INFLUENCE OF BACKGROUND NOISE SPATIAL COHERENCE ON HIGH RESOLUTION PASSIVE METHOD

by

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ABSTRACT

New signal processing methods for passive listening have appeared recently : they are called "high resolution" methods because they have theoretically a better resolving power than conventional or adaptive beamforming. But they need additional knowledge on background noise structure, and particularly on ambient noise for low frequency. These methods are based on the estimation of the spectral density matrix of the signal received on the array and on the utilization of its eigenvectors and eigenvalues. This matrix contains the background noise spectral density matrix for which a good parametrized model is needed. Performances estimated by simulations show on one hand the interest of high resolution methods compared to conventional methods, and on the other hand their sensitivity to the background noise model which is used.

1 - INTRODUCTION

One of the main functions of an underwater passive listening system is the determination of the number of sources present in the medium, as well as their characteristic parameters. For that, one uses noises radiated by the sources which are received on the sensors of an array. The basic function is array processing which is traditionally carried out by conventional beamforming. Effort towards performance improvement has led first to adaptive array processing. It provides a gain which is limited by the signal to noise ratio of the sources. Recently, studies have been done on new methods which are more powerful [1-2-3] : they are called high resolution methods. But they need an additional hypothesis on the medium.

Adaptive array processing, as conventional beamforming, needs hypothesis only on the sources : they are point like, perfectly spatially coherent, and the wavefront shape from a source is a known function of the source position (the transfer function of the sensors is also supposed known). Let $\vec{r}(t)$ be the vector representing the signal received on the K sensors of the array. The correlation matrix of the received signal is :

$$C(\tau) = E\left[\vec{r}(t)\ \vec{r}^{\,+}(t+\tau)\right] \quad (1)$$

where E() denotes expectation and $\vec{r}^{\,+}$ is the conjugate transpose of $\vec{r}$. With the above hypotheses, the spectral density matrix, which is the Fourier transform of $C(\tau)$, of a source alone is equal to :

$$\Gamma(f) = \gamma(f)\ \vec{D}(f)\ \vec{D}^{+}(f) \tag{2}$$

where $\gamma(f)$ is the spectral density of the received signal and $\vec{D}(f)$ the source position vector, composed of the transfer functions between the source and each sensor, normalized by the transfer function between the source and a reference point of the array. This matrix is rank one, which is characteristic of its perfect spatial coherence.
The additional hypothesis needed by the high resolution methods concerns the spatial coherence of the background noise. In general, background noise is supposed to be statistically independent between the sensors [1] or suppressed [2]. High resolution methods have better performances than those of adaptive array processing thanks to that hypothesis. They suppose also that the noisefield can be resolved : the number N of sources is less than the number K of sensors. The spectral density matrix of the received signals is equal to (sources and background noise are statistically independent) :

$$\Gamma(f) = \Gamma_B(f) + \Gamma_S(f) = \sigma(f)\ I + \sum_{i=1}^{N} \gamma_i(f)\ \vec{D}_i(f)\ \vec{D}_i^{+}(f) \tag{3}$$

$\Gamma_B(f)$ and $\Gamma_S(f)$ are respectively the spectral density matrices of the background noise and of the sources, I is the spatial coherence matrix of the background noise which is the identity matrix and $\sigma(f)$ its spectral density, $\vec{D}_i(f)$ and $\gamma_i(f)$ are respectively the position vector and the spectral density of the i^{th} source.

2 - HIGH RESOLUTION METHOD PRINCIPLES.

High resolution methods are based on eigenvalue-eigenvector decomposition of the spectral density matrix $\Gamma(f)$. An eigenvector $\vec{V}(f)$ and its corresponding eigenvalue $\lambda(f)$, are defined by the relation :

$$\Gamma(f)\ \vec{V}(f) = \sigma(f)\ \vec{V}(f) + \sum_{i=1}^{N} \gamma_i(f)\ \vec{D}_i(f) \left[\vec{D}^{+}_i(f)\ \vec{V}(f)\right] = \lambda(f)\ \vec{V}(f) \tag{4}$$

It can be shown that $\Gamma(f)$ has :

a) N eigenvectors $\vec{V}_i(f)$ ($i \in [1,N]$) that are the eigenvectors of the sources alone matrix $\Gamma_S(f)$ corresponding to the N non-zero eigenvalues $\lambda_{si}(f)$ of $\Gamma_S(f)$ the rank of which is equal to N ; the corresponding eigenvalues are equal to : $\lambda_i(f) = \lambda_{si}(f) + \sigma(f)$; these N eigenvectors are an orthogonal basis of the N dimensional source subspace spanned by the N position vectors $\vec{D}_i(f)$ of the sources ; then :

$$\sum_{i=1}^{N} \gamma_i(f)\ \vec{D}_i(f)\ \vec{D}_i^{+}(f) = \sum_{i=1}^{N} \lambda_{si}(f)\ \vec{V}_i(f)\ \vec{V}_i^{+}(f) \tag{5}$$

b) (K-N) eigenvectors $\vec{V}_i(f)$ orthogonal to the preceding ones and therefore to each position vector $\vec{D}_i(f)$:

$$\vec{V}_j^{+}(f)\ \vec{D}_i(f) = 0 \qquad (6)$$

They are a basis of the (K-N) dimensional subspace orthogonal to the source subspace. The corresponding (K-N) eigenvalues are all equal to $\sigma(f)$, and therefore smaller than the previous ones.

From the above analysis, it is deduced that :

a) The number K of sensors minus the number of minimum and equal eigenvalues gives the number of sources.

b) The position of the sources can be determined by using :

- either the source subspace and relation (5) as proposed in [1] or [2]
- or the orthogonal subspace as proposed in [3] : as the eigenvectors of this subspace are orthogonal to each source position vector, if $\vec{D}(f,\vec{\theta})$ is a position vector according to the wavefront shape of the model (including propagation and array) and corresponding to a source the position of which is denoted by $\vec{\theta}$, the scalar products : $\vec{V}_j^{+}(f)\ \vec{D}(f,\vec{\theta})$ are equal to zero for each value of $\vec{\theta}$ corresponding to the position of a source in the medium. For stability and ambiguity reasons [3], the orthogonal subspace method uses the function :

$$G(f,\vec{\theta}) = \sum_{j=1}^{K-N} |\vec{V}_j^{+}(f)\ \vec{D}(f,\vec{\theta})|^2 \qquad (7)$$

It is that separation into two subspaces, one which contains the sources, the source subspace, and the other which contains the background noise only, the orthogonal subspace, which gives to high resolution methods their main property : their asymptotic (infinite observation time) resolving power is infinite, that is to say two sources can be resolved even if they are very close together and very weak ; it is no longer limited by the input signal to noise ratio $\gamma_i(f)/\sigma(f)$ as it is for adaptive array processing.

The above properties are valid only for an infinite observation time. In practice, the observation time is limited and only an estimation $\hat{\Gamma}(f)$ of $\Gamma(f)$ is available. Therefore, only estimation of the source parameters can be obtained. The minimum eigenvalues of $\hat{\Gamma}(f)$ are not exactly equal but show a spread that depends upon the observation time. Thus the determination of the number of sources is an actual detection test with detection and false alarm probabilities : it must be noticed that this detection test of the number of sources does not need the wavefront shape knowledge.

Eigenvectors are also estimates of the actual ones and only estimates of the source positions are obtained. In particular, orthogonal subspace eigenvectors are not exactly orthogonal to the source position vectors and $G(f,\vec{\theta})$ (relation (7)) exhibits minimum instead of zeroes. Fig. 1 shows an example obtained by simulation. Receiving array is linear, composed of 12 equispaced sensors. There are two sources at infinity in the medium with bearings 0° and-5.3° and spatially incoherent background noise. Sources are in the same plane as array. The figure shows versus bearing the output spectral density of adaptive array processing γ_A and the inverse of $G(f,\vec{\theta})$, which exhibits spikes at the source position instead of minimum. The spectral density matrix is estimated by :

$$\hat{\Gamma}(f) = \frac{1}{P} \sum_{i=1}^{P} \vec{X}_i(f)\, \vec{X}_i^{+}(f) \qquad (8)$$

where $\vec{X}_i(f)$ is the discrete Fourier transform of the vector signal $\vec{r}(t)$ computed on a time duration equal to T. When the parameter P, that is to say the observation time, increases, the adaptive array response remains about the same while the response of $G(f,\vec{\theta})$ is improved : the spikes become more narrow.

The physical limitations to the asymptotically infinite resolving power of high resolution methods are given by the imperfect knowledge and the fluctuations of the background noise spatial coherence and of the source wavefront shape. Influence of the background noise spatial coherence is examined now.

3 - INFLUENCE OF BACKGROUND NOISE SPATIAL COHERENCE [4]

In fact, at sea, the background noise has not a spatial coherence matrix equal to an identity matrix. It is composed of several components : the sea noise, generated by the wind at the surface, the flow noise, generated along the array, and the traffic noise at the low frequencies. Generally, the electronic noise of the hydrophone channels is negligible.

So in the general case, the spectral density matrix of the background noise can be written :

$$\Gamma_B(f) = \sigma(f)\, J(f) \qquad (9)$$

where $\sigma(f)$ is the spectral density and $J(f)$ the spatial coherence matrix of the background noise.

In that case, the properties of the eigenvectors and eigenvalues of the spectral density matrix stated in the preceding section, are no more valid. For example, in order to illustrate the phenomenon, the very simple case where the background noise is reduced to surface noise only [5] can be considered. The cross-spectral density between two sensors distant of d is equal to :

$$\gamma_d(f) = \sigma(f)\, \frac{2^m\, m!}{(2\pi f d/c)}\, J_m(2\pi f d/c) \qquad (10)$$

where c is the sound velocity in the sea, $J_m()$ the Bessel function of order m, and m a modelling parameter.

The signals received on a linear array of 12 equally spaced sensors have been simulated. Bearing only is considered. Table 1 shows the eigenvalues obtained for a noisefield composed of background noise only with a parameter m equal to zero, and for a length ℓ betwen two adjacent sensors equal to 0.3 or 0.7 wavelength. If the spatial coherence matrix of the background noise was an identity matrix according to the hypothesis, all the eigenvalues would be equal : Table 1 shows significant differences

$\ell=0.3\lambda$	2.45	2.26	1.34	1.29	1.13	1.10	1.05	1.00	0.32	0.05	0.004	2.10^{-4}
$\ell=0.7\lambda$	2.67	2.22	1.38	1.11	0.76	0.61	0.58	0.55	0.54	0.53	0.52	0.52

Table 1 : Eigenvalues of spatially correlated background noise

Fig. 2 presents the diagram versus bearing θ of the eigenvector corresponding to the maximum eigenvalue of the spectral density matrix estimate $\hat{\Gamma}(f)$. The noisefield is composed of background noise and of one source with bearing 5.4° and signal to noise ratio : -15 dB. The length between two adjacent sensors is half a wavelength. In this case, the eigenvector $\vec{V}_M(f)$ corresponding to the maximum eigenvalue of the theoretical spectral density matrix $\Gamma(f)$ is equal to the source direction vector. Therefore, its diagram defined by :

$$D(\theta) = |\vec{D}^+(f,\theta)\ \vec{V}_M(f)|^2$$

is equal to the classical beamforming diagram. Three diagrams are drawn on Fig. 2 : they are obtained for background noise the spatial coherence parameter m of which is equal to 0.5, 0.2 and 0.8. For 0.5 the spatial coherence matrix of the background noise is equal to the identity matrix according to the hypothesis. The diagram is effectively good for m=0.5. Diagrams show weak differences for m=0.2 and a completely different shape for m=0.8.

Fig. 3 presents results obtained with the same noisefield as for Fig. 2, but for the orthogonal subspace method : expression (7). $G^{-1}(f,\theta)$ is drawn instead of $G(f,\theta)$ as in Fig. 1, in order to see spikes instead of minimum. The diagrams $G^{-1}(f,\theta)$ has been drawn for the same background noise spatial coherence parameters as before. The best result is obtained for m=0.5, that is to say when the spatial coherence of the actual background noise is equal to the spatial coherence of the model.

Fig. 4 presents results obtained also with the orthogonal subspace method. The noisefield is composed of background noise and of two sources with bearings : 0° and 4°, and signal to noise ratio : -10 dB. Three diagrams $G^{-1}(f,\theta)$ are drawn versus bearing θ for the same background noise spatial coherence parameters : 0.5, 0.2 and 0.8. There also the best result is obtained for m=0.5.

The few results given in that section show clearly that the background noise spatial coherence has to be known as exactly as possible.

4 - ADAPTIVITY TO BACKGROUND NOISE SPATIAL COHERENCE.

When the background noise spatial coherence matrix is not equal to the identity matrix :

$$\Gamma_B(f) = \sigma(f)\ J(f)$$

it can be shown [4] that if J(f) is known, the problem can be solved. As J(f) is hermitian, positive definite, it is possible to define a matrix C(f) such that :

$$C(f)\ J(f)\ C^{+}(f) = I \tag{11}$$

Therefore, the spectral density matrix $\Gamma(f)$ transformed by $C(f)$ is equal to :

$$\Gamma_C(f) = C(f)\ \Gamma(f)\ C^{+}(f) = \sigma(f)\ C(f)\ J(f)\ C^{+}(f) + \sum_{i=1}^{N} \gamma_i(f) C(f) \vec{D}_i(f) \vec{D}_i^{+}(f) C^{+}(f) \tag{12}$$

As : $C(f)\vec{D}_i(f)$ is a vector, which can be written $\vec{D}_{ci}(f)$, the transformed matrix $\Gamma_c(f)$ is equal to :

$$\Gamma_c(f) = \sigma(f)\ I + \sum_{i=1}^{N} \gamma_i(f)\ \vec{D}_{ci}(f)\ \vec{D}_{ci}^{+}(f) \tag{13}$$

Therefore, the background noise has been spatially whitened, and the eigenvectors and the eigenvalues of the transformed spectral density matrix $\Gamma_c(f)$ have the properties stated in the theory (section 2). Of course, to recover the source positions, a transformed source position vector model $\vec{D}_c(\vec{\theta},f)$ must be used, in expression (7) in particular :

$$\vec{D}_c(\vec{\theta},f) = C(f)\ \vec{D}(\vec{\theta},f) \tag{14}$$

So, the problem is solved when $J(f)$ is known.

In fact, it has been shown [6] that it is not necessary to know exactly the spatial coherence matrix of the background noise : it is only necessary that it may be modalized as a function of unknown parameters. In this case, the background noise spectral density matrix model can be written :

$$\Gamma_B(f,\vec{m}) = \sigma(f)\ J(f,\vec{m}) \tag{15}$$

where $\sigma(f)$ is unknown and $\vec{m}$ represents the unknown parameter.

Let $C(f,\vec{m})$ be the matrix such that :

$$C(f,\vec{m})\ J(f,\vec{m})\ C^{+}(f,\vec{m}) = I \tag{16}$$

Let $\vec{m}_o$ be the parameter values of the actual background noise. If the eigenvalues of the transformed received signal spectral density matrix :

$$\Gamma_c(f,\vec{m}) = C(f,\vec{m})\ \Gamma(f)\ C^{+}(f,\vec{m}) \tag{17}$$

are computed versus $\vec{m}$, when $\vec{m}$ is equal to $\vec{m}_o$, (K-N) eigenvalues become equal and the other N are greater. In practice, of course, only an estimation $\hat{\Gamma}(f)$ of $\Gamma(f)$ is available. If the eigenvalues diagram of the matrix :

$$\hat{\Gamma}_c(f,\vec{m}) = C(f,\vec{m})\ \hat{\Gamma}(f)\ C^{+}(f,\vec{m}) \tag{18}$$

is drawn versus m, when m is near m_o, a focusing spot is observed which is as much sharp as the estimate $\hat{\Gamma}(f)$ is better. The value of $\vec{m}$ corresponding to that focusing spot gives estimates of the background noise parameter $\vec{m}_o$ and of the number of sources together. Eigenvectors and eigenvalues corresponding to that value of $\vec{m}$ are used for source parameters estimation.

In order to show the ability of the method, simulations have been conducted, using the same array and noisefield as in the preceding section: the background noise spatial coherence model (expression (10)) has one unknown parameter : m. The noisefield is composed of two sources with bearings : 0° and 4°, and signal to noise ratio : -10 dB (as for Fig. 4), and the background noise spatial coherence parameter is equal to 0.5. Fig. 5 presents the plots versus m of the eigenvalues of the theoretical spectral density matrix (a), and of the matrix $\hat{\Gamma}(f)$ estimated with P = 80 (expression (8)) (b), and P=400 (c) (P is the number of instantaneous spectra integrated to estimate $\hat{\Gamma}(f)$). The focusing spot happens for m=0.5 : it becomes sharper when P, that is to say the observation time, is increasing. The two higher eigenvalues which indicate the presence of the two sources are clearly seen.

This eigenvalue diagram is therefore an interesting mean to estimate the background noise spatial coherence parameters.

5 - CONCLUSION

The interest of high resolution methods is clearly demonstrated by their asymptotical performances. But in order to be used in practice, it is absolutely necessary to have a good parametrized model for the spatial coherence of the background noise and in particular of the ambient noise.

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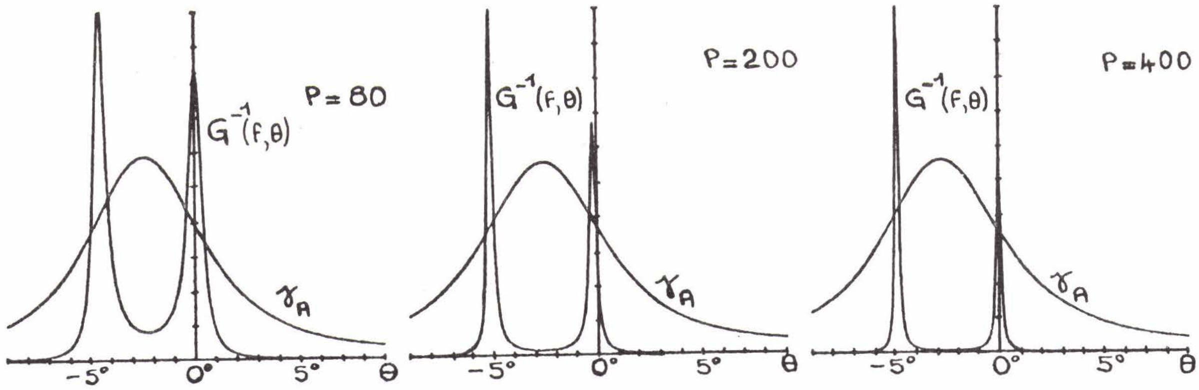

FIG. 1 ADAPTIVE ARRAY OUTPUT POWER γ_A AND INVERSE ORTHOGONAL SUBSPACE HIGH RESOLUTION METHOD RESPONSE $G^{-1}(f,\theta)$ VERSUS BEARING θ, FOR SPATIALLY INCOHERENT BACKGROUND NOISE AND TWO SOURCES WITH BEARINGS: 0° AND -5.3°, AND SIGNAL TO NOISE RATIO: 0 dB

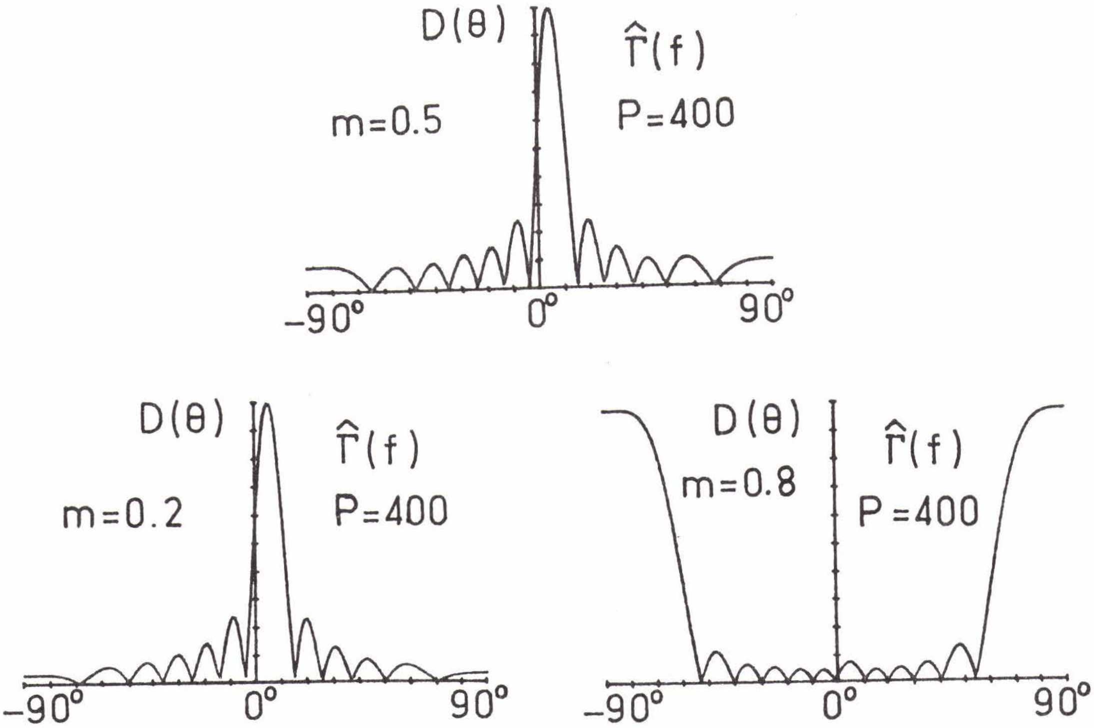

FIG. 2 MAXIMUM EIGENVECTOR DIAGRAMS VERSUS BEARING θ FOR SPATIALLY COHERENT BACKGROUND NOISE WITH SPATIAL COHERENCE PARAMETER m AND ONE SOURCE WITH BEARING: 5.4°, AND SIGNAL TO NOISE RATIO: -15 dB. For $m = 0.5$, background noise is spatially incoherent.

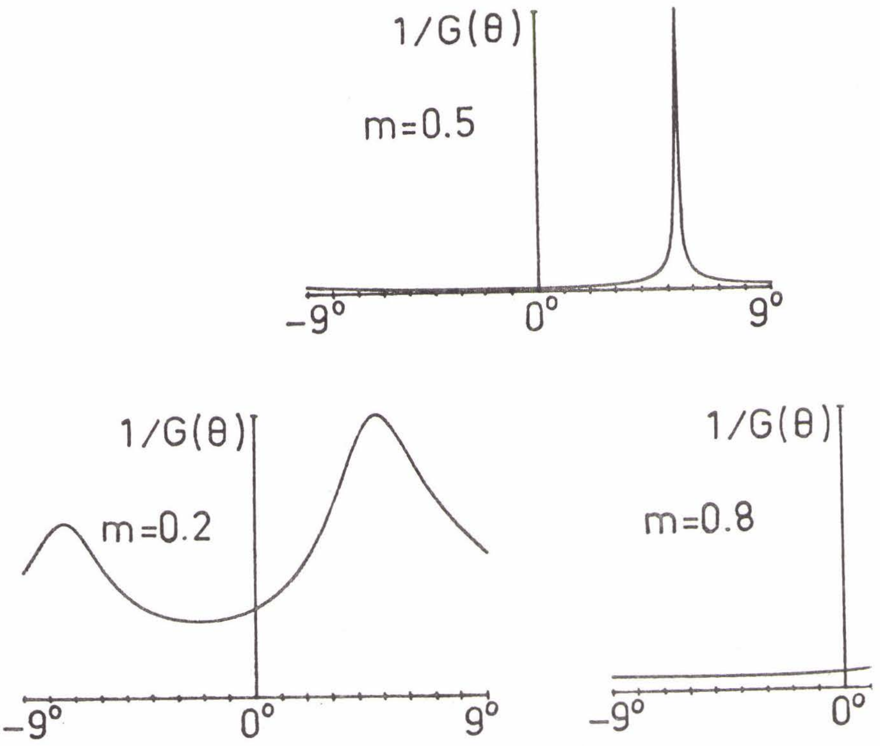

FIG. 3 INVERSE OF ORTHOGONAL SUBSPACE HIGH RESOLUTION METHOD RESPONSE $G^{-1}(f,\theta)$ VERSUS BEARING θ FOR THE SAME NOISE FIELD AS IN FIG. 2.

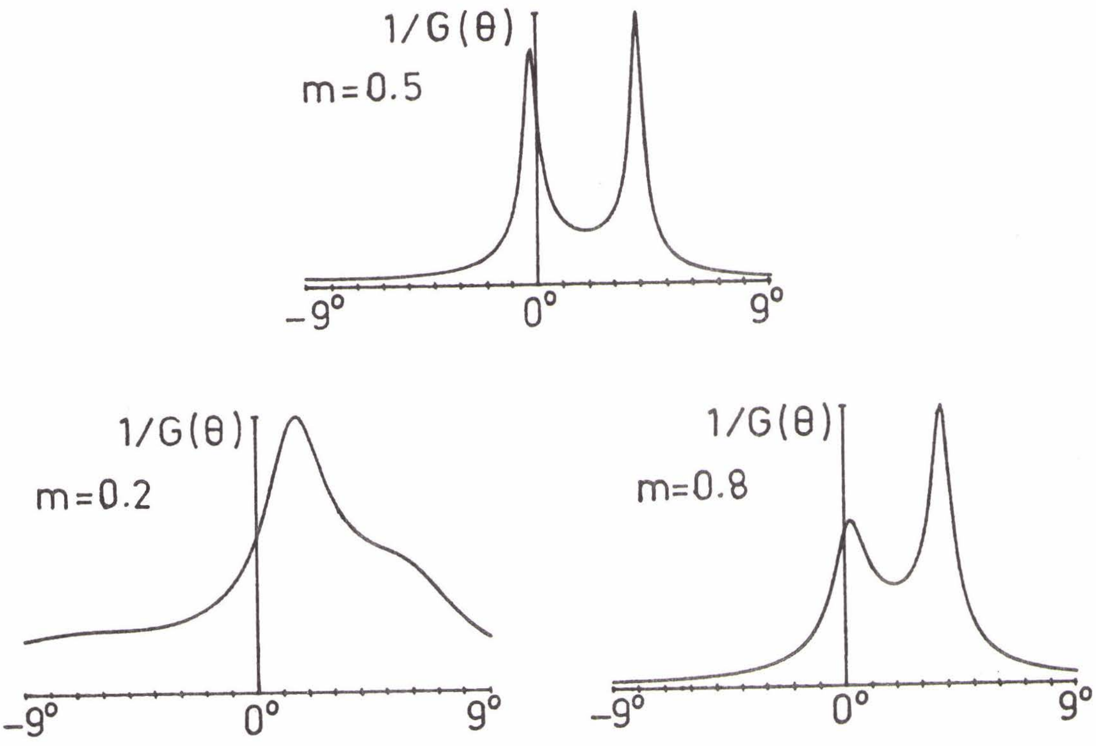

FIG. 4 INVERSE OF ORTHOGONAL SUBSPACE HIGH RESOLUTION METHOD RESPONSE $G^{-1}(f,\theta)$ VERSUS BEARING θ FOR THE SAME BACKGROUND NOISE AS IN FIG. 2 AND TWO SOURCZS WITH BEARINGS: 0° AND 4°, AND SIGNAL TO NOISE RATIO: -10 dB.

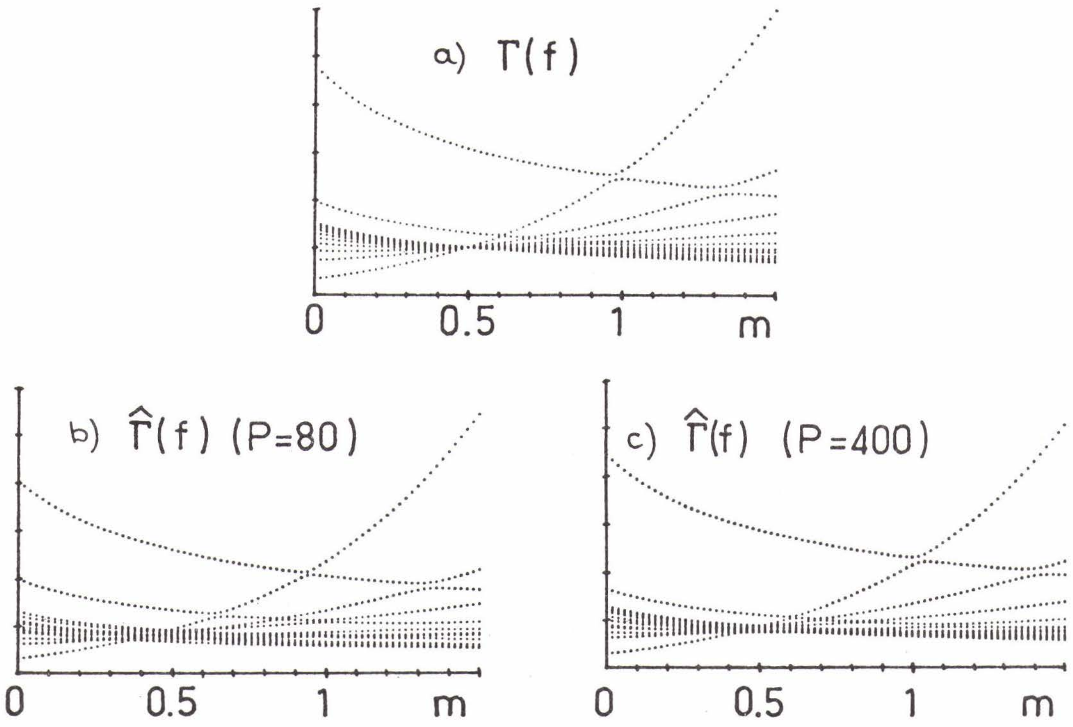

FIG. 5 EIGENVALUE DIAGRAMS FOR THE SAME NOISE FIELD AS IN FIG. 4 FOR THE THEORETICAL SPECTRAL DENSITY MATRIX: a), AND FOR THE SPECTRAL DENSITY MATRIX ESTIMATE FOR TWO VALUES OF THE OBSERVATION TIME (proportional to P): b) and c).