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MODELLING ACOUSTO-ELASTIC WAVEGUIDE/OBJECT SCATTERING WITH THE RALEIGH HYPOTHESIS

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Modelling acousto-elastic waveguide/object scattering with the Rayleigh hypothesis

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Jan L. Spoelstra

Director

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Modelling acousto-elastic waveguide/object scattering with the Rayleigh hypothesis

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Executive Summary:

The detection of mines in shallow water is highly dependent on the acoustic propagation conditions in the water column and in the bottom. Thus, in order to understand and improve mine detection and classification in shallow water environments, it is important to be able to accurately model propagation and scattering. Often the ocean may have range-dependent interfaces, both top and bottom, and thus in the modelling it is important to be able to account for range-dependent propagation/scattering in conjunction with the object scattering.

In this memorandum an approximate, but efficient, and often very accurate method is presented for modelling object scattering in range-dependent shallow water waveguides. Numerical results are compared with other established numerical codes for waveguide propagation examples and the memorandum is concluded with an example of object scattering in a waveguide with both rough upper and lower surfaces. It may be possible with further work to improve the generality of the method and also improve the approximations made.

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Modelling acousto-elastic waveguide/object scattering with the Rayleigh hypothesis

John A. Fawcett

Abstract: In this memorandum the plane wave scattering matrices for a range-dependent seabed and upper free surface are computed using the Rayleigh hypothesis at the interfaces. These matrices can then be used to compute the pressure field within a waveguide bounded by these surfaces; they can also be easily combined with the scattering matrix characterizing an object. Computational examples are given for range-dependent acousto-elastic waveguides and the fields compared with those computed by other methods. An example with a steel cylinder in the waveguide is also shown.

Keywords: Rayleigh hypothesis, scattering, elastic

Contents

1 Introduction

In an earlier paper [1] we described how scattering in a range-dependent waveguide could be computed by using a plane-wave decomposition method. However, it was not specified how to compute the scattering matrices for the range-dependent surfaces, although for the case of a sloping interface, a simple shifting matrix yielded accurate results [1]. In [2] perturbation theory was used to compute the transmission matrix for a rough seabed. In this memorandum we use the Rayleigh hypothesis, that is we make assumptions about the scattered field consisting solely of up or down-going waves, to solve efficiently the scattering problem at rough interfaces. The Rayleigh hypothesis has been used previously and studied numerically and theoretically (see, for example [3],[4]) in the context of scattering from rough surfaces. In computational work, using a methodology almost identical to that of this memorandum, Koketsu et al [5] used the Rayleigh hypothesis in conjunction with invariant embedding to consider scattering and transmission through range-dependent layers for some geophysical problems. In this memorandum, we consider propagation geometries and frequencies more related to underwater acoustics waveguides to investigate the potential of this method.

The Rayleigh hypothesis makes assumptions about the vertical plane wave components of the scattered wavefield, but there is no division with respect to forward and backscatter directions (right- and left-going energy). Thus, backscattered energy is automatically included in the numerical solution. Using the Rayleigh hypothesis to solve waveguide propagation/scattering problems is certainly not the only, nor, perhaps, the most efficient method for our computational examples, but it does handle three difficult problems in underwater acoustics modelling in an unified fashion: (1) range dependence (2) elasticity (3) backscatter. When combined with the invariant embedding method [1],[5] it can handle any number of rough interfaces and also the problem of a scattering object in the waveguide.

In this memorandum we consider the water column to be isovelocity. In fact, this is probably not necessary. If the refractive index squared is linear near the interfaces, then one could use Airy functions rather than plane waves to describe the vertical propagation of the energy; or, perhaps, simply taking the profile to be isovelocity near the scattering interface or object would be sufficiently accurate. The propagation of the wavefield between a set of interfaces, when the intervening sound speed profile varies vertically, could be described by a propagator matrix and the theory described below could be easily modified to handle these cases.

2
Theory

We first consider a single range-dependent interface between water and an underlying sediment. In the water the incident compressional potential is taken to have the form of a plane-wave,

$$\phi_p^{inc} = \exp(i(k_x x + \gamma z)) \tag{1}$$

and we will write for the scattered field in the water column,

$$\phi^w = \sum_{n=-N}^{N} a_n \exp(i(k_n x + \gamma_{n,w} z)). \tag{2}$$

In the sediment, the compressional and shear potentials have the form

$$\phi^{sed} = \sum_{n=-N}^{N} b_n \exp(i(k_n x - \gamma_{n,p} z)), \tag{3}$$

and

$$\psi^{sed} = \sum_{n=-N}^{N} d_n \exp(i(k_n x - \gamma_{n,s} z)) \tag{4}$$

where

$$\begin{aligned} \gamma_{n,w} &\equiv \sqrt{\omega^2/c_w^2 - k_n^2} \\ \gamma_{n,p} &\equiv \sqrt{\omega^2/c_p^2 - k_n^2} \\ \gamma_{n,s} &\equiv \sqrt{\omega^2/c_s^2 - k_n^2}. \end{aligned} \tag{5}$$

Here c_w and c_p denote the compressional sound speeds of the water and sediment respectively; c_s denotes the sediment shear speed. The approximation we have made is that we have **assumed** that the scattered wavefield consists only of an upgoing spectrum in the water and only a downgoing spectrum in the sediment.

In order to determine the unknown coefficients we will invoke the conditions of continuity of normal displacement and the normal component of traction at the interface as well as the vanishing of the tangential component of traction. We impose this continuity at K=2N+1 discrete horizontal points along the interface (i.e., the number of plane waves used in the expansions of Eqs.(2) to (4)); the spacing is set at $\Delta x = \pi/\Delta k$, where Δk is determined from the relationship

$$\Delta k = \frac{k_{max}}{N} \tag{6}$$

and k_{max} is the largest horizontal wavenumber considered computationally (chosen to be larger than ω/c_w). Thus, there is a system of $3K$ equations in $3K$ unknowns, which when solved yield the scattering coefficients. If this same system is solved for various incident wavenumbers k_n then a generalized reflection matrix R^{du}, and transmission matrices for the compressional and shear potentials can be computed. Computationally, this means performing the LU decomposition of the system matrix once and solving for multiple incident fields (right hand sides). For the upper pressure release surface of the waveguide, there is only one set of unknown coefficients corresponding to down-going plane waves for the compressional potential in the water; these coefficients are determined by the condition that the scattered plus incident pressure should be equal to zero along this surface. For some computations, we will take this surface to be flat, in which case the upper reflection matrix is diagonal with the diagonal elements having the value of -1.

It can be shown [1] that the vectors of upgoing and downgoing plane-wave components in the water, for a receiver below the source is given by

$$\begin{aligned}\vec{p}_u &= (I - R^B R^U)^{-1} R^B (\vec{p}_d^{\,inc} + R^U \vec{p}_u^{\,inc}) \\ \vec{p}_d &= \vec{p}_d^{\,inc} + R^U \vec{p}_u^{\,inc} + R^U \vec{p}^{\,u},\end{aligned} \tag{7}$$

with similar expressions for a receiver above the source. Here we have used the notation that R^U is the generalized reflection matrix for the upper surface and R^B the matrix for the lower interface. The matrix I is the identity matrix and $\vec{p}_u^{\,inc}$ and $\vec{p}_d^{\,inc}$ are the vectors of upgoing and downgoing plane-wave coefficients generated by the source. Taking the mean level of the lower interface to be $z = 0$, R^U is defined to contain the exponential phase factor to take the plane waves up to the top surface from $z = 0$ and the scattered waves back down again.

In the formulation discussed above, the Rayleigh hypothesis was invoked for the scattered field at the interface. In fact, we can include downgoing plane waves in the expansion of Eq.(2) by using the condition at the upper interface to relate the upgoing and downgoing coefficients. For example, if the upper pressure release surface is flat then we can include upgoing and downgoing waves in our expansion with the condition that the coefficient a'_n for the downgoing component is related to the upgoing coefficient a_n by

$$a'_n = -\exp(2i\gamma_{n,w}L)a_n \tag{8}$$

where L is the depth of the waveguide, so that the expansion in the water column can still be written entirely in terms of a_n. The downgoing and upgoing planewaves are then both used in the water column expansion set, when the continuity equations are solved at the range-dependent interface; thus, the coefficient matrix for these equations is different than that formed when only upgoing waves are used in the expansion. We denote the reflection matrix computed from the solution of this new set as $\tilde{R}^B$, and we can write

$$\begin{aligned} \vec{p}_u &= \tilde{R}^B(\vec{p}_d^{\,inc} + R^U\vec{p}_u^{\,inc}) \\ \vec{p}_d &= R^U\vec{p}_u + (\vec{p}_d^{\,inc} + R^U\vec{p}_u^{\,inc}). \end{aligned} \tag{9}$$

where for the case of a flat upper surface R^U is diagonal with diagonal elements equal to $-\exp(2i\gamma_{n,w}L)$.

It is also possible to include a scattering object in the waveguide by specifying its plane-wave scattering matrix and combining this matrix with the waveguide matrices [1]. In [1] it was shown how to analytically compute the scattering matrix for a cylinder.

3

Numerical implementation and examples

The numerical implementation of the Rayleigh hypothesis for modelling waveguide propagation is straightforward. We take a complex-valued contour of wavenumbers of the form

$$k(t) = t - i\epsilon \sin(t\pi/\alpha) \tag{10}$$

for $-k_{max} \leq t \leq k_{max}$. The parameter ϵ is a small number (e.g. $\epsilon = f/20000$ seems to work well) and α is a number slightly bigger than k_{max}. This contour is then sampled at $2N$ discrete points. These same wavenumbers are used in the Rayleigh expansions at the interfaces. The reflection and transmission matrices for the interfaces are computed and the wavefield in the waveguide constructed using Eq.(7) or possibly Eq.(9). The relationship between the wavenumber discretization (or equivalently, the number of wavenumbers) and the spatial extent of the computation approximately obeys the Nyquist sampling restriction.

3.1 Low-frequency propagation

In this first example we consider a waveguide 220 m deep with a ridge, $h(x)$, modelled by

$$h(x) = 160\cos(x\pi/2000); \quad -1000 < x < 1000, \quad h(x) \equiv 0, \quad \text{otherwise.} \tag{11}$$

A 25-Hz source is located 10 m below the sea surface. The sediment in this example was taken to be fluid with compressional speed $V_p = 2000$ m/s and density $\rho = 1.5$ g/cm^3. In Fig. 1 we show the two-dimensional field computed by the Rayleigh-method and the parabolic equation code FEPE [6] (the field is shown in terms of power in decibel units).

For this example, we used the formulation which includes downgoing plane waves in the expansion of the reflected wavefield in the water column; however, there is very little difference between the results of this and the upgoing only formulation. As can be seen, the agreement between the 2 methods is excellent. However, a slight problem with the Rayleigh expansion can be seen for the transmitted field at the peak of the bathymetry where the field values are too high. This problem may be due to the fact that only downgoing waves are assumed in the sediment, whereas in

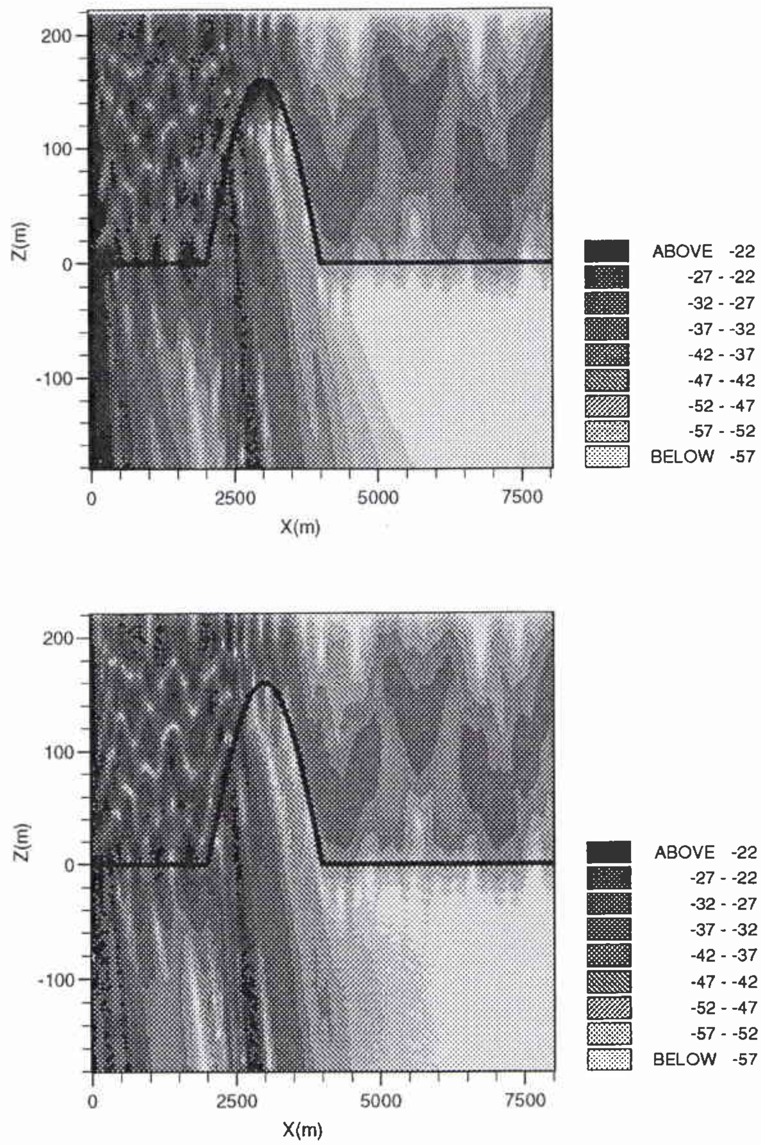

Figure 1: *A comparison of the two-dimensional fields computed by the Rayleigh hypothesis (upper) and FEPE (lower)*

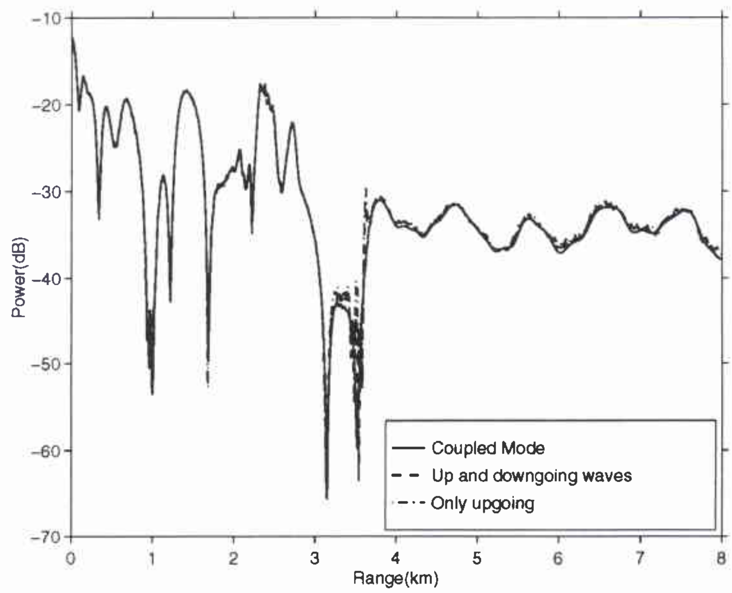

Figure 2: *A comparison of the transmission loss as computed by the Rayleigh method and COUPLE*

terms of rays, a near horizontal ray in the water column could be sufficiently bent upwards by transmission into the sediment to be upgoing in direction.

In Fig. 2 we show a comparison between the transmission loss for a receiver located at 135 m depth as computed by COUPLE[7], using the fully coupled normal mode solution, and by the Rayleigh method with both of the formulations. There is excellent agreement between all 3 curves.

3.2 *Elastic computations*

We now consider an elastic sediment case. As a straightforward benchmark case, we first consider the bottom to be flat; the bottom compressional speed is V_p = 2000 m/s, the bottom shear speed is V_s = 800 m/s, and ρ = 1.5 g/cm^3. In Fig. 3 a comparison of the field as computed by the wavenumber integration code OASES [8] and the Rayleigh method are presented. As can be seen the transmission loss curves are essentially identical. This example is not a test of the Rayleigh hypothesis, since the interface is flat, but does show that our matching conditions at the discrete spatial points yield accurate reflection coefficients.

In Fig. 4 a ridge 100 m high with a horizontal extent of 2000 m is introduced with the elastic properties as above. The first two plots show the compressional potential (scaled by the density) as computed by the elastic parabolic equation code FEPES

[9] and the Rayleigh method respectively. The last plot shows the field for the same bathymetry but for a shear speed of 10 m/s; the amount of energy making it past the ridge is significantly higher in this case.

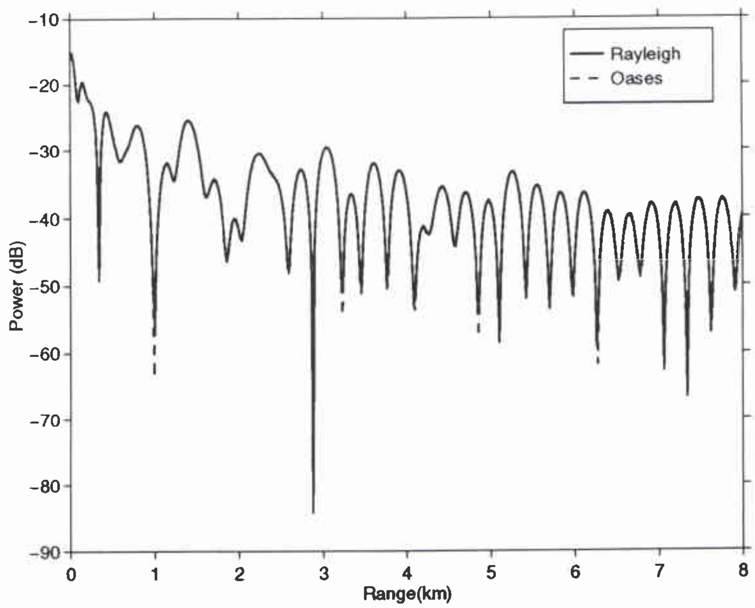

Figure 3: *A comparison of the transmission loss for an elastic bottom as computed by the Rayleigh method and OASES*

3.3 Rough seabed and rough free surface

In this example the free surface has the form:

$$h_{top}(x) = 0.5 \sin(\pi x/40) \tag{12}$$

and the bottom surface has the form

$$h_{bot}(x) = 5.0 \cos(\pi x/40). \tag{13}$$

The sediment has the elastic parameters V_p =2400 m/s, V_s =800 m/s, and ρ = 2.0 g/cm^3. The compressional potential is shown in Fig. 5 for a 500-Hz point source located at 70 m depth. As can be seen the top and bottom have caused significant scattering within the waveguide.

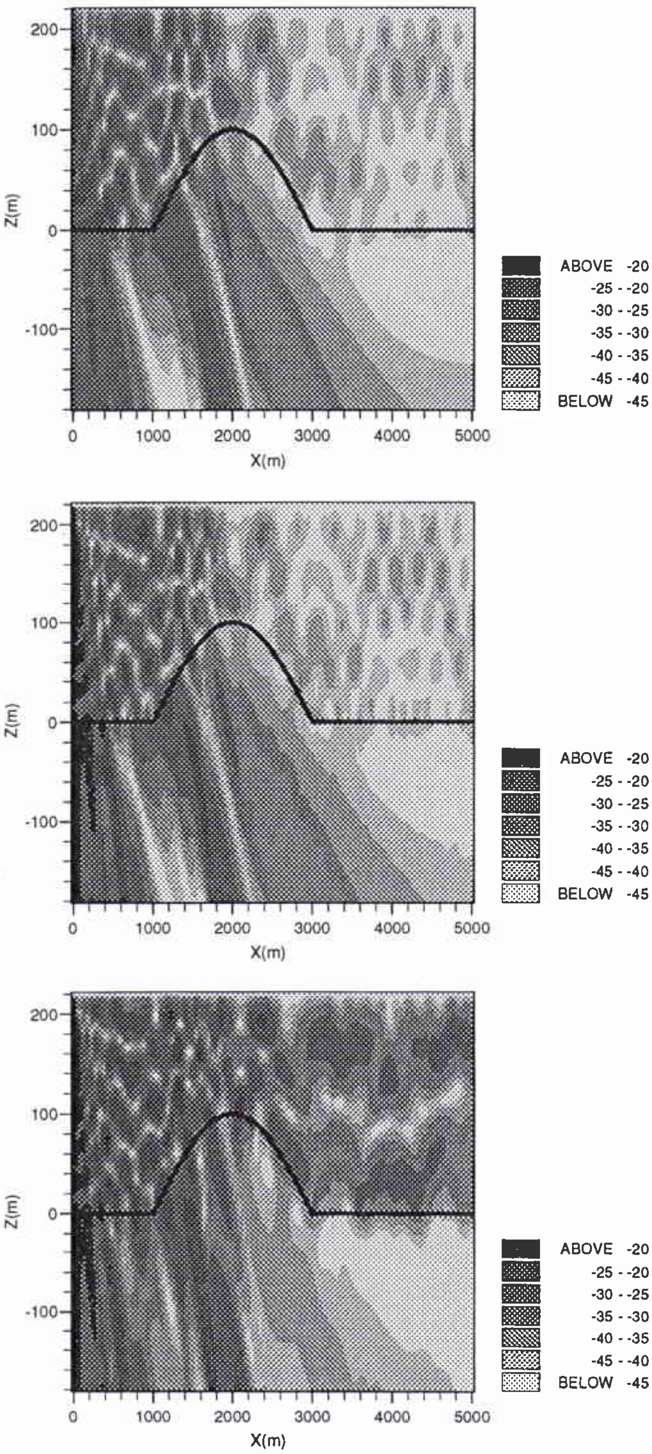

Figure 4: *A comparison of the two-dimensional fields computed by FEPES(top) and the Rayleigh method for the elastic sediment(middle) and for V_s = 10 m/s(bottom).*

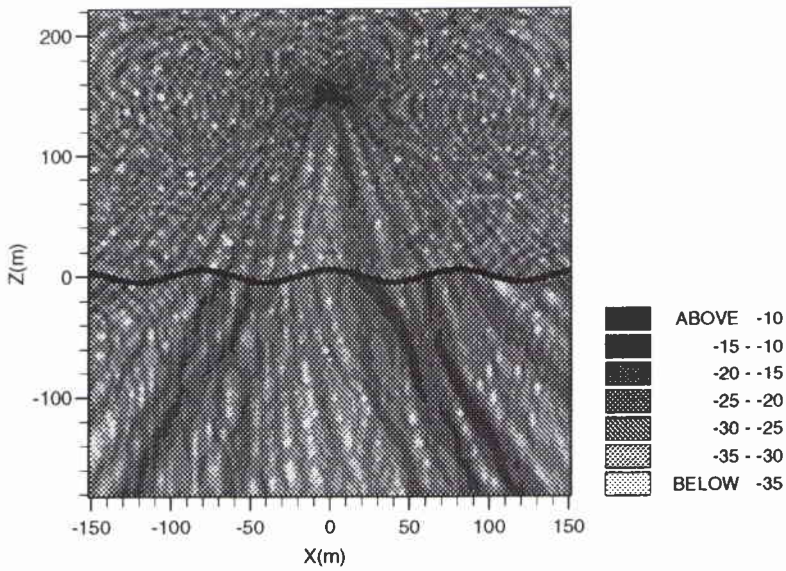

Figure 5: *Two-dimensional field (shown in decibel units) for a rough elastic bottom and rough free surface for a 500-Hz point source*

3.4 Rough seabed, rough free surface, and scattering object

We now include a 3m-radius, solid steel cylinder in the waveguide. The full elasticity of the cylinder is accounted for by solving the fluid/elastic cylinder problem in free-space for the coefficients of the scattered Hankel functions; these coefficients are then used in the definition of the cylinder's plane-wave scattering matrix [1],[2]. The point source is located 20 m to the left and above the cylinder. The source is located 60 m below the upper surface. In Fig. 6a we show a 50 $\times$ 50 m section of the field scattered by the cylinder in free-space. Note that the spectral representation of the field along the horizontal axis passing through the centre of the cylinder has poor convergence.

The shadow zone behind the cylinder is evident and the interference pattern between the incident and reflected fields can be seen on the front side of the cylinder. In Fig. 6b the rough upper free surface has been added. This significantly changes the appearance of the field and the shadow zone behind the cylinder has more energy entering into it, although a shadow zone region can still be discerned. Finally, in Fig. 6b the bottom surface has been added in. This has the effect of distorting the wavefield, but the mean characteristics of the field are the same as those of Fig. 6c.

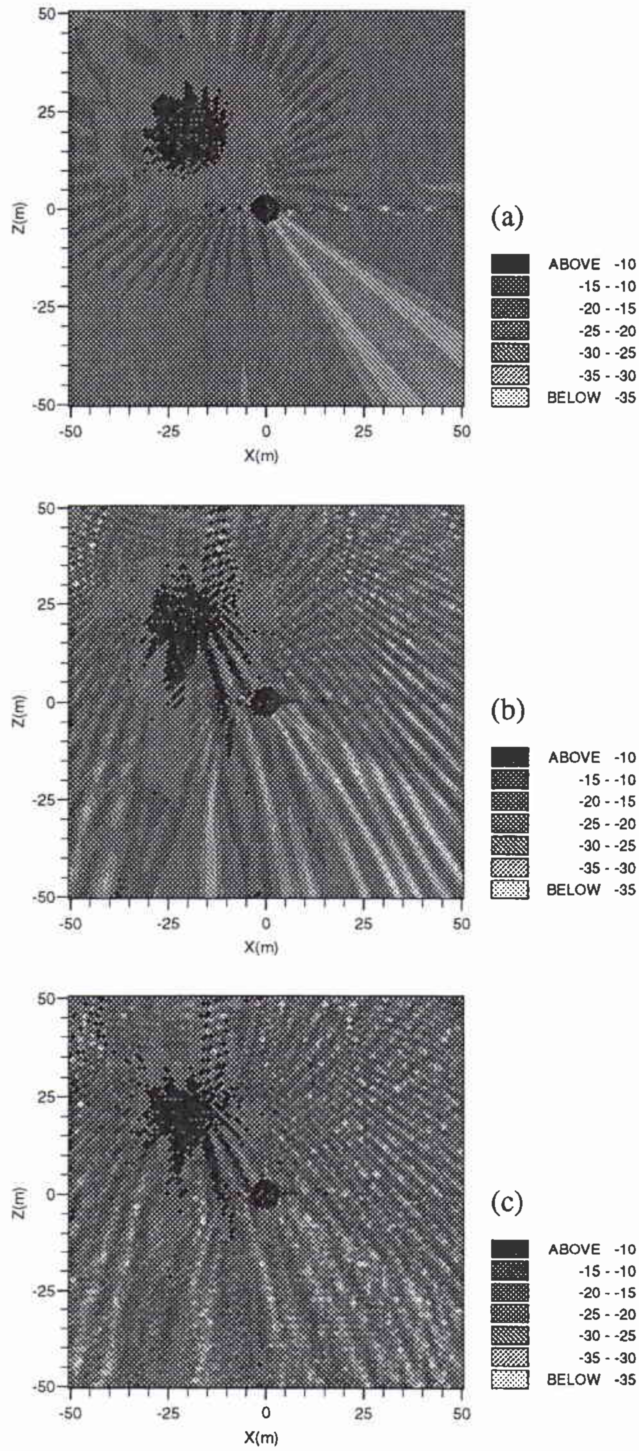

Figure 6: *Scattering from a 3m-radius steel cylinder for (a) free space (b) rough surface above (no bottom) (c) rough surface above and below for 500 Hz. The sections of the waveguide displayed do not include the surfaces, but the effect of the surfaces can be seen.*

4
Summary

We have shown that using the Rayleigh hypothesis allows one to accurately model acousto-elastic waveguides with significant range dependence. Also a scattering object can be easily included in the waveguide. There are situations of very strong range dependence where the method has some problems. Whether it is possible to improve the method in these cases by applying various constraints on the solution[4] or by using different expansion sets would be an interesting area of research. Computationally, the method is efficient, particularly for the 500-Hz examples, where the solution for these smaller regions (but with significant variation of the top and bottom surfaces) took only about 262 seconds on an ALPHAStation 500.

The examples we consider all had an isovelocity water column; this restriction could be relaxed by using, for example, Airy function expansions in the vertical coordinate and accounting for a vertical profile in between the lower and upper surfaces of the water column.

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Abstract

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Rayleigh hypothesis – scattering – elastic

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