EFFECT OF SHIP MOTION ON SONAR DETECTION PERFORMANCE

by

J.G. Schothorst
RVO/TNO
The Hague, The Netherlands

ABSTRACT

For an unstabilized transducer, the effect of ship motion on sonar echoes is twofold:

1. Attenuation caused by the rotational movement of the transducer in the vertical plane. The results show that the different loss figures are periodic in range. The average loss (in dB) increases about linearly with the amplitude (in degrees) of the swinging transducer.

2. Distortion caused by the irregular translational movement of the transducer in the direction of the target. In order to arrive at realistic figures for attenuation and distortion, actual ship movements are being measured. Some samples of recorded ship movement data are presented.

1.0 INTRODUCTION

In this paper a brief discussion will be given about the effect of ship motion on the detection performance of a sonar. Two effects will be dealt with: attenuation of sonar echoes due to pitch and roll of a ship with an unstabilized sonar beam, and distortion of sonar echoes due to movements of a sonar transducer in the horizontal plane.

For the case that the different types of ship motion are sinusoidal, theoretical results will be derived for the attenuation, doppler shift, frequency spread and optimum pulse length. These theoretical results will be applied to some samples of actual ship motion data.

2.0 ATTENUATION

In high seas, sonar echoes received by an unstabilized transducer will be attenuated, since the sonar beam moves with the pitch and roll of the ship.

In order to calculate this attenuation, only the movement in the vertical plane through transducer and target has to be considered. The loss in echo strength is namely due to the fact that the direction of the main beam in the vertical plane may differ from the direction in which the submarine is located. Depending on the elevation angle of the transducer at the time of transmission a considerable amount of transmitted energy may miss the target. Also energy reflected by the target may arrive at the transducer when it points in a direction differing from the target direction.

In calculation the loss in echo strength, the following assumptions were made:

a. the main beam of the transducer in the vertical plane is the same for transmission and reception and has a sin x/x - shape. (see formula for the beam shape on fig. 1, in which φ equals elevation angle and B = 3 dB beamwidth)

b. the angle of the sound ray to and from the target with the horizontal plane is close to zero

c. the movement of the transducer in the vertical plane through transducer and target can be described by a sinusoidal function of time with an amplitude A less than 1.2 B (see expression for elevation angle φ on fig.1. This sinusoidal movement of the elevation angle should be regarded as the main Fourier component of the total pitch/roll motion of the ship.

Combination of the expressions for beam shape and elevation angle results in the expression for the instantaneous attenuation f(t). The attenuation of a sonar echo however is a product of the attenuation at the time of transmission t_o and the attenuation at the time of reception $t_o + 2R/C$, in which R = range (m) and C = speed of sound in water = 1500 m/s. The resulting loss in echo strength $l(t_o,R)$ depends both on time of transmission and range. Generally the time of transmission is randomly distributed with respect to the motion of the ship, in which case the mean loss in echo strength can be obtained by averaging $l(t_o,R)$ over t_o. In addition to this mean value also the extreme values of $l(t_o,R)$ have been calculated in order to obtain a measure for the spread in loss of echo strength.

A result of these calculations is presented in fig. 2 for the case A/B = 0.8. The loss figures are plotted here as a function of R/TC, in which T = time period of periodical movement (s). A second horizontal axis with the range in km applies to a period T ≈ 10.6 seconds.

A first observation of fig. 2 shows that the result is periodical in range. The range period (equals TC/4) is of the order of 4 km. Fig. 2 shows further that depending on the time of transmission, the loss may be as low as the minimum curve or as high as the maximum curve. In order to diminish this spread it might be advantageous in certain circumstances to transmit only when the axis of the main beam is directed horizontally. The curve "HOR. TRANSMISSION" indicates for this case the loss of echo strength as a function of range.
Without such or other precautions high losses may be encountered (up to 20 dB for an amplitude of 8° and a beamwidth of 10° as is shown by fig.2).

In high seas, large amplitudes for pitch and roll are not uncommon. This is shown by fig. 3 and 4, which present some extreme values for pitch and roll for a frigate in sea state 6. On the basis of this information it may be concluded that in high sea conditions the detection performance of an unstabilized sonar will be severely affected by the pitch and roll of the ship.

3.0 DISTORTION

The distortion of sonar signals is caused by fluctuations in the distance between sonar and target. For the calculation of the effect of these fluctuations the following assumptions were made:

a. the angle of the sound ray to and from the target with the horizontal plane is close to zero
b. the movement of the transducer in the horizontal plane can be described by a sinusoidal function of time (see expression for X on fig. 5).

The assumed sinusoidal movement of the transducer should be regarded again as the main Fourier component of the total horizontal ship motion. Since this kind of ship motion is generally measured by means of accelerometers, the effect of it will be expressed in terms of acceleration in stead of displacement or velocity.

A measure for the distortion of sonar signals can quite easily be determined with the aid of the doppler shift, which is caused by the horizontal motion of the ship. As indicated on fig. 5 the doppler shift Δf is proportional to the instantaneous speed V in a particular direction as well as to the carrier frequency f_o. The doppler shift of a sonar echo is the sum of of this doppler shift Δf at the time of transmission t_o and at the time of reception $t_o + 2R/C$. The resulting doppler shift Δf_s is a function of several parameters, including the time of transmission t_o and the range R.
The maximum value of the doppler shift Δf_s with respect to t_o has been plotted in fig. 5 as a function of the range R. Depending on the time of transmission t_o the doppler shift may be as low as zero or as high as the plotted curve. The overall maximum of this doppler shift occurs at range intervals of TC/2 meter ($\approx$ 8 km).

A measure for the distortion of the sonar pulse can be obtained from the frequency spread during the pulse. This frequency spread has been derived from the calculated doppler shift Δf_s for the case that the pulse length $\tau \ll T$, the period of ship motion. In this case the frequency spread during the pulse can be approximated by the difference between the doppler shifts Δf_s at the end $(t_o + \tau)$ and at the start (t_o) of the pulse (see fig. 6).
The resulting expression for the frequency spread W shows again a dependance on the time of transmission t_o and the range R. The maximum value of W with respect to t_o has been plotted in fig. 6. Depending on the actual time of transmission, the frequency spread may be as low as zero or high as the plotted curve. The overall maximum value for this frequency spread occurs at range intervals of TC/2 meter (≈ 8 km).

In this brief account of the effect of ship motion, attention is only paid to the "worst case" situation, for which the values for doppler shift and frequency spread are maximum. For this reason fig. 7 summarizes the expressions for the maximum values of doppler shift and frequency spread.

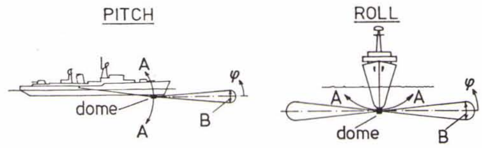
From the maximum doppler shift-formula an expression has been derived for the maximum error in the estimated target radial speed. As could be expected beforehand, this maximum error is equal to the maximum speed of the fluctuation in range between ship and target.

From the frequency spread-formula an expression can be derived for the optimum length of a sonar pulse. As a result of the frequency spread, the sonar pulse will be distorted and a correlation loss will occur in a coherent sonar detector. For the worst case situation of fig. 7 the frequency spread increases linearly with the pulse length τ. Because of this fact also the correlation loss will in the first instance increase with τ. For this reason, the signal to noise ratio S/N will show an optimum when it is regarded as a function of the pulse length τ. It might be expected that the optimum value of τ is of the order of the inverse of the frequency-spread. Fig. 7 shows an expression for the approximated optimum value of τ.
The actual optimum value appears to be very close. This actual value for the optimum pulse length has been obtained from a plot of computed results of S/N (see fig. 8). Beyond the optimum pulse length, the signal to noise ratio drops quite rapidly for increasing values of τ. For a

pulse length of twice the optimum one, the signal to noise ratio S/N is already 10 dB below its maximum value.

The "worst case expressions" of fig. 7 are finally applied to the ship motion data of fig. 9. In this figure typical amplitude spectra for horizontal accelerations are plotted for a destroyer in sea state 3. For this reasonable sea state condition the maximum amplitude of the acceleration is quite low (0.12 m/s^2). By consequence the error in the estimated target radial speed is not yet large.
The effect of frequency spread for this mild condition seems to be a more important factor in the case of coherent detection. Fig. 9 indicates that the maximum pulse length in this case is already limited to a value smaller than 1 second. In high sea conditions this restriction will be much more severe.
According to a rough estimate of ship motion in high sea conditions, the amplitude of acceleration in the horizontal plane may well be a factor 10-20 larger. Accordingly, the maximum error in estimated target radial speed will increase to 1.5 - 3 m/s and the maximum pulse length for f_o = 7000 Hz will be limited in this case to 0.18 - 0.25 seconds.

On the basis of the results discussed in this paper it may be concluded that in high sea conditions the motion of the ship may cause a severe degradation of the sonar detection performance. Accordingly, the effect of ship motion should be given due attention in the design and operation of a sonar system.



BEAM SHAPE : $\dfrac{\sin(2.8\frac{\varphi}{B})}{2.8\frac{\varphi}{B}}$

$$\varphi = A \sin 2\pi \frac{t}{T}$$

ATTENUATION :

$$f(t) = \left[\frac{\sin(2.8\frac{A}{B}\sin 2\pi\frac{t}{T})}{2.8\frac{A}{B}\sin 2\pi\frac{t}{T}}\right]^2$$

ATTENUATION OF SONAR ECHO

$$l(t_o, R) = f(t_o)\, f(t_o + \frac{2R}{C})$$

$$l(R) = \frac{1}{T_o}\int_o^{T_o} l(t_o, R)\, dt_o$$

$$L = 10 \log l$$

FIG. 1

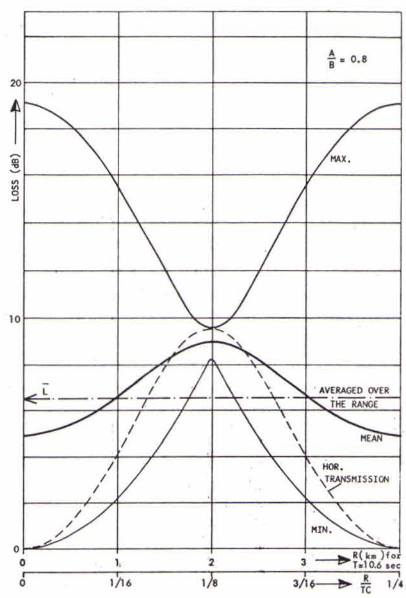


FIG. 2

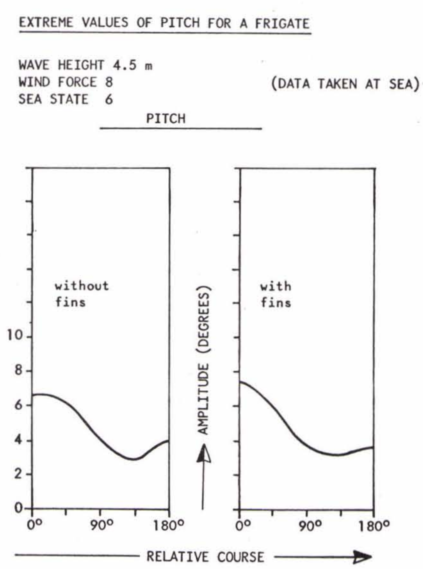


FIG. 3

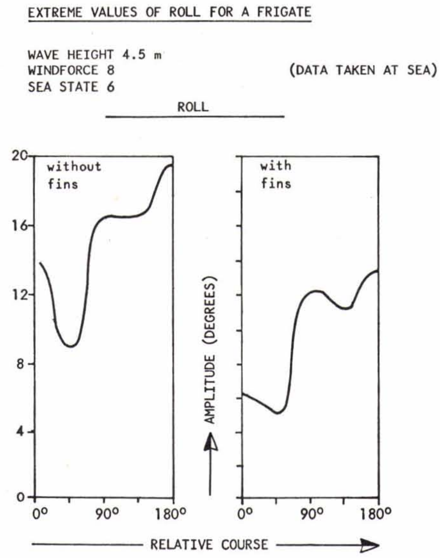
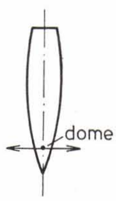


FIG. 4

TRANSVERSE ACCELERATION

LONGITUDINAL ACCELERATION

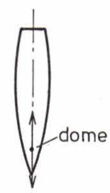


$$x = \hat{x} \sin \frac{2\pi}{T} t$$

$$V = \hat{V} \cos \frac{2\pi}{T} t$$

$$a = -\frac{2\pi \hat{V}}{T} \sin \frac{2\pi}{T} t$$

$$\text{with } \hat{a} = \frac{2\pi \hat{V}}{T}$$



doppler shift $\Delta f = \frac{V f_0}{C} = \frac{\hat{V} f_0}{C} \cos \frac{2\pi}{T} t$

doppler shift of sonar echo :

$$\Delta f_S = \Delta f(t_0) + \Delta f\left(t_0 + \frac{2R}{C}\right)$$

or

$$\Delta f_S = \frac{\hat{a} T f_0}{C} \cdot \cos \frac{2\pi}{T} \frac{R}{C} \cdot \cos \frac{2\pi}{T}\left(t_0 + \frac{R}{C}\right)$$

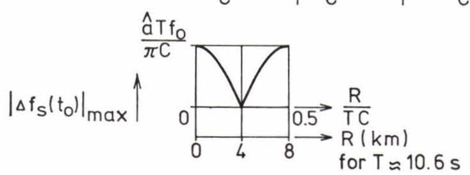


FIG. 5

FREQUENCY SPREAD DURING SONAR PULSE τ

$$W = \Delta f_S(t_0 + \tau) - \Delta f_S(t_0) \qquad (\tau << T)$$

or

$$W = -\frac{2\hat{a}\tau f_0}{C} \cdot \cos \frac{2\pi}{T} \frac{R}{C} \cdot \sin \frac{2\pi}{T}\left(t_0 + \frac{R}{C} + \frac{\tau}{2}\right)$$

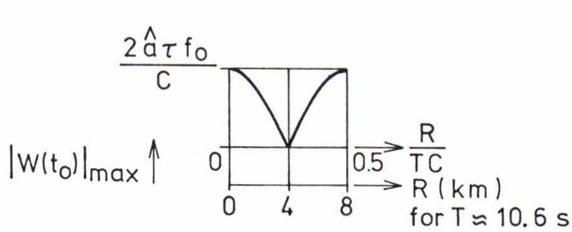


FIG. 6

"WORST CASE" SITUATION

max. doppler shift $|\Delta f_s|_{max} = \dfrac{\hat{a} T f_o}{\pi c}$

max. error in estimated target radial speed :

$$|\Delta V|_{max} = |\Delta f_s|_{max} \cdot \frac{c}{2 f_o} = \frac{\hat{a} T}{2\pi} \quad (= \hat{V})$$

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max. frequency spread $|W|_{max} = \dfrac{2 \hat{a} \tau f_o}{c}$

approx. for τ_{opt} : $\tau \approx \dfrac{1}{|W|_{max}} \rightarrow \tau = 0.7 \sqrt{\dfrac{c}{\hat{a} f_o}}$

actual optimum : $\tau_{opt} = 0.6 \sqrt{\dfrac{c}{\hat{a} f_o}}$

FIG. 7

COHERENT DETECTION OF TONE PULSE

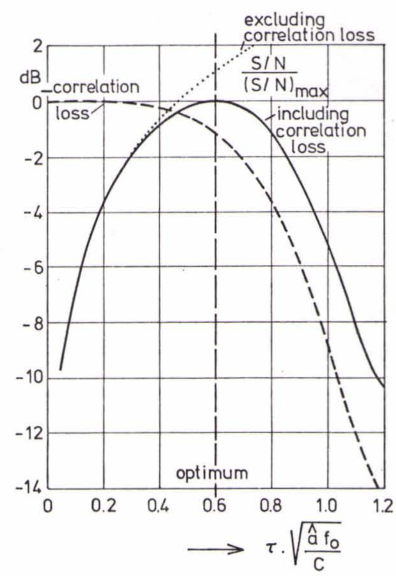
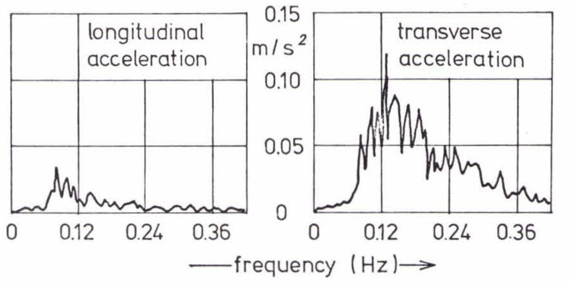


FIG. 8

TYPICAL AMPLITUDE SPECTRA FOR HORIZONTAL ACCELERATONS OF A DESTROYER

(data taken at sea) Wave height : 1 - 2 m
Wind force : 5
Sea state : 3



maximum amplitude : $\hat{a} = 0.12$ m/s² at T = 8 s :

max. error in estimated target radial speed : 0.15 m/s
optimum pulse length at 7000 Hz : 0.8 s

FIG. 9