Retrodirective Array Performance Case Studies and Implications for Mine Countermeasures

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Abstract

The use of focused acoustic fields for the mechanical destruction of fluid-borne targets is applied in the medical field for the comminution of kidney stones by lithotripsy. Inhomogeneities in the propagation medium are of practical concern to both the medical and the ocean acoustics communities. The use of phase conjugation to compensate for medium inhomogeneities and multipath effects has been demonstrated for linear acoustics in both medical and underwater contexts. The feasibility of focusing an intense acoustic field onto waterborne targets has implications for mine countermeasures because of the possibility of remotely neutralizing mines. The demonstration of the concept in actual mine countermeasure applications has yet to be realized. We present the results of a numerical study which investigates the performance of linear arrays using time reversal as a means of focusing acoustic fields onto targets in an arbitrary medium modeled after a shallow water channel. The simulation investigates the cases where jitter exists in the initial phase of the time signals. The existence of tight focal widths, narrower than the free-space diffraction limit prediction, sometimes called "super-focusing", was observed for simulations containing small-scale inhomogeneities.

1. Introduction

Time reversal acoustics has a range of medical and underwater applications [1, 2]. The concept of time reversal is an extension of phase conjugation theory, which is known to hold for linear fields in reciprocal media [3, 4]. Remarkable focusing capabilities can be achieved using phase-conjugate arrays in almost arbitrary media by using a time reversal system [5]. However, a number of common phenomena can degrade the performance of a phase conjugate array, including:

- Nonstationary medium
- Limited signal capture resolution or dynamic range
- Nonlinear medium
- Nonlinear acoustical or electrical transfer functions

It is unlikely that perfect time reversal can be practically implemented in a system where the difference between transmitted and received signal levels can exceed 60 dB due to the resolution and dynamic range limitations of current systems. Uncertainties in amplitude, time-domain waveform, and phase will result in a general degradation of the performance of the phased array. The present study shows the results of computer simulations which illustrate the effects of uncertainties introduced into a signal in the receive-transmit operation of an otherwise ideal retrodirective system. The study of the performance and feasibility of an actual retrodirective system can be initially approached using such modeling to get an estimate of the most detrimental factors that might reduce an array's focusing ability.

The problem of building a phase conjugation device capable of operating in the ocean at the intensities required for mine neutralization is far more complex than a simple model problem. Foremost perhaps is the fact that the modeled system is a linear time-invariant system. An actual usable mine countermeasure (MCM) system will certainly require going to such high intensities that nonlinear properties of the transduction devices and the propagation medium will become important. However, the current model is a good first step towards quantifying some of the permissible uncertainties for effective operation of such systems. The present study presents the results of simulations that compare unfavorable test cases to an idealized reference focusing case. The unfavorable test scenario introduces stochastic jitter into the time-domain signals captured by each of the array transducers. Also, a comparison of focal zone width is done with the free-space diffraction limit, showing the "super-focusing" discussed in [6].

2. Signal Phase Error

Random stochastic jitter is introduced into the time-domain signals that the array elements record during the receive mode. The jitter is meant to model an initial phase error, which can result from limited-resolution data acquisition electronics in the receive and/or transmit modes. The jitter is calculated for each array element individually as a time delay in the *initial* phase of each signal. Because the present study seeks to define a starting point for evaluating the relative effects of the more important parameters described that reduce the effectiveness of the time reversal array, only the simplest cases are studied. For example, one could have introduced jitter into each cycle of the wavetrains, or into each time step. In addition, amplitude jitter could be included. The simulations in this study only introduce jitter into the initial phase information of the pulses because the latter effects should be investigated in the category of pulse shape uncertainties, or nonlinear behavior, which is not addressed here. Ultimately, one would combine several of the above effects together to try and deduce what the performance of such a system might look like in the presence of several debilitating factors, but that would make the analysis of the results more complicated.

The jitter is given in terms of a fraction of the narrow-band signal's period. For a time-domain signal at array element k of the form $p_k(t)$, we introduce a time delay, δt, so that

$$p_k(t) \rightarrow p_k(t + \delta t_k). \tag{1}$$

The base source waveform is a sinusoidal envelope. The uncertainty is introduced for each of the elements independently, padding the leading (jitter) time, δt, with zeros. The jitter's duration is computed randomly for the k^{th} element from the narrow-band period, τ_0, the maximum error, A, as a fraction of 2π of the base wavelength for a run, and a random multiplier, σ_k,

$$\delta t_k = \sigma_k A \tau_0. \tag{2}$$

The random variable, σ_k, can range from zero to unity, and is different for each element, but the maximum possible jitter for any array element is A for a given simulation. Of course, the jitter can be defined in other ways, and could be thought of as being due to two processes: one during to the receive mode, and the other during to the transmit mode of the array.

3. Description of Problem and Solution

A vertical, 64-harmonic-element, equally-spaced linear array with an aperture of $25.6m$ is located in the center of a shallow water channel. A $2kHz$ narrow-band point source is located $51.2m$ away from the array as shown in figure 1.

3.1. The Solution Method

The linear inhomogeneous acoustic wave equation is the governing equation for all simulations presented here,

$$\nabla \cdot \left(\frac{\nabla p}{\rho} \right) - \frac{1}{\rho c^2} \frac{\partial^2 p}{\partial t^2} = 0, \tag{3}$$

with the primary variable being the acoustic pressure, $p(\mathbf{r}, t)$.

The wave equation is solved in the time domain using a two-dimensional second-order accurate (in space and time) finite-difference time-domain (FDTD) code. The calculations are carried out on a rectangular grid space of dimensions 1024 horizontal by 512 vertical mesh points. Absorbing boundary conditions were used at the extreme upper and lower edges of the computational domain to simulate an extended spatial region for visual clarity, although this is not necessary, as the time reversal method is especially useful for situations where multiple paths and scattering exist.

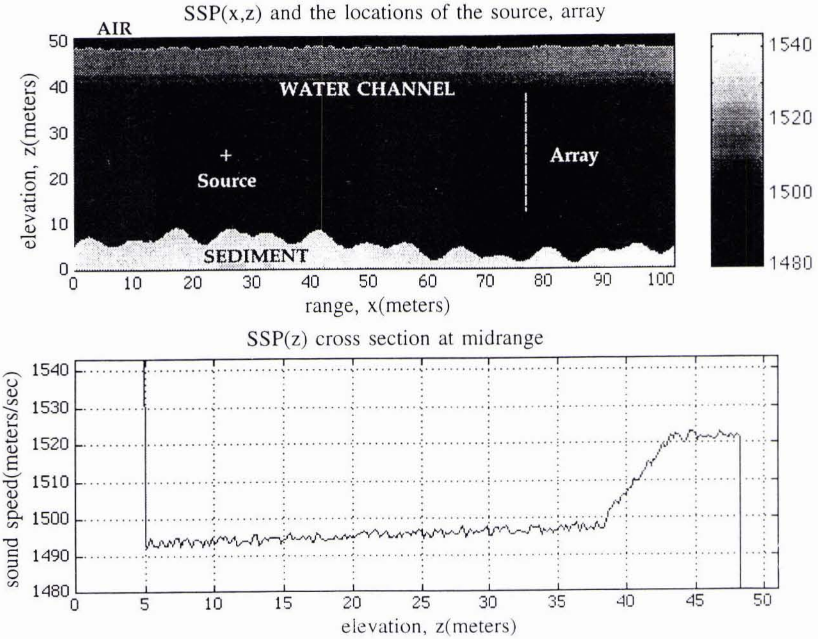

Figure 1: The sound speed profile used in the simulations. The extreme values for sediment and air are truncated to better illustrate the profile in water.

3.2. The Propagation Medium

The wave equation (3) is solved in an inhomogeneous medium modeled as air above a water channel approximately $50m$ deep with a (fast) fluid sediment below it. The sound speed profile (SSP) is fashioned after data given in [7] and [8]. A graphical representation of the SSP is shown in figure 1. Although the basic SSP profile is similar to that used in many studies, it serves to illustrate the physical concepts only, and is not meant to be an accurate oceanographic representation of the SSP of any actual body of water. The properties used for $20°C$ air were a homogeneous sound speed of $343m/sec$ and a density of $1.2kg/m^3$. The sound speed in the water channel was a function of depth below the surface, with inhomogeneities added to that profile. The sediment also had inhomogeneities built on top of a DC sound speed of $1650m/sec$, and a DC density of $1860kg/m^3$. Spectral statistics were not considered for the present study. The density field was obtained in a similar manner to complement the SSP. Inhomogeneities in the water and sediment are in the form of fluctuating regions of excess sound speed and density. Furthermore, a fine random component is added to the sound speed and density to give some fine structure.

The shapes of the air-water and water-sediment interfaces are composed of combinations of sinusoids with small, local, random fluctuations. Again, the intention is to provide simulations in a non-uniform medium with rough interfaces and not to model any oceanographic spectra at this time.

4. Results

The results of the simulations indicate that the time reversal method is in fact a good choice for focusing linear acoustic waves onto a target in an inhomogeneous medium with multipath effects and rough boundaries. Two snapshots are shown for the reference (ideal) case run. Figure 2 shows the pressure field during the receive mode of the array in the top panel, and the instant of maximum focus onto the source during the transmit mode of the array in the lower panel.

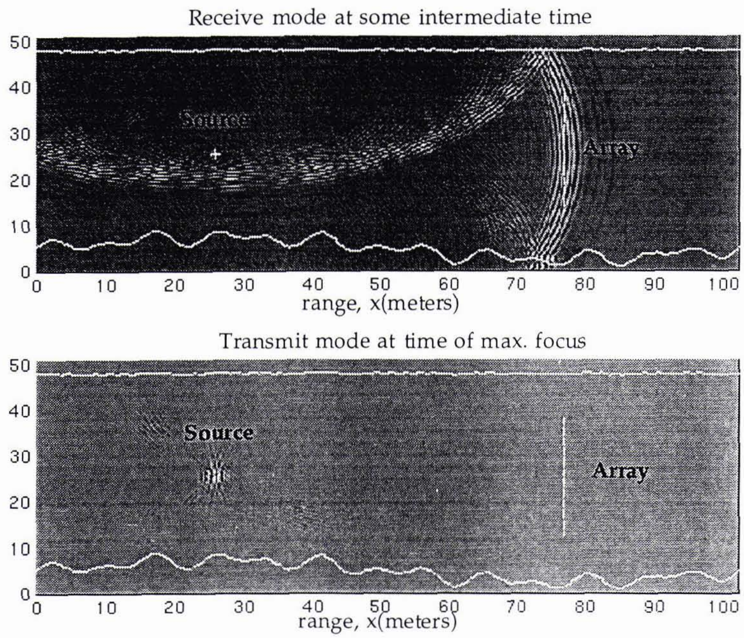

Figure 2: The acoustic pressure field. The top frame shows the pressure after some time from leaving the source during the array's receive mode operation. The lower frame shows the pressure after the array's transmit mode, when the maximum pressure occurs at the source location.

4.1. Phase Jitter

Random time-domain jitter is introduced in the form of zero padding leading the initial phase signal from each array element. Figure 3 shows the pressure field in dB around the location of the source at the instant of maximum focusing. The values are referenced to the maximum pressure (at the source's location usually). It was found that the location of the focus maximum remained near the original location of the source. The reason is that for many-element arrays the focal shift would tend to average out to its original value because the error has a zero mean. On the other hand, significant degradation in focus quality was observed for jitter exceeding about one-tenth of a wave period. As expected, the initial phase of the waveforms was shown to be very important to the focusing ability of the array. Simulations were conducted with jitter than ranged from $0.1(2\pi)$ up to a full 2π of a period. The $-3dB$ points did not show any appreciable spreading from case to case, but the magnitude of the acoustic pressure for the cases with large jitter was far reduced and spread over a large region of the channel, resulting in significant focusing degradation for jitters greater than 0.1 to 0.2 of one period (figure 3).

4.2. Super-focusing

The inhomogeneities in the propagation medium can act to enlarge the effective aperture of the array. This is because the small differences in the index of refraction act as localized sources which are distributed throughout the medium. Since the focal zone's FWHM spot size in free-space is given by the diffraction limit as

$$w = 1.2\ \lambda z/a, \tag{4}$$

the width is inversely proportional to the aperture. In our case the predicted width at $-3dB$ should be approximately $1.8m$ in free-space. In fact we observe that the $-3dB$ points occur at $\pm 0.3m$ from the source location. Simulations in homogeneous media using the same code have shown that the code does follow the predicted focal width in the absence of inhomogeneities [12].

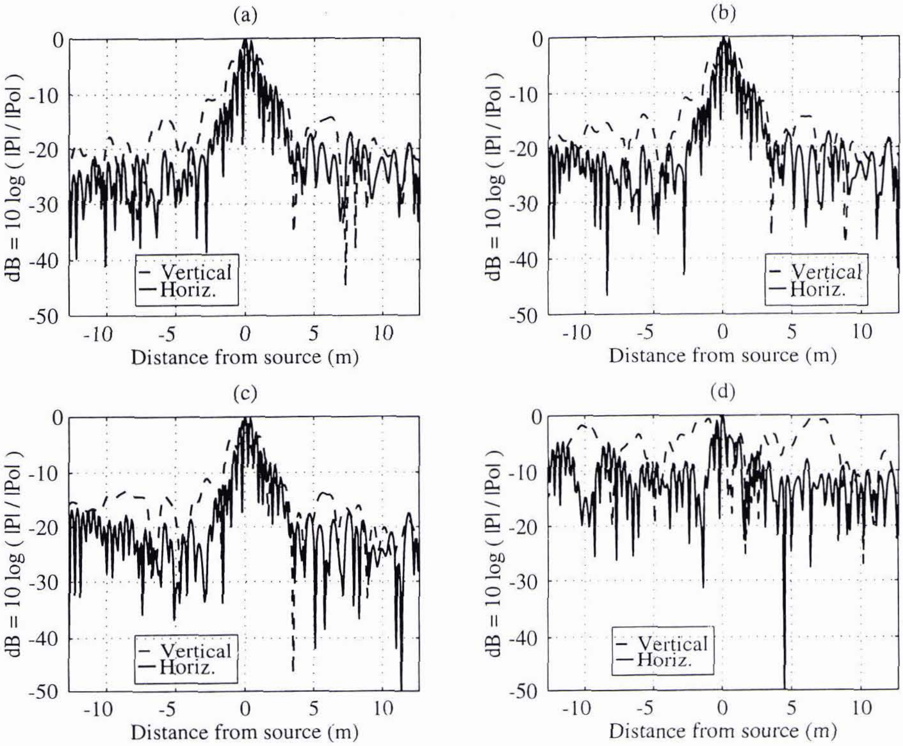

Figure 3: Slices through the source position at the time of maximum focusing for various jitter conditions. The dashed lines denote the vertical slices (parallel to the array), while the solid lines denote the horizontal slices (perpendicular to the array). Panels are for (a) No jitter (reference case), (b) Max. jitter = $0.1(2\pi)$, (c) Max. jitter = $0.2(2\pi)$, (d) Max. jitter = 2π.

Experiments by Derode, *et al.* [5], have also demonstrated this effect in the laboratory for high-order multiple scattering.

5. Conclusions

The premise of using phase conjugate arrays for the focusing of intense acoustic fields onto a remote waterborne target has implications for MCM system design. The possibility of remote neutralization of pressure-sensitive mines would be an asset to the MCM arsenals that exist today [11]. The concept has been demonstrated in theory and in the laboratory for ultrasonic frequencies in medical applications. Ocean field experiments in shallow water have been conducted recently by Kuperman, *et. al.*, (unpublished). These experiments were conducted at $400Hz$, and would not encounter the difficulty with electronic jitter that high frequency arrays would suffer from. However, the results obtained so far from experimental [5] and theoretical [6] groups show a remarkable robustness when using the time reversal technique with multiple scattering and reflection in random media.

In this report we used a linear acoustics model to simulate propagation through a shallow water channel with an inhomogeneous sound speed profile as well as a rough surface and bottom. The results for cases that are expected to degrade the focusing ability of a time reversal array by altering the initial phase information are given. Initial phase timing is corrupted by some jitter introduced to model time-domain electronic system uncertainties affecting the relative phases of the transmitted array element waveforms.

The array's focusing appears to hold up well under these circumstances for jitter up to 0.1 to 0.2 of the narrow-band period. Above a value of 0.2 period results in significant loss of focus. The location of maximum pressure remained at the location of the source because the jitter is a zero-mean random variable. indicating that initial phase of the returned signals is more important than the details of the waveform phase shape. This is encouraging, since the data aquisition of a *broad-band* time-reversal signal, and the translation of that signal into a corresponding high-intensity array pressure output is unlikely with current technology. This is especially true if the array consists of elements whose bandwidth is significantly greater than PZT transducers.

Other factors not studied here that are detrimental to the focusing of phase conjugate arrays also need to be investigated. Chief among these is the nonlinear behaviour of the medium and the transducer and electronics' transfer functions, which will become important for high intensity or broad-band signals. The nonlinearity of the transduction process under real ocean MCM conditions is certain to play an important role that needs to be studied theoretically and in the laboratory before any definitive conclusions can be made regarding the feasibility of constructing an actual MCM phase-conjugate system.

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