing of interest to the Army. There were participants from US Government agencies, industry, and academia. The Workshop was supported financially by ARO via a grant to the University of Tennessee. The web site for the Workshop is <https://quantum.utk.edu/workshop/>.

Participation was by invitation only. The workshop attracted a diverse set of participants which included mathematicians, computer scientists, logicians, chemists, and physicists (both theorists and experimentalists). They came from various US Government agencies (ARL, AFRL, AFSOR, NIST, ORNL, SANDIA, LANL), industry (Microsoft, IonQ), and academic institutions (University of Tennessee, University of Maryland, Harvard University, Tulane University, Virginia Polytechnic Institute and State University, Johns Hopkins University, New Mexico State University, University of California Santa Barbara, University of Illinois at Chicago, Macquarie University, University of Pennsylvania, University of Arizona).

This was a successful Workshop that attracted leading researchers who engaged in stimulating discussions. The state of the art of mathematical tools based on recent mathematical advances, such as CQM, TQFTs, HoTT, the Quon picture language, and higher category theory, which are expected to provide insights into quantum information processing, was discussed at the Workshop. Discussions among participants centered around questions such as: How do we draw inspirations from classical information processing in applying category theories? How do we frame theorem proving in the context of quantum computation? How do we connect HoTT to quantum physics? What is the relationship between path-integral quantization and higher categories? The discussions involved a synergy between mathematicians and other theorists, as well as experimentalists who can take advantage of mathematical tools, and provide feedback regarding the physical realization of mathematical tools. The Workshop was an important event toward the advancement of research in the field, and a catalyst in the development of mathematical tools related to quantum information processing of interest to the Army.

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Report on “Workshop on Higher Category Approach to Certifiably Correct Quantum Information Processing Systems”

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Summary

This is a report on the “Workshop on Higher Category Approach to Certifiably Correct Quantum Information Processing Systems” held at the Sheraton College Park North Hotel, Beltsville, MD, on Monday, February 4, 2019. It attracted mathematicians, computer scientists, logicians, chemists, and physicists (both theorists and experimentalists) who discussed their research and participated in stimulating discussions on the development of mathematical tools that could advance synergies between higher category theory, topological quantum field theories, and quantum information processing of interest to the Army. There were participants from US Government agencies, industry, and academia. The Workshop was supported financially by ARO via a grant to the University of Tennessee. The web site for the Workshop is <https://quantum.utk.edu/workshop/>.

Participation

Participation was by invitation only. The workshop attracted a diverse set of participants which included mathematicians, computer scientists, logicians, chemists, and physicists (both theorists and experimentalists). They came from various US Government agencies (ARL, AFRL, AFSOR, NIST, ORNL, SANDIA, LANL), industry (Microsoft, IonQ), and academic institutions (University of Tennessee, University of Maryland, Harvard University, Tulane University, Virginia Polytechnic Institute and State University, Johns Hopkins University, New Mexico State University, University of California Santa Barbara, University of Illinois at Chicago, Macquarie University, University of Pennsylvania, University of Arizona).

List of talks

George Siopsis (University of Tennessee)	<i>Hybrid Quantum Classical Algorithms</i>
Norbert Matthias Linke (University of Maryland)	<i>Quantum-classical hybrid algorithms on a small trapped-ion quantum computer</i>
Emily Riehl (Johns Hopkins University)	<i>A type theory for synthetic infinity-categories</i>
John Harding (New Mexico State University)	<i>Decompositions in quantum mechanics</i>
Arthur Jaffe (Harvard University)	<i>A New Look at Pictures in Quantum Information</i>

Michael Mislove (Tulane University)	<i>Semantic Models of Quantum Programming Languages: Recursion in Linear / Nonlinear Models</i>
Shawn Xingshan Cui (Virginia Polytechnic Institute and State University)	<i>On topological quantum computing</i>
Zhenghan Wang (University of California Santa Barbara & Microsoft)	<i>Is the quantum model of computation unique?</i>
Louis H Kauffman (University of Illinois at Chicago)	<i>Topological Computing, Knots, Braids, and Higher Categories</i>
Jacek Jakowski, Oak Ridge National Laboratory	<i>Quantum chemistry benchmark for quantum computing</i>
Paul Gustafson, AFRL	<i>Dimension as a quantum statistic and applications</i>
Carl Alexander Miller, University of Maryland	<i>Picture-proofs for quantum cryptography based on categorical quantum mechanics</i>
Robert Rand, University of Maryland	<i>Formally Verifying Quantum Protocols</i>
Omar Shehab, IonQ	<i>Verification and certification of noisy intermediate-scale quantum computing</i>

Army relevance

The various talks at the workshop and the discussions among participants centered around higher category theory, topological quantum field theories (TQFTs), topological quantum information, and simulation of materials with topological properties for topological quantum computing (TQC), as long-term goals of research of interest to the Army. These are important mathematical problems of interest to defense organizations in general and the Army in particular: quantum algorithms are powerful tools to simulate quantum systems and play an important role in decision theoretic framework in adversarial situations. Moreover, due to error resistant features, topological quantum information processing is paramount to defense applications in general and specifically the Army that has significant investment in quantum information processing. Developing verifiable systems in parallel is an important endeavor since the field of quantum information is still at an early stage of development, and so incorporating the process of verification is pragmatic.

Discussion

*[Workshop participants' names are in **bold** characters.]*

The workshop explored synergies between higher category theory, TQFTs, and quantum information processing. Speakers addressed questions such as: how to apply (topological) quantum field theory to quantum information processing in the context of higher category theory; how to draw inspirations from classical information processing in applying category theories; how to frame theorem proving in

the context of quantum computation; how to connect homotopy type theory to quantum physics; the relationship between path-integral quantization and higher categories. Two main approaches have emerged: categorical quantum mechanics pioneered by Abramsky and Coecke, and the recent Quon topological picture-language for quantum information developed by **Jaffe** and collaborators. Results from these investigations will provide insights in quantum information processing and in areas of pure mathematics and logic, such as quantum probability theory, TQFTs, and Homotopy Type Theory (HoTT).

All existing tools for quantum information processing and quantum computation are low-level. The standard formulation of quantum mechanics is not sufficient for an understanding of the interplay of classical and quantum information processing. We need a higher-level approach.

There is a plethora of models of quantum computation that correspond to different physical platforms: quantum circuits (discrete variables) [1], continuous variables with both classical and quantum manipulations primarily on photonic systems [2], topological quantum computing [3], one-way (measurement based) quantum computing [4,5], adiabatic quantum computing [6] (physically realized by D-wave systems based on quantum annealing [7]), quantum-classical hybrid methods with near-term applicability [8], etc. Quantum algorithms cannot be transported between different platforms in general. We need a higher-level language.

A high-level language for quantum programs has been provided by the categorical quantum mechanics (CQM) research program which was initiated by Abramsky and Coecke [9]. CQM verifications have been given for programs such as the Steane code and cluster state arguments, and can be extended to a higher-categorical setting, developing Baez's work on a categorified notion of Hilbert space [10,11]. These methods were developed to fully topological formal verification of quantum programs and applied to quantum communication [12]. It is of interest to analyze quantum algorithms using topological formal verification and compare with CQM methods.

TQC is of particular interest because it promises to provide better control of errors. For applications to distributed computing and communication protocols, one needs to understand interactions with space-time structure which underpins the importance of understanding TQFTs. They provide intuitive descriptions of topological invariants, and potentially robust physical realizations of quantum information processing.

The study of topological invariants has been a fertile ground for interactions between mathematicians and physicists. An important example is the Jones polynomial which is a characteristic polynomial for a knot in three dimensions. Two knots are inequivalent if they have different Jones polynomials. However, the mathematical definition of the Jones polynomial is not manifestly three-dimensional. Witten provided a path-integral definition of the Jones polynomial using a three-dimensional Chern-Simons quantum field theory based on a non-abelian gauge field [13]. Even though Witten's expression does not admit a rigorous mathematical definition, it provides an intuitive geometric definition of a topological invariant. The Jones polynomial is also related to other aspects of Mathematical Physics, such as integrable lattice statistical mechanics, and two-dimensional conformal field theory and associated representations of braid groups. A quantum algorithm for computing it was found by **Kauffman** and Lomonaco [14], and was implemented using Nuclear Magnetic Resonance (NMR) [15]. At the workshop, **Kauffman** discussed a research program based on quantum knots developed by him and Lomonaco [16,17] which can be

useful in robust quantum information processing. He reviewed relationships between knots, braids and topological computing with an eye towards relationships with categories and higher categories. He also discussed anyonic topological computing — the Fibonacci model and braiding representations using Majorana fermions, how categories are related to knot invariants and to the categorification of knot invariants, and how one can, in principle, write quantum algorithms for categorified invariants such as Khovanov homology. The relationships with higher categories that he discussed are those directly related to knot diagrams, to categorifications and to higher-dimensional knots such as embedded surfaces in four-dimensional space and braiding of anyons represented by closed loops and knots.

As discussed by **Siopsis**, an important physical realization of quantum systems, including TQFTs, is provided by continuous variables (CVs) based primarily on photonic systems. CVs offer the exciting possibility of realizing quantum information processing with existing technology. In addition to Gaussian gates which have already been implemented, one needs to construct a non-Gaussian element (gate). Such gates have been proposed using the GKP scheme [18], the MFF scheme [19], and by **Siopsis** and collaborators [20]. The last one was used in an algorithm to simulate scattering amplitudes in quantum field theory [21,22], which had certain advantages over its discrete-variable (DV) counterpart [23,24], as well as algorithms for quantum machine learning [25] to be compared with their DV counterparts [26,27,28]. For practical applications, one needs to perform quantum error correction which is still in its infancy for CV systems, and will be a significant component of **Siopsis's** future research. Even though non-Gaussian errors need only Gaussian elements to correct [29,30], Gaussian error correction requires the use of non-Gaussian elements [31]. Moreover, algorithms require the use of eigenstates of quadrature operators which can only be approximately physically realized as squeezed states. Information is encoded on cat states (superpositions of coherent states) [18,32]. They can be shown to be robust against errors through implementation within circuit QED using Quantum Stochastic Differential Equations [33]. In general, they need to obey Knill-Laflamme conditions [34]. A promising alternative is a quantum error suppression scheme, such as dynamical decoupling (bang-bang control) [35,36], which has already been successfully applied to nuclear spin systems [37], photonic qubits [38], and DV quantum computing [39]. It will be interesting to explore the possibility of applying it (or a variant thereof) toward error suppression in CV quantum information processing.

A very interesting and elegant approach to categorical quantum mechanics based on a TQFT was recently proposed by **Jaffe** and collaborators and presented by **Jaffe** and **Liu** at the Workshop. The model is a topological variant of tensor networks and provides charged string braiding representation of multi-qudit gates. The mathematical equivalence of the model to the categorical formulation allows one to work with intuitive diagrams that facilitate the visualization of the flow of quantum information. It is based on pictorial representations of parafermion Pauli matrices [40]. It leads to a new way to design protocols, e.g., multi-partite quantum communication protocols [41]. **Jaffe** and collaborators have found two ways to represent qudits: two-dimensional charged strings (*two-string* model), and pairs of particle-antiparticle charged strings (*four-string* model) embedded into the interior of a three-manifold [42]. The latter leads to composites of three-manifolds and strings (*quons*) whose description is similar to topological field theories (three-dimensional TQFTs with lower-dimensional defects [43]). The quon formulation leads to a topological interpretation of the C^* -Hopf algebra relations, which are widely used in tensor networks, a simple three-dimensional representation of the CNOT gate which plays a central role in information processing, and a three-dimensional topological protocol for teleportation.

Riehl discussed interesting new methods in higher category theory she and collaborators have developed [44,45,46]. This is work that relates to quantum logic. The connection between quantum information and univalent foundations is largely unexplored territory. Modalities, as have recently been incorporated into HoTT, should be useful in differentiating between the processing of quantum vs classical information. The goal is ultimately allowing practitioners to reason synthetically about monoidal $(\infty,1)$ - and $(\infty,2)$ -categories based on sound and complete pictorial calculi for the fully extended field theories. Formidable technical higher homotopical/categorical work needs to happen before that could be achieved because in this context we no longer have commutative diagrams but homotopy coherent ones that the higher coherence data cannot just be swept under the rug. Surprisingly and remarkably, **Riehl** and Shulman recently discovered that a simplicial extension of HoTT has a sound interpretation into an ∞ -topos containing Rezk's complete Segal spaces as objects [44]. Thus, in this frame, type theory can be used to prove new theorems about $(\infty, 1)$ -categories, such as a new dependent generalization of the ∞ -categorical Yoneda lemma. **Riehl's** goal is to extend this synthetic framework to one that allows us to reason about symmetric monoidal $(\infty, 1)$ -categories and achieve something similar in the $(\infty, 2)$ -case.

Wang discussed his work on TQFTs and how it relates to the topological quantum computing (TQC) program at Microsoft [47,48]. TQC solves the fragility of the qubits at the hardware level by using topological invariants of quantum systems. Information is encoded nonlocally into topological invariants that spread into local quantities. Nature provides such topological invariants in topological phases of matter, such as the fractional quantum Hall liquids and topological insulators. The topological invariant for TQC is the ground state degeneracy in topologically ordered states with nonabelian anyons. Anyon systems are algebraically modeled by modular tensor categories [49,50] which encode 2+1—dimensional TQFTs and their dual 1+1—dimensional conformal field theories.

TQC was further discussed by **Cui** who illustrated the concept with an important class of anyons, namely, metaplectic anyons, and showed that braidings of anyons assisted by certain topologically protected measurements is universal for quantum computing [51,52]. The interest in metaplectic anyons arises from the potential physical realization in fractional quantum Hall systems. **Cui** also talked about the application of TQFTs in topology and gave a new invariant of smooth 4-manifolds of state-sum type.

Harding discussed connections between quantum logic and categorical quantum mechanics [53,54] focusing on decompositions. This is an approach to the foundations of quantum mechanics that takes focus from the structure of Hilbert spaces and places it on what one does with Hilbert spaces. For nearly any type of structure, such as a set or topological space, its direct decompositions form an orthomodular coset that allows basic aspects of quantum mechanics. This includes treatment of experiments, observables, and dynamics. These ideas extend to objects in any category with well-behaved finite products. Further aspects of quantum mechanics require additional features on the structure. The existence of a norm allows development of states, probabilities of outcomes, and expected values of observables. The existence of a tensor, or monoidal structure, allows for composite systems.

Mislove discussed his work on defining semantic models for high-level functional quantum programming languages that support recursion. He was inspired by Rios's and Selinger's language Proto-Quipper-M, a finitary circuit description language for quantum programs. The models were based on Benton's work on linear / nonlinear models for the Λ -calculus, adding an enriched structure needed to define recursion. He described his constructions, and the issues that were the most challenging, which

led to models supporting recursive terms and recursive types with soundness and computational adequacy under appropriate hypotheses. With a focus on circuit description languages, there was no notion of execution of a quantum program. For that, one needs to add dynamic lifting, so that measurements are included in the models.

Gustafson discussed several useful interpretations of the categorical dimension of objects in a braided fusion category, as well as some conjectures demonstrating the value of quantum dimension as a quantum statistic for detecting certain behaviors of anyons in topological phases of matter. He concluded that objects in braided fusion categories with integral squared dimension have distinctive properties. He illustrated these ideas in the case of metaplectic modular categories.

Formal visual reasoning in quantum information not only makes some proofs clearer, but also enables new proofs that would have otherwise been very difficult. **Miller** and collaborators started with a known visual language from categorical quantum mechanics and added new features to it in order to make a language that is amenable to quantum cryptography. They gave a diagrammatic proof of a new result (parallel self-testing of the GHZ state) and also re-proved a known result (linear randomness expansion implies unbounded randomness expansion). They computer-verified the main sequence of the second proof using the Globular software package.

Rand presented QWIRE, a tool for writing quantum programs and proving them correct. QWIRE allows us to describe quantum circuits and translate those circuits into functions on quantum states, whether represented as unit vectors or density matrices. We can then prove that these functions behave as desired, using a variety of reasoning techniques. He discussed existing approaches to verifying quantum algorithms using QWIRE as well as ongoing work to verify error-prone quantum computations.

Shehab, Jakowski and **Linke** concentrated on noisy intermediate-scale quantum computers. **Shehab** discussed verification and certification of noisy intermediate-scale quantum computing. He demonstrated a toy nuclear physics problem he and collaborators solved on a trapped-ion quantum computer with reasonable accuracy, and discussed potential ways of verifying and certifying the outcome of this computation. **Jakowski** presented a quantum chemistry benchmark which included alkali metal hydrides with various basis sets and leveraged the variational eigensolver, various error mitigation strategies and the XACC (eXtreme-scale ACCelerator) software framework. XACC follows a coprocessor machine model that is independent of the underlying quantum computing hardware, thereby enabling quantum programs to be defined and executed on a variety of QPU types through a unified application programming interface. Results provide a relevant baseline for future improvement of the underlying hardware. **Linke** focused on the experimental side of quantum computing. The first applications of near-term quantum computers will likely involve optimization tasks in a quantum-classical hybrid approach, where the quantum hardware is combined with a classical optimization algorithm. **Linke** presented recent results from such a hybrid system based on a trapped-ion quantum computer. The experimental system is comprised of a chain of $^{171}\text{Yb}^+$ ions with individual Raman beam addressing and individual readout. This fully connected processor can be configured to run any sequence of single- and two-qubit gates, making it an arbitrarily programmable quantum computer [53]. They combined this device with a classical optimizer which varied the quantum circuit parameters and interpreted the results. This hybrid system was used to demonstrate the training of shallow circuits for generative modelling [54]. They contrasted the use of different classical strategies for the training task,

in particular particle-swarm and Bayesian optimization. Methods for scaling up this system were also discussed.

Conclusion

This was a successful Workshop that attracted leading researchers who engaged in stimulating discussions. The state of the art of mathematical tools based on recent mathematical advances, such as CQM, TQFTs, HoTT, the Quon picture language, and higher category theory, which are expected to provide insights into quantum information processing, was discussed at the Workshop. Discussions among participants centered around questions such as: How do we draw inspirations from classical information processing in applying category theories? How do we frame theorem proving in the context of quantum computation? How do we connect HoTT to quantum physics? What is the relationship between path-integral quantization and higher categories? The discussions involved a synergy between mathematicians and other theorists, as well as experimentalists who can take advantage of mathematical tools, and provide feedback regarding the physical realization of mathematical tools. The Workshop was an important event toward the advancement of research in the field, and a catalyst in the development of mathematical tools related to quantum information processing of interest to the Army.

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