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Quantum Interferometer Based on Multiphoton Entanglement for Enhanced Detection

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A theoretical architecture for a very wideband, highly sensitive radio frequency (RF) detector is being created. Rydberg atoms combined with quantum interferometry based on linear combinations of M and N states form the basis of this new detector. Closed form expressions have been derived showing the effectiveness of this procedure. Adaptive optics techniques were developed offering significant enhancements. Entanglement greatly reduces the optical frequency phase error and the number of photons required while maximizing visibility. This will ultimately yield a highly sensitive very wideband RF detector. Numerical results show the effectiveness of these combined approaches.

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EXECUTIVE SUMMARY

A theoretical architecture for an ultra-wideband, highly sensitive radio frequency (RF) detector is being created. The detector will combine Rydberg atoms and optical interferometry based on quantum entanglement. It should offer detection from 4GHz to 50GHz. Use of entanglement will offer detection sensitivity far exceeding RF detection schemes based on Rydberg atoms created by others.

The initial phase, June 8 – Sept. 30, 2020, emphasized research related to the optical frequency quantum interferometer and the multiphoton entangled states. The research emphasized the utility of a multiphoton entangled state referred to as a linear combination of *M* and *N* states (LCMNS).

Mathematical analysis was conducted related to novel quantum entangled states useful for interferometry. For interferometry applications, these states offer a huge improvement in phase sensitivity, while greatly reducing the number of photons used. Shot noise limited interferometers will offer a phase error of at least $1/\sqrt{N}$, where N is the number of photons used. For large N, there can be vibration of optical elements reducing the sensitivity of the interferometer. The approach based on entanglement presented here, even in the presence of loss reduces the number of photons used by roughly a factor of $1/\sqrt{N}$, while yielding enhanced phase sensitivity. The new interferometer is an improvement on a previous interferometer based on LCMNS. LCMNS have been shown to be much more robust in the presence of environmental effects than plain *M* and *N* states or N00N states. The new interferometer described in this paper uses adaptive optics techniques to adjust the coefficients of the LCMNS, simultaneously minimizing phase error, maximizing visibility and greatly reducing the number of photons used. Closed form expressions for the minimum phase error and visibility were derived as well as expressions relating the two quantities. Closed form constraints for optimal coefficients were derived. Adaptive optics procedures for realizing the coefficients were considered. Expressions were derived explicitly relating the fundamental parameters of the model space to each other. Environmental effects are included using open systems theory. Relationships to previous research results were discussed showing the superiority of the current approach. The work yielded high quality numerical results.

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QUANTUM INTERFEROMETER BASED ON MULTIPHOTON ENTANGLEMENT FOR ENHANCED DETECTION

1. INTRODUCTION

Existing radio frequency (RF) detectors require antennae of various sizes to detect electromagnetic emissions at different frequencies. These systems can be highly sensitive to interference, jamming and may be subject to natural electromagnetic effects, e.g. solar flares. They also lack the sensitivity to detect many low powered signals.

A new class of detectors based on Rydberg atoms has been proposed [1-3]. These systems would permit detection over a wider range of frequencies (up to four octaves, i.e. C-band to Q-band or 4 to 50 GHz) and be much more resistant to electromagnetic interference than classical systems. [1, 2]

The Rydberg antennae would consist of a vapor of Rydberg atoms [1-3]. These are atoms in a highly excited state. The highly excited state makes the atoms especially sensitive to RF radiation.

It has been realized [1] that the atoms in the vapor cell can be illuminated by a laser at a particular frequency, this places the atoms in a state where they can't absorb additional energy, so that photons from a second laser will pass through the vapor, rendering the gas effectively transparent. The critical frequency at which the gas is transparent changes if it interacts with RF radiation. This will produce a flickering of the second laser beam. The illuminated vapor cell becomes an optical RF detector, without requiring additional metallic components, e.g. wires, antenna rods or circuitry.

The Rydberg system has been experimentally tested. The system was successful in receiving pulsed and modulated RF fields. Experiments include the detection of both AM and FM broadcast. [1]

Anderson [1] has advanced that the vapor cell approach should be damage resistant if they are impacted by powerful electromagnetic emissions, e.g. solar flares due to the lack of circuitry in their design. They may also be more resistant to microwave attacks than classical architectures. Anderson has also advanced that the flexibility of the design permits the atomic system to operate within whatever frequency band is desired. This may give it a greater resistance to electronic warfare.

The detector cells are on the order of millimeters in size. It has been advanced that further minimization is possible. Supporting systems are still large, precluding the use of these atomic systems in small structures, e.g. an automobile dashboard. Anderson indicates that over the next two years it may be possible to reduce the size of the supporting system down to the size of a suitcase. [1]

A recent paper [2] explores increasing the RF detector sensitivity of these atomic detectors by introducing an interferometer. Initial results are promising. The limitations on the atomic detector's RF sensitivity is imposed by the phase error on the interferometer. The best interferometer phase errors obtained were three orders of magnitude greater than the shot noise limit.

If the interferometer uses N non-entangled photons, then the smallest possible phase error is given by the shot noise limit, $\delta\phi = 1/\sqrt{N}$. [4, 5] The phase error can always be reduced by increasing the number of photons in the interferometer, but after a certain point the elements of the interferometer can be heated resulting in damage through cracking or additional vibrations that can contribute to an increase in phase error.

It is known that if the photons used in the optical interferometer are entangled then the phase error can be greatly reduced. If N photons are entangled to form a path entangled N00N state then in the absence of loss the phase error is given by the Heisenberg limit, $\delta\phi = 1/N$. [4, 5] Unfortunately, N00N states of this kind are fragile and can be particularly vulnerable to loss. [4, 5]

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It has been demonstrated that a generalization to the N00N state, the *M* and *N* states [6-8] are more robust when faced with loss mechanisms than N00N states. It has been further demonstrated that linear combinations of *M* and *N* states significantly increase robustness while offering smaller phase errors. In a recent peer reviewed journal article [8] a linear combination of *M* and *N* states exhibited a robustness measure four times greater than that of the associated *M* and *N* state and 11 times greater than that of the associated N00N state. Therefore, the linear combination of *M* and *N* states is much more robust than competing types of entangled states. [8]

In the same paper [8], for the same systems as above, the linear combination of *M* and *N* states had a phase error 11 times smaller than the corresponding *M* and *N* state and 30 times smaller than the associated N00N state. The use of *M* and *N* states or linear combination *M* and *N* states should offer significant improvements in the measured phase ultimately significantly improving the RF sensitivity of the atomic detector.

A recent Naval Research Laboratory (NRL) program yielded the optical phase error improvements based on entanglement discussed above. The proposed program will leverage experience obtained during past NRL programs [7-26] ultimately creating an atomic receiver system with enhanced sensitivity.

The initial phase of NISE funding for this project, June 8 – Sept. 30, 2020, emphasized research related to the optical frequency quantum interferometer and the multiphoton entangled states. The research developed methods of generating high quality LCMNS using adaptive optics procedures to select coefficients in wave function expansions that offer significant utility.

It has been observed that the use of quantum mechanically entangled particles, e.g., photons for measurement processes can give rise to phenomena known as super sensitivity and super resolution [4, 7, 8, 27]. Super sensitivity refers to a measurement with an error less than the shot noise limit and sometimes as small as the Heisenberg limit (HL). Super resolution refers to measurements that are better than the Rayleigh diffraction limit. A paper by the author [28], to be published discusses closed form expressions for the optimal detection operator and its square as well as visibility, and minimum phase error as a function of the resolution for linear combinations of *M* and *N* states (LCMNS), plain *M* and *N* states (PMNSs) and N00N states [5, 6, 7, 8, 11] This yields an optimization criterion for determining the coefficients for a LCMNS. A closed form solution for these coefficients has been derived. The coefficients have been shown to offer high phase sensitivity, i.e. a small phase error while using only a small number of photons. These coefficients can be obtained using adaptive optics procedures. Correction of this kind reduces the interferometric measurements sensitivity to internal loss mechanisms, improving phase measurement while permitting the interferometer to have low complexity. Results related to interferometers and interferometer sensors, using quantum entanglement are provided.

Quantum mechanically entangled photonic states referred to as N00N states in a lossless environmental can yield phase measurements using a Mach-Zehnder interferometer (MZI) that reach the Heisenberg limit [4-6, 9, 27, 29, 30]. N00N states are very fragile under loss processes [6-9, 11, 27]. This fragility can lead to phase errors for the N00N state when dealing with *N* photons for large *N* that can exceed the phase error for *N* separate photons [8, 27]. Even with their fragility N00N states can be effectively used for some applications, e.g. miniaturization [29], and space-based activities [29]. N00N states can be effectively used in the presence of loss for LADAR range finding if a correction procedure is applied [9] or the ranges to be determined are small and loss is low [9]. The correction procedure for N00N states [9] requires significant amounts of environmental information, and for examples considered in the literature [9], even with the correction procedure the range error was generally improved by no more than a factor of two when compared to four separate photons propagating through an atmosphere with randomly varying attenuation.

It has been shown that a different type of entangled photonic state, the plain *M* and *N* state is more robust under propagation in a lossy environment [6-8] than a comparable N00N state. At the same time plain *M* and *N* states offer high visibility, i.e. a high value of the maximum expectation of the detection operator and low values of the minimum phase error. It has also been shown that instead of using plain *M* and *N* states, by using linear combinations of *M* and *N* states, the requirement for Fock state generation, a difficult constraint is removed [4, 5]. It has been demonstrated that not only do LCMNS permit Fock state generation to be avoided, but they also offer higher resolution, higher visibility and lower values of the minimum phase error than N00N or PMNS for propagation in an environment with loss [7, 8]. The extra robustness that LCMNS offer should provide significant utility for sensors based on quantum entanglement. Use of LCMNS and the adaptive optics procedure for determining coefficients permits a significant improvement in phase error while reducing the overall transmitter and detector complexity by reducing the required number of photons.

This paper has the following organization. In section 2 plain and linear combinations of *M* and *N* states are introduced. The density matrix and reduced density matrix for LCMNS derived in a previous paper are reviewed. In section 3 convenient forms of the optimal measurement operator A and its square A^2 are reviewed. Closed form expressions for the expectation of A, the visibility, i.e., the maximum expectation of A, the expectation of A^2, the phase error and the minimum phase error are reviewed. The detection operator's ability to raise and lower in pairs, the number of photons in an *M* and *N* state is established. The model space and constraints over which the detection operator is optimal are defined as well as the mathematical form of the coefficients in the LCMNS expansions. Section 4 discusses a representative collection of computational results illustrating the superiority of LCMNS. The optimal resolution for the maximum visibility and the minimum phase error is found. For visibility, comparisons between LCMNS and N00N states are provided. Section 4 discusses an extension of these ideas obtained by determining the coefficients of the LCMNS using adaptive optics procedures; subsequently, minimizing phase error, maximizing visibility and greatly reducing the number of photons used. Closed form expressions for the minimum phase error and visibility are discussed as well as expressions relating the two quantities. Closed form constraints for optimal coefficients are discussed. Expressions are discussed that will appear in a future publication by the author [28] explicitly relating the fundamental parameters of the model space to each other. Environmental effects are included using open systems theory. Relationships to previous research results are provided showing the superiority of the current approach. Section 5 discusses near future research directions to be pursued in the next funding cycle. Finally, section 6 provides conclusions.

2. LINEAR COMBINATIONS OF M AND N STATES

Various entangled states can be used to generate enhanced visibility and reduce phase errors compared to classical photonic states that don't use entanglement. The classical states require large numbers of photons which can contribute to loss mechanisms. This section describes various types of entangled states which offer significant utility when compared to classical photonic states and interferometers.

For an interferometer with arms "a" and "b" when the photons are prepared in a N00N state, N photons travel down arm "a" and 0 down arm "b" or vice-a-versa. It has been observed that N00N states are fragile under loss processes [9, 27]. To help deal with the effects of loss, a correction procedure has been developed [9] that permits super sensitivity to be achieved when applying quantum entanglement to LADAR over extended ranges. It is desirable to reduce the amount of correction required, since the correction algorithm will require additional information, which in turn implies additional hardware and time demands. To increase the measurement system's robustness when dealing with loss processes and reducing correction requirements, PMNSs and LCMNS are introduced in this section.

The LCMNS have coefficients that must be determined. In section 4 a discussion is provided closed form expressions to be published in a near future paper by the author [28] that yields coefficients that simultaneously minimize the phase, the number of photons used and maximize the visibility for a given value of the resolution parameter P and loss parameter R. The closed form expressions for the coefficients form the basis for an adaptive optics procedure that helps to significantly reduce the effects of internal loss within the interferometer. This adaptive approach helps to greatly reduce the number of photons required for making the phase estimate. Reducing the number of photons helps reduce the effects of internal loss, reducing power requirements and possibly contributing to improvements in combined measures of effectiveness (MOEs) related to the overall measurement systems size, weight, power and costs (SWAPc).

A PMNS is a generalization of the concept of a N00N state. For a Mach-Zehnder interferometer and a system made up of m and m' photons prepared in a plain M and N state, m photons travel along arm "a" and m' down arm "b" or vice-versa. The wave function for the plain M and N state is given in (1). It is assumed that there is a phase ϕ to be measured along arm "b",

$$\left| m :: m' \right\rangle_{ab} \equiv \frac{1}{\sqrt{2}} \left[\exp(-j \cdot m' \cdot \phi) \cdot \left| m, m' \right\rangle_{ab} + \exp(-j \cdot m \cdot \phi) \cdot \left| m', m \right\rangle_{ab} \right]. \tag{1}$$

In (1) and afterward when the symbol $\left| m :: m' \right\rangle$ appears it indicates that $m > m'$. It can be observed from (1) that for $m = N$ and $m' = 0$ that the PMNS in (1) reduces to a N00N state [5-8].

It can be observed on intuitive and theoretical grounds that a plain M and N state should be more robust under loss processes than a N00N state [6-8]. It has been noted and has been shown [5-8] that plain M and N states offer higher visibility. The visibility is the maximum absolute value of the expectation of the detection operator. This quantity is discussed in more detail in section 3.

A further generalization of the PMNS is a LCMNS. It is shown below that a LCMNS offers still higher visibility than PMNSs and provides the additional advantage that Fock states need not be constructed [4-8]. A method based on a double MZI and a cross-Kerr effect for constructing N00N states, linear combinations of N00N states, plain M and N states and linear combinations of M and N states is described in the literature [5]. A method of greatly increasing the efficiency of this entanglement procedure using electromagnetically induced transparency is discussed by Dowling [4].

A LCMNS takes the form

$$\left| \psi \right\rangle_{ab} \equiv \sum_{m,m'=0}^{N} C_{m,m'} \left| m :: m' \right\rangle_{ab}. \tag{2}$$

For a specific model space, the coefficients $C_{m,m'}; \quad m,m' = 0,1,\ldots,N$ and their normalization are discussed in section 3.

The density matrix related to $\left| \psi \right\rangle_{ab}$ is

$$\rho_{ab} \equiv \left| \psi \right\rangle_{ab\,ab} \left\langle \psi \right| = \sum_{m,m'=0}^{N} \sum_{n,n'=0}^{N} C_{m,m'} \cdot C^{*}_{n,n'} \cdot \left| m :: m' \right\rangle_{ab\,ab} \left\langle n :: n' \right|. \tag{3}$$

As discussed by Huver [6], the effects of loss along arms "a" and "b" of the MZI will be modeled by the introduction of a single beam splitter. Let the destruction operators related to the output of the effective loss beam splitter be denoted as $\hat{a}'$ and $\hat{b}'$, respectively, where

$$\hat{a}' = t_a \cdot \hat{a} + r_a^* \cdot \hat{a}_V \,, \tag{4}$$

and

$$\hat{b}' = t_b \cdot \hat{b} + r_b^* \cdot \hat{b}_V \,. \tag{5}$$

The quantities $t_u = \sqrt{T_u} \cdot \exp(j \cdot \phi_u)$ and $r_u = \sqrt{R_u} \cdot \exp(j \cdot \psi_u)$ for $u = a \quad or \quad b$ are the complex transmission and reflectance coefficients, respectively along arms "a" and "b". The destruction operators $\hat{a}$ and $\hat{b}$ destroy photons along arms "a" and "b", respectively. The destruction operators $\hat{a}_\nu$ and $\hat{b}_\nu$ destroy environmental noise photons along arms "a" and "b", respectively.

In the literature [7,8], it is shown that the reduced density matrix denoted as $\hat{\rho}_{a'b'}$ takes the form

$$\hat{\rho}_{a'b'} = tr_V(\rho_{a'b'}) = \sum_{m,m'=0}^{N} \sum_{n,n'=0}^{N} \sum_{k=0}^{\min(m,n)} \sum_{l=0}^{\min(m',n')} \exp[-j \cdot (m'-n') \cdot \phi] \cdot Q_{k,l}^{m,m',n,n'} |m-k, m'-l\rangle_{ab\,ab}\langle n-k, n'-l| \,, \tag{6}$$

where

$$Q_{k,l}^{m,m',n,n'} \equiv \frac{1}{2} \cdot \sqrt{\binom{m}{k} \cdot \binom{m'}{l} \cdot \binom{n}{k} \cdot \binom{n'}{l}} \cdot T_a^{\frac{m+n-2k}{2}} \cdot T_b^{\frac{m'+n'-2l}{2}} \cdot R_a^k \cdot R_b^l \cdot \exp[j \cdot (n-m) \cdot \phi_a] \cdot \exp[j \cdot (n'-m') \cdot \phi_b] \cdot \left[C_{m,m'} \cdot C_{n,n'}^* + C_{m',m} \cdot C_{n',n}^* + C_{m,m'} \cdot C_{n',n}^* + C_{m',m} \cdot C_{n,n'}^*\right] \tag{7}$$

It is useful to observe that

$$Q_{k,l}^{n,n',m,m'} = \left(Q_{k,l}^{m,m',n,n'}\right)^* \tag{8}$$

Substitution of (8) into (6) under the dummy indices exchanges $m \leftrightarrow n$ and $m' \leftrightarrow n'$ gives

$$\hat{\rho}_{a'b'} = tr_V(\rho_{a'b'}) = \frac{1}{2} \sum_{m,m'=0}^{N} \sum_{n,n'=0}^{N} \sum_{k=0}^{\min(m,n)} \sum_{l=0}^{\min(m',n')} \Big[\exp[-j \cdot (m'-n') \cdot \phi] \cdot Q_{k,l}^{m,m',n,n'} |m-k, m'-l\rangle_{ab\,ab}\langle n-k, n'-l| + \exp[j \cdot (m'-n') \cdot \phi] \cdot \left(Q_{k,l}^{m,m',n,n'}\right)^* |n-k, n'-l\rangle_{ab\,ab}\langle m-k, m'-l| \Big]. \tag{9}$$

This symmetrized form is useful, and it points up the required Hermitian property of the density operator.

3. THE OPTIMAL DETECTION OPERATOR FOR A LINEAR COMBINATION OF M AND N STATES

In this section a detection operator that is optimal for LCMNS subject to a set of constraints is reviewed. It has been shown [7, 8] that detection operator can be viewed as increasing or decreasing the number of photons in a state by an amount P, where P is referred to as the resolution parameter, a parameter that determines the resolution when imaging with LCMNS.

$$\langle A \rangle = V \cdot \cos(P \cdot \phi) \tag{10}$$

where V is the visibility which is defined to be the maximum absolute value of the expectation of A, i.e.

$$V \equiv \langle A \rangle \equiv \left| \langle \psi | A | \psi \rangle \right|_{\phi=0} \tag{11}$$

where, $|\psi\rangle$ indicates the state of interest, e.g. a N00N state or a LCMNS.

To determine the optimal form for A for a LCMNS with respect to a particular model space it will be necessary to define some of the properties of $C_{m,m'}$ $\;for\;$ $m, m' = 1,2,\ldots,N$. It is convenient to specify a model space with the following properties:

$$C(i,j) \equiv C_{i,j}; \quad for \quad i,j = 1,2,\ldots,N \tag{12}$$

for $i = 1,2$ and γ_i a normalization constant that forces $|\psi\rangle_{ab}$ in (2) to have unit norm,

$$M_{U,1} \le M_{L,2}\,, \tag{13}$$

$$C(p,q) \equiv C_2(p) \cdot C_1(q)\,. \tag{14}$$

Also assume that

$$M_{L,2} = P + M_{L,1} \tag{15}$$

$$M_{U,2} = P + M_{U,1} < 2P\,. \tag{16}$$

It follows from (12-16) that for the coefficients $C_{m,m'}$ to be nonzero, that $M_{L,1} < m' < M_{U,1}$ and $M_{L,2} < m < M_{U,2}$. States $|m'\rangle$ such that $M_{L,1} < m' < M_{U,1}$ will be referred to as being in the lower band and states $|m\rangle$ where $M_{L,2} < m < M_{U,2}$ will be referred to as being in the upper band.

$$\delta\phi \cong \Delta A \Big/ \left| \frac{\partial \langle A \rangle}{\partial \phi} \right|\,. \tag{17}$$

The spread in A is defined as

$$\Delta A \equiv \sqrt{\langle A^2 \rangle - \langle A \rangle^2}\,. \tag{18}$$

In the literature [7, 8] it is shown that

$$A = \sum_{q_1=0}^{M_{U,1}} \sum_{q_2=0}^{M_{U,1}} \left[|q_1, q_2 + P\rangle\langle q_1 + P, q_2| + |q_1 + P, q_2\rangle\langle q_1, q_2 + P| \right]. \tag{19}$$

and

$$A^2 = \sum_{q_1=0}^{M_{U,1}} \sum_{q_2=0}^{M_{U,1}} \left[|q_1, q_2 + P\rangle\langle q_1, q_2 + P| + |q_1 + P, q_2\rangle\langle q_1 + P, q_2| \right]. \tag{20}$$

where

$$V = \Theta(a) \cdot \Theta(b) \tag{21}$$

$$\Theta(u) \equiv \sum_{m=M_{L,1}}^{M_{U,1}} C_1(m) \cdot C_2(m+P) \cdot \Lambda_u^{m,m+P} \tag{22}$$

$$\Lambda_u^{m,n} \equiv \sum_{k=0}^{\min(m,n)} \left[\binom{m}{k} \cdot \binom{n}{k} \right]^{1/2} \cdot T_u^{\frac{m+n-2k}{2}} \cdot R_u^k \tag{23}$$

and $u = a$ or b. Also,

$$\langle A^2 \rangle = \frac{1}{2} \cdot \sum_{m=M_{L,1}}^{M_{U,1}} \left[C_2(m+P) \right]^2 \cdot \left(\Omega_a^{m,P} + \Omega_b^{m,P} \right) \tag{24}$$

where

$$\Omega_u^{m,P} \equiv \sum_{k=0}^{m} \binom{m+P}{k} \cdot T_u^{m+P-k} \cdot R_u^k \tag{25}$$

where $u = a$ or b.

$$\delta\phi \cong \sqrt{\langle A^2 \rangle / V^2 - 1 + \sin^2(P \cdot \phi)} \Big/ \left[P \cdot |\sin(P \cdot \phi)| \right]. \tag{26}$$

The minimum phase error denoted as $\delta\phi_{\min}$ takes the form

$$\delta\phi_{\min} \cong \sqrt{\langle A^2 \rangle} \Big/ (P \cdot V). \tag{27}$$

The form of $\delta\phi_{\min}$ in (27) permits the minimum phase error to be calculated for N00N states, PMNSs and LCMNS. It is straightforward to show that (27) reproduced the known form of $\delta\phi_{\min}$ for N00N states [7, 8, 27]. The expression for $\delta\phi_{\min}$ for N00N states or *N* separate photons, i.e. *N* non-entangled photon [7, 8, 27], can be easily shown to be

$$\delta\phi_{\min} \cong \sqrt{\frac{1}{2} \cdot \left(\frac{1}{T_a^{q_1}} + \frac{1}{T_b^{q_1}} \right)} \Big/ P^{q_2}, \tag{28}$$

where $q_1 = P$ and $q_2 = 1$ for N00N states, and $q_1 = 1$ and $q_2 = 1/2$, for P separate photons. Observe in the limit of no loss along arms "a" and "b" the phase error for a N00N state with P photons reduces to $\delta\phi = 1/P$, i.e., the HL. In the limit of no noise, the phase error for P separate photons reduces to the shot noise limit (SNL) [7, 8, 27, 30] i.e. $\delta\phi = 1/\sqrt{P}$. For the computations below, for a PMNS consisting of $m + m'$ photons, other authors [6] considered the HL to be $1/(m + m')$ and the SNL, $1/(m + m')^{1/2}$.

4. COMPUTATIONAL RESULTS

This section offers computational results showing the effectiveness of the optimized coefficient sets used for the adaptive optics procedure. Minimum values of phase errors and number of photons used as well as maximum values of the visibility are simultaneously found. These extrema are shown to be local extrema, i.e. different extremal results can be found for the same inputs.

Increasing visibility and reducing phase error can significantly contribute to improved detection. Improving the visibility and phase error while reducing the number of photons required may offer significant advantages, ultimately reducing the size, weight, power requirements and cost (SWAPc) of the system. This is a subject for future research.

Increasing the number of photons is shown to offer improved results. The number of photons required for a given phase error and resolution drops off sharply as the loss mechanisms in the environment are reduced. Open systems theory is used to model the environmental impact on the principle quantum system.

The expression in (27) yields a form of the minimum phase error that is minimum for a given set of coefficients. The expression in (27) is a function of the environmental parameter, R, which characterizes loss as well as the model space parameters M_{L1}, M_{L2} and P.

Under a specific selection of coefficients LCMNS have offered significant performance improvements over PMNSs, and N00N states in a paper by Smith [8]. In one example in this paper LCMNS yielded a visibility four times that of the associated PMNS and 11 times that of the related N00N state. The minimum phase error experienced even more dramatic improvements when using LCMNS for this example. The minimum phase error for a LCMNS was 11 times smaller than that of the associated PMNS and 30 times smaller than the related N00N state.

It should be noted that no attempt was made in this previous paper [8] to optimize over the coefficient set. Selection of an optimal coefficient set yields even more dramatic improvements as observed below.

Optimal forms of the coefficients in the LCMNS have been derived in closed form by the author of this paper and will appear in a future publication [28]. These coefficients yield a global or local minimum of the phase error, i.e. a smallest value of the phase error for all coefficient sets, which is denoted as $(\delta\phi_{\min})_{\min}$. Closed form expressions that yield the minimum values of M_{L1} and M_{L2} as a function of P and R have also been derived. These expressions permit the minimum number of photons to be used while simultaneously minimizing the phase error and maximizing the visibility.

An adaptive optics procedure for making the necessary measurements to determine the coefficients, wave functions and hence the minimum phase and maximum visibility will be described in a future paper [28] by the author. This paper will also discuss closed form results relating the minimum number of photons required to the resolution parameter *P* and the environmental loss parameter *R*. These closed form expressions derived in [28] are fundamental to the operation of the adaptive optics procedure.

It should be recalled that minimizing the optical phase error will ultimately improve detection of the RF waveform using procedures described in section 1. Keeping the number of photons used minimum greatly reduces effects that degrade performance like vibrating optical elements.

Some numerical results that show the utility of multiphoton entanglement combined with the corrective procedure follow. These are just examples, by selecting larger values of *P*, the resolution parameter smaller values of phase error are yielded. Large values of *P* frequently mean larger values of M_{L1} and M_{L2} increasing the overall number of photons. Increasing the number of photons also increases the complexity of the transmitter and detector. The absolute limit on the correction process is still a subject of research and will be investigated during the next funding period.

For the first numerical example, for input values of $P=1000$, $R=0.01$, the adaptive procedure yields the model space parameters $M_{L1}=1009$ and, $M_{L2}=1077$. This results in values of $\left(\delta\phi_{\min}\right)_{\min} = 0.0054$ and $V=0.1866$. So, for a total of up to $2M_{L2}+P = 3154$ photons, $\left(\delta\phi_{\min}\right)_{\min} \cong 0.005 \text{ radians} = 0.3^{\circ}$. Improved values of the phase error and visibility can be obtained by increasing *P*. This is a subject of a future publication.

This LCMNS with $P=1000$ has a minimum phase error more than *30* times smaller than for a N00N state with a *1000* photons, i.e. the N00N state analogous to this LCMNS. Increasing the number of photons in the N00N state to *3200* produces a minimum phase error of more than *3000*, due to the N00N state's inherent super Beer's law describing loss. So, use of a LCMNS offers a significant advantage over N00N states.

Increasing the number of photons used makes the minimum phase error smaller. For the case where $R=0.01$, $P=1000$, $M_{L1}=4162$, and $M_{L2} = 4267$, the minimum phase error takes the value $\left(\delta\phi_{\min}\right)_{\min} = 0.0017$ radians and the visibility, $V=0.5822$. So, for a maximum total number of photons $2M_{L2}+P = 8534$ photons, $\left(\delta\phi_{\min}\right)_{\min} \cong 0.002$ radians $\cong 0.12^{\circ}$. So, phase error and visibility can be improved by increasing *P* or M_{L1} and M_{L2}. It should be observed that this example uses the same input of *P* and *R* as the one above and achieves improved values of the phase error and visibility. This is an example of the optimization procedure finding local extrema.

Decreasing the value of *R* reduces the number of photons required. For input values of $R=0.001$, and $P=1000$, values of $M_{L1}=288$ and $M_{U1}=317$ were found with $\left(\delta\phi_{\min}\right)_{\min} = 0.0014$ radians and $V=0.7257$. Observe that $P=1000$ and $R=0.01$ implies $PR=10$ so in a sense an average of *10* photons are lost during transmission, propagation and measurement. For $P=1000$ and $R=0.001$, $PR=1$, so an average of one photon is lost during this process. For the *10*-photon loss case, the system exhibits some sensitivity to the difference M_{U1}-M_{L1}. The sensitivity is reduced for the one photon case. The one photon loss case is not very sensitive to M_{L1}, which is reasonable since the system would rarely depart from a number of photons near M_{U1}.

Another way of looking at the difference between results for $R=0.01$, $P=1000$ and $R=0.001$ and $P=1000$ is that the overall value of $M_{UI}=4267$ for the $R=0.01$ case and $M_{UI}=317$ for the $R=0.001$. So, a smaller loss parameter means that the value of M_{UI} is reduced by a factor of more than *13*.

5. NEAR FUTURE RESEARCH TOPICS

In the next funding cycle of the NISE program various research topics will be considered. These include, but are not limited to:

1. Additional research on the optical interferometer to determine limits of resolution, visibility, and phase error as a function of the complexity of the transmitters, detector and control elements.
2. Theoretical architectures for the optical transmitter, control, and detector components.
3. Exploration of the relationship between various operators and detector components likely using the factorization approaches of Reck et al. [31] and/or Clementine et al. [32]. These approaches offer algorithms for automatically representing unitary operators in terms of beam splitters, phase shifters and other optical elements.
4. Research related to the selection of the best Rydberg atoms for this application.
5. Theoretical architectures for integration of the Rydberg component with the optical interferometer enhanced by multiphoton entanglement and adaptive optics techniques.
6. Exploration of how combining hyper-entanglement with multiphoton entanglement will offer significant improvement [12, 24, 25]

6. CONCLUSIONS

A theoretical architecture for an ultra-wideband, highly sensitive RF detector is being created. The detector will combine Rydberg atoms and optical interferometry based on quantum entanglement. It should offer detection from 4GHz to 50GHz. Use of entanglement will offer detection sensitivity far exceeding RF detection schemes based on Rydberg atoms created by others.

The initial NISE funded research period, June 8 – Sept. 30, 2020, emphasized research related to the optical frequency quantum interferometer and the multiphoton entangled states. The research emphasized the utility of a multiphoton entangled state referred to as a linear combination of *M* and *N* states (LCMNS).

A quantum interferometer using multiphoton entanglement is under development. It uses adaptive optics techniques for selection of coefficients for LCMNS. The novel interferometer offers huge improvements in phase sensitivity over related interferometers, while greatly reducing the number of photons used. Shot noise limited interferometers will offer a phase error of at least $1/\sqrt{N}$, where N is the number of photons used. For large N, there can be vibration of optical elements reducing the phase sensitivity of the interferometer. The approach based on entanglement presented here, even in the presence of loss can reduce the number of photons used by roughly a factor of $\sqrt{N}$, while yielding enhanced phase sensitivity. The new interferometer is an improvement on a previous interferometer based on linear combination of *M* and *N* states. LCMNS have been shown to be much more robust in the presence of environmental effects than other approaches. For an LCMNS with $P=1000$ the minimum phase error is more than *30* times smaller than for a N00N state with a *1000* photons, i.e. the N00N state analogous to this LCMNS. Increasing the number of photons in the N00N state to *3200* produces a N00N state minimum phase error more than four orders of magnitude higher than that of the LCMNS, due to the N00N state's inherent super Beer's law describing loss. So, increasing the number of photons in the N00N state caused the results to deteriorate. So, use of a LCMNS offers a significant advantage for increasing the phase sensitivity of an interferometer based on multiphoton entanglement. It offers the ability to simultaneously minimize phase error, minimize the number of photons used reducing vibration of optical elements and hence loss and maximize the visibility. Increasing visibility can significantly contribute to improved detection and may help reduce the size, weight, power and cost requirements of the detector. This is a subject for future research.

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