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COUPLED-MODE MODELLING OF ACOUSTIC SCATTERING FROM THREE-DIMENSIONAL, AXISYMMETRIC OBJECTS

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Executive Summary:

A better understanding of the three-dimensional scattering of acoustic energy from objects will lead to a significant improvement in our ability to detect and classify mines. These mines may be totally within the water column but are often partially or fully buried in sediment. Thus, it is important to be able to model the effects of burial on the scattered energy.

In this memorandum a computational approach to the modelling of scattering from three-dimensional azimuthally symmetric objects is described. Cylinders, cones, and spheres are examples of azimuthally symmetric objects, the shapes of which do not vary azimuthally about an interior central axis. Many mine shapes fall into this category. In the numerical examples presented, the method is applied to the case of a cylindrical disk surrounded by water and also to the case of the disk partially buried in an underlying sediment. The method is simplified by using the fluid approximation for the scattering object and surrounding environment; that is, only the compressional velocities and the densities are modelled. In the future, it would be desirable to extend the method to include the effects of shear energy and to model more realistic mine shapes.

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Coupled-mode modelling of acoustic scattering from three-dimensional, axisymmetric objects

John A. Fawcett

Abstract:

In this memorandum a coupled-mode method for computing the wavefield scattered by three-dimensional, axisymmetric objects is presented. The method provides a unified approach to solving three-dimensional scattering problems for axisymmetric objects in free-space, in a waveguide, or partially or fully buried in a basement. Numerical computations of scattering from finite cylinders in free space and embedded between two half-spaces are presented. Also, the method is used to compute the scattering from an infinitely long cylinder for which analytical results are available. The fluid approximation is used for both the object and the surrounding medium.

Contents

1 Introduction

Coupled modes [1-3] have been used by previous authors for the computation of the acoustic wavefield in two-dimensional range-dependent waveguides. The theory of coupled modes has also been applied to the acoustic scattering from three-dimensional azimuthally symmetric features [4-9]. In this type of three-dimensional geometry, the full three-dimensional scattering problem is broken down into a sequence of two-dimensional problems; a two-dimensional scattering problem for each angular Fourier component. Each of these two-dimensional problems can then be solved using the coupled-mode approach. Reference [5] considers scattering from a cylindrical elastic inclusion in a bounded elastic waveguide. Reference [6] uses Fourier decomposition in conjunction with a Boundary Integral Equation Method (BIEM), but uses the results from coupled mode computations for comparison with the BIEM results. The coupled-mode solution for scattering in an oceanic waveguide with azimuthally symmetric bathymetry is considered in [7] and references [8-9] consider a cylindrical island in the oceanic waveguide.

In this memorandum we use the azimuthally symmetric coupled-mode approach to compute the scattering from three-dimensional objects; in particular, a finite-length acoustic cylinder. We will consider two cases; when the object is surrounded by a homogeneous acoustic free-space and when the object is embedded between two half-spaces (i.e, partially buried). When we describe the technique used to solve these two types of problems, it will be apparent that the same methodology can be used to solve scattering problems for an object totally buried, above an interface, and within a water column bounded above by a pressure release surface. Thus this approach provides an uniform approach to a variety of object scattering problems.

The medium is divided into two main range sections - that occupied by the cylinder and the cylinder's exterior. Within the interior domain, the medium is taken to be vertically symmetric about the line $z = 0$ (see Fig.1) and consists of 3 layers, an upper homogeneous space, a layer with the cylinder's compressional velocity and density, and a lower homogeneous space. The exterior medium consists of two halfspaces with the same parameters as the upper and lower media in the interior domain; however, the interface between the two spaces, z_i, in the exterior domain may not necessarily be at $z = 0$. If, in fact, the upper and lower spaces have the same parameters, a free-space environment is modelled. Although we consider cylinders in this memorandum, more general azimuthally symmetric objects can be approximated by a sequence of cylindrical rings. In this case, instead of a single coupling matrix at a particular radius, there are coupling equations at each radial interface. This is analagous to the bathymetric staircase approximation which is used in, for example, reference [2].

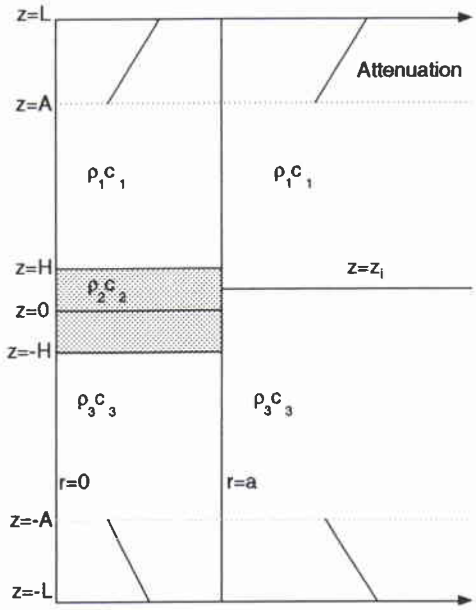

Figure 1: *Schematic diagram of range-dependent environment used to model object scattering*

In order to make the computational domain finite, we impose boundary conditions on large but finite values of z, $z = \pm L$. Then, in order to nullify the effects of these artificial boundaries we artificially add an attenuation profile to the medium near these boundaries. In order to compute the modal quantities for the resultant velocity profile we use Galerkin's method with the modes for the media without attenuation serving as the trial functions. This is the approach which was used by Evans and Gilbert [3].

There are closed-form solutions for the acoustic scattering from infinitely long cylinders and we compare the results computed from our method with these solutions. We present the results of some three-dimensional full field computations for penetrable finite cylinders in free space and for cylinders partially buried in an underlying half-space.

2 Theory

In this section we first describe, in general, the coupled-mode approach to azimuthally symmetric scattering problems. We then discuss how we determine the modes and eigenvalues for the problem using Galerkin's method and finally we discuss how we compute the intermodal integrals.

2.1 Azimuthally symmetric coupled-mode approach

Let us consider a 3-dimensional coordinate system (r, θ, z) with the origin of the coordinate system being at the centre of a finite length cylinder with radius a. We consider space divided into two radial sections; the interior section $0 < r < a$ and the exterior section $r > a$. The velocity and density profiles within the interior region are denoted as $c^{in}(z)$ and $\rho^{in}(z)$ and those for the exterior region as $c^{ex}(z)$ and $\rho^{ex}(z)$. In general, we will use the notation 'ex' to refer to quantities exterior to the radial extent of the cylinder and 'in' for the interior quantities. The profiles for the interior region contain a vertical section corresponding to the cylinder's velocity and density values. The Helmholtz equation for a point source at (r_s, θ_s, z_s) can be written

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial P}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 P}{\partial \theta^2} + \frac{\partial^2 P}{\partial z^2} + \frac{\omega^2}{c^2(r,\theta,z)}P = \frac{\delta(r-r_s)\delta(z-z_s)\delta(\theta-\theta_s)}{2\pi r_s} \quad (1)$$

where at $r = a$, the field must satisfy the continuity conditions

$$P^{in} = P^{ex}; \qquad P_r^{in}/\rho^{in}(z) = P_r^{ex}/\rho^{ex}(z). \quad (2)$$

The cylinder also has continuity conditions at its top and bottom edges, but these conditions will be satisfied by the modal functions we use for the interior region.

Due to the azimuthal symmetry of the cylinder we can write the solution of Eq.(1) in the form

$$P(r,\theta,z) = \sum_{\nu=0}^{\infty} \Theta_\nu(r,z)\epsilon_\nu \cos\nu(\theta-\theta_s) \quad (3)$$

where $\epsilon_\nu = 1$ for $\nu = 0$ and $\epsilon_\nu = 2$ otherwise. The partial differential equation for $\Theta_\nu(r,\theta)$ is

$$\frac{1}{r}\frac{\partial}{\partial r}(\frac{r\partial\Theta_\nu}{\partial r}) + \frac{\partial^2\Theta_\nu}{\partial z^2} + (\frac{\omega^2}{c^2(r,z)} - \frac{\nu^2}{r^2})\Theta_\nu = \frac{\delta(r-r_s)\delta(z-z_s)}{2\pi r_s} \tag{4}$$

and the boundary conditions for $\Theta_\nu(r,z), \nu = 0,\ldots,\infty$ are those of Eq.(2). Equation(4) now defines a two-dimensional range-dependent scattering problem with the angular order entering through the term $-\nu^2/r^2$. This equation can be solved in a variety of ways; in this memorandum we utilize the coupled mode approach.

First we truncate the infinite medium by imposing the boundary conditions

$$\Theta_\nu(r, z = L) = 0; \qquad \frac{\partial\Theta_\nu(r, z = -L)}{\partial z} = 0 \tag{5}$$

We then minimize the effect of these boundary conditions by introducing artificial attenuation profiles near these boundaries. However, if we wish to model a pressure release surface at the top of a waveguide we would not add an attenuating profile near the top boundary. We seek a solution for $\Theta_\nu(r,z)$ in terms of the vertical modes for the 2 media; $\phi_n(z)$ will denote a member of the set of modes for the exterior of the cylinder and ψ_n a member of the set of modes for the interior. We can write

$$\Theta_\nu^{ex}(r,z) = \sum_{n=1}^{N}[a_n\phi_n(z)H_\nu(k_n^{ex}r) + \Gamma_n\phi_n(z)J_\nu(k_n^{ex}r)] \tag{6a}$$

where the coefficients a_n are to be determined and

$$\Gamma_n = \frac{i}{4}H_\nu(k_n^{ex}r_s)\phi_n(z_s) \tag{6b}$$

in the case of a point source, i.e., Γ_n are the modal expansion coefficients of the known incident field. For $r < a$ we write

$$\Theta_\nu^{in}(r,z) = \sum_{n=1}^{N} b_n\psi_n(z)J_\nu(k_n^{in}r). \tag{6c}$$

The modes $\phi_n(z)$ satisfy the vertical eigenvalue equation

$$\frac{d^2\phi_n(z)}{dz^2} + [\frac{\omega^2}{(c^{ex})^2(z)} + iQ(z)]\phi_n(z) = (k_n^{ex})^2\phi_n(z) \tag{7}$$

with the appropriate continuity conditions at medium discontinuities and the boundary conditions of Eq.(5). In Eq.(7) we have explicitly indicated the presence of an attenuation profile $Q(z)$. There is a similar eigenvalue/eigenfunction problem for the interior eigenfunctions $\psi_n(z)$.

Using the mode set from Eq.(7), the system of coupled mode equations is formulated. First we write for the continuity of pressure at $r = a$,

$$\sum_{n=1}^{N} b_n \psi_n(z) J_\nu(k_n^{in} a) - a_n \phi_n(z) H_\nu(k_n^{ex} a) = \sum_{n=1}^{N} \Gamma_n \phi_n(z) J_\nu(k_n^{ex} a) \tag{8a}$$

and for the continuity of the density-normalized radial derivative of pressure,

$$\sum_{n=1}^{N} b_n \frac{\psi_n(z)}{\rho^{in}(z)} J_{\nu,r}(k_n^{in} a) - a_n \frac{\phi_n(z)}{\rho^{ex}(z)} H_{\nu,r}(k_n^{ex} a) = \sum_{n=1}^{N} \Gamma_n \frac{\phi_n(z)}{\rho^{ex}(z)} J_{\nu,r}(k_n^{ex} a). \tag{8b}$$

Multiplying Eq.(8a) by $\psi_m(z)/\rho^{in}(z)$ for $m = 1, ..., N$ and integrating from $z = -L$ to $z = L$ we obtain

$$b_m J_\nu(k_m^{in} a) - \sum_{n=1}^{N} C_{nm} a_n H_\nu(k_n^{ex} a) = \gamma_m \tag{9a}$$

where

$$C_{nm} \equiv \int_{-L}^{L} \frac{1}{\rho^{in}(z)} \phi_n(z) \psi_m(z) dz \tag{9b}$$

and

$$\gamma_m \equiv \sum_{n=1}^{N} J_\nu(k_n^{ex} a) \Gamma_n \int_{-L}^{L} \frac{1}{\rho^{in}(z)} \phi_n(z) \psi_m(z) dz. \tag{9c}$$

Similarly, we multiply Eq.(8b) by $\psi_m(z)$ for $m = 1, ..., N$ and integrate from $z = -L$ to $z = L$ to obtain

$$b_m J_{\nu,r}(k_m^{in} a) - \sum_{n=1}^{N} D_{nm} a_n H_{\nu,r}(k_n^{ex} a) = \Sigma_m \tag{10a}$$

where

$$D_{nm} \equiv \int_{-L}^{L} \frac{1}{\rho^{ex}(z)} \phi_n(z) \psi_m(z) dz \tag{10b}$$

and

$$\Sigma_m \equiv \sum_{n=1}^{N} J_{n,r}(k_n^{ex} a) \Gamma_n \int_{-L}^{L} \frac{1}{\rho^{ex}(z)} \phi_n(z) \psi_m(z) dz. \tag{10c}$$

Thus we have a $2N \times 2N$ system of equations for the coefficient vector $(b_n; a_n)$ of the form

$$\begin{pmatrix} J_\nu(k_n^{in} a)\delta_{n,m} & C_{nm} H_\nu(k_n^{ex} a) \\ J_{\nu,r}(k_n^{in} a)\delta_{n,m} & D_{nm} H_{\nu,r}(k_n^{ex} a) \end{pmatrix} \begin{pmatrix} b_n \\ a_n \end{pmatrix} = \begin{pmatrix} \gamma_m \\ \Sigma_m \end{pmatrix} \tag{11}$$

where $\delta_{n,m}$ denotes the Kronecker delta function. The solution of this system yields the modal coefficients for both the interior and exterior modal sets for the νth angular order.

2.2 Computation of modes and eigenvalues

We first consider the computation of the modes and eigenvalues for the case in which there is no added attenuation profile, $Q(z) = 0$ in Eq.(7) - these modes serve as the basis set for the modes when there is an attenuation profile. The medium in the interior cylindrical region consists of the layer corresponding to the object, and upper and lower bounding halfspaces. In the upper portion of the medium we take the solution to be proportional to

$$\sin(\sqrt{(\omega^2/c_1^2 - k_n^2)}[z - L]); \tag{12a}$$

in the lower part of the medium we take the solution to be proportional to

$$\cos(\sqrt{(\omega^2/c_3^2 - k_n^2)}[z + L]). \tag{12b}$$

Within the vertical layer corresponding to the cylinder $-H \leq z \leq H$, we consider the solution to have the form

$$a \sin(\sqrt{(\omega^2/c_2^2 - k_n^2 z)}) + b \cos(\sqrt{(\omega^2/c_2^2 - k_n^2 z)}). \tag{12c}$$

Thus there are 4 unknown coefficients and there are 4 equations derived from the continuity of p and p_z/ρ at $z = \pm H$. This system of homogeneous equations for the coefficients has only the zero solution except at the values of k_n for which the determinant of the system is zero. Numerically, we search along the real line to locate these values of k_n.

The eigenvalue/mode problem for the exterior medium is similar. The modes have the form of Eq.(12a) in the upper medium and that of Eq.(12b) in the lower medium. The eigenvalue problem is reduced to satisfying the standard continuity conditions at the interface $z = z_i$.

2.3 The Galerkin method

Thus far in the discussion, we have assumed that we can determine the modes and eigenvalues for Eq.(7). This is not a trivial problem when there is attenuation in the problem, as in this case the eigenvalues are complex and it is no longer possible to simply search for the eigenvalues along the real line. Instead of searching for complex-valued eigenvalues we will use Galerkin's method to approximate the eigenvalues and eigenfunctions of the problem.

We wish to find modal solutions to Eq.(7),

$$\frac{d^2\phi_n(z)}{dz^2} + [\frac{\omega^2}{c^2(z)} + iQ(z)]\phi_n(z) = k_n^2\phi_n(z) \tag{13}$$

In subsection 2.2 we discussed finding the solutions to

$$\frac{d^2\tau_n(z)}{dz^2} + \frac{\omega^2}{c^2(z)}\tau_n(z) = \hat{k}_n^2\tau_n(z) \tag{14}$$

with the appropriate boundary conditions and interface continuity conditions. We use these solutions to construct the solution to Eq.(13); we take

$$\phi_n(z) = \sum_{k=1}^{N} a_k^n \tau_k(z). \tag{15}$$

Substituting the expression of Eq.(15) into Eq.(13) we obtain

$$\sum_{k=1}^{N} a_k^n \frac{d^2\tau_k(z)}{dz^2} + [\frac{\omega^2}{c^2(z)} + iQ(z)]a_k^n\tau_k(z) = k_n^2 a_k^n \tau_k(z) \tag{16}$$

Multiplying Eq.(16) by $\tau_m(z)/\rho(z)$ and integrating with respect to z from $z = -L$ to $z = L$ and using Eq.(14) we obtain the eigenvalue/eigenvector problem

$$A_{m,k}a_k^n = \lambda_n a_m^n \tag{17}$$

where the (m,k) element of A is given by

$$A_{m,k} = \int_{-L}^{L} \frac{1}{\rho(z)}\tau_m(z)\tau_k(z)iQ(z)dz + \delta_{m,k}\hat{k}_m^2. \tag{18}$$

A standard eigenvalue/eigenvector decomposition of the matrix A yields the vectors of coefficients $a_k^n, k = 1, ..., N$ of the trial functions for the mode $\phi_n(z)$ and the corresponding eigenvalues. In Eq.(18) we choose $Q(z)$ to be zero everywhere, except for the 2 intervals $z = [A, L]$ and $z = [-L, -A]$. Within these intervals we take $Q(z)$ to be linearly increasing from 0 to a user-input value for $z = \pm L$. However, as mentioned previously, we can retain either of the boundary conditions of Eq.(5) by using $Q(z) = 0$ near the appropriate boundary.

2.4 Computation of inter-modal projections

In subsection 2.1 above, it can be seen that it is necessary to compute integrals of the form

$$\int_{-L}^{L} \frac{1}{\rho}\phi_n(z)\psi_m(z)dz. \tag{19}$$

We have shown how to compute the modes ϕ_n and ψ_m in terms of simpler modal functions using Galerkin's method. We now describe an efficient analytic procedure for computing the integrals of the form of Eq.(19). Let

$$\phi_n(z) = \sum_{k=1}^{N} q_k^n \tau_k(z) \tag{20a}$$

and

$$\psi_n(z) = \sum_{k=1}^{N} r_k^n \Omega_k(z). \tag{20b}$$

Then Eq.(19) can be expressed as

$$\int_{-L}^{L} \frac{1}{\rho}\phi_n(z)\psi_m(z)dz = \sum_{i=1}^{N}\sum_{j=1}^{N} q_i^n b_j^m \int_{-L}^{L} \frac{1}{\rho(z)}\tau_i(z)\Omega_j(z)dz. \tag{21}$$

The integrals of Eq.(21) are straightforward to evaluate analytically as the functions $\tau_i(z)$ and $\Omega_j(z)$ are composed of simple sine and cosine functions. We can express Eq.(21) in terms of matrices

$$\int_{-L}^{L} \frac{1}{\rho}\phi_n(z)\psi_m(z)dz = ACB^T \tag{22}$$

where

$$A_{i,j} \equiv q_j^i : \quad B_{i,j} \equiv r_j^i : \quad C_{i,j} \equiv \int_{-L}^{L} \frac{1}{\rho(z)}\tau_i(z)\Omega_j(z)dz. \tag{23}$$

The profile $\rho(z)$ in Eq.(23) may be either $\rho^{in}(z)$ or $\rho^{ex}(z)$ depending on whether one requires the projections of Eq.(9c) or (10c).

In summary, we construct the modes for the two environments, which have an artificial attenuation profile, by Galerkin's method. This method allows us to express the modes of these environments in terms of simpler modal functions. We compute the inter-modal integrals in terms of these simpler modes. Having computed the matrix elements for Eq.(11) we can compute the interior and exterior modal coefficients for the νth angular order for the cylindrical system. Repeating this for several values of ν we can construct the three-dimensional solution by Fourier synthesis. It is important to note that the modal computations and projections etc. do not depend upon the value of ν and these computations are only done once.

2.5 Computation of the acoustic field

The solution of Eq.(11) yields the νth angular modal coefficients ($b_j^\nu, j = 1, ..., N$) for the interior pressure field and the coefficients a_j^ν for the scattered field in the exterior region. The total pressure field within the interior region can be computed by

$$p^{in} = \sum_{\nu=0}^{N_B} \epsilon_\nu \sum_{j=1}^{N} b_j^\nu J_\nu(k_j^{in} r)\psi_j(z)cos[\nu(\theta - \theta_s)] \tag{24}$$

and in the exterior

$$p^{ex} = \sum_{\nu=0}^{N_B} \epsilon_\nu \sum_{j=1}^{N} a_j^\nu H_\nu(k_j^{ex} r)\phi_j(z)cos[\nu(\theta - \theta_s)] + p^{src}, \tag{25}$$

where p^{src} is the incident field upon the cylinder due to the source and would, in fact, be the total pressure field in the absence of the cylinder. We require a computational expression for this incident field. It is possible to add in the angular/modal expansion for the incident field at each point of computation. However, we instead use simpler representations of the incident field. For example, for a surrounding homogeneous space we use

$$p^{src} = \frac{exp(i\omega/c|\vec{x}_r - \vec{x}_s|)}{4\pi|\vec{x}_r - \vec{x}_s|} \tag{26a}$$

where $\vec{x}_r$ and $\vec{x}_s$ are the three-dimensional position vectors of the receiver and source respectively. In the case of a two-halfspace medium, it is more difficult to compute the incident field; however in terms of our approximate mode set we can write that

$$p^{src} \approx \sum_{j=0}^{N} \frac{i}{4}\phi_j(z) H_0(k_j|\vec{r}_r - \vec{r}_s|) \tag{26b}$$

where $\vec{r}_r$ and $\vec{r}_s$ are the two-dimensional position vectors in the (r,θ) plane of the receiver and source respectively. The computational advantages of using a source representation of the form of Eq.(26b) have been discussed in [8].

3

Numerical Examples

3.1 An infinite cylinder

As a test case for the coupled mode method we consider an infinitely long cylinder with acoustic parameters $c_c = 5950 m/s$ and $\rho = 7700 kg/m^3$ with a radius of 0.5 m. These correspond to the compressional parameters of steel. The surrounding homogeneous medium has parameters $c_0 = 1500 m/s$ and $\rho = 1000 kg/m^3$. In order to obtain an analytical solution to this problem, we perform a Fourier Transform with respect to z to obtain the two-dimensional problem

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial P}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 P}{\partial \theta^2} + (\frac{\omega^2}{c^2(r,\theta,z)} - k_z^2)P = \frac{\delta(r-r_s)\delta(\theta-\theta_s)}{2\pi r_s}e^{ik_z z_s}. \quad (27)$$

For each value of k_z we can write the exterior scattered pressure field, $\tilde{p}^{sc}$ in the form

$$\tilde{p}^{sc}(r,\theta;k_z) = \sum_{n=0}^{N} H_n^1(\gamma r)\alpha_n \cos[n(\theta-\theta_s)] \quad (28a)$$

where

$$\gamma \equiv \sqrt{\omega^2/c_0^2 - k_z^2} \quad (28b)$$

and α_n are determined by the continuity conditions of Eqs.(2). Similarly, the interior solution has the form

$$\tilde{P}(r,\theta;k_z) = \sum_{n=0}^{N} J_n(\gamma^{in} r)\beta_n \cos n(\theta-\theta_s). \quad (29)$$

The three-dimensional field is then constructed by performing the numerical quadrature of the wavenumber integral,

$$P(r,\theta,z) = 2\int_{\zeta} \tilde{P}(r,\theta,k_z) cos(k_z(z-z_s))dk_z \quad (30a)$$

where ζ represents an integration contour in the complex plane (for Re$(k_z) > 0$). In our computations we used

$$\zeta(t) = t - i\tanh(4t/k_{max})/10 \quad (30b)$$

where we consider $0 \leq t \leq k_{max} = 2\omega/1500$

In Fig.2a, we show the coupled-mode solution in the plane $z = -1m$ for a 1500-Hz source located at a range of 10 m from the cylinder at $z = 1$ m. Thus the source is off the righthand side of the plot and we are considering a horizontal section of the field. The circular cross-section of the cylinder is indicated in the figure and the axis of symmetry (the cylinder's axis) is perpendicular to the plane of the page. In Fig.2b the corresponding analytic solution is shown. As can be seen, the agreement between the coupled-mode and analytic solutions is excellent. For the coupled mode solution, we used 2 homogeneous media; one for the exterior fluid and one with the cylinder's parameters as the interior medium. The minimum and maximum values of z for the computational domain are $z = \pm 22$m. The attenuation profiles start at $z = \pm 12$m and the attenuation increases linearly from 0.0 dB/λ to 5.47 dB/λ at the boundaries. We used 11 azimuthal functions and 141 vertical modes.

3.2 A cylindrical disk in free space

We now consider a finite cylinder with the same acoustic parameters as above. The radius is 0.5 m and the total length is also 0.5 m. The point source is located 10.0 m in range from the cylinder and at z=0.25 m ($\theta = 0$) (i.e., the vertical location of the source is level with the top of the cylinder and is off to the right of the cylinder). We compute the total pressure field in a 4m $\times$ 4m grid in a vertical section, the x-z plane, (i.e., $\theta = 0, \pi$) around and in the cylinder and display the results in Fig.3 for frequencies 750 Hz, 1500 Hz, and 3000 Hz. We are now considering a vertical section of the field, the cylinder appears as a rectangle in Fig.3 and the axis of symmetry of the cylinder is the z-axis. There is significant scattering from the cylinder in all three cases. There is an interference pattern on the incident side of the cylinder, resulting from the interference of the backscattered and incident energy. As would be expected this pattern is more rapidly oscillating in the case of the higher frequencies. Also, there is a shadow zone behind the cylinder. This zone is more sharply defined in the case of the higher frequencies although some energy is still evident behind the cylinder. For the 750 Hz computation we used 100 modes and 7 azimuthal terms, for 1500 Hz, 140 modes and 11 azimuthal terms, and for the 3000 Hz example 210 modes and 17 azimuthal terms.

3.3 A cylindrical disk embedded in a half-space

We now consider the cylinder of the above example buried 90% in a lower halfspace. Instead of a point source, a vertical array of phased sources (17 sources at 0.25 m

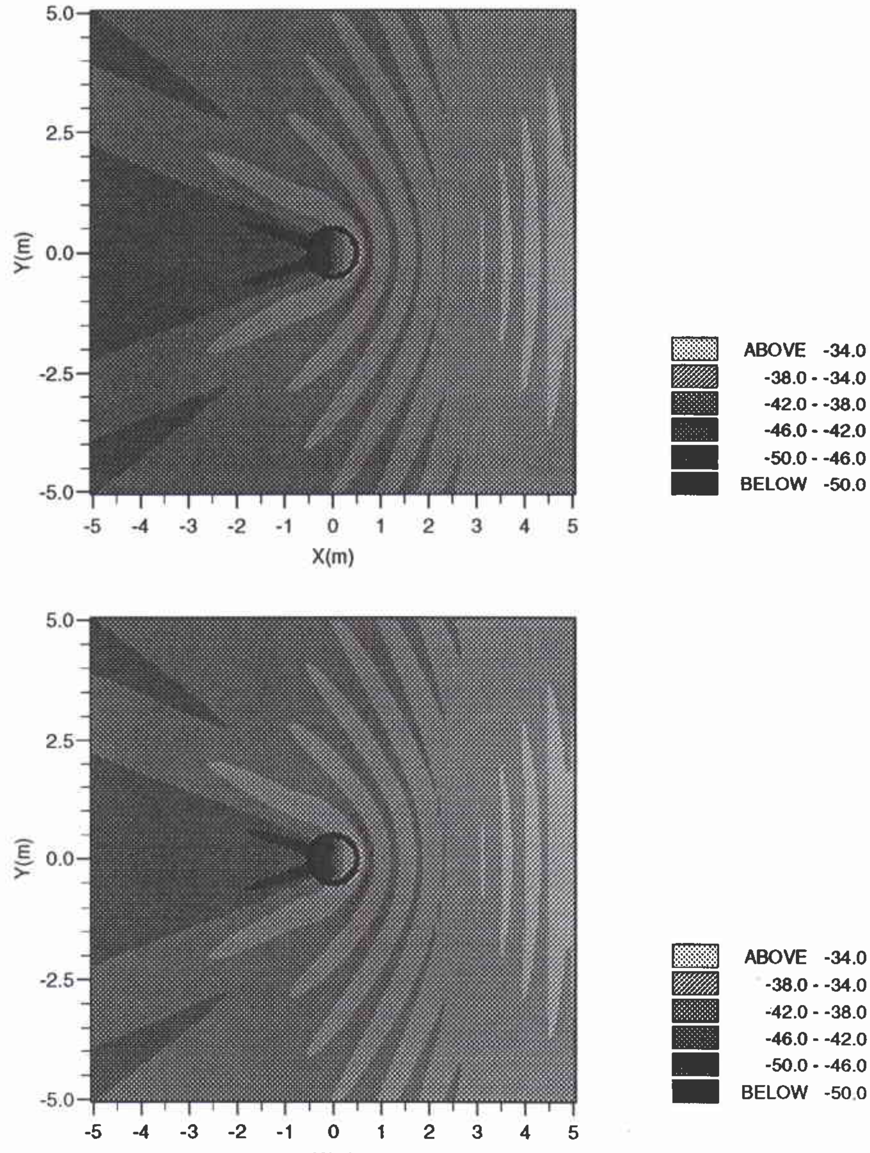

Figure 2: *Analytic and couple mode solutions for scattering from an infinite cylinder (total field is shown)*

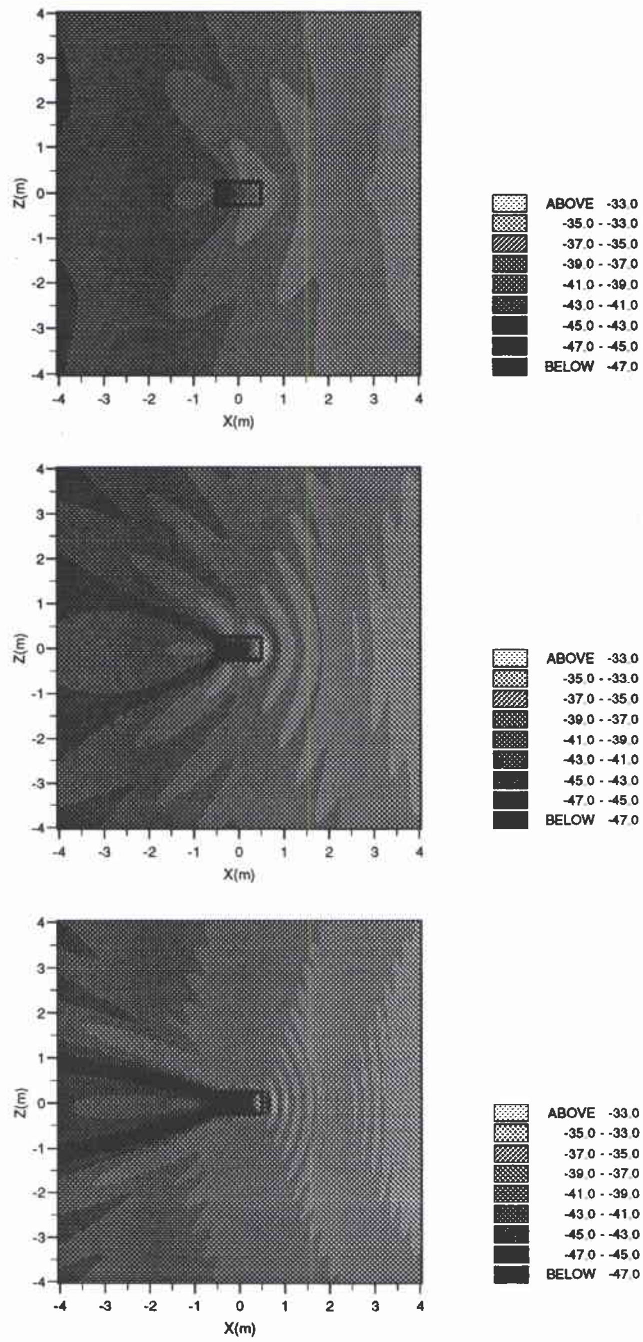

Figure 3: *Scattering from cylindrical disk for 750Hz, 1500Hz, and 3000 Hz (total field is shown). This is a vertical section of the pressure field (y=0)*

spacing or 4 wavelengths for 1500 Hz) is used. In the first example we consider the centre of the source at a height of 6 m above the centre of the cylinder and 1m to the right; hence the source beam is almost normally incident upon the cylinder. The bottom half-space has a sound speed of 1550 m/s and a density of 1100 kg/m^3. The two-dimensional plot of the total pressure field in the x-z plane is shown in Figure 4. Because the energy is almost normally incident, there is considerable penetration into the bottom. A beam-like structure is evident in the shadow area of the cylinder. In the backscattered direction there is a complex interference pattern caused by the interaction of the incident beam, the reflection of the beam from the interface, and the field scattered by the cylinder.

In the second example, the centre point of the source array is located at a horizontal angle of 20° with respect to the cylinder centre. The sources are then phased to produce an incident beam of 20°, directly incident upon the cylinder. The sediment sound speed is 1800 m/s for this example and the density 1500kg/m^3 and hence the incident angle is subcritical. The interference between the incident and the energy scattered from the cylinder is evident in the backscatter direction and a shadow zone behind the cylinder can be seen in the sediment. The field in the water column, in the forward scatter direction, is dominated by the reflection of the incident beam off the interface.

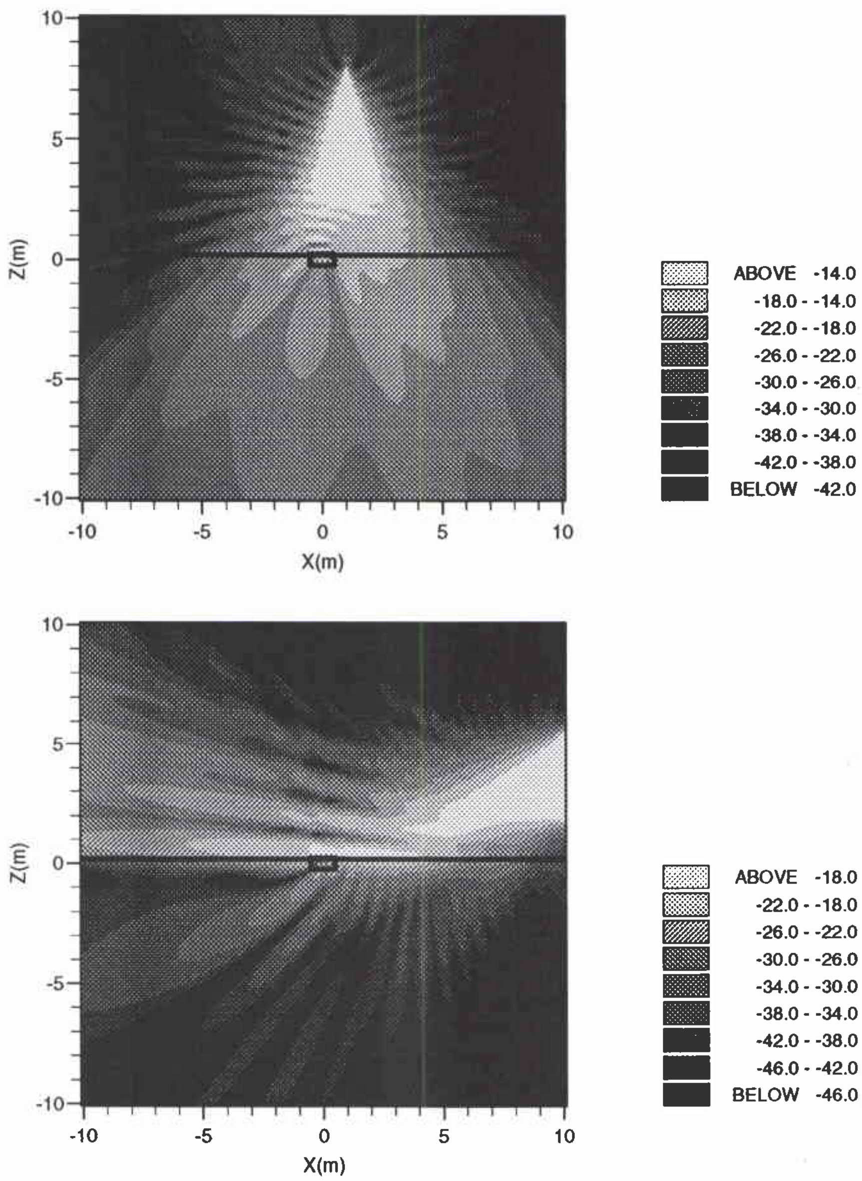

Figure 4: *Analytic and coupled mode solutions for scattering from a cylindrical disk partially buried(total field is shown). Vertical sections (y=0) of the pressure field are shown. For the upper figure the energy is almost normally incident and the sediment has a sound speed* $C_p = 1550m/s$ *and* $\rho = 1100kg/m^3$; *for the bottom figure the energy has a grazing angle of 20°* *and the sediment has a sound speed of* $C_p = 1800m/s$ *and* $\rho = 1500kg/m^3$.

4
Summary

We have shown that coupled-modes can be effectively used to solve acoustic three-dimensional object scattering problems when the object and the surrounding environment are azimuthally symmetric. We used Galerkin's method to compute eigenfunctions and eigenvalues for vertical velocity profiles with added attenuation profiles. In this memorandum, the scattering objects considered were finite cylinders; however more complex structures including shelled cylinders can be considered by coupling together additional cylindrical domains.

The approach described in this memorandum can be used to model the object scattering in a variety of background environments; free-space, half-space, buried and partially buried, and waveguides. It should be possible to use coupled elastic modes [5] to solve analagous scattering problems for elastic objects. In this memorandum, we considered scattering problems for frequencies up to 3000 Hz - the frequency could be increased at the expense of computing more modes and having larger coupling matrices. Alternatively, for free-space or half-space scattering problems, it might be possible in some cases to reduce the vertical extent of the surrounding space, thereby reducing the number of required modes.

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Title

Coupled-mode modelling of acoustic scattering from three-dimensional axisymmetric objects

Abstract

In this memorandum a coupled-mode method for computing the wavefield scattered by three-dimensional, axisymmetric objects is presented. The method provides a unified approach to solving three-dimensional scattering problems for axisymmetric objects in free space, in a waveguide, or partially or fully buried in a basement. Numerical computations of scattering from finite cylinders in free space and embedded between two half-paces are presented. Also, the method is used to compute the scattering from an infinitely long cylinder for which analytical results are available. The fluid approximation is used for both the object and the surrounding medium.

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