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TIME-EVOLUTION MODELLING OF SEAFLOOR SCATTER, PART 1: THEORY

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Jan L. Spoelstra
Director

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Executive Summary:

Acoustic scattering from the seafloor is a complex phenomena. A good understanding of the underlying physical process is essential to reliable remote assessment of seafloor parameters and improving MCM performance modelling.

With the recent emphasis on higher frequencies, new parameters such as seafloor roughness and sediment inhomogeneities are significant factors with concomitant effects on modelling sophistication.

This report describes a new approach for the prediction of scatter from the seafloor interface and volume in the time-domain. The report is divided into two parts. Part I describes the theoretical background of the model, part II describes the implementation and the experimental verification.

Traditionally, scattering models have focused on predicting the backscatter strength, which is basically a measurement of the energy scattered back from a unit area at a given angle. The main limitation in this approach is that all the information coming from the time-domain signal is reduced to a single number, and both phase and shape information of the original signal are lost. Here, a different approach consisting of modelling the full time-series has been chosen.

The model has been tested against simplified, but theoretically known situations and against data recorded with a parametric sonar. The results of the comparison are encouraging, and they demonstrate that the main interaction mechanisms with the seafloor are accounted for. Future work will include comparison with more data recorded from different sonar systems and with better ground truth knowledge. The use of the model for reliable remote seafloor classification will also be studied.

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Time-evolution modelling of seafloor scatter. Part I: Theory

E. Pouliquen, O. Bergem, N.G. Pace

Abstract: A time-series model for acoustic seafloor backscattering is described. The method expresses analytically the elementary time-backscattered response of every seafloor surface and every seafloor volume infinitesimal element. For chosen geometric, acoustical and acquisition parameters, they are summed to produce in the time-domain a realization of the reverberated time pressure field received at the source. The approach is based on the Kirchhoff approximation for the seafloor interface backscattering and on the Small Perturbation theory for the seafloor volume. It only accounts for single backscattering mechanisms of the compressional wave with the seafloor. The model is implemented using calculated height fields for the water-sediment interface and the distributed seafloor volume inhomogeneities. The analytical description of the model and its limitations is described in this report.

Keywords: seafloor backscattering o surface scattering o volume scattering o kirchhoff o small perturbation o time domain

Contents

1 Introduction

Growing interest in seafloor characteristics in shallow water is accompanied by an expectation for improved accuracy in the estimation of seafloor parameters. In order to achieve satisfactory characterization, the physics of the scattering mechanisms on and in the seafloor have to be understood. It is particularly important to evaluate the importance of each contribution from the seafloor surface and from the seafloor volume. The sole consideration of the total backscattering strength does not allow a detailed study of the scattering mechanisms. One way to achieve that, is to observe the whole time series of the backscattered signal. Each time series depends on geo-acoustic properties such as local density, local sound-speed, local attenuation, roughness spectrum, correlation lengths and on the geometry of measurement such as the water height of the source from the seafloor surface, the orientation of the source, and on the source characteristics (pulse characteristics, directivity pattern). When an acoustic pulse is interacting with a seafloor, all these parameters affect the backscattering process, resulting in a complex time backscattered signal. Intuitively, the seafloor surface contribution should be at the beginning of the returned echo and the seafloor volume contribution should affect the tail of the echo more. But it is impossible to separate these two contributions with a time cut-off approach. Intuitively again, a hard seafloor would have a stronger seafloor surface return than a soft bottom. But this is also contingent on seafloor surface roughness or seafloor volume inhomogeneities. The geometry of measurement also has an important effect on the backscattered time signal. One should expect an extension of the time signal duration as the water height between the source and the seafloor surface increases. A simple calculation shows that the extension is not linear. Another element which strongly affects the signal, is the directivity pattern and orientation of the source. For each ping, it not only conditions the insonified area but it also weights acoustic contributions from each direction. Side lobes, for example, are often the origin of significant acoustic backscattering. If one requires a close-to-reality representation of the scattering phenomena, it is not possible to approximate the directivity pattern with a cone, or with an exponential function. Other factors are the pulse characteristics (pulse shape, spectrum and duration) which affect the type of scattering mechanisms, geometric or Rayleigh, the expected amount of coherent and incoherent signal and average shape of the signal.

These factors demonstrate the need to model the backscattered pressure field in the time domain for various geometric, physical and acoustical configurations in order to (1) understand the processes occuring during echo-sounding, (2) to define the limits of a possible quantitative and qualitative characterisation method and (3) to articulate a possible methodology for a reliable and efficient seabed classification.

2

Model approach

The modelling of a backscattered time signal is a hybrid process, using a classical approach for the physical interaction of acoustic wave and seafloor on a small scale. On a larger scale, it combines a full description of the source characteristics and of the geometry of the measurement with the use of an artificially-generated seafloor the main geo-acoustic parameters of which are completely controled. The combination of these two different approaches allows the synthesis of most of the physical phenomena which determine the time backscattered signal. More specifically, the model combines analytical expressions of the elementary seafloor surface and elementary seafloor volume time pressure field response, with a stochastic description of the seafloor. The expression of the time backscattered pressure from an elementary seafloor surface is based on the Kirchhoff approximation (described in Section 3), and the expression for the time backscattered pressure from an elementary volume on Small Perturbation theory (described in Section 4). Extension to larger seafloor surfaces and volumes is effected by stochastic realizations of the seafloor surface and volume features. Each stochastic sample of the bottom is realized for a given set of statistical properties of the seafloor surface and the volume: the seafloor surface is characterized principally by RMS height (roughness) and by parameters describing the horizontal features (correlation lengths for example). The volume is characterized by a RMS variation and vertical and horizontal correlation lengths of the density and sound speed variations around mean values. This approach is new, in the sense that the physics of the interaction of sound waves with the seafloor is included with a description of the environment, in order to compute a time evolution of the seafloor backscattered signal.

Some constraints and limitations have been set in order to reduce the dimension of the framework and to allow some approximations. The model has to at least be valid in the angular region, close to normal incidence but extension to higher angles of incidence could be envisaged in later studies. The frequency of the transmitted signal has to be high enough and the seafloor surface smooth enough to allow the use of the Kirchhoff approximation [1]. Equally, the characteristics of the inhomogeneities should vary slightly around their mean values to allow the use of Small Perturbation theory (Section 3 and 4). As the above conditions are not too restrictive, the model is valid for a wide range of acoustic instruments including echo-sounders, swath (at least for the most vertical beams) and parametric systems.

3

Seafloor surface scattering

The main condition for the validity of the Kirchhoff approximation is the following: if the roughness of the surface is sufficiently smooth in the horizontal dimension in order to avoid shadowing effects and multiple reflection on the surface, one can assume that the sound reflection on each point of the surface is equivalent to a reflection by a tangent plane located at this considered point, **R**. This condition is expressed by Brekhovskikh [1] in terms of the necessity of having a minimal interface radius of curvature r_c at a given incidence θ_{inc} and wave number k_0:

$$2k_0 r_c \cos(\theta_{inc}) \gg 1 \tag{1}$$

Even close to normal incidence, this condition might seem too strict for some applications described later. Actually, the reason for this condition is to avoid a large amount of sharp edges or sharp points having a small radius of curvature creating edge or shadow effects. In the case of our study, we will assume the validity of the tangent plane approximation even for small wavenumbers providing that the seafloor surface correlation length l is greater than the wavelength ($l \geq \lambda$ condition from Thorsos [2]). This hypothesis is debatable but makes sense in the case of smooth bottoms, i.e. from mud to sand at normal incidence, and the experimental agreement displayed later on seems to justify this approach. In addition, at the computation stage, it is possible to check and reject the acoustic rays which are affected by a possible shadow effect or that are impinging on the bottom with a beyond critical local angle of incidence γ (see figure 1).

Assuming that these conditions are verified, the boundary conditions at the point **R** on the interface are the following (Brekhovskikh [1], Clay & Medwin [3]):

$$p_r(\mathbf{R}) = \Re_{01}(\mathbf{R})p_i(\mathbf{R}) \tag{2}$$

$$\frac{\partial}{\partial n}p_r(\mathbf{R}) = -\Re_{01}(\mathbf{R})\frac{\partial}{\partial n}p_i(\mathbf{R}) \tag{3}$$

$\frac{\partial}{\partial n}$ is the normal derivative operator. $\Re_{01}$ is the local water-sediment plane wave reflection coefficient at the point **R**. We consider a source located at a height H ($\mathbf{P} = \{0, 0, H\}$). This source is at first emitting a CW wave of frequency f, at a source

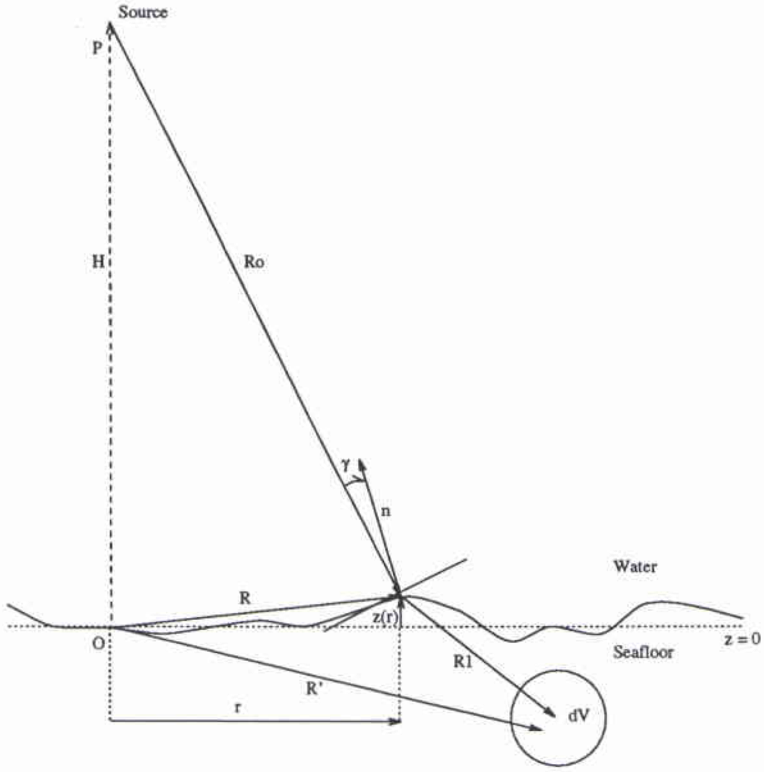

Figure 1 Geometry of measurement.

level p_0 with a transmitted directivity $D_i(\mathbf{R})$ given by the direction of $\mathbf{R} - \mathbf{P}$ (Fig. 1). The water column above the seabed interface is assumed to be homogeneous with a constant sound velocity profile. The vector $\mathbf{n}$ is the vector normal to the surface at $\mathbf{R}$. In this case, following the same demonstration as Chotiros in [4], the incident pressure field received at the point $\mathbf{R}$ takes this form:

$$p_i(\mathbf{R}) = p_0 G_i(\mathbf{R}) D_i(\mathbf{R}) \tag{4}$$

where $G_i(\mathbf{R})$ is the Green function for the incident pressure field:

$$G_i(\mathbf{R}) = \frac{e^{-jk_0 R_0}}{R_0} \tag{5}$$

with $R_0 = |\mathbf{R_0}|$. Using Eq. (2) and Eq. (4), assuming $e^{2\pi jft}$ time dependance, we can express the reflected pressure $p_r(\mathbf{R})$ just above the surface:

$$p_r(\mathbf{R}) = p_0 G_i(\mathbf{R}) D_i(\mathbf{R}) \Re_{01}(\mathbf{R}) \tag{6}$$

Also, the expression of the normal displacement velocity of the reflected field v_r at the point $(\mathbf{R})$ can be expressed by [3]:

$$\frac{\partial}{\partial n} p_r(\mathbf{R}) = -\bar{\rho_0} \frac{\partial}{\partial t} v_r(\mathbf{R}) \tag{7}$$

$\bar{\rho_0}$ is the water mean density. Using Eq. (3) and Eq. (4), the left hand side term of Eq. (7) gives:

$$\frac{\partial}{\partial n} p_r(\mathbf{R}) = -p_0 D_i(\mathbf{R}) \Re_{01}(\mathbf{R}) \frac{\partial}{\partial n} (G_i(\mathbf{R})) \tag{8}$$

A closer look at $\frac{\partial}{\partial n} (G_i(\mathbf{R}))$ shows that:

$$\begin{aligned} \frac{\partial}{\partial n} (G_i(\mathbf{R})) &= \nabla[(G_i(\mathbf{R})].\mathbf{n} \\ &= (\frac{\mathbf{R_0}.\mathbf{n}}{R_0}) \frac{\partial}{\partial R_0} \left(\frac{e^{-jk_0 R_0}}{R_0} \right) \end{aligned} \tag{9}$$

Since $k_0 R_0 \gg 1$ in our case (interface in the far field region), we can approximate Eq. (9) by:

$$\begin{aligned} \frac{\partial}{\partial n}(G_i(\mathbf{R})) &= -(\frac{\mathbf{R_0}.\mathbf{n}}{R_0}) j k_0 G_i(\mathbf{R}) \\ &= j \cos(\gamma(\mathbf{R})) k_0 G_i(\mathbf{R}) \end{aligned} \tag{10}$$

with

$$(\frac{\mathbf{R_0}.\mathbf{n}}{R_0}) = -\cos(\gamma(\mathbf{R}))) \tag{11}$$

$\gamma(\mathbf{R})$ is the angle made between the incident direction and the normal vector $\mathbf{n}$ to the surface at $\mathbf{R}$. Using Eq. (6), the first term of Eq. (7) equals:

$$\frac{\partial}{\partial n} p_r(\mathbf{R}) = -j \cos(\gamma(\mathbf{R})) k_0 p_r(\mathbf{R}) \tag{12}$$

The right hand side term of Eq. (7) gives:

$$-\bar{\rho_0} \frac{\partial}{\partial t} v_r(\mathbf{R}) = -j \bar{\rho_0} \bar{c_0} k_0 v_r(\mathbf{R}) \tag{13}$$

with $\bar{c_0}$ being the average sound speed of water. Combining Eq. (7), Eq. (12) and Eq. (13), we obtain an expression of the local displacement v_r at $\mathbf{R}$:

$$v_r(\mathbf{R}) = \frac{\cos(\gamma(\mathbf{R}))}{\bar{\rho_0} \bar{c_0}} p_r(\mathbf{R}) \tag{14}$$

Morse [5] gives an expression of the radiated pressure at the point P due to the element $d\mathbf{S}_\mathbf{R}$ located at $\mathbf{R}$:

$$dp_s(\mathbf{P}) = j \frac{\bar{\rho_0} \bar{c_0} k_0}{2\pi} v_r(\mathbf{R}) G_r(\mathbf{R}) D_r(\mathbf{R}) d\mathbf{S}_\mathbf{R} \tag{15}$$

where $G_r(\mathbf{R})$ is the Green function of the reflected pressure field and $D_r(\mathbf{R})$ the receiving directivity of the source (located in $\mathbf{P}$). Equation (15) can be expanded using Eq. (6) and Eq. (14):

$$dp_s(\mathbf{P}) = j\frac{k_0 \cos(\gamma(\mathbf{R}))}{2\pi}p_0 \times G_i(\mathbf{R})G_r(\mathbf{R})D_i(\mathbf{R})D_r(\mathbf{R})\Re_{01}(\mathbf{R})d\mathbf{S_R} \quad (16)$$

The product $D_i(\mathbf{R})D_r(\mathbf{R})$ is the gain of the emitter-receiver in the direction defined by $\mathbf{R}$. Rigorously, $D_i(\mathbf{R})D_r(\mathbf{R})$ should be included with the frequency dependent terms but the directivity pattern will be assumed to be constant over the considered bandwidth of the used signal. The product $G_i(\mathbf{R})G_r(\mathbf{R})$ contains the propagation loss in the water column. It also contains the phase variation during the propagation. Since we are simply considering a monostatic case (source and receiver at the same location), $G_i(\mathbf{R}) = G_r(\mathbf{R})$ and this product can be replaced by $G(\mathbf{R})^2$. It is also assumed that the reflection coefficient $\Re_{01}(\mathbf{R})$ is frequency invariant over the bandwidth of the transmitted pulse. Then, separating the frequency dependent terms of (16) and putting them into curly brackets, we obtain:

$$dp_s(\mathbf{P}, f) = j\frac{\cos(\gamma(\mathbf{R}))}{\bar{c_0}}p_0 \times D_i(\mathbf{R})D_r(\mathbf{R})\Re_{01}(\mathbf{R})\{fG(\mathbf{R}, f)^2\}d\mathbf{S_R} \quad (17)$$

In the case of a non CW signal, $E(f)$ being the spectrum of the transmitted pulse $e(t)$, one can write:

$$dp_s(\mathbf{P}, f) = j\frac{\cos(\gamma(\mathbf{R}))}{\bar{c_0}}p_0 \times D_i(\mathbf{R})D_r(\mathbf{R})\Re_{01}(\mathbf{R})\{fG(\mathbf{R}, f)^2E(f)\}d\mathbf{S_R} \quad (18)$$

Until now, the contribution of an elementary surface $d\mathbf{S_R}$ is expressed at a given frequency f. This expression is not directly generalisable to a larger scale by integrating each elementary pressure field over the whole insonified seafloor surface (S) before switching to the time domain. Switching into the time domain can be done at this point using the inverse Fourier transform. Analyticaly, $dp_s(\mathbf{P}, t)$ is first expressed thus:

$$dp_s(\mathbf{P}, t) = \mathcal{F}^{-1}\{dp_s(\mathbf{P}, f)\} = j\frac{\cos(\gamma(\mathbf{R}))}{\bar{c_0}}p_0 \times D_i(\mathbf{R})D_r(\mathbf{R})\Re_{01}(\mathbf{R})\mathcal{F}^{-1}\{fG(\mathbf{R}, f)^2E(f)\}d\mathbf{S_R} \quad (19)$$

where $\mathcal{F}^{-1}$ represents the inverse Fourier transform operator. Let us define $\mathcal{K}_s(\mathbf{R},t)$ as being equal to $\mathcal{F}^{-1}\{fG(\mathbf{R},f)^2E(f)\}$ and simplify it:

$$\mathcal{K}_s(\mathbf{R},t) = \int_{-\infty}^{+\infty} fG(\mathbf{R},f)^2E(f)e^{2\pi jft}df \tag{20}$$

Then, using Eq. (5), we find:

$$\begin{aligned}\mathcal{K}_s(\mathbf{R},t) &= \frac{1}{R_0^2}\int_{-\infty}^{+\infty} fE(f)e^{2\pi jf(t-\frac{2R_0}{\bar{c_0}})}df \\ &= \frac{1}{R_0^2}\mathcal{F}^{-1}\{fE(f)e^{-2\pi jf(\frac{2R_0}{\bar{c_0}})}\}\end{aligned} \tag{21}$$

So,

$$\mathcal{K}_s(\mathbf{R},t+\frac{2R_0}{\bar{c_0}}) = \frac{1}{R_0^2}\mathcal{F}^{-1}\{fE(f)\} \tag{22}$$

Knowing that for a continuous and differentiable function e we have for the n^{th} derivative $e^{(n)}$:

$$\frac{e^{(n)}(\mathbf{R},t)}{(2\pi)^n} = \mathcal{F}^{-1}\{j^n f^n E(f)\} \tag{23}$$

We obtain in this case:

$$\mathcal{K}_s(\mathbf{R},t) = \frac{1}{2\pi jR_0^2}e^{'}(t-\frac{2R_0}{\bar{c_0}}) \tag{24}$$

where $e^{'}(t)$ stands for the first time derivative of the transmitted pulse $e(t)$. Then, equation (19) becomes:

$$\begin{aligned}dp_s(\mathbf{P},t) &= \frac{\cos(\gamma(\mathbf{R}))}{2\pi\bar{c_0}R_0^2}p_0 \\ &\quad\times(D_i(\mathbf{R})D_r(\mathbf{R}))\,\Re_{01}(\mathbf{R})e^{'}(t-\frac{2R_0}{\bar{c_0}})dS_{\mathbf{R}}\end{aligned} \tag{25}$$

The equation obtained above gives a rather simple and interesting expression for the time pressure field response received at **P** from an elementary surface for a given pulse shape in the range of the validity of the Kirchhoff approximation. On a larger scale, the backscattered pressure signal received at the source from the surface S on the bottom is given by the following integral which adds up the time contribution from each element of surface $d\mathbf{S_R}$:

$$p_s(\mathbf{P}, t) = \int_{(S)} dp_s(\mathbf{P}, t) \tag{26}$$

Unfortunately, this integral is not analyticaly expressable without making rough assumptions and has to be numerically computed using a pre-defined surface height field.

4 Seafloor volume scattering

The analytical construction of the time pressure field received at **P** from an elementary volume is based on Ivakin's volume scattering approach [6]. The expression of the local volume backscattered pressure is based on Chernov's Small Perturbation theory [7] [8]. This theory considers both sound speed and density variation in the volume. A detailed and updated formulation of this theory can be found in [9]. The construction of this time pressure field follows the same principles as for the seafloor surface modelling but deals in this case with a 3D scattering field.

First, assuming that we have an incident pressure field $p_t(\mathbf{R})$ transmitted through the surface and still allowing the tangent plane approximation [3], we can write:

$$p_t(\mathbf{R}) = (1 + \Re_{01}(\mathbf{R}))p_i(\mathbf{R}) \tag{27}$$

$$\frac{\partial}{\partial n}p_t(\mathbf{R}) = (1 + \Re_{01}(\mathbf{R}))\frac{\partial}{\partial n}p_i(\mathbf{R}) \tag{28}$$

Then, as in Eq. (6), the pressure field of the transmitted wave through the water-sediment interface takes this form at the point **R**:

$$p_t(\mathbf{R}) = p_0 G_i(\mathbf{R}) D_i(\mathbf{R}) \Im_{01}(\mathbf{R}) \tag{29}$$

with $\Im_{01}(\mathbf{R}) = 1 + \Re_{01}(\mathbf{R})$ being the local water-sediment plane wave transmission coefficient.

Following Yamamoto [9] by assuming that sound speed and density variations are small compared to their actual respective mean values, in the backscattering case, the solution to the propagation equation in a medium containing variations of sound speed and density can be written as follows:

$$dp_v(\mathbf{R}) = \frac{k_1^2}{2\pi}\mu(\mathbf{R}')p_t(\mathbf{R}')G(\mathbf{R},\mathbf{R}')d\mathbf{V_R} \tag{30}$$

Here, k_1 is the average wave number in the sediment and $\mu(\mathbf{R}')$ being defined as the degree of inhomogeneities at the location $\mathbf{R}' = \mathbf{R} + \mathbf{R}_1$ (Fig. 1):

$$\mu(\mathbf{R}') = \gamma_c(\mathbf{R}') + \gamma_\rho(\mathbf{R}') \tag{31}$$

γ_c and γ_ρ are respectively the relative fluctuation of the sound speed and of the density at the same location:

$$\gamma_c(\mathbf{R}') = \frac{c_1(\mathbf{R}') - \bar{c_1}}{\bar{c_1}} \tag{32}$$

$$\gamma_\rho(\mathbf{R}') = \frac{\rho_1(\mathbf{R}') - \bar{\rho_1}}{\bar{\rho_1}} \tag{33}$$

Here, $\bar{c_1}$ and $\bar{\rho_1}$ are respectively the average sound speed and density in the entire insonified volume. For core data analysis, it is possible to evaluate separately γ_c and γ_ρ. These core analyses (as noted by Hamilton [10] [11] and Yamamoto [9] for example) have shown that these two variables were not independent. As an approximation, it is possible to choose a relation of proportionality between γ_c and γ_ρ. This assumption is not always valid but as density variation and sound speed variation approximations are difficult, it may be appropriate to use global estimation of the degree of inhomogeneities of the volume $\mu(\mathbf{R}')$ when separate measurements of γ_c and γ_ρ are not available. Considering now Eq. (30), the resulting pressure field created at the elementary volume $d\mathbf{V_R} = d\mathbf{S_R}dR_1$, can be expressed as follows:

$$p_t(\mathbf{R}') = p_0 G_i(\mathbf{R}) D_i(\mathbf{R}) \Im_{01}(\mathbf{R}) e^{-jk_0 \bar{n_1} R_1 - \frac{\beta R_1}{2}} \tag{34}$$

Following Ivakin et al. [6], spherical spreading in the sediment is neglected because propagation in the sediment is strongly attenuated due to the absorption coefficient β. However, this hypothesis may not be valid in the critical angle region. Rigorously, $n_1(\mathbf{R}')$, the refractive index at the location $\mathbf{R}'$ in the sediment should also be used in the propagation loss term, but the averaged refractive index $\bar{n_1}$ is used instead over the first meters of the sediment, disregarding the possible refraction effect within the sediment itself. According to Ivakin [6], the Green function $G(\mathbf{R}, \mathbf{R}')$ of equation Eq. (30) can be approximated by:

$$G(\mathbf{R}, \mathbf{R}') = e^{-jk_0 \bar{n_1} R_1 - \frac{\beta R_1}{2}} \tag{35}$$

Combining Eq. (35) and Eq. (30) the following expression of the backscattered pressure field just under the surface is obtained:

$$dp_v(\mathbf{R}) = \frac{k_0^2 n_1^2(\mathbf{R}')}{2\pi}\mu(\mathbf{R}')p_t(\mathbf{R})e^{-2jk_0\bar{n_1}R_1-\beta R_1}d\mathbf{V_R} \tag{36}$$

Then, back above the surface using again the tangent plane approximation with Eq. (29) and Eq. (36), $dp_v(\mathbf{R})$ becomes:

$$\begin{aligned} dp_v(\mathbf{R}) &= \frac{k_0^2 n_1^2(\mathbf{R}')}{2\pi}\mu(\mathbf{R}')p_0 G_i(\mathbf{R})D_i(\mathbf{R}) \\ &\times \Im_{01}(\mathbf{R})\Im_{10}(\mathbf{R})e^{-2jk_0\bar{n_1}R_1-\beta R_1}d\mathbf{V_R} \end{aligned} \tag{37}$$

By analogy with what was done for the interface backscattering at a given frequency f, the pressure field received at the source point P takes the form:

$$\begin{aligned} dp_v(\mathbf{P},f) &= \frac{k_0^2 n_1^2(\mathbf{R}')}{2\pi}\mu(\mathbf{R}')p_0 D_i(\mathbf{R})D_r(\mathbf{R}) \\ &\times \Im_{01}(\mathbf{R})\Im_{10}(\mathbf{R})G_i(\mathbf{R})G_r(\mathbf{R})e^{-2jk_0\bar{n_1}R_1-\beta R_1}d\mathbf{V_R} \end{aligned} \tag{38}$$

In the case of backscattering, $G_i(\mathbf{R})G_r(\mathbf{R}) = G(\mathbf{R})^2$ is the squared Green's function $(1/R_0^2)e^{-2jk_0R_0}$. For a non CW signal of spectrum $E(f)$, we consider that the attenuation β in the seafloor is frequency dependent as it is suggested by Clay & Medwin [3] (pp 260):

$$\beta = \alpha|f| \tag{39}$$

where α is the attenuation coefficient which is assumed to be fixed at a given porosity. Equation (38) becomes:

$$\begin{aligned} dp_v(\mathbf{P},t) &= \frac{2\pi n_1^2(\mathbf{R}')}{\bar{c_0}^2}\mu(\mathbf{R}')p_0 \\ &\times D_i(\mathbf{R})D_r(\mathbf{R})\Im_{01}(\mathbf{R})\Im_{10}(\mathbf{R}) \\ &\times \mathcal{F}^{-1}\left(e^{-\alpha|f|R_1}f^2G(\mathbf{R})^2e^{-2jk_0\bar{n_1}R_1}E(f)\right) \\ &\times d\mathbf{V_R} \end{aligned} \tag{40}$$

where the attenuation loss term $e^{-\alpha|f|R_1}$ appears with the other frequency dependent terms. Let us define $\Gamma_{(v)}(\mathbf{R},t)$ as the following:

$$\Gamma_{(v)}(\mathbf{R}, f) = \mathcal{F}^{-1}\left(e^{-\alpha|f|R_1}(f^2 G(\mathbf{R}, f)^2 e^{-2jk_0\bar{n_1}R_1} E(f) e^{2\pi jft})\right) \tag{41}$$

Let us also define $\mathcal{K}_v(\mathbf{R}, t)$ as the following:

$$\mathcal{K}_v(\mathbf{R}, t) = \int_{-\infty}^{+\infty} f^2 G(\mathbf{R}, f)^2 e^{-2jk_0\bar{n_1}R_1} E(f) e^{2\pi jft} df \tag{42}$$

Expressing $G(\mathbf{R})^2$, we find:

$$\mathcal{K}_v(\mathbf{R}, t) = \frac{1}{R_0^2}\int_{-\infty}^{+\infty} f^2 E(f) e^{2\pi jf(t-2(\frac{\bar{n_1}R_1+R_0}{\bar{c_0}}))} df \tag{43}$$

By looking at (23), we can write:

$$\mathcal{K}_v(\mathbf{R}, t) = -\frac{1}{(2\pi)^2 R_0^2} e''(t - 2(\frac{\bar{n_1}R_1 + R_0}{\bar{c_0}})) \tag{44}$$

Fourier transform properties allow us to express $\Gamma_v(\mathbf{R}, t)$ as follows:

$$\Gamma_v(\mathbf{R}, t) = \mathcal{F}^{-1}\left(e^{-\alpha|f|R_1}\right) * \mathcal{K}_v(\mathbf{R}, t) \tag{45}$$

with $*$ being the convolution operator. Equation (44) gives an analytical expression of $\mathcal{K}_v(\mathbf{R}, t)$ in the time domain. The first part of the right hand term of Eq. (45) is also analyticaly expressable in the time domain:

$$\mathcal{F}^{-1}\left(e^{-\alpha|f|R_1}\right) = \frac{1}{\pi}\frac{\frac{\alpha R_1}{2\pi}}{(\frac{\alpha R_1}{2\pi})^2 + t^2} \tag{46}$$

Then, Eq. (40) becomes:

$$\begin{aligned} dp_v(\mathbf{P}, t) &= \frac{-n_1^2(\mathbf{R}')}{2\pi R_0^2 \bar{c_0}^2}\mu(\mathbf{R}')p_0 \\ &\times\ D_i(\mathbf{R})D_r(\mathbf{R})\Im_{01}(\mathbf{R})\Im_{10}(\mathbf{R}) \\ &\times \left(\frac{1}{\pi}\frac{\frac{\alpha R_1}{2\pi}}{(\frac{\alpha R_1}{2\pi})^2 + t^2} * e''(t - 2(\frac{\bar{n_1}R_1 + R_0}{\bar{c_0}}))\right) \\ &\times\ d\mathbf{V_R} \end{aligned} \tag{47}$$

This expression is less simple than equation Eq. (25) because it contains a convolution operation but it accounts reasonably well for the frequency content of the time backscattered signal from an elementary volume. From this last expression, the backscattered pressure signal received at the source from a volume (V) located under the seafloor surface (S) is given by the integral:

$$p_v(\mathbf{P}, t) = \int_{(V)} dp_v(\mathbf{P}, t) \tag{48}$$

As for the surface contribution, this last expression is not analyticaly computable. It should be numerically computed using a pre-defined volume inhomogeneities field.

5

Interface and volume scattering integral

The total pressure field received at P from the sea-bottom is the sum of the pressure field from the surface Eq. (26) and the pressure field from the volume Eq. (48):

$$\begin{aligned} p(\mathbf{P},t) &= p_s(\mathbf{P},t) + p_v(\mathbf{P},t) \\ &= \int_{(S)} dp_{(s)}(\mathbf{P},t) + \int_{(V)} dp_{(v)}(\mathbf{P},t) \end{aligned} \quad (49)$$

An example of the construction of the surface and volume time series is presented in Fig. (2) in the case of normal incidence. At a given time t, it shows which part of the seafloor surface and of the seafloor volume is contributing at the source $\mathbf{P}$.

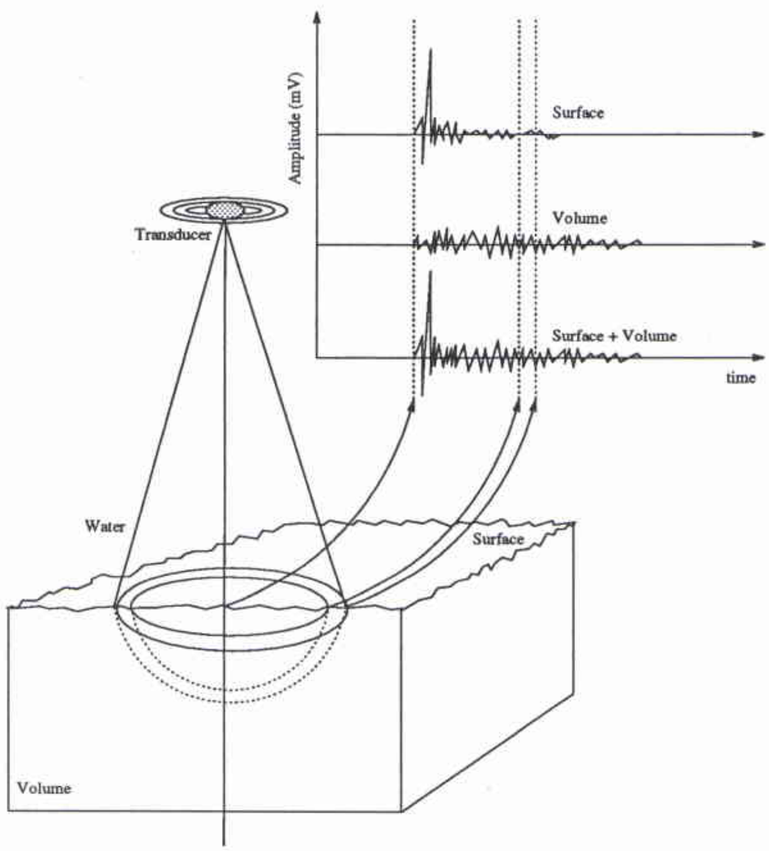

Figure 2 Construction of the time series.

6

Conclusion

The prime objective of this model is to understand the physical processes occuring during echo-sounding. To account as accurately as possible for the main factors influencing the the time series. The domain of validity of this model is not completely defined. The pertinence of the Kirchhoff approximation for extreme cases such as low frequency or moderate-to-high incident angles is still subject to debate. Similarly, the Small Perturbation theory may not be applicable to highly inhomogeneous sediments. Another limitation of this model is that shear waves, interface waves, slow waves and multiple scattering are not taken into account as these phenomena are not easily measurable at sea and their significance is often negligible in high frequency applications. Gassy sediment creating non linear effects which may be of particular importance is not accounted for in this model. These limitations are largely compensated for by the possibility of generating the whole time series backscattered from the seafloor for a well described configuration, instead of focusing on the unpredictable and little understood average quantity of the seafloor backscattering strength. For many years, backscattering strength has been measured and modelled for various seafloor types but results are not consistent. For example, there is not a one-to-one relation between a seafloor and its corresponding evolution of the backscattering strength as a function of the incident angle at a given frequency. The error in the ground truthing, in the backscattering strength measurement and in the modelling is too important to allow the sole use of backscattering strength for classification or characterization purposes. Both surface and volume contributions must be taken into account. The modelling of the time pressure field is one way to achieve this.

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Title

Time-evolution modelling of seafloor scatter, Part 1: Theory

Abstract

A time-series model for acoustic seafloor backscattering is described. The method expresses analytically the elementary time-backscattered response of every seafloor surface and every seafloor volume infinitesimal element. For chosen geometric, acoustical and acquisition parameters, they are summed to produce in the time-domain a realization of the reverberated time pressure field received at the source. The approach is based on the Kirchhoff approximation for the seafloor interface backscattering and on the Small Perturbation theory for the seafloor volume. It only accounts for signal backscattering mechanisms of the compressional wave with the seafloor. The model is implemented using calculated height fields for the water-sediment interface and the distributed seafloor volume inhomogeneities. The analytical description of the model and its limitations is described in this report.

Keywords

seafloor backscattering – surface scattering – volume scattering – Kirchhoff – small perturbation – time domain

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