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Impact of uncertain environmental knowledge on the shallow-water transfer function

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Impact of uncertain environmental knowledge on the shallow-water transfer function

D.F. Gingras

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Impact of uncertain environmental knowledge on the shallow-water transfer function

D.F. Gingras

Executive Summary: Acoustic source localization (target localization) using an array of sensors is a problem of interest for sonar signal processing. Incorporation of the acoustic channel model into target localization array processing has been shown to perform quite well in many situations. The ability to localize in range, depth and bearing is extremely attractive. An issue of concern is the strong dependence of performance on precise knowledge about the environmental parameters. The objective of this memorandum is to characterize the effect of uncertainty about environmental knowledge on channel-model localization methods for the shallow-water application.

In the analysis, the acoustic channel transfer function, i.e. channel response *vs* frequency, is employed as the primary tool to evaluate the sensitivity to uncertainty about environmental knowledge. A broadband channel transfer function error measure is defined and evaluated as a function of perturbation of water sound speed, sound speed in the bottom, channel depth and bottom attenuation.

An example is provided which indicates that for some shallow-water applications the channel transfer function is quite sensitive to perturbation of the water sound speed, subbottom sound speed and the channel depth. Results are illustrated for winter and summer environments with and without a sediment layer.

This memorandum serves as a precursor to future work on the development of channel model-dependent localization methods which are 'robust', that is, the performance is less sensitive to uncertainty about environmental knowledge.

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Impact of uncertain environmental knowledge on the shallow-water transfer function

D.F. Gingras

Abstract: It is assumed that the acoustic channel can be modeled as a linear time-invariant space-variant filter. In this case, from linear systems theory, it is known that the filter output, that is the predicted source signal replica at the sensor location, is formed by the convolution of the source signal with the filter impulse response function. The acoustic channel impulse response function is a function of the source location, the sensor location and the environmental parameters. In this analysis we use the channel transfer function, the fourier transform of the channel impulse response function, as a function of frequency to evaluate the effect of environmental parameter uncertainty. In the course of this work we establish a measure, referred to as the 'transfer function error measure' which provides an estimate of the average error for the channel transfer function due to environmental parameter uncertainty as a function of receiver depth. This error measure, which is also a function of the source location, is used to characterize the sensitivity of propagation model-based array processors to uncertainty in environmental knowledge such as water sound speed, bottom sound speed, bottom attenuation and channel depth. For simple canonical shallow-water channels, winter and summer profiles, results are presented which illustrate the effect of uncertain environmental knowledge on the transfer function error measure.

Keywords: ○ environmental uncertainty ○ matched-field processing ○ normal modes ○ shallow water ○ transfer function

Contents

1
Introduction

The localization of an acoustic source in an underwater environment using measurements from an array of sensors is often a problem of interest for sonar signal processing. Source localization is usually accomplished through a correlation process whereby a vector of sensor outputs are correlated with a vector of predicted source signal replicas. The source signal replica vector is constructed by assuming the source signal is known then predicting the effect of the underwater channel on the source signal as received at the sensor locations as a function of source location parameters. The regions of the source signal parameter space where this correlation is large are taken to be the estimated source locations.

For many years the only channel effect that array signal processing incorporated into the signal replica prediction was propagation time delay. More recently the work of Hinich [1] and Bucker [2], which uses a deterministic full field propagation model to predict the replica vectors, has been applied [3]. This type of array signal processing, which is a relatively straightforward extension of traditional array processing, is often referred to as 'matched-field' processing. It is, in fact, an example of a more general class of signal processing methods referred to as model-based signal processing. As with any model-based signal processing method the performance is highly dependent on the accuracy of the knowledge about the actual system being modeled and the accuracy of the model implementation. That is, if the model represents the system accurately then the performance will be quite good, but if the model does not accurately represent the system the overall performance may be significantly degraded. In fact, it is possible that the performance of the model-based processor may be substantially inferior to that of a processor which uses no *a priori* information.

There are, at least, three major considerations affecting the performance of propagation model-based localization processors, they are: (1) ***system parameters*** such as the array configuration, the source frequency and bandwidth, (2) ***environmental parameters*** such as the channel depth, water sound speed profile, sediment properties and subbottom properties, and (3) ***environmental uncertainty*** that is uncertainty in the knowledge about the above environmental properties. In this memorandum only the third consideration, environmental uncertainty, is addressed. A major goal was the development of methods to evaluate the sensitivity to environmental uncertainty independent of the system parameters.

For the model-based localization problem the knowledge required consists of detailed environmental parameters such as sound speed structure, density and attenuation in both the water and bottom. In any real ocean area the uncertainty in knowledge about the environmental parameters may be significant, since they may vary both deterministically and stochastically as a function of space and time. Since the signal replica vectors are constructed using a numerical solution of a second-order differential equation the relationship between uncertainty in knowledge about the environmental parameters and the performance of the array processor is highly non-linear and difficult to predict. We make the assumption, which is true for most channels, that if the environmental knowledge is accurate the numerical model can accurately predict the complex transfer function. Thus, the goal of this memorandum is to provide insight into the relationship between uncertainty in knowledge about the environmental parameters and its effect on the predicted signal replica vectors used in model-based array processing.

Often it is very reasonable to assume that the acoustic channel can be modeled as a linear time-invariant space-variant filter. In this case, from linear systems theory, we know that the filter output, that is the predicted source signal replica at the sensor location, is formed by the convolution of the source signal with the filter impulse response function. The acoustic channel impulse response function is a function of the source location, the sensor location and the environmental parameters. The channel impulse response function is also referred to as the Green's function. The channel transfer function, the fourier transform of the channel impulse response function, for an acoustic channel between any two points in the channel can be computed using a numerical propagation model and the set of environment parameters. In the work reported herein the SACLANTCEN Normal-mode Acoustic Propagation (SNAP) model has been used to compute the channel transfer function [4].

The channel transfer function, magnitude and phase, as a function of frequency is used to analyze the effect of environmental parameter uncertainty on the accuracy of the predicted signal replica vectors. In the course of this work we also establish a measure, referred to as the 'transfer function error measure' which provides an estimate of the average error for the channel transfer function due to environmental parameter uncertainty as a function of receiver depth. This error measure, which is also a function of the source location, is used to characterize the sensitivity of propagation model-based array processors to uncertainty in environmental knowledge. Since this measure is evaluated as a function of depth, for any depth, it provides a measure which is not dependent on an assumed array configuration.

There has been a number of papers that have dealt with the effect of uncertainty about environmental knowledge on the detection and localization performance of matched-field processors [5–9]. References [5–8] discuss this problem for shallow water applications, in general, analyzing the effects of environmental and system parameter uncertainty on processor performance at a single frequency for one or a small number of source locations. Reference [9], deals primarily with sound speed uncertainty in the water column for a deep water case, again for a single frequency and a small number of source locations. The effect of environmental parameter uncertainty on matched-field processing is strongly dependent on the assumed source location [10] yet none of the previous papers have dealt with this issue. In contrast to most previous papers on this subject this memorandum examines the sensitivity to environmental parameter uncertainty for broadband sources and uses the frequency dependent channel transfer function, which is a function of source and receiver location, as the performance measure.

The end goal of this work is to characterize the effect of uncertainty about environmental knowledge on the channel transfer function so that the effect of this uncertainty can be incorporated into the development of a processor which is robust to the uncertainty. It is expected that a robust processor will improve matched-field detection and localization performance in the presence of uncertainty about environmental knowledge.

This memorandum is organized as follows: Section 2 provides the form of the signal model, with assumptions, and the form of the matched-field processor in terms of the channel transfer function. Section 3 provides a normal mode representation for the channel transfer function and the mathematical representation for the transfer function error measures. Section 4 illustrates the effect of perturbation of environmental parameters on the channel transfer function for simple shallow water channels. Finally, in Sect. 5, some conclusions are presented.

We remark that evaluation of the sensitivity of the channel transfer function to environmental parameter uncertainty is of course an intermediate measure. A more global measure would be the sensitivity of the model-based processor output power ratio as discussed in [10]. But, evaluation of the output power ratio includes the effect of the array configuration and the processing algorithm. Inclusion of these factors, while more complete, makes it difficult to isolate and understand the effect of the environmental parameter uncertainty. Since the output power of the conventional processor is a linear function of the transfer function predictions, errors in these predictions will directly impact the processor output power. It is expected that evaluation of the channel transfer function sensitivity to environmental parameter uncertainty will provide considerable insight into the development of model-based processors that are tolerant to environmental parameter uncertainty.

Throughout this memorandum vectors are denoted by boldface lowercase letters, matrices are denoted by boldface uppercase letters and $\mathbf{u}^*$ indicates conjugate transpose of the vector $\mathbf{u}$.

2
Preliminaries

We assume throughout that we are working in a horizontally-stratified acoustic waveguide or channel which is characterized by a depth varying sound speed and certain boundary conditions. A cylindrical coordinate system $\{r, \theta, z\}$ is used with the depth axis, the z-axis, passing through the receivers. The field is assumed to be independent of azimuthal angle, θ. Furthermore, we assume that the propagation of acoustic signals through the channel can be described by the linear, inhomogeneous, scalar wave equation. Since the wave equation for small-amplitude acoustic signals is linear we can represent the channel as a linear time-variant, space-variant filter. Assume that the signal-plus-noise observations at a receiver location denoted by the vector $\mathbf{v}_j$ is a broadband random process denoted by $y_{\mathbf{v}_j}(t)$ and that this process consists of a stationary random signal component emitted at some location $\boldsymbol{\beta} = (r, z)$ convolved with the channel impulse response function plus stationary random noise. In general, the channel impulse response function $h_{\mathbf{v}_j}(\boldsymbol{\beta}, \boldsymbol{\theta}; t)$ is a linear time-variant space-variant function of the receiver location $\mathbf{v}_j$, the source location $\boldsymbol{\beta}$, and a vector of channel parameters $\boldsymbol{\theta}$ which contains the environmental properties for the specific acoustic channel. The environmental parameters usually consist of at least the sound speed *vs* depth in the water column, the thickness, density, attenuation and sound speed *vs* depth for the sediment and subbottom density, attenuation and sound speed. For the analysis considered herein the environmental parameters are assumed to be range-independent and the channel is assumed to be time-invariant.

The channel filter between any two locations in the channel $\mathbf{v}_j$ and $\boldsymbol{\beta}$ is characterized in the time domain by the impulse response function $h_{\mathbf{v}_j}(\boldsymbol{\beta}, \boldsymbol{\theta}; t)$ or in the frequency domain by the transfer function $h_{\mathbf{v}_j}(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega)$. Now, for convenience, since the receivers are located on the depth axis at the origin of the coordinate system let the subscript j denote the receiver depth coordinate.

Assume that there is a single broadband source plus additive noise, the observations at the jth sensor are modeled by the convolution plus noise

$$y_j(t) = u(t) * h_j(\boldsymbol{\beta}, \boldsymbol{\theta}; t) + n_j(t) \tag{1}$$

for $t = 0, 1, \ldots, T-1$ and $1 \leq j \leq L$, where $\boldsymbol{\beta}$ is the vector indicating the location of a broadband source, j the index for vector $\mathbf{v}_j$ which denotes the location of the jth sensor, $u(t)$ the random process radiated by the source, $h_j(\boldsymbol{\beta}, \boldsymbol{\theta}; t)$ the channel impulse response function for the channel between the source at $\boldsymbol{\beta}$ and the sensor at $\mathbf{v}_j$ (the channel is a function of the parameter vector $\boldsymbol{\theta}$ which represents knowledge about the environment), and $n_j(t)$ the additive noise at the jth sensor.

The source process $u(t)$ is a broadband zero-mean stationary gaussian process, uncorrelated with the additive noises with spectral density $\phi_u(\omega)$. While $u(t)$ is not strictly defined as a bandlimited process we assume that a majority of its power is in a finite band of width B Hz. The additive noises $\{N_j(t)\}$ are broadband zero-mean correlated stationary gaussian processes with cross-spectral density matrix $\mathbf{Q}(\omega)$.

The sensor outputs are observed for a common time interval of T s, which is long compared to the random process correlation times, long compared to the propagation time of the source process across the array and greater than the duration of the channel impulse response. Since the observation interval is finite we represent the observations using a finite fourier transform representation. The normalized fourier coefficients $\{y_j(\omega),\ \omega \in B\}$ are given by

$$y_j(\omega) = (1/\sqrt{T}) \sum_{t=0}^{T-1} y_j(t) \exp(-i\omega t). \tag{2}$$

We assume that the impulse response function is such that

$$\sum_{t=-\infty}^{\infty} (1 + |t|)|h(\boldsymbol{\beta}, \boldsymbol{\theta}; t)| < \infty, \tag{3}$$

then the fourier transform representation for the impulse response or transfer function is given by

$$h(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega) = \sum_{t=-\infty}^{\infty} h(\boldsymbol{\beta}, \boldsymbol{\theta}; t) \exp(-i\omega t). \tag{4}$$

Define the channel transfer function vector $\mathbf{h}(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n)$ to be a unit norm vector

$$\mathbf{h}(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n) = \frac{(h_1(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n), h_2(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n), \ldots, h_L(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n))^T}{|(h_1(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n), h_2(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n), \ldots, h_L(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n))|}. \tag{5}$$

Under the assumption that the observation interval T is long, compared to the correlation lengths of the signal and noise processes, we make the usual assumption that the fourier coefficients are uncorrelated across frequency. Furthermore we assume that the noise processes are spatially uncorrelated with the same spectrum, let $\mathbf{Q}(\omega_n) = \phi_n(\omega_n)\mathbf{I}$, then a derivation of the likelihood ratio detector [11] yields the familiar 'Bartlett' likelihood ratio or matched filter processor, i.e.

$$\Lambda(\boldsymbol{\beta}) = \sum_{n=1}^{N} |\gamma(\omega_n)\mathbf{h}^*(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n)\mathbf{y}(\omega_n)|^2, \tag{6}$$

where

$$|\gamma(\omega_n)|^2 = \frac{\phi_u(\omega_n)/\phi_n(\omega_n)}{\phi_u(\omega_n) + \phi_n(\omega_n)}. \tag{7}$$

Since the observations $\mathbf{y}(\omega)$ are random the detection statistic $\Lambda(\boldsymbol{\beta})$ will be random, thus we use a mean detection statistic $\overline{\Lambda}(\boldsymbol{\beta}) \equiv E[\Lambda(\boldsymbol{\beta})]$. Define the covariance matrix of the observation vectors to be $\mathbf{R}(\omega) \equiv E[\mathbf{y}(\omega)\mathbf{y}^*(\omega)]$ then the mean detection statistic becomes

$$\overline{\Lambda}(\boldsymbol{\beta}) = \sum_{n=1}^{N} |\gamma(\omega_n)|^2 \mathbf{h}^*(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n)\mathbf{R}(\omega_n)\mathbf{h}(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n). \tag{8}$$

We see from Eq. (8) that the channel transfer function vector $\mathbf{h}(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n)$ between an assumed source position $\boldsymbol{\beta}$ and the sensor locations $\{\mathbf{v}_j\}$ $j = 1, \ldots, L$ provides the detailed information about the acoustic channel which is integral to matched-field processing. We remark that our ability to accurately predict the channel transfer function, which in turn is strongly dependent on the accuracy of the environmental knowledge, is a major factor affecting the performance of matched-field processors.

3
Channel transfer function

3.1. TRANSFER FUNCTION REPRESENTATION

The channel filter between any two locations in the channel $\mathbf{v}_j$ and $\boldsymbol{\beta}$, $\boldsymbol{\beta} = (r, z)$, is characterized in the time domain by the impulse response function $h_j(\boldsymbol{\beta}, \boldsymbol{\theta}; t)$ or in the frequency domain by the transfer function $h_j(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega)$. Since most numerical solutions of the wave equation are developed under the assumption that the source is a point source radiating at a single frequency we define the channel transfer function over a set of discrete frequencies, $\omega = \omega_n (n = 1, \ldots, N)$. The channel transfer function, or Green's function, can be expanded in terms of a complete set of normal modes [12]. In general the expansion will consist of a discrete sum plus an integral over the continuous modes. For our purposes we consider only the discrete spectrum since the continuous spectrum makes a small contribution beyond the near field of the source. In this case the channel transfer function can be represented by a normal-mode series of the form

$$h_j(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega) = \sum_{m=1}^{M} \phi_m(z)\phi_m(z_j)H_0^{(1)}(\kappa_m r), \tag{9}$$

where the set of normalized mode eigenfunctions $\{\phi_m\}$ and the set of mode eigenvalues $\{\kappa_m\}$ are solutions of the equations

$$\frac{\mathrm{d}^2\phi_m(z)}{\mathrm{d}z^2} + \left[\left(\frac{\omega}{c(z)}\right)^2 - \kappa_m^2\right]\phi_m(z) = 0, \tag{10a}$$

$$\int_0^\infty \rho(z)\phi_n(z)\phi_m(z)\,\mathrm{d}z = \delta_{n,m}, \tag{10b}$$

and boundary conditions. $\delta_{n,m}$ is the Kronecker delta function. The sound speed $c(z)$, the density $\rho(z)$, and the boundary conditions essentially define the environmental parameter vector $\boldsymbol{\theta}$. $H_0^{(1)}$ is the zeroth-order Hankel function of the first kind. While not displayed it should be noted that the eigenvalue solution of the wave equation is strongly dependent on the source frequency ω. For a detailed discussion of the normal-mode solution of the wave equation and its dependence on the environmental parameters see, for example, Tolstoy and Clay [13].

The numerical calculations for the channel transfer function were performed using the SACLANTCEN Normal-mode Acoustic Propagation (SNAP) model [4]. This model was chosen because the shallow-water ocean environment can be modeled in a very realistic fashion. The environment is defined in terms of a half-space divided into three layers: a water-column layer, a layer of sediment, and a semi-infinite subbottom layer. The SNAP environmental description allows: a depth-variable sound speed, with constant density and volume attenuation, in the water column; a depth-variable sound speed, with constant density and attenuation, in the sediment; and a depth-independent sound speed, density and attenuation in the subbottom. The SUPERSNAP version used herein incorporates an improved algorithm for finding the modal eigenvalues and eigenfunctions, the PULSE option, which provides multiple frequency results, was used to evaluate the transfer function as a function of frequency.

3.2. QUANTIFYING TRANSFER FUNCTION VARIABILITY

Environmental parameter uncertainty, more precisely uncertainty in the knowledge about the true ocean environmental parameters, generally arises due to the stochastic nature of the ocean and due to errors in the environmental parameter measurements. This uncertainty should be handled by treating $\boldsymbol{\theta}$ as a multivariate random vector and by employing stochastic propagation models. Due to the nonlinear relation between the environmental parameters and the transfer function it does not appear possible to analytically evaluate the effect of these random variables on the solution of the wave equation. In this memorandum we use numerical methods and treat $\boldsymbol{\theta}$ as an 'imprecisely known' but deterministic vector; we evaluate the effect of environmental parameter uncertainty in terms of nominal and perturbed environmental parameter vectors.

Quantifying the effect of environmental parameter uncertainty on the performance of matched-field processing is difficult because the effect is a function of many factors, such as the perturbation of the environmental parameters, the source location, the source frequency, the configuration and the placement of the receive array. Most previous work quantified the effect of perturbation of environmental parameters on the matched-field surface peak-to-sidelobe ratio or localization error for a single source at a single frequency using one array configuration. While this type of analysis is enlightening it is limited in overall scope. Thus, other means for quantifying the effect of environmental parameter uncertainty on the performance of matched-field processing are required. In this memorandum we have developed the use of the channel transfer function for quantifying the effect. We have chosen to use the channel transfer function because as seen in the previous section the channel transfer function represents all aspects of the channel effects on the received signal. It is exactly the difference between the nominal transfer function (i.e. the one which is assumed to be true) and some perturbation of the nominal transfer function, caused by uncertain knowledge about the environmental parameters, which produces the performance degradation observed with matched-field processing [10]. We will denote the nominal environmental parameter vector by $\boldsymbol{\theta}_0$ and a perturbed environment vector by $\boldsymbol{\theta}_i$ where i identifies a particular perturbation in environmental parameters from a set of expected perturbations. Furthermore we use $h_j(\boldsymbol{\beta}, \boldsymbol{\theta}_0; \omega)$ to denote the nominal transfer function and $h_j(\boldsymbol{\beta}, \boldsymbol{\theta}_i; \omega)$ to denote a perturbed transfer function. Figure 1 illustrates the modulus and phase for a nominal transfer function and for a transfer function calculated with a small perturbation of the water column sound speed. Plots of this type are quite useful but suffer from the same problem that previous methods have in that one plot illustrates the effect for only one source–receiver combination. Our goal is to understand the role of environmental parameter uncertainty on the transfer function, that is on the error

$$h_j(\boldsymbol{\beta}, \boldsymbol{\theta}_0; \omega) - h_j(\boldsymbol{\beta}, \boldsymbol{\theta}_i; \omega)$$

as a function of source–receiver location. Since this is a complex-valued function of frequency we use the following averaged real-valued measure

$$J_i(\mathbf{v}_j; \boldsymbol{\beta}) = (1/N) \sum_{n=n_1}^{n_2} |h_j(\boldsymbol{\beta}, \boldsymbol{\theta}_0; \omega_n) - h_j(\boldsymbol{\beta}, \boldsymbol{\theta}_i; \omega_n)|^2, \tag{11}$$

where N is the number of frequency samples in the band B, ω_{n_1} the lower band limit, ω_{n_2} the upper band limit, and $\mathbf{v}_j$ the receiver location. $J_i(\mathbf{v}_j; \boldsymbol{\beta})$ provides a measure of the broadband error between the nominal and perturbed transfer functions which can be evaluated as a function of source location $\boldsymbol{\beta}$ and receiver location $\mathbf{v}_j$. Figure 2 illustrates an example of the transfer function error measure $J_i(\mathbf{v}_j; \boldsymbol{\beta})$ evaluated for a nominal and perturbed environment. $10 \log J_i(\mathbf{v}_j; \boldsymbol{\beta})$ is plotted as a function of source location on the range/depth plane. Note there is considerable variation in this measure over source location on the range/depth plane especially as a function of range. As illustrated in the next section this measure is quite useful for quantifying the effect of uncertainty in knowledge about environmental parameters.

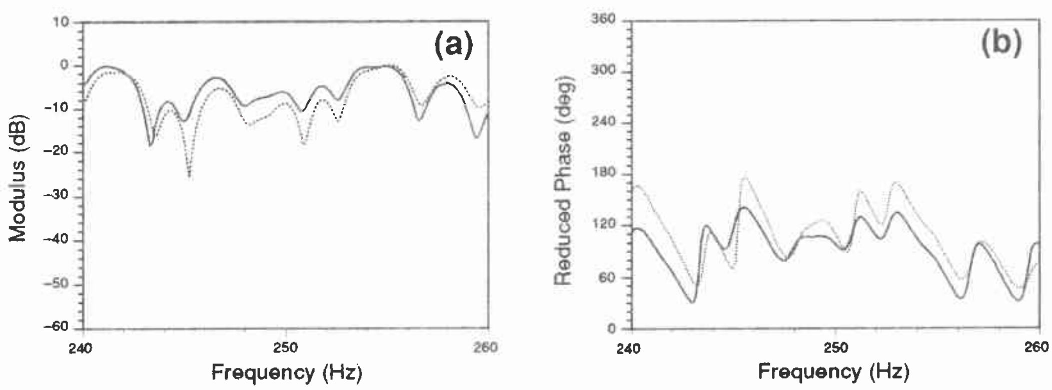

Figure 1 *Complex channel transfer function for shallow water example, receiver at 30 m, source at 60 m depth and 16 km range; (a) modulus, (b) phase ; (solid line) nominal environment, (dotted line) perturbed environment.*

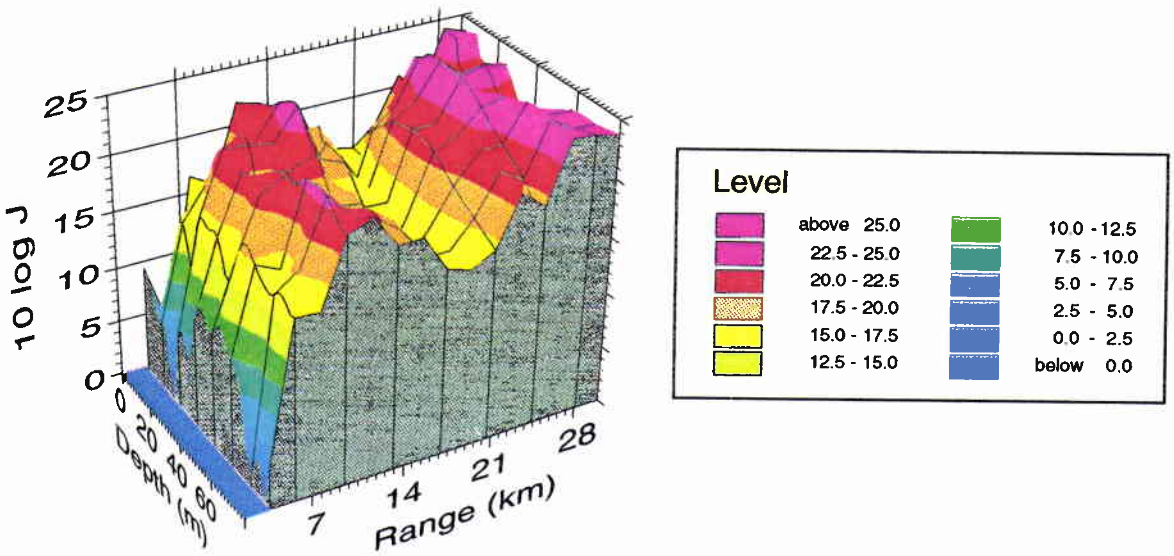

Figure 2 *Transfer function error measure $J_i(\mathbf{v}_j;\beta)$ for a nominal and perturbed environment for the frequency band 240–260 Hz.*

4

Canonical shallow-water example

As shown in Sect. 2 the channel transfer function $h_j(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n)$ between an assumed source position $\boldsymbol{\beta}$ and the sensor location $\mathbf{v}_j$ provides the detailed information about the acoustic channel which is required by a model-based processor for source detection and localization, see Eq. (8). It was shown in Sect. 3 that the channel transfer function $h_j(\boldsymbol{\beta}, \boldsymbol{\theta}; \omega_n)$, evaluated by solving the wave-equation, is strongly dependent on the channel environmental parameters. The environmental parameters consist of the sound speed *vs* depth in the water column, the thickness, density, attenuation and sound speed *vs* depth for the sediment and density, attenuation and sound speed for the subbottom.

In this section we investigate the sensitivity of the channel transfer function to perturbation of environmental parameters. We are using the channel transfer function to investigate the effect of uncertainty in knowledge about the true ocean environmental parameters on the performance of model-based processing. The assumption being that if the nominal transfer function (i.e. computed with the 'true' parameters) and the perturbed transfer function (i.e. computed with the perturbed parameters) are 'close' then there will be little degradation in processor performance due to the environmental parameter uncertainty.

The purpose of this section is to *illustrate* the use of the channel transfer function error measure for evaluating sensitivity to uncertainty about environmental knowledge. The particular conclusions proposed for the 'canonical' environments should not be construed to apply to all shallow-water environments. For any particular shallow-water application the methods of Sect. 3 should be applied to evaluate the sensitivity to uncertainty about environmental knowledge for that unique environment.

A simple 'canonical' shallow-water channel is employed to illustrate the effect of environmental parameter uncertainty. Figure 3 illustrates the canonical environments that were used. The winter environment, Fig. 3a, is the same as that used by Hamson and Heitmeyer [7], a slightly upward-refracting sound-speed profile, no sediment layer, medium to hard bottom and no shear. Figure 3b illustrates the cannonical summer environment, an isovelocity layer down to 20 m, a thermocline from 20 m to 40 m, below 40 m the profile is identical to the winter profile. In Figs. 3c,d a 6 m layer of sediment was included for both profiles. The sediment characteristics are typical of a site in the Mediterranean which has been studied by SACLANTCEN [14]. This sediment layer is very absorbent and highly attenuates the higher order modes. The four environments are referred to as winter no sediment (WN),

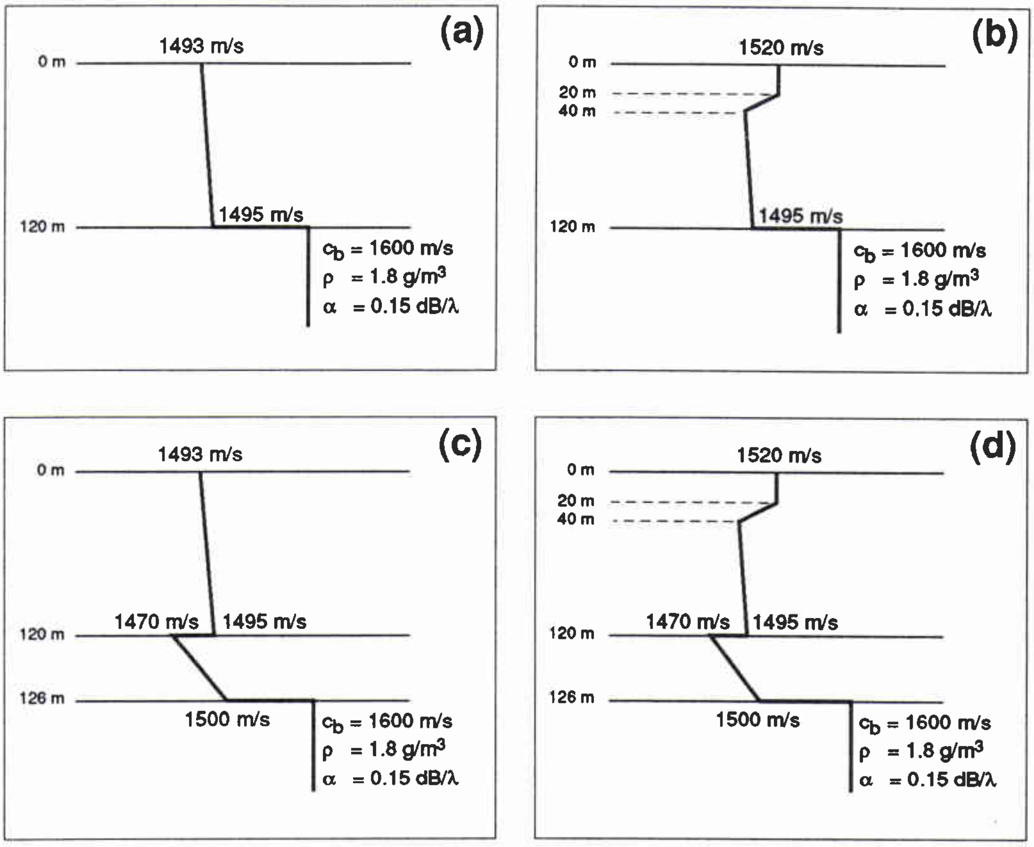

Figure 3 *The canonical shallow-water environments; (a) winter no sediment (WN), (b) summer no sediment (SN), (c) winter with sediment (WY), (d) summer with sediment (SY).*

summer no sediment (SN), winter with sediment (WY) and summer with sediment (SY). The subbottom characteristics are constant for the four cases. Figure 4 provides a description of the four canonical environments in terms of transmission loss (TL) *vs* range and depth for a 250 Hz source at a depth of 30 m. It can be seen by comparing the four plots that the TL *vs* range is about the same for the winter and summer profiles and that when the sediment is included the TL *vs* range increases more rapidly. At 250 Hz both environments support 14 modes without the sediment layer and 15 modes with the sediment layer.

In the course of the analysis, discussed in this section, certain parameters were kept constant. The range/depth plane consisted of 2–30 km in range and 10–80 m in depth. The resolution used was 2 km in range and 2 m in depth. The frequency resolution used in the fourier decomposition was 0.122 Hz. The frequency band was 240–260 Hz.

In the following subsections the transfer function error measure $J_i(\mathbf{v}_j;\boldsymbol{\beta})$, Eq. (11), is used to evaluate the effect of uncertainity in knowledge about environmental parame-

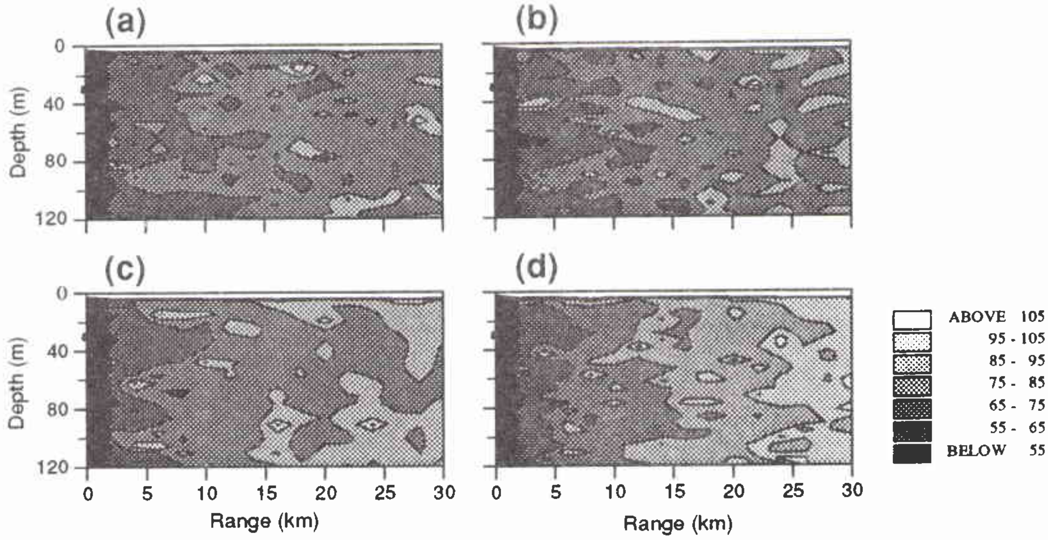

Figure 4 *Transmission loss vs range and depth for a 250 Hz source at 30 m; (a) winter no sediment (WN), (b) summer no sediment (SN), (c) winter with sediment (WY), (d) summer with sediment (SY).*

ters. In order to approximately characterize the relationship between the magnitude of the transfer function error measure and the effect of this error on matched-field localization three examples of environmental mismatch were computed. The nominal environment was that of Fig. 3a (WN), the array consisted of ten sensors spaced at 10 m with the top sensor at 10 m depth. Figures 5a,b illustrate the case for a source location of 60 m depth and 16 km range. In Fig. 5a there is no mismatch and the largest sidelobe peak is ~4–6 dB down from the source peak. In Fig. 5b, where the environmental mismatch is such that the error measure at the source location was ~10–12, it is seen that the largest sidelobe peak is only ~2–4 dB down from the source peak. In Fig. 5c the no mismatch case is illustrated for a source located at 60 m depth and 20 km range. Figure 5d illustrates the same case with environmental mismatch, the transfer function error measure at the source location was ~15–17. The effect of this magnitude of transfer function error is fairly serious, the highest sidelobe peak is at the same level as the source peak. In Fig. 5e the no mismatch case is illustrated for a source located at 60 m depth and 28 km range. Figure 5f illustrates the same case with environmental mismatch, the transfer function error measure at the source location was > 25. The effect of this magnitude of transfer function error is very serious, the highest sidelobe peak is at the same value as the source peak, and the source peak is no longer at the correct source location.

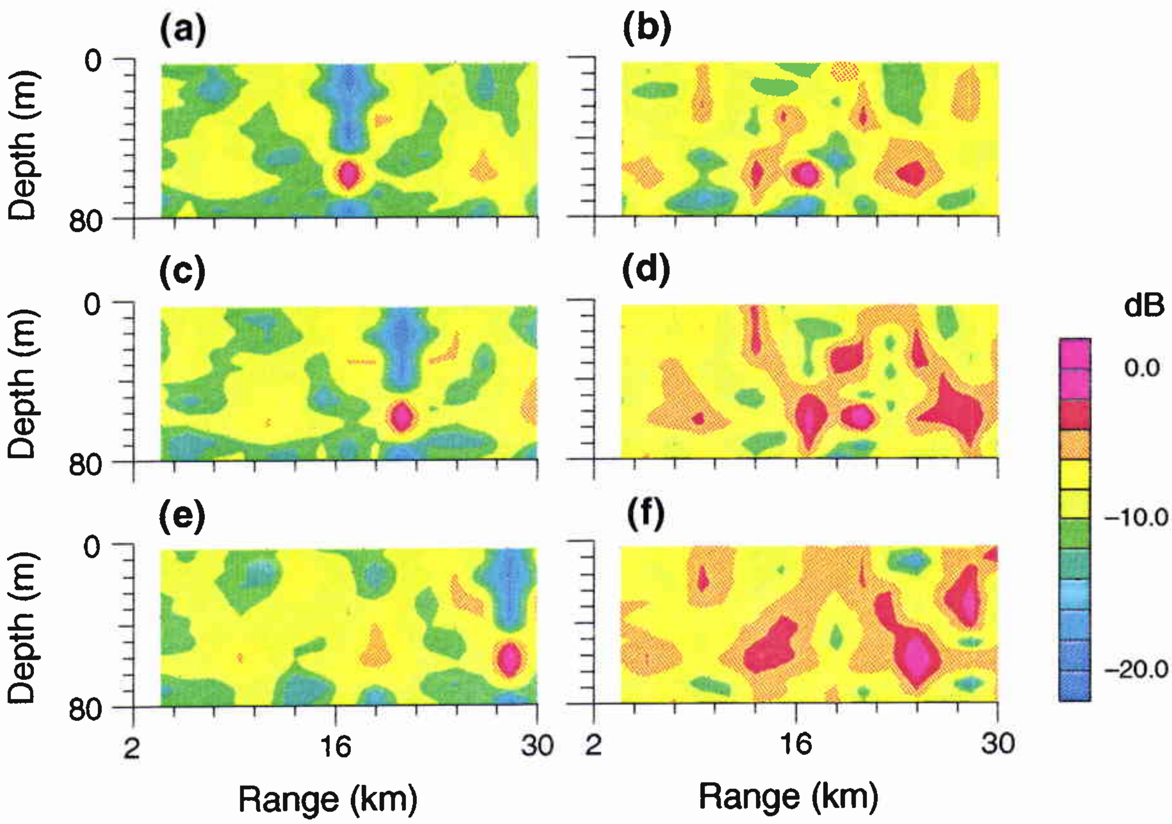

Figure 5 *Matched-field range/depth ambiguity surfaces illustrating the impact of environmental mismatch; (a) source at 60 m and 16 km, perfect knowledge, (b) source at 60 m and 16 km, error measure about 10 to 12, (c) source at 60 m and 20 km, perfect knowledge, (d) source at 60 m and 20 km, error measure about 15 to 17, (e) source at 60 m and 28 km, perfect knowledge, (f) source at 60 m and 28 km, error measure about 25.*

4.1. PERTURBATION OF WATER SOUND SPEED

Figure 6 illustrates the modulus and phase of the complex transfer function for three source locations at one receiver location as a function of frequency for the nominal and perturbed environment. The perturbation was produced by reducing the surface sound speed by 1 m/s (0.06%). The transfer function phase plots are in 'reduced phase' that is, the complex transfer function at each frequency ω has been divided by the factor $\exp(-i\omega r/c_{\text{ref}})$ where r is the source range and c_{ref} is a single sound speed value, c_{ref} was set to 1493 m/s. The phase of the resulting complex number was evaluated and plotted as a function of frequency. The receiver is located at a depth of 30 m. Figures 6a,b are the result for a source at 30 m depth and 6 km range. In this case the moduli are quite close across the frequency band and the phase variations are similar across frequency with a constant difference of $\sim 130°$. In Figs. 6c,d the source is located at 30 m in depth and 12 km in range, at this range the difference between the nominal and perturbed transfer functions are somewhat smaller for both the modulus and phase. In Figs. 6e,f the source is at 30 m depth and 24 km range, it is seen that the difference between the moduli

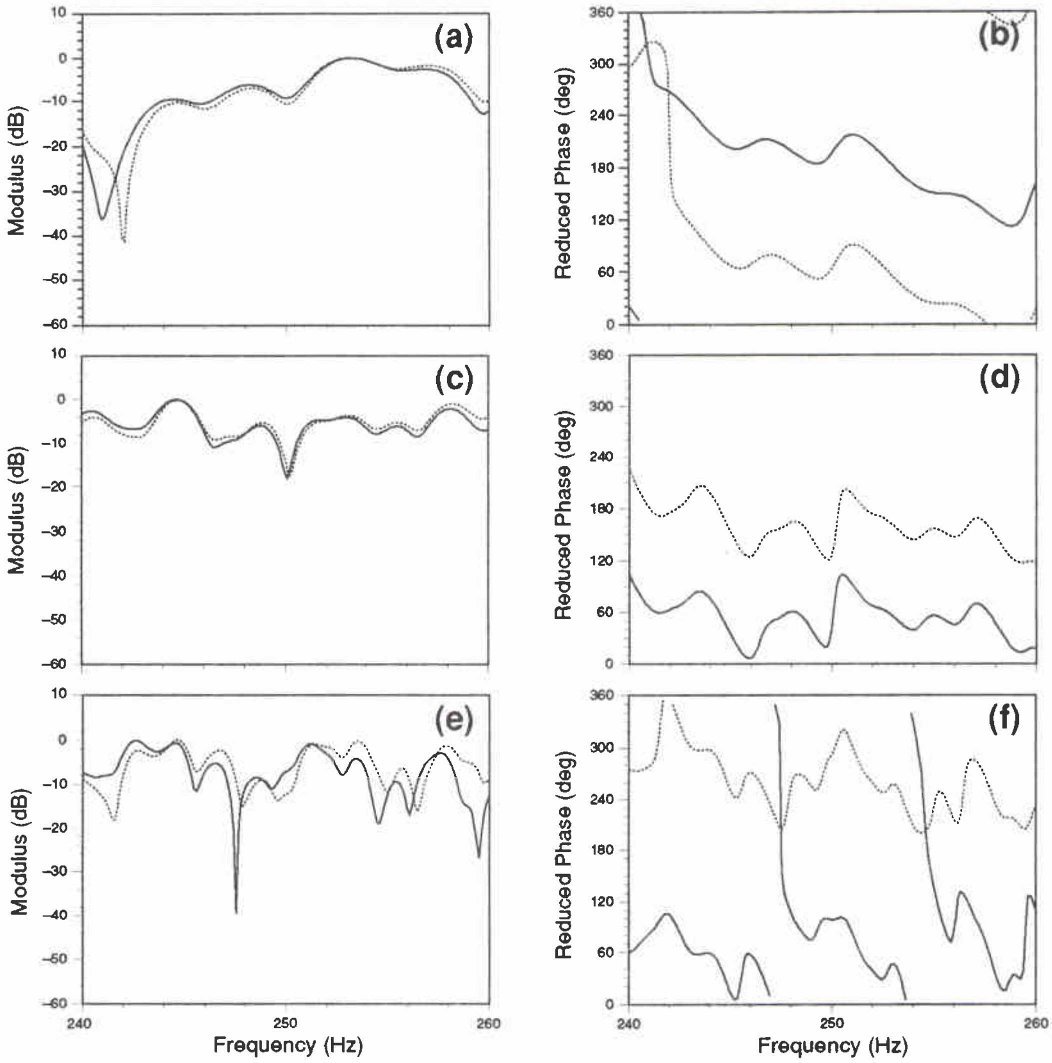

Figure 6 *Transfer function modulus and phase for nominal environment (solid line) and for perturbed environment (dotted line) for winter no sediment (WN) with a receiver at 30 m for three source locations: (a) 30 m depth and 6 km range, (b) 30 m depth and 12 km range, (c) 30 m depth and 24 km range. Perturbation is of the water sound speed; -1 m/s at the surface.*

is larger than the other two cases and difference between the phases is quite large. The transfer function modulus and phase plots of Fig. 6 illustrate that, in general, the effect of sound-speed perturbation varies considerable with source location on the range/depth plane.

In Fig. 7 the perturbation of the surface value of the water sound speed is examined further. Figure 7 illustrates the transfer function error measure for each of the four canonical environments of Fig. 3. The perturbation is a decrease of the surface sound speed value by 1 m/s. For the winter profiles the surface sound speed is perturbed

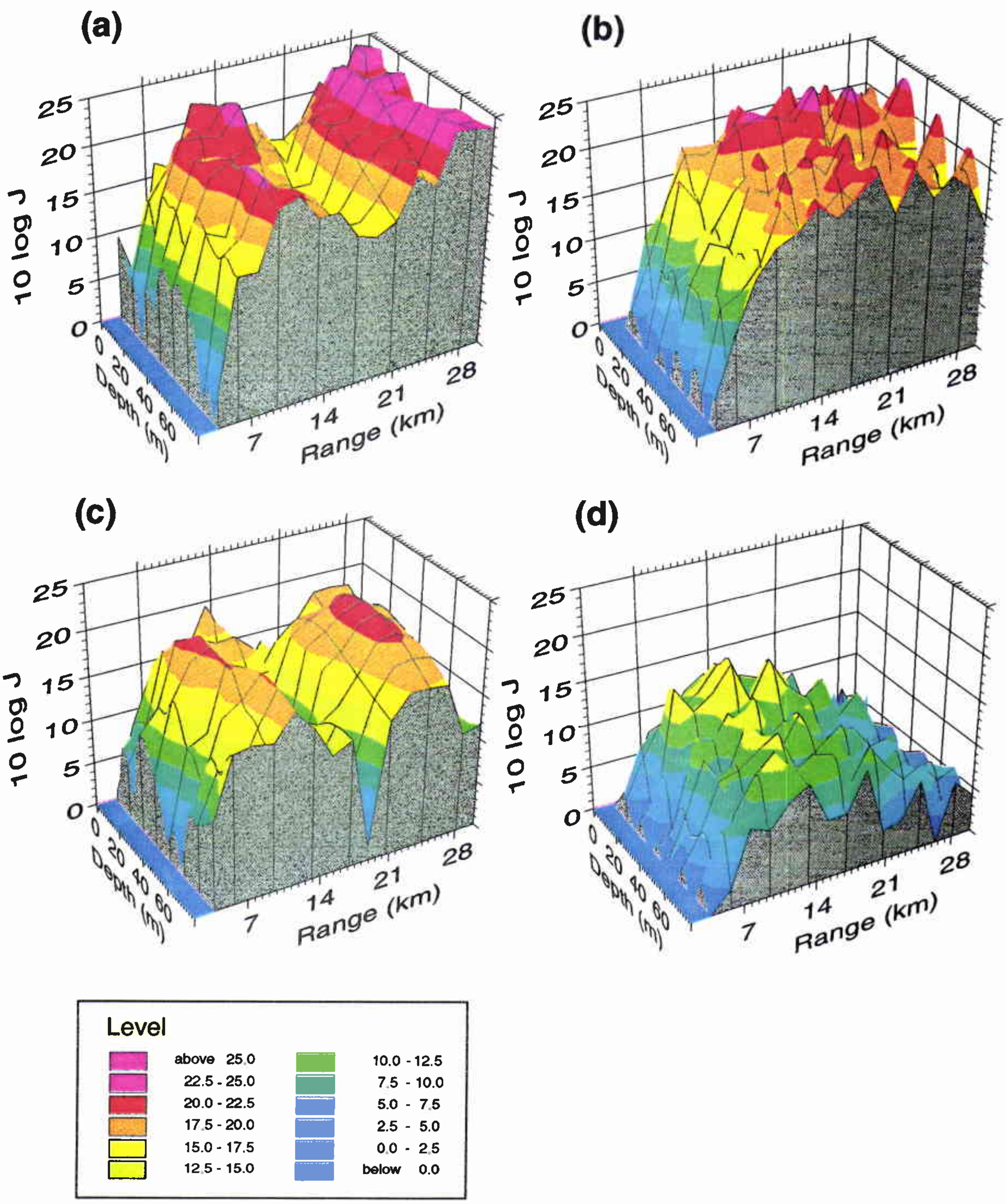

Figure 7 *Transfer function error measure $J_i(\mathbf{v}_j;\beta)$ for nominal and perturbed environment, perturbation is of water sound speed at the surface, -1 m/s; (a) winter no sediment (WN), (b) summer no sediment (SN), (c) winter with sediment (WY), (d) summer with sediment (SY).*

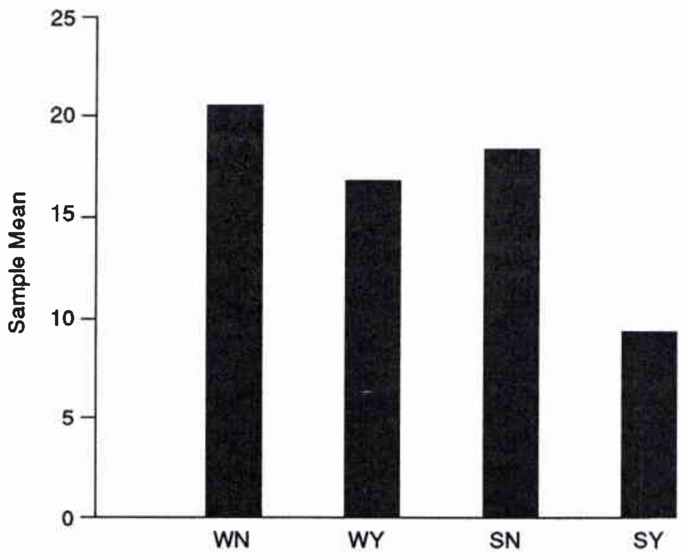

Figure 8 *Sample mean of transfer function error measure $J_i(\mathbf{v}_j;\boldsymbol{\beta})$ evaluated over the range/depth plane for nominal and perturbed environment for the four canonical environments: winter no sediment (WN), winter with sediment (WY), summer no sediment (SN), and summer with sediment (SY). Perturbation is of the water sound speed; -1 m/s at the surface.*

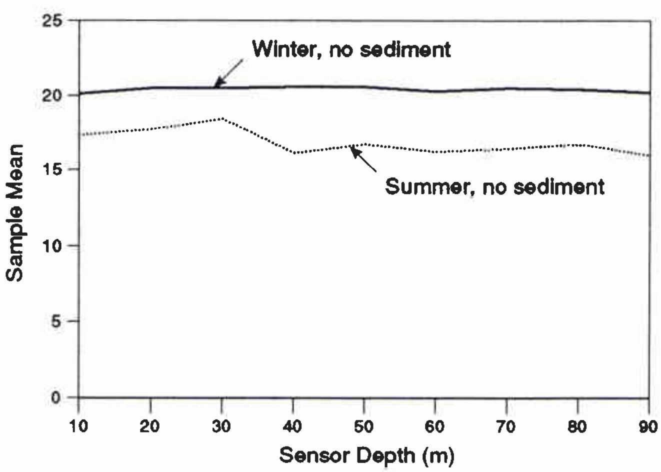

Figure 9 *Sample mean of the transfer function error measure $J_i(\mathbf{v}_j;\boldsymbol{\beta})$ evaluated over the range/depth plane for a nominal and perturbed environment as a function of receiver depth for the winter (WN) and summer (SN) environments without sediment. Perturbation is of water sound speed, -1 m/s, at the surface.*

from 1493 to 1492 m/s. For the summer profiles the sound speed of the isovelocity layer is perturbed from 1519 to 1518 m/s. It is seen that the effect of this relatively small perturbation is quite large, i.e. peak error measure values between 20–25, for both the winter and summer profiles. In Fig. 5 it was seen that transfer function error measure values of this magnitude can have a very large impact on the matched-field localization. The addition of the sediment layer produces only a small effect for the winter profile but has quite a large effect for the summer profile especially at the longer ranges, see Figs. 7c,d. This is due to the fact that the sediment is highly absorbent and thus the higher order modes are highly attenuated, especially at the longer ranges.

A useful summary measure is the sample mean of the transfer function error measure calculated over the entire range/depth plane. Figure 8 illustrates the sample mean for each of the transfer function error surfaces of Fig. 7, that is, for a perturbation of surface sound speed of −1 m/s for the four canonical environments. The addition of the sediment layer to the winter profile reduces the sample mean of the transfer function error measure. For the summer profile the addition of the sediment layer reduces the sample mean of the transfer function error measure substantially. From Fig. 8 it is seen that the transfer function error is greatest for the winter no sediment case (WN) and the smallest for the summer with sediment case (SY).

The results of Figs. 6–8 are for a single receiver depth of 30 m. Figure 9 illustrates performance as a function of receiver depth, in this case the perturbed environment is again −1 m/s for the surface sound speed. For a number of receiver depths the transfer function error measure $J_i(\mathbf{v}_j; \boldsymbol{\beta})$ was evaluated over the range/depth plane for the winter (WN) and summer (SN) environments. For each receiver depth the sample mean of the error measure $J_i(\mathbf{v}_j; \boldsymbol{\beta})$ was evaluated over the range/depth plane and the results are illustrated in Fig. 9. The sample mean of the error measure is almost constant over receiver depths from 10–90 m for both the winter and summer environments. The summer result shows a slight decrease in the sample mean for sensors located below the thermocline. These results indicate that for these canonical shallow-water environments the error measure is not strongly dependent on the receiver depth. This lack of dependence on receiver depth may not hold for other shallow-water environments.

4.2. PERTURBATION OF BOTTOM SOUND SPEED

Figure 10 illustrates the effect on the transfer function error measure of perturbations of the bottom sound-speed. This figure illustrates the results for a bottom sound speed perturbation of −20 m/s (1.25%) for the four canonical environments. Note first that for the winter no-sediment case (WN) Fig. 10a, the peak value of the error measure at the longer ranges is substantially less than that for the water sound-speed perturbations of Fig. 7. For the winter cases (WN and WY) the structure of the error measure as a function of range and depth is quite smooth, it is almost constant over depth for all ranges and increases monotonically with increasing range. Figures 10c,d illustrate the situation when the sediment layer is included. For both cases the transfer function error measure is reduced substantially when the sediment layer is included. The previous discussion about the attenuation of the higher order modes applies in this case also.

Figure 11 also illustrates the effect of bottom sound-speed perturbation. The sample mean of the transfer function error measure evaluated over the range/depth plane for a perturbation of −20 m/s for the four canonical environments is illustrated. The results for the winter and summer profile are quite similar. A large decrease in the error measure is noted for the two environments with the sediment layer. Comparing Figs. 8 and 11 we see that, for the canonical shallow water environments with sediment, a 1.25% perturbation of bottom sound speed has considerably less

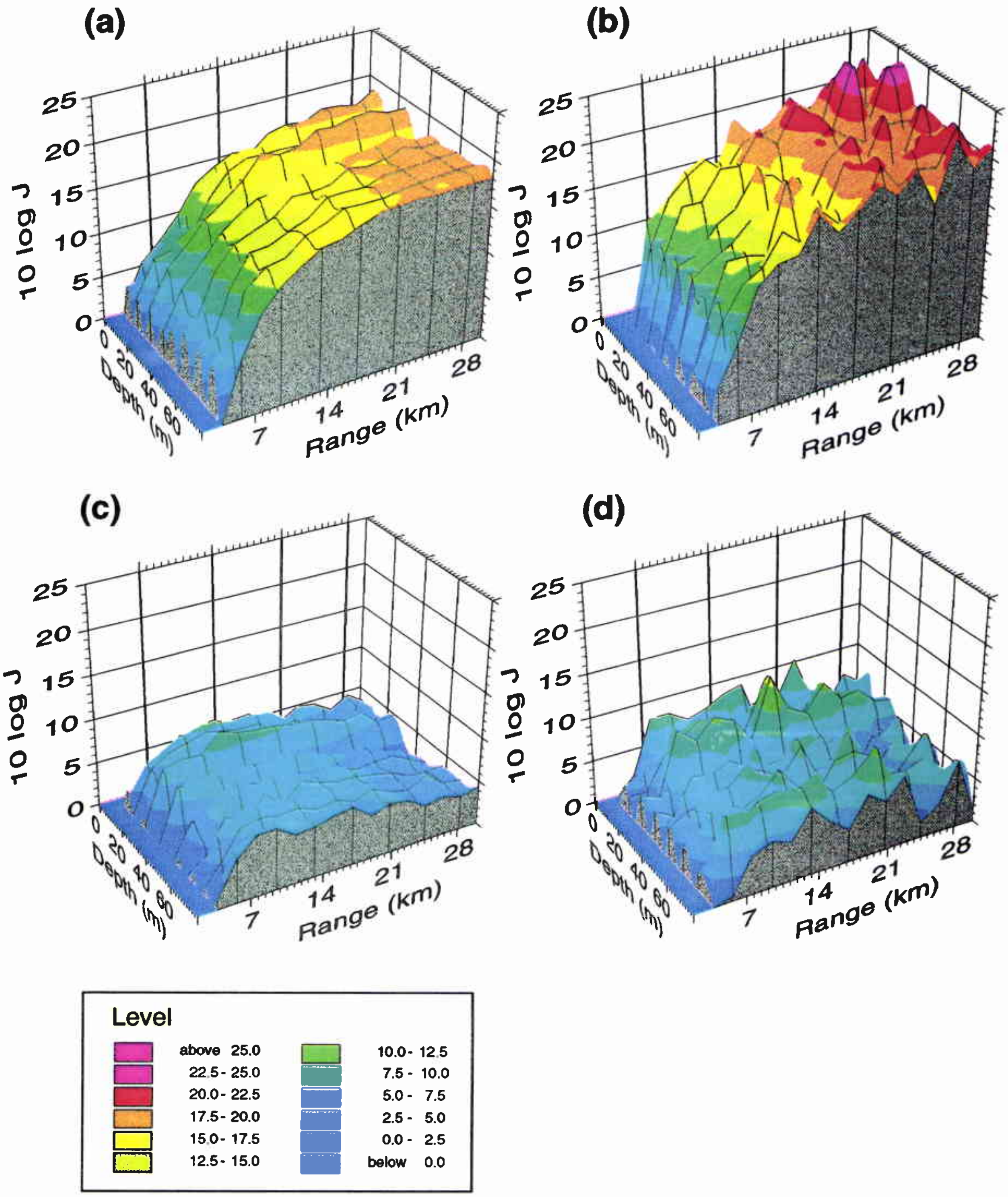

Figure 10 *Transfer function error measure $J_i(\mathbf{v}_j;\beta)$ for nominal and perturbed environment, perturbation is of bottom sound speed, -20 m/s; (a) winter no sediment (WN), (b) summer no sediment (SN), (c) winter with sediment (WY), (d) summer with sediment (SY).*

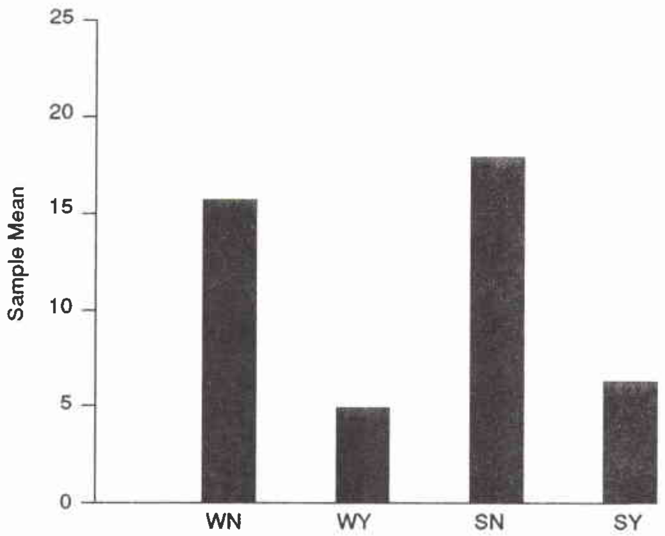

Figure 11 *Sample mean of transfer function error measure $J_i(\mathbf{v}_j; \beta)$ evaluated over the range/depth plane for nominal and perturbed environment for the four canonical environments; winter no sediment (WN), winter with sediment (WY), summer no sediment (SN), and summer with sediment (SY). Perturbation is of the bottom sound speed; -20 m/s.*

effect on the channel transfer function error than the 0.06% perturbation of water sound speed.

4.3. PERTURBATION OF CHANNEL DEPTH

Figure 12 illustrates the effect of perturbation of the channel depth, the nominal depth was 120 m. Figure 12 illustrates the effect of a +1 m perturbation of bottom depth (0.8%) for the four canonical environments. As seen from the figure the effect is quite large for the two cases without sediment, on the order of 20–25 for the peak values. For the two environments with sediment it is seen that the peak error measure values are on the order of 12–17.

Figure 13 illustrates the sample mean of the transfer function error measure evaluated over the range/depth plane for a perturbation of +1 m for the four canonical environments. The results for the winter and summer profile are similar. A large decrease in the error measure is noted for the two environments with the sediment layer.

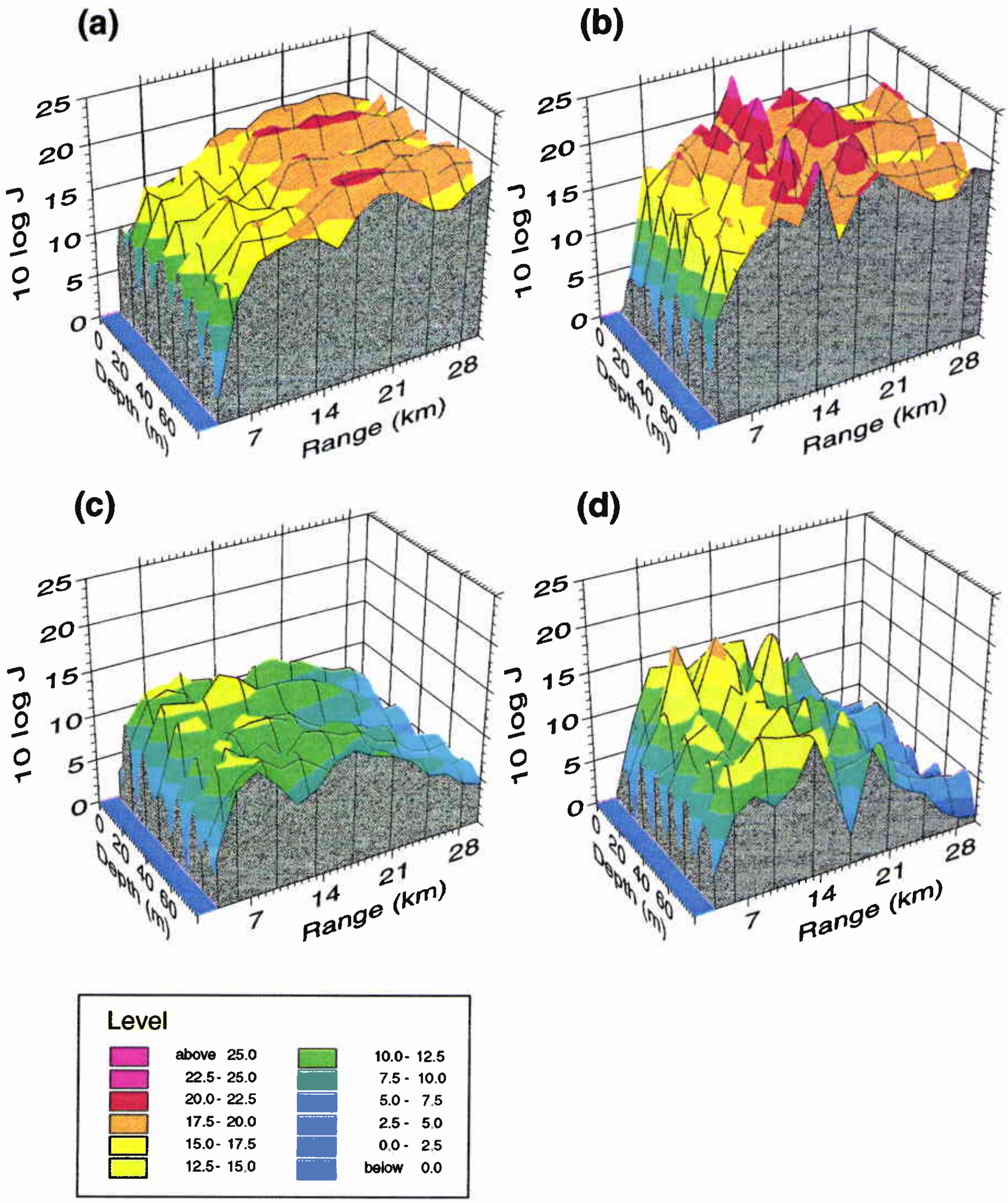

Figure 12 *Transfer function error measure $J_i(\mathbf{v}_j;\beta)$ for nominal and perturbed environment, perturbation is of channel depth, +1 m; (a) winter no sediment(WN), (b) summer no sediment (SN), (c) winter with sediment (WY), (d) summer with sediment (SY).*

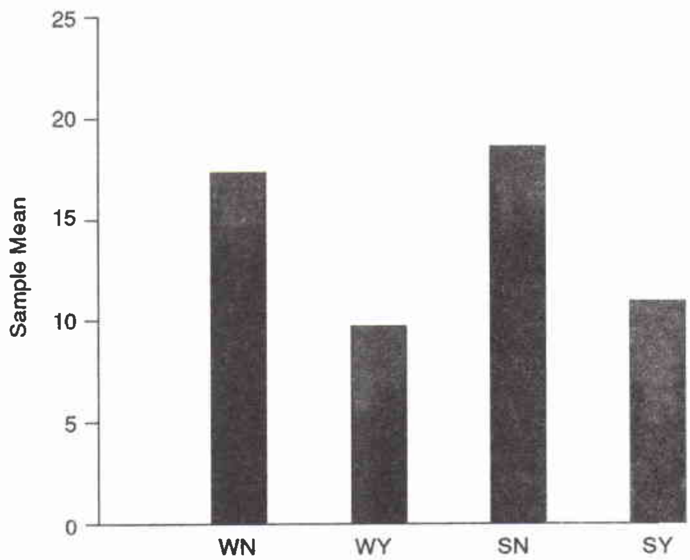

Figure 13 *Sample mean of transfer function error measure $J_i(\mathbf{v}_j; \boldsymbol{\beta})$ evaluated over the range/depth plane for nominal and perturbed environment for the four canonical environments; winter no sediment (WN), winter with sediment (WY), summer no sediment (SN), and summer with sediment (SY). Perturbation is of the channel depth; 1 m.*

4.4. PERTURBATION OF BOTTOM ATTENUATION

Figure 14 illustrates the effect of perturbation of the bottom attenuation (compressional), the nominal value was 0.15 dB/λ, the figure illustrates the effect of a perturbed value of 0.06 dB/λ (60%) for the two environments without sediment. The transfer function error measure for the two environments with the sediment layer was essentially zero. The effect of bottom attenuation perturbation, on the order of 60%, on the channel transfer function is small when compared to the 0.06% perturbation of the water sound speed. Also, the effect of perturbation of the bottom attenuation is much less than of the 1.25% perturbation of the bottom sound speed.

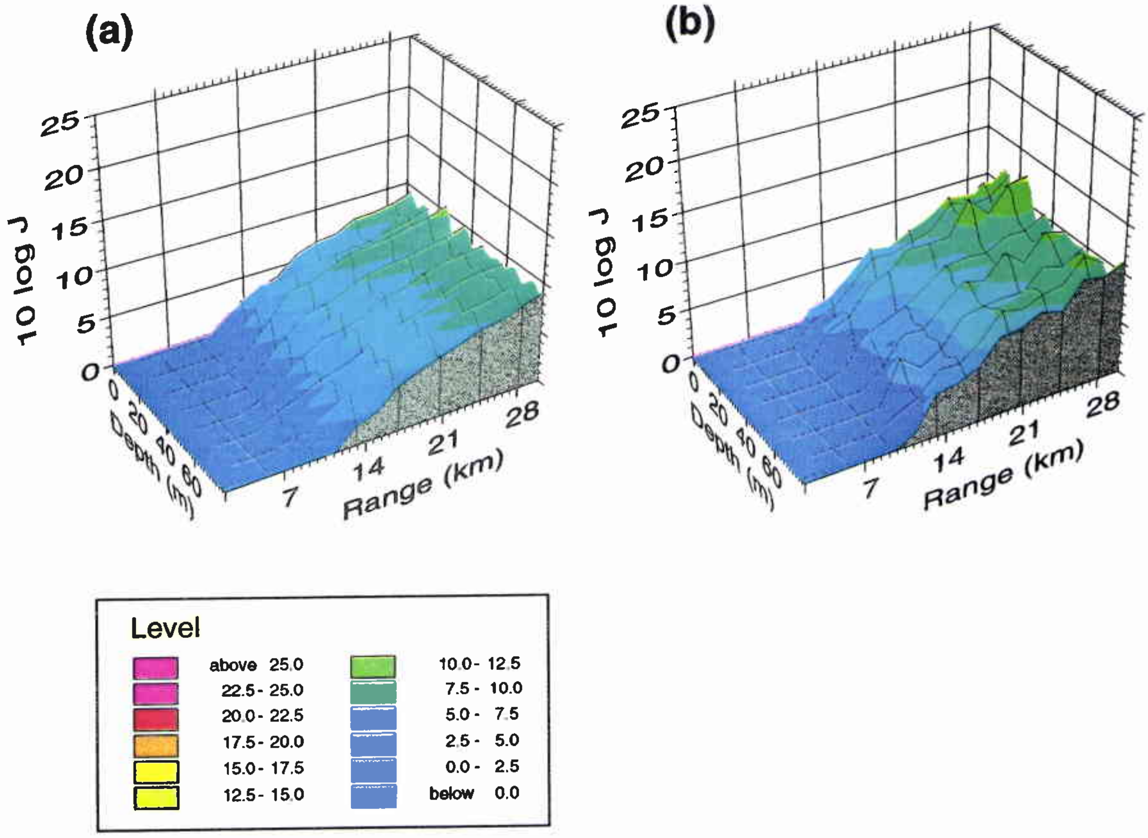

Figure 14 *Transfer function error measure $J_i(\mathbf{v}_j;\beta)$ for nominal and perturbed environment, perturbation is of bottom attenuation, attenuation = 0.06; (a) winter no sediment (WN), (b) summer no sediment (SN).*

5
Discussion

As previously remarked, evaluation of the sensitivity of the channel transfer function to environmental parameter uncertainty is only an intermediate measure of sensitivity. Complete evaluation of the sensitivity of model-based processors to parameter uncertainty is extremely difficult since it depends not only on the environmental parameters but also on the system parameters. In this memorandum an effort has been made to isolate the analysis from the system parameters and emphasize the effect of the environmental parameters individually. A global evaluation of sensitivity over many environmental and system parameters is possible to compute but it is probably impossible to present the results in a comprehensible manner.

As remarked in the introduction, most previous papers which have dealt with the parameter uncertainty problem for model-based processing have chosen a single environment, a single or a small set of system parameters and a single source location. Clearly, since the pressure field is highly variable over the range/depth plane, the processor sensitivity to environmental parameter uncertainty is going to be strongly dependent on the source location.

In this memorandum we have attempted, through the introduction of the channel transfer fuction error measure, to provide a method for evaluating the sensitivity of model-based processors to uncertainity about environmental knowledge which is independent of the system parameters. In most applications it will be the environmental parameters which are the most difficult to precisely know, thus the ability to evaluate the sensitivity independent of the more controllable system parameters will be highly useful.

For the simple canonical environments, used in the example, it can be seen that both with and without the sediment layer the sensitivity to uncertainty about the water channel sound speed is quite large. Uncertainty in the channel depth caused a large difference between the nominal and perturbed transfer functions. In this case the presence of the highly absorbent sediment layer significantly reduced the impact of the environmental uncertainty. Uncertainty in the bottom sound speed also caused a fairly large difference between the nominal and the perturbed transfer functions. Again the presence of the sediment layer significantly reduced the impact of the environmental uncertainty. Perturbation of the bottom attenuation was seen to have a small effect on the channel transfer function for the environments without the sediment layer and no effect for the environments with the sediment layer. Figure 15 provides a comparison of the magnitude of the channel transfer function error measure for the four types of environmental uncertainty that were examined.

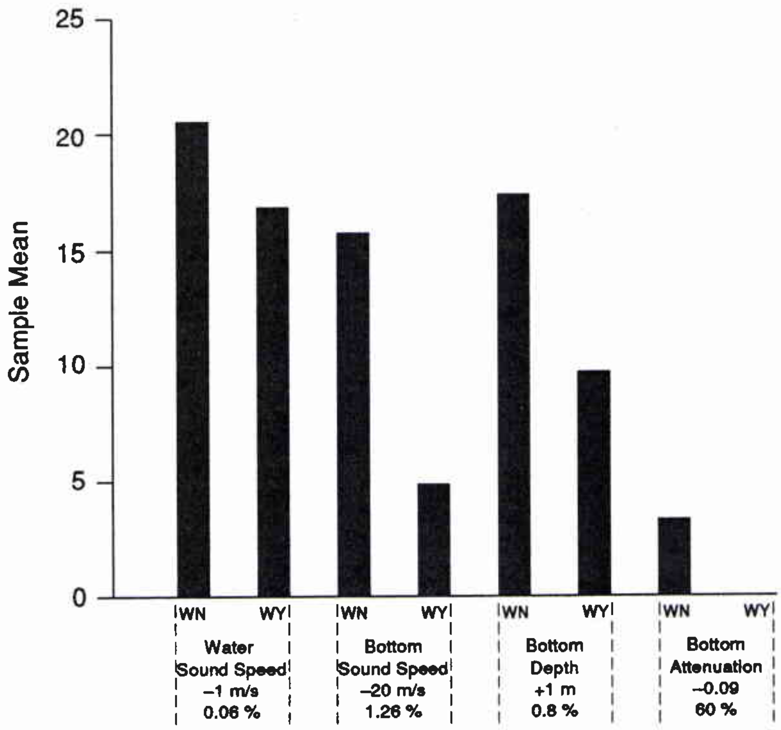

Figure 15 *Sample mean of transfer function error measure $J_i(\mathbf{v}_j;\beta)$ evaluated over the range/depth plane for nominal and perturbed environment for winter no sediment (WN) and winter with sediment (WY), for the four types of environmental perturbation; water surface sound speed, bottom sound speed, channel depth and bottom attenuation.*

It should be noted that while the apparent reduction in sensitivity to uncertainty about environmental knowledge due to the sediment layer may provide a performance gain, this gain may be offset by a corresponding performance loss due to environmental considerations. That is, with the sediment layer there are fewer modes propagating to the longer ranges, thus the localization process has less information about the source location and the localization performance may be poor even when the environmental knowledge is exactly known [7].

The broadband channel transfer function error measure has been demonstrated to be a very valuable tool for understanding the sensitivity of model-based processing to uncertainty in knowledge about environmental parameters independent of the system parameters.

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Title

Impact of uncertain environmental knowledge on the shallow-water transfer function

Abstract

It is assumed that the acoustic channel can be modeled as a linear time-invariant space-variant filter. In this case, from linear systems theory, it is known that the filter output, that is the predicted source signal replica at the sensor location, is formed by the convolution of the source signal with the filter impulse response function. The acoustic channel impulse response function is a function of the source location, the sensor location and the environmental parameters. In this analysis we use the channel transfer function, the fourier transform of the channel impulse response function, as a function of frequency to evaluate the effect of environmental parameter uncertainty. In the course of this work we establish a measure, referred to as the 'transfer function error measure' which provides an estimate of the average error for the channel transfer function due to environmental parameter uncertainty as a function of receiver depth. This error measure, which is also a function of the source location, is used to characterize the sensitivity of propagation model-based array processors to uncertainty in environmental knowledge such as water sound speed, bottom sound speed, bottom attenuation and channel depth. For simple canonical shallow-water channels, winter and summer profiles, results are presented which illustrate the effect of uncertain environmental knowledge on the transfer function error measure.

Keywords

, environmental uncertainty, matched-field processing, normal modes, shallow water, transfer function

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