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ROBUST NORMALIZATION ALGORITHMS FOR LOW-FREQUENCY ACTIVE SONAR SIGNALS

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David L. Bradley
Director

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Executive Summary: Normalization is the process of forming a signal-to-noise ratio (SNR) from the raw signal power received by a sonar system. It is considered to be an essential part of sonar systems, especially under non-alerted conditions, and it may contribute significantly to the overall effectiveness of a surveillance system. However, the effectiveness of the normalization process is reliant upon an accurate estimate of the noise only reference; under real circumstances, the received data in general contains both noise and signal elements that can be difficult to separate. This report considers two algorithms for providing an adaptive estimate of the noise only reference for the normalization processors of active sonar signals.

A 'soft' background noise estimation approach rather than a selective rejection of false echoes is chosen. Owing to their specific design, these algorithms are particularly good candidates to normalize the data stream before applying post-processing methods based on ping history. Their performances and their robustness against signal components in the reference cells are examined. Recommendations about the implementation of the algorithms are derived. The results obtained from applying the algorithms to a real data set show that they perform well.

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Robust normalization algorithms for low-frequency active sonar signals

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Abstract: Two normalization processors of active sonar signals are presented. Due to their specific design, these algorithms are particularly good candidates to normalize the data stream before applying post-processing methods based on ping history. Their performances and their robustness against signal components in the reference cells are examined. Some recommendations about the implementation of the algorithms are derived. Results on a real data set show that they perform well.

Keywords: LFAS – low frequency active sonar – robust normalization

Contents

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1 Introduction

Normalization has always been a controversial subject mainly due to a misunderstanding of its role in the signal processing chain. It does not improve performance in the sense that it does not modify the receiver operating curves (ROC) of the receiver (cf. [1]); however its contribution to the overall effectiveness of any surveillance system is essential. The resources (e.g. the cpu and the memory space of an automatic alert system or the concentration of an operator watching a display) must be allocated uniformly in space (range and bearing) and in time (ping history) to avoid local overloads which are detrimental to the surveillance of the whole zone. By generating a constant false alarm distribution, the normalization process achieves this task, which is necessary to operate a detector in a non-alerted context. This function might be totally irrelevant in another context such as in tracking or in alerted detection conditions. Also the fact that normalization is very often associated with thresholding may imply an incapability to detect low signal-to-noise ratio echoes. This is not true: the normalized time series can be considered without any thresholding and if the background noise power is correctly estimated the performance will not be degraded.

The background noise definition is the key to any normalization method: to normalize we need to make an assumption about the probability density function (pdf) of the noise. In our case we have chosen to define the background noise as the Gaussian noise component at the input of the receiver. The behavior of the normalization as regards other components of the noise (e.g. non-Gaussian reverberation) is an essential factor in the choice of the algorithm.

The objective of this report is to present and describe two specific normalization algorithms. These have been designed with the following requirements in mind:

1. To be used for normalizing the output of active sonar receivers based on the envelope matched filter (EMF), whether followed by a post-integration process or not.

2. To be as 'soft' as possible, i.e. not to attempt to reject selectively deterministic false alarms such as bottom or coastal features. We want to retain all the voltage excursions which are 'significantly' greater than the background noise in the immediate vicinity. The reason is that we do not want to make a detection decision on a one-ping basis. Therefore the algorithm should be well adapted to normalize multi-ping displays or to reduce the input data stream of multi-ping post-processing algorithms.

3. To be as 'natural' as possible in order not to introduce artifacts into the normalized time series which could be difficult to handle by a multi-ping classification algorithm.

4. To be ‘low cost’: i.e. not to degrade the detection performance of the receiver. As we said before this is theoretically proven for any normalization method assuming that the noise power estimation is perfect (which is never the case).

5. To be as simple as possible in order to be used in real time applications.

Since each beam and Doppler channel is affected in its own particular way by the reverberation, each channel must be treated in its own right. Although this requirement does not preclude the processing in one channel to be partly controlled by the signals in adjacent channels, we restrict the present discussion to a process without channel coupling.

As shown in [1], the pdf of the noise samples to be normalized is in the most general case (including a post-integration stage) a chi-square law with an even number of degrees of freedom $N_{df}=2K$, where K represents the number of performed post-integrations (e.g. K=1 means no post-integration).

Note The post-integration might consist of either a summation of the peaks of the EMF time series inside a window in range or a summation of the EMF peaks over several pings along a predetermined track. The case of the split replica correlation process can also be treated as a special case of post-integration [1].

We consider the general case of a chi-square N_{df} degrees of freedom distributed noise time series (z) defined as the sum of the squares of N_{df}=(2K) independent, zero-mean Gaussian random variables having the same variance (σ^2). Therefore we have

$$z = \sum_{k=1}^{N_{df}} x_k^2 \text{ with } x_k \sim N(0,\sigma^2) \qquad \text{(I)}$$

and it is possible to show (cf. [1], Annex 2) that:

$$\begin{aligned} M_0 &= E(z) = N_{df}\,\sigma^2 \\ V_0 &= Var(z) = 2N_{df}\,\sigma^4 = \frac{2M_0^2}{N_{df}} \end{aligned} \qquad \text{(II)}$$

where E( ) and Var( ) stand respectively for expectation and variance, and $N(0,\sigma^2)$ is the Gaussian probability density function (pdf) of the independent random variables x_k.

Thus the normalized time series (z_n) is easily deduced from (z) as follows:

$$z_n := \frac{z}{\sigma^2} = \frac{z}{M_0} N_{df} \text{ and } pdf(z_n) = \chi^2_{(N_{df})} \qquad \text{(III)}$$

In practice, since σ^2 is not known, an estimate $\widehat{\sigma^2}$ deduced from $\widehat{E(z)}$, the estimated expectation of z, is used to perform the normalization:

$$z_n = \frac{z}{\widehat{\sigma^2}} \text{ and } \widehat{\sigma^2} = \frac{\widehat{E(z)}}{N_{df}} \qquad \text{(IV)}$$

The objective of the normalization process is basically to provide an adaptive estimation of E(z). Among the various methods we have been testing, we have chosen to present two algorithms whose performances are very close to the requirements quoted above. Even if they achieve similar results, their different implementations could be a major factor when deciding which one to choose.

2
Logarithmic normalization

2.1. INTRODUCTION

The concept of logarithmic normalization has been used in many different ways and it is possible to find in the literature several related papers [2–4]. The principle of the logarithmic normalization lies in the fact that the estimation of the expectation of z is deduced from the expectation of the logarithmic values of the noise time series as shown in the following section.

2.2. EXPECTATION OF THE LOGARITHM OF A CHI-SQUARE, N_{df} DEGREES OF FREEDOM DISTRIBUTED RANDOM VARIABLE

We consider the general case described in Sect. 1, using the same notations as in (I), (II), (III) and (IV).

Let us consider the random variable lz_e as the logarithmic value of z. We have

$$lz_e = \log_e(z) = \log_e(\frac{z_n M_0}{N_{df}}) = \log_e(z_n) + \log_e(M_0) - \log_e(N_{df})$$

and

$$M_e = E(lz_e) = E\{\log_e(z_n)\} + \log_e(M_0) - \log_e(N_{df}) \qquad (1)$$

To deduce M_0 as a function of M_e we need to know first what is $E\{\log_e(z_n)\}$, the mean of the logarithm of a chi-square, distributed random variable with N_{df} degrees of freedom.

Considering the moment generating function of $\log_e(z_n)$ given in [5], Chap. 7, Sect. 6, p. 181 and [6], Chap. 1, Sect. 5:

$$M(r) = E\{e^{r \log_e(z_n)}\} = E\{z_n^r\} = 2^r \frac{\Gamma(\frac{N_{df}}{2}+r)}{\Gamma(\frac{N_{df}}{2})} \qquad (2)$$

According to [7], Chap. 11, p. 425, where Γ is the gamma function (cf. [8], Chap. 6) we deduce the moment about zero of order p (m_p):

$$m_p = \int \{\log_e(z_n)\}^p \, pdf(z_n)\, dz_n = \frac{d^p(M)}{dr^p}(r=0) \qquad (3)$$

For p=1 we have

$$E\{\log_e(z_n)\} = m_1 = \frac{d(M)}{dr}(r=0) = \log_e(2) + \frac{\dfrac{d\{\Gamma(\frac{N_{df}}{2})\}}{dr}}{\Gamma(\frac{N_{df}}{2})} = \log_e(2) + \psi(\frac{N_{df}}{2}) \quad (4)$$

where ψ is the digamma (or psi) function (cf. [7], 6.3.1).

From the recurrence formula of the gamma function it is possible to show that (cf. [7], 6.4.6)

$$\psi(a+1) = \psi(a) + \frac{1}{a} \text{ and } \psi(1) = -\gamma$$

where γ is the Euler constant equal to 0.57721 56649... (cf. [7], 6.1.3).

Since we have assumed that N_{df} is an even integer, i.e. $N_{df} = 2K$, we derive

$$\psi(\frac{N_{df}}{2}) = \psi(K) = \{\sum_{i=1}^{K-1} \frac{1}{i}\} - \gamma$$

and

$$E\{\log_e(z_n)\} = \log_e(2) + \{\sum_{i=1}^{K-1} \frac{1}{i}\} - \gamma \quad (5)$$

2.3. ESTIMATION OF M_0

From (1) and (5)

$$M_e = \log_e(M_0) + \{\sum_{i=1}^{K-1} \frac{1}{i}\} - \gamma - \log_e(K) \quad (6)$$

or

$$M_0 = \exp(M_e)\, K \exp(\gamma - \{\sum_{i=1}^{K-1} \frac{1}{i}\}) \quad (7)$$

Instead of the natural logarithm lz_e and due to the decibel definition, it is more convenient to consider the random variable lz_{10} as the decimal logarithmic value of the random variable z. Then we have

$$lz_{10} = \log_{10}(z) = \frac{\log_e(z)}{\log_e(10)} \text{ thus } M_{10} = E(lz_{10}) = \frac{M_e}{\log_e(10)}$$

From (7)

$$M_0 = 10^{M_{10}} K \exp(\gamma - \{\sum_{i=1}^{K-1} \frac{1}{i}\}) \tag{8}$$

We estimate M_e (and M_{10}) as

$$\begin{aligned} \widehat{M_e} &= \frac{1}{P} \sum_{i=1}^{P} \log_e(z_i) \\ \widehat{M_{10}} &= \frac{1}{P} \sum_{i=1}^{P} \log_{10}(z_i) \end{aligned} \tag{9}$$

where P is the number of the noise-only reference samples chosen in a window on each side of the point to be normalized. By not removing the noise cell no gap is introduced, additonally, we do not remove the point itself from the noise-only reference samples thus easing the estimation process (the influence of non-noise only samples in the reference window will be studied in Subsect. 2.5). We then deduce the estimate of M_0 using (7) or (8).

Note, it is not necessary to take the square of the envelope matched filter. We can use the rms data and deduce directly the estimate of M_e (and M_{10}):

$$\begin{aligned} \widehat{M_e} &= \frac{2}{P} \sum_{i=1}^{P} \log_e(z_i^{1/2}) \\ \widehat{M_{10}} &= \frac{2}{P} \sum_{i=1}^{P} \log_{10}(z_i^{1/2}) \end{aligned}$$

2.4. PERFORMANCE OF THE LOGARITHMIC ESTIMATION OF M_0

The variances of lz_e and lz_{10} are easily deduced from the variance of $\{\log_e(z_n)\}$:

$$\begin{aligned} Var(lz_e) &= Var\{\log_e(z_n)\} \\ Var(lz_{10}) &= \frac{1}{\log_e^2(10)} Var\{\log_e(z_n)\} \end{aligned}$$

Following the same method as in Subsect. 2.2 we have

$$Var\{\log_e(z_n)\} = E\{\log_e^2(z_n)\} - E\{\log_e(z_n)\}^2 \tag{10}$$

with

$$E\{\log_e^2(z_n)\} = m_2 = \frac{d^2(M)}{dr^2}(r=0)$$

From (2) and (3) we obtain

$$m_2 = \{\log_e(2)\}^2 + 2\log_e(2)\ \psi(\frac{Ndf}{2}) + \frac{\frac{d^2\Gamma}{dr^2}(\frac{Ndf}{2})}{\Gamma(\frac{Ndf}{2})}$$

From the ψ function definition (cf. [7], 6.3.1) we have

$$\psi(r) = \frac{\frac{d\Gamma}{dr}}{\Gamma} \quad \text{then} \quad \frac{\frac{d^2\Gamma}{dr^2}}{\Gamma} = \frac{d\psi}{dr} + \psi^2$$

and therefore from (4) and (10)

$$\mathrm{Var}\{\log_e(z_n)\} = \frac{d\psi}{dr}(\frac{Ndf}{2} = K)$$

Using the general recurrence formula (cf. [7], 6.4.2 and 6.4.6)

$$\psi^{(n)}(r+1) = \psi^{(n)}(r) + (-1)^n n!r^{-n-1} \quad \text{for } n=0,1,2,\ldots$$

$$\text{with} \quad \psi^{(n)}(1) = (-1)^{n+1} n!\xi(n+1) \quad \text{for } n=1,2,\ldots \quad \text{and } \psi(1) = -\gamma$$

where ξ is the Riemann Zeta function (cf. [7], Chap. 23), we deduce

$$\frac{d\psi}{dr}(\frac{Ndf}{2}) = \frac{d\psi}{dr}(K) = \xi(2) - \{\sum_{i=1}^{K-1} \frac{1}{i^2}\}$$

with $\xi(2)=1.64493$ ... (cf. [7], p. 811).

Hence
$$\mathrm{Var}\{\log_e(z_n)\} = \xi(2) - \{\sum_{i=1}^{K-1} \frac{1}{i^2}\} \qquad (11)$$

The variance of the logarithmic mean estimate M_e as defined in (9) is then deduced:

$$\mathrm{Var}(\widehat{M_e}) = \frac{1}{P}\mathrm{Var}\{\log_e(z)\} = \frac{1}{P}(\xi(2) - \{\sum_{i=1}^{K-1} \frac{1}{i^2}\}) \qquad (12)$$

where we recall that P is the number of reference samples.

In the case of small fluctuations of lz_e, the variance of the derived M_0 estimate can be approximated with the differential function of (7):

$$\mathrm{Var}(\widehat{M_0}) \approx E\{(d\widehat{M_0})^2\} = M_0^2\, E\{(d\widehat{M_e})^2\} \approx M_0^2\ \mathrm{Var}(\widehat{M_e})$$

Then we deduce

$$\mathrm{Var}(\widehat{M_0}) = \frac{M_0^2}{P}\{\xi(2) - \{\sum_{i=1}^{K-1} \frac{1}{i^2}\}\} \qquad (13)$$

The variance V_{0nv_av} of an M_0 estimate based on an average of the natural values of z would be

$$V_{0nv\_av} = \frac{Var(z)}{P} = \frac{M_0^2}{P.K} \qquad (14)$$

The ratio in dB between the variances (13) and (14) of these two different ways to estimate M_0 is given as a function of N_{df} in Fig. 1. We see that the logarithmic estimation is always better than the natural-value-average estimation except for N_{df}=2 (2 dB difference). This can be easily explained by the shape of the chi-square pdf around zero as a function of N_{df}. The higher N_{df}, the lower the pdf around zero (see Fig. 2, N_{df}=1, 2, 3 and 4). Then since the values close to zero generate important variations in the logarithmic values it is normal to find a higher variance of the logarithmic normalization when N_{df}=2.

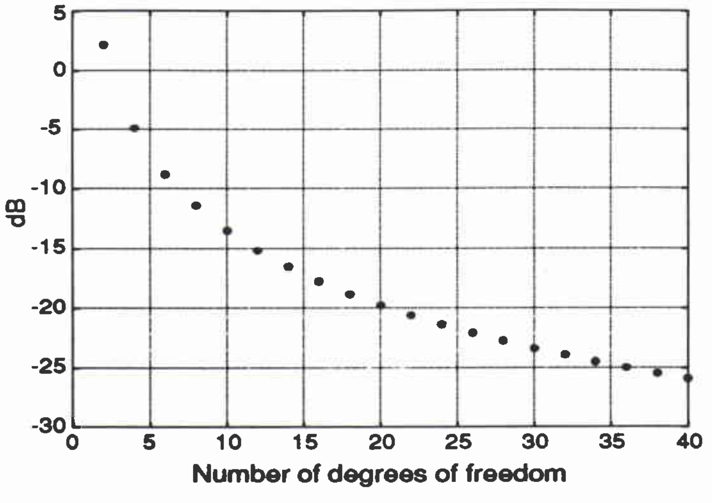

Figure 1 *Ratio in dB between the variances of the M_0 estimates based either on the logarithmic or on the natural value expectations.*

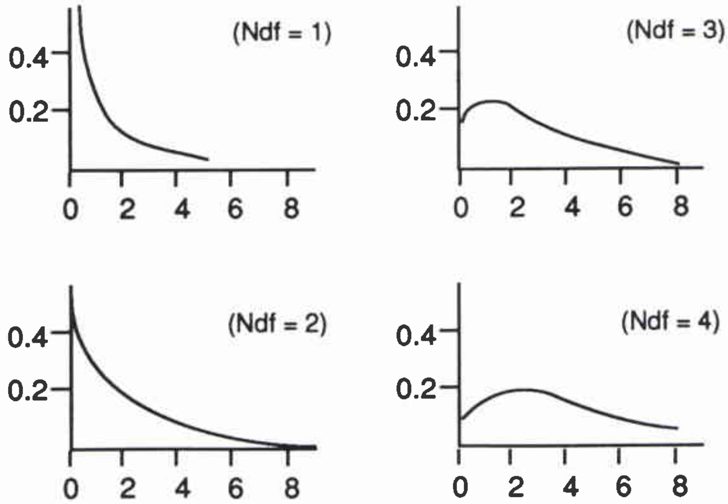

Figure 2 Chi-square density function (from Johnson , Kotz and Kemp, [6] p. 169).

To counter the low precision of the noise power estimate for $N_{df}=2$ one should use more samples for the logarithmic normalization than for the natural value in the proportion 1: $\xi(2)$.

We have chosen a different way to solve this problem: to retain as reference samples the local maxima (i.e. peak values). The theoretical calculations then become cumbersome and a Monte Carlo simulation gives the bias correction. From the 100000 simulations, $N_{df}=2$, we obtained

$$\frac{e^{E(\log_e(z_n,\ \text{local maxima}))}}{E(z_n)} = \frac{10^{E(\log_{10}(z_n,\ \text{local maxima}))}}{E(z_n)} \approx 1.50 \quad (15)$$

This choice is not necessarily better: as we will see in the next subsection, the main advantage of the logarithmic normalization is its natural robustness against signal components in the reference. Using only local maxima reduces the number of points in the noise-only reference window and tends to decrease the robustness of the algorithm as will now be shown.

2.5. INFLUENCE OF NON NOISE-ONLY SAMPLES IN THE REFERENCE WINDOW

2.5.1. Induced bias in the noise power estimation

The influence of some high-powered non-noise-only samples (above 0 dB SNRout) in the reference window (e.g. due to the echo of any kind of object including the target itself) is reduced due to the compression of the dynamic range using the logarithmic scale.

Let us consider the particular but frequent case, $N_{df}=2$, when there is no post-integration after the envelope matched filter. Then (8) becomes

$$M_{10} = \log_{10}(M_0) - \frac{\gamma}{\log_e(10)} \quad (16)$$

Let us assume that Q samples inside the P-sample wide window are not noise-only. We have

$$o_i = z_i + s_i \quad \text{for } 1 \le i \le Q$$
$$o_i = z_i \quad \text{for } Q+1 \le i \le P$$

where o_i is the observed time series in the reference window, z_i is the noise time series we want to estimate the mean, and Q peak signal components s_i with identical values equal to s are corrupting Q over P samples.

We assume that the signal component is strong enough so the following approximation applies:

$$\frac{s}{M_0} >> 1 \quad \text{so} \quad \log_{10}(z_i + s) \approx \log_{10}(s)$$

We deduce from (9)

$$E(\widehat{M_{10}}) = M_{10} + \frac{Q}{P}\{\log_{10}(s) - M_{10}\}$$

which, according to (16), is equivalent to

$$E(\widehat{M_{10}}) = M_{10} + \frac{Q}{P}\{\log_{10}(s) - \log_{10}(M_0) + \frac{\gamma}{\log_e(10)}\}$$

The bias is then

$$\text{Bias}_{M_{10}} = \frac{Q}{P}\{\log_{10}(s) - \log_{10}(M_0) + \frac{\gamma}{\log_e(10)}\}$$

The signal-to-noise ratio estimate in dB of any peak value can be derived from the M_0 estimate:

$$\widehat{SNR_{dB}} \approx 10\,\{\log_{10}(\text{Peak Value}) - \log_{10}(M_0)\}$$

(cf. [1], Subsect. 5.3. for the SNR definition after EMF).

According to (16) we have

$$\widehat{\log_{10}(M_0)} = \widehat{M_{10}} + \frac{\gamma}{\log_e(10)}$$

Table 1 *Effect of signal components in the noise-only referenc (logarithmic normalization)*

Q/P (%)	SNR_S (dB)	$Bias_{SNRdB}$ (dB)
10	10	-1.3
20	10	-2.5
30	10	-3.7
50	10	-6.3
10	20	-2.2
20	20	-4.5
30	20	-6.7
50	20	-11.3

Hence the bias of the estimation of $\log_{10}(M_0)$ is equal to the bias of the estimation of M_{10} and the SNR_{dB} bias is

$$Bias_{SNRdB} = - 10 \frac{Q}{P} \{\log_{10}(s) - \log_{10}(M_0) + \frac{\gamma}{\log_e(10)}\}$$

Defining SNR_S as the SNR in dB of each of the Q perturbing signal components we derive

$$Bias_{SNRdB} = - \frac{Q}{P} \{SNR_S + \frac{10\,\gamma}{\log_e(10)}\} \qquad (17)$$

The SNR is underestimated and we give in Table 1 the bias for different values of the ratio Q over P and for two values of SNR_S equal to 10 and 20 dB.

2.5.2. Comparison with a natural value average estimation

A natural value average estimate of M_0 would be

$$\widehat{M_0} = \frac{1}{P} \sum_{i=1}^{P} z_i$$

With the same assumptions as in Subsect. 2.4.1 we derive the effect of Q signal components in a P-sample wide reference window:

$$E(\widehat{M_0}) \approx M_0 + \frac{Q}{P} (s - M_0) \approx M_0 \,(1 + \frac{Q}{P}\, 10^{SNR_S/10})$$

and

$$Bias_{SNRdB} \approx -10 \log_{10}(1 + \frac{Q}{P}\, 10^{SNR_S/10}) \qquad (18)$$

We give in Table 2 the bias for the same values of the ratio Q over P and of SNR_S as in Subsect. 2.4.1.

Table 2 *Effect of signal components in the noise-only reference (natural value normalization)*

Q/P (%)	SNR_S (dB)	$Bias_{SNRdB}$ (dB)
10	10	-3.0
20	10	-4.8
30	10	-6.0
50	10	-7.8
10	20	-10.4
20	20	-13.2
30	20	-14.9
50	20	-17.1

We see that the logarithmic normalization performs well for low and high signal-to-noise ratio estimations and in all cases it gives a much better estimate than natural value normalizations. It is wise to recommend the choice of the window size such that the Q over P ratio is lower than 30%.

2.6. RESULTS

2.6.1. Check of the theoretical results

As a check on the theoretical results, Monte Carlo simulations have been made, generating a random variable, sum of the square of two independent Gaussian random variables (i.e. N_{df}=2) with the same variance (σ^2). In Tables 3a and 3b the measured mean and variance of lz_e and lz_{10}, together with their predicted values, are given.

Table 3a *Simulation results: measured mean and variance of lz_e*

σ^2	M_0	$M_e=\log_e(M_0)-\gamma$	$V_e=\xi(2)$	$\widehat{M_e}$	$\widehat{V_e}$
1	2	0.1159	1.6449	0.1112	1.6422
2	4	0.8091	1.6449	0.81842	1.6727
5	10	1.7254	1.6449	1.7289	1.6607
10	20	2.4185	1.6449	2.4461	1.5873
100	200	4.7211	1.6449	4.7275	1.6450

Table 3b *Simulation results: measured mean and variance of* lz_{10}

σ^2	M_0	$M_{10} = \log_{10}(M_0) - \frac{\gamma}{\log_e(10)}$	$V_{10} = \frac{\xi(2)}{\log_e^2(10)}$	$\widehat{M_{10}}$	$\widehat{V_{10}}$
1	2	0.0503	0.3102	0.0483	0.3097
2	4	0.3514	0.3102	0.3554	0.3155
5	10	0.7493	0.3102	0.7509	0.3132
10	20	1.0503	0.3102	1.0623	0.2994
100	200	2.0503	0.3102	2.0531	0.3103

2.6.2. Results on real data

The algorithms have been tested on a very large number of real data. However we will give the results considering only one representative beamformed matched filtered (without post-integration) time series acquired with the Low Frequency Active Sonar of the SACLANT Undersea Research Centre during a sea trial on board NRV *Alliance*. We require that the normalization algorithm be robust enough to perform well in any kind of situation and do not address the experimental context of the acquired data.

The results are presented in the form of four figures:

1. The original rms amplitude time series.

2. The normalized rms amplitude time series.

3. The background noise rms power estimate.

4. The signal-to-noise ratio estimate in dB.

2.6.2a. Without local maxima filtering of the noise-only reference samples (i.e. advantage to robustness)

We apply the logarithmic normalization algorithm described in Subsect. 2.1 with a reference window equivalent to ±50 local maxima. No local maxima filtering is performed. Results are given in Figs. 3–6. We see that the background noise is well estimated and that all the interesting features of the original time series have been kept in the normalized one.

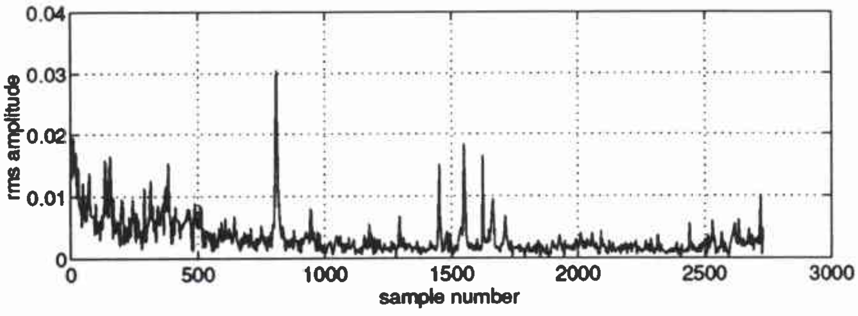

Figure 3 *Original rms time series.*

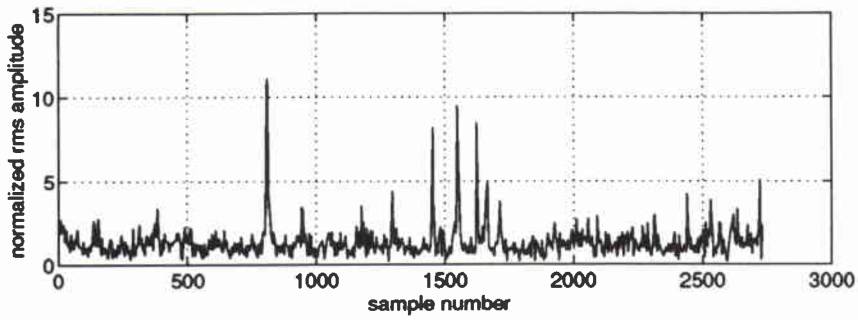

Figure 4 *Normalized rms time series – without local maxima filtering*

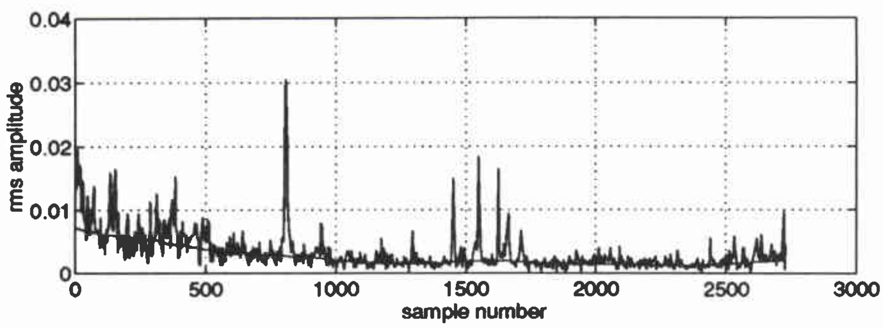

Figure 5 *Rms time series and background noise estimate – without local maxima filtering.*

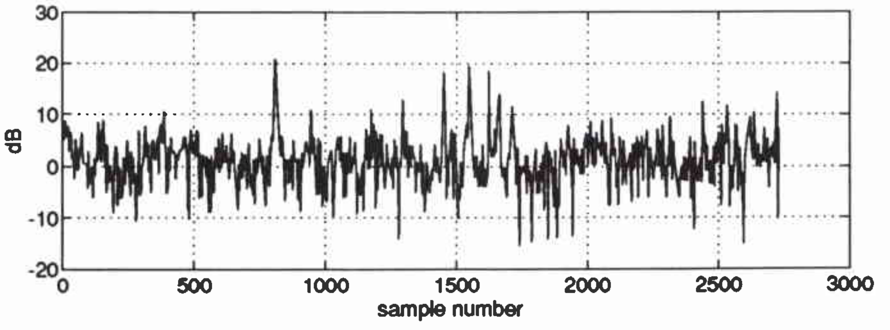

Figure 6 *Signal-to-noise ratio estimate – without local maxima filtering.*

2.6.2b. Using only the local maxima inside the noise-only reference samples (i.e. advantage to precision)

Here we apply a local maxima filter on the reference samples before estimating the logarithmic mean using formula (15) to deduce the noise power. Results are given in Figs. 7–10. The background noise estimate is a little bit lower than in the previous case especially in the first samples where the reverberation is predominant; however this difference is marginal. Hence the local maxima filter does not seem to be essential and since it costs some cpu time it might be better to just consider the simpler logarithmic normalization without local maxima filtering.

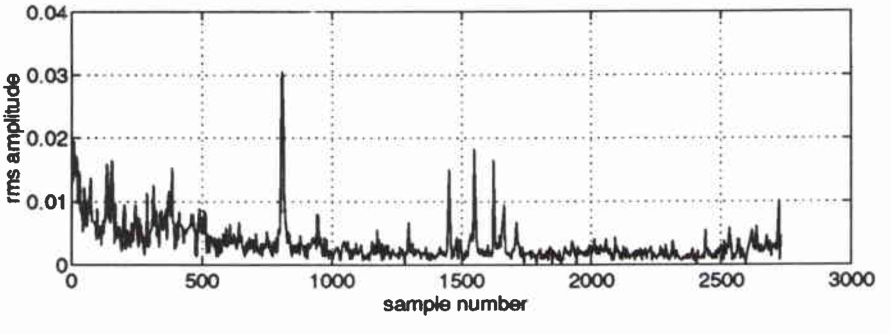

Figure 7 *Original rms time series.*

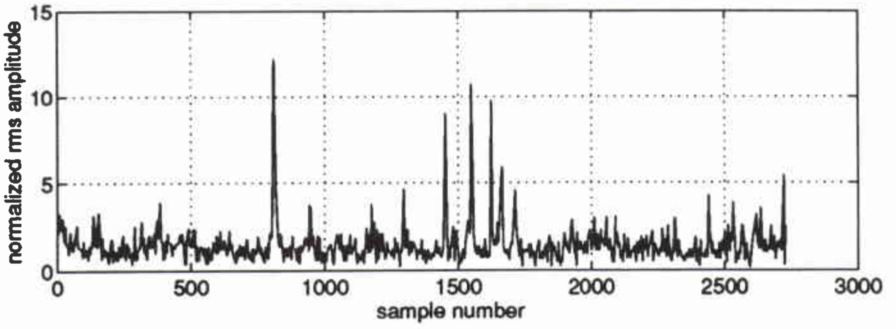

Figure 8 *Normalized rms time series – with local maxima filtering.*

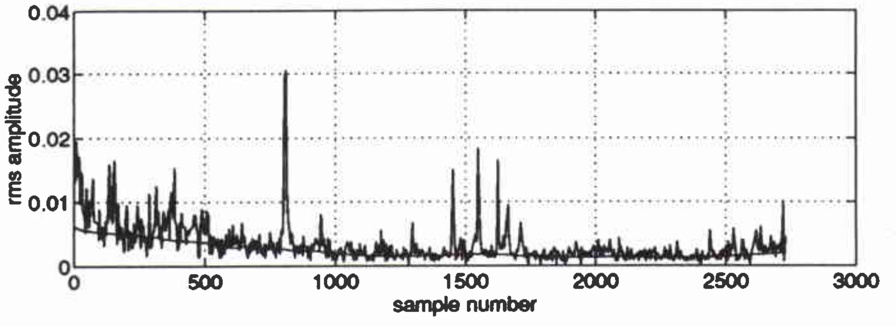

Figure 9 *Rms time series and background noise estimate with local maxima filtering.*

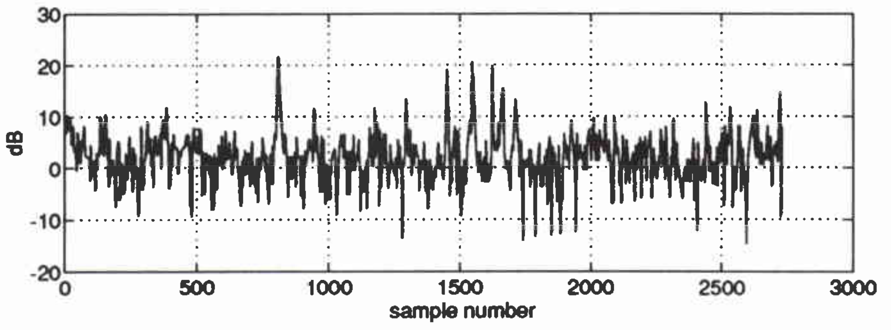

Figure 10 *Signal-to-noise ratio estimate – with local maxima filtering.*

3
Order statistics normalization

3.1. INTRODUCTION

The estimation of the expectation of z is now deduced from the order-statistics of the samples chosen in a noise-only reference window around the point to normalize. This subject has been widely addressed in the literature especially with regard to the constant false alarm rate receiver for radar detection [9–13].

A window of m noise-only reference samples is considered. These m samples are sorted into an ascending order (hence the name 'order statistics' normalization) and only the rth sample of the ordered sequence is kept. The m-r strongest signals in the reference cells do not therefore unduly bias the estimate of the noise power, and thus give improved performance in the presence of extraneous echoes.

The expectation of z is deduced from the expectation of the rth-order statistics as we will now show.

3.2. EXPECTATION OF THE rTH-OVER-m ORDER STATISTICS OF A CHI-SQUARE, N_{df} DEGREES OF FREEDOM DISTRIBUTED RANDOM VARIABLE

Again we consider the general case described in Sect. 1, using the same notations as in (I), (II), (III) and (IV).

Let us define the random variable $z_{\text{rth-over-m}}$ as the rth-over-m statistics of z and $M_{\text{rth-over-m}}$ its expectation. Since multiplying by a constant factor does not change the order, we have:

$$z_{\text{rth-over-m}} = (z_n)_{\text{rth-over-m}} \frac{M_0}{N_{df}}$$
$$M_{\text{rth-over-m}} = E(z_{\text{rth-over-m}}) = \frac{M_0}{N_{df}} E\{(z_n)_{\text{rth-over-m}}\} \tag{19}$$

To deduce M_0 from $M_{\text{rth-over-m}}$, we need to know first what is $E\{(z_n)_{\text{rth-over-m}}\}$, the mean of the rth-over-m statistics of a chi-square, N_{df} degrees of freedom random variable. A considerable amount of work has been done in evaluating the lower moments of order statistics [14], [15]. The pdf of the rth smallest among m independent random variables each having a chi-square distribution is given in [5].

It is possible to express the mean of the rth-over-m statistics as a finite sum of terms involving factorials. These expressions are cumbersome and we restrict our discussion to the particular case, $N_{df}=2$, when there is no post-integration after EMF. The same method, however, could be applied for any value of N_{df}.

According to [5], we deduce

$$\text{pdf}\{(z_n)_{\text{rth-over-m}}\}\ (x=\chi^2_{N_{df}=2}) = \frac{1}{2}\frac{m!}{(r-1)!\ (m-r)!}\ (1-e^{-x/2})^{r-1}e^{-(m-r+1)x/2} \tag{20}$$

and

$$E\{(z_n)_{\text{rth-over-m}}\} = \frac{1}{2}\frac{m!}{(r-1)!\ (m-r)!}\int_0^{\infty} x\ (1-e^{-x/2})^{r-1}e^{-(m-r+1)x/2}\ dx$$

Note, for m=1 and r=1 we obtain pdf(x)=1/2 exp(-x/2) which is consistent with the pdf of a chi-square random variable with two degrees of freedom.

Using the binomial decomposition of $(1-e^{-x/2})^{r-1}$ and the integral result

$$\int x\ e^{kx}\ dx = \frac{e^{kx}}{k}\ x - \frac{e^{kx}}{k^2}$$

we derive

$$E\{(z_n)_{\text{rth-over-m}}\} = 2\frac{m!}{(r-1)!\ (m-r)!}\sum_{k=0}^{r-1}\binom{r-1}{k}(-1)^k\frac{1}{(n-r+1+k)^2} \tag{21}$$

where $\binom{r-1}{k} = \frac{(r-1)!}{k!\ (r-1-k)!}$ are the binomial coefficients.

Note, as a check of formula (21), we obtain for m=8 and r=1, $E\{(z_n)_{\text{min-over-8}}\}=1/4$ from which we deduce the well-known result of the minima-over-8 normalization frequently used for image processing applications (cf. [16]):

$$E(z_{\text{min-over-8}}) = \frac{M_0}{2}\ E\{(z_n)_{\text{min-over-8}}\} = \frac{M_0}{8}$$

3.3. ESTIMATION OF M_0

Then from (19) we deduce M_0:

$$M_0 = \frac{N_{df}}{E\{(z_n)_{\text{rth-over-m}}\}}\ M_{\text{rth-over-m}} \tag{22}$$

We have chosen to estimate $M_{\text{rth-over-m}}$ without any averaging. (It might be better to average but this requires processing the same time series twice: first to find $z_{\text{rth-over-m}}$ and second to average.) Hence

$$\widehat{M_{rth\text{-}over\text{-}m}} = z_{rth\text{-}over\text{-}m} \tag{23}$$

3.4. PERFORMANCE OF THE ORDER STATISTICS ESTIMATION OF M_0

The variance of $z_{rth\text{-}over\text{-}m}$ is easily deduced from the variance of $(z_n)_{rth\text{-}over\text{-}m}$:

$$\mathrm{Var}(z_{rth\text{-}over\text{-}m}) = \left\{\frac{M_0}{N_{df}}\right\}^2 \mathrm{Var}\{(z_n)_{rth\text{-}over\text{-}m}\} \tag{24}$$

As in the previous subsection, we restrict our discussion to the particular case $N_{df}=2$. Following the same method as in Subsect. 3.2 we have

$$\mathrm{Var}\{(z_n)_{rth\text{-}over\text{-}m}\} = E\{(z_n)^2_{rth\text{-}over\text{-}m}\} - (E\{(z_n)_{rth\text{-}over\text{-}m}\})^2$$

with

$$E\{(z_n)^2_{rth\text{-}over\text{-}m}\} = \frac{1}{2}\frac{m!}{(r-1)!\,(m-r)!}\int_0^\infty x^2\,(1-e^{-x/2})^{r-1} e^{-(m-r+1)x/2}\,dx$$

Using the binomial decomposition of $(1-e^{-x/2})^{r-1}$ and the integral result

$$\int x^2 e^{kx}\,dx = \frac{e^{kx}}{k}x^2 - 2\frac{e^{kx}}{k^2}x + 2\frac{e^{kx}}{k^3}$$

we derive

$$E\{(z_n)^2_{rth\text{-}over\text{-}m}\} = 8\,\frac{m!}{(r-1)!\,(m-r)!}\sum_{k=0}^{r-1}\binom{r-1}{k}(-1)^k\frac{1}{(m-r+1+k)^3} \tag{25}$$

From (22), (23) and (24), considering the non-averaged estimate of $M_{rth\text{-}over\text{-}m}$, we deduce the variance of the M_0 estimate:

$$\mathrm{Var}(\widehat{M_{rth\text{-}over\text{-}m}}) = \mathrm{Var}(z_{rth\text{-}over\text{-}m}) = \left\{\frac{M_0}{N_{df}}\right\}^2 \mathrm{Var}\{(z_n)_{rth\text{-}over\text{-}m}\}$$

and

$$\mathrm{Var}(\widehat{M_0}) = \frac{N_{df}^2}{E\{(z_n)_{rth\text{-}over\text{-}m}\}^2}\,\mathrm{Var}(\widehat{M_{rth\text{-}over\text{-}m}}) \text{ thus :}$$

$$\mathrm{Var}(\widehat{M_0}) = M_0^2\,\frac{\mathrm{Var}\{(z_n)_{rth\text{-}over\text{-}m}\}}{E\{(z_n)_{rth\text{-}over\text{-}m}\}^2} = M_0^2\,\{\frac{E\{(z_n)^2_{rth\text{-}over\text{-}m}\}}{E\{(z_n)_{rth\text{-}over\text{-}m}\}^2} - 1\} \quad (26)$$

For a given noise power (i.e. a given M_0) we see that the variance of the estimate is only a function of the ratio $E\{(z_n)^2_{rth\text{-}over\text{-}m}\}/E\{(z_n)_{rth\text{-}over\text{-}m}\}^2$ (always greater than one). Then the optimal choice of r among the m possible values is the one which minimizes this ratio. We have plotted on Fig. 11 the different values of this ratio for the particular case m=20. We see that we reach an optimum for r=17. A generalization of this result is possible, and it is shown in [17], [18] that performance is generally optimized if r is set to about 7m/8.

However this optimized value of r is not very interesting since it only allows 12% (one-eighth) of the samples in the noise-only reference to be non-noise-only before the rth value is incorrectly a non-noise-only value. As shown in Fig. 11, in practice a value around r=3m/4 provides better resilience to several extraneous echoes while maintaining acceptable loss under ideal conditions (cf. [10–11]). Since our main objective is to keep all the details of the signal structure and considering the flatness of the minimum of the curve of Fig. 11, we have chosen to fix the value of r equal to 7m/10 in order to allow a percentage of non-noise-only samples in the noise-only reference equal to 30% before the noise only estimator degrades. Therefore from now on only the seventieth percentile (70%) normalization will be considered, although it would be easy to deduce from the previous developments the performance for any value of r.

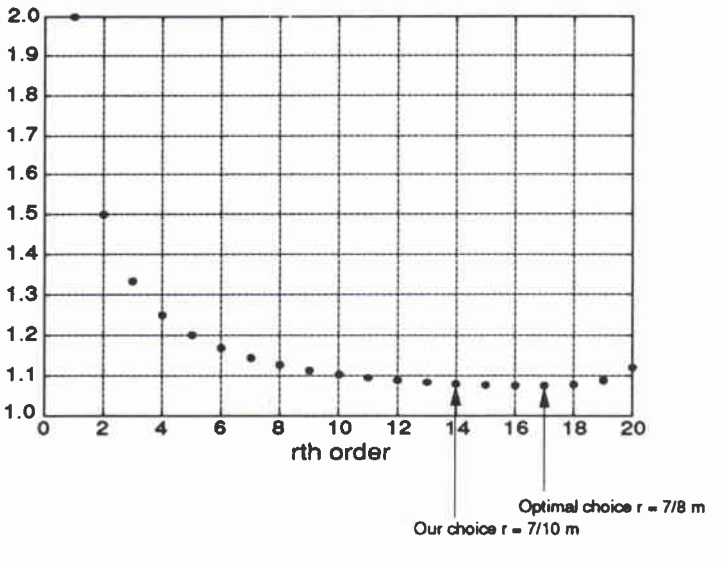

Figure 11 *Optimal choice of the ratio r over m in the particular case m=20.*

3.5. THE SEVENTIETH PERCENTILE ESTIMATE OF M_0

From (21) we see that the mean $E\{(z_n)_{\text{rth-over-m}}\}$ depends on m and r and not only on the ratio r-over-m. We give in Table 4 the value of $E\{(z_n)_{\text{rth-over-m}}\}$ for different values of m and r having the same ratio equal to 70%.

Table 4 *Expectation of the rth-over-m statistics with a r-over-m constant ratio*

m	r	$E\{(z_n)_{rth/m}\}$
10	7	2.1913
20	14	2.2955
50	35	2.3620*
100	70	2.3848*
500	350	2.4033*
1000	700	2.4056*

**For these values it was practically impossible to use the formula (21) because of important round-off errors due the large values of m and r. Thus we obtained these results from a numerical quadrature using an adaptive recursive Newton method.*

We see that the variations of $E\{(z_n)_{\text{rth-over-m}}\}$ as a function of m and r are small (less than 0.5 dB). We will assume in the following that the mean of the seventieth percentile statistics of a chi-square, two degrees of freedom random variable is approximately equal to 2.4. Therefore we can legitimately define $M_{70}=E\{(z)_{70}\}$ the mean of the seventieth percentile statistics of z with no further precision about m and r. Thus $E\{(z_n)_{70}\}\approx 2.4$ and we deduce easily M_0 from M_{70} as

$$M_0 = M_{70}\frac{2}{E\{(z_n)_{70}\}} \approx M_{70}\frac{2}{2.4} \tag{27}$$

We have deduced from the results of $E\{(z_n)_{\text{rth-over-m}}\}$ given in Table 4 and from (25) the corresponding variances (see Table 5).

Table 5 *Variance of the rth-over-m statistics with a constant r-over-m ratio*

m	r	$\text{Var}\{(z_n)_{rth/m}\}$
10	7	0.7546
20	14	0.4191
50	35	0.1785*
100	70	0.0912*
500	350	0.0185*
1000	700	0.0094*

**For these values it was practically impossible to use the formula (21) because of important round-off errors due to the large values of m and r. Thus we obtained these results from a numerical quadrature using an adaptive recursive Newton method.*

From (26) we deduce the variance of the M_0 estimate

$$\text{Var}(\widehat{M_0}) \approx \left(\frac{2}{2.4}\right)^2 \text{Var}(\widehat{M_{70}}) \approx M_0^2 \frac{\text{Var}\{(z_n)_{\text{rth-over-m}}\}}{(2.4)^2}$$

We see (cf. Table 5) that contrary to the mean, the variance cannot be considered as constant as a function of the ratio r-over-m. Logically the greater m is, the smaller the variance is. The variance of an M_0 estimate based on the average of the natural value of P samples is

$$\text{Var}(\widehat{M_0}) = \frac{M_0^2}{P} \quad \text{with } N_{df}=2$$

We give in Table 6 the number of samples P required by the natural value average estimation to achieve the same variances as given in Table 5.

Table 6 *Number of samples of the natural value average normalization*

m	r	P
10	7	8
20	14	14
50	35	32
100	70	63
500	350	311
1000	700	613

As a rule of thumb we see that the variance of the rth-over-m order statistic estimate is equivalent to the variance of the natural value average estimate when the sizes of the noise-only reference windows are in the ratio m to r (this does not mean that both methods are equally interesting since the main advantage of using the order statistics normalization is its robustness against non-noise-only samples in the noise-only reference). So the price to pay for the robustness brought by the order statistics normalization is the size of the reference window whose 'effective' length is reduced to r.

3.6. INFLUENCE OF NON-NOISE-ONLY SAMPLES IN THE REFERENCE WINDOW

The main advantage of using order statistics normalization is their natural robustness against strong signal components in the noise-only reference. If the ratio Q over P defined in Subsect. 2.4. is lower than 1-(r-over-m) (i.e. 30% in our case), the signal components have very little influence on the order statistics whatever their power. Hence the bias of the noise power estimation will be very low. We see that we end up with the same recommendations as in Subsect. 2.5.2, i.e. to choose the window size in such a way that the Q over P ratio is lower than 30%. Note that the robustness is excellent as regards the power of the corrupting signal components (what counts is only the Q over P ratio) but the loss in performance is drastic when the Q over P ratio is higher than 30%.

3.7. RESULTS

3.7.1. Check of the theoretical results

As a check of the theoretical results, Monte Carlo simulations have been made by generating a random variable, the sum of the square of two independent Gaussian random variables (i.e. $N_{df}=2$) with the same variance (σ^2). In Table 7 we give the measured mean and variance of $z_{70=14/20}$ with m=20 and r=14 together with their predicted values.

Table 7 *Simulation results*

σ^2	M_0	Theoretical		Measured	
		$M_{70=14/20}=\frac{2.2955}{2}M_0$	$V_{70=14/20}=0.4191(\frac{M_0}{2})^2$	$\widehat{M_{70}}$	$\widehat{V_{70}}$
1	2	2.2955	0.4191	2.3032	0.4292
2	4	4.5910	1.6764	4.5933	1.6547
5	10	11.4775	10.477	11.4652	10.2761
10	20	22.9550	41.910	22.9732	41.8663
100	200	229.5500	4191.0	229.7028	4200.3

3.7.2. Results on real data

Using the same time series and the same presentations of the results as in Subsect. 2.6.2 we apply the seventieth percentile order statistical normalization algorithm described in Subsect. 3.5. with a reference window equivalent to ±50 local maxima. Results are given in Figs. 12–15. We see that the background noise is well estimated. The method is somewhat less adaptive than the logarithmic normalization ('steps' in the background noise estimate).

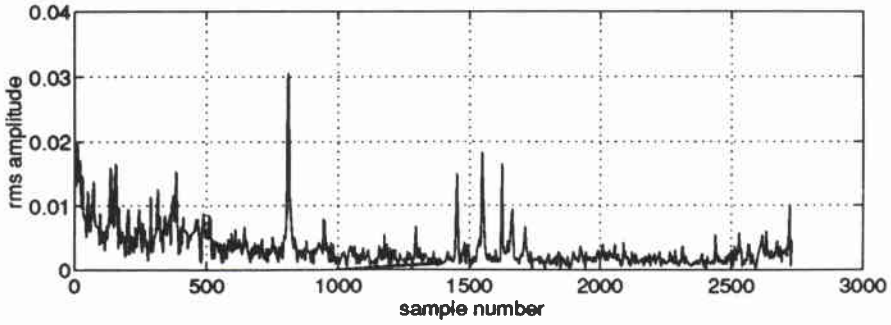

Figure 12 *Original rms time series.*

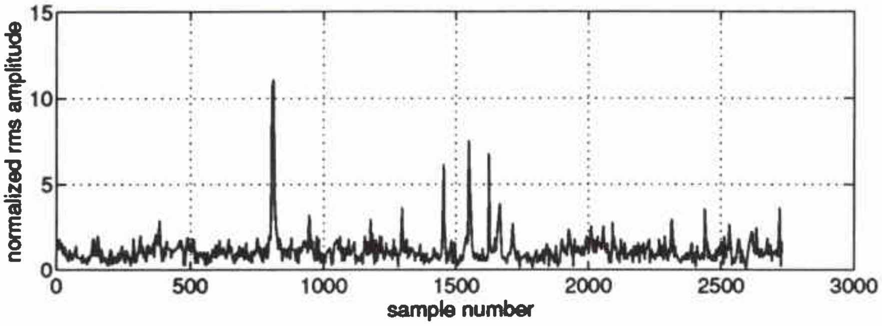

Figure 13 *Normalized rms time series.*

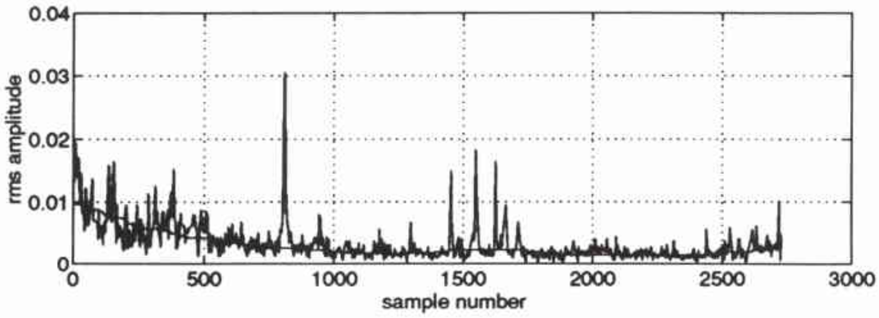

Figure 14 *Rms time series and background noise estimate.*

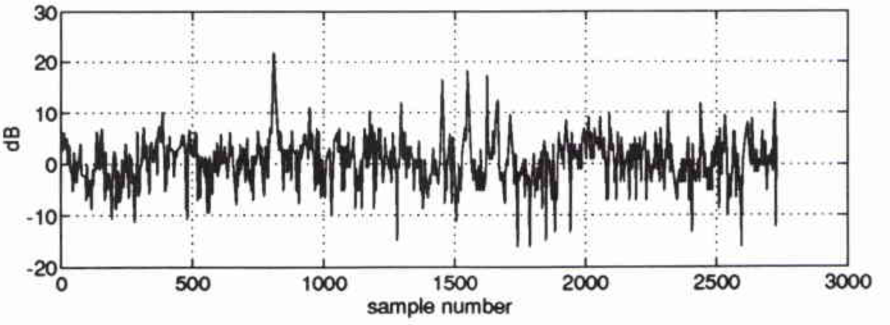

Figure 15 *Signal-to-noise ratio estimate.*

4 Conclusions

Two normalization algorithms applied after envelope matched filtering sonar data have been described. It has been shown how these algorithms are good candidates to reduce the data stream especially when post-processing methods based on the ping history are applied.

These algorithms meet the requirements described in the introduction:

1. The background noise power is well estimated with little bias. In consequence all the interesting features are kept, even the low signal-to-noise ratio ones. The simple way we choose the noise-only reference (e.g. no gap) has two major advantages:

   a) The normalization is robust (the theoretical results are close to real data).

   b) No artifacts are added in the normalized time series (adding some artificial detections).

2. The cost of these normalizations in terms of a possible loss of performance is extremely low.

3. Implementation of the order statistics normalization is more complicated than logarithmic normalization which, due to its extreme simplicity, makes it a more attractive algorithm for real time applications.

The precision of the noise power estimates has been analyzed and compared with the precision achieved by the classical natural value average estimator. Even if in some circumstances, the precision might not be as good as the natural value average estimator, this is a minor theoretical drawback compared to the advantage brought by the increased robustness of these algorithms in the treatment of real data signals. As a result they have been used with advantage for improving multi-ping displays.

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TDKK Turkey	5
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SACLANT	3
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