AN EXPERIMENTAL VERIFICATION OF A GEOMETRIC ACOUSTIC APPROXIMATION

by

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This paper sets out to show that a very simple ray treatment may be used to give a surprisingly good estimate of a sound field, even in conditions where the criteria for the use of geometric acoustics are apparently not fulfilled.

Experimental Arrangement [Fig.1]

A hollow thin-walled air-filled steel cylinder was immersed in water with its axis horizontal. Its dimensions were 4 ft long by 1 ft diameter by 0.05" wall thickness. The ends were capped to make it watertight. The cylinder could be rotated about a vertical axis through its centre, while in a horizontally-travelling continuous plane wave sinusoidal acoustic field. Three frequencies, 4.8 kHz, 10 kHz and 19.2 kHz, were used.

To investigate the field round the cylinder a probe hydrophone was mounted on a bracket as shown in Fig. 1, its position with respect to the cylinder being determined by the dimensions x and y. Both x and y could be varied. For each position of the probe hydrophone a response curve was plotted by rotating the cylinder through an angle γ about the vertical axis; $\gamma = 0$ corresponding to the situation in which the axis of the cylinder was parallel to the incident wavefront.

A selection of the amplitude/azimuthal angle plots for a number of combinations of x and y, and for each of the three frequencies, is shown in Fig. 2 (dashed lines).

Theory

At the lowest frequency the cylinder is only 4 wavelengths long by 1 wavelength diameter. This is in the intractable region where neither the small nor the large wavelength approximations apply. Although an exact wave-theoretical solution is known for a cylinder of infinite length, it was feared that this series solution might be only slowly convergent, and as simplicity was a prime requirement an attempt was made to use a crude ray theory.

Simplifying assumptions were necessary, viz:

(a) That the cylinder is of infinite length.

(b) That on reflection at the cylinder wall the phase change appropriate to a small pencil of incident energy could be calculated as that which would be obtained on reflection from a plane parallel thin sheet of steel of a plane wave at the same angle of incidence.

(c) That any ray is reflected specularly at the surface.

(d) That shear waves in the cylinder wall may be neglected.

In these circumstances it is easily shown that the sound field at the hydrophone will arise from the coherent addition of the intensities associated with the incident ray and a unique reflected ray arriving at the hydrophone.

Denote the radius of the cylinder by a, and the wall thickness by δ. The hydrophone position will be given by the polar coordinates r, Φ referred to the cylinder [in Fig. 1 $r^2 = (x+6)^2 + y^2$; $\tan \Phi = y/(x+6)$]. Let the wavelengths in water and in steel be λ and λ_1 respectively, the corresponding densities being ρ_1 and ρ_2. The problem is now essentially a simple but somewhat tedious exercise in coordinate geometry.

The first step is to compute an auxiliary angle ψ from the transcendental equation

$$\sin(2\psi - \Phi) = (a/r)\sin\psi \qquad \text{[Eq. 1]}$$

(ψ is in fact the cylindrical angular coordinate defining the position of the point of specular reflection). It should be noted that this angle is independent of the azimuthal angle γ.

We now compute separately the amplitude and phase of the reflected intensity at the hydrophone.

If the incident amplitude is 1 and the reflected amplitude A, we assume that A^2 = ratio of the cross-section of a pencil of rays before reflection to the cross-section of the same pencil at the hydrophone. The result is that

$$A^2 = \frac{(a/r)\cos\psi}{(a/r)\cos 2\psi + 2\operatorname{cosec}\psi \sin(\Phi - \psi)} \,. \qquad \text{[Eq. 2]}$$

It will be noted that A never exceeds 1 (since the reflected pencil always diverges). As $r \to a$, $A \to 1$, as we should expect. For $r >> a$, $A^2 \to (a/r)\cos(\Phi/2)$; this is exactly the result obtained by the Fresnel zone approach. The amplitude A is independent of γ.

The relative phase of the reflected ray has two components. The first arises from the path difference to the hydrophone. This turns out to be

$$\Delta = 2r\cot\psi\cos\psi \;\sin(\Phi - \psi)\cos\gamma \,. \qquad \text{[Eq. 3]}$$

The second phase change is that due to specular reflection. Assuming Rayleigh reflection for stratified three media (water, steel, air), the reflected ray suffers a change of phase ϵ, but is unaltered in amplitude. The complete expression for this change of phase assumes a different form according to whether the critical angle is or is not exceeded. However, in the circumstances considered, where $2\pi\delta/\lambda << 1$, it can be shown that the approximation

$$\tan(\epsilon/2) = \frac{\sec\gamma \sec\psi}{2\pi} \frac{\rho_1}{\rho_2} \frac{\lambda}{\delta} \qquad \text{[Eq. 4]}$$

holds throughout the complete range. It will be noted that for this thin sheet the value of λ_1 (i.e. a knowledge of the speed of sound in steel) is irrelevant.

Expressions 1 to 4 are easily calculable, and the results are plotted in Fig. 2. The amplitude scale has been normalized so that the free-field response of the hydrophone is unity, and it should be noted that this is a linear scale.

Discussion of Results

Considering the crudeness of the theory the agreement with experiment is, in general, gratifying, particularly in view of the fact that there are no fitted parameters in the theory. It is not easy to ascertain, indeed, whether the differences observed arise from the deficiencies of the theory or the inevitable errors in the experimental measurements. Two sources of error are, for example, the disturbance of the sound field by the bracket/hydrophone combination (the hydrophone was a cylinder approximately $\frac{1}{2}'' \times \frac{1}{4}''$), and the difficulty of ensuring accuracy in maintaining the exact geometry (which can have a large effect on the phase relationships).

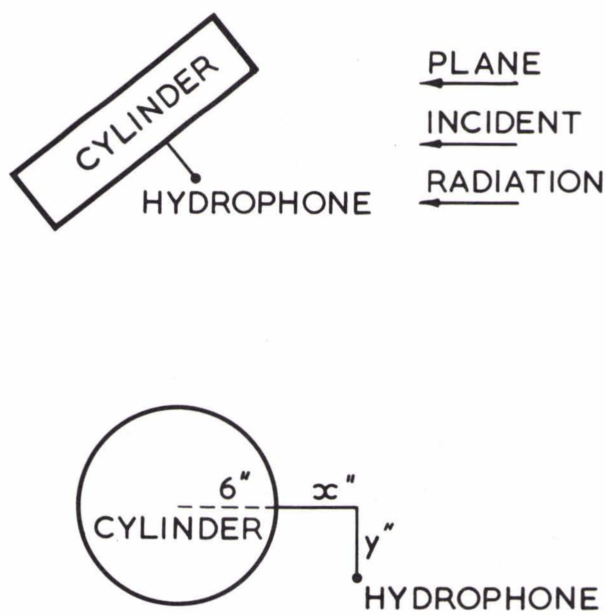
There is some suggestion that some of the unpredicted oscillations of the experimental results about the theoretical curve may arise from multipath effects, perhaps arising from shear wave stimulation, in that the positions of peaks and nulls correspond to paths in the steel having the appropriate time delays. The amplitudes of these oscillations are, however, slight in general, suggesting that these secondary effects are reasonably insignificant.

CONCLUSION

It is concluded that even in limiting conditions an extremely crude theory may give a surprisingly accurate answer. It is difficult to lay down criteria for the validity of such approaches, but when combined with an experimental validation they may be very useful.

DISCUSSION

The author acknowledged that his predictions might have been improved by incorporating phase shifts and delays as treated for example by Brekhovskikh, and also creeping rays; but that his work was done quite some time ago, predicated on simplicity, and without the benefit of a computer, and that the results were quite encouraging as they were.



EXPERIMENTAL ARRANGEMENT

FIG. 1

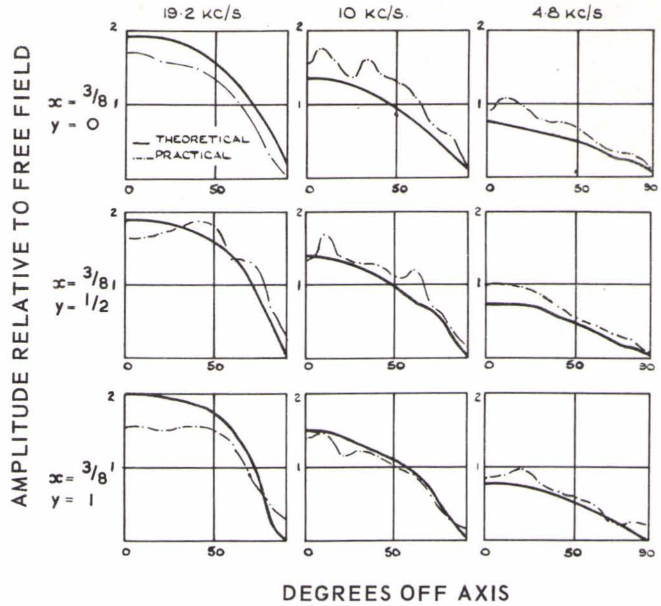


FIG. 2a

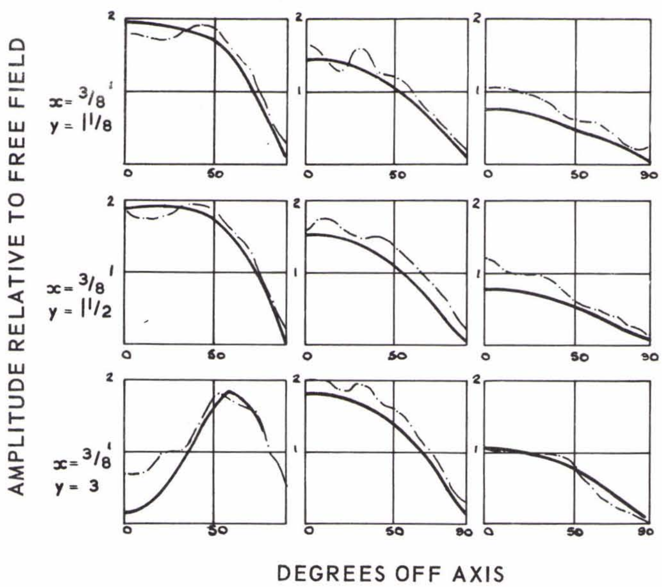


FIG. 2b

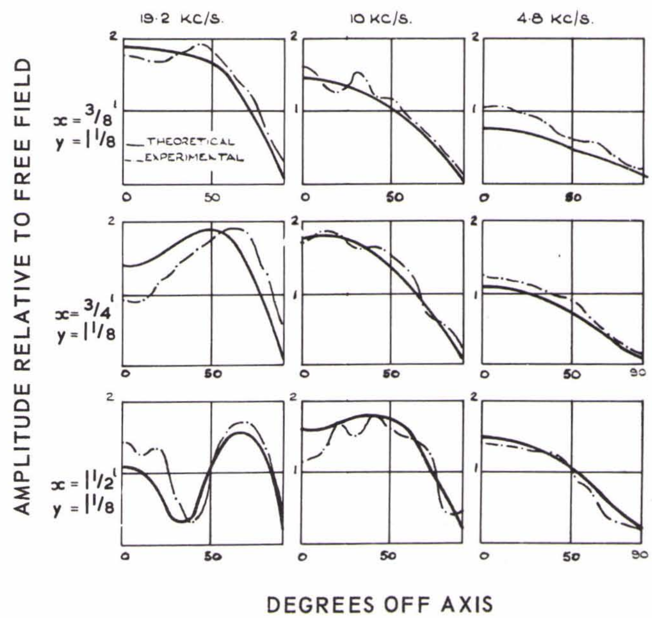


FIG. 2c