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**SACLANT UNDERSEA
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MEMORANDUM**



**SPECTRAL INTEGRAL REPRESENTATIONS
OF SCATTERING FROM
VOLUME INHOMOGENEITIES**

K. D. LePage, H. Schmidt

July 1999

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**Spectral integral representations
of scattering from
volume inhomogeneities**

Kevin D. LePage and Henrik Schmidt

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Executive Summary: The OASES code for modeling broad-band acoustic pulse propagation in fluid-elastic stratified media has been enhanced to model scattering from the subbottom. The code now offers the ability to model reverberation time series caused both by rough surface and volume scattering processes in arbitrary waveguides. Since reverberation is often viewed as noise which limits the ability to detect and classify targets, this enhancement offers an increased capability for conducting monostatic system performance studies for ASW and MCM operations. The code also offers an enhanced ability to model reverberation time series which have been collected in SACLANTCEN experiments with the purpose of understanding the mechanisms and environmental parameters which control the reverberation process.

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Kevin D. LePage and Henrik Schmidt

Abstract: In-situ measurements of scattering strength can be obtained by analyzing the early-time, high angle reverberation from bottom and sub-bottom features. In order to provide insight into the mechanisms which cause bottom reverberation, and to their distinguishing characteristics, it is necessary to have a capability for modeling both the rough surface and the volume scattering mechanisms. For high-angle, early-time backscatter, the most appropriate approach is to use a spectral integral representation, which naturally includes the continuous spectrum important for this angular regime. A rough surface scattering theory developed earlier in this framework has provided important insights into wave scattering and penetration physics at the seafloor. Here a consistent representation for the subbottom scattering is developed and implemented in the same spectral integral code, OASES.

Keywords: reverberation ◦ backscatter ◦ volume scattering ◦ seabed inhomogeneity

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1

Introduction

Given a model of the seafloor as locally stratified with superimposed roughness at layer interfaces and sound speed and density inhomogeneities distributed between, a combination of mechanisms may be hypothesized to contribute to reverberation: rough surface scattering from the interfaces themselves, and volume scattering from the layers. In order to understand the distinguishing characteristics of these two scattering mechanisms, it is necessary to have some modeling capability, however rudimentary, which as far as possible handles the two mechanisms consistently. Since the underlying physics of the waveguide contributes significantly to the structure of the observed reverberation, it is important for any scattering theory to have the ability to include the stratification of the water column and the bottom. Also, since measurements of the backscattering strength of the seafloor are often conducted using the early time backscatter as a function of angle, the theory must properly treat steep incident and scattering angles. Both of these requirements are satisfied by scattering theories which are developed around a spectral integral representation of the incident, and by extension, scattered fields. This approach lends itself to the incorporation of arbitrary waveguide effects, and inherently treats high angle propagation properly.

There has been significant progress in the modeling of volume scattering from sediments in recent years, much of it based around simplified or approximate spectral integral representations of the Green's function. A complete survey of the field would be formidable, however a few references stand out as being particularly relevant to the work presented here. Hines [1, 2] has developed a model which treats volume scattering from both slow and fast sediments with three dimensional correlation functions in closed form, using an approximate Green's function which treats the refracted and the local (forced) backscattering paths found in fast sediments. This work is closest in spirit to the present approach since a substantial effort is directed towards modeling the Green's function accurately. The approximate Green's function and some simplifying assumptions allow Hines to obtain closed form expressions for the scattered intensity. Here an exact Green's functions is used and consequently the results are obtained as realizations. Hines work is unique because earlier work has treated volume scattering from sediments which are uncorrelated vertically, first in the isovelocity case [3, 4], and more recently for sediments with upward refracting sound speeds [5]. While the bottom model in the latter case may be more realistic than the isovelocity sediment used by Hines, the assumption of ver-

tical incoherence of the scattering volume seems restrictive. The effects of upward refraction have also been studied in detail by Holland *et al.*, [6], who showed that even weak initial gradients can significantly impact the scattering characteristics of discrete scatterers within the sediment layer. More complicated bottoms can also be treated by Tracey *et al.* [7], who have characterized the effects of volume scattering on long range propagation in a waveguide. This work also assumes that the sediment inhomogeneities are uncorrelated vertically, and the modal approach taken in this study does not conveniently handle the high angle scattering important at early time. Other studies of note have been directed toward evaluating the effects of sediment elasticity [8, 9] and toward developing a unified approach to modeling volume and rough surface backscatter [10, 11]. Elasticity is ignored in the present work, but one of the goals is to be able to investigate the uniqueness of surface and volume scattering mechanisms within the ocean waveguide environment.

A rough surface perturbation scattering theory for arbitrary waveguides has been developed systematically in the OASES code [12, 13] over the last several years [14]. Arbitrary stratification and elastic properties are handled, properly accounting for the underlying waveguide physics, and the rough surface scattering itself is treated as a first order perturbation, including elastic terms and slope terms at boundaries and even including three dimensional scattering from anisotropically rough two dimensional surfaces in some versions of the code [15]. Although the results are valid only for surfaces excursions small relative to the vertical wavelengths, the predictions from the code have proven very useful for understanding a variety of underwater acoustic phenomena, including reverberation statistics [7, 14], sound penetration [16] and the anisotropic nature of waveguide reverberation [15]. At SACLANTCEN the OASES waveguide reverberation code is in the processes of being applied to the analysis of bottom backscattering data collected on a vertical array in the Ligurian Sea [17], where it has already been found to be capable of qualitatively reproducing the time-angle dependency of the experimental backscattering data. Indeed it was the desire to investigate the relative significance of scattering due to seabed roughness and bottom inhomogeneities for this data set that motivated the development of a consistent small perturbation volume scattering theory within the OASES code.

2

Theory

The spectral integral representation of scattering from either a rough seabed or a sediment volume is equivalent to the decomposition of the scattered field into a horizontal wavenumber spectrum of up-going plane waves originating either at the seabed itself or at the volume scatterers within the bottom. As will be shown below, scattering from volume inhomogeneities can be represented by a spatial distribution of virtual point sources. Within the spectral integral framework the spectral kernel for these sources are easily derived analytically [18], with the kernels above the source depths representing upward propagating plane waves. Consequently a spectral integral representation of the reverberant field in the water column is achieved simply by spatially integrating over the virtual source distribution.

For modeling volume scattering, the wave equation is expanded in the sound speed and density fluctuations, and the total field is decomposed into a mean, or coherent, component p , and a scattered component p_s , which is assumed to be of the order of the perturbations. Formal averaging and collection of terms of equal order in the perturbation then leads to separate equations for the mean and scattered fields, totally analogous to the perturbational treatment of surface roughness [7, 10].

The ocean is here assumed to be a horizontally stratified or range-independent waveguide, where the average geo-acoustic properties depend only on depth. Also, the analysis will here be limited to 2-dimensional scattering in plane geometry. For sound speed fluctuations (assuming constant density in the scattering layer) the Helmholtz equation for the harmonic field of time dependence $\exp -i\omega t$ is, away from the sources, [1, 3]

$$\left\{ \nabla^2 + k_b(z)^2 \left(1 - 2 \frac{\Delta c}{c}(x', z') \right) \right\} (p + p_s) = 0 \quad (1)$$

Here $k_b(z) = \omega/c_b(z)$ is the depth-dependent medium wavenumber. Assuming that the non-dimensional sound speed perturbation $\Delta c/c$ is of the same order as the scattered field, the averaging procedure leads to the inhomogeneous wave equation for the scattered field

$$\left\{ \nabla^2 + k_b^2(z) \right\} p_s(x, z) = 2 \frac{\Delta c}{c}(x', z') k_b^2 p(x', z'), \quad (2)$$

the solution to which is

$$p_s(x, z) = 2k_b^2 \int dx' \int dz' G(x - x', z; z') \frac{\Delta c}{c}(x', z') p(x', z'), \quad (3)$$

where $G(x - x', z; z')$ is the Green's function for the stratified medium, which for a range-independent waveguide is dependent on the horizontal separation between the source and receiver, and their depths. Similarly, for density fluctuations, the wave equation is to first order in the non-dimensional density fluctuation

$$\left\{ (\rho + \Delta\rho) \nabla \cdot \left(\rho^{-1} \left(1 - \frac{\Delta\rho}{\rho} \right) \nabla \right) + k_b^2(z) \right\} (p + p_s) = 0, \quad (4)$$

which for background density constant in the layer yields the following inhomogeneous wave equation for the scattered field

$$\left\{ \nabla^2 + k_b^2(z) \right\} p_s(x, z) = \left(\nabla \frac{\Delta\rho}{\rho} \right) \cdot \nabla p(x', z'), \quad (5)$$

with the solution

$$p_s(x, z) = \int dx' \int dz' G(x - x', z; z') \left(\nabla \frac{\Delta\rho}{\rho}(x', z') \right) \cdot \nabla p(x', z') \quad (6)$$

2.1 Volume scattering in iso-velocity medium

In a bottom halfspace with constant background wavenumber k_b , the integral representation the unperturbed field p , consisting solely of down-going waves below a single interface at depth $z = 0$, is

$$p(x', z') = \int dk \tilde{p}(k) T_{12}(k) e^{ik_b z'} e^{ikx'}, \quad (7)$$

where $\tilde{p}(k)$ is the downgoing incident wave spectrum in horizontal wavenumber k above the interface. $T_{12}(k)$ is the transmission coefficient into the lower halfspace, and k_{bz} is the vertical wavenumber, defined as

$$\begin{aligned} k_{bz} &= \sqrt{k_b^2 - k^2} & k &\leq k_b \\ &= i\sqrt{k^2 - k_b^2} & k &> k_b, \end{aligned} \quad (8)$$

ensuring that the radiation condition is satisfied for $z' \rightarrow \infty$. The gradient of the incident field, required in Eq. (6), is easily found from Eq. (7) as

$$\nabla p(x', z') = \int dk (ik\mathbf{e}_x + ik_{zb}\mathbf{e}_z) \tilde{p}(k) T_{12}(k) e^{iz'k_{bz}} e^{ikx'}. \quad (9)$$

Similarly, the Green's function from the scatterers to the depth $z = 0^-$ just above the interface is [18]

$$G(x - x', 0^-; z') = -\frac{i}{4\pi} \int dq q_{bz}^{-1} T_{21}(q) e^{iz'q_{zb}} e^{iq(x-x')}, \quad (10)$$

where q is the scattered horizontal wavenumber, q_{bz} is the corresponding vertical wavenumber in the bottom, and $T_{21}(q)$ is the spectral transmission coefficient from the lower halfspace to the upper halfspace.

Using the representations (7), (9) and (10) in Eqs. (3) and (6) and integrating over x' (see Appendix A for details) then yields an integral representation the scattered field. For sound speed inhomogeneity the result for a receiver at depth $z < 0$ in the water is

$$p_s(x, z) = -ik_b^2 \int dk \tilde{p}(k) T_{12}(k) \int dq q_{bz}^{-1} T_{21}(q) e^{iqx} e^{-izq_{wz}} \int dz' \frac{\widetilde{\Delta c}}{c}(q - k, z') e^{iz'(q_{bz} + k_{bz})}, \quad (11)$$

where $\widetilde{\Delta c}/c$ represents the horizontal Fourier transform of the sound speed perturbation, and q_{wz} is the vertical wavenumber in the water. Assuming the sediment to be lossless ($\Im\{k_b\} \equiv 0$), the contribution from the *propagating* components of the incident and scattered fields ($[k, q] \leq k_b$) to the depth integral of Eq. (11) can be simply evaluated by a two-dimensional Fourier transform of the sound speed inhomogeneities, with the result

$$p_s(x, z) = -2\pi i k_b^2 \int dk \tilde{p}(k) T_{12}(k) \int dq \frac{e^{-izq_{wz}}}{q_{bz}} T_{21}(q) \frac{\widetilde{\Delta c}}{c}(q - k, -q_{bz} - k_{bz}) e^{iqx}, \quad (12)$$

For lossy sediments, and for cases where the contribution from the *evanescent* components ($[k, q] > k_b$) of the incident and scattered field is significant, the scattered field no longer has a direct interpretation in terms of the depth Fourier transform of the sound speed inhomogeneities, and the depth integral in Eq. (11) must be evaluated explicitly.

For density inhomogeneities the spectral integral representation of the scattered field is

$$p_s(x, z) = -i \frac{k_b^2}{2} \int dk \tilde{p}(k) T_{12}(k) \int dq \frac{e^{-izq_{wz}}}{q_{bz}} T_{21}(q) e^{iqx} \int dz' \left(\frac{k^2 - kq}{k_b^2} + \frac{ik_{bz}}{k_b^2} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta \rho}}{\rho}(q - k, z') e^{iz'(q_{bz} + k_{bz})} \quad (13)$$

Ignoring loss, the depth-derivatives and the depth integral may also here be formulated as a two-dimensional Fourier transform for the propagating incident and scattered spectrum, yielding

$$p_s(x, z) = -\pi i \int dk \tilde{p}(k) T_{12}(k) \int dq \frac{e^{-izq_{wz}}}{q_{bz}} T_{21}(q) e^{iqx} \left(k_b^2 - kq + k_{bz}q_{bz} \right) \frac{\widetilde{\Delta \rho}}{\rho}(q - k, -q_{bz} - k_{bz}). \quad (14)$$

Once again, the depth integrals must be explicitly evaluated using Eq. (13) when the vertical wavenumbers are complex, i.e. for the evanescent components of the incident and scattered fields, and when the sediment is lossy.

Notice that the wavenumber dependent factor in the kernel of Eq. (14) reaches its maximum for k and q large and of opposite sign, corresponding to horizontal backscatter, or for k_{bz} and q_{bz} large, corresponding to vertical backscatter. This enhanced backscatter is a distinguishing characteristic of scattering from density inhomogeneities.

The theory presented in this section has several restrictions. The three most important are that it is a two dimensional theory, that small perturbations in sound speed and density have been assumed, and that the single scattering approximation has been taken. The first assumption can be eliminated in an analogous way to which two dimensional rough surface scattering results have recently been extended to three dimensions using the Fourier spectral approach [15]. The multiple scattering effects between scatterers can be important and have been the subject of recent investigations [19, 20]. The present work does not attempt to address the effects of multiple scattering between the individual scatterers, but does include multiple scattering of the scattered field at the waveguide boundaries.

2.2 Scattering from layers with background sound speed gradient

Sediment gradients can substantially influence the characteristics of volume scattering contributions [5, 6]. In order to treat volume scattering from such sediments, it is possible to replace the spectral integral representations for the incident field in Eq. (7) and the scattered field Green function in Eq. (10) with their equivalent forms for layers with *pseudo-linear* sound speed profile, $k_b^2(z) = az + b$. For such a medium the depth separated Helmholtz equation

$$\left\{ \frac{\partial^2}{\partial z^2} + (k_b^2(z) - k^2) \right\} p = 0$$

may be written [18] as an Airy partial differential equation

$$\left\{ \frac{\partial^2}{\partial Z^2} - Z \right\} p = 0,$$

where

$$Z(z, k) = a^{-2/3} (k^2 - (az + b)).$$

In this case two independent vertical eigenfunctions are the Airy functions

$$Ai(Z(z, k)),$$

which have the characteristic of a standing wave with a turning point at depth $z = -b/a$, and the linear combination [21]

$$\begin{aligned} & Ai(Z(z, k)) - iBi(Z(z, k)) \\ &= 2e^{-2\pi i/6} Ai(Z(z, k)e^{2\pi i/3}), \end{aligned}$$

which have the characteristic of a wave propagating from the turning point depth, and therefore satisfying the radiation condition in the direction of decaying sound speed. For propagation into scattering layers with positive sound speed gradients, Eq. (7) takes the form

$$p(x', z') = \int dk \tilde{p}(k) \tilde{T}_{12}(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} e^{ikx'}, \quad (15)$$

while for propagation into layers with negative gradients, the result is

$$p(x', z') = \int dk \tilde{p}(k) \tilde{T}_{12}(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(0, k)e^{2\pi i/3})} e^{ikx'}, \quad (16)$$

where $\tilde{T}_{12}$ is the transmission coefficient into a halfspace with sound speed gradient $c(z) = \omega/\sqrt{az + b}$.

Using the integral representation for the Green's function in this type of refracting medium, e.g. derived in Ref. [18], the field scattered into the upper halfspace by a virtual source in an upward refracting sediment is

$$\begin{aligned} G(x - x', 0^-; z') = \int dq \left\{ \frac{\tilde{T}_{21}(q)}{4\pi} e^{iq(x-x')} \right. \\ \left. \frac{2a^{-1/3} Ai(Z(z', q)) Ai(Z(0, q)e^{2\pi i/3})}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))} \right\}, \quad (17) \end{aligned}$$

while for a downward refracting sediment the field radiated into the upper halfspace is, immediately above the interface at $z = 0$

$$\begin{aligned} G(x - x', 0^-; z') = \int dq \left\{ \frac{\tilde{T}_{21}(q)}{4\pi} e^{iq(x-x')} \right. \\ \left. \frac{2a^{-1/3} Ai(Z(z', q)e^{2\pi i/3}) Ai(Z(0, q))}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))} \right\}. \quad (18) \end{aligned}$$

These equations replace Eq. (10) for refractive media.

2.3 Volume scattering in general layered media

The volume scattering formulations derived above for a fluid halfspace can be easily generalized to horizontally stratified media within the *Direct Global Matrix* (DGM) approach to the solution of the depth-separated wave equation in such environments [18]. Consequently they are straightforwardly implemented into the OASES code for seismo-acoustic propagation in arbitrarily stratified ocean waveguides [13]. In analogy to the handling of physical sources, the contributions from the virtual scattering sources within a layer are introduced by superimposing their free-field radiation with a set of homogeneous solutions, the amplitudes of which are found by satisfying the boundary conditions of continuous stress and particle velocity at all interfaces in the stratification. As is the case for physical sources, only upgoing waves contribute above the virtual sources, while below only downgoing waves contribute.

Adopting the Born approximation [22], the up- and downgoing waves of the unperturbed (or mean,) field are used to determine the wavenumber spectrum of upgoing scattered waves at the upper interface and downgoing scattered waves at the lower interface using the integral expressions derived above, but without accounting for the transmission through the interfaces. The up- and downgoing mean field amplitudes $\tilde{a}^{\pm}(k)$ in each layer are provided by first solving for the field excited by the desired source distribution in the absence of any scattering. For a layer of thickness D , the upgoing scattered field at wavenumber q at the upper interface then is, treating sound speed and density inhomogeneity scattering together,

$$\begin{aligned} \tilde{p}_s(q, 0^+) = & -\frac{i}{2q_{bz}} \int dk \int dz' \tilde{a}^+(k) e^{iz'(q_{bz}-k_{bz})} e^{iDk_{bz}} \\ & \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq - ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta \rho}}{\rho}(q-k, z') \right) \\ & - \frac{i}{2q_{bz}} \int dk \int dz' \tilde{a}^-(k) e^{iz'(q_{bz}+k_{bz})} \\ & \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq + ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta \rho}}{\rho}(q-k, z') \right), \end{aligned} \quad (19)$$

while at the lower interface the downgoing scattered field is

$$\begin{aligned} \tilde{p}_s(q, D^-) = & -\frac{i}{2q_{bz}} \int dk \int dz' \tilde{a}^+(k) e^{i(D-z')(q_{bz}+k_{bz})} \\ & \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq - ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta \rho}}{\rho}(q-k, z') \right) \\ & - \frac{i}{2q_{bz}} \int dk \int dz' \tilde{a}^-(k) e^{iz'(k_{bz}-q_{bz})} e^{iDq_{bz}} \\ & \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq + ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta \rho}}{\rho}(q-k, z') \right), \end{aligned} \quad (20)$$

The interface conditions satisfied by OASES at fluid interfaces are the continuity of pressure and vertical velocity. The volume scattering contributions to the boundary conditions for pressure at each interface are given directly by Eqs. (19) and (20). The corresponding contributions to the velocity boundary condition at the upper and lower interfaces are

$$\begin{aligned}\tilde{v}_{\mathbf{n}_s}(q, 0^+) &= -\frac{i}{2\omega\rho} \int dk \int dz' \tilde{a}^+(k) e^{iz'(q_{bz}-k_{bz})} e^{iDk_{bz}} \\ &\quad \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq - ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta\rho}}{\rho}(q-k, z') \right) \\ &\quad - \frac{i}{2\omega\rho} \int dk \int dz' \tilde{a}^-(k) e^{iz'(q_{bz}+k_{bz})} \\ &\quad \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq + ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta\rho}}{\rho}(q-k, z') \right) \quad (21)\end{aligned}$$

and

$$\begin{aligned}\tilde{v}_{\mathbf{n}_s}(q, D^-) &= \frac{i}{2\omega\rho} \int dk \int dz' \tilde{a}^+(k) e^{i(D-z')(q_{bz}+k_{bz})} \\ &\quad \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq - ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta\rho}}{\rho}(q-k, z') \right) \\ &\quad + \frac{i}{2\omega\rho} \int dk \int dz' \tilde{a}^-(k) e^{iz'(k_{bz}-q_{bz})} e^{iDq_{bz}} \\ &\quad \times \left(2k_b^2 \frac{\widetilde{\Delta c}}{c}(q-k, z') + \left(k^2 - kq + ik_{bz} \frac{\partial}{\partial z'} \right) \frac{\widetilde{\Delta\rho}}{\rho}(q-k, z') \right) \quad (22)\end{aligned}$$

The inhomogeneous boundary conditions for the scattered field coming from a refractive layer are given in Appendix C.

The implementation into OASES of this formulation is straightforward. First, the standard code is used to determine the up- and downgoing plane wave amplitudes of the mean, or coherent, field. In a second pass, these amplitudes are then used together with a random realization of the perturbation and Eqs. (1) - (18) to provide a virtual source contribution at the upper and lower boundary of the scattering layer, and the standard DGM solution followed by the evaluation of the wavenumber integral then yields the realization of the scattered field. The generality of the formulation has been maintained in the implementation, allowing for combined sound speed and density inhomogeneities with arbitrary two-dimensional spatial distributions. These distributions correspond to two-dimensional Gaussian or power-law spectra with user specified anisotropy in the vertical plane.

3

Results

In order to validate the volume scattering formulation and the implementation into OASES, plane wave scatter cross sections from a layer of scatterers embedded in a uniform background sound speed of 1500 m/s were computed and compared to theoretical closed form solutions (derived in Appendix B). The results are shown in Fig. 1 for 360 and 3600 Hz respectively at various angles of incidence. Both the OASES results and the closed form results were frequency averaged over a 200 Hz bandwidth, in order to smooth the OASES results, which are computed for one realization of scatterers. The results indicate that the OASES implementation closely approximates the known closed form solution, with the forward scattered resonance condition and the angular dependence faithfully reproduced. The remaining disagreement, in general less than 2 dB, may be attributed to details of the OASES implementation which are inconsistent with the plane wave and infinite aperture scattering hypothesis of the theoretical result. These are: 1) OASES uses a complex frequency formulation to reduce temporal aliasing [18], which causes the frequency domain solution to experience a small decay in the plane wave amplitude as it propagates through the layer, and 2) the use of a finite depth scattering layer to approximate the exact Bragg scattering resonance, on which the closed form theory depends. The finite aperture has the effect of slightly increasing backscattering cross section, as the sidelobes of the forward scattered resonance can pollute the backscattered response. Of course for actual scattering simulations from scattering layers of finite thickness, this is a real effect.

3.1 Example 1: Volume scattering from a slow isospeed layer

The first example illustrates broad band volume scattering from a thick and slow sediment layer with the properties shown in Fig. 2. The sound speed in the sediment is lower than in the water and the subbottom, which together with a density contrasts creates a waveguide for the scattered field. The scatterers have a horizontal correlation length of 10 m, a vertical correlation length of 2 m, a zero degree skew angle, and are distributed according to a power law with a fractal dimension of 2.5 [23]. The rms non-dimensional sound speed perturbation is 1/1500 and the rms non-dimensional density perturbation is 7/1500. The perturbations are 100% correlated. The environment is ensonified by a line source which is situated at a height of 28 m above the sediment-water interface. The source has a Hanning

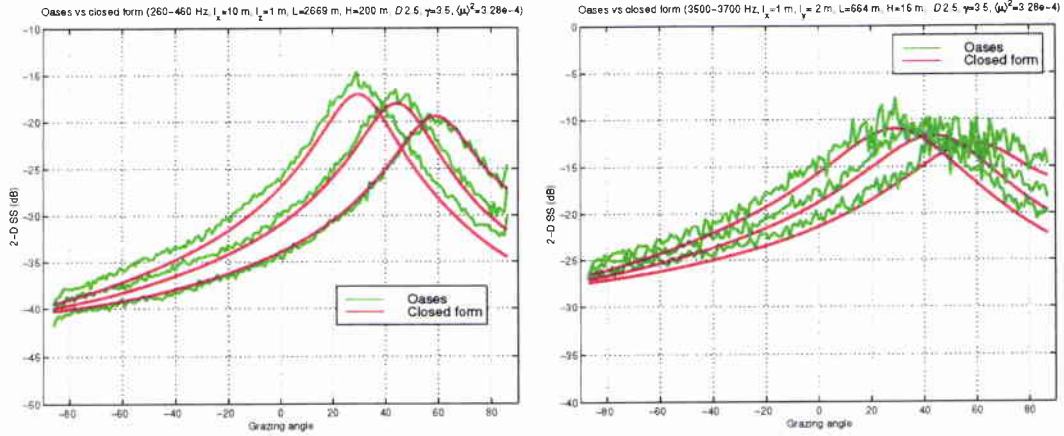


Figure 1 Benchmark results for the spectral integral representation of volume scattering from sound speed and density inhomogeneities. The red curves are closed form values for the two dimensional volume scattering strength and the green curves are OASES results. Both curves are averaged over a 200 Hz band. The panel on the left shows results for 30, 45 and 60 degrees incidence at 360 Hz. The correlation length scales are $l_x = 10$ m and $l_z = 2$ m, with a dip angle $\phi = 0$ from horizontal. The right panel illustrates the results at 3600 Hz, with $l_x = 1$ m and $l_z = .2$ m. In both examples the non-dimensional variance of the sound speed inhomogeneities is $3.28E - 4$, and the density and sound speed inhomogeneities are 100% correlated with a proportionality constant $\gamma = \frac{1}{2} \frac{\Delta \rho}{\rho} \frac{c}{\Delta c} = 3.5$.

shaded spectrum with a center frequency of 500 Hz and 200 Hz of bandwidth. To simplify the interpretation of results, the source is prevented from exciting negative wavenumbers. The resulting scattered field, shown in the form of snapshots of the envelope in dB in Fig. 3 at various times after the shot, is therefore asymmetric, which eases the identification of the forward and backscattered components of the field. With this type of incident field, scattered energy seen to the left of the source is *by definition* backscattered, while the field to the right of the source may be either forward or backscattered.

The results show a predictable but interesting evolution in time and space. To the far right of the scattered field at any particular time step is the “forward scattered” field which is the strongest component. The only portion of this forward scattered component which is truly forward scattered, *i.e.* traveling in the same direction as the incident field, is the scattered field right along the sediment water interface. The scattered waves form a cylindrically spreading field which is most intense and time compressed near the sediment-water interface, while becoming more spread out in time at steeper scattering angles. More detailed analysis shows that the

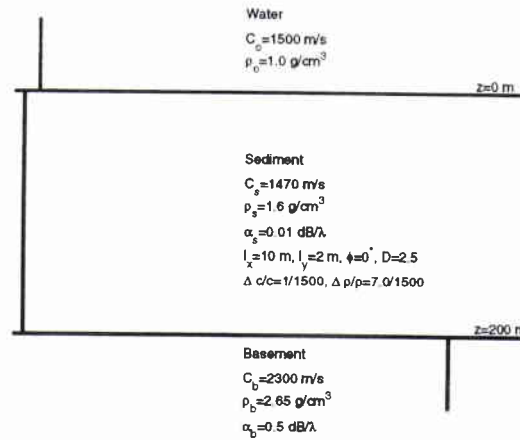


Figure 2 The environment for Example 1. A slow sediment layer 200 m thick is sandwiched between the upper halfspace of water and a dense, fast basement.

vertical extent of the most intense part of the scattered field is defined by the vertical correlation length scale of the scatterers; if the vertical correlation length scale is very short then more vertical “smearing” of the forward scattered energy is expected.

Behind the forward scattered field is a band of high scattered energy the temporal extent of which is defined by the difference in the total path distance between the scattering from the top and the bottom of the scattering layer, divided by the average sound speed. In this case, since the sound speed in the sediment is almost the same as in the water column, the result is that for receivers at zero range the thickness of this band corresponds to twice the thickness of the sediment layer, or 400 m. The thickness decreases as the receiver range is increased, consistent with the fact that the travel time difference between the two paths decreases.

To the left of the source, the field is purely backscattered. The backscattered field is characterized by much lower levels of energy, which are determined by the correlation length scales of the scatterers and their spectral characteristics. For scatterers with correlation lengths small with respect to the wavelengths of the incident field, backscattered levels are higher. For scatterers correlated on scales very long with respect to an incident wavelength, the backscattered levels are much lower, with the level depending on the correlation characteristics of the scatterers. Since in this example the scatterers have a correlation length which is large with respect to a wavelength horizontally ($l_x \equiv 10\text{m} > \lambda \equiv 3\text{m}$), the backscattered levels at low grazing angles are observed to be lower than they are at higher angles, where the wavelength is long with respect to the vertical correlation length of the scatterers

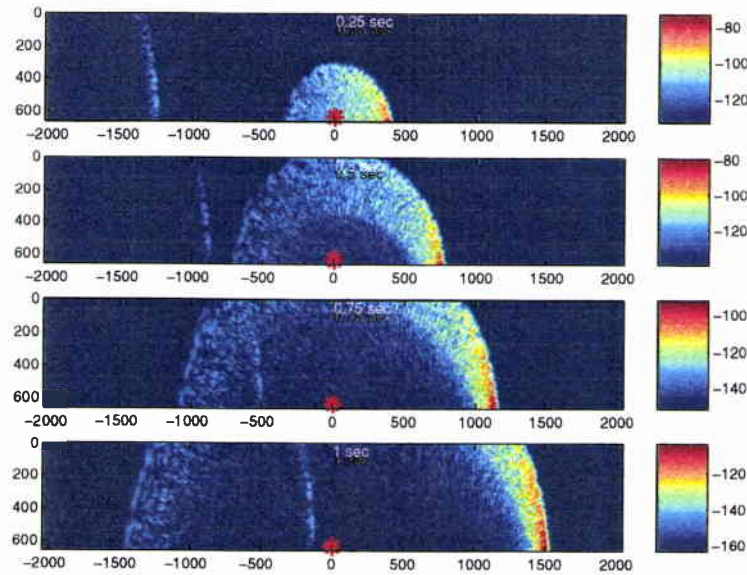


Figure 3 *Spatial snapshots of the evolution of the scattered field for Example 1. The center frequency is 500 Hz with 200 Hz of bandwidth. The source (*) is placed 28 m above the water-sediment interface, and is designed to radiate only right-traveling waves. Hence the asymmetrical nature of the result. The region to the left of the source by definition has only left-traveling waves; to the right of the source both left and right-traveling waves exist. The region of strongest scattering is the forward scattered regime, to the far right and near the sediment-water interface.*

($\lambda > l_z \equiv 2\text{m}$.) This effect would be accentuated for scatterers with Gaussian spatial distribution properties. Finally, note that there is a slight asymmetry to the arrival times of the backscattered field. This asymmetry is caused by the fact that the backscattered field is retarded relative to the forward scatter field by a spatial extent roughly corresponding to the twice the spatial extent $c\tau$ of the incident pulse. This characteristic asymmetry in the scattered field is most easily observed in the bottom panel of Fig. 3 near the seabed.

If the scattering layer has the same background properties as the enclosing upper and lower halfspaces, then the features just described complete the description of the scattered field. However, if there is partial reflection of the scattered field at the upper and lower interface, multiple scattered arrivals will exist. The amplitude of the multiples rapidly decays due to the smaller than unity reflection coefficients at the interfaces, and the geometrical (cylindrical) spreading of the scattered wavefronts.

Since a finite reflection coefficient exists in this example at both the upper and lower

interface, the scattered field calculations show late time components consistent with the reverberation of the incident and scattered field within the sediment layer. The scattered multipath is observable as reduced amplitude arrivals following thick band of primary scattered arrivals in Fig. 3.

The numerical artifact seen propagating from the left to the right in all panels of Fig. 3 is the time aliased forward scattered field. By the time this artifact reaches the source it suffers a 54 dB attenuation due to the use of a complex frequency offset of $\Delta\omega$ in the OASES calculations. It is important in any Fourier synthesis application to be aware of aliasing features. In wave propagation problems, these aliasing features propagate like late time waves and their effects have to be ameliorated by the use of the complex frequency during the calculation of the Green functions [13] and by restricting attention to the valid time window of the synthesis $T_o \leq T \leq T_o + 2\pi/\Delta\omega$ [18]. When complex frequency techniques are not used time aliasing can only be avoided by ensuring the time window is long enough to contain the entire response, requiring $T_o \equiv 0$ and $\Delta\omega \ll 2\pi/\tau$, where τ is the maximum temporal extent of the response of the waveguide, including scattering, for the desired receiver position. By using complex frequency integration, the requirements to $\Delta\omega$ can be reduced significantly [18].

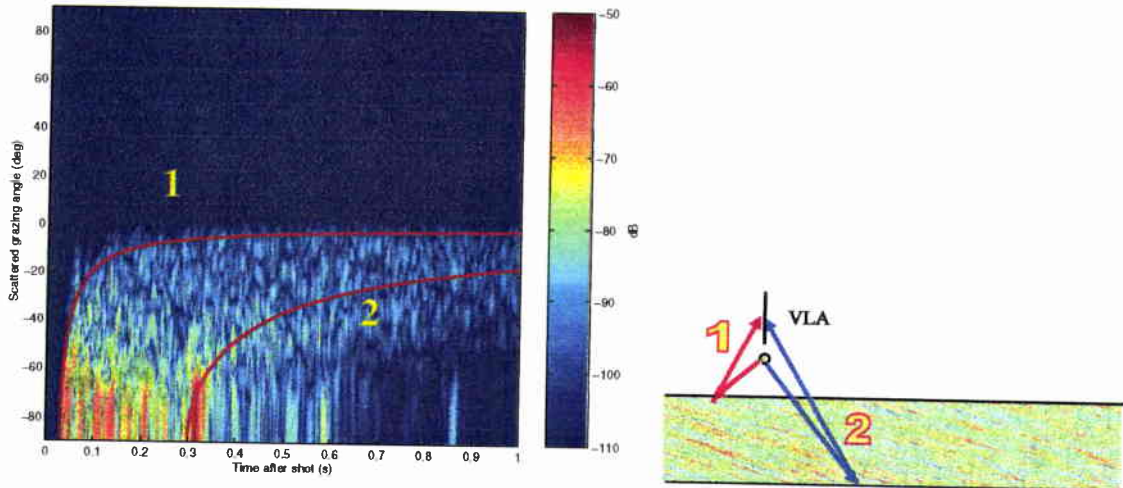


Figure 4 Time-angle plot of scattered field received on a VLA at $r = 0$ for the environment of Example 1. Two traces are superimposed, the upper red curve represents the time-angle evolution of returns from scatterers on the sediment-water interface (ray 1 on the ray diagram,) while the lower red curve represents return from the sediment-basement interface (ray 2 on the ray diagram.) Scattered returns which lie below the lower curve are scattered multipath from within the sediment layer.

The interpretation of the results is aided significantly by array processing of the

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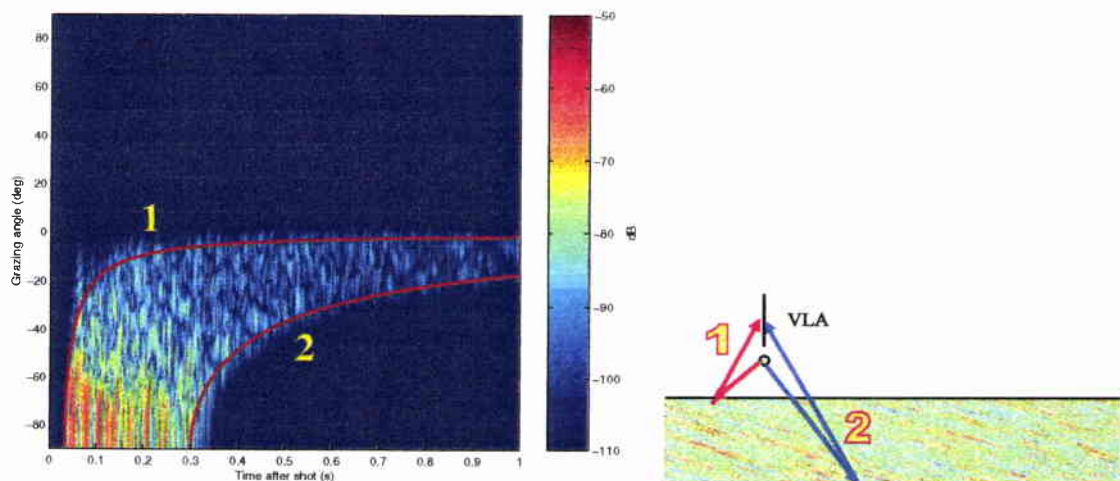


Figure 5 *Time-angle plot of volume scattering in a homogeneous background. Here all the scattered energy lies between the scattering angle trajectories for the top (ray 1 on the ray diagram) and the bottom of the scattering layer (ray 2.) There is no multipath.*

synthetic data. Fig. 4 shows the response of a Hanning shaded Bartlett beamformer applied to a 50 m aperture vertical array placed 25 meters above the sediment. Overlaid on the figure are the time-angle trajectories of hypothetical scatterers placed on the water-sediment interface (the upper curve, evolving to low angles more quickly in time,) and the sediment-basement interface (lower curve). The ray-paths associated with these two trajectories are shown schematically in the lower diagram, with (1) identifying the scattering from inhomogeneities close to the seabed, and (2) corresponding to scattering originating close to the basement. The primary scattered field lies between these two trajectories. The thickness of the band in angle is determined by the thickness of the scattering layer. The abruptness of the temporal evolution is determined by the proximity of the source and the receiver array to the sediment water interface. The later scattered multiples fill in the lower right quadrant in the time-angle plane. With homogeneous background properties this region would be empty, as is shown in Fig. 5. The simplicity of the interpretation relies to some degree on the assumption that the sediment layer has the same sound speed as the upper halfspace.

For comparison, Fig. 6 shows the beamformer response for the same background medium, but where the scattering is generated by seabed roughness. The most obvious difference is the distinct arrival nature of the roughness scattering, as opposed to the angle-smeared nature of the volume scattering result. In the absence of the sediment-basement interface, only the uppermost beam-time trajectory (1) would

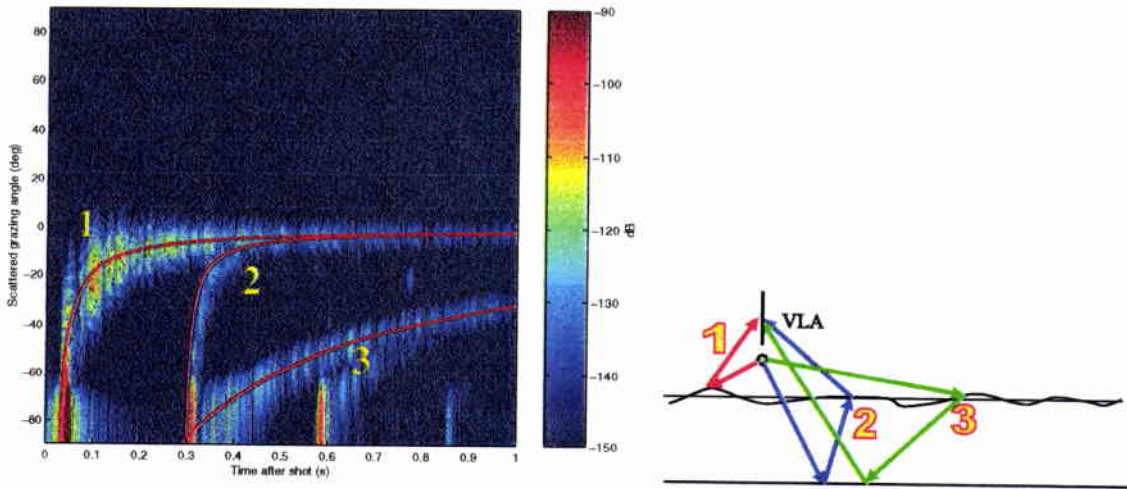


Figure 6 Time-angle plot for rough surface scattering for the environment in Example 1. There are three main branches to the response. The most quickly evolving trajectory (corresponding to ray 1 on the ray diagram) is the direct backscatter from the water-sediment interface. The second trajectory (corresponding to ray 2) is the scattered field excited by the basement reflected wave, while the lower right trajectory (ray 3) is the downward scattered field from the direct blast which has reflected off the basement and come back to the receiver.

exist, representing scattering directly from the seabed back to the receiving array. In the presence of the sediment-basement interface, two additional trajectories are evident. The trajectory (2) is caused by scattering of the basement-reflected incident field when it passes through the sediment-water interface, while the slowest evolving trajectory (3) is the downward scattered energy which has been reflected by the basement and propagated back into the upper halfspace. These results together with those shown in Fig. 4 confirm that volume and surface scattering mechanisms should be resolvable in experiments which follow the basic configuration of a monostatic source receiver geometry with a vertical aperture, as long as the vertical aperture is sufficient to resolve the sediment layer thickness, as has been proposed by Holland [17].

3.2 Example 2: Volume scattering from a sediment layer with a strong gradient

It is well established that associated with the gradual consolidation process significant gradients may exist in the geoacoustic properties near the seabed. In spite of this most previous models of scattering from the sediment volume, with the exception of the work by Mourad *et al.* [5] and Holland *et al.* [6], has ignored the

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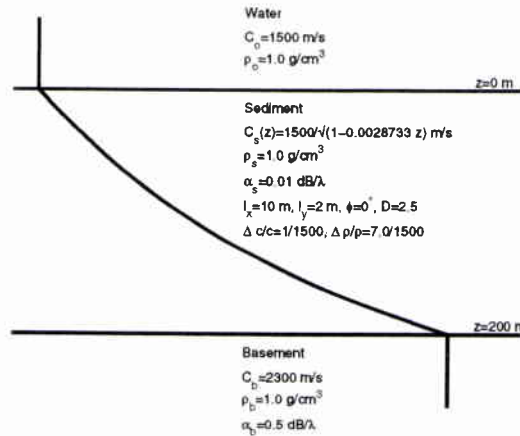


Figure 7 *The environment for Example 2. On a homogeneous background of uniform density, a increase in sound speed from 1500 m/s to 2300 m/s occurs over a depth of 200 m. There is 0.01 dB/ λ attenuation in the sediment and 0.5 dB/ λ in the basement.*

associated refractive effects. By implementing the present formulation into OASES, the effect of such gradients and their interaction with adjacent layers may be investigated. The following example is used to illustrate this capability of the model and to demonstrate the significance of such refractive features to the spatial and temporal characteristics of the backscattering. The environmental model of the previous example is modified to have a strong gradient of 4 s^{-1} in the sediments, as shown in Fig. 7. Further, to emphasize the gradient effects, the density is initially held constant throughout the stratification, and the sound speed is assumed continuous at the two interfaces, although results will also be shown where these restrictions have been removed.

In the presence of strong gradients it is necessary to incorporate the waveguide effects for both the incident and scattered fields in the analysis. The refraction of the incident field turns out to be extremely important for understanding and interpreting the scattered field results. This is because in strongly upward refracting sediments the incident field may not ensonify the entire sediment layer. Instead the bulk of the incident field turns and exits the sediment layer at a range defined by the thickness of the layer and its gradient. In this example this range is only 750 m from the source. The strong gradient causes upward refraction which is responsible for re-directing propagating incident and scattered energy with shallow enough grazing angles (in this case less than roughly 50°) into the upper halfspace. At the exit point of the deepest diving incident energy, the forward scattered field overtakes

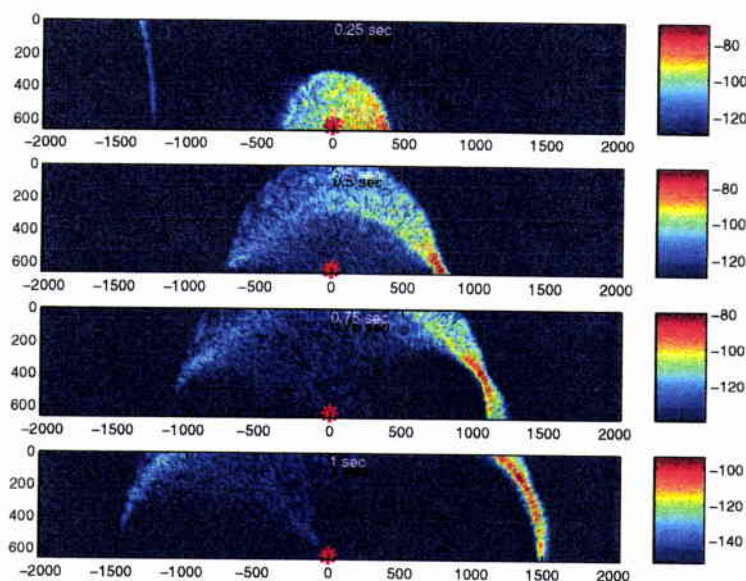


Figure 8 *Spatial snapshots of the evolution of the scattered field in Example 2. The deep diving incident field entering the sediment at grazing angles less than 50 degrees turns and exits the sediment layer at a range of approximately 750 m. The forward scattered field is therefore comprised of two strong contributions, one associated with grazing incidence along the sediment-water interface and the second associated with downward scattered energy which is turned by the strong gradient in the sediment and exits upward. The backscattered field suffers focusing, so that beyond times of 0.9 s, corresponding to the lowest panel, the backscattered field in the vicinity of the source has been refocused and is less evident at nearby receiver positions. This refocusing of the backscattered energy to the source position at late time is a unique phenomena of scattering from layers with strong gradients.*

the grazing incidence forward scattered field traveling along the sediment water interface, causing a focusing phenomena in both the incident field, and the scattered field, as can be seen for the lower three panels of Fig. 8. This strong focusing causes a strong amplitude peak of the forward scattered energy in the vicinity of the exit point of the incident field.

At later times the gradient-related phenomena become even more interesting. In cases with negligible reflection at the upper and lower interfaces of the sediment layer, there are two spatial regions where scattered energy is excluded from propagating. The first is in the forward scattered direction directly behind the region of strong focusing, while the second is in the backscattered direction at low grazing angles following the backscattered halo. The cause of the insignificant scattering amplitudes

in these regions is the strong effect of the sediment gradient on the scattered field, which must travel back down through the sediment before it can be observed at the source range. From the lowermost panel of Fig. 8, it is clear that at approximately 1 s after the source firing, the backscattered field is exiting the sediment layer only in the vicinity of the source range, at a grazing angle of approximately 50° . In fact a travel time of about 0.9 s is consistent for round-trip deep refracted backscatter from the scattering volume near the sediment-water interface at the range of 750 m, where the incident field exits the sediment and radiated into the upper halfspace.

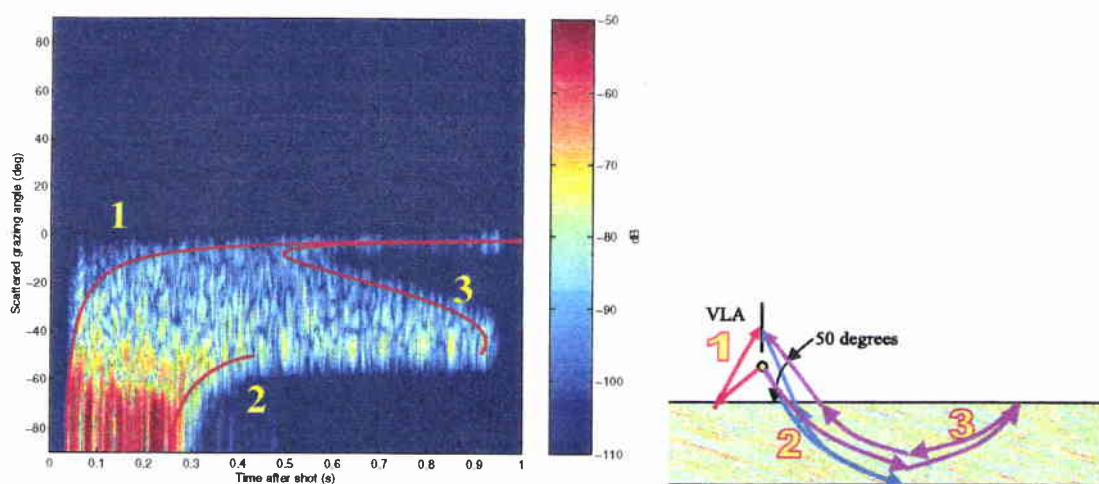


Figure 9 Time-angle plot for backscatter received at $r = 0$ in Example 2. For times less than approximately 0.5 s, the scattered energy is confined between the time-angle trajectory of scatterers on the sediment-water interface (ray 1,) and the scatterers on the bottom of the sediment layer (ray 2.) At later times the direct-path backscatter from the uppermost scatterers persists (ray 1,) but the bulk of the response is confined between the time-angle trajectory of scatterers on the top of the sediment layer which are observable as deep refracted backscatter (ray 3) and the angle of total internal reflection of the sediment, which is approximately 50 degrees. At approximately 0.9 s, the deep refracted backscatter cuts off as all downward propagating backscattered energy escapes into the lower halfspace for scatterers beyond the corresponding range of approximately 750 m. These scattering characteristics are very distinguishable from those of an iso-speed scattering layer.

As in the previous example, the background waveguide effects become particularly evident by using array processing techniques. Using the same vertical array geometry as before, the Bartlett beamformer response is shown in Fig. 9, together with a ray diagram identifying new refracted trajectories. An angular focusing of the scattered field at later times for sediments with strong sound speed gradients is evident. For times beyond approximately 0.5 s after the source firing, the scattered field separates

into two branches in the time-angle plane. The most significant branch of the late scattering response lies between the time-angle trajectory of scatterers at the top of the sediment layer which are observed through a deep refracted path, (3,) and the angle of total internal reflection of the sediment layer, which in this case is approximately 50° . At approximately 0.9 s, the deep refracted backscatter from the uppermost scatterers within the sediment layer arrive at this angle, and is sharply cut off for later times. The upper branch of the late response, associated with the forced backscatter of the shallowest volume scatterers within the sediment, is a much weaker contributor to the total scattered response at late time, which conforms to the upper boundary of the trajectory (1) familiar from the iso speed sediment scattering in the first example. Beyond 0.9 s, this is the only backscattering path observable.

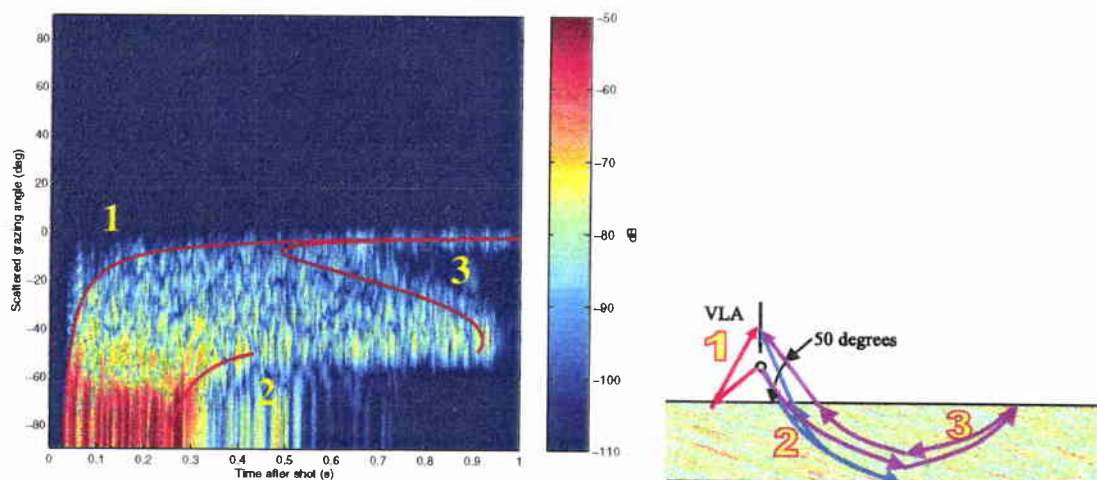


Figure 10 Time-angle plot for Example 1 with the sound speed gradient in the sediment of Example 2. Here the effects of late-time resonance of the scattered field within the sediment layer are evident in the lower right-hand side of the plot. Otherwise the results are nearly identical to those in Fig. 9.

At early time the lower boundary of the backscatter angular response is determined by the time-angle trajectory of scatterers at the bottom of the sediment layer (2.) These scatterers no longer contribute to the backscatter after approximately 0.45 s, which is why this trajectory is shown cut off. At later times progressively shallower scatterers are excluded from contributing to the to the deep refracted path (3,) until finally the scatterers at the top of the layer are excluded and the response cuts off. Comparison of these results with the iso speed results in Fig. 4 clearly indicates that in the absence of interface discontinuities, the late time bifurcation and cutoff of the scattering is a dominant characteristic of scattering from a thick sediment with a strong sound speed gradient. The presence of this behavior, confirmed by

the overlayed refractive raytrace results, is a unique identifying characteristic of upward refracting sediments. Provided further analysis confirms the observability and robustness of this phenomenon under real ocean conditions, it may provide opportunities for 'through-the-sonar' environmental assessment concepts.

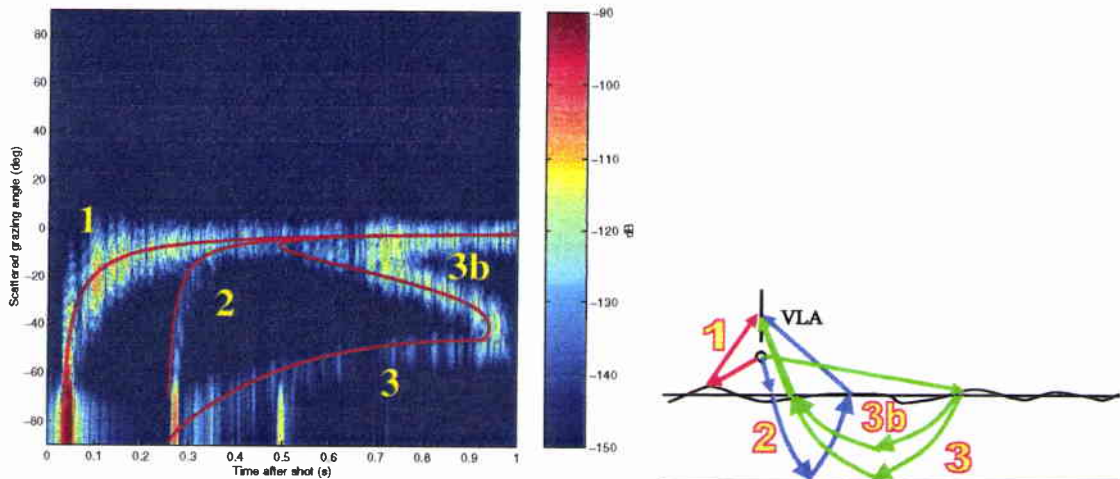


Figure 11 Time-angle plot of rough surface scattering from the environment of Example 1 with the layer sound speed gradient from Example 2. Three branches of the backscattered response correspond to those for the iso-speed layer in Fig. 6, but the arrival times of the reflected branches (rays 2 and 3) are significantly modified by the refractive properties of the waveguide. In addition a new upper branch to the refracted arrival (ray 3b) is found. This branch corresponds to backscattered paths which have a turning point within the sediment and therefore do not reflect off the sediment-basement interface.

The combined effects of gradients and realistic interface contrasts are illustrated in Fig. 10, showing the beam-time response from the scattering layer in the case where the density contrasts from Example 1 are restored. In this case multipath high angle arrivals at intermediate time are observed persisting for grazing angles greater than 50° for times between 0.3 and 0.6 s. However, at later times the bifurcation and cutoff associated with the strong gradient is still the dominant feature.

The analogous result for rough surface scattering in the presence of the gradient is shown in Fig. 11. The overlayed time trajectories are similar to those in Fig. 6, but indicate the differences that refraction can make. While the general characteristics of the rough surface scattering result in Fig. 6 persist, an additional branch of arrival (3,) denoted (3b,) is noted. This branch corresponds to deep refracted backscattering. This path suffers an internal reflection, corresponding to a ray turning point, within the sediment layer. As with volume scattering, the presence of

the deep refracted branch is an identifying characteristic of rough surface scattering from interfaces of strongly refractive sediments.

4

Conclusions

By implementing a perturbation formulation for scattering from volume inhomogeneities within the DGM framework of the OASES code, a versatile and general simulation capability has been developed for seabed scattering in general stratified ocean waveguides, with consistent handling of 2-D scattering from interface roughness and sound speed and density inhomogeneities within iso-velocity and gradient layers. The new modeling capability has been validated by comparison with closed form expressions for the far-field scattering cross section, and the numerical efficiency of the code is comparable to the existing rough surface scattering capability.

Results show that the background characteristics of the waveguide can be as important as the scattering characteristics of the volume scatterers themselves in determining the temporal and angular characteristics of the scattered field. Multipath propagation of the scattered field can be an important effect, whether it is the reverberation of the scattered field within the scattering layer, or the more general refractive properties of the waveguide between the scattering volume and the source and receiver, which are often important in real waveguides.

The layered media formulation presented here has several advantages over a traditional two-halfspace approach. The ability to include free surface effects when evaluating experimental configurations or results is a valuable asset, even though for clarity such effects were not included in the examples presented here. Also, cases where volume scatterers are dispersed through a general layered media can be evaluated more easily with the present approach. Finally, the treatment of layer gradients is more appropriate under the multilayered approach. This is because half-spaces with pseudo-linear gradients have the unphysical property that all energy, no matter how high the propagation angle, is returned from the depth $z = -a/b$, where the sound speed becomes infinite. With the multilayered approach, gradient layers can be truncated and a radiation condition applied before the caustic depth is reached.

The results shown here indicate that with the volume scattering extension OASES has become a more flexible tool for evaluating scattering and reverberation scenarios in realistic ocean environments. Results show that volume scattering is quite distinguishable for surface scattering for the cases investigated. On the other hand it is likely that the two mechanisms become increasingly difficult to separate from

one another as the pulse width becomes large with respect to the layer thickness. However, such effects can now be explored in a unified model framework, with the flexibility that affords.

The modeling described here is the simplest generally used to describe volume reverberation. However, essential characteristics, including two-dimensional, anisotropic roughness correlation structure of the scatterers, as well as sound speed and density scattering, and scattering in refractive layers, have all been treated so as to make the model as useful as possible to those wishing to design experiments or evaluate scenarios or data sets.

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Annex A

Evaluation of integral transform representations of volume scatter

The spectral representations of the incident and scattered field are given in Eqs. (9) and (10). The spectral representation of the volume inhomogeneities themselves is

$$\frac{\Delta c}{c}(x', z') \equiv \int dl \frac{\widetilde{\Delta c}}{c}(l, z') e^{ilx'}. \quad (23)$$

The scattered field in the water at depth $z < 0$ caused by scatterers at depth z' is then given as sum of two integrals over the scatterer distributions x' , representing forward and backward scatter, respectively, and the wavenumbers of the incident and scattered fields and of the scatterer distribution, k, q and l , respectively,

$$\begin{aligned} p_s(x, z) = & -\frac{ik_b^2}{2\pi} \int_{-\infty}^x dx' \int dq \frac{T_{21}(q)}{q_{bz}} e^{iq_{bz}z'} e^{-iq_{wz}z} e^{iq(x-x')} \\ & \int dk \tilde{p}(k) T_{12}(k) e^{ik_{bz}z'} e^{ikx'} \int dl \frac{\widetilde{\Delta c}}{c}(l) e^{ilx'} \\ & - \frac{ik_b^2}{2\pi} \int_x^{\infty} dx' \int dq \frac{T_{21}(q)}{q_{bz}} e^{iq_{bz}z'} e^{-iq(x-x')} \\ & \int dk \tilde{p}(k) T_{12}(k) e^{ik_{bz}z'} e^{-iq_{wz}z} e^{ikx'} \int dl \frac{\widetilde{\Delta c}}{c}(l) e^{ilx'}. \end{aligned} \quad (24)$$

Equation (24) may be rearranged as follows

$$\begin{aligned} p_s(x, z) = & -\frac{ik_b^2}{2\pi} \int dq \int dk \int dl \frac{T_{12}(k) T_{21}(q)}{q_{bz}} \tilde{p}(k) e^{i(q_{bz}+k_{bz})z'} e^{-iq_{wz}z} e^{iqx} \frac{\widetilde{\Delta c}}{c}(l) \\ & \int_{-\infty}^x dx' e^{i(k+l-q)x'} \\ & - \frac{ik_b^2}{2\pi} \int dq \int dk \int dl \frac{T_{12}(k) T_{21}(q)}{q_{bz}} \tilde{p}(k) e^{i(q_{bz}+k_{bz})z'} e^{-iq_{wz}z} e^{-iqx} \frac{\widetilde{\Delta c}}{c}(l) \\ & \int_x^{\infty} dx' e^{i(k+l+q)x'}. \end{aligned} \quad (25)$$

The integrals over x' in Eq. (25) may be evaluated as follows

$$\begin{aligned} \int_x^{\infty} e^{iqx'} dx' & \equiv \lim_{\sigma \rightarrow \infty} \int_x^{\infty} e^{-x'^2/2\sigma^2} e^{iqx'} dx' \\ & \equiv \lim_{\sigma \rightarrow \infty} \sqrt{\pi\sigma^2/2} e^{-q^2\sigma^2/2} \left(1 - \sqrt{1 - e^{-x^2/2\sigma^2}} \right) \\ & \equiv \pi \delta(q), \end{aligned} \quad (26)$$

thus the integrals over x' and subsequently l in Eq. (25) may be evaluated to yield

$$\begin{aligned}
 p_s(x, z) = & -\frac{ik_b^2}{2} \int dq \int dk \frac{T_{12}(k)T_{21}(q)}{q_{bz}} \tilde{p}(k) e^{i(q_{bz}+k_{bz})z'} e^{-iq_{wz}z} e^{iqx} \frac{\widetilde{\Delta c}}{c} (q-k) \\
 & - \frac{ik_b^2}{2} \int dq \int dk \frac{T_{12}(k)T_{21}(q)}{q_{bz}} \tilde{p}(k) e^{i(q_{bz}+k_{bz})z'} e^{-iq_{wz}z} e^{-iqx} \frac{\widetilde{\Delta c}}{c} (-q-k)
 \end{aligned} \quad (27)$$

Since $T_{21}(q) \equiv T_{21}(-q)$, Eq. (27) may be written

$$p_s(x, z) = -ik_b^2 \int dq \int dk \frac{T_{12}(k)T_{21}(q)}{q_{bz}} \tilde{p}(k) e^{i(q_{bz}+k_{bz})z'} e^{-iq_{wz}z} e^{iqx} \frac{\widetilde{\Delta c}}{c} (q-k) , \quad (28)$$

which is the result in Eq. (11). The result in Eq. (13) for density inhomogeneities is achieved in a similar manner.

Annex B

Expressions relating integral transform representations of volume scattering to theoretically derived volume scattering cross-sections

Given an upward propagating spectrum of plane waves $\tilde{s}$ excited by volume scatterers, the pressure received at grazing angle θ and range r is

$$\begin{aligned} p(r, \theta) &= \int dk \tilde{s}(k) e^{ik_{wz}r \sin \theta} e^{ikr \cos \theta} \\ &= \int dk \tilde{s}(k) e^{i\Theta(k)}. \end{aligned} \quad (29)$$

This integral may be evaluated asymptotically about the stationary phase point

$$\begin{aligned} 0 &\equiv \frac{\partial \Theta}{\partial k} \\ &= -k k_{wz}^{-1} r \sin \theta + r \cos \theta \\ \Rightarrow k &= k_w \cos \theta. \end{aligned} \quad (30)$$

At the stationary point, the curvature of the phase function Θ is evaluated as

$$\begin{aligned} \frac{\partial^2 \Theta}{\partial k^2} &= -k_{wz}^{-1} r \sin \theta + k^2 k_{bz}^{-3} r \sin \theta \\ &= \frac{r}{k_w \sin^2 \theta}. \end{aligned} \quad (31)$$

Thus Eq. (29) may be approximately evaluated as

$$\begin{aligned} p(r, \theta) &\simeq \int dk \tilde{s}(k_w \cos \theta) e^{i\Theta(k_w \cos \theta)} e^{i\frac{1}{2}(k - k_w \cos \theta)^2 \frac{\partial^2 \Theta}{\partial k^2}} \Big|_{k=k_w \cos \theta} \\ &= e^{ik_w r} \tilde{s}(k_w \cos \theta) e^{i\pi/4} \int dk e^{-\frac{k^2}{2} \frac{\partial^2 \Theta}{\partial k^2}} \\ &= e^{ik_w r} \tilde{s}(k_w \cos \theta) e^{i\pi/4} \sqrt{2\pi / \frac{\partial^2 \Theta}{\partial k^2}}. \end{aligned} \quad (32)$$

The scattering cross section in the far field for 2-D geometry is [24]

$$\sigma_{ss} \equiv \frac{I(r, \theta) r}{I_i L D},$$

where L is the length of the scattering volume and d is it's depth. Using Eq. (32), the cross section for a spectrum of upgoing planewaves $\tilde{s}$ scattered by a unity amplitude incident plane wave ($I_i \equiv 1$) is

$$\sigma_{ss}(\theta) = \frac{2\pi |\tilde{s}(k_w \cos \theta)|^2 k_w \sin^2 \theta}{LD}. \quad (33)$$

For scattered spectra $\tilde{S}_n$ which have been evaluated numerically on a discrete grid with spacing Δk , Eq. (33) takes the form

$$\sigma_{ss}(\arccos(n\Delta k/k_w)) = \frac{|\tilde{S}_n|^2 (k_w^2 - (n\Delta k)^2)}{k_w D}, \quad (34)$$

since in this case $\tilde{S}_n$ must be reduced to the approximate spectral density

$$\hat{\tilde{s}}(n\Delta k) \simeq \tilde{S}_n / \Delta k,$$

and the length of the scattering volume is also related to the spectral resolution by the relation

$$L \equiv 2\pi / \Delta k.$$

The closed for solution for the volume scattering cross section of a 2-D volume follows the 3-D derivation of Yamamoto [25]. The scattered pressure at receiver vector $\mathbf{x}$ is given as the surface integral over the depth and range of the scattering volume

$$\begin{aligned} p(\mathbf{x}) = & \frac{i}{4} \int_S dS H_o^{(1)}(k_w |\mathbf{x} - \mathbf{x}'|) \left\{ \left(2k_w^2 \frac{\Delta c}{c}(\mathbf{x}') + k_w^2 \frac{\Delta \rho}{\rho}(\mathbf{x}') \right) p_i(\mathbf{x}') \right. \\ & \left. + \nabla \cdot \left(\frac{\Delta \rho}{\rho}(\mathbf{x}') \nabla p_i(\mathbf{x}') \right) \right\}. \end{aligned} \quad (35)$$

Equation (35) may be rewritten in a form which does not require the evaluation of the gradient of the density fluctuations $\Delta \rho$. Utilizing Green's first identity

$$\begin{aligned} & \int_S dS \nabla \cdot \left(\frac{\Delta \rho}{\rho}(\mathbf{x}') \nabla p_i(\mathbf{x}') \right) H_o^{(1)}(k_w |\mathbf{x} - \mathbf{x}'|) \\ & + \frac{\Delta \rho}{\rho}(\mathbf{x}') \nabla p_i(\mathbf{x}') \cdot \nabla H_o^{(1)}(k_w |\mathbf{x} - \mathbf{x}'|) \\ & = \oint_C \frac{\Delta \rho}{\rho}(\mathbf{x}') \mathbf{n} \cdot \nabla p_i(\mathbf{x}') H_o^{(1)}(k_w |\mathbf{x} - \mathbf{x}'|), \end{aligned} \quad (36)$$

and recognizing that in the absence of sources and away from the density inhomogeneities, the line integral $\oint_C dC \mathbf{n} \cdot \nabla p_i \equiv 0$, we obtain the equivalent expression

$$\begin{aligned} p(\mathbf{x}) = & \frac{i}{4} \int_S dS \left\{ 2k_w^2 \frac{\Delta c}{c}(\mathbf{x}') + k_w^2 \frac{\Delta \rho}{\rho}(\mathbf{x}') \right\} p_i(\mathbf{x}') H_o^{(1)}(k_w |\mathbf{x} - \mathbf{x}'|) \\ & - \frac{i}{4} \int_S dS \frac{\Delta \rho}{\rho}(\mathbf{x}') \nabla p_i(\mathbf{x}') \cdot \nabla H_o^{(1)}(k_w |\mathbf{x} - \mathbf{x}'|). \end{aligned} \quad (37)$$

To obtain the scattering cross section, we utilize the large argument approximations for the Hankel functions

$$H_o^{(1)}(k_w |\mathbf{x} - \mathbf{x}'|) \simeq e^{ik_w |\mathbf{x} - \mathbf{x}'| - i\pi/4} \sqrt{2/\pi k_w |\mathbf{x} - \mathbf{x}'|},$$

and the Fraunhofer approximation for their argument

$$k_w |\mathbf{x} - \mathbf{x}'| \simeq k_w r - k_w \mathbf{e}_r \cdot \mathbf{x}'$$

and for the incident field

$$p_i(\mathbf{x}') \simeq e^{ik_w \mathbf{e}_i \cdot \mathbf{x}'}$$

Together these approximations transform Eq. (10) to the form

$$p(\mathbf{x}) = \frac{ik_w^2}{4} e^{ik_w r + \pi/4} \sqrt{2/\pi k_w r} \int dS \left\{ 2 \frac{\Delta c}{c}(\mathbf{x}') + \frac{\Delta \rho}{\rho}(\mathbf{x}') (1 - \mathbf{e}_r \cdot \mathbf{e}_i) \right\} e^{ik_w (\mathbf{e}_i - \mathbf{e}_r) \cdot \mathbf{x}'}. \quad (38)$$

Squaring Eq. (38) and taking the formal ensemble average, we obtain the average intensity

$$\langle I(r, \mathbf{e}_r, \mathbf{e}_i) \rangle = \frac{k_w^4}{2\pi} \frac{\Gamma^2}{k_w r} \int_S dS \int_{S'} dS' \langle \Delta c(\mathbf{x}') \Delta c(\mathbf{x}'') \rangle e^{i(\mathbf{e}_i - \mathbf{e}_r) \cdot (\mathbf{x}' - \mathbf{x}'')}, \quad (39)$$

where

$$\Gamma = 1 + \frac{1}{2} \frac{\Delta \rho}{\rho} \frac{c}{\Delta c} (1 - \mathbf{e}_r \cdot \mathbf{e}_i),$$

and the density and sound speed perturbations are assumed 100% correlated, *i.e.*

$$\frac{\Delta \rho}{\rho} = f\left(\frac{\Delta c}{c}\right).$$

Assuming spatial homogeneity and sufficient scattering volume to resolve the spectrum, Eq. (39) can be expressed in terms of the sound speed inhomogeneity spectrum

$$\langle I(r, \mathbf{e}_r, \mathbf{e}_i) \rangle = 2\pi k_w^3 \Gamma^2 S(k_w (\mathbf{e}_i - \mathbf{e}_r)) DL/r, \quad (40)$$

and expected value of the scattering cross section for a unit *intensity* plane wave is then

$$\langle \sigma_{ss} \rangle = 2\pi k_w^3 \Gamma^2 S(k_w (\mathbf{e}_i - \mathbf{e}_r)). \quad (41)$$

Annex C

Volume scattering in general layered media from inhomogeneities within a refracting layer

For upward refracting media, the inhomogeneous boundary condition at the upper interface due to sound speed inhomogeneities alone (equivalent to the first line of Eq. (19) is

$$\begin{aligned} \bar{p}_s(q, 0^+) = -ik_b^2 \int dk \int dz' \left(\bar{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \bar{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \quad (42) \\ \frac{\widetilde{\Delta c}}{c}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)) Ai(Z(0, q)e^{2\pi i/3})}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))}, \end{aligned}$$

while for upper interface density inhomogeneity scattering term one obtains

$$\begin{aligned} \bar{p}_s(q, 0^+) = -\frac{i}{2} \int dk \int dz' (k^2 - kq) \left(\bar{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \bar{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \\ \frac{\widetilde{\Delta \rho}}{\rho}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)) Ai(Z(0, q)e^{2\pi i/3})}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))} \\ -\frac{i}{2} \int dk \int dz' \left(\bar{a}^-(k) \frac{Ai'(Z(z', k))}{Ai(Z(0, k))} + \bar{a}^+(k) \frac{Ai'(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \quad (43) \\ \frac{\widetilde{\Delta \rho'}}{\rho}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)) Ai(Z(0, q)e^{2\pi i/3})}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))}. \end{aligned}$$

At the lower interface the vertical velocity inhomogeneous boundary condition associated with sound speed inhomogeneities is

$$\begin{aligned} \bar{p}_s(q, D^-) = -ik_b^2 \int dk \int dz' \left(\bar{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \bar{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \quad (44) \\ \frac{\widetilde{\Delta c}}{c}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)e^{2\pi i/3}) Ai(Z(D, q))}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))}, \end{aligned}$$

while for density inhomogeneity scattering it is

$$\begin{aligned}
 \tilde{p}_s(q, D^-) = & -\frac{i}{2} \int dk \int dz' (k^2 - kq) \left(\tilde{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \\
 & \frac{\widetilde{\Delta\rho}}{\rho}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)e^{2\pi i/3}) Ai(Z(D, q))}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))} \\
 & -\frac{i}{2} \int dk \int dz' \left(\tilde{a}^-(k) \frac{Ai'(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai'(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \\
 & \frac{\widetilde{\Delta\rho'}}{\rho}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)e^{2\pi i/3}) Ai(Z(D, q))}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))}.
 \end{aligned} \quad (45)$$

The four expressions for the inhomogeneous vertical velocity boundary condition, commensurate with Eqs. (42) through (45), are, for the upper interface sound speed inhomogeneity term

$$\begin{aligned}
 \tilde{v}_{\mathbf{n}_s}(q, 0^+) = & \frac{k_b^2}{\omega} \int dk \int dz' \left(\tilde{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \\
 & \frac{\widetilde{\Delta c}}{c}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)) Ai'(Z(0, q)e^{2\pi i/3})}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))},
 \end{aligned} \quad (46)$$

while for density inhomogeneity scattering term the vertical velocity inhomogeneity at the upper interface is

$$\begin{aligned}
 \tilde{v}_{\mathbf{n}_s}(q, 0^+) = & \frac{1}{2\omega} \int dk \int dz' (k^2 - kq) \left(\tilde{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \\
 & \frac{\widetilde{\Delta\rho}}{\rho}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)) Ai'(Z(0, q)e^{2\pi i/3})}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))} \\
 & + \frac{1}{2\omega} \int dk \int dz' \left(\tilde{a}^-(k) \frac{Ai'(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai'(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \\
 & \frac{\widetilde{\Delta\rho'}}{\rho}(q - k, z') \frac{2a^{-1/3} Ai(Z(z', q)) Ai'(Z(0, q)e^{2\pi i/3})}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))}.
 \end{aligned} \quad (47)$$

At the lower interface the vertical velocity inhomogeneity caused by downgoing scattered waves from sound speed inhomogeneities is

$$\tilde{v}_{\mathbf{n}_s}(q, D^-) = \frac{k_b^2}{\omega} \int dk \int dz' \left(\tilde{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \quad (48)$$

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$$\frac{\widetilde{\Delta c}}{c}(q-k, z') \frac{2a^{-1/3} Ai(Z(z', q)e^{2\pi i/3}) Ai'(Z(D, q))}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))},$$

while for density inhomogeneity scattering the inhomogeneous vertical velocity at the lower interface is

$$\begin{aligned} \tilde{v}_{\mathbf{n},}(q, D^-) &= \frac{1}{2\omega} \int dk \int dz' (k^2 - kq) \left(\tilde{a}^-(k) \frac{Ai(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \\ &\quad \frac{\widetilde{\Delta \rho}}{\rho}(q-k, z') \frac{2a^{-1/3} Ai(Z(z', q)e^{2\pi i/3}) Ai'(Z(D, q))}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))} \\ &+ \frac{1}{2\omega} \int dk \int dz' \left(\tilde{a}^-(k) \frac{Ai'(Z(z', k))}{Ai(Z(0, k))} + \tilde{a}^+(k) \frac{Ai'(Z(z', k)e^{2\pi i/3})}{Ai(Z(D, k)e^{2\pi i/3})} \right) \quad (49) \\ &\quad \frac{\widetilde{\Delta \rho'}}{\rho}(q-k, z') \frac{2a^{-1/3} Ai(Z(z', q)e^{2\pi i/3}) Ai'(Z(D, q))}{Ai'(Z(z', q)) Ai(Z(z', q)e^{2\pi i/3}) - Ai'(Z(z', q)e^{2\pi i/3}) Ai(Z(z', q))}. \end{aligned}$$

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Abstract <i>In-situ</i> measurements of scattering strength can be obtained by analyzing the early-time, high angle reverberation from bottom and sub-bottom features. In order to provide insight into the mechanisms which cause bottom reverberation, and to their distinguishing characteristics, it is necessary to have a capability for modelling both the rough surface and the volume scattering mechanisms. For high-angle, early-time backscatter, the most appropriate approach is to use a spectral integral representation, which naturally includes the continuous spectrum important for this angular regime. A rough surface scattering theory developed earlier in this framework has provided important insights into wave scattering and penetration physics at the seafloor. Here a consistent representation for the subbottom scattering is developed and implemented in the same spectral integral code, OASES.		
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