

Lifetime Analysis of Aircraft Sensor Windows Using NASA STD 5018 Procedure

by
Daniel C. Harris
Chemistry Division
Research and Development Group
and
Bernadett Bodnar
Weapons, Sensors, Seekers, and Guidance Division
Weapons and Energetics Department

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**NAVAL AIR WARFARE CENTER WEAPONS DIVISION
CHINA LAKE, CA 93555-6100**



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FOREWORD

This report applies NASA STD 5018 guidelines to the design of a 0.42-meter-diameter circular, single-pane sensor windows for a manned aircraft. Windows made of fused silica, multispectral zinc sulfide, and BK7 glass are considered. A proof pressure test is determined for each window to provide a safe design life.

This report was reviewed for technical accuracy by Sung Choi.

Approved by
A. VAN NEVEL, *Head*
Research and Development Group
29 September 2020

Under authority of
W. S. DILLON
RDML, U.S. Navy
Commander

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EXECUTIVE SUMMARY

This report describes a method to predict a safe design life for single-pane sensor windows on an aircraft flying at altitudes up to 45,000 feet with a maximum pressure difference of 10.3 pound-force per square inch (lbf/in²) between the cabin and the outside. The circular windows are made of fused silica, multispectral zinc sulfide, or BK7 glass with a diameter of 16.61 inches and a thickness of 1.063 inches. They are supported by compliant O-rings located 0.256 inch from the window edge. The design stress (also called the service stress) is the maximum tensile stress of ~5 megapascals (MPa) (~700 pounds per square inch (psi)) on the face of the window resulting from the 10.3 psi pressure difference.

Design rules in our analysis are taken from the U. S. National Aeronautics and Space Administration (NASA) STD 5018, "Strength Design and Verification Criteria for Glass, Ceramics, and Windows in Human Space Flight Applications" (Reference 1). Window design life is calculated with the exponential law for slow crack growth, which is more conservative than the widely used power law. Some alternative design procedures are also considered, including a 3-parameter Weibull equation with a lower threshold for failure in BK7 glass.

Prior to installing a window, a proof pressure test is conducted with the window mounted in a fixture that simulates the stress distribution in flight. The proof stress must be at least three times greater than the service stress. We assume that fracture is controlled by flaws (microscopic fractures) on the surface of the window, not defects in the volume of the window. From fracture mechanics, we calculate the depth of the largest flaw that could survive the proof test without failure. The design life is then set at ¼ of the time calculated for that deepest (= weakest) flaw to propagate to failure if it were held at the maximum service stress for its entire life.

The proof test should be conducted for the shortest possible time under the driest possible conditions to minimize crack propagation during the test. Moisture enhances the rate of crack propagation in the window materials of interest. A conservative design life is calculated from the worst-case crack propagation rate observed with test coupons at 100% humidity or immersed in water.

The distribution of expected window strengths is computed with Weibull statistics from strengths of test coupons in equibiaxial flexure tests. Crack propagation rates are generally measured by dynamic fatigue of test coupons ground and polished in a manner that mimics the large window as closely as possible. In dynamic fatigue, coupons are subjected to flexure stress over a range of constant stress rates until they break. The higher the stress rate, the less time the coupon has before it fails, and the less the cracks can grow. Therefore, higher stress rates lead to higher observed strengths. The worst-case probability of failure of a full-size window in a proof pressure test is calculated from the Weibull distribution of window strengths and from the worst-case rate of crack propagation during the proof test. The proof test should be optimized to give a suitably long design life with a

low probability of failure in the proof test. Windows are designed with a thickness that enables proof test goals to be achieved.

For a fused silica window with a service stress (σ_a) of 4.82 MPa and a proof stress (σ_p) of 14.7 MPa (giving a proof stress ratio $\sigma_p/\sigma_a = 3.05$), the design life in Table ES-1 is either near 45 years (yr) with crack growth parameters taken from one source or 1 to 2 yr with crack growth parameters taken from a different source. Both sources are regarded as reliable. The difference might arise from different test methods or from the materials that were tested. The design life is expressed as a number of safe flight hours at the service stress. One year of design life is 8,760 flight hours. The calculated probability of failure in the proof test with worst-case slow crack growth during the proof test is <0.1%. If the proof stress is raised from 14.7 to 15.7 MPa, the design life increases by a factor of 2.5 to 3 because the maximum initial flaw depth remaining from the proof test decreases from 800 to 700 microns (μm).

TABLE ES-1. Fused Silica Window Design Life From NASA STD 5018 Design Rules With Exponential Crack Growth Law.

Window and Parameters	Maximum Initial Flaw Depth, μm	Design Life, yr
Service stress (σ_a) = 4.82 MPa; proof stress (σ_p) = 14.7 MPa; (σ_p/σ_a) = 3.05; $Y = \sqrt{\pi}$		
Wiederhorn crack growth parameters Table 4 line 1	800	45
Salem crack growth parameters Table 4 line 2	800	1.1
Salem crack growth parameters Table 4 line 3	800	2.0

For a multispectral zinc sulfide window with the same dimensions as the fused silica window, the service stress is 5.12 MPa. The design life in Table ES-2 is in the range 670 to 2,870 hours (h) with proof stress ranging from 15.36 MPa ($\sigma_p/\sigma_a = 3.00$) to 17.00 MPa ($\sigma_p/\sigma_a = 3.32$). The probability of failure in the proof test with worst-case slow crack growth during the proof test ranges from 1.5 to 2.8%. If the window and proof test chamber are rigorously dry, there might be no crack growth during the proof test and the probability of failure would be reduced to 0.3 to 0.5%.

TABLE ES-2. Zinc Sulfide Window Design Life From NASA STD 5018 Design Rules With Exponential Crack Growth Law.

Proof Pressure Ratio σ_p/σ_a	Proof Stress, MPa	Maximum Flaw Size From Proof Test, μm	Design Life, h	With Worst-Case Slow Crack Growth	
				Minimum Strength to Survive Proof Test, MPa	Proof Test Probability of Failure, %
Service stress (σ_a) = 5.12 MPa; $Y = \sqrt{\pi}$					
3.000	15.36	498	670	20.74	1.5
3.125	16.00	539	1,350	21.66	1.9
3.320	17.00	477	2,870	23.11	2.8

NASA STD 5018 uses a fracture mechanics geometric factor $Y = \sqrt{\pi}$, which applies to a crack whose length on the surface is much longer than its depth beneath the surface. An argument can be made for using a geometric factor closer to $Y = 2/\sqrt{\pi}$, which applies to a semicircular crack whose depth (radius) is half as great as its length (diameter) on the surface. Selecting this alternate value of Y increases the predicted design lives in Table ES-2 to 1940, 3320, and 7080 h.

For a BK7 glass window with the same dimensions as the fused silica window, the service stress is 4.90 MPa. The design life in Table ES-3 is only 8 h with a proof stress ratio $\sigma_p/\sigma_a = 3.00$. However, increasing the proof stress ratio to $\sigma_p/\sigma_a = 3.88$ gives a design life of 4,800 h with a worst-case probability of failure in the proof test of 0.7%.

TABLE ES-3. BK7 Glass Window Design Life From NASA STD 5018 Design Rules With Exponential Crack Growth Law.

Proof Pressure Ratio σ_p/σ_a	Proof Stress, MPa	Maximum Flaw Size From Proof Test, μm	Design Life, h	With Worst-Case Slow Crack Growth	
				Minimum Strength to Survive Proof Test, MPa	Proof Test Probability of Failure, %
Service stress (σ_a) = 4.90 MPa; $Y = \sqrt{\pi}$					
3.00	14.7	737	8	26.8	0.2
3.67	18.0	669	1,500	33.5	0.5
3.88	19.0	600	4,800	35.5	0.7
3.92	19.2	588	6,000	35.9	0.7

Probability of failure in the proof test for the three window materials was calculated from the 2-parameter Weibull Equation ES-1 fit to observed strengths of test coupons.

2-parameter Weibull equation:
$$P_f = 1 - e^{-\left(\frac{A_e}{A_o}\right)\left(\frac{\sigma}{\sigma_o}\right)^m} \quad (\text{ES-1})$$

where P_f = probability of failure

σ = applied stress

A_o = unit area $\equiv$ 1 square centimeter (cm^2)

σ_o = Weibull scale factor = strength for 1 cm^2 area of material in uniaxial stress

m = Weibull modulus (a measure of spread of strength distribution)

A_e = effective area in tension (several cm^2 for coupons and several hundred cm^2 for windows)

Figures ES-1, ES-2, and ES-3 show probability of failure versus stress for fused silica, Cleartran® (multispectral) zinc sulfide, and BK7 glass.

3-PARAMETER WEIBULL EQUATION

For BK7 and some other glasses, it has been demonstrated that well-prepared material behaves as if there is a lower threshold of stress below which mechanical failure is not observed. The threshold is ascribed to an upper limit of flaw depths if the material is manufactured well. The rationale is that “for surfaces generated mechanically with grinding tools consisting of many small diamonds suspended in a brass carrier,” a “maximum crack depth must exist” (Hartmann, Reference 2). To take advantage of a well-finished surface with a lower threshold for failure, the edges must be finished well enough so that they are not the source of failure of the window. For this purpose, BK7 window edges must be fine ground and etched with hydrofluoric acid to remove edge damage.

Even for a material capable of exhibiting a lower threshold for failure, many exceptions can erase the lower threshold. For example, an abrasive particle from a tool can detach and be dragged across a polished surface to leave a deep scratch that weakens the material beyond the expected lower threshold. At least 100 coupons of material need to be tested to demonstrate the existence of a statistically meaningful failure threshold. No amount of coupons ever guarantees that the next window that is polished will have the same “perfect” finish as test coupons exhibiting a lower failure threshold.

When plotted in a logarithmic manner, the same data fit to a 2-parameter Weibull equation with no lower threshold of failure in Figure ES-3 is recast in Figure ES-4 to suggest a lower threshold of failure.

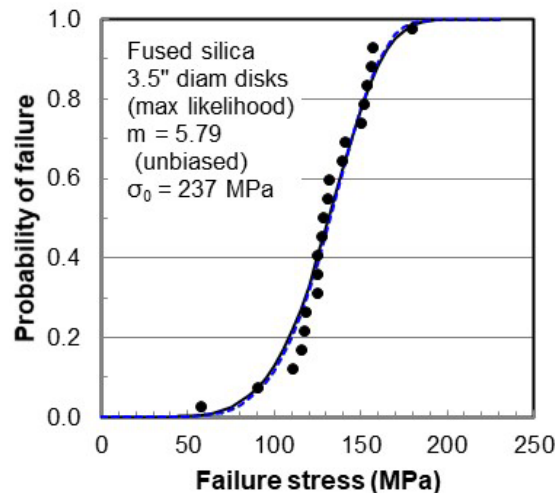


FIGURE ES-1. Strength and Weibull parameters for Heraeus Infrasil 302 fused silica tested at 50% relative humidity in air at 23°C at stress rate $\approx 4 \text{ MPa/s}$.

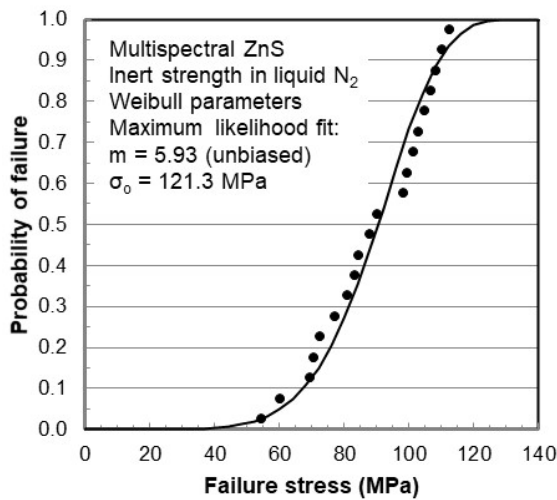


FIGURE ES-2. Inert strength and Weibull parameters for Cleartran® ZnS 4-point flexure bars (Type C in ASTM C1161). Stress rate ≈ 3.4 MPa/s.

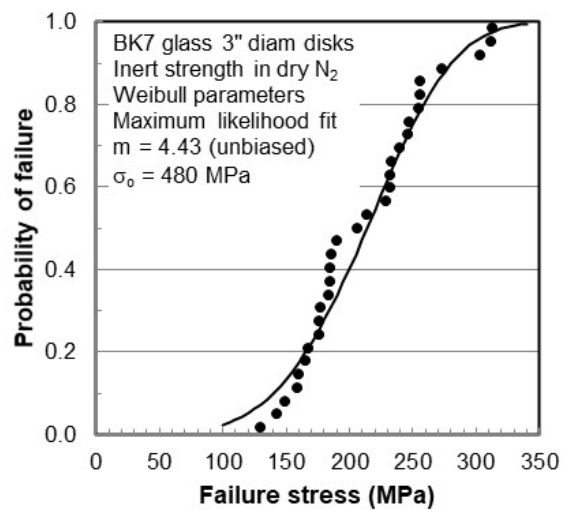


FIGURE ES-3. Inert strength and 2-parameter Weibull equation parameters for BK7 glass disks. Stress rate ≈ 7.7 MPa/s.

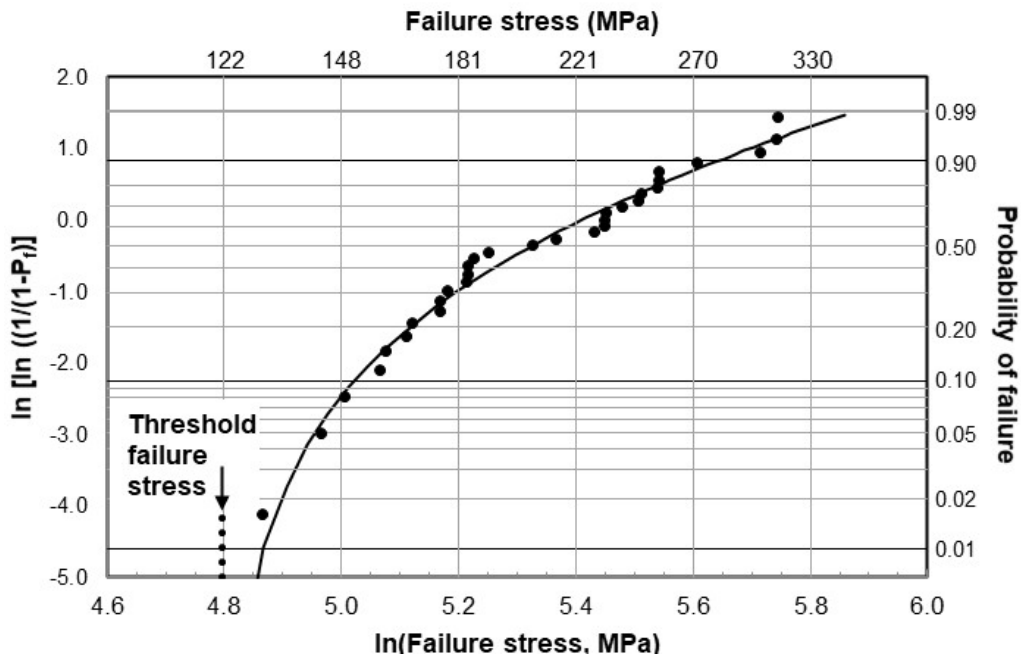


FIGURE ES-4. When plotted in an appropriate logarithmic manner, data from Figure ES-3 for BK7 glass suggest that there exists a lower threshold for failure near $e^{4.8} = 120$ MPa, below which failure does not occur. If there were no threshold strength, the nearly linear upper right part of the curve would continue to the left as a straight line with no lower limit for failure stress. (Data from Reference 3.)

If we accept that there could be a threshold for failure in BK7 glass, we could believe that the deepest flaw in a pristine window will not fail below the threshold of 120 MPa in Figure ES-4. Choosing a safety factor of 2, we could suppose that a pristine window has a strength of at least 60 MPa. The design life of the window would be determined by the time required for the deepest flaw to propagate to failure under a service stress of 4.90 MPa. The depth of that 60-MPa-strong flaw would be 72 μm according to NASA STD 5018 design rules and that flaw would give a design life $>10^5$ yr at a design stress of 4.90 MPa.

It would be foolhardy to trust human life to the presumed existence of a lower threshold for failure. However, we illustrate in the BK7 portion of this document how the likelihood of a lower threshold can allow a strenuous proof test that provides a longer design life with little expectation of failure in the proof test.

The curve in Figure ES-4 is described by the 3-parameter Weibull Equation ES-2 in which m , σ_θ , and k are analogous to m , σ_o , and (A_e/A_o) of the 2-parameter Equation ES-1. The parameter σ_t in Equation ES-2 is the threshold stress below which failure does not occur.

$$\text{3-parameter Weibull equation: } P_f = 1 - e^{-k\left(\frac{\sigma - \sigma_t}{\sigma_\theta}\right)^m} \quad (\text{ES-2})$$

EXPONENTIAL LAW VERSUS POWER LAW FOR PREDICTING DESIGN LIFE

Figure ES-5 shows 126 measured crack growth rates in fused silica from the seminal work of Wiederhorn in the study of crack growth in glass. Crack growth was monitored by direct observation of crack length versus time. In Wiederhorn's early work, he fit the observed crack velocity versus stress intensity factor to an exponential equation of the form $v = v_o e^{\beta K_I}$, where v is crack velocity, K_I is the stress intensity factor, and v_o and β are empirical parameters. Later workers found it more convenient to fit the same data to a power law of the form $v = A^*(K_I/K_{Ic})^n$, where A^* and n are empirical parameters and K_{Ic} is the critical stress intensity factor for failure.

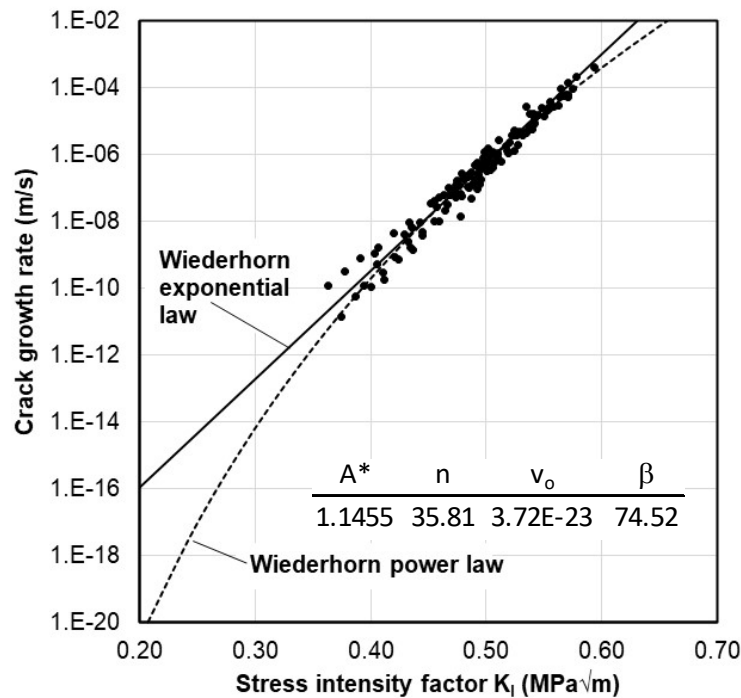


FIGURE ES-5. Comparison of exponential law $v = v_o e^{\beta K_I}$ and power law $v = A^* (K_I/K_{Ic})^n$ fit to 126 data points for crack growth in fused silica near 25°C in air at 100% relative humidity. The units of A^* and v_o are m/s. Parameters are from lines 1 and 4 of Table 4. Data from Reference 4 were digitized for this analysis. The slope and intercept of the digitized data fit to the equation $K_I = a + b \cdot \ln(v)$ were within 0.5% and 0.2% of the values reported by Wiederhorn.

We see in Figure ES-5 that both crack growth laws fit observed data nearly equally well in the region that is measured. However, for a stress intensity factor below $K_I = 0.4 \text{ MPa}\sqrt{\text{m}}$, the two equations diverge. The power law predicts lower crack growth rate than the exponential law at low stress intensity factor. The power law and exponential law derived from the same data set predict markedly different design lives because most of the lifetime of a window occurs when the stress intensity factor is below $0.4 \text{ MPa}\sqrt{\text{m}}$.

Table ES-4 compares design lives for the same fused silica window predicted by the two crack growth laws fit to the same observations in Figure ES-5. The power law predicts a lifetime 100 times longer than the exponential law. For fused silica crack growth measured by Salem at NASA, the power law lifetimes are one order of magnitude greater than those from the exponential law.

TABLE ES-4. Fused Silica Window Design Life
From Exponential and Power Laws.

Slow Crack Growth Parameters	Design Life, yr
Service stress (σ_a) = 4.82 MPa; proof stress (σ_p) = 14.7 MPa; initial flaw depth = 800 μm	
Wiederhorn exponential law crack growth parameters Table 4 line 1	45
Wiederhorn power law crack growth parameters Table 4 line 4	4,500
Salem exponential law crack growth parameters Table 4 line 2	1.1
Salem exponential law crack growth parameters Table 4 line 3	2.0
Salem power law crack growth parameters Table 4 line 5	26
Salem power law crack growth parameters Table 4 line 6	13

In general, we do not know which (if either) law applies to a particular material at low stress intensity factor. NASA STD 5018 states that the design life shall be calculated from the exponential law. This is a conservative design rule that we follow in our analysis.

The strong appeal of the power law is that it can be integrated to give analytical formulas for crack growth during a proof test and crack growth in dynamic fatigue measurements. The exponential law must be integrated numerically. Both laws are approximately equivalent to each other in the stress intensity factor range ~ 0.4 to $\sim 0.55 \text{ MPa}\sqrt{\text{m}}$ for fused silica in Figure ES-5. This stress intensity factor range applies to dynamic fatigue measurements and to crack growth in proof tests. However, for design life, the exponential law is more conservative than the exponential law by more than a factor of 10.

Appendix A in this report gives a method to derive the exponential law from the power law obtained from dynamic fatigue measurements. Appendixes B and C explain how to use both crack growth laws to calculate the strength loss during a proof test and in dynamic fatigue testing. Appendix D shows how to use the maximum likelihood method to find the parameters for the 3-parameter Weibull equation.

I. WINDOW OVERVIEW

This report describes the process by which the design life of aircraft sensor windows can be estimated from the rate at which microscopic cracks in the window propagate under the influence of service stress in the window. As cracks propagate, the window becomes weaker. The design life of the window is an approximate number of flight hours during which the window retains a large margin of mechanical safety for continued operation. Most aircraft windows have inner and outer panes for safety in case one pane breaks. Sensor windows have a single pane for optimal imaging. Failure of a sensor window would endanger human life, so the design should be conservative.

Figure 1 shows a cross section of a circular window mounted in a metal frame with compliant circular O-rings on each face for support. Higher pressure inside the cabin and lower pressure outside the aircraft create tensile stress on the outside face of the window. The maximum pressure difference is $\Delta P = 0.710$ bar (10.3 pound-force per square inch (lbf/in²)). Figure 2 shows radial and hoop components of tensile stress for fused silica from equations in Appendix E. The stresses are equal at their maximum value at the center of the window.

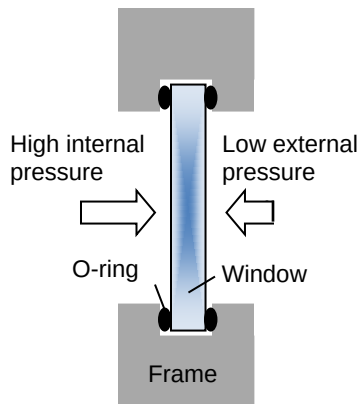


FIGURE 1. Aircraft sensor window with high pressure inside the cabin and low pressure outside.

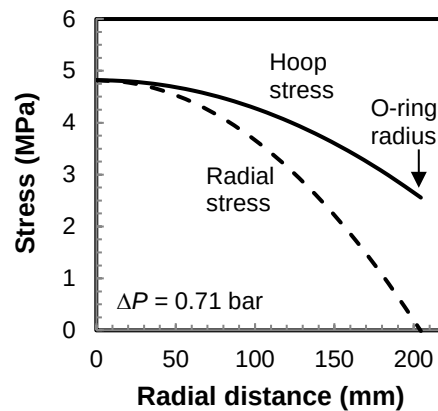
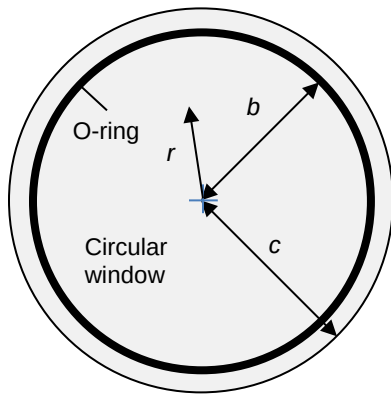


FIGURE 2. Tensile stress on outside face of fused silica window decreases with increasing distance from the center of the window.

For imaging at visible wavelengths, we consider a window made of fused silica (SiO_2), such as Corning 7979 or 7980 glass. We also present a design for Schott borosilicate crown glass BK7. For infrared imaging, we evaluate multispectral zinc sulfide, a polycrystalline ceramic material with trade names including Cleartran® and water-clear. Window dimensions for all three materials are the same and are shown in Figure 3.



Window radius: $c = 8.305 \text{ in.} = 210.95 \text{ mm}$
 O-ring radius: $b = 8.049 \text{ in.} = 204.44 \text{ mm}$
 Window thickness: $d = 1.0625 \text{ in.} = 26.99 \text{ mm}$
 Design pressure: $\Delta P = 10.3 \text{ psi} = 0.710 \text{ bar}$

FIGURE 3. Window dimensions. Radial stress is in the plane of the window parallel to any radial vector, $\vec{r}$. Hoop stress is circumferential stress in the plane of the window.

II. NASA STD 5018 WINDOW DESIGN FOR HUMAN FLIGHT APPLICATIONS

To calculate the design life of the aircraft sensor window, we choose to employ design rules embodied in the U.S. National Aeronautics and Space Administration (NASA) Technical Standard (STD) 5018 “Strength Design and Verification Criteria for Glass, Ceramics, and Windows in Human Space Flight Applications” (Reference 1). Much of our analysis can be taken as an example of the application of the NASA guidelines to the design of a specific window. We do not mean to imply that other approaches—which could be less conservative—are not applicable to the same design problem. The degree of conservatism in the design is a subjective decision.

Principal equations describe (1) the growth rate of microscopic cracks under the influence of stress, (2) a relationship between stress intensity factor at the crack tip and the depth of the crack, and (3) design life of the window.

NASA STD 5018 Section 4.7.3 states that the exponential law Equation 1a for crack growth rate shall be selected. We show in Appendix F that the exponential law is more conservative in its design life prediction. The power law for crack growth rate is given in Equation 1b. It is widely used in fracture mechanics calculations and lifetime prediction because it can be integrated analytically to provide closed-form equations for window design life and for strength loss in proof tests. The more conservative exponential law must be integrated numerically to provide window lifetime. For most window materials, we do

not have experimental measurements to validate either crack growth law at a stress intensity factor below $\sim 0.3 \text{ MPa}\sqrt{\text{m}}$, so we choose the more conservative exponential law prediction.

Crack growth velocity (v): $v = v_0 e^{\beta K_I}$ (*Exponential law*) (1a)

$v = da/dt$, where a = flaw depth and t = time

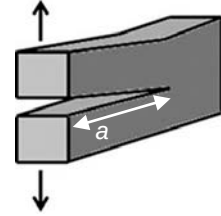
K_I = stress intensity factor

v_0 and β are measured material parameters

Alternate growth velocity equation: $v = A^* \left(\frac{K_I}{K_{Ic}} \right)^n$ (*Power law*) (1b)

A^* and n are measured material parameters

K_{Ic} = critical stress intensity factor for catastrophic failure



In fracture mechanics, the stress intensity factor near the tip of a flaw in a brittle material is related to the applied stress, and the depth and geometry of the flaw.

Relation between stress intensity factor (K_I) and flaw depth (a):

$$K_I = U \cdot \sigma \cdot (\pi a / Q)^{1/2} = U \sigma Y \sqrt{a} \quad (2)$$

σ = stress

a = flaw depth

U = uncertainty factor from Table 1

Q = shape factor = 1 for long flaws (length/depth ≥ 20) ($Q = 1$ is assumed)

$Y = \sqrt{\pi/Q}$ (geometric factor based on crack shape)

TABLE 1. Uncertainty Factor U for Equation 2 From NASA STD 5018^a.

Design Life	Uncertainty Factor U	
> 1 year (yr) (>8,760 hours (h))	1.1	← $U = 1.1$ for expected design life > 1 yr
1 month to 1 yr (720–8,760 h)	1.2	
1 week to 1 month (168–720 h)	1.3	
< 1 week (<168 h)	1.4	

^aUncertainty factor U is from the NASA procedure, not from fracture mechanics. U multiplies the stress σ to account for greater uncertainty of the stress in shorter missions.

The uncertainty factor U multiplies the stress σ to account for wider possible variation in stress as the design life becomes shorter. The longer the service life, the more the variations can average out and the smaller the uncertainty factor applied to the stress. In practice, we estimate the design life and choose the value of U for that design life. If the calculated design life comes out in a different category from what we chose in Table 1, we adjust the value of U and repeat the calculation until the value of U and the design life are compatible.

The stress (σ) for a proof pressure test is computed with Equation 2 by first selecting the maximum allowed flaw depth (a) to ensure a desired window design life. For this calculation, NASA STD 5018 specifies that K_I on the left side of Equation 2 shall be

$$K_I = K_{Ic(average)} + 1 \text{ standard deviation in } K_{Ic} \text{ measurement} \quad (2a)$$

NASA STD 5018 Section 4.7.6 states that experience shows that an invisible crack tail extends past a visible crack to approximately twice the visible crack depth, making the total flaw depth three times the visible flaw depth. For example, if the maximum allowable flaw depth for Equation 2 is 750 microns (μm), then the maximum allowable visible flaw depth is $\frac{1}{3}(750 \mu\text{m}) = 250 \mu\text{m}$.

Design Life. NASA STD 5018 sets the design life of a window as one quarter of the time to failure (Appendix G) calculated with Equation 3:

Time to failure (t) static fatigue life:

$$t = \left(\frac{2}{U^2 Y^2 \sigma^2} \right) \int_{K_{Ii}}^{K_{Ic}} \left(\frac{K_I}{v} \right) dK_I \quad (3)$$

Design life = $t/4$

K_I = stress intensity factor

σ = tensile stress acting to open cracks taken as maximum design stress, σ_a

U = uncertainty factor in Equation 2

Y = geometric factor = $\sqrt{\pi/Q}$

v = flaw growth velocity = da/dt

K_{Ii} = initial stress intensity factor

K_{Ic} = critical stress intensity factor for catastrophic failure

NASA STD 5018 states that K_{Ic} for the upper limit of integration in Equation 3 shall be

$$K_{Ic} = K_{Ic(average)} - 3 \text{ standard deviations in } K_{Ic} \text{ measurement}$$

III. FUSED SILICA WINDOW DESIGN

Corning 7980 fused silica is useful in the wavelength range 0.25 to 2.0 microns (μm). Some of its mechanical properties are listed in Table 2. Infrared grade Corning 7979 fused silica has one tenth of the hydroxyl (OH) content of Corning 7980 and can be used up to 2.6 μm . In Table 2, “inert conditions” are a rigorously dry environment to prevent moisture-assisted crack growth.

TABLE 2. Mechanical Properties of Fused Silica.

Property	Corning Fused Silica
Young’s modulus E (GPa)	73
Poisson’s ratio ν	0.16
Fracture toughness (critical stress intensity factor):	
K_{Ic} (MPa $\sqrt{\text{m}}$) ^a	0.75, 0.78 ^d
Standard deviation	0.03, 0.06
K_{Ic} (MPa $\sqrt{\text{m}}$) ^b	0.77 ^e
Standard deviation	0.02
K_{Ic} (MPa $\sqrt{\text{m}}$) chosen for this analysis ^c	0.77 (0.04)

^aReference 5.

^bReference 6.

^cInert K_{Ic} is measured in a rigorously dry atmosphere to minimize slow crack growth.

^dCorning 7980 fused silica.

^eCorning 7940 fused silica. This glass was discontinued and replaced by Corning 7980.

WINDOW DESIGN SERVICE STRESS (σ_a)

Consider the window geometry in Figures 2 and 3 with uniform pressure P_o applied to one face. Both faces are supported by O-rings, and the window is otherwise free to flex. The maximum stress at the center of the tensile face of the window is

$$\text{Maximum stress: } \sigma_{\max} = \frac{3P_o b^2}{8d^2} \left[(1 - \nu) \frac{b^2}{c^2} + 2(1 + \nu) \right] + \sigma_s \quad (4)$$

where ν is Poisson’s ratio and $\sigma_s = P_o(3 + \nu)/[4(1 - \nu)]$ is a small stress due to overhang of the window beyond its support. For the dimensions of the aircraft window in Figure 3 with the maximum design pressure $P_o = 10.3 \text{ psi} = 0.710 \text{ bar}$, maximum service stress is

$$\text{Fused silica maximum service stress } \sigma_a = 4.82 \text{ MPa}$$

WINDOW DESIGN PROOF STRESS (σ_p)

The proof pressure test stress (σ_p) applied before a window is put into service (and periodically thereafter) establishes the expected design life of the window in the absence of new damage. The principle of the proof test is that if a window survives an applied stress up to a selected level, then the largest flaw that can be left in the window after the test must be smaller than a flaw that would fail in the proof test. Any larger flaw would have led to catastrophic failure in the proof test. The time for that largest flaw left after the proof test to grow to failure under the influence of the service stress establishes the expected design life of the window. NASA STD 5018 takes the design life to be one fourth of the expected time for the largest flaw to grow big enough to fail.

We select a proof stress with Equation 2 by first selecting the maximum allowed flaw depth (a). On the left side of Equation 2, we set the stress intensity factor K_I equal to the critical stress intensity factor for failure (K_{Ic}) plus one standard deviation in the measurement of K_{Ic} .

Design stress intensity factor with values from Table 2:

Fused silica $K_{Ic}(\text{design}) = K_{Ic}(\text{average}) + 1 \text{ std dev} = 0.77 + 0.04 = 0.81 \text{ MPa}\sqrt{\text{m}}$

NASA STD 5018 Section 4.7.7 states “Vehicle/element windowpanes shall be designed by first choosing the limiting flaw depth, i.e., maximum initial design flaw depth.” Chart 15 of the NASA class handout shows a range of 0.003 to 0.009 inch (76 to 229 μm) for the actual depth of flaws (a in Equation 2, including the visible flaw and the invisible flaw extending to three times the depth of the visible flaw) typically observed from handling damage, with the caveat that “larger flaws result when untrained hands are involved.”

For our analysis, we choose a design limiting flaw depth of $a = 800 \mu\text{m}$ (0.0315 inch) or $a = 700 \mu\text{m}$ (0.0276 inch) to obtain a proof stress σ_p approximately three times greater than the service stress σ_a . Our goal is to set the proof stress high enough to ensure adequate design life but not so high that it will cause undue damage to the window in the proof test. Equation 2 then gives us the window design proof stress σ_p :

$$\textbf{Proof stress } (\sigma_p): \quad K_{Ic}(\text{design}) = U \cdot \sigma_p \cdot (\pi a / Q)^{1/2} \Rightarrow \sigma_p = \frac{K_{Ic}(\text{design})}{U(\pi a / Q)^{1/2}} \quad (2)$$

$$\text{For } 800 \mu\text{m}: \quad 0.81 \text{ MPa}\sqrt{\text{m}} = 1.1 \cdot \sigma_p \cdot (\pi \cdot [800 \times 10^{-6} \text{ m}] / 1)^{1/2} \Rightarrow \sigma_p = 14.7 \text{ MPa}$$

$$\sigma_p / \sigma_a = (14.7 \text{ MPa}) / (4.82 \text{ MPa}) = 3.05$$

$$\text{For } 700 \mu\text{m}: \quad 0.81 \text{ MPa}\sqrt{\text{m}} = 1.1 \cdot \sigma_p \cdot (\pi \cdot [700 \times 10^{-6} \text{ m}] / 1)^{1/2} \Rightarrow \sigma_p = 15.7 \text{ MPa}$$

$$\sigma_p / \sigma_a = (15.7 \text{ MPa}) / (4.82 \text{ MPa}) = 3.26$$

The uncertainty factor $U = 1.1$ from Table 1 was used in the calculation because we expect the design life to be >1 year. If the analysis does not produce a design life >1 year, we need to go back through the design process with a different value of U .

NASA STD 5018 Section 4.9.4 states that the quotient (proof stress/service stress, σ_p/σ_a) shall be the larger of the value we just calculated or 3.0. Our calculated values are $\sigma_p/\sigma_a = 3.05$ and 3.26. At NASA, up to 30% of all windowpanes fail their proof tests. NASA states that many failures originate at the window edge where there might be deeper flaws than on the face and where there can be bending stresses from the window frame. The design process described here does not account for window edge failure, which can only be assessed by an appropriate proof test that applies significant stress at the edge of the window.

WEIBULL STRENGTH PARAMETERS FOR FUSED SILICA

We choose to write the Weibull equation for probability of window failure in the form

Weibull equation:
$$P_f = 1 - e^{-\left(\frac{A_e}{A_o}\right)\left(\frac{\sigma}{\sigma_o}\right)^m} \quad (5)$$

where P_f = probability of failure

σ = applied stress

σ_o = Weibull scale factor = strength for 1 cm² area of material in uniaxial stress

m = Weibull modulus (a measure of spread of strength distribution)

A_e = effective area of window

A_o = unit area $\equiv 1$ cm²

For the “pressure-on-ring” window geometry in Figures 2 and 3, the effective area* is given by ASTM C1683 as

Effective area:
$$A_e = \frac{4\pi(1+\nu)}{1+m} \left(\frac{b}{c}\right)^2 \left[\frac{2c^2(1+\nu) + b^2(1-\nu)}{(3+\nu)(1+3\nu)} \right] \quad (6)$$

where b and c are dimensions in Figure 3, ν is Poisson’s ratio (Table 2), and m is the Weibull modulus. For a window with effective area A_e , the expected mean strength is

$$\text{Expected mean strength} = \sigma_o \left(\frac{A_o}{A_e}\right)^{1/m} \Gamma\left(1 + \frac{1}{m}\right) \quad (7)$$

where Γ is the gamma function of the argument $(1 + 1/m)$. In a Microsoft Excel® spreadsheet, the gamma function is called with the statement “=Gamma(1+1/m)”.

* A window of effective area A_e has the same probability of failure as a window with geometric area = A_e loaded in uniaxial tension.

For equibiaxial flexure tests of disks with stress rates of 6 or 4 MPa/s in ambient humidity and temperature, we have three data sets analyzed by the method in ASTM C1239. These are summarized in Table 3.

TABLE 3. Weibull Parameters for Fused Silica Coupons and Full-Size Window.

Parameter	Fused Silica Data Sets		
	Figure 3	Figure 4 Left	Figure 4 Right
	Detrio ^a	Weak NCAR Silica ^b	Strong NCAR Silica ^c
Weibull modulus (m)	10	5.8	10.7
Weibull scale factor (σ_0) (MPa)	120	237	302
Aircraft window effective area A_e (cm ²) ^d	368	596	346
Aircraft window expected mean strength (MPa)	63	73	167

^aSiO₂ data from Reference 7. The material that was tested was General Electric Type 124 fused quartz immersed in distilled water. Stress rate $\approx$ 6 MPa/s.

^bHeraeus Infrasil 302 fused silica data from National Center for Atmospheric Research, Boulder, Colorado. Weibull parameters for 21 weakest samples from 32 total samples with a bimodal strength distribution. Stress rate $\approx$ 4 MPa/s. Tested at 50% relative humidity in air at 23°C.

^cHeraeus Infrasil 302 fused silica data from National Center for Atmospheric Research, Boulder, Colorado. Weibull parameters for 12 strongest samples from 32 total samples with a bimodal strength distribution. Stress rate $\approx$ 4 MPa/s. Tested at 50% relative humidity in air at 23°C.

^dAircraft window radius 8.305 inches, thickness 1.0625 inches, O-ring radius 8.049 inches.

Three sets of fused quartz disks with radii of 12.7, 38.1, and 114.3 millimeters (mm) reported by Detrio in 2008 exhibited well-behaved Weibull curves, one of which is shown in Figure 4. Weibull parameters for this material in Table 3 are an “eyeball” composite of the values for the three sets of disks.

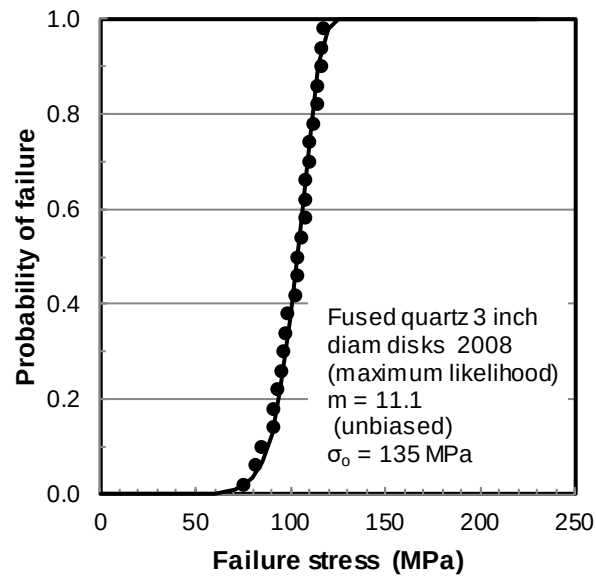


FIGURE 4. Weibull plot for equibiaxial flexure testing of fused quartz disks with a radius of 38.1 mm and thickness of 3.96 mm. Support radius = 28.58 mm and load radius = 14.29 mm. Stress rate $\approx 6 \text{ MPa/s}$ at $\sim 20^\circ$ in distilled water. Data from Reference 7. The material that was tested was General Electric Type 124 fused quartz.

Equibiaxial flexure strength tests of 32 disks of Heraeus Infrasil 302 fused silica in Figure 5 were not well behaved. However, the lowest strength 21 of 32 disks had a well-behaved Weibull curve in Figure 6 (left), and the highest strength 12 of 32 disks had a well-behaved Weibull curve in Figure 6 (right). Disk number 21 was included in both populations for analysis. Resulting Weibull parameters are listed in Table 3.

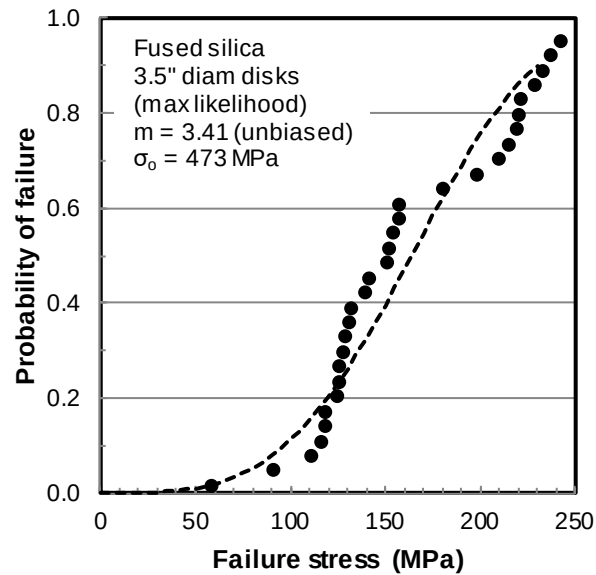


FIGURE 5. Weibull plot for equibiaxial testing of 32 Heraeus Infrasil 302 fused silica disks with radius = 44.6 mm and thickness = 5.72 mm and scratch/dig finish specification of 40/20 over 90% of the clear aperture. Load radius = 14.60 mm and support radius = 32.38 mm. Stress rate ≈ 4 MPa/s. Load and support rings were machined from Delrin. Tests conducted at 50% relative humidity in air at 23°C. Data from National Center for Atmospheric Research, Boulder, Colorado.

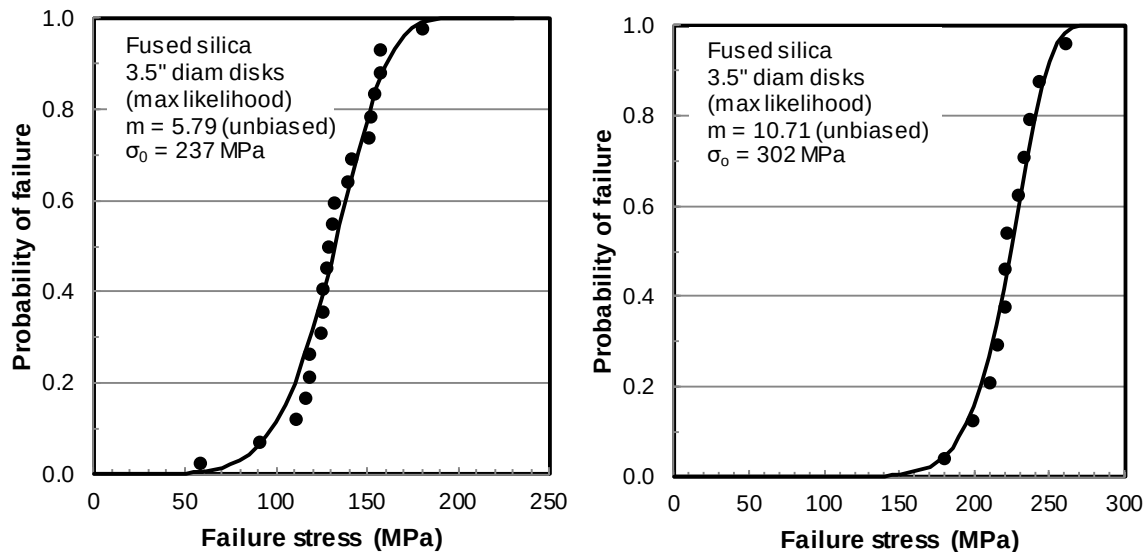


FIGURE 6. *Left:* Weibull plot for equibiaxial testing of 21 lowest-strength Heraeus Infrasil 302 fused silica disks from Figure 5. *Right:* Weibull plot for 12 highest strength disks from Figure 5. Resulting Weibull parameters for each set are listed in Table 3.

SLOW CRACK GROWTH PARAMETERS

Direct measurements of slow crack growth rate in fused silica are shown in Figure 7. Results fit to the equation $K_I = a + b \ln(v)$ are shown in lines 1 to 3 of Table 4, where they are also recast into the equivalent form $v = v_0 e^{\beta K_I}$, which is Equation 1a, the exponential slow crack growth law.

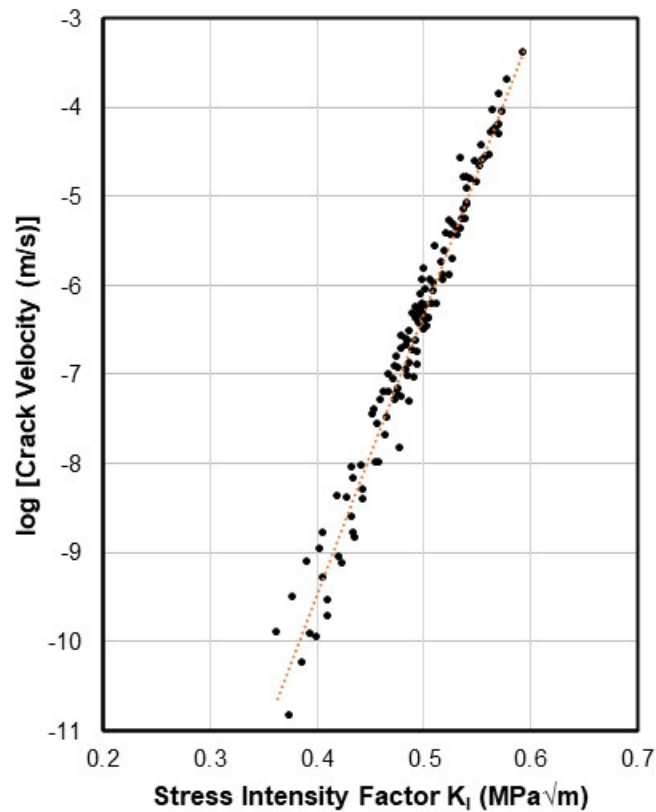


FIGURE 7. Slow crack growth in fused silica (Corning 7940) near 25°C in air at 100% relative humidity measured at the U.S. National Bureau of Standards. Data digitized and redrawn from Reference 8.

Table 4 lists crack growth rates from Figure 7 and crack growth rates measured by J. A. Salem at NASA. The exponential laws from both sources were used to calculate design life later in Table 5.

TABLE 4. Fused Silica (Corning 7940) Slow Crack Growth Parameters at Room Temperature at 100% Humidity or Immersed in Water.

Fit to Exponential Law	$K_I = a + b \ln(v)$		$v = v_o e^{\beta K_I}$	
	a (MPa√m)	b (MPa√m)	v_o (m/s)	$\beta(1/[MPa\sqrt{m}])$
1. Wiederhorn double cantilever ^a	0.6931	0.01342	3.716×10^{-23}	74.52
Ground disks broken in equibiaxial flexure at different stress rates:				
2. Salem flexure tests ^b			4.81×10^{-19}	54.3
3. Salem flexure tests ^b			3.33×10^{-20}	61.5
Fit to Power Law	$v = A^* \left(\frac{K_I}{K_{Ic}} \right)^n$ with $K_{Ic} = 0.75 \text{ MPa}\sqrt{\text{m}}$:			
			A^* (m/s)	n
4. Wiederhorn double cantilever ^c			1.145	35.8
Ground disks broken in equibiaxial flexure at different stress rates:				
5. Salem fit to all data points ^b			0.0773	28
6. Salem fit to medians for each stress rate ^b			0.0567	27

^aData from Reference 4.^bUnpublished data from J. A. Salem, NASA Glenn Space Flight Center. Samples were tested in water.^cPower law fit by Harris and Bodnar to digitized data from Figure 7.

Figure 8 superimposes BK7 glass and fused silica crack growth rates. Growth rates that determine the design life of a window are in the range $\sim 10^{-15}$ to 10^{-7} meters per second (m/s). In the measured region 10^{-11} to 10^{-7} m/s, fused silica crack growth rate is ~ 100 times slower than BK7 crack growth rate, giving fused silica ~ 100 times greater lifetime than BK7.

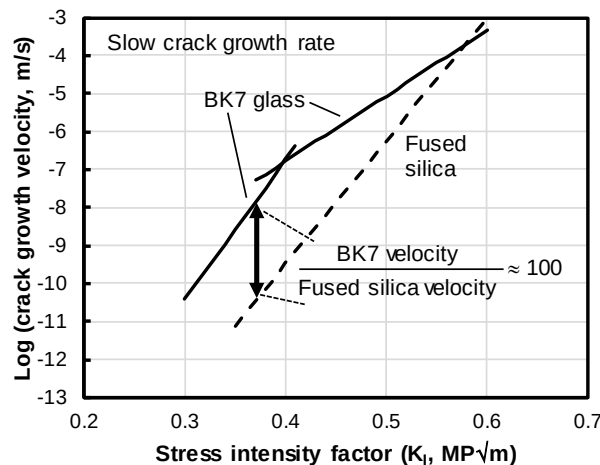


FIGURE 8. Comparison of crack growth rates of BK7 and fused silica at 100% humidity. Data from Reference 8.

FUSED SILICA WINDOW LIFETIME

NASA STD 5018 states that the design life for a window is one fourth of the calculated time to failure. For a window that experiences a constant service stress σ_a , the time to failure is the accumulated time that can be spent with stress σ_a applied to the window.

To find the time to failure, we substitute Equation 1a for crack velocity into Equation 3 for time to failure and numerically integrate from the initial value of K_{Ii} to the final value ($K_{Ic} - 3 \times \text{standard deviations in } K_{Ic}$), at which catastrophic failure occurs. Choosing the maximum allowed crack depth of 800 or 700 μm in Table 5 determines the initial stress intensity factor (and the proof test stress) from Equation 2. We used stress intensity factor increments of $\Delta K_I = 0.00005 \text{ MPa}\sqrt{\text{m}}$ for evaluating the integral. Most of the time before failure occurs with crack growth velocities less than 10^{-11} m/s —below experimentally measured crack velocities in Figure 7.

An equivalent way to calculate window time to failure is to take equal time steps Δt . In the first step we start with $a_1 = 800 \text{ }\mu\text{m}$, from which we calculate K_I from which we calculate crack growth velocity v_1 . For the next step, we set $a_2 = 800 \text{ }\mu\text{m} + v_1 \Delta t$. From the new value a_2 , we calculate new values of K_I , v_2 , and $a_3 = a_2 + v_2 \Delta t$. We continue this process until the time at which $K_I > K_{Ic}$, which is the window time to failure.

TABLE 5. Fused Silica Window Design Life With Proof Pressure Test in Figure 9.

Window and Parameters				Maximum Initial Flaw Depth, μm	Design Life, yr ^a
<i>Design life of fused silica window in Figure 3 with O-ring 0.256 inch from edge</i>					
Service stress (σ_a) = 4.82 MPa; proof stress (σ_p) = 14.7 MPa; (σ_p/σ_a) = 3.05					
Wiederhorn crack growth parameters Table 4 line 1 ^b				800	45
Salem crack growth parameters Table 4 line 2 ^c				800	1.1
Salem crack growth parameters Table 4 line 3 ^c				800	2.0
Service stress (σ_a) = 4.82 MPa; Proof stress (σ_p) = 15.7 MPa; (σ_p/σ_a) = 3.26					
Wiederhorn crack growth parameters Table 4 line 1 ^b				700	150
Salem crack growth parameters Table 4 line 2 ^c				700	2.5
Salem crack growth parameters Table 4 line 3 ^c				700	5.4
Maximum Initial Flaw, μm	Crack Growth Parameters in Table 3	Weibull Parameters		Probability of Failure in Proof Test	
		m	σ_o (MPa)	Neglecting Crack Growth ^d	Including Crack Growth ^e
<i>Probability of failure of fused silica window with proof stress profile in Figure 9</i>					
800	Wiederhorn line 1	10.0	120	3×10^{-7}	9×10^{-6}
800	Wiederhorn line 1	5.8	237	6×10^{-5}	4×10^{-4}
800	Wiederhorn line 1	10.7	302	3×10^{-12}	1×10^{-10}
800	Salem line 2	5.8	237	6×10^{-5}	3×10^{-4}
700	Wiederhorn line 1	5.8	237	9×10^{-5}	7×10^{-4}
700	Salem line 2	5.8	237	9×10^{-5}	5×10^{-4}

^aDesign life = 25% of time to failure computed with Equation 3 using the exponential crack growth law, Equation 1a. Other parameters used are $K_{Ic}(\text{design}) = K_{Ic}(\text{average}) + 1 \text{ std dev}$ and $Y = \sqrt{\pi}$.

^bReference 4.

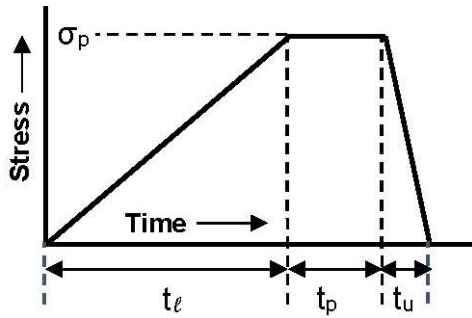
^cUnpublished data from J. A. Salem, NASA Glenn Space Flight Center.

^dCalculated from Weibull parameters and no slow crack growth in proof test.

^eCalculated from Weibull parameters with slow crack growth in proof test using exponential crack growth law:-

PROOF TEST

Figure 9 shows the stress profile used for proof test calculations in the lower half of Table 5. This profile is plausible for testing a large aircraft window. Stress is applied at a constant rate during loading time t_ℓ , then held constant for time t_p at proof stress σ_p , and then released at constant rate for unloading time t_u . The window should be dry and tested in a dry atmosphere for the shortest possible time to minimize crack growth during the proof test.



$$\text{Loading: } \sigma(t) = \frac{t}{t_\ell} \sigma_p \quad (8)$$

$$\text{Peak stress: } \sigma(t) = \sigma_p \quad (9)$$

$$\text{Unloading: } \sigma(t) = \left(1 - \frac{t - t_\ell - t_p}{t_u}\right) \sigma_p \quad (10)$$

FIGURE 9. Stress profile used in Table 5 for calculating probability of failure in proof test with slow crack growth. Times are $t_\ell = 10$ s, $t_p = 1$ s, and $t_u = 2$ s.

NASA data shows that, in a rigorously dry environment, the strength of fused silica is 50% greater than the strength when tested in water. The difference in strength arose from slow crack growth in the wet environment. There was no measurable crack growth in the dry environment. These observations highlight the importance of drying the window prior to a proof test, maintaining a dry environment during the proof test, and minimizing the time at high stress while controlling pressure to attain no less and no more than the intended pressure.

Appendix B derives equations for loss of strength by a window during a proof test. For the proof test in Figure 9, the power law Equation 1b produces Equation 11 for strength loss:

$$\text{Power law strength loss in proof test:} \quad S_f^{n-2} = S_i^{n-2} - \frac{Y^2 A^* (n-2)}{2K_{Ic}^2} \sigma_p^n \left[t_p + \frac{t_\ell + t_u}{n+1} \right] \quad (11)$$

where S_i is the initial strength of the window and S_f is the final strength after the proof test.

The exponential law Equation 1a gives a strength loss that must be found by numerical integration of Equation 12

Exponential law strength
loss in proof test:

$$S_f^{-2} = S_i^{-2} + \frac{\gamma^2}{K_{lc}^2} v_o \int_0^t e^{\beta K_{lc} \sigma} dt \quad (12)$$

where σ in the integral is the instantaneous applied stress and S is the instantaneous strength of the window. Both σ and S are functions of time. We can write an equation for the time dependence of σ , but we cannot write an analytical expression for the time dependence of S . It is necessary to integrate Equation 12 in a series of short steps, during which the value of S is kept constant in each short step but updated to a new value in the next step. The final strength S_f from one step is the new strength S_i and the new strength S for the next step. The integration is carried out for the applied stress profile and times in Figure 9.

In general, Equations 11 and 12 produce similar predictions for strength loss in the proof test, so there is no reason not to use the simpler Equation 11. The general finding is that, for a given proof stress, there is a minimum initial window strength required for survival of the proof test. Strength lost near that threshold strength for survival is significant. If the initial strength of the window is just a few MPa above the threshold for survival, strength loss in the proof test is generally small.

Strength loss in the proof test is overestimated in our calculations because a proof test is conducted in a dry atmosphere and the slow crack growth parameters in Table 4 were measured in 100% humidity or in water. The moist environment promotes faster crack growth and is deliberately chosen to provide a conservative estimate of window lifetime.

The lower half of Table 5 shows calculated probabilities of failure in the proof test. This probability should lie between the two extremes of no crack growth during the test and worst-case crack growth that would occur in a moist environment. In both cases, the probability of failure is calculated with Weibull Equation 5 for the full-size window with its effective area A_e computed with Equation 6. For no crack growth, the probability of failure is the probability that the strength of the full-size window is less than or equal to the proof stress σ_p . With slow crack growth, the probability of failure is the Weibull probability that the strength of the full-size window is less than or equal to the minimum initial strength S_i required to survive the proof test.

As an example of finding the probability of failure in the proof test, suppose that the minimum strength required to survive a proof test is calculated to be 30 MPa. The middle column of Weibull data in Table 3 gives the most conservative (largest) probability of failure. With effective area $A_e = 596 \text{ cm}^2$, we find the probability of failure with Equation 5:

$$P_f = 1 - e^{-\left(\frac{A_e}{A_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^m} = 1 - e^{-\left(\frac{596 \text{ cm}^2}{1 \text{ cm}^2}\right)\left(\frac{30 \text{ MPa}}{237 \text{ MPa}}\right)^{5.8}} = 0.004 \quad (5)$$

FUSED SILICA DESIGN LIFE ESTIMATE IS CONSERVATIVE FOR AIRCRAFT FLIGHT ALTITUDE

We now provide evidence that window lifetime of fused silica on an aircraft flying at altitudes of 30,000 or 40,000 feet will be roughly 100 times longer than lifetimes calculated in Table 5 for room temperature with a humid environment.

Stress-induced crack growth in fused silica requires breaking Si–O–Si chemical bonds and reaction of the broken bonds with water create Si–OH groups on the fresh surface of the crack. Rates of reactions generally increase with increasing temperature. However, fused silica is unusual in that its crack growth rate *decreases* with increasing temperature in the measured range of 25 to 300°C (Reference 9). Because water is involved in crack formation, the crack growth rate also depends on the vapor pressure of water in the surrounding atmosphere, with growth rate slowing as the vapor pressure of water decreases. For high-altitude flight, the temperature and vapor pressure of water are both lower than at the surface of the earth. These two factors have opposite effects on slow crack growth in fused silica.

The combined effect of temperature and water vapor pressure on slow crack growth rate v (m/s) for fused silica in the range 25 to 300°C is described by the equation (Reference 9)

$$V = \frac{v_I v_{II}}{v_I + v_{II}} \quad (13)$$

where v_I is the crack growth rate in the low stress intensity factor region where most of the life of a window occurs and v_{II} is the almost-constant growth rate at higher stress intensities near the end of window lifetime. Both v_I and v_{II} depend on temperature and water vapor pressure:

$$v_I = \exp[-A + BK_I + CP_{H_2O}^{1/3}/T^3] \quad (14)$$

$$v_{II} = \frac{FK_I^2 P_{H_2O}}{\ln(GT) \cdot T^{1/2}} \quad (15)$$

where K_I is stress intensity factor at the crack tip, P_{H_2O} is vapor pressure (Pa) of water in the atmosphere, T is temperature (K), and the parameters are $A = 59$, $B = 80 \text{ (MPa}\sqrt{\text{m}})^{-1}$, $C = 2.3 \times 10^7 \text{ K}^3 \text{ Pa}^{-1/3}$, $F = 4 \times 10^{-17} \text{ K}^{1/2} / (\text{Pa}^3 \cdot \text{s})$, and $G = 0.681 \text{ K}^{-1}$. Note that the units of B and F require K_I to be expressed as $\text{MPa}\sqrt{\text{m}}$ in Equation 14, and as $\text{Pa}\sqrt{\text{m}}$ in Equation 15.* The validated range for Equations 14 and 15 is 25 to 300°C. We extrapolate to calculate crack growth rates at low temperature for high-altitude flight.

* The plus sign between the second and third terms in Equation 14 was missing in the original journal article. The article's author T. I. Suratwala confirmed that Equation 14 written here is correct.

We use the slow crack growth rate v from Equation 13 in the window lifetime integral of Equation 3 to compute design life ($= \frac{1}{4}$ of time to failure). Results are shown in Table 6. Water vapor pressure in Table 6 is the pressure of water in equilibrium with ice at the indicated atmospheric temperatures. These vapor pressures correspond to 100% relative humidity at each altitude. We see that the design life that was as little as 1 to 2 years in Table 5 for room temperature crack growth in a humid environment increases to hundreds of years in the stratosphere where the window will spend most of its service life.

TABLE 6. Fused Silica Window Design Life at Low Temperature
With 100% Humidity.

Altitude, ft	Temperature		100% humidity P_{H_2O} (Pa)	Design Life (yr) With 700 or 800 μm Largest Flaw Depth Left From Proof Test	
	K	$^{\circ}\text{C}$		800 μm	700 μm
7,600	273	0	610	1.1	3.9
12,600	263	-10	260	4.8	17.9
17,700	253	-20	103.5	19.6	72.7
22,700	243	-30	38.1	70	260
27,800	233	-40	12.9	217	805
32,900	223	-50	3.94	584	2,160
49,000	216.6	-56.5	1.75	1,010	3,750
---	203	-70	0.26	2,670	9,900

IV. MULTISPECTRAL ZINC SULFIDE WINDOW DESIGN

INTRODUCTION

The multispectral zinc sulfide sensor window has the same dimensions as the fused silica window in the previous section of this report. Multispectral zinc sulfide transmits radiation from the visible to long wave infrared regions up to a wavelength of $\sim 10\text{ }\mu\text{m}$. Fused silica transmits from ultraviolet up to short-wave infrared wavelengths of 2 or 3 μm , depending on the purity of the fused silica.

The design life of the fused silica window in the previous section was derived as prescribed by NASA STD 5018. The design life in Table 5 was not less than 1.1 year $\approx 9,600$ flight hours, which is more than adequate for a practical window.

A zinc sulfide window does not have as much mechanical margin of safety as fused silica. NASA STD 5018 design rules give a design life of $\sim 3,000$ hours or less. Therefore, our analysis for zinc sulfide design life will follow two parallel paths. One will be the conservative NASA procedure laid out for fused silica. The other path will employ a less conservative value of the geometric factor Y in Equation 2. The less conservative value of Y can be justified as being a more realistic representation of commonly observed semicircular or semi-elliptical surface flaw shapes in many window materials, including zinc sulfide. The value $Y = \sqrt{\pi} = 1.77$ prescribed by NASA STD 5018 applies to a linear flaw, such as a scratch, whose depth is shallow in comparison with its length. The alternative value $Y = 2/\sqrt{\pi} = 1.13$ that we will also evaluate applies to a semicircular “half penny” surface flaw. During crack growth, a semi-elliptical flaw tends to grow toward a semicircular shape because the stress intensity factor at the tip of the minor axis of the ellipse is higher than the stress intensity factor at the major axis (References 10 and 11).

Our alternate design approach will only deviate from the NASA STD 5018 guidelines in choosing $Y = 2/\sqrt{\pi} = 1.13$ instead of $Y = \sqrt{\pi} = 1.77$. Other design rules will be the same for both approaches. In particular, design life will be calculated with the exponential crack growth law Equation 1a, not the power law Equation 1b. The exponential law is more conservative than the power law in predicting design life. The design life predicted by the exponential law is shorter than the design life predicted by the power law. However, both laws are equivalent in their predictions of probability of failure in a proof test because the stress intensity factor in the proof test is high enough so the two laws nearly coincide.

WINDOW DIMENSIONS AND ZINC SULFIDE MECHANICAL PROPERTIES

Dimensions of the zinc sulfide aircraft window are the same as in Figure 3, and the design pressure difference across the window is also $\Delta P = 10.3\text{ psi} = 0.710\text{ bar}$. Mechanical properties of zinc sulfide are listed in Table 7.

TABLE 7. Mechanical Properties of Zinc Sulfide.

	II-VI Material ^a		Raytheon ^b		Rohm&Haas ^c
	ZnS Multispectral	ZnS Standard	ZnS Multispectral	ZnS Standard	ZnS Multispectral
Young's modulus (E) (GPa)	91	79	88	74	89
Poisson's ratio (ν)			0.32	0.29	0.33
K_{Ic} (MPa $\sqrt{m}$) in dry N ₂	0.74	0.82			
Standard deviation	0.14	0.02			

^aNASA measurements, 2016; Reference 6.^bReference 12.^cReference 13.

We adopt the following values for multispectral ZnS: Young's modulus $E = 89$ GPa, Poisson's ratio $\nu = 0.33$, $K_{Ic} = 0.74 \pm 0.05$ MPa $\sqrt{m}$ (K_{Ic} in dry nitrogen). For Equation 2a for a proof pressure test, we choose $K_{Ic} = 0.74 + 0.05 = 0.79$ MPa $\sqrt{m}$. For the upper limit of integration in Equation 3, we choose $K_{Ic} = 0.74 - 3(0.05) = 0.59$ MPa $\sqrt{m}$.

WINDOW DESIGN SERVICE STRESS (σ_a)

Equation 4 gives the maximum stress at the center of the tensile face for the window in Figures 2 and 3 with uniform pressure $P_o = \Delta P = 10.3$ psi = 0.0710 MPa applied to one face. For multispectral zinc sulfide, this maximum stress is $\sigma_a = 5.12$ MPa, which is taken as the service stress for calculation of design life.

WEIBULL STRENGTH PARAMETERS FOR MULTISPECTRAL ZINC SULFIDE

We use Weibull Equation 5 with the effective area of the circular aircraft window given by Equation 6. The "inert strength" of multispectral zinc sulfide (Cleartran® from Morton) measured with 4-point flexure bars immersed in liquid nitrogen at the University of Dayton Research Institute in 1993 is shown in Figure 10. "Inert strength" means conditions were chosen for minimal slow crack growth.

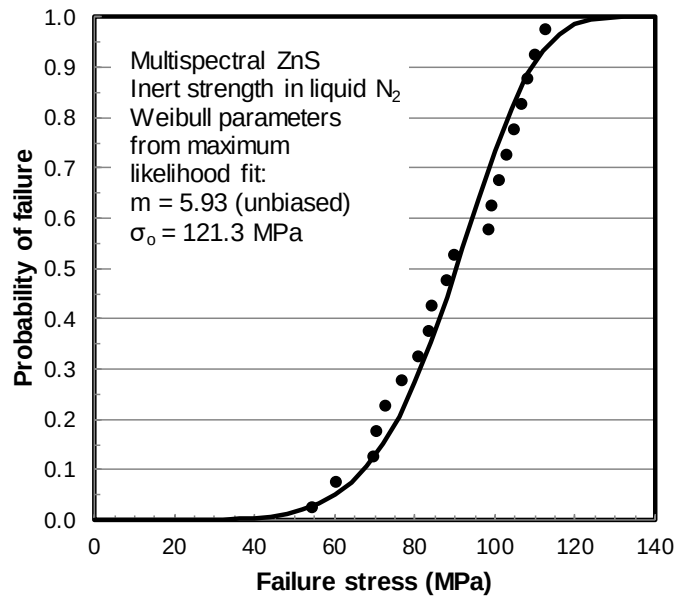


FIGURE 10. Inert strength of Cleartran® multispectral zinc sulfide immersed in liquid nitrogen. Type C 4-point flexure bars (ASTM C1161) had a length of 90 mm, width $b = 8.00$ mm, thickness $d = 6.00$ mm, support span $L_{\text{support}} = 80.0$ mm, and load span $L_{\text{load}} = 40.0$ mm. The crosshead speed of 0.021 mm/s produced a load rate of ~ 3.4 MPa/s. Fracture origins of 90% of specimens were found on the polished tensile face. (Data from Reference 14.)

Table 8 summarizes Weibull parameters for inert strength. With an aircraft window effective area of 503 cm² in tension, the expected mean inert strength is just 39 MPa.

TABLE 8. Weibull Parameters ASTM C1239 Analysis of Multispectral Zinc Sulfide.

	Multispectral ZnS
Weibull modulus (m)	5.9
Weibull scale factor (σ_0) (MPa)	121
4-Point flexure coupon effective area (cm ²)	4.06
Aircraft window effective area (cm ²)	503
Aircraft window expected mean inert strength (MPa)	39

Aircraft window radius 8.305 inches, thickness 1.0625 inches, O-ring radius 8.049 inches.

SLOW CRACK GROWTH PARAMETERS FROM ZINC SULFIDE: DYNAMIC FATIGUE MEASUREMENT

For flexure tests conducted at a constant stress rate $\dot{\sigma}$ (MPa/s), the fracture strength S_f (MPa) is related to the inert strength S_i by Equation 16 (Reference 15), which is derived in Appendix C.

$$\begin{array}{l} \text{Strength degradation} \\ \text{during flexure test} \end{array} \quad S_f^{n+1} = B(n+1)S_i^{n-2}\dot{\sigma} \quad (16)$$

where n is the crack velocity exponent in Equation 1b, and the slow crack growth constant A^* in Equation 1b is relate to B by:

$$A^* = \frac{2K_{Ic}^2}{BY^2(n-2)} \quad (17)$$

We take the geometric factor Y to be 1.28 for the test coupons. This value of Y corresponds to a surface flaw with a semi-elliptical shape having a ratio flaw depth/flaw length on surface $(a/2c) \approx 0.42$. Observed flaws have a range of shapes.

Figure 11 shows University of Dayton Research Institute dynamic fatigue strength measurements giving slow crack growth rate for Cleartran zinc sulfide. The table gives stress at 50% probability of failure for sets of 15 or 16 4-point bend bars tested at 21°C at load rates of 0.0174 to 82.7 MPa/s. To ensure worst-case crack growth, the tensile face of each disk was immersed in water during the test. Inert strength at the bottom of the table is the 50% failure probability stress from the Weibull curve in Figure 10 measured with disks immersed in liquid nitrogen. The strength at liquid nitrogen temperature was independent of stress rate over a factor of 1000 variation in stress rate.

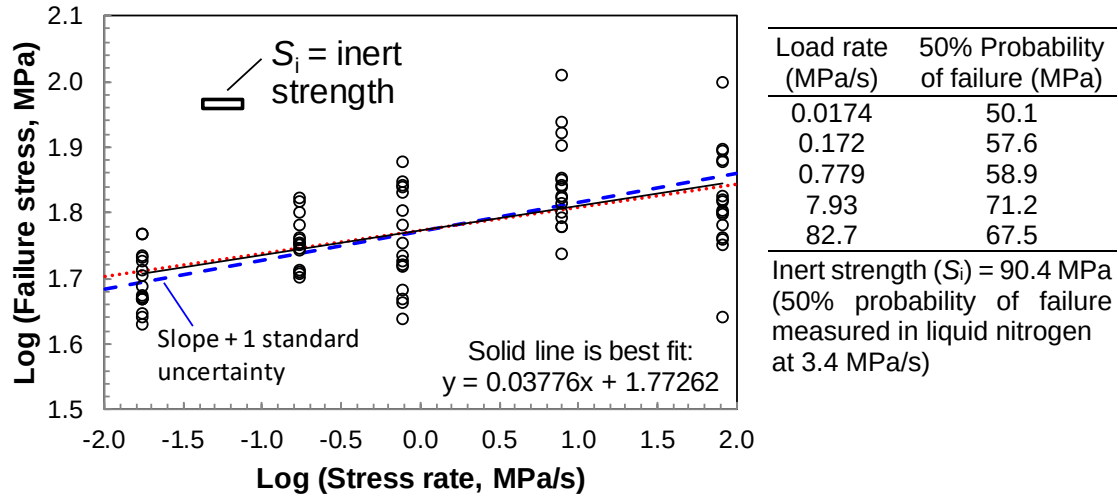


FIGURE 11. Dynamic fatigue measurement for Cleartran zinc sulfide coupons immersed in water at 21°C. Slope and intercept are least-squares linear regression to all 79 experimental points. Dashed line has one standard uncertainty added to slope. Dotted line has one standard uncertainty subtracted from slope. Dashed line slope gives shortest window design life. Other details are in caption of Figure 10. (Data from Reference 14.)

Derivation of slow crack growth parameters from dynamic fatigue measurements follows the procedure in ASTM C1368. Figure 11 has logarithmic axes because stress rate spans four orders of magnitude. Taking the logarithm of both sides of Equation 16 produces a linear equation:

$$\underbrace{\log(S_f)}_y = \left(\frac{1}{n+1}\right) \underbrace{\log(\dot{\sigma})}_x + \left(\frac{1}{n+1}\right) \log(B[n+1]S_i^{n-2}) \quad (18)$$

$$\text{slope} \equiv \alpha = \left(\frac{1}{n+1}\right) \quad (19)$$

$$y\text{-intercept} \equiv \beta = \left(\frac{1}{n+1}\right) \log(B[n+1]S_i^{n-2}) \quad (20)$$

The least-squares slope (0.03776) and intercept (1.7726) in Figure 11 are computed for stress rate in MPa/s and fracture strength in MPa. The crack velocity exponent n in Equation 1b is computed from the slope α :

$$\text{Slope } (\alpha) = 0.03776 = \left(\frac{1}{n+1}\right) \Rightarrow n = \frac{1}{\alpha} - 1 = \frac{1}{0.03776} - 1 = 25.49 \quad (21)$$

The parameter B in Equation 18 is calculated from the slope ($\alpha = 0.03776$), intercept ($\beta = 1.7726$), and the inert strength ($S_i = 90.4$ MPa, which we take as the 50% probability of failure for the 4-point flexure test coupons tested in liquid nitrogen).

$$\beta = \left(\frac{1}{n+1}\right) \log(B[n+1]S_i^{n-2}) \Rightarrow B = \frac{\alpha(10^{\beta/\alpha})}{S_i^{\left(\frac{1}{\alpha}-3\right)}} = 0.3836 \text{ MPa}^2\cdot\text{s} \quad (22)$$

The University of Dayton Research Institute report cited in Figure 11 gives $B = 1.28$ from the same data set. We use $B = 0.3836$ for our window lifetime calculations.

The crack growth rate parameter A^* in Equation 1b is computed from Equation 23 from the values $B = 0.3836$, $n = 25.49$, $Y = 1.28$, and inert $K_{Ic} = 0.74 \text{ MPa}\sqrt{\text{m}}$:

$$A^* = \frac{2K_{Ic}^2}{BY^2(n-2)} = \frac{2(0.74^2)}{(0.3836)(1.28^2)(25.49-2)} = 0.07419 \text{ m/s} \quad (23)$$

Equation 1b for crack growth velocity therefore becomes

$$v = A^* \left(\frac{K_I}{K_{Ic}}\right)^n = (0.07419 \text{ m/s}) \left(\frac{K_I}{0.74 \text{ MPa}\sqrt{\text{m}}}\right)^{25.49} \quad (24)$$

Table 9 gives the slope and intercept for dynamic fatigue measurements and derived crack growth parameters. The table shows the effect of adding one standard uncertainty (standard deviation of the mean) to the slope of the best-fit line in Figure 11. This new line is shown by the dashed line in Figure 11. The purpose of adding one standard uncertainty is to evaluate the sensitivity of design life to statistical variations in the dynamic fatigue measurement. We find below that design life is sensitive to the slope of the dynamic fatigue measurements.

Appendix A shows how we derived exponential law parameters v_0 and β for Equation 1a from power law parameters for Equation 1b by least squares fitting of the exponential law to the power law in the range $K_I = 0.533 \cdot K_{Ic}$ to $K_I = 0.733 \cdot K_{Ic}$. We judge by inspection of crack growth rate measurements by Wiederhorn for several glass materials that the power law and exponential law overlap in this approximate range of stress intensity factor. Figure 12 shows excellent agreement between observed dynamic fatigue strength and prediction with Equation C-6 in Appendix C using exponential law parameters in Table 9.

TABLE 9. Slow Crack Growth Parameters for Multispectral ZnS
Derived From Figure 11.

	Dynamic Fatigue Analysis			Power Law			Exponential Law	
	Slope α	Intercept β	B	A	n	$A^*(\text{m/s})$	$v_o(\text{m/s})$	β
Least squares	0.03776	1.7726	0.3836	159.6	25.49	0.07419	4.27×10^{-18}	54.69
+1 standard slope uncertainty	0.04361	1.7726	1.990	12.44	21.93	0.01686	1.79×10^{-16}	47.06

Parameters for dynamic fatigue data analysis: Inert $K_{Ic} = 0.74 \text{ MPa}\sqrt{\text{m}}$; $Y = 1.28$.

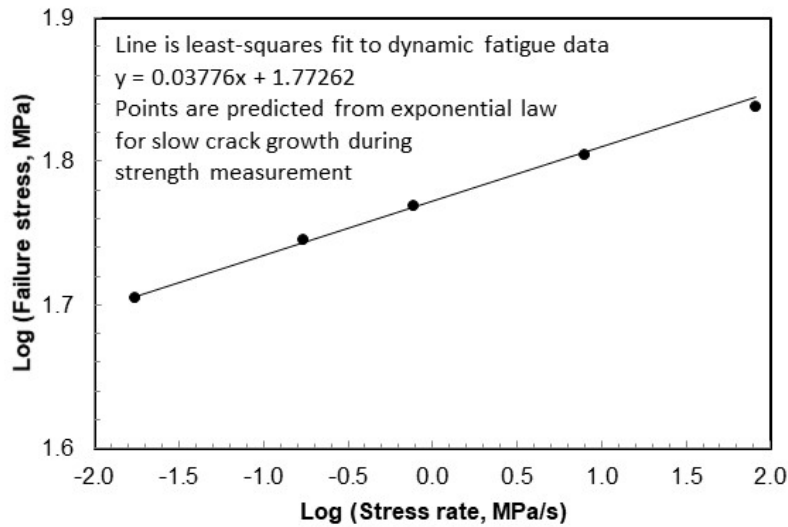


FIGURE 12. Prediction of dynamic fatigue strength from slow crack growth. Line is from best least-squares fit to dynamic fatigue strengths in Figure 11. Points are *predicted* strengths from Equation C-6 in Appendix C using exponential law slow crack growth parameters in Table 9.

WINDOW PROOF STRESS (σ_p) FOR DESIRED DESIGN LIFE BY NASA STD 5018 PROCEDURE

The proof stress (σ_p) applied before a window is put into service establishes the design life for the window. Our procedure is to insert a trial proof stress σ_p into Equation 2 to find the depth a of the largest flaw that can survive the proof test. For a proof test, we use $K_{Ic} + 1$ standard deviation $\approx 0.74 + 0.05 = 0.79 \text{ MPa}\sqrt{\text{m}}$ on the left side of Equation 2. We do not know what value of the uncertainty factor U to use in Table 1 until we calculate the design life in a later step. To start the calculation, we guess a value of U and then later adjust it to the correct value based on the calculated design life.

The aircraft window has a maximum tensile stress $\sigma_a = 5.12 \text{ MPa}$ for a pressure difference of $\Delta P = 10.3 \text{ psi}$ across the window. For illustration, we select a proof pressure ratio $\sigma_p/\sigma_a = 3.125$, giving a stress of $\sigma_p = (3.125)(5.12) = 16.0 \text{ MPa}$ in Equation 2:

$$K_I = 0.79 \text{ MPa}\sqrt{\text{m}} = U \cdot \sigma_p \cdot (\pi a/Q)^{1/2} = U \cdot (16.0 \text{ MPa}) \cdot (\pi a/Q)^{1/2}$$

Inserting the NASA value $Q = 1$ and guessing $U = 1.2$, we find crack depth $a = 539 \text{ }\mu\text{m}$ as the deepest flaw that could be present and still survive the proof pressure test.

Now we put the window into service at the maximum tensile stress of $\sigma_a = 5.12 \text{ MPa}$. The initial maximum stress intensity factor with a $539 \text{ }\mu\text{m}$ flaw depth is

$$K_I = U \cdot \sigma_a \cdot (\pi a/Q)^{1/2} = (1.2)(5.12 \text{ MPa}) \cdot (\pi(539 \times 10^{-6} \text{ m})/1)^{1/2} = 0.2528 \text{ MPa}\sqrt{\text{m}}$$

The initial crack velocity is computed with Equation 1a using the parameters v_0 and β from the first line of Table 9:

$$v = v_0 \cdot e^{\beta K_I} = 4.27 \times 10^{-18} \cdot e^{54.69 \cdot 0.2528} = 4.31 \times 10^{-12} \text{ m/s}$$

Time to failure is computed by numerical evaluation of the integral in Equation 3:

$$t = \left(\frac{2}{U^2 Y^2 \sigma^2} \right) \int_{K_{Ii}}^{K_{Ic}} \left(\frac{K_I}{v} \right) dK_I = \left(\frac{2}{(1.2)^2 \pi (5.12)^2} \right) \int_{0.2528}^{0.59} \left(\frac{K_I}{v} \right) dK_I = 1.94 \times 10^7 \text{ s}$$

with $\sigma = 5.12 \text{ MPa}$, and the limits of integration $K_{Ii} = 0.2528 \text{ MPa}\sqrt{\text{m}}$ and $K_{Ic} = K_{Ic} - 3$ standard deviations in $K_{Ic} = 0.74 - 3(0.05) = 0.59 \text{ MPa}\sqrt{\text{m}}$. Numerical integration gives $t = 1.94 \times 10^7 \text{ s} = 5387 \text{ h}$. The NASA design life is one fourth of the time to failure $= (5387 \text{ h})/4 = 1347 \text{ h}$. In Table 1, we see that $U = 1.2$ for design life in the range 720–8760 h, so we guessed the right value of U . If we had guessed the wrong value, we would repeat the calculation from the beginning with the correct value of U .

We find that a proof pressure ratio $\sigma_p/\sigma_a = 3.125$ provides a design life of 1,347 h. We can then vary the proof pressure ratio until we obtain a desired design life, or until we reach the maximum proof stress that we are willing to risk.

Table 10 gives design life computed with the exponential slow crack growth rate law Equation 1a prescribed by NASA STD 5018. For comparison, Table 11 shows *20 to 60 times longer* design life computed with the power law slow crack growth Equation 2a. Both crack growth laws give similar crack growth rates at stress intensities of $K_{Ic} > 0.4 \text{ MPa}\sqrt{\text{m}}$. In the absence of knowing which crack growth law is valid at low stress intensity factor for zinc sulfide, we choose the more conservative exponential law.*

The exponential law in Table 10 gives a design life of 1,300 to 2,900 h for proof stresses of 16 to 17 MPa. However, if we add one standard uncertainty to the slope of the dynamic fatigue graph in Figure 11, the design life decreases to 200 to 400 h for proof stresses of 16 to 17 MPa in the lower half of Table 10. Design life is highly sensitive to the uncertainty of the slope in the dynamic fatigue measurements in Figure 11.

Tables 12 and 13 show the effect of changing the geometric factor Y for crack stress intensity factor in Equations 2 and 3. The value $Y = \sqrt{\pi}$, which was used for Tables 10 and 11, is specified by NASA STD 5018 and corresponds to a long, shallow crack shape. For Tables 12 and 13, we changed the value to $Y = 2/\sqrt{\pi}$, which corresponds to a semicircular flaw shape. The semicircular flaw shape is often observed at failure origins after a coupon has been broken.

For the exponential crack growth law Equation 1a and for the power law Equation 1b, decreasing Y from $\sqrt{\pi}$ to $2/\sqrt{\pi}$ increases the design life by approximately a factor of 2.5. Compare Table 12 with Table 10 and compare Table 13 with Table 11 to see this effect on design life.

* We do not know which slow crack growth law (1a) or (1b) applies to zinc sulfide at low stress intensity. In a study of four different materials by static fatigue, two obeyed the exponential law at low stress intensity, and two obeyed the power law (Reference 16). Static fatigue experiments should distinguish between the power law and the exponential law for a given material. We are not aware of static fatigue measurements on zinc sulfide. In static fatigue, the lifetime of a component subjected to a *constant* stress is measured. Static fatigue can be a time-consuming process.

TABLE 10. Design Life of ZnS Aircraft Window With Exponential Law Crack Growth and Geometric Factor $Y = \sqrt{\pi}$ in Equations 2 and 3.

Proof Pressure Ratio σ_p/σ_a	Proof Stress, MPa	Max Flaw Size From Proof Test, μm	Design Life (h) Exponential Law Parameters v_o and β	U (Eq. 2)	No Crack Growth	With Crack Growth	
					Proof Test Probability of Failure	Min. Required Strength S_i , MPa	Probability of Failure
Design life with best least squares slope fit to dynamic fatigue measurements in Figure 11							
3.000	15.36	498	670	1.3	0.003	20.74	0.015
3.125	16.00	539	1,350	1.2	0.003	21.66	0.019
3.320	17.00	477	2,870	1.2	0.005	23.11	0.028
Design life with least squares slope plus one standard uncertainty for dynamic fatigue in Figure 11							
3.000	15.36	498	140	1.3	0.003	20.26	0.013
3.125	16.00	459	220	1.3	0.003	21.17	0.017
3.320	17.00	407	420	1.3	0.005	22.60	0.025

Notes for both sections of Table. Proof test: loading time 2 seconds (s), dwell 0.5 s, unload 2 s.

K_{Ic} (for proof test flaw size) = $0.79 \text{ MPa}\sqrt{m}$.

K_{Ic} (for crack growth during lifetime and during proof test) = $0.74 \text{ MPa}\sqrt{m}$.

Y (geometric factor) = $\sqrt{\pi}$ for proof test and design life calculations.

Upper table: $v_o = 4.27 \times 10^{-18} \text{ m/s}$; $\beta = 54.69$. Lower table: $v_o = 1.79 \times 10^{-16} \text{ m/s}$; $\beta = 47.06$.

TABLE 11. Design Life of ZnS Aircraft Window With Power Law Crack Growth and Geometric Factor $Y = \sqrt{\pi}$ in Equations 2 and 3.

Proof Pressure Ratio σ_p/σ_a	Proof Stress, MPa	Max Flaw Size From Proof Test, μm	Design Life (h) Exponential Law Parameters v_o and β	U (Eq. 2)	No Crack Growth	With Crack Growth	
					Proof Test Probability of Failure	Min. Required Strength S_i , MPa	Probability of Failure
Design life with best least squares slope fit to dynamic fatigue measurements in Figure 11							
3.000	15.36	696	15,000	1.1	0.003	20.39	0.014
3.125	16.00	641	40,000	1.1	0.003	21.31	0.018
3.320	17.00	568	165,000	1.1	0.005	22.76	0.026

Proof test: loading time 2 s, dwell 0.5 s, unload 2 s.

K_{Ic} (for proof test flaw size) = $0.79 \text{ MPa}\sqrt{m}$.

K_{Ic} (for crack growth during lifetime and during proof test) = $0.74 \text{ MPa}\sqrt{m}$.

$Y = \sqrt{\pi}$ for proof test and design life calculations; Power law parameters: $A^* = 0.07419 \text{ m/s}$; $n = 25.49$.

TABLE 12. Design Life of ZnS Aircraft Window With Exponential Law Crack Growth and Geometric Factor $Y = 2/\sqrt{\pi}$ in Equations 2 and 3.

Proof Pressure Ratio σ_p/σ_a	Proof Stress, MPa	Max Flaw Size From Proof Test, μm	Design Life (h) Exponential Law Parameters v_o and β	U (Eq. 2)	No Crack Growth	With Crack Growth	
					Proof Test Probability of Failure	Min. Required Strength S_i , MPa	Probability of Failure
Design life with best least squares slope fit to dynamic fatigue measurements in Figure 11							
3.000	15.36	1,443	1,940	1.2	0.003	20.10	0.13
3.125	16.00	1,330	3,320	1.2	0.003	21.00	0.016
3.320	17.00	1,178	7,080	1.2	0.005	22.40	0.024
Design life with least squares slope plus one standard uncertainty for dynamic fatigue in Figure 11							
3.000	15.36	1,229	345	1.3	0.003	19.55	0.011
3.125	16.00	1,133	545	1.3	0.003	20.43	0.014
3.320	17.00	1,178	1,218	1.2	0.005	21.81	0.020

Notes for both sections of Table. Proof test: loading time 2 s, dwell 0.5 s, unload 2 s.

K_{Ic} (for proof test flaw size) = $0.79 \text{ MPa}\sqrt{m}$.

K_{Ic} (for crack growth during lifetime and during proof test) = $0.74 \text{ MPa}\sqrt{m}$.

Y (geometric factor) = $2/\sqrt{\pi}$ for proof test and design life calculations.

Upper table: $v_o = 4.27 \times 10^{-18} \text{ m/s}$; $\beta = 54.69$. Lower table: $v_o = 1.79 \times 10^{-16} \text{ m/s}$; $\beta = 47.06$.

TABLE 13. Design Life of ZnS Aircraft Window With Power Law Crack Growth and Geometric Factor $Y = 2/\sqrt{\pi}$ in Equations 2 and 3.

Proof Pressure Ratio σ_p/σ_a	Proof Stress, MPa	Max Flaw Size From Proof Test, μm	Design Life (h) Exponential Law Parameters v_o and β	U (Eq. 2)	No Crack Growth	With Crack Growth	
					Proof Test Probability of Failure	Min. Required Strength S_i , MPa	Probability of Failure
Design life with best least squares slope fit to dynamic fatigue measurements in Figure 11							
3.000	15.36	1,717	38,000	1.1	0.003	19.61	0.011
3.125	16.00	1,582	98,000	1.1	0.003	20.51	0.014
3.320	17.00	1,402	410,000	1.1	0.005	21.90	0.021

Proof test: loading time 2 s, dwell 0.5 s, unload 2 s.

K_{Ic} (for proof test flaw size) = $0.79 \text{ MPa}\sqrt{m}$.

K_{Ic} (for crack growth during lifetime and during proof test) = $0.74 \text{ MPa}\sqrt{m}$.

$Y = 2/\sqrt{\pi}$ for proof test and design life calculations; Power law parameters: $A^* = 0.07419 \text{ m/s}$; $n = 25.49$.

Figure 13 shows the effect on design life when selecting different crack growth laws and different values for Y . The power law gives design lives that are 20 to 50 times greater than design lives predicted with the exponential law. Selecting Y to be $2/\sqrt{\pi}$ instead of $\sqrt{\pi}$ increases design life by approximately a factor of 2.5.

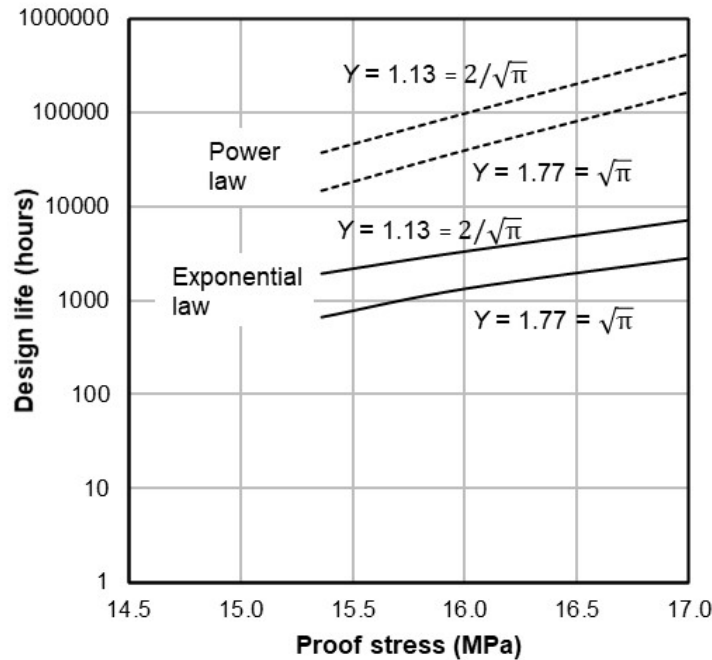


FIGURE 13. Design life of multispectral zinc sulfide window calculated with either the exponential crack growth law Equation 1a or the power law Equation 1b. In each case, the geometric factor was chosen to be either $\sqrt{\pi} = 1.77$ corresponding to a shallow linear flaw or $2/\sqrt{\pi} = 1.13$ corresponding to a semicircular flaw.

There is no “right” value for Y , but the choice of the semicircular flaw shape with $Y = 2/\sqrt{\pi}$ could be more commonly observed than the linear flaw shape with $Y = \sqrt{\pi}$. NASA specifies designing with the more conservative value $Y = \sqrt{\pi}$.

We do not know which crack growth law (exponential or power) applies to zinc sulfide at low stress intensities governing design life. Again, NASA specifies using the more conservative choice, which is the exponential law. If we knew that zinc sulfide follows the power law at low stress intensity factor, we would choose the power law.

MINIMIZING PROBABILITY OF WINDOW FAILURE IN PROOF TEST

A large zinc sulfide window is expensive and the lead time for procurement is not less than 2 years. We do not want to risk breaking the window in the proof test.

A schematic proof pressure profile was shown in Figure 9. The proof pressure profile for Tables 10 through 13 has a loading time of 2 s, a dwell time at peak pressure of 0.5 s, and an unloading time of 2 s. (These times are different from what was modeled for fused silica in Figure 9.) A window must be rigorously dried in the proof pressure chamber prior to testing so that crack growth is minimal during the test, which should be conducted as rapidly as feasible. The proof pressure apparatus should be calibrated by pressurizing an aluminum surrogate window and measuring strain to show that pressure can be applied accurately and rapidly in a controlled manner without overshooting the pressure.

It might be difficult to achieve a controlled 2 s load time and a 0.5 s dwell time at peak stress. Therefore, we calculated the minimum strength required to survive the proof test if dwell time at peak stress is increased from 0.5 to 1 to 2 s. Table 14 shows the probability of failure in the proof test with worst-case crack growth if the window were exposed to water during the test. The probability of failure in the proof test increases from 2.8% to 3.7% as the time at peak stress increases from 0.5 to 2 s. It is clearly desirable to minimize the time at peak stress.

TABLE 14. Effect of Dwell Time at Peak Proof Stress on the Minimum Initial Strength of Zinc Sulfide Required to Survive the Proof Test.

Time at Peak Stress, s	Proof Stress, MPa	Design Life, h	Minimum Initial Strength Required to Survive Proof Test S_i , MPa	Probability of Failure in Proof Test, %
0.5	17.00	2870	23.11	2.8
1.0	17.00	2870	23.60	3.2
2.0	17.00	2870	24.14	3.7

Parameters for calculations are the same as in upper half of Table 10.

V. BK7 GLASS WINDOW DESIGN WITH 3-PARAMETER WEIBULL EQUATION

INTRODUCTION

Large BK7 glass windows have flown several thousand flight hours without incident for 30 years in the Navy P-3 aircraft. Large BK7 windows designed by Fuller et al. (Reference 3) have flown for many years on Gulfstream jet aircraft. Our present analysis combines the NASA STD 5018 design rules with additional insight from the 3-parameter Weibull equation for BK7 to achieve a safe design including the essential proof pressure test.

Early in our BK7 window design work, we used Weibull strength parameters derived from glass with a “commercial polish” that had a mean strength of just 68 MPa. In the present analysis, we adopt the Fuller (Reference 3) data set obtained from well-finished BK7 coupons with a mean strength of 212 MPa. In order to take advantage of the high strength of the polished face of a window, the edges must be finished well enough so that they are not the source of failure of the window. For this purpose, window edges must be fine ground and etched with hydrofluoric acid to remove edge damage. We expect that with high strength on the face of the window and carefully finished edges, windows will have a long design life and will tolerate a high proof stress without failure.

Our BK7 analysis is divided into two main sections. One analysis uses a 3-parameter Weibull equation with a threshold stress below which failure does not occur, but incorporates a proof test as well. The other analysis uses NASA STD 5018 design rules with design life based on proof testing. Probability of failure in the proof test is calculated with a 2-parameter Weibull equation, which has no lower threshold stress for failure.

WINDOW DIMENSIONS AND BK7 GLASS MECHANICAL PROPERTIES

Dimensions of the BK7 aircraft window are the same as in Figure 3, and the design pressure difference across the window is $\Delta P = 10.3 \text{ psi} = 0.710 \text{ bar}$. Mechanical properties of BK7 glass are listed in Table 15.

TABLE 15. Mechanical Properties of BK7 Glass.

	Schott BK7 Glass
Young's modulus E (GPa)	82
Poisson's ratio ν	0.21
Fracture toughness:	
K_{Ic} (MPa√m) ^a	0.94, 0.92
Standard deviation in K_{Ic}	0.04, 0.03
K_{Ic} (MPa√m) ^b	0.98 (0.04)
K_{Ic} (MPa√m) chosen for this analysis	0.95 (0.04)
K_{Ic} (MPa√m) for flaw depth calculation Equations 2 and 2a ^c	0.99

^aReference 5.^bReference 6.^c K_{Ic} for flaw depth calculation in Equation 2 = Average K_{Ic} + 1* standard deviation = 0.95 + 0.04 = 0.99 MPa√m.

3-PARAMETER WEIBULL EQUATION WITH LOWER-LIMIT THRESHOLD FOR STRENGTH

A common version of the Weibull equation (Reference 17) that describes observed distributions of strength of many glass and ceramic materials has two adjustable parameters:

Weibull equation with two parameters m and σ_θ

$$P_f = 1 - e^{-\left(\frac{\sigma}{\sigma_\theta}\right)^m} \quad (25)$$

where P_f is the probability of failure at stress σ . The two parameters adjusted to fit the observed strength distribution are the Weibull modulus m and the *characteristic strength* σ_θ , as shown at the left side of Figure 14. We often recast the equation into the form

Weibull equation with Weibull scale factor σ_o

$$P_f = 1 - e^{-\left(\frac{A_e}{A_o}\right)\left(\frac{\sigma}{\sigma_o}\right)^m} \quad (26)$$

in which A_e is the effective surface area in tension (Appendix E), A_o is taken as 1 cm², and σ_o is called the *Weibull scale factor*. The relation between σ_o and σ_θ is

$$\sigma_o = \sigma_\theta \left(\frac{A_e}{A_o}\right)^{1/m} \quad (27)$$

In 2-parameter Weibull Equations 25 and 26, m is related to the breadth of the distribution: a narrow distribution gives large m . The characteristic strength and scale factor are related to mean strength, with higher σ_θ or σ_o corresponding to high strength.

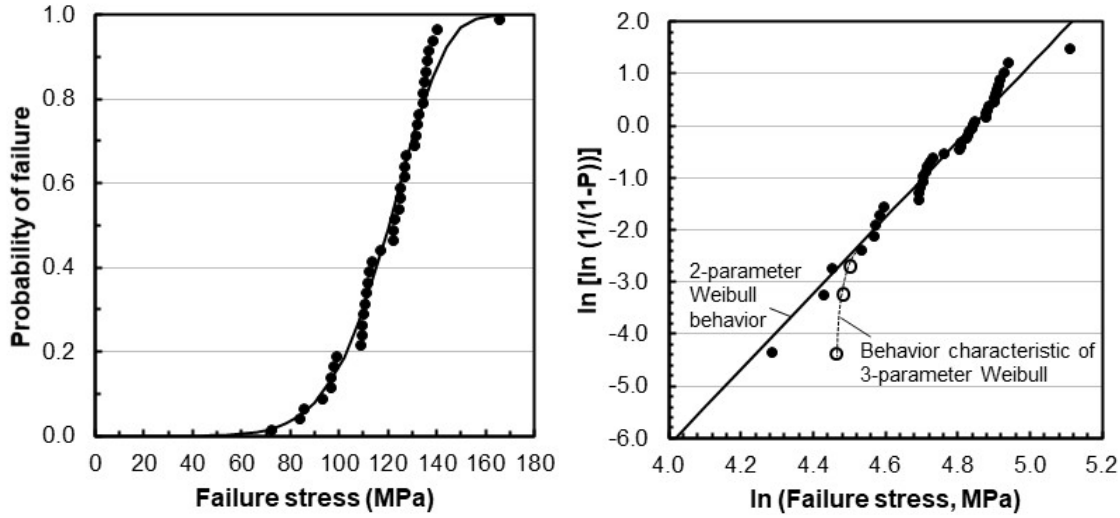


FIGURE 14. Example of 2-parameter Weibull behavior. *Left:* Strengths of 40 disks of transparent yttria fit to Equation 25 with $m = 7.04$, and $\sigma_\theta = 126.5$ MPa. *Right:* Same data in logarithmic plot of Equation 28. Three open circles are fictitious data whose behavior deviates from the straight line and suggest that there could be a threshold stress below which failure does not occur.

Equation 25 can be cast into the linear form in Equation 28:

Linear form of Weibull Equation 25

$$\ln \left[\ln \left(\frac{1}{1-P_f} \right) \right] = m \cdot \ln \sigma - m \cdot \ln \sigma_\theta \quad (28)$$

A graph of $\ln \left[\ln \left(\frac{1}{1-P_f} \right) \right]$ versus $\ln \sigma$ is fit by a straight line with slope m and intercept $-m \cdot \ln \sigma_\theta$. Data from Figure 14 (left) are plotted in linear form with Equation 28 at the right side of Figure 14. Conformance to the straight line tells us that Equation 25 describes the data.

For some materials, the low-strength points in the panel at the right side of Figure 14 deviate from the straight line and approach a lower limit below which failure is not observed. We call this limit the threshold stress, σ_t . BK7 glass strengths in Figure 15 exhibit threshold strength behavior and can be fit to the empirical Equation 29:

3 – Parameter Weibull equation with threshold strength σ_t

$$P_f = 1 - e^{-\left(\frac{\sigma - \sigma_t}{\sigma_\theta} \right)^m} \quad (29)$$

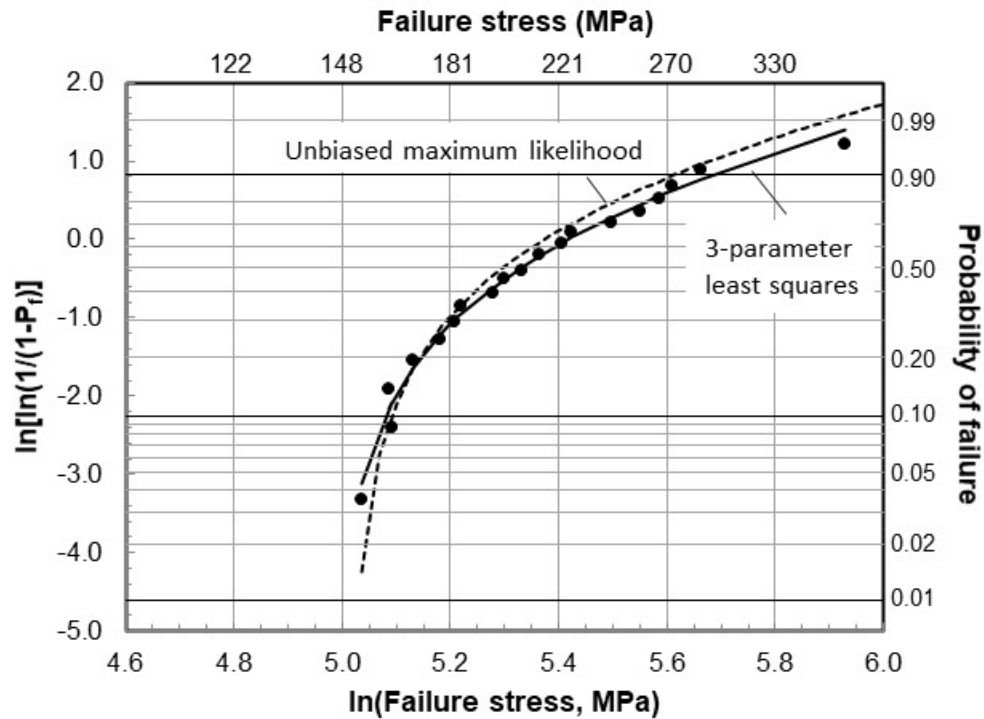


FIGURE 15. BK7 glass “D64 & Etched” strength from Schott AG. Note logarithmic axes as in the right side of Figure 14. Points lie on a straight line if they obey the 2-parameter Weibull Equation 25. The curves conform to 3-parameter Weibull Equation 29, with a threshold strength $\sigma_t = 150$ MPa. “D64 & Etched” coupons were coarse ground with a bonded diamond tool with $64\ \mu\text{m}$ maximum grain size and then etched to remove the ground surface and subsurface flaws. Only small flaws remain after etching. Weibull parameters are derived in Figure D-3 in Appendix D. (Data from Figure 9 of Reference 18. Weibull parameters derived by Dan Harris. Weibull parameters listed in Table 1 of TIE-33 are not correct.)

Figure 16 shows BK7 glass strength from Fuller (Reference 3). The left-hand graph with linear axes gives no hint of a threshold strength. Logarithmic axes in the right-hand graph would give a straight line for 2-parameter Weibull behavior. Instead, we see curvature suggesting there is a lower threshold strength near $e^{4.8} \approx 120$ MPa. Table 16 lists Weibull parameters for BK7 glass with various surface conditions. Figure 17 plots the probability of survival ($= 1 - \text{probability of failure}$) for four distributions in Table 16.

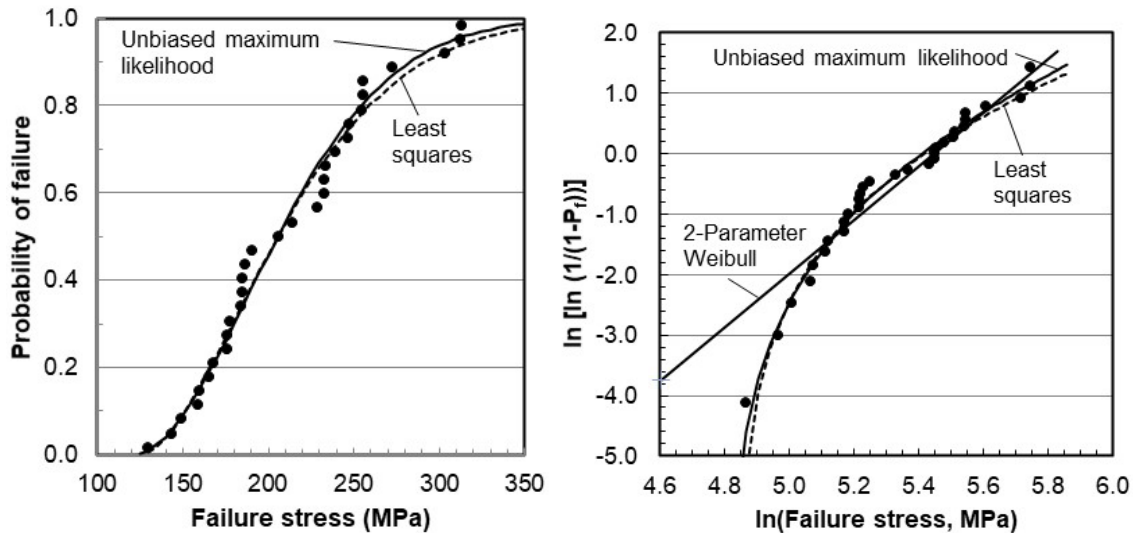


FIGURE 16. *Left:* Inert strengths of 31 polished BK7 glass disks fit to 3-parameter Weibull Equation 29 by method in Appendix D. Inert strength was measured in a dry nitrogen atmosphere with a load rate of 445 N/s. *Right:* When plotted with logarithmic axes of Equation 28, curvature suggests there is a threshold strength. (Data from Reference 3.)

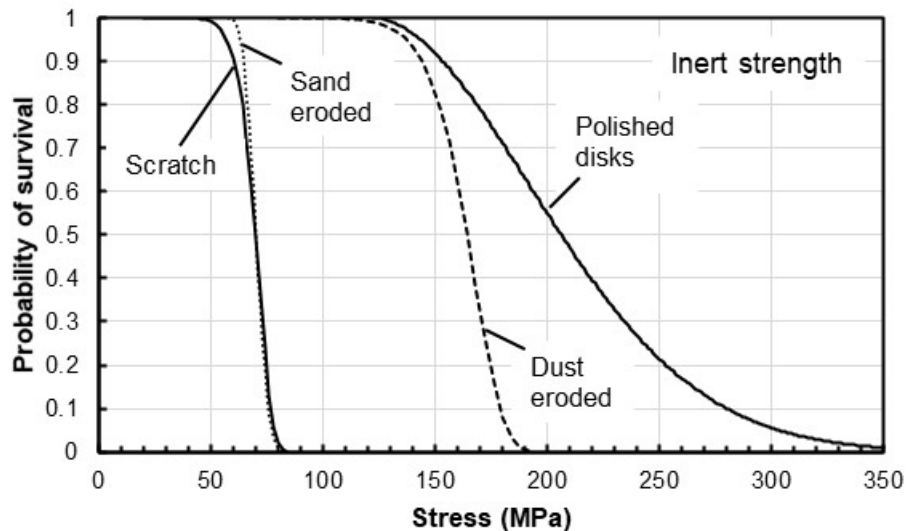


FIGURE 17. Three-parameter Weibull inert strength distributions for equibiaxial flexure test disks with Fuller (Reference 3) parameters from Table 16.

TABLE 16. Weibull Parameters in Equation 29 for Inert Strength of BK7 Glass in Figures 16, 17, and 15 With Different Surface Conditions.

Surface Condition	Threshold Strength		Weibull Exponent m
	σ_t , MPa	σ_θ , MPa	
Fuller as-polished ^a	121.7	102.3	1.91 (1.83 unbiased)
Fuller dust-eroded ^b	20	148	12.4
Fuller sand-eroded ^c	57	14	3.3
Fuller scratched ^d	0	72	12.8
Schott D64 coarse grind & etched ^e	151.8	63.7	1.36 (1.26 unbiased)

^aFor the Fuller study, 76.2-mm-diameter glass disks with a thickness of 5.3 mm were finished flat to within a wedge of <38 μm . Flat faces had <2 waves of irregularity at 0.63 μm with 80/10 scratch dig surface quality. Three steps of controlled grinding removed at least 203, 91, and 38 μm with abrasive particle sizes of 30, 12, and 5 μm , respectively. Edges were beveled with a 320 or finer grit diamond tool and etched with 40% HF for 15 min. (From Reference 3.)

^bErosion with dust at 234 m/s at impact angle of 15 degrees from specimen surface.

^cErosion with sand at 29 m/s at impact angle of 90 degrees from specimen surface.

^dScratched with Vickers diamond indenter to leave a single scratch 20 to 25 mm long and 50 to 100 μm wide near the center of the disk.

^eSchott data were digitized from Figure 9 of Schott Glass Technical Information Document TIE-33 (Reference 18). Weibull parameters derived by Dan Harris. Weibull parameters for BK7 in Table 1 of TIE-33 are not correct.

OUTLIERS HAPPEN

Peter Hartmann of Schott AG argues in support of designing with a threshold strength if such a threshold is observed (Reference 2). Schott measured strengths of statistically relevant numbers (100 or more) of test coupons to observe a lower limit threshold for strength, when that limit exists (References 18, 19, 20). Figures 15 and 16 (*right*) show curvature expected for the 3-parameter Weibull equation with a threshold strength for BK7 glass. These data sets have only 19 and 31 coupons, but the curvature is sufficiently pronounced to make the threshold evident.

Hartmann states that “with only 20 specimens ... differentiation [between 2-parameter and 3-parameter Weibull behavior] is not possible. This requires 100 specimens per set, the more the better” (Reference 20). It is generally prohibitively expensive to test such large numbers of coupons. Also, results retain some ambiguity because outliers may need to be removed from the data for legitimate reasons. We will always be uncertain of whether one particular window machined in the same manner as the test coupons is weaker than the threshold strength for the coupons.

Hartmann provides several specific examples of outliers observed in the preparation of large groups to coupons from ZERODUR® glass ceramic and explains the origins of the outliers (Reference 20). In one case, five low-strength outliers were observed in a set

of ~90 coupons that were ground as a group on a single platen. The outliers all came from one edge region in which the grinding tool entered the plate from outside the plate. When outliers were removed from the data set, the remaining coupons conformed to a clear 3-parameter Weibull distribution with a threshold strength of 68 MPa. Four of the five weak outliers had strengths in the range 50 to 60 MPa—well below the “threshold” strength. Another cause of an outlier was an abrasive particle breaking off from the grinding tool and creating a scratch as the particle was dragged along the smooth surface of the work piece.

A conclusion we draw from these examples is that any given window could have undetected damage that makes it weaker than the threshold strength derived from a large set of coupons machined in an “identical” manner to the window. We can never have full confidence that any particular window must be stronger than the “statistically sound” threshold strength derived from coupon tests.

HARTMANN’S DESIGN PHILOSOPHY FOR MATERIAL WITH A LOWER THRESHOLD FOR STRENGTH

Hartmann states that “for surfaces generated mechanically with grinding tools consisting of many small diamonds suspended in a brass carrier,” a “maximum crack depth must exist” (Reference 2). There is evidence for this statement with ZERODUR® glass ceramic and ULE® glass (Reference 21). If a maximum flaw depth (a) exists, we infer from fracture mechanics that there will be a minimum strength (σ):

$$\begin{array}{l} \text{Relation of strength} \\ \text{to flaw depth} \end{array} \qquad K_{Ic} = \sigma Y \sqrt{a} \qquad (30)$$

In Equation 30, K_{Ic} is the fracture toughness of the glass and Y is a geometric factor in the approximate range 1 to 2.

Hartmann proposes a “deterministic method to calculate lifetime or design strength based on the use of the experimentally determined minimum breakage stress σ_t for well defined surface conditions.”

- The glass is considered to begin its service life with a strength no lower than the threshold strength σ_t measured on coupons ground and polished in the exact same manner as a full-size window.
- From the threshold strength, Equation 30 tells us the depth of the largest flaw in the material.
- Using measured worst-case slow crack growth parameters for the glass in 100% humidity or immersed in water, the lifetime of the window is calculated as the time required for the deepest flaw to propagate to failure if the window is held at the design stress.

- The threshold strength of the window is independent of the size of the window, as long as the full-size window can be finished in an identical manner to test coupons. We hope that the depth of subsurface damage in the window should be the same as in the test coupons.

We now show how to use Hartmann's suggestions to find the design life for a BK7 aircraft window. We recognize that we cannot blindly assume that the full-size window has the same maximum flaw depth as the test coupons. Our procedure will combine conservative application of Hartmann's design philosophy with proof testing guidelines from NASA STD 5018 to ensure the safety of the window.

FLAW DEPTH RANGE IN BK7 GLASS

Figure 18 relates strength to flaw depth for BK7 glass. Measured strengths of BK7 in Figures 15 and 16 are roughly in the range 300 to 150 MPa, corresponding to flaw depths of 3 to 13 μm . Observed threshold strengths (σ_t) of BK7 in the range 150 to 20 MPa in Table 16 correspond to maximum flaw depths of 13 to 700 μm . For optically polished BK7, the threshold strength near 120 MPa corresponds to a maximum flaw depth of 20 μm .

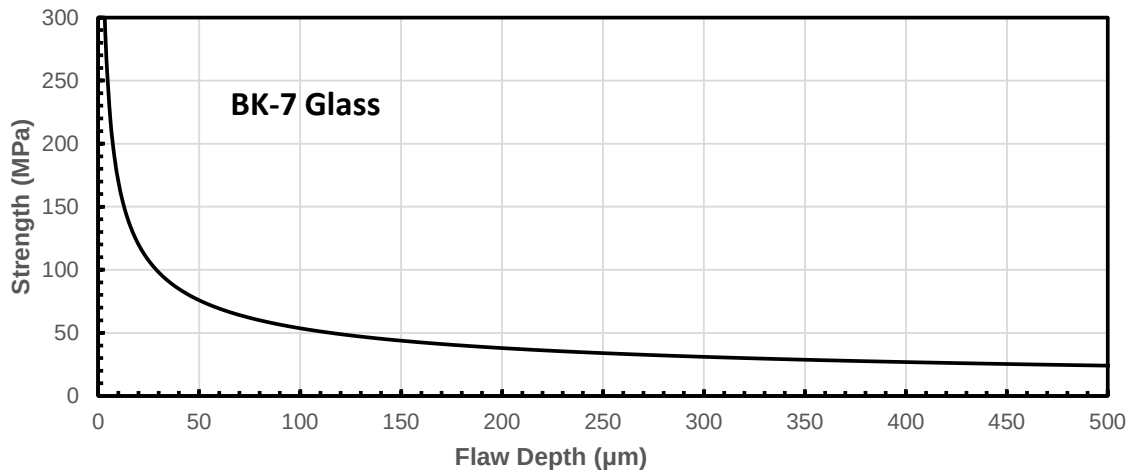


FIGURE 18. Strength versus flaw depth from Equation 30 with $K_{Ic} = 0.95 \text{ MPa}\sqrt{m}$, and $Y = \sqrt{\pi}$.

Lower threshold strengths than 120 MPa are observed in Table 16 for coupons that have been intentionally damaged by dust or sand erosion or scratching that could be encountered in service. We assume that significant erosion damage or scratches will be observed by visual inspection after each flight and that a damaged window will be removed from service or refinished and recertified with a proof pressure test before it is allowed back into service. Thorough inspection of a window after it has flown and before it is allowed to fly again is an integral part of NASA's protocol. NASA STD 5018 states that inspection must be able to detect flaws that have a visible depth of at least 15 μm , which NASA infers corresponds to an actual depth of 45 μm . Experience at NASA is consistent with the lower 2/3 of a flaw being invisible by inspection. In Figure 18, a flaw depth of 45 μm corresponds to a strength of 80 MPa for BK7 glass. That is, a NASA inspection is supposed to be able to observe a 15 μm visible flaw depth corresponding to an actual depth of 45 μm , which would lower the strength of BK7 glass to 80 MPa.

BK7 WINDOW DESIGN USING 3-PARAMETER WEIBULL EQUATION WITH THRESHOLD STRENGTH

Step 1: Measure threshold strength and other Weibull parameters. It is implicit in this design method that enough coupons must be studied to show that the strength distribution conforms to the 3-parameter Weibull Equation 29. Prepare biaxial strength test coupons with a procedure and tools that replicate those which will be used to make the full-size window. Biaxial flexure coupons with a large polished face are the most natural shape that can be polished in a similar manner to a full-size window. It is implicit in making the full-size window that edges must be finished well enough so that they are not the source of failure of the window. Edge behavior cannot be readily replicated in coupons. Coupon failures originating at edges are identified by inspection of the fracture pattern and omitted from the data set.

For our window design, we use data from 31 test coupons in Figure 16. Appendix D shows how to extract the following parameters for Weibull Equation 29 from the raw data:

$$\sigma_t = 121.7 \text{ MPa} \quad \sigma_\theta = 102.3 \text{ MPa} \quad m = 1.91 \text{ (1.83 unbiased)}$$

Step 2: Establish design service stress (σ_a) for full-size window. For the dimensions of the aircraft window in Figure 3 with the maximum design pressure of 10.3 psi, Equation 4 tells us that the maximum service stress is

$$\text{BK7 glass maximum service stress } \sigma_a = 4.90 \text{ MPa}$$

Step 3: Select design threshold strength. Although the threshold strength was measured from coupons in step 1, we apply a safety factor to the threshold strength for design purposes. With a threshold strength of 121.7 MPa and a service stress of just 4.90 MPa, we can afford to use a comfortable safety factor of 2 in the threshold strength. That is, we will knock down the threshold strength from 121.7 MPa to $\frac{1}{2}(121.7) \approx 60$ MPa.

Step 4: Calculate size of largest initial flaw. NASA STD 5018 for window design prescribes that Equation 2 be used for the relationship between stress intensity factor, stress, and flaw depth. The critical stress intensity factor K_{Ic} on the left side of Equation 2 shall be

$$K_{Ic} = K_{Ic(\text{average})} + 1 \text{ standard deviation in } K_{Ic} = 0.95 + 0.04 = 0.99 \text{ MPa}\sqrt{\text{m}}$$

with K_{Ic} taken from Table 15. From the threshold strength $S = 60$ MPa (after applying a safety factor of 2), Equation 31 gives us the deepest flaw depth (a) in the glass:

$$\begin{aligned} K_{Ic} &= U \cdot S \cdot (\pi a / Q)^{1/2} \\ 0.99 \text{ MPa}\sqrt{\text{m}} &= 1.1 \cdot 60 \text{ MPa} \cdot (\pi a / 1)^{1/2} \Rightarrow a = 71.6 \text{ }\mu\text{m} \end{aligned} \quad (31)$$

For design, we set the depth of the largest flaw in the window at 71.6 μm prior to service.

Step 5: Calculate time to failure for largest flaw. For service stress $\sigma_a = 4.90$ MPa, the stress intensity factor K_{Ii} at the tip of the deepest flaw is

$$K_{Ii} = U\sigma(\pi a / Q)^{1/2} = 1.1 \cdot 4.90 \text{ MPa} \cdot [\pi \cdot (71.6 \times 10^{-6} \text{ m}) / 1]^{1/2} = 0.08084 \text{ MPa}\sqrt{\text{m}}$$

The time for the flaw to propagate to failure at constant stress σ_a is

$$\text{Time to failure (t) static fatigue life:} \quad t = \left(\frac{2Q}{U^2 \pi \sigma_a^2} \right) \int_{K_{Ii}}^{K_{Ic}} \left(\frac{K_I}{v} \right) dK_I \quad (32)$$

where the upper limit of integration is $K_{Ic(\text{average})} - 3 \text{ standard deviations} = 0.95 - 3(0.04) = 0.83 \text{ MPa}\sqrt{\text{m}}$ with numbers from Table 15.

Table 17 gives slow crack growth parameters for BK7 glass for the exponential law on lines 1 and 2 and for the power law on line 3. The two lines for the exponential law apply to faster and slower ranges of crack growth velocities. Figure 19 shows observed crack growth velocities, as well as plots from the three lines Table 17.

TABLE 17. Slow Crack Growth Parameters for BK7 Glass in 100% Humidity.

Exponential Law ^a		$K_I = a + b \ln(v)$		$v = v_0 e^{\beta K_I}$	
		a (MPa√m)	b (MPa√m)	v_0 (m/s)	β (1/[MPa√m])
1. BK7 glass ^b	$v > 10^{-6.5}$ m/s	0.7946	0.0253	2.291×10^{-14}	39.53
2. BK7 glass ^b	$v < 10^{-6.5}$ m/s	0.5826	0.01177	3.184×10^{-22}	84.96
Power law (coupons in distilled water)		$v = A^* \left(\frac{K_I}{K_{Ic}} \right)^n$		$K_{Ic} = 0.95 \text{ MPa}\sqrt{\text{m}}$	
3. BK7 glass ^c	$v > 10^{-8}$ m/s	$A^* = 6 \text{ m/s}$		$n = 19.86$	

^aThe two equations $K_I = a + b \ln(v)$ and $v = v_0 e^{\beta K_I}$ are equivalent. $K_I = a + b \ln(v)$ is convenient for fitting experimental crack growth data and $v = v_0 e^{\beta K_I}$ is convenient for lifetime calculations.

^bData from Reference 4. BK7 data were later fit to two lines instead of one in Reference 8.

^cPower law assuming $Y = 1.28$ derived by Dan Harris from unpublished Vickers-indented biaxial flexure disk dynamic fatigue experiments from Reference 3. Data preserved and provided to us by E. R. Fuller and G. D. Quinn 25 years after the experiments were done.

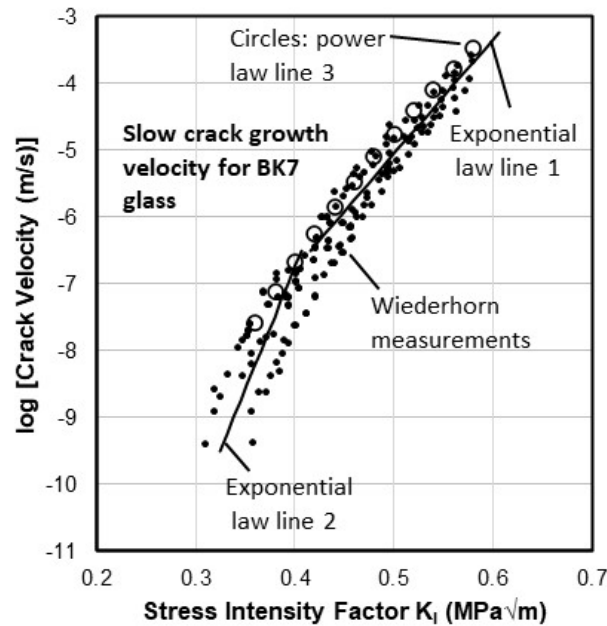


FIGURE 19. BK7 slow crack growth in humid environment. (Data from Reference 4). Lines and circles are from equations in Table 17. Circles calculated with $Y = 1.28$.

Next, we substitute the exponential law from Table 17 line 2 into the integrand of Equation 32 and carry out a numerical integration to find the time to failure for the largest flaw $t = 7.81 \times 10^{13} \text{ s} = 2.48 \times 10^6 \text{ y}$. We predict that it will take more than 2 million years for the crack to grow to failure at a constant service stress of $\sigma_a = 4.90 \text{ MPa}$.

Step 6: Calculate design life. NASA STD 5018 defines the design life as $\frac{1}{4}$ of the time to failure:

$$\text{Design life} = \frac{1}{4} (\text{time to failure}) = \frac{1}{4} (2.48 \times 10^6 \text{ y}) = 6.19 \times 10^5 \text{ y}$$

The line in Figure 20 shows that design life is a strong function of service stress σ_a . Not shown on the graph, the design life would increase by a factor of 17.5 if we doubled the assumed threshold strength from 60 to 120 MPa for a design stress of $\sigma_a = 4.90$ MPa. Three square points on the graph are window lifetimes calculated by Fuller (Reference 3) with a procedure substantially different from ours. His predicted lifetimes are roughly similar to ours at the same service stress.

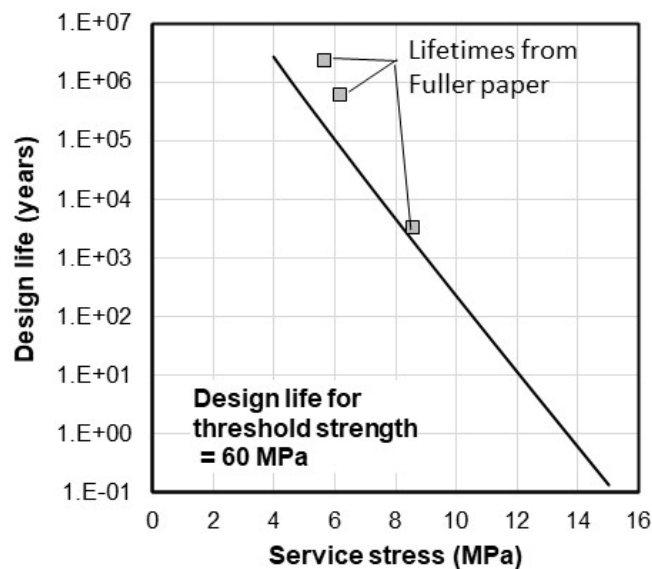


FIGURE 20. Dependence of design life on service stress for BK7 window assuming threshold strength is 60 MPa ($= \frac{1}{2}$ of measured σ_t). Squares are window lifetimes from Fuller (Reference 3).

In summary, the Hartmann design approach accepts (with caution) that the finished window has a threshold strength below which failure does not occur. We applied a safety factor of 2 to the threshold strength. The window lifetime is the time it will take the deepest flaw—whose depth is computed from the threshold strength—to propagate to failure when subjected to the service stress. NASA sets the design life to be one quarter of the calculated time to failure.

PROOF PRESSURE TEST TO VERIFY MINIMUM STRENGTH OF A WINDOW

A proof pressure test is a critical step to demonstrate the airworthiness of an individual window before it should be allowed to fly. The proof pressure test ensures that there are no

flaws in the window deeper than deepest flaw that would fail in the proof test. With the concept of a lower failure threshold in the 3-parameter Weibull equation, the proof test is not required to establish the size of the deepest flaw. The failure threshold tells us the depth of the deepest flaw. If the failure threshold is high, the deepest flaw could be smaller than the size we could certify with a proof pressure test because the pressure would need to be greater than what we are willing or able to apply. BK7 with a threshold strength of 120 MPa (or knocked down to 60 MPa) fits this description.

NASA STD 5018 sets the minimum proof stress σ_p to be at least three times the service stress σ_a . We follow this practice to make sure that no outlying window goes into service. With expectations from the threshold strength that the maximum flaw depth in the window is smaller than what would be discovered by the proof stress, the probability of failure of the window in the proof test is negligible.

The BK7 window suffers imperceptible damage during the proof test if its initial strength is actually 60 MPa ($= \frac{1}{2}$ of threshold strength σ_t). With an initial strength $S_i = 60$ MPa, numerical integration of Equation 12 predicts a strength loss $S_i - S_f = 0.0002$ MPa in the 14.7 MPa proof test for the proof pressure profile in Figure 9. If we raised the proof test stress to $\sigma_p = 5\sigma_a = 24.5$ MPa, strength loss in the proof test would be only 0.09 MPa. If the initial strength were only $S_i = 40$ MPa, strength loss in the 14.7 MPa proof test would be just 0.006 MPa. Estimates of strength loss are conservative because crack growth parameters are measured at 100% humidity, whereas the proof test is conducted in a dry atmosphere in which crack growth is expected to be orders of magnitude slower.

When threshold strength behavior is demonstrated in coupons, we can pursue designs that were not previously feasible because of an overly high expected probability of failure in the proof test. Manufacturing defects might still occur, but they can be screened out by a proof test.

AREA SCALING OF STRENGTH FOR 3-PARAMETER WEIBULL EQUATION

For the 2-parameter Weibull Equation 25, the effective area in tension can be incorporated to give Equation 26. With the 2-parameter Weibull distribution, a necessary consequence is that the greater the effective area of a window, the lower is its expected mean strength because there is a greater probability of finding a critical flaw in a larger area.

For the 3-parameter Weibull Equation 29, there is a threshold strength σ_t below which the probability of failure of the window is (ideally) zero, regardless of the size of the window. That is, a large window is expected to have the same threshold strength as small test coupons if the window and the coupon can be finished in a similar manner.

The sensitivity to area scaling for the 3-parameter Weibull distribution is expressed in Equation 33 taken from Fuller (Reference 3):

3 – Parameter Weibull
equation with area scaling

$$P_f = 1 - e^{-k \left(\frac{\sigma - \sigma_t}{\sigma_\theta} \right)^m} \quad (33)$$

where k is the effective area defined in an analogous manner to A_e for the 2-parameter Weibull equation:

Effective area:

$$k = \frac{1}{A_o} \int \left(\frac{\sigma - \sigma_t}{\sigma_\theta} \right)^m dA \quad (34)$$

The factor k is expressed in multiples of the effective area of test coupons, which is A_o in Equation 34. It is possible that the stress distribution in the test coupon will be different from that of the large window. The integral in Equation 34 accounts for that difference.

In their aircraft window design study, Fuller (Reference 3) noted that “the scaling factor, k , is extremely difficult to estimate accurately.” However, they were able to use plausible magnitudes such as $k = 30$ and $k = 50$ for different windows to make approximate predictions. Figure 21 shows that area scaling to $k = 50$ and $k = 100$ sharply narrows the range of strength above the threshold strength of $\sigma_t = 122$ MPa. In the limit of large k , all windows fail at stress = σ_t . The expected mean strength of small windows is greater than the expected mean strength of large windows because small windows have a significant population above σ_t in Figure 21. Large windows have little population above σ_t . The key point for predicting minimum window lifetime is that neither small nor large windows have an initial strength less than σ_t .

Although the minimum strength of a window is expected to be independent of window diameter, design life decreases with increasing window diameter because tensile stress in a window increases with window diameter if the applied pressure is constant. Slow crack growth rate, which determines window design life, increases as the tensile stress increases.

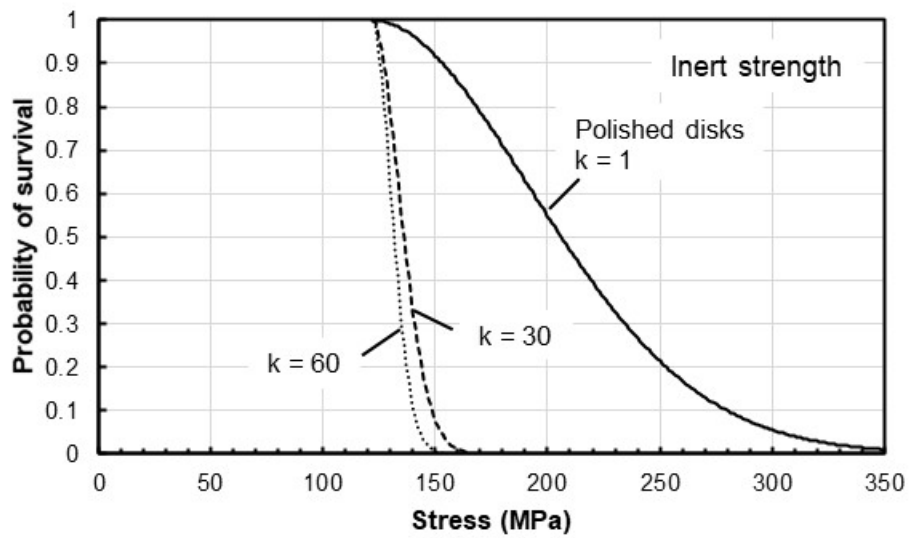


FIGURE 21. Three-parameter Weibull inert strength distributions calculated for large windows with $k = 30$ and $k = 60$. The value of k is (effective area of window)/(effective area of 3-inch test disk). Inert strength is measured in a rigorously dry atmosphere with a rapid load rate of 445 N/s to minimize slow crack growth.

VI. BK7 GLASS WINDOW DESIGN WITH 2-PARAMETER WEIBULL EQUATION

Until now, we accepted that there exists a threshold stress for BK7 below which it does not fail. The physical basis for a threshold stress is that there is a maximum flaw depth resulting from grinding and polishing of the window and coupons and they are assumed to be equal.

Now we take a different approach that does not postulate a threshold stress. Instead, we rely on a proof pressure test to establish the minimum strength of the window. We assume that the minimum strength of the window is no greater than the proof stress. In this case, the maximum flaw depth in the window is that which would survive the proof stress. This flaw is larger than $71.6 \mu\text{m}$ derived earlier when the minimum strength of the window was taken as $\frac{1}{2} \sigma_t = 60 \text{ MPa}$. In the absence of assuming there is a threshold stress for failure, we fall back to NASA STD 5018 standard for our design.

With the 3-parameter Weibull equation, the probability of failure of the window is zero below the threshold stress. If we do not accept that there is a threshold stress, we revert to using a 2-parameter Weibull equation to predict probability of failure at low stress.

Step 1: Establish design service stress (σ_a) for full-size window.

$\sigma_a = 4.90 \text{ MPa}$ as before

Step 2: Derive parameters for 2-parameter Weibull Equation 26. Strengths of the 31 BK7 coupons in Appendix D (Figure D-1) were re-analyzed to derive parameters $m = 4.434$ (unbiases) and $\sigma_\theta = 232.2 \text{ MPa}$ for the 2-parameter Weibull Equation 25. Effective area $A_e = 25.09 \text{ cm}^2$ was computed for equibiaxial flexure disks with Equation E-4 in Appendix E. The Weibull scale factor $\sigma_o = 480 \text{ MPa}$ for Equation 26 was computed with Equation 27.

The effective area $A_e = 707 \text{ cm}^2$ of the BK7 aircraft window in Figure 3 was computed with Equation 6. From A_e , the expected mean strength of the window is 99.5 MPa from Equation 7. Results are summarized in Table 18.

TABLE 18. 2-Parameter Weibull Parameters Derived
From Fuller (Reference 3) Raw Data.

Parameter	Value for BK7 glass
Weibull unbiased modulus (m) ^a	4.43
Weibull scale factor (σ_o) (MPa)	480
Aircraft window effective area A_e (cm ²)	707
Aircraft window expected mean strength (MPa)	99.5

^aOur Weibull modulus $m = 4.43$ derived by maximum likelihood method from Fuller data in Appendix D does not agree with $m = 5.133$ value given by Fuller.

Area scaling for a large window ($A_e = 707 \text{ cm}^2$) with a low Weibull modulus ($m = 4.43$) is shown in Figure 22 by the straight line in the left half of the graph. The BK7 coupons with an effective area of 25 cm^2 have a mean strength of 212 MPa. The window has a mean strength of 99.5 MPa. The coupons have a 10% probability of failure near 140 MPa. The window has a 10% probability of failure near 65 MPa.

Recall that the 3-parameter Weibull Equation 29 for both the coupons and the large window has a threshold strength near 120 MPa below which no failure is expected. There is a stark difference between the 2-parameter and 3-parameter Weibull predictions.

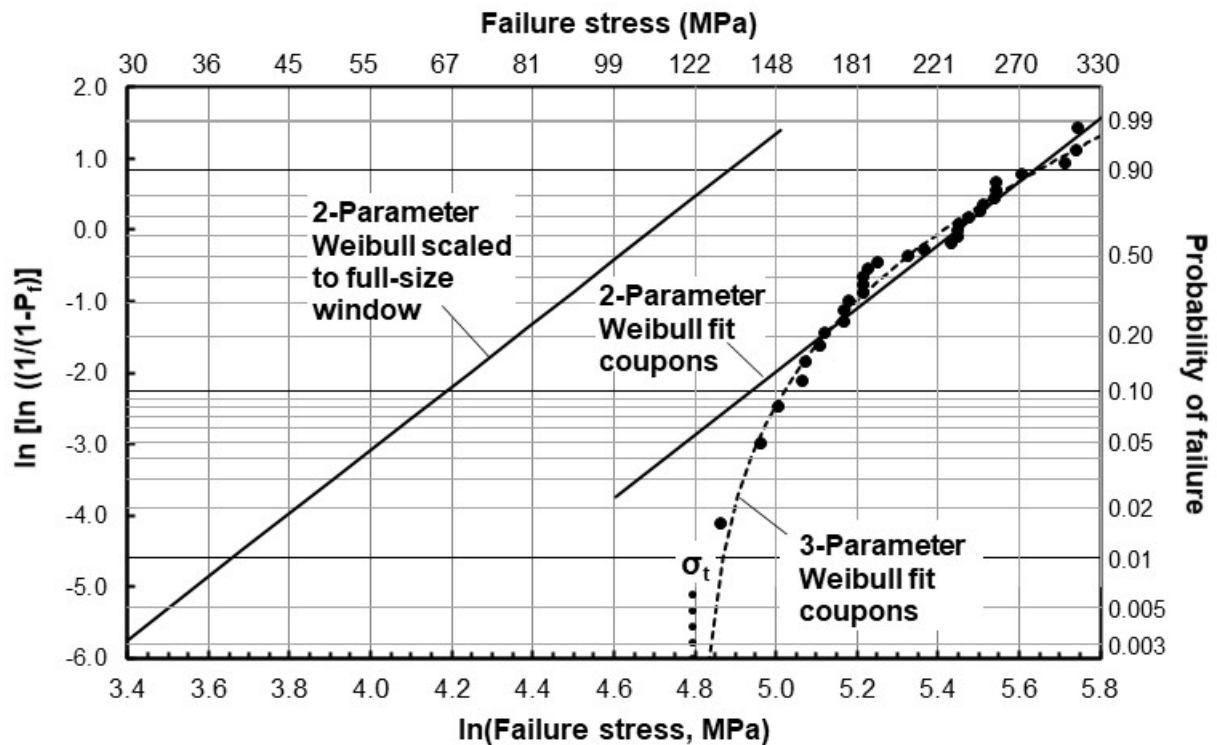


FIGURE 22. Weibull curves for BK7 glass. Two right-hand curves are fit to 31 BK7 glass test coupons from Fuller (Reference 3). Points are measured strengths. The curve through the points is the maximum likelihood fit to Equation 29 with $\sigma_t = 120.8 \text{ MPa}$, $\sigma_\theta = 99.5 \text{ MPa}$, and $m = 1.90$ (unbiased). The straight line is the maximum likelihood fit to Equation 25 with $\sigma_\theta = 232.2 \text{ MPa}$ and $m = 4.434$ (unbiased) plotted in the logarithmic form of Equation 28. Left-hand line is 2-parameter Weibull Equation 26 scaled for full-size BK7 window with effective area of 707 cm^2 , $\sigma_o = 480 \text{ MPa}$, and $m = 4.434$.

Step 3: Calculate design life for chosen values of proof stress. We select a trial proof stress and calculate the resulting design life. If the window survives the proof test, then there is a known maximum flaw depth that could be present in the window after the test. A window with a larger flaw would have broken in the test. We then calculate the time required for that maximum flaw depth to propagate to failure when held at the design service stress $\sigma_a = 4.90$ MPa. The design life is one quarter of the time to failure. Table 19 gives results for chosen proof stress levels.

TABLE 19. Design Life From Selected Proof Stress
for BK7 at Design Stress = 4.90 MPa.

Proof Stress, MPa	Maximum Flaw Depth, μm	Design Life (h) With Uncertainty Factor U for Equation 3 From Table 1 ^a	Minimum Initial Window Strength (MPa) to Survive Proof Test in 100% Humidity ^b	Fraction Expected to Fail Proof Test in 100% Humidity ^c
14.7	737	8 h ($U = 1.4$)	26.8	0.002
18.0	669	1,500 h ($U = 1.2$)	33.5	0.005
19.0	600	4,800 h ($U = 1.2$)	35.5	0.007
19.1	594	5,400 h ($U = 1.2$)	35.7	0.007
19.2	588	6,000 h ($U = 1.2$)	35.9	0.007
19.6	671	10,800 h ($U = 1.1$)	36.8	0.008
20.0	645	16,000 h ($U = 1.1$)	37.54	0.009
22.0	533	96,000 h ($U = 1.1$)	41.8	0.014
24.5	430	580,000 h ($U = 1.1$)	47.1	0.024

^aCalculated with slow crack growth parameters in Table 17 line 2 for 100% humidity.

^bCalculated with slow crack growth parameters in Table 17 line 1 for 100% humidity.

^cProbability that window has the strength in column 4. Calculated from 2-parameter Weibull equation parameters in Table 18 for full-size window.

As an example, we compute each quantity in the row for the 20.0 MPa proof stress. To find the maximum flaw depth that can remain after a 20.0 MPa proof test, we set strength $S = 20$ MPa in Equation 35. We do not know what value of uncertainty factor U from Table 1 to use until after calculating the design life. With the hope that the design life will be > 1 yr (> 8760 h), we choose $U = 1.1$ for a trial calculation.

$$K_{Ic} = U \cdot S \cdot (\pi a / Q)^{1/2} \quad (35)$$

$$0.99 \text{ MPa}\sqrt{\text{m}} = 1.1 \cdot 20 \text{ MPa} \cdot (\pi a / 1)^{1/2} \Rightarrow a = 645 \mu\text{m}$$

We find that the largest flaw that could be present after the 20 MPa proof test is 645 μm , which is much larger than 71.6 μm derived from a threshold failure stress of $\frac{1}{2} \sigma_t = 60$ MPa from the 3-parameter Weibull equation.

Now we use Equation 3 to compute the time to failure of the window with an initial flaw depth of 645 μm held at a constant stress of $\sigma_a = 4.90 \text{ MPa}$. The initial stress intensity factor K_{Ii} at the tip of the flaw is computed with Equation 2:

$$K_{Ii} = U \cdot \sigma \cdot (\pi a / Q)^{1/2} \quad (2)$$

$$= 1.1 \cdot 4.90 \text{ MPa} \cdot [\pi \cdot (645 \times 10^{-6} \text{ m}) / 1]^{1/2} = 0.2426 \text{ MPa}\sqrt{\text{m}}$$

The equation for the time to failure looks like this:

$$\text{Time to failure (t) static fatigue life:} \quad t = \left(\frac{2Q}{U^2 \pi \sigma_a^2} \right) \int_{K_{Ii}}^{K_{Ic}} \left(\frac{K_I}{v} \right) dK_I \quad (3)$$

with the upper limit of integration $K_{Ic(\text{average})} - 3 \text{ std dev} = 0.95 - 3(0.04) = 0.83 \text{ MPa}\sqrt{\text{m}}$.

During the vast majority of the life of the window, the stress intensity factor is low and crack growth velocity $\ll 10^{-6.5} \text{ m/s}$, so we use crack growth parameters in Table 17, line 2 to find v in the integrand of Equation 3. Numerical integration of Equation 3 gives time to failure $t = 2.32 \times 10^8 \text{ s} = 6.44 \times 10^4 \text{ h}$. The design life is $\frac{1}{4} t = 16\,000 \text{ h}$ in Table 19.

Step 4: Calculate minimum initial window strength required to survive proof test and probability of failure in proof test. Slow crack growth during the proof test weakens the window being tested. We do not want to risk losing an expensive window with a long procurement time in the proof test. Table 19 shows calculated results from slow crack growth in the last two columns. Both the window design life and crack growth in the proof test are calculated with crack growth parameters measured for BK7 glass in 100% humidity, which is a worst case for rapid crack growth.

The minimum initial strength of a window that can survive the 20 MPa proof test is computed with Equation 12, which is derived in Appendix B.

$$\text{Loss of strength in proof test} \quad S_f^{-2} = S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o \int_0^t e^{\beta K_{Ic} S} dt \quad (12)$$

In Equation 12, S_f is the final strength after the proof test, S_i is the initial strength before the proof test, and S is the strength at any instant during the proof test. Y and K_{Ic} are from Equation 6 relating strength to flaw depth. We used $Y = 1.95$ and $K_{Ic} = 0.95 \text{ MPa}\sqrt{\text{m}}$ in Equation 12. The parameters v_o and β are from the exponential law for slow crack growth in line 1 of Table 17. We choose parameters from line 1 for $v > 10^{-6.5} \text{ m/s}$ because the proof test causes significant crack growth during the test. The rate of crack growth at high stress intensity factor is in the high range ($> 10^{-6.5} \text{ m/s}$), not the low range on line 2 of Table 17 ($< 10^{-6.5} \text{ m/s}$).

For this example, the proof stress profile in Figure 9 has peak stress $\sigma_p = 20 \text{ MPa}$ with a 10 s loading time, 1 s at peak stress, and 2 s unloading time. In applying Equation 12 to

strength loss during the proof test, we discover that any window whose initial strength is less than 37.54 MPa will break during the test (giving $S_f < 0$). Figure 23 shows the evolution of strength for a window whose initial strength is 37.54 MPa. The final strength after the proof test is 28.18 MPa.

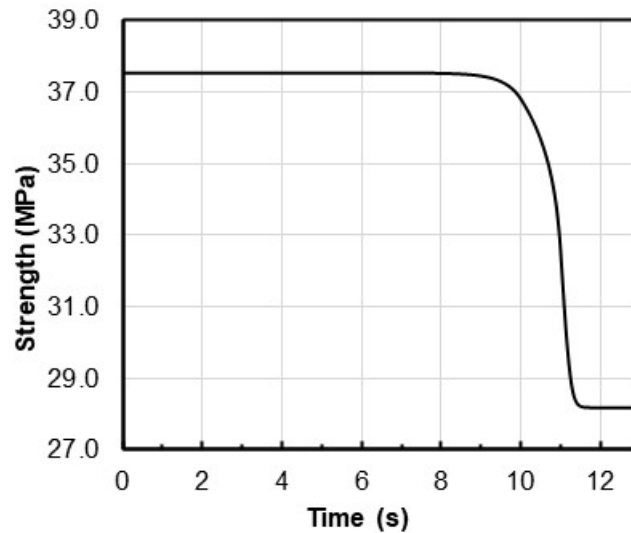


FIGURE 23. Calculated loss of strength in a BK7 window during a 20 MPa proof pressure test whose profile is shown in Figure 9 with times $t_\ell = 10$ s, $t_p = 1$ s, and $t_u = 2$ s. Initial window strength was taken as 37.54 MPa. Strength loss occurs mainly during the 1 s peak stress between 10 and 11 s, with some loss during loading and unloading. Crack growth parameters in Table 17, line 1 (100% humidity) were used.

A proof test should be conducted under the driest feasible conditions with the shortest practical times for loading, holding, and unloading the pressure. Crack growth parameters used to calculate strength loss in Figure 23 were from measurements in 100% humidity, so Figure 23 overestimates strength loss.

How bad is the overestimate of strength loss in Figure 23? We do not know of slow crack growth rates for BK7 glass in the absence of moisture. But, there is a measurement for the closely related glass UBK7, whose composition is compared with BK7 in Appendix H. In Appendix H, we estimate that *slow crack growth in UBK7 in vacuum at 293 K is 10^{20} times slower than in 100% humidity*. We therefore expect no significant crack growth for BK7 in a rigorously dry atmosphere and no significant strength loss in a well-executed proof test.

If there were slow crack growth and the minimum initial strength to survive the 20 MPa proof test were really 37.54 MPa, what is the probability that a full-size window could have a strength below 37.54 MPa? We answer this question by inserting 37.54 MPa for stress σ in the 2-parameter Weibull Equation 26 for the full-size window with parameters from Table 18.

Weibull equation with
Weibull scale factor σ_0

$$P_f = 1 - e^{-\left(\frac{A_e}{A_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^m} \quad (26)$$

$$P_f = 1 - e^{-\left(\frac{707 \text{ cm}^2}{1 \text{ cm}^2}\right)\left(\frac{37.54 \text{ MPa}}{480 \text{ MPa}}\right)^{4.43}} = 0.009$$

If there is slow crack growth in the proof test equal to that which would occur at 100% humidity, we predict that 0.9% of full-size BK7 windows will break in the proof test. The failure rates in Table 19 range from 0.2% to 2.4% as proof stress increases. However, if the proof test is conducted in a rigorously dry atmosphere, we expect slow crack growth to be negligible and the failure rate should be far below 1% for all proof stress levels in Table 19.

Which proof stress in Table 19 should we choose? If we want a design life greater than 1 yr, the 19.6 MPa proof pressure test is a good choice. There might be some reservations executing this test. The pressure required to generate 19.6 MPa of stress in the aircraft window is 41.2 psi. With the area of the window in Figure 1, the force on the window is 8,400 pounds. Test fixture hardware and seals must survive this force and this pressure. Decreasing the proof pressure has rapidly diminishing returns. If we lower the proof stress by 3% to 19.0 MPa, the design life decreases by 45% from 16,000 to 4,800 h. If it is practical to execute, we would select the 19.6 MPa proof stress. The strong dependence of design life on proof stress should alert you to the fact that small uncertainty in proof stress gives large uncertainty in design life.

For calculating window lifetime, the 19.6 MPa proof pressure test in Table 19 is equivalent to assuming that the window strength is just 19.6 MPa. The 10 800 h design life of a 19.6-MPa-strong window is *half a million times less* than the design life in Figure 20, which assumed that the window strength is 60 MPa (= ½ of measured σ_t).

We assume that slow crack growth rate in service is the rate observed in 100% humidity at room temperature. For an airplane flying in the stratosphere, the crack growth rate should be a few orders of magnitude slower because of the low outside temperature and the low vapor pressure of water in the cold atmosphere.

EFFECT OF TEMPERATURE ON SLOW CRACK GROWTH

Crack growth is a chemical reaction involving breaking of chemical bonds. Rates of reactions generally increase with increasing temperature. Slow crack growth parameters in Table 17 apply near 298 K. The outside window face (tensile face) temperature at an

altitude of 40,000 feet is ~ 220 K ($\sim -53^\circ\text{C}$). Table H-2 in Appendix H gives temperature-dependent crack growth parameters *in vacuum* for borosilicate crown II glass, also called UBK7, and which is similar in composition to BK7. The temperature range is valid for 298 to 668 K. Crack growth in moisture is faster than crack growth in vacuum. Extrapolation to low temperature suggests that the crack growth rate at 220 K might be four orders of magnitude slower than crack growth at 300 K, with a corresponding increase of several orders of magnitude in aircraft window life.

Calculated crack growth rates with Equation H-1 in Appendix H Table H-2.

$$\begin{aligned}\text{For stress intensity factor } K_I &= 0.8 \text{ MPa}\sqrt{\text{m}} \quad (8 \times 10^5 \text{ Pa}\sqrt{\text{m}}) \\ \text{Crack growth rate (v)} &= 1.3 \times 10^{-6} \text{ m/s at 400 K} \\ &= 1.6 \times 10^{-9} \text{ m/s at 300 K} \\ &= 9 \times 10^{-14} \text{ m/s at 220 K}\end{aligned}$$

$K_I = 0.8 \text{ MPa}\sqrt{\text{m}}$ was chosen because it is in the range of measured data.

At a lower stress intensity factor, $\frac{\text{growth rate at 220 K}}{\text{growth rate at 300 K}}$ is predicted to be much less than at 300 K

COMPARING 2- AND 3-PARAMETER WEIBULL EQUATIONS FOR BK7 GLASS DESIGN

For applications in which human life depends on the integrity of the glass component, we could never trust a 3-parameter Weibull equation design without a proof test to confirm that the component has adequate strength. With the 2-parameter Weibull equation, we needed a proof stress nearly four times the design stress to obtain a 1-year design life at a service stress of 4.9 MPa in Table 19. If human life is not at stake, the 3-parameter Weibull equation allows us to design a component that must withstand greater stress.

For example, if we believe that the window has a “safe” threshold strength of at least 60 MPa, we could employ a 20 MPa proof test for a component whose design stress is 10 MPa. Worst-case crack growth predicts that the window needs an initial strength of 37.5 MPa to survive the 20 MPa proof stress. The 2- or 3-parameter Weibull equations predict a negligible probability of failure in a 20 MPa proof test. The threshold strength of 60 MPa implies a maximum flaw size of 71.6 μm in Equation 31. With a 10 MPa design stress and a 71.6 μm initial flaw size, the initial stress intensity factor is $0.165 \text{ MPa}\sqrt{\text{m}}$ from Equation 2 and the design life from Equation 3 is 220 yr. With the 2-parameter Weibull equation in Table 19, a 20 MPa proof test gave a design life of 1.8 yr for a design stress of 4.9 MPa. The 3-parameter Weibull equation allowed us to double the design stress and simultaneously increase design life 100 $\times$. These design differences depend on what we believe to be true about window strength and how much risk we are willing to take in trusting the design.

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NOMENCLATURE

°C	degrees Celsius
ASTM	American Society for Testing and Materials
cm ²	square centimeter
ft	feet
h	hours
K	Kelvin
lbf/in ²	pound-force per square inch
m	meters
m/s	meters per second
mm	millimeters
MPa	megapascals
NASA	National Aeronautics and Space Administration
OH	hydroxyl
Psi	pounds per square inch
s	seconds
STD	Standard
yr	years
σ	stress
σ_a	service stress
σ_p	proof stress
μm	microns

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Appendix A

**DERIVING EXPONENTIAL LAW SLOW CRACK GROWTH PARAMETERS
FROM POWER LAW PARAMETERS**

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Exponential law for flaw growth velocity (v):

$$v = v_o e^{\beta K_I} \quad (\text{A-1a})$$

$v = da/dt$, where a = flaw depth and t = time

K_I = stress intensity factor

v_o and β are measured material parameters

Power law for flaw growth velocity (v):

$$v = A^* \left(\frac{K_I}{K_{Ic}} \right)^n \quad (\text{A-1b})$$

A^* and n are measured material parameters

K_{Ic} = critical stress intensity factor for mechanical failure

The ASTM C1368 dynamic fatigue method for measuring slow crack growth rate in ceramics is to measure the strength of multiple replicate coupons of a material at different stress rates ($\dot{\sigma}$) spanning ~four orders of magnitude, such as 0.002, 0.02, 0.2, 2, and 20 MPa/s. The slower the stress rate, the more time there is for cracks to grow during the strength measurement, and the lower is the strength. In terms of the power law Equation A-1b for crack growth, the measured strength σ_f (“f” stands for “final” strength) is related to the initial strength σ_i (“i” stands for “initial” or “inert” strength) of the material by the equation

$$\sigma_f^{n+1} = B(n+1)\sigma_i^{n-2}\dot{\sigma} \quad (\text{A-2})$$

where n is the exponent in the power law (A-1b), B is related to A^* in Equation A-1b by $B = 2K_{Ic}^2/[A^*(n-2)Y^2]$, and Y is the geometric factor from the fracture mechanics relation between stress intensity factor K_I , stress σ , and crack length a : $K_I = Y\sigma\sqrt{a}$. Derivation of Equation A-2 uses the approximation $\sigma_f^{n-2} \ll \sigma_i^{n-2}$. The derivation of Equation A-2 is given in Appendix C.

To measure the power law slow crack growth parameters n and A^* in Equation A-1b, a graph is constructed showing the logarithm of measured strength σ_f versus the logarithm of stress rate $\dot{\sigma}$. From the slope and intercept and separately measured inert strength σ_i , values of n and B are computed. Then A^* is calculated from B . ASTM C1368 does not provide a method to derive the exponential law parameters v_o and β .

Figure F-1 shows digitized slow crack growth rate measurements for fused silica taken from work by Wiederhorn. We used the full data set to derive the power law and exponential law, with results shown in the Figure F-1.

To derive exponential law parameters from power law parameters, we chose to find a least-squares fit of the equation $v = v_o e^{\beta K_I}$ to data points from the equation $v = A^*(K_I/K_{Ic})^n$ in the range $K_I = 0.40$ to $0.55 \text{ MPa}\sqrt{m}$ in Figure F-1. This range spans crack velocities from 10^{-9} to 10^{-5} m/s in the measured points. Much of the damage from crack growth during strength measurements goes through this range of velocities.

For other materials, we choose to fit the exponential equation to the power law equation in the range $K_I = 0.533K_{Ic}$ to $0.733K_{Ic}$ —a range in which we estimate that the power law and exponential law should overlap.

Figure A-1 illustrates the following steps used to find exponential law parameters from power law parameters for fused silica:

1. Calculate 16 equally spaced power law points for fused silica in the interval $K_I = 0.40$ to $0.55 \text{ MPa}\sqrt{\text{m}}$. For other materials, use the range $K_I = 0.533K_{Ic}$ to $0.733K_{Ic}$.
2. Guess values for $\log(v_o)$ and β for the exponential law.
3. Use unweighted least-squares refinement to adjust the values of $\log(v_o)$ and β to achieve the best fit of the exponential law equation to the power law points.

Figure F-1 shows divergence of the power law and exponential law from each other over a wide range of stress intensity factor K_I . The exponential law provides a more conservative estimate of window lifetime than does the power law because the stress intensity factor during most of the lifetime of a window is very low where the two curves diverge. For slow crack growth during a proof test, the exponential and power laws give similar predictions because the stress intensity factor in the proof test is in the range where the two curves overlap in Figure A-1.

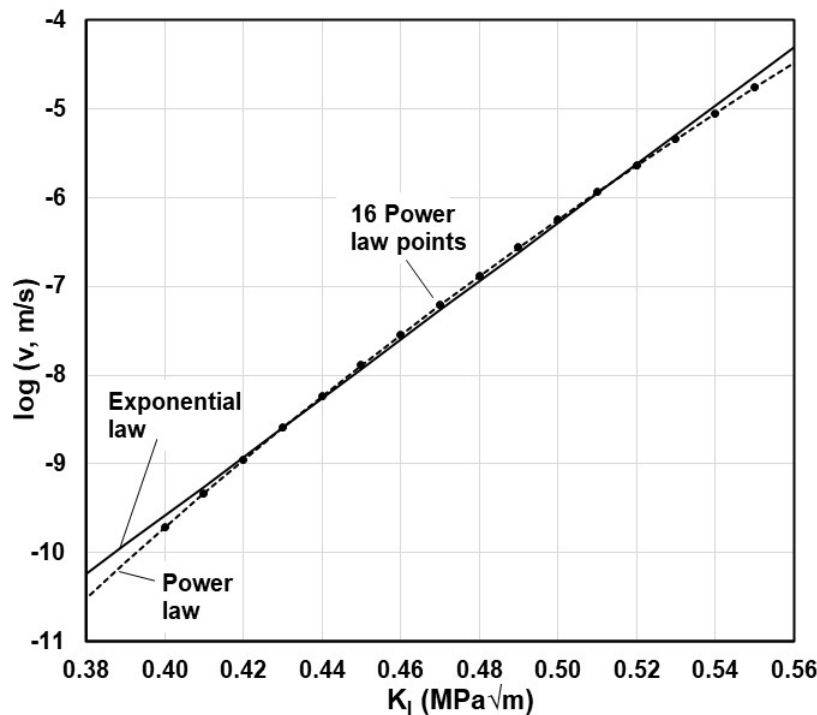


FIGURE A-1. Unweighted least-squares fit of exponential law $v = v_o e^{\beta K_I}$ to 16 points from the power law $v = A^* (K_I/K_{Ic})^n$.

Power law points in Figure A-1 were computed for fused silica from $v = A^*(K_I/K_{Ic})^n$ with $A^* = 1.14554$ m/s, $n = 35.805$, and $K_{Ic} = 0.75$ MPa $\sqrt{m}$. Exponential law parameters derived from Figure A-1 by fitting to 16 points differ by 1% from those derived from fitting all 126 measured points in Figure F-1.

Exponential Law Parameter	From Fitting 16 Points in Figure A-1	From Fitting All Points in Figure F-1
$\log(v_o)$	-22.768	22.592
β	75.910	74.946

RECOMMENDATION

We recommend the following procedure to derive exponential law parameters for other materials from power law parameters obtained from ASTM C1368 constant stress-rate strength testing:

1. For a test material whose critical stress intensity factor is K_{Ic} , calculate 16 equally spaced power law points in the interval $K_I = 0.40(K_{Ic}/0.75)$ to $0.55(K_{Ic}/0.75)$ MPa $\sqrt{m}$. This range of stress intensity factor is the same fraction of the critical stress intensity factor of fused silica chosen for fitting in Figure A-1. The critical stress intensity factor of fused silica is 0.75 MPa $\sqrt{m}$.
2. Estimate values for $\log(v_o)$ and β for the exponential law for the test material.
3. Use unweighted least-squares refinement to adjust the values of $\log(v_o)$ and β to achieve the best fit.

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Appendix B

**STRENGTH LOSS BY SLOW CRACK GROWTH
DURING PROOF TEST**

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Prior to use, a window might be subjected to a proof pressure test to demonstrate that the window can withstand its intended load plus an additional safety margin. NASA STD 5018 (dated 8/12/2011) directs that the proof pressure load shall be at least three times greater than the expected maximum service load. Under stress during a proof test and during service, microscopic cracks in a window can grow in size and thereby weaken a window. We now derive equations to assess the loss of strength of a window as a result of proof testing.

SLOW CRACK GROWTH EQUATIONS

Two empirical equations are often used to fit the observed growth of flaws in ceramics and glass under stress.

Exponential law for flaw growth velocity (v): $v = v_0 e^{\beta K_I}$ (B-1)

$v = da/dt$, where a = flaw depth and t = time

K_I = stress intensity factor at crack tip

v_0 and β are measured material parameters

Power law for flaw growth velocity: $v = A^* \left(\frac{K_I}{K_{Ic}} \right)^n$ (B-2)

A^* and n are measured material parameters

K_{Ic} = critical stress intensity factor for catastrophic failure

The exponential law was used in the earliest studies by Wiederhorn at the U.S. National Bureau of Standards. The power law was adopted in subsequent work because its mathematical form is simpler to integrate to find the length of a growing crack.

POWER LAW EQUATION FOR STRENGTH LOSS BY SLOW CRACK GROWTH DURING PROOF TEST

Our starting point is fracture mechanics Equation B-3 relating stress intensity factor (K_I) to stress (σ) and Equation B-4 relating critical stress intensity factor (K_{Ic}) to strength (S):

$$K_I = Y\sigma\sqrt{a} \quad (B-3)$$

$$K_{Ic} = YS\sqrt{a} \quad (B-4)$$

where a is the flaw depth and Y is a geometric factor approximate in the range 1 to 2. The quotient of Equations B-3 and B-4 allows us to express the ratio K_I/K_{Ic} as a ratio of the stress on the material to the strength of the material:

$$\frac{K_I}{K_{Ic}} = \frac{Y\sigma\sqrt{a}}{YS\sqrt{a}} = \frac{\sigma}{S} \quad (B-5)$$

B-3

From Equation B-5, we can rewrite the power law for crack velocity (da/dt) in terms of stress and strength:

$$\frac{da}{dt} = A^* \left(\frac{K_I}{K_{Ic}} \right)^n = A^* \left(\frac{\sigma}{S} \right)^n \quad (\text{B-6})$$

where A^* and n are measured parameters.

Now square Equation B-4, isolate S^2 , and take the derivative of each side with respect to time:

$$S^2 = \frac{K_{Ic}^2}{Y^2 a} \quad (\text{B-7})$$

$$2S \frac{dS}{dt} = -\frac{K_{Ic}^2}{Y^2} a^{-2} \frac{da}{dt} \quad (\text{B-8})$$

In taking the derivative in Equation B-8, we made the approximation that Y is not a function of time. Then substitute $A^* \left(\frac{\sigma}{S} \right)^n$ from Equation B-6 for da/dt in Equation B-8:

$$2S \frac{dS}{dt} = -\frac{K_{Ic}^2}{Y^2} a^{-2} A^* \frac{\sigma^n}{S^n} \quad (\text{B-9})$$

From Equation B-4 we can write $a^2 = K_{Ic}^4 / (Y^4 S^4)$ and substitute this expression for a^2 into Equation B-9. Then rearrange to isolate S on the left side and σ on the right side of the equation:

$$2S \frac{dS}{dt} = -\frac{K_{Ic}^2}{Y^2} \frac{Y^4 S^4}{K_{Ic}^4} A^* \frac{\sigma^n}{S^n} \Rightarrow S^{n-3} dS = -\frac{Y^2 A^* \sigma^n}{2K_{Ic}^2} dt \quad (\text{B-10})$$

Then integrate Equation B-10 from initial strength S_i to final strength S_f and from time $t = 0$ to time t :

$$\int_{S_i}^{S_f} S^{n-3} dS = -\frac{Y^2 A^*}{2K_{Ic}^2} \int_0^t \sigma^n dt$$

Strength loss from
power law for slow
crack growth:

$$S_f^{n-2} = S_i^{n-2} - \frac{Y^2 A^* (n-2)}{2K_{Ic}^2} \int_0^t \sigma(t)^n dt \quad (\text{B-11})$$

Equation B-11 allows us to calculate how a window with initial strength S_i degrades to a final strength S_f by slow crack growth during the application of stress in a proof test.

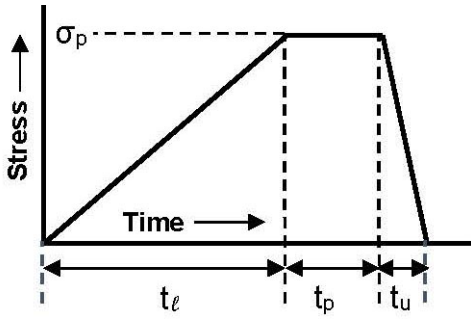


FIGURE B-1. Proof stress profile.

Consider a proof test with linear loading and unloading of a window with the stress profile in Figure B-1. Stress is applied at a constant rate from time $t = 0$ to time $t = t_\ell$ (loading time). Then constant proof stress σ_p is held for time t_p . The stress during each phase of the test can be written

$$\text{Loading: } \sigma(t) = \frac{t}{t_\ell} \sigma_p \quad (\text{B-12})$$

$$\text{Peak stress: } \sigma(t) = \sigma_p \quad (\text{B-13})$$

$$\text{Unloading: } \sigma(t) = \left(1 - \frac{t - t_l - t_p}{t_u}\right) \sigma_p \quad (\text{B-14})$$

With these functions of stress, the integral in Equation B-11 is found to be

$$\int_0^t \sigma(t)^n dt = \sigma_p^n \left[t_p + \frac{t_\ell + t_u}{n+1} \right] \quad (\text{B-15})$$

Substituting the right side of Equation B-15 into Equation B-11, we can now write Equation B-16 relating the strength S_f at the end of the proof test to the initial strength S_i before the proof test, if slow crack growth is described by power law Equation B-2:

$$S_f^{n-2} = S_i^{n-2} - \frac{Y^2 A^* (n-2)}{2K_{Ic}^2} \sigma_p^n \left[t_p + \frac{t_\ell + t_u}{n+1} \right] \quad (\text{B-16})$$

EXAMPLE: USING POWER LAW EQUATION B-16 FOR STRENGTH LOSS BY SLOW CRACK GROWTH DURING PROOF TEST

Consider parameters in Table B-1 for slow crack growth for borosilicate glass BK7 and fused silica in a moist environment. Suppose that we conduct a proof test of a large fused silica window with peak stress $\sigma_p = 14.7$ MPa with the stress profile above with a loading time of $t_\ell = 10$ s, a hold time of $t_p = 1$ s at peak stress, and an unloading time of $t_u = 2$ s. Let us use the following parameters: $Y = 1.95$, $A^* = 0.0773$ (Table B-1, line 6), $n = 28$ (Table B-1, line 6), and $K_{Ic} = 0.75$ MPa $\sqrt{\text{m}}$. The term $\frac{Y^2 A^* (n-2)}{2K_{Ic}^2}$ in Equation B-16 is equal to $6.793 \text{ s}^{-1} \text{ MPa}^{-2}$. Suppose that the window has an initial strength $S_i = 20$ MPa prior to the proof test. The strength at the end of the proof test is found with Equation B-16:

$$\begin{aligned} S_f^{n-2} &= S_i^{n-2} - \frac{Y^2 A^* (n-2)}{2K_{Ic}^2} \sigma_p^n \left[t_p + \frac{t_\ell + t_u}{n+1} \right] \\ S_f^{28-2} &= (20 \text{ MPa})^{28-2} - (6.793 \text{ s}^{-1} \text{ MPa}^{-2})(14.7 \text{ MPa})^{28} \left[1 + \frac{10+2}{28+1} \right] \\ S_f &= 19.113 \text{ MPa} \end{aligned} \quad (\text{B-17})$$

The window strength is reduced by 4.4% from 20 MPa to 19.113 MPa in this example because of slow crack growth during the proof test.

TABLE B-1. Slow Crack Growth Parameters at 100% Humidity or in Water.

Fit to Exponential Law		$K_I = a + b \ln(v)$		$v = v_0 e^{\beta K_I}$	
		a (MPa√m)	b (MPa√m)	v_0 (m/s)	β (1/[MPa√m])
1. BK7 glass ^a	$v > 10^{-6.5}$ m/s	0.7946	0.0253	2.291×10^{-14}	39.53
2.	$v < 10^{-6.5}$ m/s	0.5826	0.01177	3.184×10^{-22}	84.96
3. Fused silica (Corning 7940) ^a		0.6931	0.01342	3.716×10^{-23}	74.52
NASA ground disks broken in equibiaxial flexure test at different stress rates::					
4. Fused silica (Corning 7940) ^b				4.81×10^{-19}	54.3
5. Fused silica (Corning 7940) ^b				3.33×10^{-20}	61.5
Fit to Power Law		$v = A^* \left(\frac{K_I}{K_{Ic}} \right)^n$ with $K_{Ic} = 0.75$ MPa√m:			
				A^* (m/s)	n
NASA ground disks broken in equibiaxial flexure at different stress rates:					
6. Fused silica (Corning 7940) fit to data points ^b				0.0773	28
7. Fused silica (Corning 7940) fit to medians ^b				0.0567	27

^aData from Reference B-1. BK7 data were later fit to two lines instead of one in Reference B-2. Samples were tested in air at 100% humidity.

^bUnpublished data from J. A. Salem, NASA Glenn Space Flight Center. Samples were tested in water.

What is the likelihood that a fused silica window will have an initial strength of $S_i = 20$ MPa and therefore lose 4.4% of its strength in the proof test at $\sigma_p = 14.7$ MPa? For the 18-inch-diameter window with properties listed on the next page, Weibull Equation B-18 gives the probability of observing a failure stress σ . Failure stress is the same as the strength of the window in a rapid test with no slow crack growth.

Weibull equation:
$$P_f = 1 - e^{-\left(\frac{A_e}{A_o}\right)\left(\frac{\sigma}{\sigma_o}\right)^m} \quad (B-18)$$

where P_f = probability of failure

σ = stress

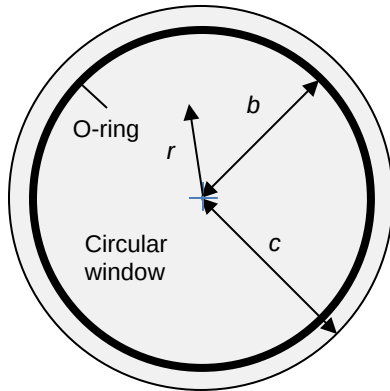
σ_o = Weibull scale factor = 237 MPa = strength for 1 cm² area in uniaxial stress

m = Weibull modulus = 5.8

A_e = effective area of window = 656 cm²

A_o = unit area $\equiv 1$ cm²

The Figure B-2 depicts a circular fused silica window chosen as an example. The window has a radius of $c = 9$ inches = 228.6 mm and is supported by an O-ring with a radius of $b = 8.5$ inches = 215.9 mm. The window thickness is $d = 1.1$ inches = 27.94 mm. The maximum design pressure on the window is 10 psi = 0.0689 MPa, which produces a tensile stress of 4.80 MPa at the center of the window face.



Fused silica window for sample calculations. Mechanical properties are

Poisson's ratio (ν) = 0.16
 Young's modulus (E) = 73 MPa
 Weibull modulus (m) = 5.8
 Weibull scale factor (σ_0) = 237 MPa
 Effective area in tension (A_e) = 656 cm²

FIGURE B-2. Circular fused silica window.

Table B-2 summarizes calculations of strength loss in the proof test using the power law strength loss Equation B-16 and the Weibull strength distribution Equation B-18. We calculated in Equation B-17 that a window with an initial strength of $S_i = 20$ MPa would lose 4.4% of its strength in the proof test at 14.7 MPa with the proof stress profile in Figure B-1. Table B-2 tells us that a window with an initial strength of $S_i = 19.72$ MPa would suffer catastrophic mechanical failure in the proof test. The probability of finding such a window is 0.00036 in the last column of Table B-3. A window with an initial strength of 25 MPa would only lose 0.008% of its strength in the proof test. Approximately 99.9% ($1 - 0.00142$ in the last column of Table B-2) of windows are expected to have a strength above 25 MPa.

TABLE B-2. Calculated Performance of Large Fused Silica Window in Proof Pressure Test with Power Law Parameters $A^* = 0.0773$ m/s and $n = 28$.

S_i	S_f		Weibull
Initial	final	% strength	probability
strength (MPa)	strength (MPa)	lost	of initial
19.71959	0	100	0.00036
20	19.113	4.435	0.00039
21	20.826	0.829	0.00052
22	21.949	0.232	0.00067
23	22.984	0.070	0.00087
24	23.994	0.025	0.00112
25	24.998	0.008	0.00142
26	25.999	0.004	0.00178

EXPONENTIAL LAW EQUATION FOR STRENGTH LOSS BY SLOW CRACK GROWTH IN PROOF TEST

We now derive an expression similar to Equation B-11 from the exponential law Equation B-1: $da/dt = v = v_0 e^{\beta K_I}$. Beginning with Equation B-8, we substitute Equation B-1 for da/dt :

$$2S \frac{dS}{dt} = -\frac{K_{Ic}^2}{Y^2} a^{-2} \frac{da}{dt} \quad (B-8)$$

$$2S \frac{dS}{dt} = -\frac{K_{Ic}^2}{Y^2} a^{-2} v_0 e^{\beta K_I} \quad (B-19)$$

From Equation B-4, we substitute $a^2 = K_{Ic}^4 / (Y^4 S^4)$ into Equation B-19

$$2S \frac{dS}{dt} = -\frac{K_{Ic}^2}{Y^2} \frac{Y^4 S^4}{K_{Ic}^4} v_0 e^{\beta K_I} \Rightarrow S^{-3} \frac{dS}{dt} = -\frac{1}{2} \frac{Y^2}{K_{Ic}^2} v_0 e^{\beta K_I} \quad (B-20)$$

From Equation B-5, we can substitute $K_I = K_{Ic} \frac{\sigma}{S}$ into the exponent of Equation B-20:

$$S^{-3} \frac{dS}{dt} = -\frac{1}{2} \frac{Y^2}{K_{Ic}^2} v_0 e^{\beta K_{Ic} \frac{\sigma}{S}} \Rightarrow S^{-3} dS = -\frac{1}{2} \frac{Y^2}{K_{Ic}^2} v_0 e^{\beta K_{Ic} \frac{\sigma}{S}} dt \quad (B-21)$$

and then integrate both sides of the equation

$$\int_{S_i}^{S_f} S^{-3} dS = -\frac{Y^2 v_o}{2K_{Ic}^2} \int_0^t e^{\beta K_{Ic} \frac{\sigma}{S}} dt$$

to get the form that will be useful to us:

*Strength loss from
exponential law for
slow crack growth:*

$$S_f^{-2} = S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o \int_0^t e^{\beta K_{Ic} \frac{\sigma}{S}} dt \quad (B-22)$$

On the right side of Equation B-22, both stress (σ) and strength (S) are functions of time. For the example below, we chose a maximum proof stress of 18.57943 MPa. For the proof test profile in Figure B-2, Equations B-12, B-13, and B-14 give the time dependence of σ . As in the example above, loading time is 10 s, peak stress time is 1 s, and unloading time is 2 s.

To find the time dependence of the strength of the window during the proof test, we perform a step-by-step numerical integration of Equation B-22 in the spreadsheet excerpt in Table B-3. Formulas for each cell of the first two rows are given below the spreadsheet.

In the first row, the first time increment in column F is selected as 0.001 s. The initial strength in the first row in column G is chosen to be 27 MPa in this example. Other input parameters for the spreadsheet in Table B-3 are shown below:

Max proof stress = 18.57943	MPa
Y =	1.95
K _{Ic} =	0.77 MPa√m
S _i =	27 MPa

	v _o	β	
W	3.72E-23	74.52	Wiederhorn

In each successive line of the spreadsheet, the initial strength in column G is taken as the final strength in column M from the previous line. Every line of the spreadsheet uses a slightly different value of S_i for its calculations. The strength S in the integrand on every line of the spreadsheet is set equal to the initial strength S_i on each line of the spreadsheet.

Figure B-3 shows the loss of strength of a fused silica window computed in Table B-3 with the proof test profile shown above with a peak proof stress of 18.58 MPa in the plateau from 10 s to 11 s. Note some loss of strength between 9 and 10 s and some loss of strength between 11.0 s (dashed vertical line) and 11.1 s. Strength losses before and after the plateau highlight the importance of keeping the loading and unloading times as short as practical.

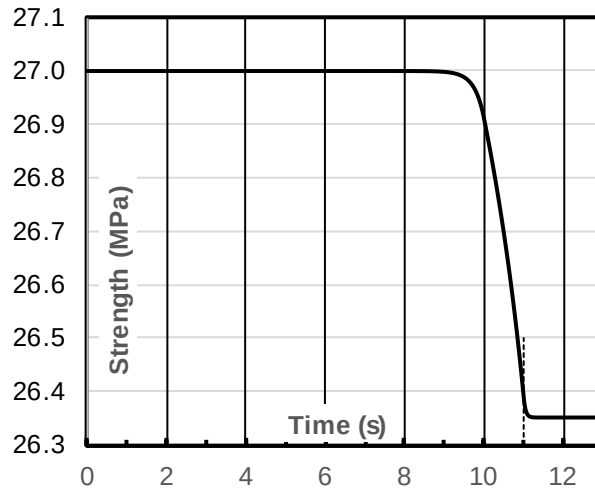


FIGURE B-3. Loss of strength of a fused silica window during a proof test reaching a peak proof stress of 18.58 MPa

TABLE B-3. Spreadsheet for Calculating Strength of Window as a Function of Time in Proof Test using Exponential Crack Growth Velocity Law by Integrating Equation B-22.

F	G	H	I	J	K	L	M
Δt increment =		0.001	s		$\frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$	$S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$	
time (s)	S_i (MPa)	σ (MPa)	$\beta K_{Ic} \sigma / S_i$	$e^{\beta K_{Ic} \sigma / S_i \Delta t}$	$\frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$	$S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$	S_f (MPa)
0.001	27.00000	0.00186	3.949E-03	1.00E-03	2.39E-25	1.37E-03	27.00000000
0.002	27.00000	0.00372	7.897E-03	1.01E-03	2.40E-25	1.37E-03	27.00000000
0.003	27.00000	0.00557	1.185E-02	1.01E-03	2.41E-25	1.37E-03	27.00000000
0.004	27.00000	0.00743	1.579E-02	1.02E-03	2.42E-25	1.37E-03	27.00000000
0.005	27.00000	0.00929	1.974E-02	1.02E-03	2.43E-25	1.37E-03	27.00000000
0.006	27.00000	0.01115	2.369E-02	1.02E-03	2.44E-25	1.37E-03	27.00000000
9.998	26.91235	18.57571	3.961E+01	1.59E+14	3.78E-08	1.38E-03	26.91198333
9.999	26.91198	18.57757	3.961E+01	1.59E+14	3.80E-08	1.38E-03	26.91161312
10	26.91161	18.57943	3.961E+01	1.60E+14	3.82E-08	1.38E-03	26.91124126
10.001	26.91124	18.57943	3.962E+01	1.60E+14	3.82E-08	1.38E-03	26.91086921
10.002	26.91087	18.57943	3.962E+01	1.60E+14	3.82E-08	1.38E-03	26.91049697
10.998	26.39216	18.57943	4.039E+01	3.49E+14	8.32E-08	1.44E-03	26.39139770
10.999	26.39140	18.57943	4.040E+01	3.50E+14	8.33E-08	1.44E-03	26.39063199
11	26.39063	18.57943	4.040E+01	3.50E+14	8.34E-08	1.44E-03	26.38986544
11.001	26.38987	18.57014	4.038E+01	3.43E+14	8.18E-08	1.44E-03	26.38911340
11.002	26.38911	18.56085	4.036E+01	3.37E+14	8.03E-08	1.44E-03	26.38837562
12.997	26.35127	0.02787	6.069E-02	1.06E-03	2.53E-25	1.44E-03	26.35126982
12.998	26.35127	0.01858	4.046E-02	1.04E-03	2.48E-25	1.44E-03	26.35126982
12.999	26.35127	0.00929	2.023E-02	1.02E-03	2.43E-25	1.44E-03	26.35126982
13	26.35127	0.00000	3.572E-11	1.00E-03	2.38E-25	1.44E-03	26.35126982

First row formulas**F:** Time = 0.001 s**G:** S_i = 27 MPa**H:** σ = (0.001/10)*max proof stress**I:** $= \beta K_{Ic} \sigma / S_i$ **J:** $= e^{\beta K_{Ic} \sigma / S_i \Delta t}$ (Δt = 0.001 s)**K:** $= \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$ **L:** $= S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$ **M:** $S_f = 1 / \sqrt{S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}}$ **Second row formulas****F:** Time = previous time + 0.001 s**G:** $S_i = S_f$ from previous line column M**H:** σ = (0.002/10)*max proof stress**I:** $= \beta K_{Ic} \sigma / S_i$ **J:** $= e^{\beta K_{Ic} \sigma / S_i \Delta t}$ (Δt = 0.001 s)**K:** $= \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$ **L:** $= S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}$ **M:** $S_f = 1 / \sqrt{S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o e^{\beta K_{Ic} \sigma / S_i \Delta t}}$

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- B-1. National Bureau of Standards. *Fracture Mechanics Study of Skylab Windows*, by S. M. Wiederhorn and D. E. Roberts. Gaithersburg, Maryland, NBS, 31 May 1972. (National Bureau of Standards Report 10-892.)
- B-2. S. M. Wiederhorn, A. G. Evans, and D. E. Roberts. "A Fracture Mechanics Study of the Skylab Windows," in *Fracture Mechanics of Ceramics*, Vol. 2, ed. by R. C. Bradt, D. P. Hasselman, and F. F. Lange. Plenum Press, New York, 1974, p. 829.

Appendix C

**DERIVATION OF FAILURE STRENGTH
IN DYNAMIC FATIGUE TESTING**

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The ASTM C1368 dynamic fatigue method measures slow crack growth rate in ceramics by measuring the strength of replicate coupons of a material at different stress rates ($\dot{\sigma}$) spanning ~four orders of magnitude, such as 0.002, 0.02, 0.2, 2, and 20 MPa/s. The slower the stress rate, the more time there is for cracks to grow during the strength measurement, and the lower is the strength.

ASTM C1368 uses power law Equation 1b for slow crack growth rate to predict the failure stress σ_f at each stress rate $\dot{\sigma}$:

$$\sigma_f^{n+1} = B(n+1)\sigma_i^{n-2}\dot{\sigma} \quad (C-1)$$

where σ_i is the inert strength of the material measured in the absence of crack growth, n is the exponent in the power law [crack growth rate $v = A^* \left(\frac{K_I}{K_{Ic}}\right)^n$], and the constant B is related to A^* and n in the power law by $B = 2K_{Ic}^2/[A^*(n-2)Y^2]$. Here K_{Ic} is the critical stress intensity factor for failure under inert conditions (no slow crack growth) and Y is the geometric factor in the relationship between stress intensity factor K_I , stress σ , and crack length a : $K_I = Y\sigma\sqrt{a}$. The purpose of this appendix is to derive Equation C-1.

The starting point for our derivation is Equation B-11 in Appendix B:

Strength loss from power law for slow crack growth:

$$S_f^{n-2} = S_i^{n-2} - \frac{Y^2 A^* (n-2)}{2K_{Ic}^2} \int_0^t \sigma(t)^n dt \quad (B-11)$$

Equation B-11 allows us to calculate how a window with initial strength S_i degrades to a final strength S_f by slow crack growth during the application of stress in a proof test. In Equation C-1, strength is represented by the variable σ , so we rewrite Equation B-11 with σ in place of S and we also replace the collection of constants by B to get:

$$\sigma_f^{n-2} = \sigma_i^{n-2} - \frac{1}{B} \int_0^t \sigma(t)^n dt \quad (C-2)$$

In dynamic fatigue, a constant stress rate $\dot{\sigma}$ is applied to the coupon until it breaks at stress σ_f at time t_f . Stress is a function of time given by $\sigma(t) = \dot{\sigma}t$, so Equation C-2 becomes

$$\sigma_f^{n-2} = \sigma_i^{n-2} - \frac{1}{B} \int_0^{t_f} \dot{\sigma}^n t^n dt = \sigma_i^{n-2} - \frac{\dot{\sigma}^n t_f^{n+1}}{B(n+1)} \quad (C-3)$$

But the stress $\dot{\sigma}t_f$ at which the coupon breaks is the strength σ_f . So, we write $\sigma_f = \dot{\sigma}t_f$ or $t_f^{n+1} = \sigma_f^{n+1}/\dot{\sigma}^{n+1}$. Substituting this last equality into Equation C-3 gives

Strength loss from power law for slow crack growth:

$$\sigma_f^{n-2} = \sigma_i^{n-2} - \frac{1}{B(n+1)\dot{\sigma}} \sigma_f^{n+1} \quad (C-4)$$

Equation C-4 is the correct equation to predict the failure strength σ_f of a test coupon with initial (inert) strength σ_i to which is applied stress at the constant rate $\dot{\sigma}$. However, ASTM C1328 makes the approximation that $\sigma_f^{n-2} \ll \sigma_i^{n-2}$ because $\sigma_f < \sigma_i$ and n is on the order of 20 or more. For example, if $\sigma_f/\sigma_i = 0.8$ and $n = 20$, $\sigma_f^{n-2}/\sigma_i^{n-2} = 0.018$. ASTM C1328 neglects σ_f^{n-2} in Equation C-4 to write

$$\sigma_i^{n-2} \approx \frac{1}{B} \frac{\sigma_f^{n+1}}{(n+1)\dot{\sigma}} \quad \text{or} \quad \sigma_f^{n+1} \approx B(n+1)\sigma_i^{n-2}\dot{\sigma} \quad (\text{C-5})$$

which is Equation C-1. From the observed failure stress at a series of different stress rates $\dot{\sigma}$, ASTM C1368 provides equations to calculate the parameters A^* and n in power law Equation 1b.

If we choose to use the exponential law Equation 1a for slow crack growth, then Equation B-22 lets us relate the failure strength in dynamic fatigue to the initial (inert) strength of the coupon and the stress rate $\dot{\sigma}$:

$$S_f^{-2} = S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o \int_0^{t t_f} e^{\beta K_{Ic} \frac{\sigma}{S}} dt \quad (\text{B-22})$$

Substituting $\sigma(t) = \dot{\sigma}t$ gives the expression

*Strength loss from
exponential law for
slow crack growth:*

$$S_f^{-2} = S_i^{-2} + \frac{Y^2}{K_{Ic}^2} v_o \int_0^{t_f} e^{\beta K_{Ic} \frac{\dot{\sigma}t}{S}} dt \quad (\text{C-6})$$

which must be integrated numerically to predict the failure strength S_f in dynamic fatigue from the inert strength S_i , the stress rate $\dot{\sigma}$, and time to failure t_f .

For dynamic fatigue in which coupons are tested to failure, the exponential law Equation 1a and power law Equation 1b are equally valid, but the power law is easier to use because it does not require numerical integration. This statement about dynamic fatigue is in contrast to a statement about design life for a window, in which the power law gives a much longer time to failure than the exponential law predicts. Figure F-1 in Appendix F explains why the two rate laws are not equivalent for calculating the design life of a window and shows that the exponential law gives the more conservative (shorter) design life.

Appendix D

**FINDING PARAMETERS FOR THE 3-PARAMETER WEIBULL EQUATION
BY MAXIMUM LIKELIHOOD METHOD**

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The 3-parameter Weibull Equation D-1 gives the probability of failure P_f as a function of stress σ . The parameters derived from fitting measured strengths of test samples are threshold strength σ_t , characteristic strength σ_θ , and Weibull modulus m :

3-Parameter Weibull equation:
$$P_f = 1 - e^{-\left(\frac{\sigma - \sigma_t}{\sigma_\theta}\right)^m} \quad (D-1)$$

The threshold strength is the limiting low stress below which no failures are observed in a large set of tests.

The *probability density* p is the derivative $dP_f/d\sigma$. To calculate the probability density, we utilize the derivative $\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$ with $u = -\left(\frac{\sigma - \sigma_t}{\sigma_\theta}\right)^m$:

Probability density
$$p \equiv \frac{dP_f}{d\sigma} = \left(\frac{m}{\sigma_\theta}\right) \left(\frac{\sigma - \sigma_t}{\sigma_\theta}\right)^{m-1} e^{-\left(\frac{\sigma - \sigma_t}{\sigma_\theta}\right)^m} \quad (D-2)$$

The *likelihood function*, $\mathcal{L}$ in Equation D-3, for n experimental data points is the product of probability densities in Equation D-2:

Likelihood function:
$$\mathcal{L} = \prod_{i=1}^n p_i = \prod_{i=1}^n \left(\frac{m}{\sigma_\theta}\right) \left(\frac{\sigma_i - \sigma_t}{\sigma_\theta}\right)^{m-1} e^{-\left(\frac{\sigma_i - \sigma_t}{\sigma_\theta}\right)^m} \quad (D-3)$$

where the symbol Π stands for a product just as Σ stands for a sum. Units of the likelihood function are 1/MPa, which means probability per megapascal. *The maximum likelihood method seeks values of m , σ_θ , and σ_t that maximize the likelihood function* in Equation D-3.

Column B of the spreadsheet in Figure D-1 shows measured strengths of 31 BK7 glass coupons from the work of Fuller et al (Reference D-1). Strengths have been ordered from weakest to strongest. Probability of failure in column C is computed with the formula $P_f = (i - 0.5)/n$, where i is the sample number in column A and n is the total number of samples (31) in cell B44. The strengths and probabilities of failure are the x and y coordinates of the points in the Weibull graph in Figure D-2 (*left*).

	A	B	C	D	E	F	G
3							Fitting parameters
4		Likelihood function:					from Solver
5		$\mathcal{L} = \prod_{i=1}^n \left[\left(\frac{m}{\sigma_\theta} \right) \left(\frac{\sigma_i - \sigma_t}{\sigma_\theta} \right)^{m-1} e^{-\left(\frac{\sigma_i - \sigma_t}{\sigma_\theta} \right)^m} \right]$				m =	1.913700113
6						σ_t =	121.6527276
7						σ_θ =	102.2933817
8							
9		x	y		ln (likelihood product) =		-163.1176594
10		Failure	Probability				
11	Sample	stress	of failure	Components of likelihood function			Likelihood function
12	number (i)	σ (MPa)	$P_f = (i-0.5)/n$	m/σ_θ	$((\sigma - \sigma_t)/\sigma_\theta)^{m-1}$	$\exp[-((\sigma - \sigma_t)/\sigma_\theta)^m]$	D*E*F
13	1	129.83	0.0161	1.871E-02	0.099	9.921E-01	1.845E-03
14	2	143.42	0.0484	1.871E-02	0.243	9.496E-01	4.320E-03
15	3	149.33	0.0806	1.871E-02	0.303	9.213E-01	5.220E-03
16	4	158.79	0.1129	1.871E-02	0.396	8.660E-01	6.419E-03
17	5	160.17	0.1452	1.871E-02	0.410	8.571E-01	6.568E-03
18	6	165.83	0.1774	1.871E-02	0.464	8.183E-01	7.108E-03
19	7	167.69	0.2097	1.871E-02	0.482	8.049E-01	7.261E-03
20	8	175.82	0.2419	1.871E-02	0.559	7.436E-01	7.782E-03
21	9	175.96	0.2742	1.871E-02	0.561	7.425E-01	7.789E-03
22	10	177.89	0.3065	1.871E-02	0.579	7.274E-01	7.878E-03
23	11	184.03	0.3387	1.871E-02	0.636	6.784E-01	8.076E-03
24	12	184.58	0.3710	1.871E-02	0.642	6.739E-01	8.088E-03
25	13	184.65	0.4032	1.871E-02	0.642	6.734E-01	8.089E-03
26	14	186.51	0.4355	1.871E-02	0.659	6.583E-01	8.121E-03
27	15	190.79	0.4677	1.871E-02	0.699	6.234E-01	8.154E-03
28	16	206.16	0.5000	1.871E-02	0.840	4.997E-01	7.851E-03
29	17	214.50	0.5323	1.871E-02	0.915	4.357E-01	7.461E-03
30	18	228.91	0.5645	1.871E-02	1.044	3.346E-01	6.536E-03
31	19	232.57	0.5968	1.871E-02	1.077	3.111E-01	6.267E-03
32	20	232.78	0.6290	1.871E-02	1.079	3.098E-01	6.252E-03
33	21	233.67	0.6613	1.871E-02	1.087	3.043E-01	6.185E-03
34	22	239.67	0.6935	1.871E-02	1.140	2.685E-01	5.725E-03
35	23	246.50	0.7258	1.871E-02	1.200	2.313E-01	5.190E-03
36	24	247.60	0.7581	1.871E-02	1.209	2.256E-01	5.104E-03
37	25	254.98	0.7903	1.871E-02	1.274	1.901E-01	4.530E-03
38	26	255.67	0.8226	1.871E-02	1.280	1.870E-01	4.477E-03
39	27	255.74	0.8548	1.871E-02	1.281	1.866E-01	4.471E-03
40	28	272.90	0.8871	1.871E-02	1.429	1.208E-01	3.231E-03
41	29	303.69	0.9194	1.871E-02	1.693	4.914E-02	1.556E-03
42	30	312.28	0.9516	1.871E-02	1.766	3.721E-02	1.229E-03
43	31	312.90	0.9839	1.871E-02	1.771	3.646E-02	1.208E-03
44	n = 31					product =	1.442E-71

FIGURE D-1. Spreadsheet for maximum likelihood determination of Weibull parameters m , σ_t , and σ_θ for 31 disks of polished BK7 glass with observed strengths from Fuller (Reference D-1).

The three components of the likelihood function, $\frac{m}{\sigma_\theta}$, $\left(\frac{\sigma_i - \sigma_t}{\sigma_\theta}\right)^{m-1}$, and $e^{-\left(\frac{\sigma_i - \sigma_t}{\sigma_\theta}\right)^m}$ are computed in columns D, E, and F using trial (guessed) values for the parameters m , σ_t , and σ_θ in cells G5, G6, and G7. The likelihood function for each row of the spreadsheet is the product of columns D, E, and F computed in column G. The likelihood function $\mathcal{L}$ in cell G44 is the product of 31 cells G13:G43.

The optimum parameters m , σ_t , and σ_θ in cells G5, G6, and G7 will give the maximum value of the likelihood product in cell G44. However, the magnitude of cell G44 is $\sim 10^{-71}$ and the trial parameters in cells G5:G7 are of the order 1 to 100. Excel Solver does not handle this maximization, so we reduce the size of the likelihood function by taking its natural logarithm, which is displayed in cell G9. We then use Excel Solver to find the values of the parameters in cells G5:G7 to maximize the value of $\ln \mathcal{L}$ in cell G9.

We cannot be certain that optimum values are found unless we start with a variety of different trial values in cells G5:G7 and show that they all converge to the same solution. If different trial values converge to different solutions, the solution giving the largest value of $\ln \mathcal{L}$ in cell G9 is selected. However, you must be able to converge on that best solution from several different sets of starting values or else the solution might not be optimum.

The optimum value of m in cell G5 after running Solver is 1.9137. The final step is to compute the *unbiased* value of the Weibull modulus m . There is statistical bias in the value of m for small data sets (References D-2 and D-3). ASTM C1239 instructs us to multiply m from the maximum likelihood method by the *unbiasing factor* in Table D-1 for the best estimate of the Weibull modulus:

$$m_{\text{unbiased}} = m \times \text{unbiasing factor} = 1.9137 \times 0.9565 = 1.830 \quad (\text{D-4})$$

interpolated unbiasing factor from Table D-1 for $n = 31$ points $\uparrow$

TABLE D-1. Unbiasing Factors for Weibull Modulus From ASTM C1239.

Number of points	Unbiasing factor	Number of points	Unbiasing factor	Number of points	Unbiasing factor	Number of points	Unbiasing factor
5	0.700	18	0.923	42	0.968	66	0.980
6	0.752	20	0.931	44	0.970	68	0.981
7	0.792	22	0.938	46	0.971	70	0.981
8	0.820	24	0.943	48	0.972	72	0.982
9	0.842	26	0.947	50	0.973	74	0.982
10	0.859	28	0.951	52	0.974	76	0.983
11	0.872	30	0.955	54	0.975	78	0.983
12	0.883	32	0.958	56	0.976	80	0.984
13	0.893	34	0.96	58	0.977	85	0.985
14	0.901	36	0.962	60	0.978	90	0.986
15	0.908	38	0.964	62	0.979	100	0.987
16	0.914	40	0.966	64	0.980	120	0.990

Numbers in this table are fit within a root-mean-square difference of 0.0011 by the formula
unbiasing factor = $1 - 1.593145 * N^{-1.046958}$ for $5 \leq N \leq 120$

When searching for the optimum values of the threshold strength σ_t , characteristic strength σ_θ , and Weibull modulus m , it helps to estimate initial trial values in cells G5, G6, and G7. The threshold strength can be estimated well from the graph at the right side of Figure 4. A curve running through the data points is heading toward $e^{4.8} \approx 120$ MPa, which is therefore a good guess for σ_t . To estimate the other two parameters, create a graph on the spreadsheet drawing the calculated Weibull curve as you try different values of σ_θ and m . Two features to observe as you try different values of the parameters are how well the Weibull curve goes through the data points and how large is the value of the likelihood function $\mathcal{L}$ in cell G44 or $\ln \mathcal{L}$ in cell G9.

When you have reasonable estimates of the three parameters, see if Excel can find the optimum values. A common problem is that Excel varies σ_t to a trial value greater than the lowest measured strength, which gives a numerical error and stops Solver because it cannot raise a negative number to a non-integer power. To avoid this error, you can tell Solver to constrain σ_t to be less than the lowest measured strength.

The Weibull parameters derived from the data of Fuller for the inert strength of polished BK7 glass are given in Table D-2 and the Weibull curves are shown in Figure D-2. The only Weibull parameter that we use for window design in this paper is the threshold strength σ_t . The maximum likelihood method gives $\sigma_t = 121.65$ MPa. Conventional least-squares curve fitting gives $\sigma_t = 126.20$ MPa. In our design method, we apply a safety factor of 2 to either number to give a threshold strength that we round to 60 MPa.

TABLE D-2. Weibull Parameters From Maximum Likelihood and Least-Squares Fitting of Fuller Inert Strength of Polished BK7 Glass Disks.

Maximum Likelihood Values	This Work	Fuller ^a
Weibull modulus m (biased)	1.9137	1.9136
Weibull modulus m (unbiased)	1.8304	Not given
Threshold strength σ_t	121.65	121.65
Characteristic strength σ_θ	102.29	102.29
Least-Squares Values	This Work	
Weibull modulus m (there is no bias correction)	1.6287	
Threshold strength σ_t	126.20	
Characteristic strength σ_θ	99.27	

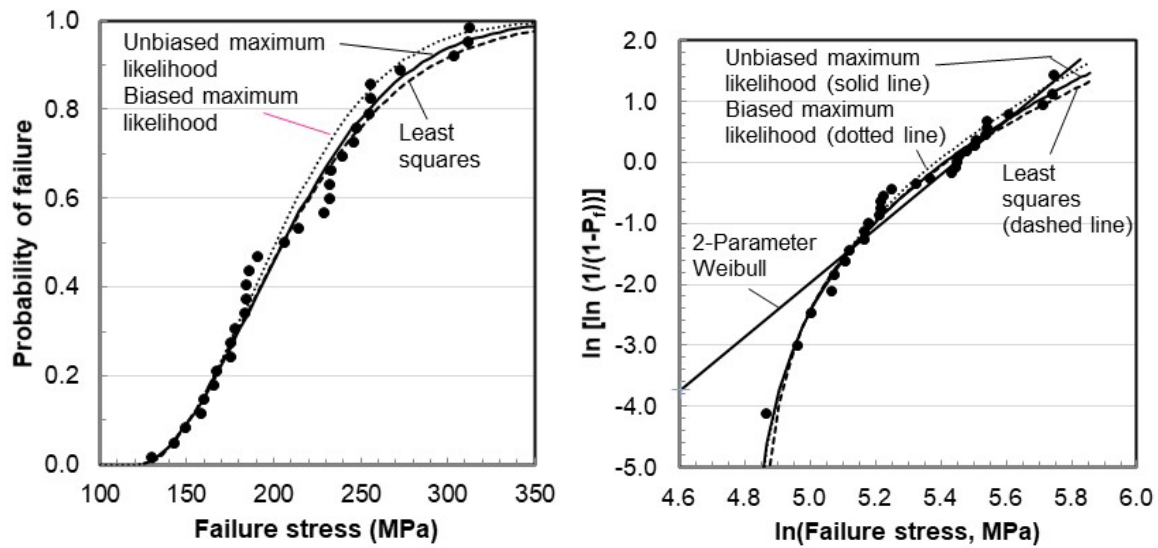
^aReference D-1.FIGURE D-2. Maximum likelihood (biased and unbiased values of m) and conventional least-squares fits to data of Fuller (Reference D-1) for inert strength of polished BK7 glass disks.

Figure D-3 shows the determination of the 3-parameter Weibull parameters for Schott BK7 glass labeled “D64 & Etched,” whose Weibull plot was shown in Figure 15. Table D-3 lists the Weibull and least-squares fitting parameters.

						From Solver	Unbiased
$\mathcal{L} = \prod_{i=1}^n p_i = \prod_{i=1}^n \left(\frac{m}{\sigma_\theta}\right) \left(\frac{\sigma_i - \sigma_t}{\sigma_\theta}\right)^{m-1} e^{-\left(\frac{\sigma_i - \sigma_t}{\sigma_\theta}\right)^m}$					m =	1.360848168	1.261506
					σ_t =	151.8	
					σ_θ =	63.69548092	
	x	y			ln (likelihood product)	-90.1929	
	Failure	Probability	D	E	F		
Sample	stress	of failure	Components of likelihood function			Likelihood function	
number (i)	σ (MPa)	Pf = (i-0.5)/n	m/ σ_θ	$((\sigma - \sigma_t)/\sigma_\theta)^{m-1}$	$\exp[-((\sigma - \sigma_t)/\sigma_\theta)^m]$	D*E*F	
1	153.99	0.026315789	2.136E-02	2.965E-01	9.898E-01	6.270E-03	
2	162.80	0.078947368	2.136E-02	5.306E-01	9.125E-01	1.034E-02	
3	162.00	0.131578947	2.136E-02	5.164E-01	9.206E-01	1.016E-02	
4	169.04	0.184210526	2.136E-02	6.240E-01	8.446E-01	1.126E-02	
5	177.83	0.236842105	2.136E-02	7.240E-01	7.439E-01	1.151E-02	
6	182.84	0.289473684	2.136E-02	7.715E-01	6.866E-01	1.132E-02	
7	184.95	0.342105263	2.136E-02	7.900E-01	6.629E-01	1.119E-02	
8	195.84	0.394736842	2.136E-02	8.753E-01	5.459E-01	1.021E-02	
9	200.05	0.447368421	2.136E-02	9.046E-01	5.040E-01	9.740E-03	
10	206.70	0.5	2.136E-02	9.478E-01	4.418E-01	8.946E-03	
11	213.57	0.552631579	2.136E-02	9.890E-01	3.832E-01	8.097E-03	
12	222.85	0.605263158	2.136E-02	1.040E+00	3.134E-01	6.965E-03	
13	226.89	0.657894737	2.136E-02	1.061E+00	2.862E-01	6.489E-03	
14	243.82	0.710526316	2.136E-02	1.142E+00	1.921E-01	4.686E-03	
15	257.34	0.763157895	2.136E-02	1.200E+00	1.369E-01	3.511E-03	
16	266.77	0.815789474	2.136E-02	1.238E+00	1.071E-01	2.833E-03	
17	272.95	0.868421053	2.136E-02	1.261E+00	9.084E-02	2.448E-03	
18	288.08	0.921052632	2.136E-02	1.316E+00	5.988E-02	1.683E-03	
19	376.08	0.973684211	2.136E-02	1.575E+00	3.904E-03	1.314E-04	
					product =	6.7566E-40	

TABLE D-3. Weibull Parameters From Maximum Likelihood and Least-Squares Fitting of Schott TIE-33 Inert Strength of Polished BK7 Glass Disks “D64 & Etched” Shown in Figure 15.

Maximum Likelihood Values	This Work	Schott TIE-33
Weibull modulus m (biased)	1.3608	---
Weibull modulus m (unbiased)	1.2615	---
Threshold strength σ_t	151.8	150.1
Characteristic strength σ_θ	63.7	---
Least-Squares Values	This Work	
Weibull modulus m (biased)	1.313	
Threshold strength σ_t	146.6	
Characteristic strength σ_θ	78.9	

Digitized data from Figure 9 of Reference D-4. Weibull parameters listed in Table 1 of TIE-33 are not correct.

REFERENCES

- D-1. E. R. Fuller, S. W. Freiman, J. B. Quinn, G. D. Quinn, and W. C. Carter. “Fracture Mechanics Approach to the Design of Glass Aircraft Windows: A Case Study,” *Proc SPIE*, Vol. 2286 (1994), p. 419.
- D-2. M. Ambrožič and L. Gorjan. “Reliability of a Weibull Analysis Using Maximum-Likelihood Method,” *J. Mater. Sci.*, Vol. 46 (2011), pp. 1862–1869.
- D-3. D. R. Thoman, L. J. Bain, and C. E. Antle. “Inferences on the Parameters of the Weibull Distribution,” *Technometrics*, Vol. 11 (1969), pp. 445–460.
- D-4. Schott Glass. “Bending Strength of Optical Glass and Zerodur®.” Germany, Advanced Optics, Schott AG, December 2015. (Technical Information Document TIE-33.)

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Appendix E

STRESS DISTRIBUTION AND EFFECTIVE AREA OF SENSOR WINDOW

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Stress distribution. Radial and hoop (circumferential) components of stress in the plane of the tensile face of the sensor window in Figure 1 of the main body of this document are given by (Reference E-1):

$$\text{Radial stress} = \frac{3\Delta P b^2}{8d^2} \left[(1 - \nu) \left(\frac{b^2}{c^2} \right) + 2(1 + \nu) - (3 + \nu) \left(\frac{r^2}{b^2} \right) \right] + \frac{(3 + \nu)\Delta P}{4(1 - \nu)} \quad (\text{E-1})$$

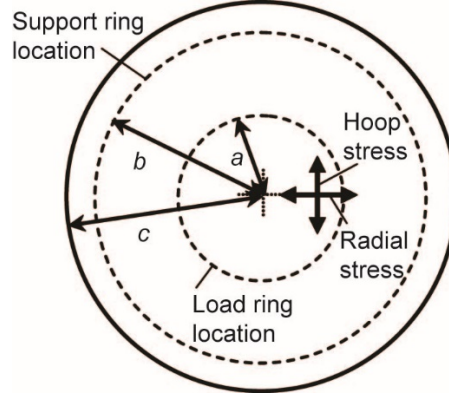
$$\text{Hoop stress} = \frac{3\Delta P b^2}{8d^2} \left[(1 - \nu) \left(\frac{b^2}{c^2} \right) + 2(1 + \nu) - (1 + 3\nu) \left(\frac{r^2}{b^2} \right) \right] + \frac{(3 + \nu)\Delta P}{4(1 - \nu)} \quad (\text{E-2})$$

where b is the O-ring radius, c is disk radius, d is disk thickness, ν is Poisson's ratio, and ΔP is the pressure difference across the window. At the center of the window, $r = 0$ and the two components of stress are equal and have their maximum value. The radial and hoop stresses are shown in Figure 2. The maximum stress in the window is taken to be the design stress:

$$\text{Design stress} = \frac{3\Delta P b^2}{8d^2} \left[(1 - \nu) \left(\frac{b^2}{c^2} \right) + 2(1 + \nu) \right] + \frac{(3 + \nu)\Delta P}{4(1 - \nu)} \quad (\text{E-3})$$

Effective area of sensor window and equibiaxial flexure test specimen. We usually describe the distribution of strength for equibiaxial flexure test disks with a 2-parameter Weibull equation having a Weibull modulus m and a scale factor σ_0 . We expect that the strength of a specimen decreases with increasing effective area, which depends on the geometric area and the stress distribution in that area. The greater the effective area, the lower is the expected mean strength because the probability of finding a weak flaw increases with area and with stress. Similarly, the expected mean strength of full-size windows also decreases with increasing effective area. Table E-1 gives formulas for the effective areas of the flexure test disk and of the sensor window.

TABLE E-1. Effective Area (A_e) in Tension for Biaxial Flexure Disk and Pressure-on-Ring Window.



Equibiaxial flexure test coupon

$$A_e = 2\pi a^2 \left\{ 1 + \frac{44(1+\nu)}{3(1+m)} \frac{5+m}{2+m} \left(\frac{b-a}{bc} \right)^2 \left[\frac{2c^2(1+\nu) + (b-a)^2(1-\nu)}{(3+\nu)(1+3\nu)} \right] \right\} \quad (E-4)$$

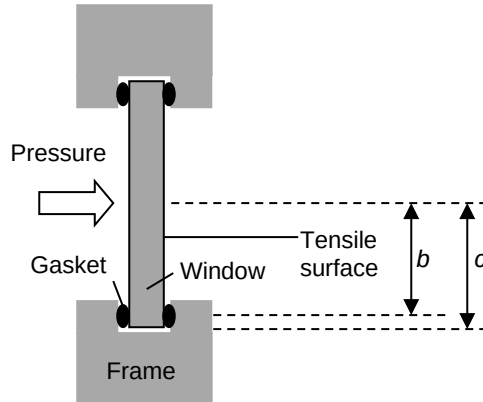
a = load radius

b = support radius

c = disk radius

m = Weibull modulus

ν = Poisson's ratio



Pressure-on-ring window (Reference E-1)

$$A_e \approx \frac{4\pi(1+\nu)}{1+m} \left(\frac{b}{c} \right)^2 \left[\frac{2c^2(1+\nu) + b^2(1-\nu)}{(3+\nu)(1+3\nu)} \right] \quad (E-5)$$

b = support radius

c = disk radius

m = Weibull modulus

ν = Poisson's ratio

REFERENCES

- E-1. J. A. Salem, and M. Adams. "The Multiaxial Strength of Tungsten Carbide," *Ceramic Eng. Sci. Proc.*, Vol. 20 (1999), pp. 459-466. See also ASTM C1683, "Standard Practice for Size Scaling of Tensile Strengths Using Weibull Statistics for Advanced Ceramics."

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Appendix F

**COMPARISON OF EXPONENTIAL LAW AND POWER LAW
EQUATIONS FOR PREDICTING WINDOW LIFETIME**

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Empirical Equations F-1a and F-1b are commonly used to fit experimental measurements relating the rate of slow crack growth to the stress intensity factor:

Exponential law:
$$v = v_o e^{\beta K_I} \quad (F-1a)$$

$v = da/dt$, where a = flaw depth and t = time

K_I = stress intensity factor

v_o and β are measured material parameters

Power law:
$$v = A^* \left(\frac{K_I}{K_{Ic}} \right)^n \quad (F-1b)$$

A^* and n are measured material parameters

K_{Ic} = critical stress intensity factor for mechanical failure

The exponential law is a conservative alternative to the power law for predicting window lifetime from slow crack growth rates. NASA STD 5018 specifies that the exponential law shall be employed for calculating window time to failure. We surmise that the reason for NASA calling out the exponential law is that it predicts shorter lifetimes than does the power law.

Figure F-1 compares exponential and power law fits to the same set of measured slow crack growth rates taken from Wiederhorn in Figure 7. The exponential and power laws approximately coincide in the range of experimental observations in Figure F-1. They diverge below (and above) the measured crack growth rates. In this case, the power law gives lower crack growth rates than the exponential law for stress intensities below $0.40 \text{ MPa}\sqrt{\text{m}}$. Most of the slow crack growth in the life of a window occurs at stress intensities below $0.40 \text{ MPa}\sqrt{\text{m}}$, so the power law predicts longer lifetimes than the exponential law in Table F-1. The power law in Figure F-1 predicts a design life 100 times longer than predicted by the exponential law (4,500 versus 45 years (yr)).

The power law is used more often than the exponential law because the power law can be integrated analytically to obtain useful equations to predict loss of strength in service or in a proof test. The exponential law requires numerical integration.

For fused silica with $K_{Ic} = 0.75 \text{ MPa}\sqrt{\text{m}}$, the power law and exponential law coincide pretty well in the range $(K_I/K_{Ic}) = 0.53$ to 0.73 ($K_I = 0.40$ to 0.55 in Figure F-1). The two laws are similar in their performance for interpreting dynamic fatigue experiments to derive slow crack growth parameters. This same range of (K_I/K_{Ic}) is relevant in severe proof tests that produce significant crack growth during the proof test. The power law is useful for dynamic fatigue and severe proof tests. For window lifetime prediction, the two laws give different results. The exponential law is the more conservative of the two.

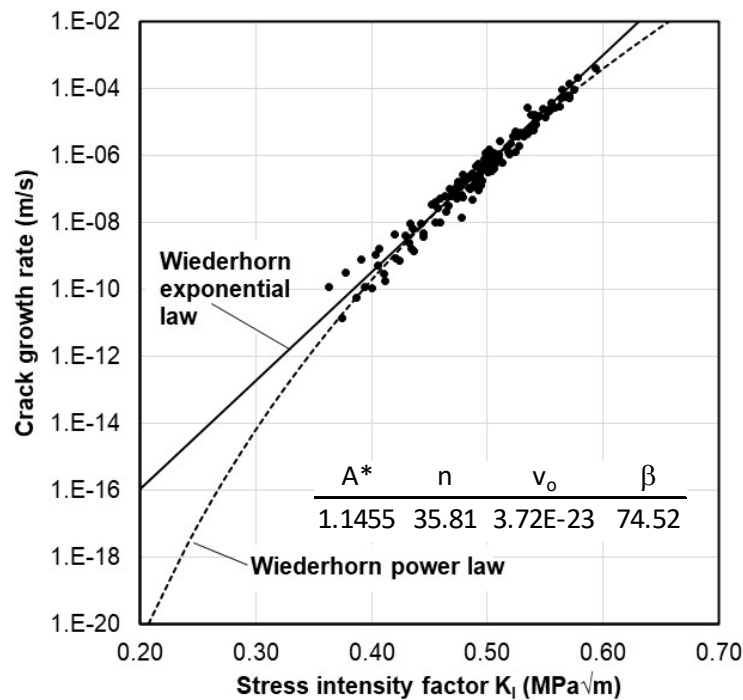


FIGURE F-1. Comparison of exponential law $v = v_o e^{\beta K_I}$ and power law $v = A^* (K_I/K_{Ic})^n$ fit to the 126-point data set for slow crack growth in fused silica near 25°C in air at 100% relative humidity. The units of A^* and v_o are m/s. Parameters are from lines 1 and 4 of Table 4. Data from Reference B-1 were digitized to conduct this analysis. Slope and intercept of the digitized data fit to the equation $K_I = a + b \ln(v)$ were within 0.5% and 0.2% of the values reported by Wiederhorn.

TABLE F-1. Fused Silica Window Design Life From Exponential and Power Laws.

Slow Crack Growth Parameters	Design Life, yr
Service stress (σ_a) = 4.82 MPa; proof stress (σ_p) = 14.7 MPa; initial flaw depth = 800 μm	
Wiederhorn exponential law crack growth parameters Table 4 line 1	45
Wiederhorn power law crack growth parameters Table 4 line 4	4,500
Salem exponential law crack growth parameters Table 4 line 2	1.1
Salem exponential law crack growth parameters Table 4 line 3	2.0
Salem power law crack growth parameters Table 4 line 5	26
Salem power law crack growth parameters Table 4 line 6	13

TABLE F-2. Fused Silica Slow Crack Growth Points Plotted in Figure F-1.

log(v, m/s)	K _I (MPa√m)	log(v, m/s)	K _I (MPa√m)	log(v, m/s)	K _I (MPa√m)	log(v, m/s)	K _I (MPa√m)	log(v, m/s)	K _I (MPa√m)
-9.9131	0.36278	-7.4032	0.45482	-7.0195	0.48553	-6.0636	0.50292	-5.2568	0.53685
-10.8416	0.37454	-7.9948	0.45546	-7.3091	0.48733	-6.3594	0.50303	-4.7917	0.53794
-9.5068	0.37784	-7.566	0.45699	-6.5302	0.48746	-6.4619	0.50434	-5.16	0.53808
-10.2425	0.38699	-7.9945	0.45927	-6.8923	0.48801	-6.3834	0.50515	-5.2686	0.53981
-9.9278	0.39405	-7.294	0.45985	-6.3307	0.48992	-6.3833	0.50557	-5.1054	0.5406
-9.1129	0.39122	-7.209	0.46404	-6.7351	0.49007	-5.9485	0.50626	-4.7974	0.54133
-9.9633	0.40041	-7.6859	0.46507	-7.0429	0.4923	-6.22	0.50763	-4.9241	0.5418
-8.9727	0.40384	-7.4985	0.46711	-6.2513	0.49285	-6.0689	0.50968	-5.0811	0.54186
-9.2985	0.40608	-7.0154	0.46735	-6.6261	0.49383	-5.9782	0.51049	-4.62	0.5485
-8.7852	0.40631	-7.2086	0.46827	-6.38	0.493	-5.5676	0.51076	-4.8151	0.54471
-9.5394	0.41082	-7.0632	0.47244	-6.44	0.496	-6.2134	0.51312	-4.8568	0.55065
-9.7326	0.41174	-7.3046	0.4738	-6.8977	0.49478	-5.754	0.5176	-4.6753	0.55354
-8.3732	0.41968	-6.9181	0.47408	-6.7648	0.49515	-5.8868	0.51849	-4.4336	0.55514
-9.0614	0.42078	-6.8032	0.4753	-6.3542	0.49542	-5.9531	0.51936	-4.6086	0.55606
-9.1336	0.42378	-7.2258	0.47588	-6.2817	0.49624	-5.6329	0.52051	-4.5721	0.55899
-8.3963	0.42941	-6.9298	0.47746	-6.1064	0.49744	-5.4214	0.52212	-4.5474	0.56237
-8.6193	0.43287	-7.1774	0.47755	-6.3116	0.49836	-5.45	0.52635	-4.2998	0.56354
-8.0456	0.43393	-7.8415	0.4778	-5.9493	0.49865	-5.7105	0.52772	-4.0401	0.56471
-8.7942	0.43463	-7.2557	0.47928	-6.2331	0.49876	-5.2883	0.52461	-4.2512	0.56733
-8.1722	0.43566	-6.5854	0.47945	-6.2209	0.4996	-5.886	0.52483	-4.2147	0.569
-8.8483	0.43676	-6.7302	0.48035	-6.3113	0.50005	-5.312	0.52842	-3.8643	0.57056
-8.0326	0.44237	-6.6875	0.48456	-6.5106	0.50097	-5.3602	0.53013	-4.3051	0.57073
-8.3101	0.44459	-6.9592	0.48466	-5.8222	0.50156	-5.4384	0.53227	-4.2084	0.57111
-8.4127	0.44463	-6.609	0.48495	-6.3898	0.50177	-5.3777	0.53521	-4.0571	0.57486
-7.4639	0.4523	-6.6391	0.48538	-6.244	0.50256	-4.5807	0.53533	-3.6943	0.57811
								-3.3968	0.59405

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Appendix G

**STRESS INTENSITY FACTOR AND WINDOW LIFETIME EQUATION
FROM FRACTURE MECHANICS**

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Figure G-1 shows a crack of depth a in a brittle material such as glass or ceramic. When tensile stress σ_{app} is applied across the crack, there is a high stress region near the crack tip.

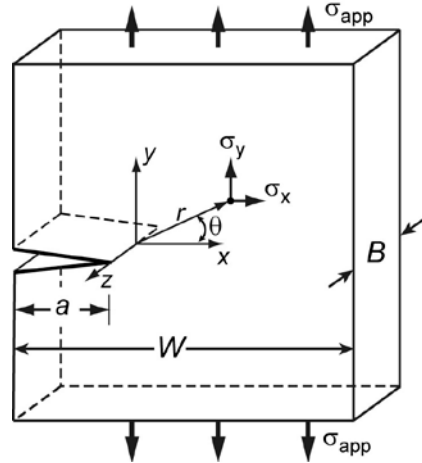


FIGURE G-1. Crack of depth a in a brittle material subjected to external stress σ_{app} .

Stress contours in the immediate vicinity of the crack tip are displayed in Figure G-2. With the polar coordinates r and θ from Figure G-1, the stress contours are described by an equation of the form

$$\text{Stress } (\sigma) = \frac{K_I}{\sqrt{2\pi r}} f(\theta) \quad (\text{G-1})$$

where $f(\theta)$ is a function of the angle θ and K_I is called the *mode factor*. Mode I refers to pulling a crack open by force perpendicular to the crack, as depicted in Figure G-1. According to Equation G-1, stress in the material becomes large as the distance (r) from the crack tip approaches 0.

Stress near the crack tip is proportional to K_I . The greater the value of K_I , the higher the stress near the crack tip. If the stress intensity factor increases to the critical value K_{Ic} , the crack opens up so rapidly that mechanical failure of the material occurs. The *critical stress intensity factor* K_{Ic} is also called the *fracture toughness* of the material. Fracture toughness measures resistance to crack propagation.

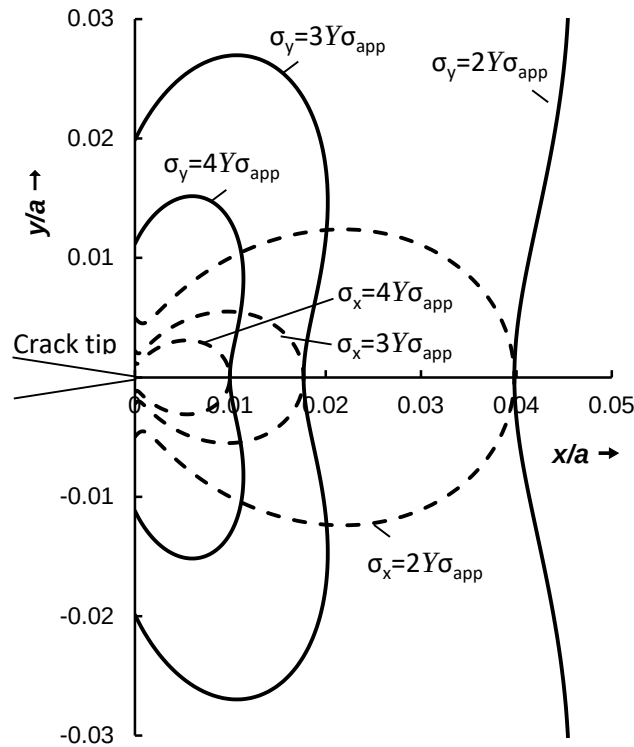


FIGURE G-2. Stress contours near crack tip shown as multiples of applied external stress. The distance 0.01 on each axis is one hundredth of the depth a of the crack.

Stress in Equation G-1 increases sharply as distance from the crack tip approaches 0. High stress could produce plastic deformation of material near the crack tip. Plastic deformation is more common in metals, but less common in brittle ceramics. Our window design is based on the approach of treating ceramic deformation as purely elastic.

If there is a crack of depth a on the surface of a material, crack velocity (v) is the rate of increase of crack depth with time (t):

$$\text{Crack velocity (v)} = \frac{da}{dt} \quad (\text{G-2})$$

In fracture mechanics, the relationship between stress intensity factor, applied stress (σ_a), and crack depth (a) is

$$K_I = \sigma_a Y \sqrt{a} \quad (\text{G-3})$$

where Y is a geometric factor, generally in the range 1 to 2, that depends on the geometry and location of the crack. If we replace K_I in Equation G-3 with the critical stress intensity factor K_{IC} , the applied stress becomes identical to the strength (S) of the material:

$$K_{Ic} = SY\sqrt{a} \quad (G-4)$$

Dividing Equation G-3 by Equation G-4, we obtain another relationship that will be useful to us for calculating crack propagation rate:

$$\frac{K_I}{K_{Ic}} = \frac{\sigma_a Y \sqrt{a}}{SY\sqrt{a}} = \frac{\sigma_a}{S} \quad (G-5)$$

For window design, we need to know how long it will take for a crack of some initial size subjected to applied stress (σ_a) to grow large enough to fracture. This question is equivalent to asking how long it will take for the initial stress intensity factor K_{Ii} to increase to the critical stress intensity factor K_{Ic} by crack growth under the influence of the applied stress.

Derivation of the lifetime equation begins with the square of Equation G-3:

$$K_I^2 = \sigma_a^2 Y^2 a \quad (G-6)$$

If crack size is small with respect to specimen size, Y is nearly constant as the crack grows. Treating Y as a constant and taking the time derivative of both sides of Equation G-6 gives

$$\frac{d}{dt}(K_I^2) = \frac{d}{dt}(\sigma_a^2 Y^2 a) \Rightarrow 2K_I \frac{dK_I}{dt} = \sigma_a^2 Y^2 \frac{da}{dt}$$

Substituting v for da/dt on the right side of the equation gives

$$2K_I \frac{dK_I}{dt} = \sigma_a^2 Y^2 v$$

Then separate the variables K_I and t :

$$\frac{2K_I dK_I}{\sigma_a^2 Y^2 v} = dt$$

Finally, integrate from $t = 0$ to $t = t_{failure}$ and from K_{Ii} at $t = 0$ to K_{Ic} at $t = t_{failure}$.

$$\begin{aligned} \int_0^{t_{failure}} dt &= \int_{K_{Ii}}^{K_{Ic}} \frac{2K_I dK_I}{\sigma_a^2 Y^2 v} \\ t_{failure} &= \frac{2}{\sigma_a^2 Y^2} \int_{K_{Ii}}^{K_{Ic}} \left(\frac{K_I}{v} \right) dK_I \end{aligned} \quad (G-7)$$

Equation G-7 allows us to calculate the time for a crack whose initial depth is $a = K_{Ii}^2 / (\sigma_a^2 Y^2)$ [from Equation G-6] to grow to failure under the constant stress σ_a .

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Appendix H

**COMPARISON OF SLOW CRACK GROWTH RATE IN VACUUM
AND IN 100% HUMIDITY**

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The purpose of this appendix is to support the case that BK7 glass crack growth rate in a dry environment is many orders of magnitude slower than crack growth rate in a wet environment. Wiederhorn has measured the temperature dependence of slow crack growth of several glasses in vacuum. He did not measure BK7, which is also called borosilicate crown I in his papers. He did measure UBK7, which is also called borosilicate crown II in his papers. The difference between the two is 1 wt% CeO₂ added to make BK7.

In Reference H-1, the following glass compositions (weight percent) are given:

	SiO ₂	B ₂ O ₃	Na ₂ O	K ₂ O	CaO	BaO	CeO ₂
BK7 Borosilicate Crown I	69	11	10	7	0.2	2	1
UBK7 Borosilicate Crown II	70	11	10	7	0.2	2	0

Table H-1 gives slow crack growth parameters for BK7 and UBK7 in air at 100% humidity at “room temperature.”

TABLE H-1. BK7 Glass and UBK7 Glass Slow Crack Growth Parameters at 100% Humidity at Room Temperature.

		$K_I = a + b \ln(v)$		$v = v_0 e^{\beta K_I}$	
		a (MPa√m)	b (MPa√m)	v_0 (m/s)	β (1/[MPa√m])
1. BK7 glass ^a	$v > 10^{-6.5}$ m/s	0.7946	0.0253	2.291×10^{-14}	39.53
2.	$v < 10^{-6.5}$ m/s	0.5826	0.01177	3.184×10^{-22}	84.96
3. UBK7 glass ^a	$v > 10^{-6.5}$ m/s	0.7872	0.02674	1.640×10^{-13}	37.40
4.	$v < 10^{-6.5}$ m/s	0.5976	0.01348	5.581×10^{-20}	74.18

^aReference H-2. Samples were tested in air at 100% humidity.

Table H-2 gives the temperature dependence of the rate of slow crack growth for UBK7 glass in vacuum. We estimate below that the crack growth rate in 100% humidity is 10^{20} times faster than the rate in vacuum at a stress intensity factor $K_I = 0.44$ MPa√m. This stress intensity factor was chosen because it is the peak K_I value in a 20 MPa proof test of a window whose initial strength is 43 MPa. It is in the measured data range at 100% humidity but far below the measured range for vacuum.

Calculated crack growth rates in UBK7 in vacuum and in air with 100% humidity

For stress intensity factor $K_I = 0.44$ MPa√m (4.4×10^5 Pa√m):

Crack growth rate in vacuum: $v = v_0 e^{bK_I/RT} e^{-E^*/RT} = 2 \times 10^{-26}$ m/s at 293 K (Table H-2)

Crack growth rate in 100% humidity: $v = 2 \times 10^{-6}$ m/s at 293 K (Table H-1 line 3)

TABLE H-2. Temperature Dependence of Slow Crack Growth for UBK7 Borosilicate Crown Glass II in Vacuum (Valid for 298 to 668 K With Crack Velocity $\approx 10^{-7.3}$ to $10^{-5.3}$ m/s).

Crack growth rate:	$v = v_o e^{bK_I/RT} e^{-E^*/RT}$			(H-1)
v = crack velocity (m/s)				
E* = activation energy (J/mol)				
v _o , b, and E* are empirical fitting parameters				
K_I = stress intensity factor (Pa√m = $\frac{N\sqrt{m}}{m^2}$) (1 MPa√m = 10 ⁶ Pa√m)				
R = gas constant = 8.31446 J/(mol·K)				
T = temperature (K)				
	E^* (J/mol)	b (SI units = $\frac{m^2 \cdot J}{N \cdot mol \sqrt{m}}$)	v_o (m/s)	
Borosilicate crown II (UBK7)	2.75×10 ⁵	0.26	740	

Crack growth rate equation and parameter values from Reference H-3.

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