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State-dependent Dynamic Networks: A System theoretic Perspective

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14. ABSTRACT The objective of this project has been the development of a foundational approach to system theoretic analysis and synthesis of state-dependent dynamic networks. State-dependent dynamic networks encode interactions amongst dynamic nodes that in turn co-evolve with the node level dynamics. Complementary to the majority of research thrusts in the area of networked systems, the research program supported by this effort aimed to refocus the spotlight on network dynamics itself, and in doing so, examine notions such as dynamic graph reachability, utilizing time-scales for analyzing state-dependent graphs, and modular algorithms for network synthesis.					
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Final Report

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The objective of this project has been the development of a foundational approach to system theoretic analysis and synthesis of state-dependent dynamic networks. State-dependent dynamic networks encode interactions amongst dynamic nodes that in turn co-evolve with the node level dynamics. Complementary to the majority of research thrusts in the area of networked systems, the research program supported by this effort aimed to refocus the spotlight on network dynamics itself, and in doing so, examine notions such as dynamic graph reachability, utilizing time-scales for analyzing state-dependent graphs, and modular algorithms for network synthesis.

Below, we provide a synopsis of our contributions. More technical details are provided in the publications that have resulted from this project.

Summary of Technical Contributions

In the following, we provide a summary of some of our contributions supported by this project.

1. One of the areas of particular interest in the project has been the role of time-scales in analyzing dynamic state-dependent networks. The motivation for this work is the realization that multiple levels of time-scales in dynamic network, not only make their dynamic behavior seemingly more complex, but also facilitate their analysis through the powerful machinery of singular perturbation theory. For example, in this project, we have examined under what conditions the evolution of dynamic edges (exemplifying time-varying interactions between the nodes), and how these interactions induce node evolution, can be analyzed using time-separation principles. The resulting analysis has highlighted deep connections between the time-scales realized at the nodes and edges of the network. Moreover, our work has highlighted how the network geometry induce yet another layer of time-scales that influence the evolution of state-dependent networks. The publications associated with this line of work include [1, 2, 3, 4, 5].
2. Network synthesis has been another facet of the research supported by this project. Network synthesis deals with constructing networks that exhibit favorable system theoretic properties. For example, one can define a notion of network level performance in terms of the \mathcal{H}_2 norm of the network in response to a disturbance— and then pose the question of what types of networks have good disturbance rejection properties while also facilitating desirable network-level connectivity. This class of problems are of great interest in applications of dynamic networks, when the information geometry underlying the coordination algorithms can be adapted to their operational environment, say, due to the presence of adversaries and network attacks. In a series of works supported by this project, we have addressed various facets of network synthesis problems, including the modular, highly efficient series-parallel network construction, how to utilize nodes' time-scales in the network synthesis problems, and how to grow networks that have favorable system theoretic properties. The publications for this line of work include [6, 7, 8]. Another facet of the network synthesis problem, that in addition to system theoretic properties, examines the role of sparsity and cardinality constraints, has been considered in our work [9].
3. In this project, we have also examined the role of signed and matrix-valued interconnections on the dynamics and control of networks. Most of the existing works have examined networks with non-negative

interactions, that when realized for example in terms of diffusion coupling, make the state of the nodes reach coordination, e.g., consensus. However, the presence of negative interactions that capture competitive interactions amongst the nodes make the resulting dynamics more intricate. This extension is important as a wide range of networks in biology and social sciences naturally contain negative interactions and their use in engineered networks can also lead to key insights into the robustness and resilience of engineered networked systems. Another variation on examining more elaborate interactions amongst the nodes in the network involve “multi-dimensional” interactions, that can be nicely captured by matrix-valued weighted networks. In a series of works supported by this project, we have thoroughly analyzed both the so-called signed and multi-dimensional weighted networks in terms of their dynamic behavior, properties of the invariant sets, and their controllability and identifiability. Another aspect of more elaborate network structure is in terms of their layering structure, when a more complex network can be viewed as being composed of layers. A natural question in this setting is whether this layering structure would allow a more modular approach to system analysis and synthesis. In this project, we have shown how one can adopt control synthesis procedures to address control design for uncertain layered networks. The publications that provide the technical details on signed, matrix-valued, and layered networks include [10, 11, 12, 13, 14].

4. During the last phase of the project, we shifted our attention to control problems on networks with a data-driven twist. This line of work has been motivated by the realization that in a number of settings, the network model, in terms of the node dynamics or their interactions, are unknown a priori and as a result, it becomes imperative to reason about influence or synthesis using the data or time-series available from the network behavior. Once we adopt this data-guided point of view, it became clear that the existing techniques to design data-guided control for complex networked systems have serious limitations. For example, one of the assumption for popular data-guided control (reinforcement learning) is the knowledge of an initial stabilizing control law for the system- this initial controller is then iterated upon for better performance. This assumption however breaks down in applications, as it often requires that the underlying system or network to be open loop stable. In a sequence of works, we have examined various aspects of the data-guided approach to network identification and control. In particular, we have shown how the Koopman operator theoretic methods can be used in network settings, and how time-series data can be used for regulation of unknown networks and systems for potentially unstable open loop scenarios. This line of work shows a great deal of promise and the PI is in the process of submitting project ideas on data-guided control inspired by this work to AFOSR/AFRL. Data-guided methods are useful in scenarios where there is an inherent uncertainty in characterizing the underlying network/system models. Another source of uncertainty in large scale networks is the presence of *other* players that aim to influence the network behavior. Game theoretic approaches provide a powerful machinery to reason about networks that are under the influence of competing groups of agents. In our work supported by the project, we have examined how efficient distributed algorithms, e.g., dual-averaging, can be adopted for multiple players settings, and how the online nature of their decision-making process can steer the network to equilibria. In particular, we have shown how network structure in distributed optimization via dual averaging facilitates the Nash solution when the network is subject to input by players with competing objectives. The publications from this line of work include [15, 16, 17].

The work supported by this project has led to a number of key insights on systems and control issues for more complex network models, including technical issues related to time-scales, signed and matrix-valued coupling, adversarial interactions, and data-guided control and identification. The project has also led to the training of a rather successful number of Ph.D. students who are currently in academia (as faculty and postdoctoral fellows– Melbourne and ETH), research labs (Honda Research Center), and during the interview process for research and development positions in industry.

Publications

The following publications have been supported by this AFOSR grant:

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