THEORY OF SCATTERING AND REFLECTION OF SOUND FROM THE SEA SURFACE

by

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Abstract:

Theoretical work on scattering and reflection of sound from the random time-varying rough sea surface is critically reviewed.
It is shown that sound scattering from the sea surface may be treated as a purely acoustic boundary value problem with the sea surface as the given boundary which may be considered as being quasi-static (linear response approximation, neglection of the fluid motion in the surface waves, subsequent inclusion of time-variations). The formulation of this boundary value problem in Fourier space (plane wave representation of the acoustic wave field) is most suited for the application of perturbation techniques and statistical averaging. The perturbation techniques which can be adapted to various limiting cases show that sound scattering from the sea surface is essentially determined by phase modulation spectra as expected for "phase objects" like the sea surface layer. After statistical averaging the corresponding functions are the Fourier transforms of the characteristic function of the joint probability density of the surface displacements. On the basis of an appropriate 1. order approximation which yields sufficiently good results over a wide range of surface roughness parameters and grazing angles the two-scale (or wave-facet) model for the scattering from the sea surface can be discussed. Shadowing corrections have to be considered separately.

I Introduction

When an acoustic signal is incident on a randomly time-varying sea surface several effects can be observed:
Part of the acoustic energy is scattered off the specular reflection direction resulting in a reflection loss and in a surface reverberation. Time- and frequency-smearing and amplitude fluctuations of the signal occur due to the time variation of the sea surface. The prediction of these properties is one of the most important problems of underwater acoustics. The theoretical description of this problem is complicated mainly by four facts:

1)
The spatial spectrum of the sea surface waves contains short wave-length ("ripple") and long wave-length components ("swell") which both, simultaneously, can be relevant for the scattering and reflection of sound. The wave-lengths of the short wave-length components are comparable with the wave-lengths of the sound used in underwater acoustics. Therefore diffraction effects have to be taken seriously - more seriously than e.g. in the case of light diffraction from standard optical gratings. This means that not only the ray approach has to be rejected but even more than that the Kirchhoff diffraction theory has to be considered critically.

2)
On the other hand, the long wave length components of the sea surface give rise to wave heights which can be much larger than the acoustic wave length. Therefore perturbation expansions of the sound field with respect to the surface elevations cannot be used straight away. If a ray approach is used shadowing and multiple reflections have to be taken into account.

3)
The sea surface is time-varying. Therefore the dynamics of the surface waves have to be considered accurately and the diffraction theory has to be formulated as a boundary value problem with time-dependent boundary conditions.

4)
The sea surface has a random character. Therefore the general diffraction theory should be formulated such that statistical averaging can be applied without difficulties. -

Several different approaches to the problem have been given in the literature. A comparison of these approaches is sometimes complicated by the fact that they are given in different representations. This may be the reason for the fact that cross connections and differences of the approximations are not very well established even in the very recent literature.

The present paper is an attempt to fill up these gaps by putting the main approximations in a uniform framework. A plane wave representation of the acoustic wave field seems to be most favored to show up the connections and differences of the approximations especially since it is quite flexible to describe various geometrical situations and to employ statistical averaging. Therefore it is used throughout this paper.

Another intention of this paper is to describe the scattering and reflection of sound from the sea surface by a most simple version of approximation which yields sufficiently good results over a wide range of surface roughness parameters and grazing angles. Such a formula may be used as a basis for further approximations, e.g. for a two-scale model for the scattering by the sea surface (swell-ripple-model).

This paper does not claim to give a complete survey of the literature on the subject. For further information and more comprehensive bibliographies we may refer to the review papers of Lysanov [1] , Fortuin [2] , Clarke [3] , Dera et al. [4] , and Hurdle [5] .

II First survey of the main approaches to the problem

The problem of scattering and reflection of sound from a surface like that of the sea has been attacked with various approaches. The main stages in the developement of these approaches may be outlined as follows. More details will be discussed later.

1. The Kirchhoff approac h

The heuristic Kirchhoff approach [6,8-10] is based on "local reflections" from locally flat boundaries. Accordingly the values of the unknown field quantity on the surface which are not primarily determined by the boundary condition (here the normal derivatives of the sound field pressure) are approximated by the corresponding values for a flat boundary (see Fig. 1).

Brekhovskikh [11] used this approximation to discuss the scattering of waves from (random) rough surfaces. Eckart [12] simplified it for the case of a random rough surface having only small slopes. Parkins [13,14] discussed the more general case and considered the effects of time-variation of the surface. Parkin's expressions have been recently worked out by Swarts and Scharf [15,16]. In the treatments of Parkins and Swarts and Scharf the Helmholtz-Kirchhoff integral for a fixed static boundary is used which means that the surface is considered as quasi-static. In order to allow for Doppler effects in the scattered field the time-variations of the surface are suplementarily taken into account via variations of the time retardation.

2. The Rayleigh approach

In the Rayleigh approach [7,10] a plane wave expansion of the wave field valid for a planely bounded half space below the boundary is schematically extended up to the boundary such that the boundary condition can be directly imposed. The crucial point is that the disturbed field is represented by outgoing waves only - even in the valleys of the rough surface (see Fig. 2).

The Rayleigh approach was applied to wave scattering from random rough surfaces by Marsh [17] and worked out for the scattering of underwater sound from the sea surface by Marsh, Shulkin and Kneale [18]. Crowther [19] introduced the concept of phase modulation spectra which is also important in a rigorous treatment of the problem (see point (7)). Recently Bachmann [20,21] has applied the Rayleigh approach within the framework of a "composite-roughness scattering model" to the problem of acoustic backscattering from the sea surface. In all these papers the Rayleigh approach was only used for static surfaces but it can easily be extended to time-varying surfaces.

3. The perturbation approach of Meecham

A perturbation approach applied to the Helmholtz-Kirchhoff integral equation for the boundary values of the unknown field quantity (here the normal derivatives of the sound field pressure) was first discussed by Meecham [22]. The starting point is the undisturbed mean boundary. Fortuin [23] applied this method to a time-varying random rough surface (similar to Parkins [13,14] for the Kirchhoff approach) by subsequently taking time retardation into account (see Fig. 3).

4. Calculations for special periodic boundaries

Analytical and subsequent numerical calculations for special shapes of static rough surfaces can be used to check the various approximations. Thus Uretsky [24] derived a matrix equation from the Helmholtz-Kirchhoff integral to study the scattering of sound from a sinosoidal surface. For the same problem Jamnejad-Dailami et al. [25] compared the results obtained from an analytical continuation technique with that obtained from the Rayleigh approach.
De Santo analysed the scattering from periodic rectangular comb structures with "solf" [26] and "hard" [27] boundaries. Watermann [28] has recently studied the reflection behaviour of a periodic interface (see further references in this latter paper).

5. Middleton's general statistical theory

Middleton [29,30] has given a general statistical theory for the transmission of a signal like an acoustic signal through an inhomogeneous medium like the ocean. Both surface and volume scattering are included, separately and together, for general source-receiver geometries, and arbitrary transmitting and receiving apertures. The inhomogeneous medium is replaced by a homogeneous one in which a random ensemble of independent point scatterers is embedded. The properties of the point scatterers, e.g. for the scattering of sound from the sea surface, is not specified in the theory of Middleton. The present paper is more concerned with the physics of the scattering process itself rather than with the other aspects of the transmission of a signal from a source to a receiver.

6. Hasselmann's non-linear wave-wave interaction approach

Hasselmann [31,32,33] has developed a general dynamic non-linear wave-wave interaction approach for random wave fields. A diagrammatic notation analogous to that of quantum field theory was introduced: one type of diagrams to describe the perturbation expansion of the field amplitudes (interaction diagrams) and a second one to summarized the energy transfer expressions (transfer diagrams) [31]. Essen and Hasselmann [34] have applied their "weak interaction approximation" to the scattering of acoustic modes in an oceanic wave guide disturbed by surface waves.

In Hasselmann's approach the acoustic wave field is built up by the modes of the undisturbed space under consideration. In this respect it is essentially equivalent to Rayleigh's approach and has the defects of the latter one.

7. Perturbation approach in the Fourier space for static random rough surfaces

Zipfel and DeSanto [35] , and simultaneously the author [36] , have developed an integral equation treatment in the Fourier space to describe wave scattering from static random rough surfaces. This treatment is the Fourier space counterpart to Meecham's perturbation approach [22] and is closely related to Uretsky's treatment for periodic boundaries [24] . In the space below the surface region the incident and the scattered wave fields are described by plane wave representations and the Helmholtz-Kirchhoff integrals along with the unknown boundary values are completely Fourier-transformed. In this plane wave scattering formalism the relation between the plane wave decomposition of the scattered field and that of the incident field is established via a scattering matrix essentially determined by phase modulation spectra as introduced by Crowther [19] .

Zipfel and DeSanto [35] have worked out a diagrammatic approach for the description of the expansions used which is similar to that of Hasselmann [31] .

8. Perturbation approach for time-varying random rough surfaces

The problem of scattering of waves from time-varying random rough surfaces has been treated with satisfactory rigour only very recently by Zipfel [37] . Zipfel has formulated the problem in terms of an integral equation in the $(\underline{k}, \omega)$ - space quite in accordance with the treatment for a static random rough surface given by Zipfel and DeSanto [35] .

Konrady [36] has attacked the problem by transforming the wave equation and the conditions for the "geometrically complicated" boundary to another partial differential equation with a "simpler" boundary condition. However, Zipfel's treatment seems to be more straightforward.

A large variety of modifications and simplifications of the first three approximations have been used in the literature. In these approaches the scattering of sound from the time-varying sea surface is treated as a boundary value problem with a quasi-static boundary for the purely acoustic wave field. Corrections to the sound field pressure due to the motion of the fluid below the surface, as comprised in Hasselmann's approach [31-33] , and those due to the motion of the surface itself, as induced in Zipfel's treatment [37] , can be disregarded for practical purposes. This will be discussed in the following chapter.

III Discussion of the basic approximations

In this chapter we will discuss the basic approximations which underlie the treatment of sound scattering from the sea surface as a boundary value problem with a quasi-static boundary for the purely acoustic wave field (use of the Helmholtz-Kirchhoff integral).

1. Basic equations

Acoustic and surface waves are governed by the hydrodynamic equations. The coupling of acoustic and surface waves is determined by the nonlinear terms in the hydrodynamic equations and the boundary conditions. To study this coupling the medium may be considered as an inviscid homogeneous one. Then, Euler's equations of motion read (see e.g. [9])

$$\dot{\underline{v}} + (\underline{v}\underline{\nabla})\underline{v} = -(\underline{\nabla}p)/\varrho + \underline{g}, \tag{1}$$

with $$\underline{\nabla}p = \left(\partial p/\partial\varrho\right)\underline{\nabla}\varrho;$$

and the continuity equation

$$\dot{\varrho} + \underline{\nabla}(\varrho\underline{v}) = 0, \tag{2}$$

where $\underline{v}$ is the velocity, p the pressure, ϱ the mass density of the fluid and $\underline{g}$ the acceleration due to gravity. For a pressure release surface like the sea surface which may be described by $z = \zeta(x, y, t)$ (see Fig. 4) the boundary condition is

$$p[z = \zeta(x, y, t)] = 0 \tag{3}$$

Omission of gravity, $\underline{g} = 0$, gives the equations for acoustic waves, while neglection of compression, $\dot{\varrho} = \underline{\nabla}\varrho = 0$, yields the equations for surface waves (gravity and capillarity waves). Thus for a homogeneous medium, linearization of (1) and (2) and elimination of $\underline{v}$ and ϱ leads to the acoustic wave equation

$$\Delta p - \ddot{p}/c_a^2 = 0, \tag{4}$$

with the dispersion relation for plane sound waves

$$\omega_a = c_a k_a , \tag{5}$$

where ω_a and k_a are the angular frequency and the wave number of a sound wave respectively, while $c_a = \sqrt{\partial p / \partial \varrho}$ is the sound velocity.

With $\dot{\varrho} = \underline{\nabla} \varrho = 0$ linearization of (1) and the boundary condition (3) leads to the dispersion relation for surface waves. For deep sea this reads

$$\omega_b^2 = k_b g + \alpha k_b^3 / \varrho , \tag{6}$$

where ω_b and k_b are the angular frequency and the wave number of a gravity-capillarity surface wave respectively and α is the specific surface tension. For the study of sound scattering from the sea surface the spectrum and the height distribution of the surface waves may be considered as given.

2. Coupling of acoustic and surface waves

The coupling of acoustic modes and surface waves is determined by the nonlinear terms in eqs. (1) and (2) and by the nonlinear terms in the expansion of the boundary condition (3) around $z = 0$. The exchange of energy between acoustic modes is described by the terms in (1) and (2) being quadratic in the acoustic field amplitudes. Rough estimates of the linear term $\dot{\underline{v}}$ and the quadratic term $(\underline{v}\,\underline{\nabla})\underline{v}$ in (1) show that the relative change $\delta v_a / v_a \approx \dot{v}_a / (v_a \omega_a)$ of the velocity amplitude v_a of a certain mode, arising from $(\underline{v}_a \underline{\nabla})\underline{v}_a \approx k_a v_a^2$ during the time $1/\omega_a$ is of the order of

$$\delta v_a / v_a = O\left\{ \frac{k_a}{\omega_a} v_a \right\} = O\left(\frac{v_a}{c_a} \right) . \tag{7}$$

Hence the purely acoustic nonlinear terms may be neglected for small acoustic amplitudes $v_a / c_a \ll 1$ (linear acoustics).

Furthermore the transfer of energy from the acoustic wave field to the surface wave field is small since the frequencies of the surface waves are small compared with those of the sound waves. Therefore the sea surface may be considered as moving independently of the incident acoustic wave field. An important consequence of this approximation is that then the scattered sound field can be described as a linear response to the surface wave field. In the following we restrict our considerations to the remaining indirect coupling of acoustic modes via surface waves.

3. Comparison of the effects of fluid motion and boundary distortion due to surface waves on the acoustic wave field

To estimate the effect of the fluid motion in the surface waves on the acoustic wave field it is again sufficient to consider the linear term $\dot{v}_a$ due to the acoustic wave field and the mixed terms in $(\underline{v}\,\underline{\nabla})\,\underline{v}$ of (1), which couple acoustic modes and surface waves. Then we find that the relative change $\delta v_a/v_a \approx (\dot{v}_a/v_a)(h/c_a)$ of the velocity amplitude v_a of an acoustic wave, arising from the fluid motion in the surface waves, $(\underline{v}\,\underline{\nabla})\underline{v} \Rightarrow (k_a+k_b)v_a v_b$ is of the order of

$$\delta v_a/v_a = O\{(k_a+k_b)v_b h/c_a\} = O\left\{k_a h\, k_b h\left(1+\frac{k_b}{k_a}\right)\frac{c_b}{c_a}\right\}, \quad (8)$$

where h is a characteristic (e.g. r.m.s.) value of the surface displacements and v_b a characteristic particle velocity in the surface waves.

To estimate the effect of boundary displacements on the acoustic wave field the boundary condition may be expanded around $z=0$: $p(\zeta) = \delta p(0) + \frac{\partial p}{\partial z}\zeta = 0$. This means that the displacement induced change in the sound field pressure at $z=0$ is of the order of

$$\delta p_a/p_a = O\{k_a h\}. \quad (9)$$

Comparing (8) and (9) and noting that $k_b/k_a \lesssim 1$, $k_b h \lesssim 1$ we see that the effect of the fluid motion on the sound field pressure is at least by a factor of the order of c_b/c_a ($\approx 10^{-3}$ for $\lambda_b = 2\pi/k_b \approx 1.5\,m$) smaller than that of the boundary displacements. Neglecting the effect of the fluid motion we arrive at a pure boundary value problem with a time-varying boundary upon an "acoustic wave ether" which may be considered to be at rest.

4. The sea surface as a quasi-static boundary

Green's theorem can be applied to determine the sound field from the values of the field quantities (sound field pressure and its normal derivative) at the boundary. Earlier papers [12-16] that used the Kirchhoff approach for analysing sound scattering from the sea surface were based on the Helmholtz-Kirchhoff integral for a temporarily fixed boundary. In order to allow for Doppler effects in the scattered field the time-variations of the surface were subsequently taken into account via variations of the time retardation.
The corrections to this procedure may be derived from Green's theorem for a

time-varying boundary. According to this theorem - which has been used only recently by Zipfel [37] - the reflection and scattering of an incident sound field at a time-varying rough surface result in a reradiated field p_s which is determined by

$$
\begin{aligned}
&4\pi p_s(\underline{r},t) = \\
&\int dt' \int d\underline{S} \left\{ p(\underline{r}_s,t') \underline{\nabla}_s G_o(\underline{r}-\underline{r}_s, t-t') - G_o(\underline{r}-\underline{r}_s, t-t') \underline{\nabla}_s p(\underline{r}_s,t') \right\} \\
&- \int dt' \int d\underline{S} \frac{\underline{v}_b(\underline{r}_s,t')}{c_a^2} \left\{ p(\underline{r}_s,t') \frac{\partial}{\partial t'} G_o(\underline{r}-\underline{r}_s, t-t') - G_o(\underline{r}-\underline{r}_s, t-t') \frac{\partial}{\partial t'} p(\underline{r}_s,t') \right\}.
\end{aligned}
\qquad (10)
$$

Here $p(\underline{r}_s, t')$ is the total sound field pressure and $\nabla_S p(\underline{r}_s, t')$ its gradient on the scattering surface $\underline{S}(t')$ while $\underline{v}_b(\underline{r}_s, t')$ is the velocity of a surface point $\underline{r}_s$. The function G_o is the infinite-space Green's function of the wave equation. It describes a spherical shock-wave according to Huygen's principle

$$\Delta G_o(\underline{r},t) - \frac{1}{c_a^2}\frac{\partial^2}{\partial t^2} G_o(\underline{r},t) = 4\pi\, \delta(\underline{r})\,\delta(t), \qquad (11)$$

$$G_o(\underline{r},t) = -\frac{\delta(t - r/c_a)}{r} = \frac{1}{4\pi^3} \iint d^3\underline{k}\, d\omega \frac{e^{i(\underline{k}\underline{r} - \omega t)}}{(\omega + i\varepsilon)^2 - k^2}. \qquad (12)$$

Zipfel [37] has used the Fourier representation for G_o on the right hand side of (12) which is quite suited to his subsequent plane wave scattering formalism. Here, we fall back to the space representation to show the connections with the commonly used integral expression for a fixed boundary. In the latter case, $\underline{v}_b = 0$, the last terms in (10) containing $\underline{v}_b$ vanish. For the terms containing the δ'-function introduced via G_o and $\underline{\nabla}_S G_o$, the t'-integration can be easily carried out (see [9] p. 847). For the term with the δ'-function introduced via $\nabla_S G_o$ a partiell integration leads to a term with a time-derivative of p. Finally the integration results in

$$
\begin{aligned}
&4\pi p_s(\underline{r},t) \\
&\int d\underline{S} \left[\frac{1}{R} \underline{\nabla}_S p(\underline{r}_s,t') - \left(\underline{\nabla}_S \frac{1}{R}\right) p(\underline{r}_s,t') + \frac{1}{c_a R} (\underline{\nabla}_S R) \frac{\partial}{\partial t'} p(\underline{r}_s,t') \right]_{t'=t-R/c_a}
\end{aligned}
\qquad (13)
$$

where $R = |\underline{r} - \underline{r}_s|$ and $[\quad]_{t'=t-R/c_a}$ means time retardation.

For a time-varying boundary, $\underline{r}_s = \underline{r}_s(t)$, the integral expression is complicated for three reasons

(1) The terms proportional to $\underline{v}_b$ do not vanish.

(2) In addition to the explicite dependence of the δ-function upon t' there is an implicite dependence upon t' via $R(t')$ resulting in $\partial R/\partial t'$-terms.

(3) When partially integrating over t', the time-dependence of the distance $R(t')$ and the boundary $\underline{S}(t')$ have to be taken into account in the derivations with respect to t'.

However, all correction terms appearing in addition to (13) are obviously of the order of the first or even higher powers of v_b/c_a. Hence such corrections should consistently be omitted if the fluid motion below the boundary is omitted.

For a pressure release surface like that of the sea, for which the pressure and its time derivative are zero over $\underline{S}$, (13) simplifies to

$$4\pi p_s(\underline{r},t) = \int d\underline{S}\left[\frac{1}{R}\underline{\nabla}_s p(\underline{r}_s,t')\right]_{t'=t-R/c_a}. \qquad (14)$$

Most of the features of the scattering pattern can also be seen from the results for the "frozen" static surface for which the stationary Helmholtz-Kirchhoff integral may be used

$$4\pi p_s(\underline{r}) = \int d\underline{S}'\,\hat{G}(\underline{r}-\underline{r}_s')\,\underline{\nabla}_s' p(\underline{r}_s') \qquad (15)$$

where $\hat{G}(\underline{r}) = \exp(ikr)/r$

is the stationary Green's function for the infinite space. An integral equation for the unknown normal derivative of p is obtained when $\underline{r}$ is shifted to the surface where the boundary condition, $p(\underline{r}_s) = p_o(\underline{r}_s) + p_s(\underline{r}_s) = 0$, can be used resulting in

$$-4\pi p_o(\underline{r}_s) = \int d\underline{S}'\,\hat{G}(\underline{r}_s-\underline{r}_s')\,\nabla_s' p(\underline{r}_s'). \qquad (16)$$

IV Field representations and scattering formulae

1. Mode representation for shallow sea

For the propagation of low frequency sound a shallow sea may be considered as a wave guide. In this case a normal-mode representation of the acoustic wave field is most suitable. Wave guide inhomogeneities like surface waves lead to a transfer of energy from the primary mode into other modes. The scattering of an arbitrary wave field can be determined from single-mode scattering by superposition of all mode contributions. For the scattering of sound by surface waves the normal-mode representation has been used in the framework of the "weak interaction approximation" by Essen and Hasselmann [34] . The use of the normal-mode representation is restricted to oceanic wave guides with boundary displacements being small compared with the acoustic wave length.

2. Plane wave representation for deep sea

For deep sea a plane wave representation of the acoustic wave field is the most appropriate one. Accordingly for a homogeneous source-free halfspace bounded by a plane $z = z_1$, the sound field can be written as

$$p(\underline{r},t) = \int d^2\underline{\varkappa} \int d\omega \; a(\underline{\varkappa},\omega)\, e^{i\underline{\varkappa}\underline{\varrho} + iK|z-z_1| - i\omega t}, \tag{17}$$

$$a(\underline{\varkappa},\omega) = \frac{1}{(2\pi)^3} \int d^2\underline{\varrho} \int dt \; p(z=z_1, \underline{\varrho}; t)\, e^{-i\underline{\varkappa}\underline{\varrho} + i\omega t},$$

where $a(\underline{\varkappa},\omega)$ is the (three-dimensional surface-time) Fourier transform of the pressure distribution $p(z=z_1, \underline{\varrho}, t)$ in the plane $z = z_1$. The two-dimensional vector $\underline{\varkappa} = k_x \underline{e}_x + k_y \underline{e}_y$ is the projection of the wave vector $\underline{k}$, $\underline{\varrho}$ that for the locus vector $\underline{r}$ on a plane $z =$ const. (see Fig. 4). $K = \sqrt{(\omega/c_a)^2 - \varkappa^2} = \pm k_z$ is the positive or negative z-component of $\underline{k}$ for $z \geq z_1$, or $z < z_1$ respectively. In order to get outgoing waves radiated into the half-space under consideration, the root of K has to be taken negative for $\omega < -|\varkappa|$, positive imaginary for $-|\varkappa| < \omega < |\varkappa|$ and positive for $\omega > |\varkappa|$ (radiation condition). This may be established by addition of a small positive imaginary quantity $i\varepsilon$ to ω as in eq. (12). If sources or diffracting boundaries lie close to the plane $z = z_1$ the integrand in (17) contains "inhomogeneous" or "evanescent" surface waves which die away exponentially with increasing distance from the plane.

Hence in the region between the source, e.g. at $z = z_0$, and the surface, $\zeta \leq z < z_0$, the primary sound field may be written as

$$p_0(\underline{r},t) = \iint d^2\underline{\varkappa}\, d\omega\, a_0(\underline{\varkappa},\omega)\, e^{i\underline{\varkappa}\underline{\xi} - ikz - i\omega t}, \tag{18}$$

while in the planely bounded region completely below the rough surface $z > \zeta_{max}$, the reradiated field may be represented by

$$p_s(\underline{r},t) = \iint d^2\underline{\varkappa}\, d\omega\, a_s(\underline{\varkappa},\omega)\, e^{i\underline{\varkappa}\underline{\xi} + ikz - i\omega t}. \tag{19}$$

The spatial (wave-number part) of the integral in (17) can also be written as an integral over an "angular spectrum of plane waves" [3,39]. As a simple example for a primary field we give that of a monochromatic omnidirectional source (spherical wave):

$$p_0(\underline{r},t) = \frac{e^{ik(r - c_a t)}}{r}, \qquad a_0(\underline{\varkappa},\omega) = \frac{i}{2\pi K}\,\delta(\omega - c_a k). \tag{20}$$

3. Scattering formulae

In the framework of the linear response approximation the scattering process can be described by a transition amplitude or scattering matrix, $T(\underline{\varkappa},\omega;\underline{\varkappa}',\omega')$ which relates the plane wave decomposition $a_s(\underline{\varkappa},\omega)$ of the scattered fields to the plane wave decomposition $a_0(\underline{\varkappa}',\omega')$ of the incident field [35-38]

$$a_s(\underline{\varkappa},\omega) = \iint d^2\underline{\varkappa}'\, d\omega'\, T(\underline{\varkappa},\omega;\underline{\varkappa}',\omega')\, a_0(\underline{\varkappa}',\omega'). \tag{21}$$

(If complex functions are used for the sound fields one must be aware that the scattering object, here the sea surface, involved in T must be described by real functions). In an asymptotic region sufficiently far removed from the surface so that the "inhomogeneous" surface waves can be neglected the integral (21) may be restricted to values of $\underline{\varkappa}'$ and ω' for which K is real. Then, (21) represents a so called reduction formula [35,37,38].

For a random surface the scattering can be described by statistical moments of T [35,37]. Thus for the coherently reflected sound field pressure we have

$$p_{coh}(\underline{r},t) =$$

$$\iiiint d^2\underline{\varkappa}\, d\omega\, d^2\underline{\varkappa}'\, d\omega'\, e^{i\underline{\varkappa}\underline{\xi}+ikz-i\omega t} \langle T(\underline{\varkappa},\omega;\underline{\varkappa}',\omega')\rangle a_0(\underline{\varkappa}',\omega') \quad (22)$$

where $\langle\ \ \rangle$ means statistical averaging.
Another most important quantity is the scattered intensity. This can be described by the surface "scattering strenth" or "scattering coefficient" q which is defined as the mean-square far-field sound presssure in unit distance R from a unit area A of the scattering surface, to the mean-square sound pressure of the incident plane wave [3,12,21,40]

$$q = \frac{R^2}{A}\langle p_s^2\rangle/|p_0|^2 = k^2\cos^2\vartheta \int d\omega\, F_T \quad (23)$$

where $F_T(\underline{\varkappa},\omega;\underline{\varkappa}_0,\omega_0) = \frac{(2\pi)^3}{A\tau}\langle T(\underline{\varkappa},\omega;\underline{\varkappa}_0,\omega_0)^2\rangle$

is the wave-number frequency spectral density due to T, τ the averaging time, and ϑ the angle between the scattering direction and the z-direction (see Fig. 4).

For the description of Doppler spectra a differential form of (23), the Doppler density of the scattering strength $Q(\omega)$, is useful [41]

$$Q(\omega) = k^2\cos^2\vartheta\, F_T; \qquad q = \int d\omega\, Q(\omega). \quad (24)$$

The scattering may also be described by a dimensionless scattering cross section per scattering area which normalizes to 1 when integrated over the solid angle [36,42]. For this the acoustic energy scattered into a solid angle element has to be referred to the energy incident on the scattering are

$$\frac{d\sigma}{d\Omega\, d\omega} = Q/\cos\vartheta_0, \quad (25)$$

where ϑ_0 is the angle of incidence.

V Discussion of the main approximations for a quasi-static surface

In chapter III we have shown that sound scattering from the sea surface can be treated as a boundary value problem with a quasi-static boundary for the purely acoustic wave field. In the following sections we compare the three main approaches for solving this boundary value problem on the basis of the plane wave representation discussed in the preceding chapter. For simplicity we consider the surface as being static since the main features of the scattering pattern can be seen from the results for the static surface. The time-variation can be subsequently taken into account by complementing the Fourier-transformations with respect to the surface coordinates with that with respect to time.

1. The Kirchhoff approach

In the Kirchhoff approach [6,8-16] the normal derivative of the sound field pressure on the surface is approximated by that occuring in the case of a reflection from the tangent plane in the point under consideration which means setting

$$\underline{\nabla}_S p = \underline{\nabla}_S (p_0 + p_S) \approx 2\, \underline{\nabla}_S p_0 \,, \tag{26}$$

where p_0 is the sound pressure of the primary wave. This implies that the surface is approximately flat over areas which are large compared with the acoustic wave length.

By introducing (26) into the Helmholtz-Kirchhoff integral (15) and by using the two-dimensional Fourier-representation, (20), for $\hat{G}$ one obtains for the wave-number part of the transition matrix defined by (21)

$$T(\underline{\varkappa},\underline{\varkappa}_0) = -\frac{1}{(2\pi)^2}\int d^2\underline{\xi}\; e^{-i(\underline{\varkappa}-\underline{\varkappa}_0)\underline{\xi} - i(K+K_0)\zeta(\underline{\xi})}\, \frac{K_0 + \underline{\varkappa}_0 \nabla_{\underline{\xi}} \zeta(\underline{\xi})}{K} \,. \tag{27}$$

The surface slope $\nabla_{\underline{\xi}}\zeta(\underline{\xi})$ involved here, can be removed by partial integration over $\underline{\xi}$ [10,13] resulting in

$$T(\underline{\varkappa},\underline{\varkappa}_0) = -\frac{\underline{k}(\underline{k}-\underline{k}_0)}{K(K+K_0)}\, A_{K+K_0}(\underline{\varkappa}-\underline{\varkappa}_0)\;; \tag{28}$$

where

$$A_K(\underline{\varkappa}) = \frac{1}{(2\pi)^2}\int d^2\underline{\xi}\, e^{-i\underline{\varkappa}\underline{\xi} - iK\zeta(\underline{\xi})} \tag{29}$$

is the "phase modulation amplitude spectrum" [19] due to the surface waves. It is the central function in all approximations and also in a rigorous treatment of the problem [35-37] .

The Kirchhoff approach is valid for smooth surfaces with radii of curvature r_c being large compared with the acoustic wave-length, for not too small grazing angles γ_0 and for scattering angles lying close to the directions of geometrical reflections. Curvature correction [36,43] show that the error is of the order of $k r_c \sin^3\gamma$ which means that the condition

$$k r_c \sin^3\gamma \gg 1 \tag{30}$$

should be fulfilled.

2. The Rayleigh approach

The essence of the Rayleigh approach [7,10,17-21] is to extend the representation (19) of the scattered field in terms of only outgoing waves right up to the boundary without changing the set of $\underline{\varkappa}$-values over which the integration is performed (no rigorous analytical continuation). Then, for the scattering of a plane wave characterized by $\underline{\varkappa}_0$, the boundary condition (3) reads

$$e^{i\underline{\varkappa}_0\underline{\xi} - iK_0\zeta} + \int d^2\underline{\varkappa}\, e^{i\underline{\varkappa}\underline{\xi} + iK\zeta}\, T(\underline{\varkappa},\underline{\varkappa}_0) = 0 \quad \text{for all } \underline{\xi}. \tag{31}$$

This integral equation for the transition matrix can be put into a more convenient form by Fourier transformation. A high flexibility of the kernel of the Fourier transformed integral equation is gained by introduction of an additional phase factor $\exp(-i\hat{K}\zeta)$ with a wave-number parameter $\hat{K}$ being arbitrary for the present. Then (31) becomes

$$A_{\hat{K}+K_0}(\underline{\varkappa}-\underline{\varkappa}_0) + \int d^2\underline{\varkappa}'\, A_{\hat{K}-K'}(\underline{\varkappa}-\underline{\varkappa}')\, T(\underline{\varkappa}',\underline{\varkappa}_0) = 0. \tag{32}$$

For various limiting cases, such as for surface waves with displacements being small compared with the acoustic wave lengths ("Rayleigh parameter"

$K_o^2 h^2 = K_o^2 \langle \zeta^2 \rangle \ll 1$) or for surface waves with wavelengths (and surface displacements) being large compared with the acoustic wave-lengths (large radii of curvature, $K r_c \gg 1$) the kernel of (32) can be put close to "diagonal" form, fitted for application of perturbation techniques, by a suitable choice of $\hat{K}$. For $\hat{K} = K_o$ the kernel is adapted to the former case while for $\hat{K} = K$ it is adapted to the latter case. The corresponding 1. order perturbation approximations are quite similar to each other

$$T(\underline{\varkappa}, \underline{\varkappa}_o) \approx -A_{2K_o}(\underline{\varkappa} - \underline{\varkappa}_o); \tag{33}$$

and

$$T(\underline{\varkappa}, \underline{\varkappa}_o) \approx -A_{K+K_o}(\underline{\varkappa} - \underline{\varkappa}_o). \tag{34}$$

For small surface displacements the errors are of the orders of $(K_o h)^2$ and $(K_o h)$ in the first and the second case respectively while for surface waves with large wave lengths (and large surface displacements) the errors are of the order of the r.m.s. slope $s = \sqrt{\langle (\nabla_\varrho \zeta)^2 \rangle}$ in both cases but somewhat smaller in the second case.

Further modifications and simplifications of the Rayleigh approach have been given in the literature. But only the general ansatz (31) should be called "Rayleigh approach". In this sense the approximations called after Rayleigh by La Casce and Tamarkin [44] and by Fortuin [2] are special approximations within the more general approach (31).

In recent years there have been a number of controversies concerning the Rayleigh approach [2,19,24,36,45]. Here we should only note that the solutions for the transition matrix resulting from (32) by application of perturbation techniques are not consistent with the original ansatz (31) [36] . This means that the scattered field cannot be adequately described as a superposition of outgoing waves only. Serious diffraction effects are due to more complicated structures of the field close to the rough surface. In this respect Hasselmann's weak interaction approach [31-33] has the same defects as the Rayleigh approach.

A rigorous perturbation treatment of the Helmholtz-Kirchhoff integral in the Fourier-space shows that for small surface displacements Rayleigh's approach is correct up to the second power of the displacements [36] . For short acoustic waves (long surface waves) it includes the Kirchhoff approach and should be as good as when the latter is curvature corrected [36]. These conditions may be summarized by

$$(K_0 h)^3 \ll 1 \quad \underline{\text{or}} \text{ otherwise} \quad (K r_c)^2 \gg 1 . \tag{35}$$

3. The perturbation approach of Meecham

The unknown normal derivative of the sound field pressure at the boundary can be calculated approximately from the Helmholtz-Kirchhoff integral equation (16) by starting with the kernel due to the undisturbed flat boundary and using a perturbation technique to account for the disturbance of this kernel. Accordingly, Meecham [22] has derived a first approximation for $\underline{\nabla}_S p$ by accounting for the perturbation of the boundary in the inhomogeneous part, $-4\pi p_0(\underline{r}_S)$, of the integral equation (16) but by neglecting it in the kernel. For the undisturbed kernel, being only a function of the spacing $(\underline{\rho} - \underline{\rho}')$, the integral equation can be simply inverted by Fourier transformation (convolution theorem). Insertion of the result for $\underline{\nabla}_S p$ into the Helmholtz-Kirchhoff integral (15) yields for the transition matrix

$$T(\underline{\varkappa}, \underline{\varkappa}_0) = -\int d^2\underline{\varkappa}' \, \frac{K'}{K} A_{K_0}(\underline{\varkappa}' - \underline{\varkappa}_0) \, A_K(\underline{\varkappa} - \underline{\varkappa}') ; \tag{36}$$

which is now determined by a convolution of two phase modulation spectra. The errors are of the order of $(K_0 h)^2$ for surface displacements being small compared with the acoustic wave-lengths, and of the order of (S^2) for acoustic wave-lengths being small compared with the surface wave-lengths. Hence we have the conditions

$$S^2 = \langle (\underline{\nabla}_\rho \zeta)^2 \rangle \ll 1 \quad \underline{\text{or}} \text{ otherwise} \quad (K h)^2 \ll 1 . \tag{37}$$

We may say that the regime of validity of (37) comprises the regimes of the 1. order approximations (33) and (34) due to the Rayleigh approach - at the expense of the higher complexity of (37).

4. Perturbation approach in the Fourier space

The formulation of the boundary value problem in the Fourier space is most suited for the application of perturbation techniques and statistical averaging [35-37] . For a static surface this means that the Helmholtz-Kirchhoff integrals (15) and (16) have to be Fourier transformed completely. The resulting integral equation for the Fourier transform of the pressure gradient on the surface has a kernel which is determined by phase modulation spectra of type (29). From the solution of this integral equation the transition matrix is, in general, obtained by additional application of an integral operator.

By suitabele transformations of the integral kernel the perturbation expansion can be adapted to various limiting cases, similar to the procedure for the Rayleigh approach as mentioned above. The corresponding 1. order results are the Kirchhoff approximation (28) or Meecham's approximation (36). The integral equation treatment in the Fourier space is also best suited to reexamine the Rayleigh approach and its defects [36] .

The 1. order results describe "single scattering processes" from the viewpoint of the perturbation theory, each being essentially determined by one phase modulation spectrum (or two of them for Meecham's approximation). The higher order terms represent "multiple scattering effects", such as resonances and shadowing, and are determined by integrals over products of phase modulation spectra.

5. The surface layer as a phase object

Comparing the 1. order results (28), (33) and (34) we notice quite similar mathematical structures. In each approximation the expression for the transition matrix consists of a phase modulation spectrum and a geometrical prefactor. Both factors are somewhat different for the different approximations in accordance with the somewhat different ranges of validity.

The occurance of phase modulation spectra in all approximations suggests to consider the surface layer as a simple "phase object" [3,8] . In fact, the pattern of the scattered field can be related completely to the transmission properties of an appropriate thin (planely bounded) phase object at the scattering surface ("generalized phase object" [3]).

The 1. order results representing "single scattering processes" correspond to the "linear phase approximation" for which the phases are linearly related to the instantaneous values of the surface displacements. The "linear phase approximation" can be adapted to various situations by suitably tracing the rays through the surface layer (see Fig. 5). For surface displacements being small compared with the acoustic wave length the ray path should be chosen such as to follow the incident and the undisturbed reflection directions ("specular" reflection from the mean surface) [3]. This approximation corresponds to the 1. order result (33) due to the Rayleigh approach. For surface waves with wave-lengths (and displacements) being large compared with the acoustic wave length the ray should be traced appropriate to "local reflections" in accordance to the Kirchhoff approach. An intermediate approximation corresponding to the 1. order result (34) of the Rayleigh approach is obtained with pathes tracing the ways from the source via the surface points to the observation point (incident direction and observation direction).

6. A simple interpolation formula

The most simple approximations (28),(33),(34),containing only one phase modulation spectrum, can be generalized by introduction of an arbitrary geometrical factor $g(\underline{k})$

$$T(\underline{\varkappa},\omega;\underline{\varkappa}_0,\omega_0) = -g(\underline{k})\, A_{K+K_0}(\underline{\varkappa}-\underline{\varkappa}_0,\omega-\omega_0)\;; \tag{38}$$

where now the Fourier-transform with respect to the surface coordinates is complemented with that with respect to time to allow for the time-variation of the sea surface

$$A_K(\underline{\varkappa},\omega) = \frac{1}{(2\pi)^3}\iint d^2\xi\, dt\, e^{-i\underline{\varkappa}\xi + i\omega t - iK\zeta(\xi,t)}\,. \tag{39}$$

Now the question arises how $g(\underline{k})$ should be chosen to yield a simple approximation which gives good results for both limiting cases, for the scattering of sound from surface waves

1) with displacements being small compared with the acoustic wave-length, and
2) with wave-lengths(and displacements) being large compared with the acoustic wave-length.

This is sufficiently well fulfilled by

$$g(\underline{k}) = \frac{2K_0}{K+K_0} = \frac{2\cos\vartheta_0}{\cos\vartheta+\cos\vartheta_0} \tag{40a}$$

resulting in the approximation [36]

$$T(\underline{\varkappa},\omega;\underline{\varkappa}_0,\omega_0) = -\frac{2\cos\vartheta_0}{\cos\vartheta+\cos\vartheta_0} A_{K+K_0}(\underline{\varkappa}-\underline{\varkappa}_0,\omega-\omega_0). \tag{40b}$$

The errors involved in this approximation are of the same order as those of Meecham's much more complicated version and its regime of validity (like that of the latter one) comprises the regimes of the 1. order results due to the Rayleigh approach. By its simple form and its wide range of validity such a formula is distinguished as a basis for the discussion of further approximations, such as a two-scale model for the sea surface. Of course a first order approximation like (40b) cannot describe shadowing effects. Therefore these have to be considered separately.

VI Applications to random surfaces

1. Statistical averaging

The approximations discussed so far imply the knowledge of the surface displacement as a function of surface coordinates and time. But, of course, no deterministic description of the sea surface is possible. Anyhow, we are only interested in appropriate averages related to the reradiated sound field, such as the coherently reflected "mean" field, the incoherently scattered intensity, the coherence function, Doppler spectra, other "second order" and possibly also "higher order" quantities (higher order correlation functions). In calculating such averages from "deterministic" approximations we may replace surface and time averages over functionals of the surface displacements by statistical averages over ensembles of surfaces.

The formulation of the boundary value problem in the Fourier space, as presented in chapter IV, is most suited for the application of statistical averaging [35-37] It shows that the surface displacements enter the moments describing the statistical properties of the sound field only through the phase modulation amplitude spectra of the form (29). Therefore these moments can be expressed by the Fourier transform of the characteristic function of the joint probability density of the surface displacements.

Usually the surface perturbation is decribed as a random process with a multivariate Gaussian displacement distribution being homogeneous with respect to surface coordinates and time. Then, for the 1. order approximations (28), (33) and (34) the reduction of the "coherently reflected" (or "specularly scattered") intensity is given by a factor [35-37] analogous to the Debye-Waller factor in crystal physics

$$\frac{I_{coh}(\underline{\varkappa}=\underline{\varkappa}_0)}{I_0(\underline{\varkappa}_0)} = \left|\left\langle e^{-2iK_0\zeta}\right\rangle\right|^2 = e^{-4K_0^2 h^2}, \qquad (41)$$

with $K_0 = k\cos\vartheta_0$;

where $h = \langle\zeta^2\rangle$ is the r.m.s. surface displacement. With this 1. order approximation the reduction is somewhat overestimated as can be seen from higher order approximations or numerical results, e.g. of Uretsky [24] or Jamnejad-Dailami [25].

The 1. order Doppler density of the scattering strength, (24), is proportional to the Fourier transform of the characteristic function of the joint probability density of surface displacements [13-16, 35-37]. Using (24) and one of the 1. order approximations, and splitting off the coherently reflected part [35,37] we can write

$$Q = \frac{K^2 g(\underline{k})^2}{(2\pi)^3} \iint d^2\Delta\varrho\, d\Delta t\, e^{i(\underline{\varkappa}-\underline{\varkappa}_0)\Delta\varrho - i(\omega-\omega_0)\Delta t} \times \tag{42a}$$
$$\times \left\{ \left\langle e^{i(K+K_0)[\zeta(\varrho+\Delta\varrho,t+\Delta t)-\zeta(\varrho,t)]} \right\rangle - \left| \left\langle e^{-i(K+K_0)\zeta} \right\rangle \right| \right.$$

with $K = k\cos\vartheta$; $\varkappa = k\sin\vartheta$;

where the geometrical factor $g(\underline{k})$ may be chosen according to (40). For a jointly normal probability density we have

$$Q = \frac{K^2 g(\underline{k})^2}{(2\pi)^3} e^{-(K+K_0)^2 h^2} \iint d^2\Delta\varrho\, d\Delta t\, e^{i(\underline{\varkappa}-\underline{\varkappa}_0)\Delta\varrho - i(\omega-\omega_0)\Delta t} \times \tag{42b}$$
$$\times \left\{ e^{(K+K_0)^2 C(\Delta\varrho,\Delta t)} - 1 \right\},$$

where $C(\Delta\varrho,\Delta t) = \langle \zeta(\varrho,t)\,\zeta(\varrho+\Delta\varrho, t+\Delta t) \rangle$ is the covariance function of the surface displacements vanishing as $C(\Delta\varrho,\Delta t) \to 0$ for $\Delta\varrho$ and/or $\Delta t \to \infty$.

A systematic analysis of the statistical moments of a sound field scattered by a randomly varying surface has been given recently by Zipfel and DeSanto [35] and by Zipfel [37]. They have developed a cluster expansion with a diagrammatic represenation similar to the Feynman expansion of quantum field theory. The cluster expansion can be partially summed to generate an integral equation for the "mean field", analogous to the Dyson equation, and an integral equation for the mutual coherence function, analogous to the Bethe-Salpeter equation.

For a given surface wave spectrum, e.g. for a Pierson-Moskowitz spectrum [52], sound scattering can be calculated, in principle with sufficient accuracy, on the basis of (42b). For this, first the covariance function $C(\Delta\varrho,\Delta t)$ must be calculated from the spectrum then Fourier transformation according to (42b) has to be applied. Since this is extremely complicated further simplifications, such as e.g. two-scale model, are opportune.-

2. Slightly rough sea

For a slightly rough sea surface [12-15, 41, 47] characterized by a small Rayleigh roughness parameter, $K_0 h = kh\cos\vartheta_0 \ll 1$, the exponentials in the integrands of (42a) and (42b) may be expanded with respect to the surface displacements and their covariance function respectively. The first nonvanishing term of this expansion is

$$Q = K^2 (K+K_0)^2 g^2 e^{-(K+K_0)^2 h^2} F_\zeta(\underline{\varkappa} - \underline{\varkappa}_0, \omega - \omega_0) ; \qquad (43)$$

where $$F_\zeta(\underline{\varkappa}, \omega) = \frac{1}{(2\pi)^3} \iint d^2\Delta\underline{\xi}\, d\Delta t\, e^{i\underline{\varkappa}\Delta\underline{\xi} - i\omega\Delta t} C(\Delta\underline{\xi}, \Delta t)$$

is the wave-number frequency spectrum of the surface waves. Considering the dispersion relation, (6), we see that for a monochromatic plane wave incident on a sinosoidal surface we get, besides the 0. order reflection, two 1. order "Bragg reflections". The Doppler spectrum of each of them consists of two discrete lines which are shifted by frequency changes $\pm\Delta\omega$ according to the "transferred transversal momentum" $\Delta\underline{\varkappa}$. For a continuous spectrum of surface waves each Fourier component produces two scattered acoustic components resulting in a continuous angular distribution of scattered intensity. In each direction the Doppler spectrum consists again of two discrete lines, which means that no Doppler <u>spread</u> occurs. According to (41) the coherently reflected wave (0. order) is reduced but not frequency shifted.

Higher order terms in the surface displacements yield higher order Bragg reflection and higher order Doppler lines. The former "1. order approximations" determined by phase modulation spectra, may be considered as results of partial summations of the present series expansion in powers of the surface displacements.

3. Very rough sea

For increasing wave lengths and amplitudes of the surface waves more and more reflection orders appear, the distances between them become smaller and smaller and higher order reflections become increasingly important while the (coherently reflected) 0. order vanishes. Asymptotically the envelope of such a large set of reflection orders may be described by the method of stationary phases [8,9] if the amplitudes of the surface waves are slowly varying in space and time compared with the variations in the sound wave. This condition is fulfilled for long surface waves with large radii of curvature, $K_0 r_c \gg 1$, usually connected with a large Rayleigh parameter $K_0 h \gg 1$. Then the displacement difference in (42a) and the covariance function in (42b) may be expanded with respect to their spatial and temporal lags, $\Delta \underline{\varrho}$ and Δt :

$$\zeta(\underline{\varrho}+\Delta\underline{\varrho}, t+\Delta t) - \zeta(\underline{\varrho}, t) \approx \underline{\nabla}_{\varrho}\zeta(\underline{\varrho}, t)\,\Delta\underline{\varrho} + \dot{\zeta}(\underline{\varrho}, t)\,\Delta t \tag{44a}$$

and [15]

$$C(\Delta\underline{\varrho}, \Delta t) \approx h^2 + \frac{1}{2}\sum_{i,j} \partial_i \partial_j C\Big|_0 \xi_i \xi_j \;; \quad \underline{\xi} = (\Delta\underline{\varrho}, \Delta t). \tag{44b}$$

Neglecting interferences of "high lights" [13,15] one obtains for the Doppler density of the scattering strength (42)

$$Q \approx \frac{K^2 q^2}{(K+K_0)^3}\, P_{\underline{\nabla}_{\varrho}\zeta, \dot{\zeta}}\left(-\frac{\underline{\varkappa}-\underline{\varkappa}_0}{K+K_0}, \frac{\omega-\omega_0}{K+K_0}\right) \tag{45}$$

where $P_{\underline{\nabla}_{\varrho}\zeta, \dot{\zeta}}(\underline{\nabla}_{\varrho}\zeta, \dot{\zeta})$ is the three-dimensional probability density function of the surface slopes and vertical velocities. Accordingly, if the surface statistic is Gaussian the angular distribution of the scattered intensity and the Doppler spectra are also Gaussian in this asymptotic approximation. Of course, an approximation of the form (45) can also be derived directly with aid of geometrical ray theory.

4. Two-scale model

Generally the spectrum of the sea surface waves contains short wave-length components ("ripple") with wave-lengths comparable with that of the sound, as well as long wave-length components ("swell") with surface displacements large compared with the acoustic wave-length. This means that neither of the two limiting cases discussed above is realized. In order to avoid extremely complicated direct evaluations of the full (unsimplified) 1. order approximations (40a) and (40b) one can use a combination of both limiting cases. This leads to the "two-scale" (or "wave-facet" or "composite roughness") model [20,21,41,46,47] in which the spectrum of the sea surface displacements is divided into a short and a long wave-length part. Sound scattering from the short waves is modulated via slope variations of the long waves.

An important question which has to be answered for this model is that after the choice of the division point. The long waves (facets) have to be chosen large compared with the acoustic wave-length though not too large so that the displacements due to the short wave-length part remain small compared with the acoustic wave-length. A plausible heuristic procedure to find an appropriate cut has been proposed by Bachmann [21] . Schwarze [41] has introduced a quantity which characterizes the "quality" of the two-scale model for a given surface wave spectrum.

A formal derivation of the two-scale model can be given on the basis of the "1. order" approximation (40a) or (40b). For this the displacement difference in (40a) or the covariance function of the displacements in (40b) are divided into short and long scale parts with division points remaining unspecified for the moment. Then, the corresponding exponentials are expanded with respect to the short scale part while the asymptotic approximation (method of stationary phase) is applied to the long scale part. A criterion for positioning the cut is provided by the higher order terms of the expansions. Accordingly the optimum position of the cut is given by the minimum of the errors due to these terms. This method which has not been worked out hitherto should be used to put the two-scale model on a more profound mathematical basis.

5. Geometrical shadowing

It has already been pointed out that the 1. order approximations discussed in chapter V cannot describe the phenomenon of shadowing of certain surface areas by other parts of the surface. Consequently, if these approximations are used shadowing corrections have to be considered additionally. They are very important for small grazing angles.

Only Gardner [48] has tried to take shadowing into acc ount directly in the Helmholtz-Kirchhoff integral by setting the unknown normal derivative of the sound field pressure at the surface equal to zero in the shadowed zones. Unfortunately his treatment contains some errors. But anyhow, it seems to be more conveniant to consider shadowing corrections in connection with the two-scale model of the sea surface.

For random surfaces the problem is to calculate the probability $S(\vartheta_0|\zeta_0,\zeta_0')$ that a point $x=0$ is illuminated with radiation at an angle of incidence ϑ_0 to the mean plane, given that the displacement is ζ_0 and the local slope is ζ_0' at this point. Wagner [49] and Smith [50] have shown that this function can be written as

$$S(\vartheta_0|\zeta_0,\zeta_0') = u(\cot\vartheta_0-\zeta_0')\exp\left\{-\int_0^\infty f(\tau)\,d\tau\right\} \tag{46}$$

where u is the unit step function and $f(\tau)\Delta\tau$ is the conditional probability that $\zeta(0)$ is shadowed by ζ in the interval $(\tau,\tau+\Delta\tau)$, given it is not shadowed by ζ in $(0,\tau)$. This latter function cannot be calculated exactly. Therefore Wagner [49] made the approximation that, for all τ, the probability that ζ crosses the ray in $\Delta\tau$ is independent of the values of ζ_0 and ζ_0' at $x=0$. An even more incisive simplification was made by Smith [50]. He approximated $f(\tau)$ by replacing $f(\tau)\Delta\tau$ with the conditional probability that $\zeta(0)$ is shadowed by ζ in $\Delta\tau$ given that it is not shadowed by ζ <u>at</u> τ. Furthermore he neglected correlations between the displacements and the slopes. But his results fit quite well to those of Wagner and those obtained by computer simulations [51].

For shadowing corrections in connection with sound scattering from the sea surface it generally suffices to know the probability that a surface point is illuminated, independent of its displacement. For Gaussian statistics Smith obtained

$$S(\vartheta_0|\zeta_0') = \frac{u(\cot\vartheta_0 - \zeta_0')}{1 + (2\pi)^{-1/2}\frac{s}{\cot\vartheta_0}\exp(-\cot^2\vartheta_0/2s^2) - \frac{1}{2}\operatorname{erfc}(\cot\vartheta_0/\sqrt{2}s)} \tag{47}$$

where s is the rms-value of ζ'. For the probability $S(\vartheta_0)$ that a point on the surface is illuminated, independent of displacement and slope, the step function u in the nominator has to be replaced with $1-\frac{1}{2}\operatorname{erfc}(\cot\vartheta_0/\sqrt{2}s)$.

Shadowing is only important for small grazing angles $\gamma_0 = \pi/2 - \vartheta_0$. Therefore, in many practical cases $S(\vartheta_0|\zeta_0')$ and $S(\vartheta_0)$ may be approximated by

$$S(\gamma_0|\zeta_0') \approx (2\pi)^{1/2}\frac{\gamma_0}{s}\,u(\gamma_0 - \zeta_0'), \tag{48a}$$

and

$$S(\gamma_0) \approx \left(\frac{\pi}{2}\right)^{1/2}\frac{\gamma_0}{s}, \qquad \text{for } \gamma_0 \ll s; \tag{48b}$$

while they may be set equal to 1 for $\gamma_0 \gtrsim s$.

For very small grazing angles, say below 5°, the tails of the probability density of the surface displacements become important. However these have not been sufficiently investigated. Therefore (48a) and (48b) become dubious. Possibly they go with a power of γ_0/s other than the first one. This effect should be further investigated theoretically as well as experimentally.

For shadowing correction of the scattering strength the scattering contributions from areas with a certain slope ζ' have to be weighted with the probability density $P_{\zeta'}(\zeta')$ and the shadowing function $S(\gamma_0|\zeta')$ due to this slope. If the scattering strength is simply multiplied by $S(\gamma_0)$ [21] the reduction of illuminated area by shadowing is somewhat overestimated.

6. Description of the sea surface

For a "fully developed" sea Pierson and Moskowitz [52] have given the frequency spectrum

$$F(\omega) = 0.0081\, g^2 \omega^{-5}\, e^{-0.74\left(\frac{g}{\omega u}\right)^4}; \qquad (49)$$

where g is acceleration due to gravity and u is the wind speed above the sea surface. Unfortunately the high frequency part of the spectrum which is important for backscattering for small grazing angles is not sufficiently well established and should be investigated further.

On the assumption of a dispersion relation of the form (6) the spectrum (49) suffices to describe an isotropic sea surface. For an anisotropic sea surface an appropriately chosen "spreading factor" must be introduced [13,34]. For details of the Doppler spectrum of scattered sound it is also important to know the ratio of surface waves running off to that running towards the receiver. This may be described by a "mixing function" as introduced by Schwarze [41].

The statistics of the surface is usually assumed to be Gaussian. But as we have pointed out already, deviations from Gaussian probability densities can be important for shadowing at small grazing angles. –

For further information on the description of the sea surface we may refer to the review paper given by Crowther [53].

VII Conclusions

The theory for the scattering and reflection of sound from the randomly varying surface of the sea has at the present time been developed to the point where appropriate approximations are available for all important situations. The perturbation theory permits a systematic examination of the errors involved. The main steps in the derivation of appropriate approximations can be summarized as follows:

1)
The sea surface may be considered as moving independently of the incident sound field. This establishes a linear response theory. The errors introduced may be neglected for commonly occuring incident acoustic intensities.

2)
The effect of the fluid motion in the surface waves on the scattered sound field may be neglected compared with the effect of the surface displacements so that sound scattering from the sea surface can be treated as a purely acoustic boundary value problem with a given time-varying boundary. The errors involved in this approximation are of the order of the ratio of the phase velocities of the surface waves to that of the sound wave.

3)
The sea surface may be considered as being quasi-static, which means that the time-variation may be taken into account supplementarily by introduction of time-dependent retardations. The corresponding errors are of the order of the ratio of a characteristic velocity of surface points to the sound velocity.

4)
In most cases appropriate 1. order approximations for the scattering from the corresponding quasi-static surface suffice for the description of sound scattering form the sea surface. The corresponding errors are of the order of the ratio of the rms radius of curvature to the acoustic wave-length, and/or of the order of the ratio of the rms surface displacement to the acoustic wave-length (Rayleigh roughness parameter).-

Though the mathematical basis for the scattering of sound from the sea surface seems now to be well established some, partly serious, problems remain:

1)

The spectrum of the sea surface waves contains short wave-length components with wave lengths comparable with that of the sound, as well as long wave-length components with surface displacements large compared with the acoustic wave-length. For given surface characteristics sound scattering can be calculated, in principle with sufficient accuracy, on the basis of the full (unsimplified) 1. order approximations being determined by the Fourier transform of the characteristic function of the joint probability density of the surface displacements. However, such calculations would be very complicated. Therefore, for practical purposes further simplifications are needed. Such a simplification is the two scala or facet model which seems to be quite suited for a relatively good, although simple description of the main effects. With this model the problem is the appropriate choice of the "facet length" or of the positioning of the cut dividing the surface wave spectrum into a short and a long wave-length part. The heuristic procedure described by Bachmann [21] and Schwarze [41] may be considered as a first step in solving the problem. It should, however, be complemented by a more mathematical treatment.

2)

Some characteristics of the sea surface which are important for sound scattering have not been sufficiently investigated. Thus the high frequency part of the surface wave spectrum governing backscattering for small grazing angles is not well-known. Moreover the tails of the probability density of surface displacements (behaviour for large displacements) which are important for shadowing at low grazing angles are also not well known. Further combined oceanographic-acoustic measurements should be made to fill up these gaps.

3)

The sea below the surface is not at all homogeneous. Thermal and bubble containing surface sublayers as well as volume scatterers have to be taken into account. Experimentally these influences complicate the extraction of surface scattering data from scattering measurements. Bachmann and de Raigniac [54] have pointed out that the large spread of surface scattering coefficients reported in literature is mainly due to the fact that such influences have generally been ignored. For the theoretical description the phase object picture seems to be the best suited to include the subsurface influences. However more detailed experimental and theoretical investigations have to be made to analyse the combined acoustic effects of surface and subsurface influences.

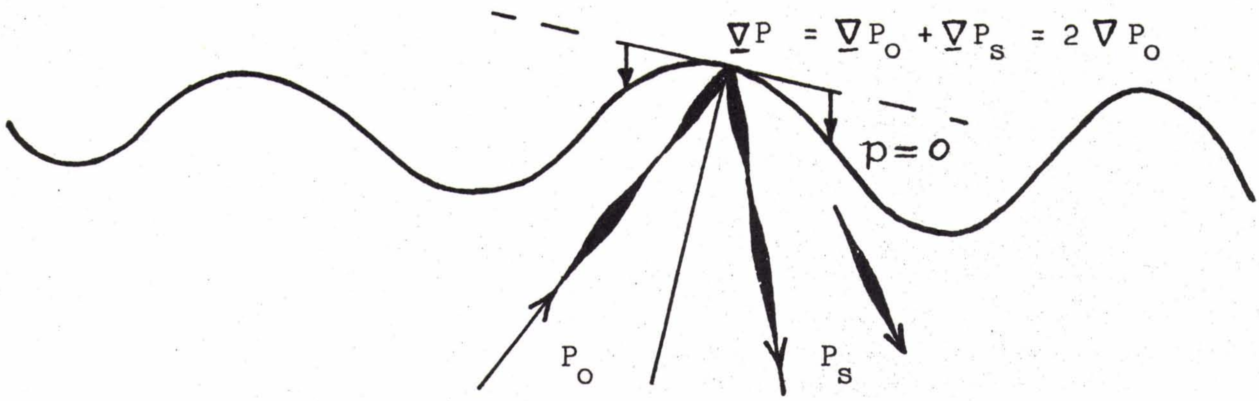

FIG. 1 SCHEMATIC REPRESENTATION OF THE KIRCHHOFF APPROACH. NORMAL DERIVATIVE OF THE SOUND FIELD PRESSURE AS FOR LOCAL REFLECTIONS FROM TANGENT PLANES.

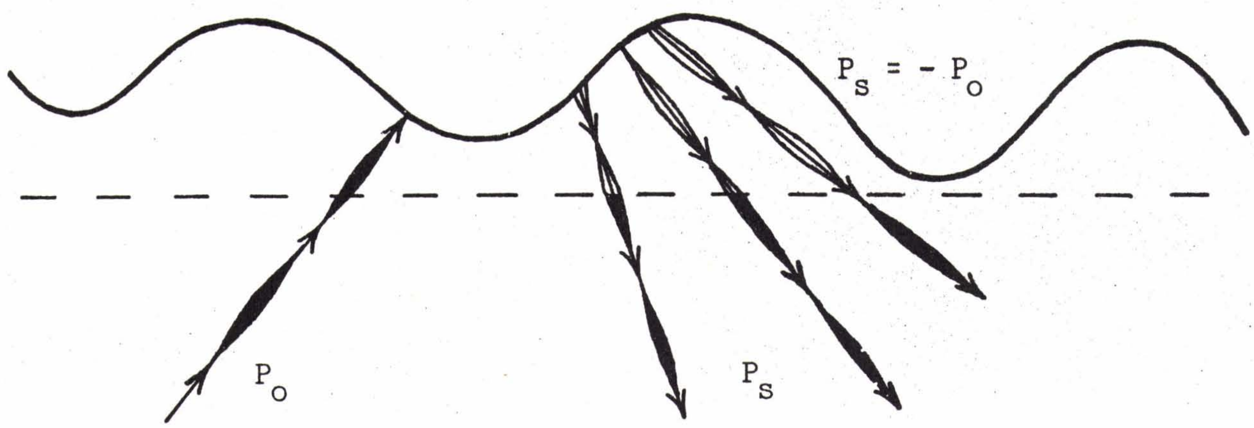

FIG. 2 SCHEMATIC REPRESENTATION OF THE RAYLEIGH APPROACH. UP TO THE SURFACE EXTENSION OF THE REPRESENTATION OF THE RERADIATED FIELD IN TERMS OF <u>OUTGOING</u> PLANE WAVES ONLY.

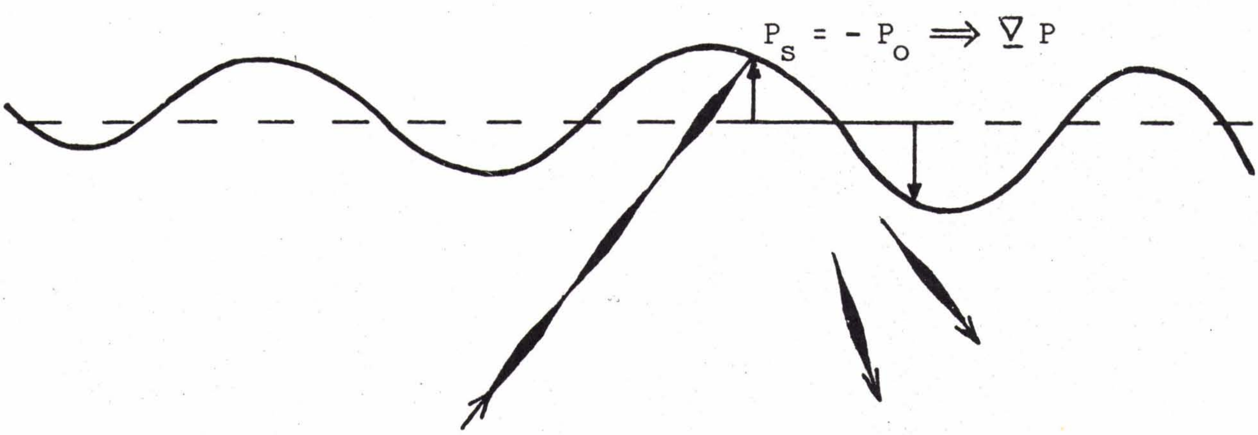

FIG. 3 SCHEMATIC REPRESENTATION OF MEECHAM'S PERTURBATION APPROACH. AVERAGE FLAT SURFACE AS STARTING-POINT FOR PERTURBATION EXPANSION.

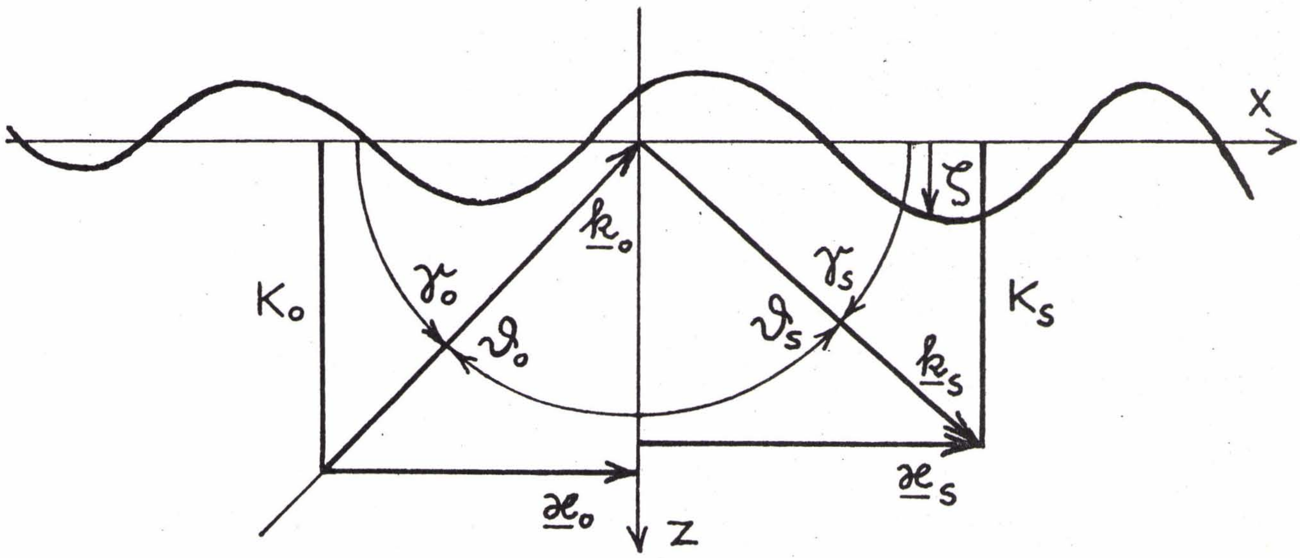

FIG.4 DESCRIPTION OF THE SCATTERING GEOMETRY.

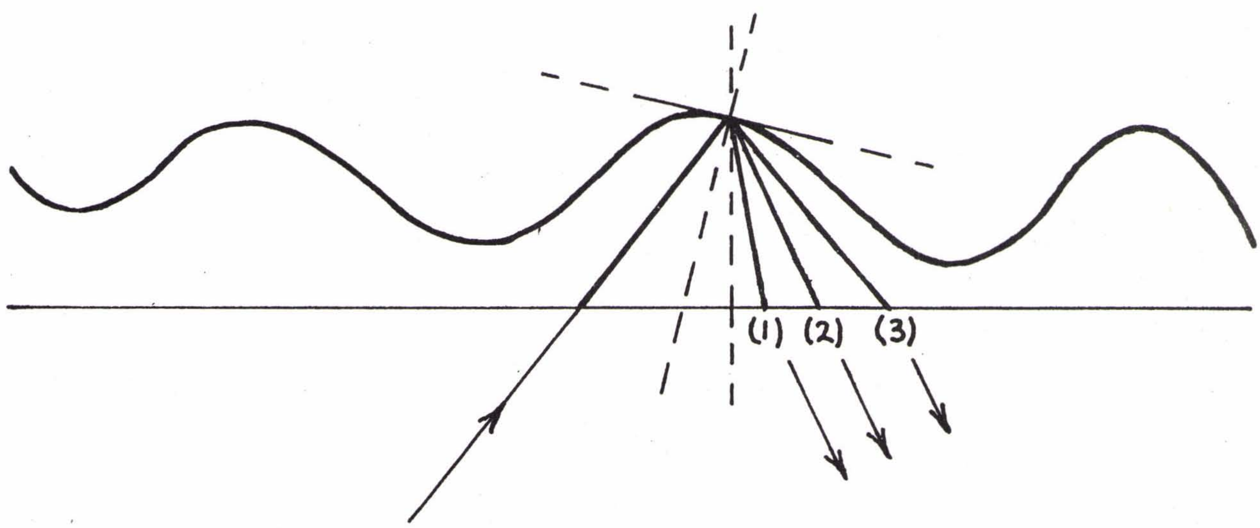

FIG. 5 THE SURFACE LAYER AS A PHASE OBJECT. RAY PATHS TRACED:
(1) according to "local reflections",
(2) along incident and scattering directions,
(3) according to specular reflection from the mean surface, corresponding to the Kirchhoff approach and the 1.Order results (34) and (33) of the Rayleigh approach, respectively.

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