TWO-DIMENSIONAL INTERNAL WAVE SPECTRA

by

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ABSTRACT

Although spatial fluctuations of the sound speed in the ocean have a variety of causes, internal waves are thought to produce most of the fluctuations over horizontal scales ranging from several tens of metres to several kilometres and vertical scales ranging from one to tens of metres. Garrett and Munk have developed a space-time in internal wave model (GM-75) which predicts temperature and velocity fluctuation spectra. This has prompted us to perform a two-dimensional experiment to provide a data set which 1) is directly applicable as input to stochastic acoustic modelling, and 2) serves to check the GM-75 model. The data is obtained from a chain towed at speeds much greater than the speed of internal waves. A spectrum in horizontal-vertical wavenumber space can then be inferred. Systematic deviations from the complete (two-dimensional) model are observed in the vertical/horizontal wavenumber model into agreement with our observed two-dimensional internal wave spectrum.

Introduction

The theme of this conference is oceanic acoustic modelling. As has just been demonstrated (Munk, 1975; Flatte, 1975), oceanic internal waves can produce significant modifications to the propagation of sound. Of necessity, an acoustic model must simplify the structure of the ocean, if for no other reason than the fact that a deterministic description of the ocean is beyond the realm of possibility--the ocean is too large and its motions are too complicated. Thanks to Garrett and Munk (1972, 1975), we now have a viable internal wave model. The Garrett and Munk models are two-dimensional models of oceanic density spectra constructed from essentially one-dimensional data. The aim of this talk is to report results of a two-dimensional wave spectrum obtained from measurements in the ocean and to relate this two-dimensional spectrum to a Garrett-Munk type model.

Before going into the details of the measurements, I think it is useful to describe to this mostly acoustically-oriented audience some of the important differences between internal waves and the more familiar acoustic waves. Equation (1) is the dispersion relation for plane acoustic wave propagation.

$$\omega^2 = k^2 c^2 \tag{1}$$

This dispersion relation is particularly simple. The medium is undispersive. Except rarely, the waves can be taken to be linear. Complications arise mostly because the sound speed c is a function of space and time. Equation (2) is the dispersion relation for plane internal wave propagation at any angle to the horizontal, k_h defining the horizontal component of wave number, and k_z the vertical component.

$$\omega^2 = \frac{N^2 k_h^2 + f^2 k_z^2}{k_h^2 + k_z^2} \tag{2}$$

Two parameters appear, both frequencies; f is the inertial frequency and N is the Väisälä frequency, defined as

$$f \equiv 2\Omega \cos\lambda \tag{3}$$

$$N^2 \equiv -\frac{g}{\rho}\frac{\partial\rho}{\partial z} \tag{4}$$

In the above equations, Ω is the rotation rate of the earth, λ is the colatitude, g is the acceleration of gravity, and ρ the density. At mid-latitudes in the upper ocean the period corresponding to f is about one day and to N is tens of minutes. Internal waves are very dispersive, both in wavelength and direction; consequently phases and groups do not travel at the same speed nor even in the same direction. Additionally, important nonlinear effects occur before a typical internal wave train propagates very far. While it might seem that the complicated nature of the medium would prevent the oceanographer from providing the acoustician with a useful model, quite the opposite is the case. It is now recognized that the entire wave spectrum is nearly always saturated, at least for certain horizontal and vertical scales and within the frequency range of the inertial and Väisälä frequencies. By "saturated" is meant that no matter how strong the "driving forces" the resulting spectrum on average is the same. Oceanographers are familiar with saturated spectra because wind-driven water waves in a wide range of wavelengths and frequencies are saturated. Unlike wind-driven water waves however, the internal waves are usually isotropic in a horizontal plane. Internal waves in the ocean appear to interact in such a way that the resulting motions are describable and repeatable in a statistical sense. This contrasts with ambient acoustic noise in the ocean, which is highly variable in space and time and where discrete sources - ships and whales, for example - are identifiable contributors.

Measurements

The Naval Research Laboratory can tow a "chain" mounted with up to 70 sensors at an average vertical spacing of 1.4 m at normal towing speed-- about 3 m/sec. We mounted thermistors at the sensor locations and have analyzed in detail data from the lowermost 32 sensors, where the average vertical spacing between thermistors was 1.27 m. The nominal towing speed was 3 m/sec, which is sufficiently fast in comparison to the phase speeds of internal waves resolved by the thermistor array so that each tow leg provides 32 closely spaced essentially instantaneous horizontal slices of the oceanic temperature field. The horizontal data is averaged over approximately 100 meters so that there are 32 x 64 sample points for a 6.7 km straight line tow. The ratio of vertical to horizontal scales resolved by the array was chosen to correspond roughly to the ratio of inertial to Brunt-Väisälä frequencies, as suggested by scale analysis of the internal wave equation. During the period August 22 to September 12, 1974, the USNS LYNCH (T-AGOR-7) was deployed in the western North Atlantic between Cape Hatteras and Bermuda. The measurements were made during four basic exercises, each consisting of a box pattern of four two legs. Relevant data relating to the exercises is summarized in Table 1. Supporting environmental data was provided by salinity/temperature/depth recorder (STD) and profiling current meter casts and parachute drogues deployed at various depths. The STD data provided estimates of the average temperature and Brunt-Väisälä frequency profiles, which are required for proper data scaling. Figure 1 shows the composite data during the first exercise. The nominal centerline tow depth was 100 m, which placed the thermistor array below the seasonal thermocline in a region where the average temperature gradient and Brunt-Väisälä frequency varied only slightly over the sampling region. Our analysis of the parachute drogue and current meter data indicates an absence of significant background current shear over the range of depths of interest here.

From the temperature records we infer an internal wave amplitude spectrum $F(k_x,k_z)$, some details of which are given by Bell, et al (1975) and are not repeated here.

As noted previously, the data base which Garrett and Munk were able to draw on in constructing their model for the internal wave spectrum was inherently one-dimensional. The one-dimensional horizontal and vertical wavenumber spectra $F(k_x)$ and $F(k_z)$ are the result of collapsing the two-dimensional spectrum $F(k_x,k_z)$ onto the respective wavenumber axes:

$$F(k_x) = \int_0^\infty F(k_x,k_z)\, dk_z \tag{5}$$

$$F(k_z) = \int_0^\infty F(k_x,k_z)\, dk_x \tag{6}$$

In Fig. 2 we have plotted the one-dimensional spectra $F(k_x)$ and $F(k_z)$ based on the thermistor chain data. The spectra are properly normalized by the Väisälä frequency for direct comparison with the Garrett and Munk model. The vertical wavenumber spectral estimates have been compensated for aliasing by extrapolating the observed spectral slopes. Because the data is averaged horizontally over scales smaller than the sampling interval, the horizontal spectrum is not aliased. Also plotted in Fig. 2 are normalized one-dimensional spectral estimates drawn from published sources (Katz 1975, Hayes, et al 1975). Clearly, our data is consistent with the historical data. The Garrett and Munk model gives a fair representation of the one-dimensional spectra.

Our empirical two-dimensional spectrum is presented at the left side of Fig. 3. The presentation is in the form of a contour map. The wave-number axes are logarithmic, and the data is represented by computer generated contours of $\log (N/N_o) F(k_x,k_z)$, the units of F being $m^2(cpm)^{-2}$. Along any curve (contour), $F(k_x,k_z)$ is constant and successive contour levels are separated by one-half an order of magnitude of F. Small scale vacillations in the contours are the result of statistical variability related to the finiteness of our data sample. The spectrum is roughly symmetric about the line $k_z = k_x N/f$, which corresponds to a (45^o) diagonal passing through the point (k_x,k_z) = (0.2 cpkm, .025 cpm). This behavior is consistent with the results of a scale analysis of the internal wave equation, which indicate that vertical and horizontal dimensions in the internal wave field should scale roughly in the ratio f/N.

The corresponding Garrett and Munk model spectrum is plotted at the right side of Fig. 3. Although the model gives a fair representation of the data for $k_x \leqslant k_z N/f$, it exhibits a trend of systematic deviation from the data for $k_x \geqslant k_z N/f$. This trend is emphasized in Fig. 4, where we have contoured the ratio of the observed spectral density to the model spectral density. Figure 4 thus summarizes the differences in the measured two-dimensional spectrum from the Garrett and Munk (1975) model.

Discussion

Recalling the dispersion relation for internal waves, Eq. (2), one may note that for relatively low frequencies, $\omega^2 \ll N^2$, the dispersion relation can be put into the form:

$$\frac{N^2 k_h^2}{f^2 k_z^2} = \frac{\omega^2}{f^2} - 1 \qquad (7)$$

Invoking horizontal isotropy we have that the frequency is constant along lines of constant k_x/k_z in measured data. The deviations between the data and the Garrett and Munk (1975) model are straight lines with the same slope. The deviation is primarily a function of frequency, rather than wavenumber _per se_. That is, the GM75 model systematically underestimates the measured spectrum as ω tends down to the inertial frequency.

The effects of frequency are perhaps better examined when gross wavenumber effects have been compensated for. Setting

$$\tan\theta = \frac{f k_z}{N k_h} \qquad (8)$$

the general GM model (Garrett and Munk 1972) may be expressed in the form

$$(k_h^2 + f^2 k_z^2/N^2)^{\frac{t+1}{2}} \quad F(k_h, k_z) = \frac{A}{Nf}\left(\frac{f}{N}\right)^t \frac{\sin\theta}{\cos^n\theta \ \sin^t\theta} \qquad (9)$$

so long as horizontal and vertical coherence scales are sufficiently large. In this form, the dependence on wavenumber magnitude is compensated for, and the resultant expression may be viewed as a function of frequency, rather than wavenumber per se. The GM75 model has the following values of t and n: $t = 2.5$; $n = -2$. The empirical two-dimensional spectrum is expressed in this form in Fig. 5, using $t = 2.5$. Also plotted in Fig. 5 is the general GM model for $t = 2.5$ and several values of n. Figure 6 is an equivalent plot, but for $t = 2$, which is seen to provide a significantly better fit to the data. It is clear from these figures that $n = -2$ does not give an adequate description of the data, and that if the general GM model is to represent this data, then the parameter n must lie between 0 and 1.

Conclusions

Thermistor chain measurements of the two-dimensional (vertical/horizontal) structure of the internal wave field in the ocean have been analyzed in detail. The results of this analysis indicate that, although the GM75 (Garrett and Munk 1975) model does not provide an adequate description of the observed two-dimensional spectrum, a slightly generalized Garrett-Munk type model does. The primary difference between the generalized GM model and the earlier versions is in the structure of the spectrum near the inertial frequency. In the generalized model, displacement spectra are characterized by a weak inertial cusp. Of course, the generalized GM model still provides an adequate description of one-dimensional spectra, and further validation and refinement of the model requires other comprehensive two-dimensional measurement efforts. Even now, however, the generalized Garrett-Munk type model with some t lying between 0 and 1 can be used in modelling some aspects of acoustic propagation.

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Table 1. Cruise data

Exercise Number	Date	Location	Depth (m)	$N/2\pi$ (per hr)	dT/dz (^{o}C/m)
1	8/25/74	34°40'N 70°20'W	100	5.4	.018
2	8/29/74	31°53'N 64°45'W	100	5.0	.031
3	9/1/74	31°50'N 64°52'W	82	6.0	.044
4	9/7/74	34°28'N 72°55'W	116	6.0	.044

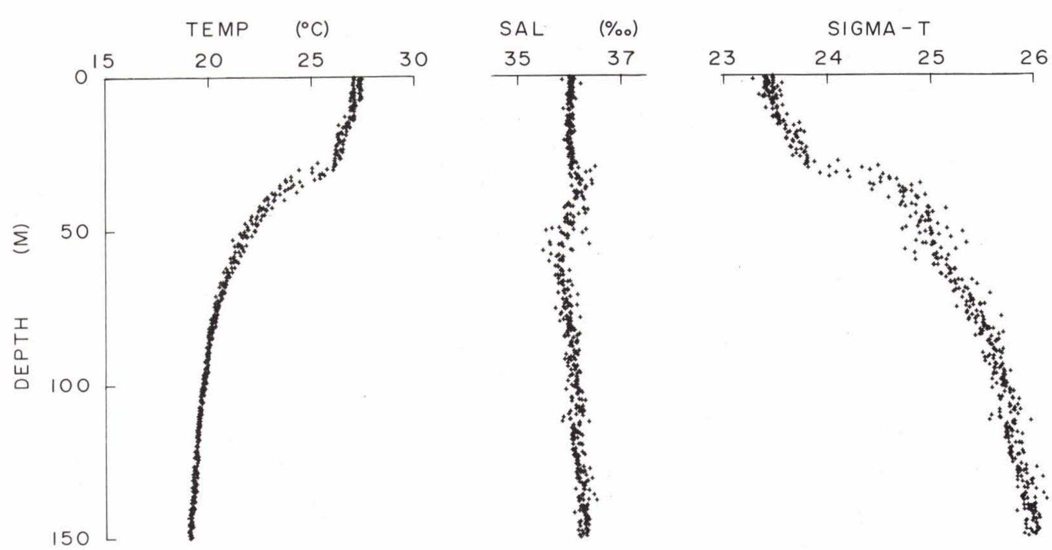

FIG. 1 COMPOSITE FROM FOUR STP STATIONS TAKEN DURING EXERCISE 1 (See Table 1)

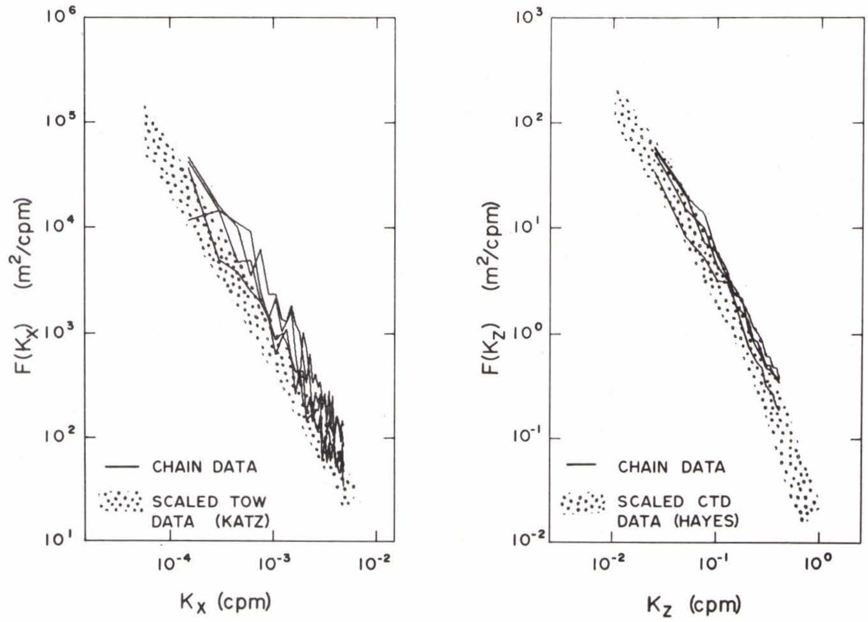

FIG. 2
ONE-DIMENSIONAL SPECTRA DERIVED FROM THE MEASURED TWO-DIMENSIONAL SPECTRUM COMPARED WITH HISTORICAL DATA

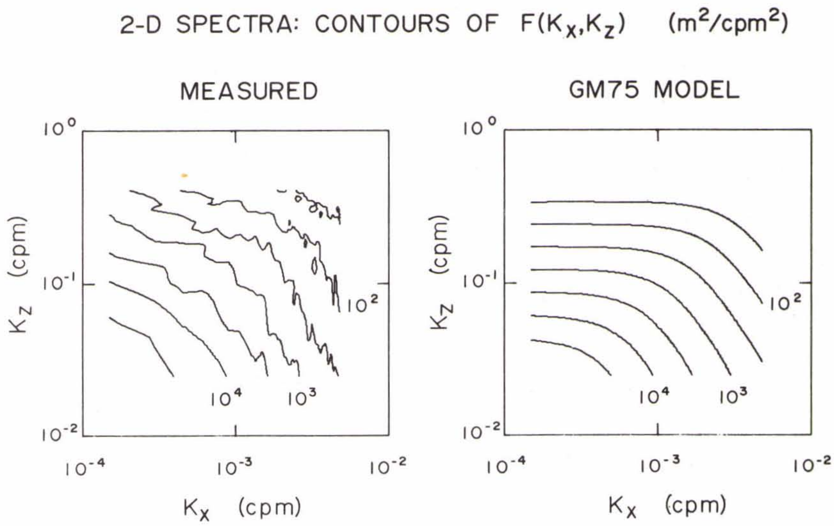

FIG. 3
AT LEFT: THE MEASURED TWO-DIMENSIONAL VERTICAL/HORIZONTAL SPECTRUM OF INTERNAL WAVE AMPLITUDE, REPRESENTED BY CONTOURS OF LOG $F(k_x,k_z)$. ALONG ANY CURVE, F IS CONSTANT; SUCCESSIVE CONTOUR LEVELS ARE SEPARATED BY $\sqrt{10}$ INCREMENTS IN SPECTRAL DENSITY. UNITS OF F ARE m^2/cpm^2

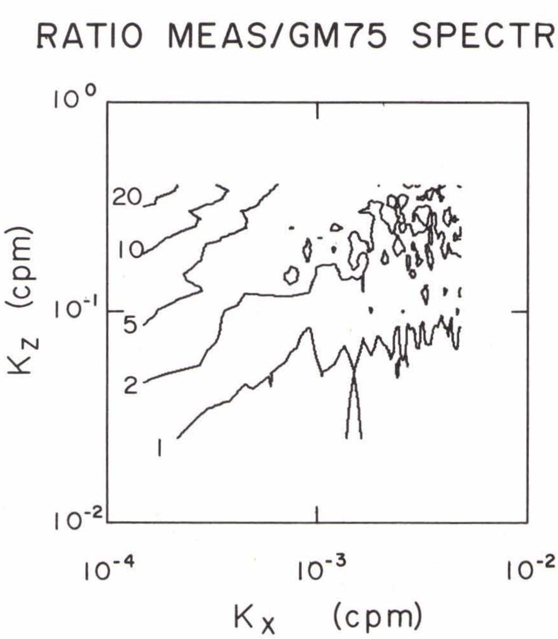

FIG. 4
THE RATIO OF THE MEASURED SPECTRUM TO THE GM75 MODEL

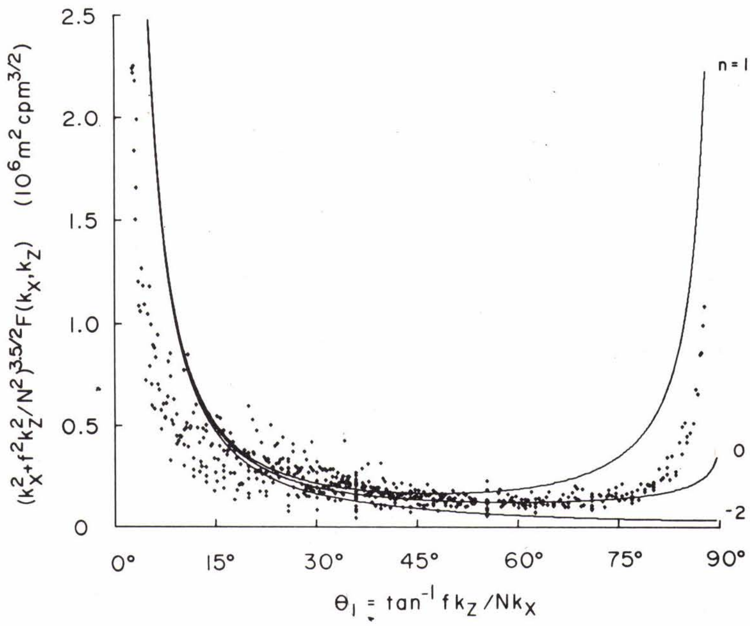

FIG. 5 THE INTERNAL WAVE SPECTRUM WITH GROSS WAVENUMBER EFFECTS ELIMINATED, ASSUMING $t = 2.5$. CURVES REPRESENT THE GENERALIZED GM MODEL FOR DIFFERENT VALUES OF n. GM75 CORRESPONDS TO n = −2.

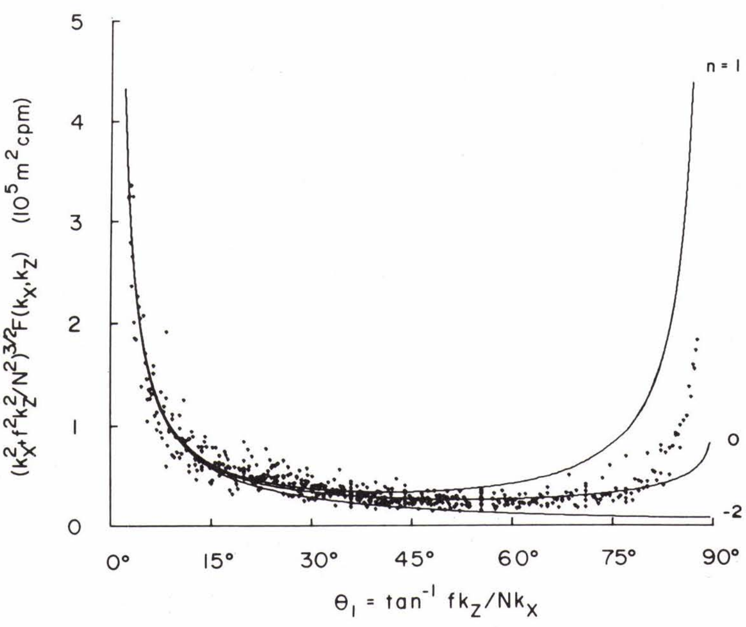

FIG. 6 SAME AS FIG. 5, BUT FOR $t = 2$. NOTE DECREASED VARIANCE OF DATA AND BETTER FIT TO MODELS.