MEASUREMENTS OF NORMAL-MODE AMPLITUDE FUNCTIONS IN A NEARLY-STRATIFIED MEDIUM*

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Abstract

In normal-mode treatments of acoustic propagation in nearly-stratified media, calculations are frequently based on the perfectly-stratified medium model, in which symmetry permits separation of the wave equation. In these treatments the assumption is made that the normal-modes adapt to local conditions and that the modes are not coupled by the changing environment. This assumption is frequently referred to as the adiabatic approximation. An experiment was performed in a shallow water area near Jacksonville, Fla., in which observations were made of individual normal modes propagating over two sloping bottom tracks. Over the first track the water depth increased from 30 m at the source to 41 m at the receiver, with the maximum bottom slope (0.3°) at the receiver. Isovelocity conditions prevailed on this track. Over the second track the water depth decreased from 120 m to 42 m with the maximum slope (0.3°-0.9°) at the source positions. The vertical sound speed gradient on this track was slightly negative at the receiver and more sharply negative at the source. In both cases the constant depth model gave the vertical pressure distributions observed, in agreement with the assumption that the modes adapt to local conditions. Results from the first track are in agreement with the adiabatic approximation, however, propagation over the second track indicates the presence of mode coupling.

Introduction

In investigations of acoustic propagation in shallow water environments, extensive use has been made of the normal mode representation of the acoustic field introduced by Pekeris.[1] The original model which treated isovelocity water overlying a uniform semi-infinite sediment has been extended to treat perfectly-stratified ocean environments,[2] in which the acoustical properties are functions of depth only. A normal-mode computer program of a perfectly-stratified medium has been developed at NRL.[3] This program models the ocean environment as two finite-thickness fluid layers overlying a semi-infinite region which can be a fluid or a shear-supporting medium. The two finite layers, which are usually taken to be water and surficial sediments, may have a velocity profile which is an arbitrary function of depth. The model subbottom is uniform. Vertical pressure distributions and group velocities of individual normal modes measured in real ocean environments agree with predictions of this computer model.[4] The locations of the at-sea experiments had been chosen to provide propagation paths which closely approximated the perfectly-stratified-medium model.

Many shallow water areas of interest exhibit horizontal as well as vertical variability, so there exists the need for models of regions with range-dependent as well as depth-dependent acoustical properties. The extensions of perfectly-stratified normal mode theory to slowly varying (almost-stratified) media by Pierce[5] and Milder[6] make use of the adiabatic approximation.

This approximation is applied by assuming that the horizontal changes with range are sufficiently gradual that the wave equation separates locally. Properties of the normal-modes, such as vertical pressure distribution, are then obtained from the constant-depth model using the local environment information. In addition, if this approximation is valid, the normal-modes propagate without coupling, i.e. without the transfer of energy among modes.

An experiment measuring attenuation of normal-modes in a tank with a sloping rubber bottom was reported[7] by Eby, Williams, Ryan, and Tamarkin in 1960. The attenuation data were in agreement with a model using the adiabatic approximation in the calculation of the attenuation coefficient.

In November 1973 an experiment was performed by NRL to measure the vertical distribution of pressure in individual normal modes propagating over a sloping bottom. This paper compares the measured distributions with those predicted by the adiabatic approximation.

Summary of Theory

The geometry used in the mathematical model for the perfectly-stratified medium is shown in Fig. 1. The ocean environment is divided into three fluid layers, each of constant density. The upper two layers may have arbitrary sound speed dependence on depth. The sound speed in the subbottom is assumed constant. A point source of unit strength and angular frequency ω is located at depth z_0. A receiver is located at depth z_1 and range r_1 from the source. The pressure at the receiver is given by

$$P\ (r,z_1,z_0) = \omega\rho_1{}^2 \left(\frac{1}{8\pi r}\right)^{\frac{1}{2}} \sum_{n=1}^{N} \frac{u_n(z_0)u_n(z_1)}{k_n^{\frac{1}{2}}} e^{i(k_n r-\omega t)-\delta_n r} \tag{1}$$

where δ_n is the modal attenuation coefficient and the $u_n(z)$ are solutions of the z-dependent part of the separated wave equation

$$\frac{d^2u_n}{dz^2} + \left(\frac{\omega^2}{c^2(z)} - k_n{}^2\right) u_n = 0 \tag{2}$$

subject to the appropriate boundary and normalization conditions. The dependence of the pressure amplitude of the nth mode on source and receiver depth is given by

$$P_n(z_1,z_0) \propto u_n(z_1)u_n(z_0) \tag{3}$$

where other parameters in Eq. 1, such as frequency and range are kept fixed. Verification of the proportionality (Eq. 3) in at-sea experiment has been reported by Ferris.[4]

If we apply the adiabatic approximation to the nearly-stratified medium case a similar expression is obtained:[8]

$$P_n(z_1,z_0) \propto u_n(z_1)u'_n(z_0) \tag{4}$$

Here u_n is the solution of Eq. 3 using the environment in the vicinity of the receiver and u'_n is obtained using the source's immediate environment. If the environments are significantly different, there will be a considerable change in the shape of the function u_n between source and receiver. The adiabatic approximation assumes that this change takes place very slowly and that energy is not exchanged among modes of propagation.

Experimental

The site of the experiment is shown in Fig. 2. A spar buoy supporting a string of 12 hydrophones was anchored at the location indicated by the numeral 1. The signals received by the hydrophone string were telemetered back to the source ship for recording. The source ship occupied the two stations to the west of the spar buoy and operated a 400 Hz source in a pulsed mode at five depths at each station. A similar set of runs was made with the buoy at location 2 and the source ship occupying the three stations to the east of the buoy. The track extending west from the buoy will be referred to as the shallow propagation path, the other track, extending east from the buoy will be called the deep propagation path. The sound speed in the sediment was determined by seismic refraction techniques.[9]

Results

Bathymetry of the shallow propagation path is shown in Fig. 3. The bottom slope at the receiver (zero range) is about 0.3°. Also shown are water sound speed profiles measured at the receiver and at the two source stations. The profiles indicate essentially isovelocity conditions along the entire track. Bathymetry and sound speed profiles for the deep propagation path are shown in Fig. 4. The water depth at the receiver is the same (42 m) as for the shallow path. On the deep path the slope increases with range from the receiver to about 0.9° at the 16 km range station. In addition, a considerable change takes place in the sound speed profile. Near the receiver the profile gradient is slightly negative. At the source stations the profile gradient is more negative, becoming increasingly negative with range from the receiver.

Data obtained from pressure distribution measurements of the first mode on the shallow path are shown in Fig. 5. At zero range a typical measured variation of pressure with receiver depth, keeping source depth and range fixed, is shown as dots. The distribution predicted from the constant depth model is indicated by the line. The vertical pressure distribution in the vicinity of the sloping bottom is seen to be accurately calculated from the constant depth model. The data plotted at 9 and 18 km indicate the variation in signal strength as the source depth is varied. The receiver depth and range are held constant in each comparison. In these cases the variation of signal strength with source depth is given by the eigenfunction calculated using the constant depth model and the environment near the source. The data are consistent with the adiabatic approximation.

The dependence of the signal strength of the second mode on source and receiver depth is shown in Fig. 6. Again the constant-depth model is seen to predict the dependence of the signal strength on the source or receiver depth when the appropriate local environment is used. In signal strength vs source depth data the absolute value of the eigenfunction is plotted since phase changes implied by axis crossings could not be measured.

Variation of first mode signal strength for propagation over the deep path is shown in Fig. 7. The vertical pressure distribution at the receiver is shown at zero range. Agreement with the eigenfunction calculated from the constant depth model is still good. Variation of the signal strength with source depth shows good agreement at the 8 km station only. At the 12 and 16 km stations stations the constant depth model predicts the first mode to be "trapped" near the bottom by the negative gradient profile. In several runs the source was placed in the upper part of the water column, where the first-mode eigenfunction is very small. If the adiabatic approximation is valid the first mode should be absent from the signal field. The first mode was observed with the relative strength indicated by the uppermost square. Agreement of the predicted and measured strength of the first mode at the 16 km station is very poor. Results for the second mode are shown in Fig. 8. Again the pressure distribution (zero range) at the receiver is predicted by the constant depth model and the receiver's immediate environment. Variation of the signal strength with source depth does not agree with predictions. Similar results shown in Fig. 9 were obtained for the third mode.

Conclusions

Measurements were made of vertical pressure distributions of individual normal modes propagating in two nearly-stratified ducts. In the first case the receiver was located over a sloping bottom. The slope (0.3°) is typical of shallow continental shelf areas. In the second case the water depth varied by a factor of 2 or 3 over the propagation path and a considerable change in sound-speed profile over the path was found. As a result there was a considerable change in the shape of the eigenfunction over the second track. In both cases the vertical pressure distribution of the modes observed was in agreement with the predictions of the constant-depth normal-mode model using the immediate environment of the receiver.

On the first path, where the change in depth was relatively small and isovelocity conditions prevailed along the entire track, the measured variation of signal strength with source depth was found to be in agreement with the predictions. Over the second path the water depth changed by a factor of two and the sound speed profile changed significantly. Here the predicted dependence of mode signal strength on source depth does not agree with the measured dependence. This disagreement may be due to the failure of the theory to calculate the excitation of the modes correctly, or it may be due to the conversion of energy from one mode to another by the environmental changes. The experimental results cannot be used to determine which of these possible causes is responsible since the signal strength of the individual modes was not measured near the source. It seems probable, however, that the excitation was calculated correctly, at least at the 8 km and 12 km stations. The bottom slope at the stations is less than at the receiver station on the first track, where local separability of the wave equation appeared to be a good approximation.

The deep propagation path exhibits a horizontal variability representative of continental shelf area waters. The indication of mode conversion along this path suggests that shallow water propagation models may need to include mode coupling effects.

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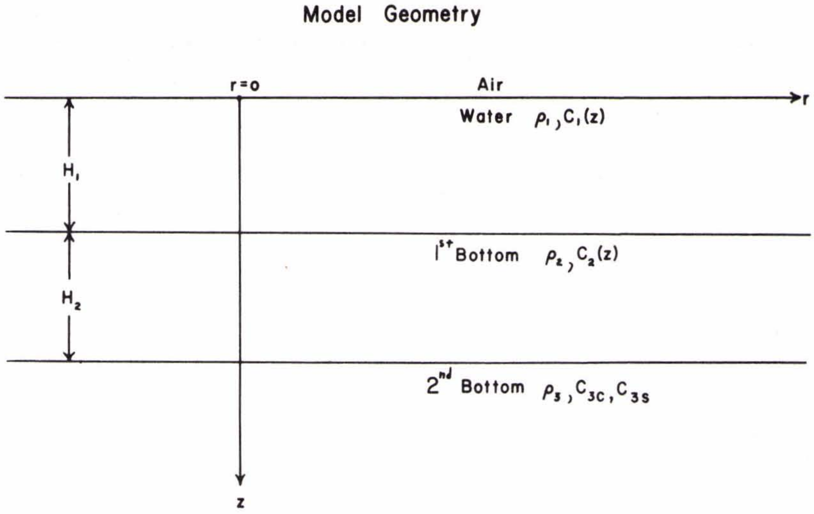
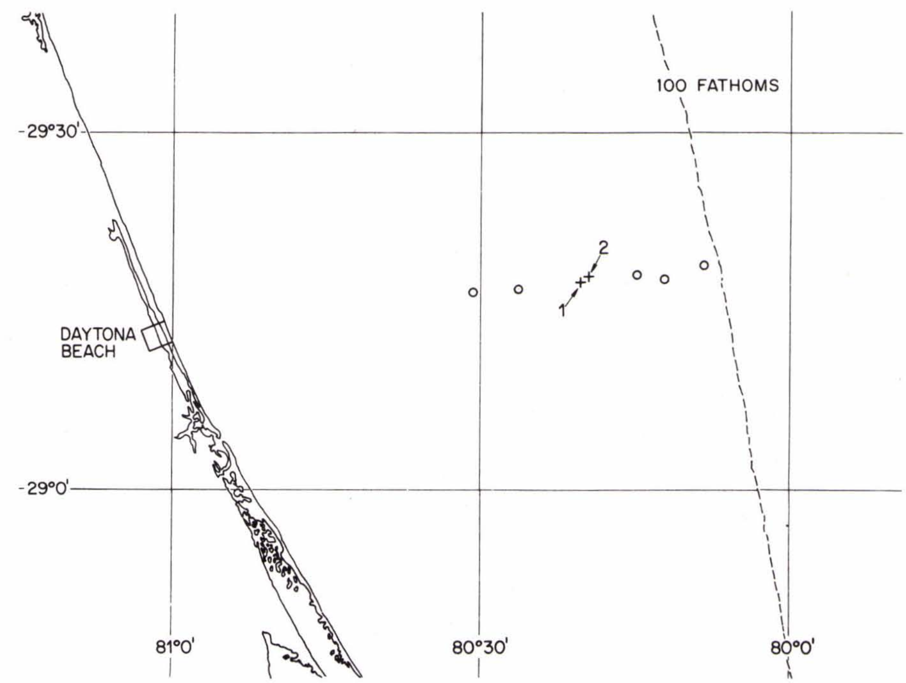


FIG. 1

FIG. 2



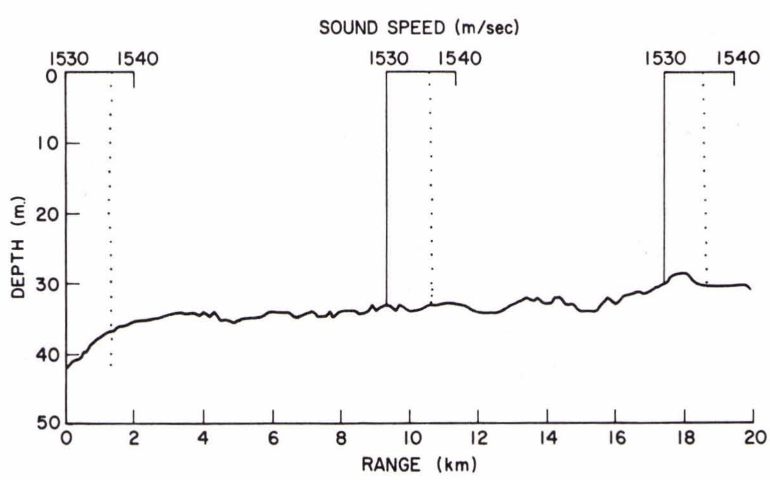


FIG. 3

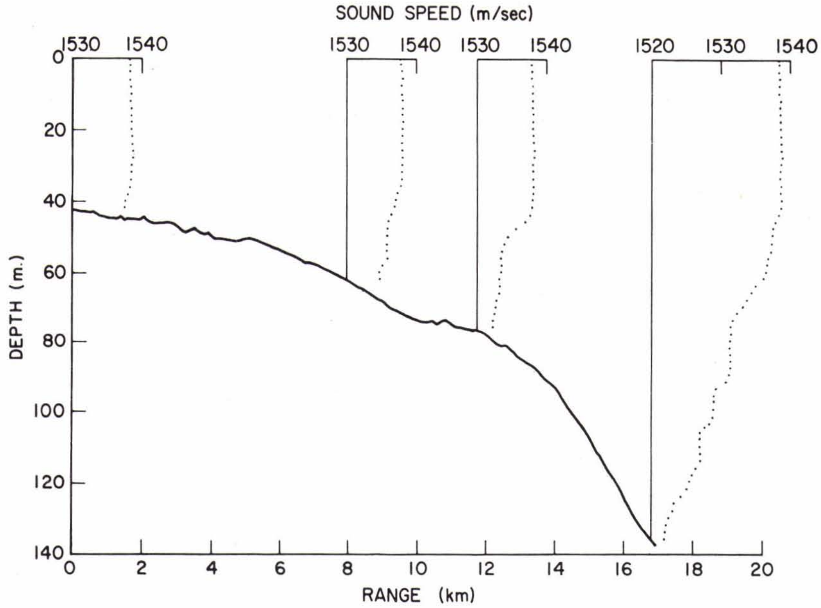


FIG. 4

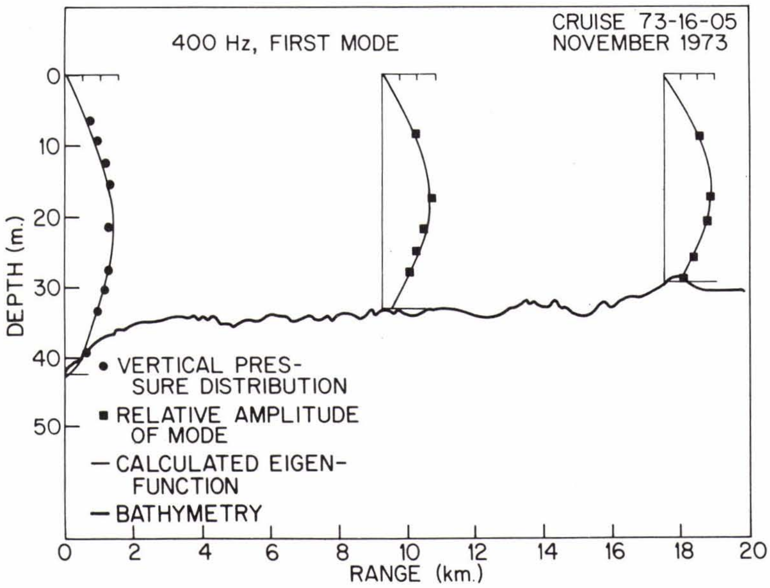


FIG. 5

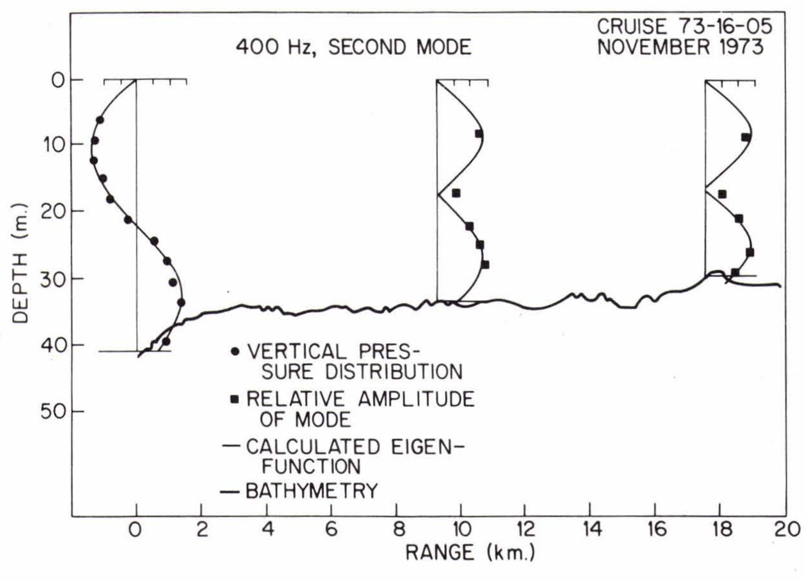


FIG. 6

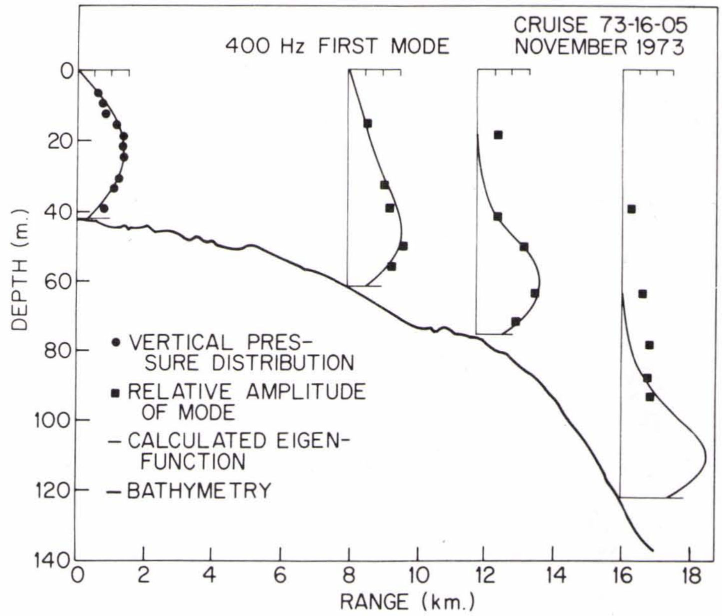


FIG. 7

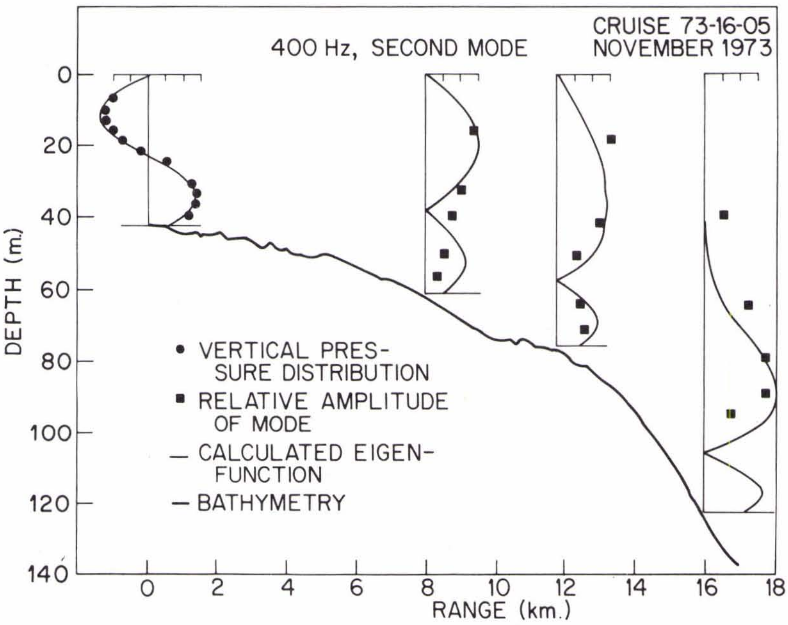


FIG. 8

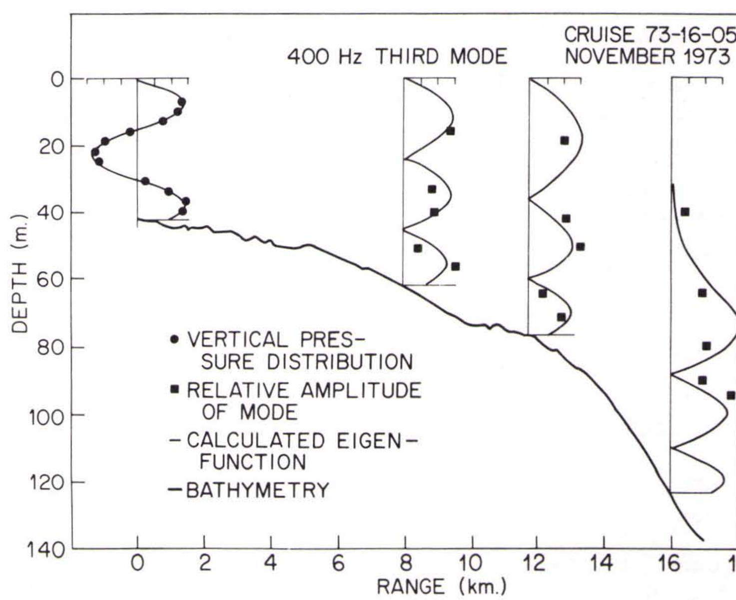


FIG. 9