

Acoustical Measurement Uncertainty for the Underwater Sound Reference Division in the Frequency Range of 3 Hz- to-1.6 kHz in Simulated Oceanic Environments

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PREFACE

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14. ABSTRACT. This report documents the uncertainty in the primary and secondary calibration of hydrophones between 3 Hz and 1,600 Hz in simulated oceanic environments at the Underwater Sound Reference Division (USRD), the National Institute of Standards and Technology (NIST) Designated Institute (DI) for acoustics: sound in water. After an overview of uncertainty estimation and propagation methods, the equipment and measurement theory for the primary coupler reciprocity calibration and secondary standing wave tube comparison calibration are described and their uncertainties developed. Sources of uncertainty are discussed in detail. Tabulated results show the combined expanded uncertainties for primary and secondary calibration of hydrophones.						
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TABLE OF CONTENTS

Section	Page
LIST OF TABLES	ii
LIST OF ABBREVIATIONS AND ACRONYMS	ii
LIST OF MATHEMATICAL SYMBOLS AND UNITS.....	ii
1 INTRODUCTION.....	1
2 UNCERTAINTY ANALYSIS METHOD OVERVIEW	2
3 PRIMARY CALIBRATION OF A HYDROPHONE	5
3.1 Measurement Theory.....	5
3.2 Type A Uncertainty Analysis	8
3.3 Type B Uncertainty Analysis	10
3.4 Combined Uncertainty	24
4 SECONDARY CALIBRATION OF A HYDROPHONE	25
4.1 Measurement Theory.....	28
4.2 Type A Uncertainty Analysis	28
4.3 Type B Uncertainty Analysis	30
4.4 Combined Uncertainty	32
5 CONCLUSION	33
6 REFERENCES.....	33

LIST OF ILLUSTRATIONS

Figure	Page
3-1 Coupler Schematic with a Projector P, a Reciprocal Transducer T, and a Hydrophone H to be Calibrated	6
3-2 Fractional Standard Deviations for each Type H48 Hydrophone Compared to the Pooled Fractional Standard Deviation	10
3-3 Voltage Measurement Block Diagram.....	19
3-4 Sensitivity Curves for H48 SN1 Measured at 40° C and 345 kPa with and without Elastomeric O-Rings in the Coupler	23
3-5 Comparison of the Sources of Type B Measurement Uncertainty in the H48 Coupler	24
4-1 Standing Wave Schematic with a Projector, P, and Reference Hydrophone, R, and the Transducer Under Test, T.....	26
4-2 Diagram of a Pressure in an Acoustic Wave in a Tube with a Closed End	27
4-3 Results for Repeated Calibration of Three Different Hydrophone Types	29

LIST OF TABLES

Table		Page
3-1	Fractional Standard Uncertainties for Coupler Reciprocity Measurements	9
3-2	Type B Uncertainties	10
3-3	Constants for the Empirical Relationship of Sound Speed in Castor Oil with Temperature and Pressure.....	14
3-4	Transducer Parameters (reference 13)	17
3-5	High Pressure 3/8 in. OD 316SS, 3 in. Long Tube Parameters	18
3-6	Combined Uncertainty for Coupler Reciprocity Calibration.....	25
4-1	Comparison Calibration – Type A Uncertainty	30
4-2	Type B Uncertainty for the Comparison Calibration.....	31
4-3	Combined Uncertainty for Comparison Calibration.....	32

LIST OF ABBREVIATIONS AND ACRONYMS

BIPM	International Bureau of Weights and Measures
CMRR	Common mode rejection ratio
CPAR	Corrective or Preventative Action Request
DI	Designated Institute
ISO	International Organization for Standardization
LOFAC	Low-Frequency Facility
NIST	National Institute of Standards and Technology
NUWC	Naval Undersea Warfare Center
NVLAP	National Voluntary Laboratory Accreditation Program
QMS	Quality Management System
RCV	Receive
RSS	Root-Sum-of-Squares
RVS	Receiving Voltage Sensitivity
U.S.	United States
USRD	Underwater Sound Reference Division

LIST OF MATHEMATICAL SYMBOLS AND UNITS

α_L	Linear Coefficient of Thermal Expansion
a	Radius (General)
b	Inner Radius of Transducer
c	Speed of Sound
C_h	Hydrophone Compliance
C_B	Coupler Boundary Compliance

LIST OF MATHEMATICAL SYMBOLS AND UNITS (Cont'd)

C_F	Fluid Compliance
C_H	Compliance of High Pressure Tube
C_T	Transducer Compliance
C_W	Coupler Wall Compliance
C_p	Capacitance of Standard Capacitor
C_r	Reciprocal Transducer Compliance
C_{sphere}	Compliance of a Sphere (General)
C_t	Total Acoustic Compliance of the Coupler System
d	Transducer Offset from Coupler Wall
e	Voltage (gGeneral)
E	Young's Modulus
e_H	Open-Circuit Voltage Across a Hydrophone
e_R	Open-Circuit Voltage Across a Reference Hydrophone
e_{xh}	Hydrophone Open-Circuit Voltage with Transducer X Driving
e_{xc}	Voltage Across Standard Capacitor with Transducer X Driving
e_{xy}	Transducer Y Open-Circuit Voltage with Transducer X Driving
e_{yh}	Hydrophone Open-Circuit Voltage with Transducer Y Driving
e_{yc}	Voltage Across Standard Capacitor with Transducer Y Driving
e_{yx}	Transducer X Open-Circuit Voltage with Transducer Y Driving
f	Frequency
F	Wavelength Correction Factor (General)
F_H	Hydrophone Wavelength Correction Factor
F_T	Reciprocal Wavelength Correction Factor
G_d	Differential Amplifier Gain
G_p	Preamplifier Gain
i	Current (General)
i_x	Current Driving Transducer X
i_y	Current Driving Transducer Y
J	Coupler Reciprocity Parameter
k	Coverage Factor or Wave Number
λ	Wavelength
L	Length
M_H	Hydrophone Sensitivity
M_R	Reference Hydrophone Sensitivity
MPa	Megapascal
n	Number of Independent Measurements
ρ	Density
p	Pressure
Pa	Pascal
r	Radius
R_d	Differential Amplifier Common Mode Rejection Ratio
s	Standard Deviation of Repeated Measurements
s_i^2	Sample Variance

LIST OF MATHEMATICAL SYMBOLS AND UNITS (Cont'd)

s_p^2	Pooled Variance
T	Temperature
u	Standard Uncertainty
u_A	Type A Standard Uncertainty
u_B	Type B Standard Uncertainty
u_c	Combined Standard Uncertainty
$u_c(y)$	Combined Standard Uncertainty of Measurement Result y
ν	Poisson's Ratio
ν_{eff}	Effective Degrees of Freedom
V	Fill Fluid Volume or Volt (Unit)
ω	Frequency (Radians)
y	Measurement Result
Y	Measurand
Z_{xh}	Transducer X to Hydrophone Transfer Impedance
Z_{xy}	Transducer X to Transducer Y Transfer Impedance
Z_{yh}	Transducer Y to Hydrophone Transfer Impedance
Z_{yx}	Transducer Y to Transducer X Transfer Impedance

ACOUSTICAL MEASUREMENT UNCERTAINTY FOR THE UNDERWATER SOUND REFERENCE DIVISION IN THE FREQUENCY RANGE OF 3 HZ-TO-1.6 KHZ IN SIMULATED OCEANIC ENVIRONMENTS

1. INTRODUCTION

How loud is an underwater sound? To answer that question and to compare different sounds, all underwater listening devices must reference a constant unit of measure. The Underwater Sound Reference Division (USRD) at the Naval Undersea Warfare Center (NUWC) Division, Newport, RI maintains the standard for that unit, the underwater acoustic pascal, for all laboratories in the United States (U.S.) as the National Institute of Standards and Technology (NIST) Designated Institute (DI) for acoustics: sound in water. Maintaining the standard requires:

- Measuring the underwater acoustic pascal with the smallest uncertainty possible
- Participating in key comparisons, or comparisons between USRD measurements and those at the standard-bearing laboratories of other countries
- Providing opportunity for U.S. laboratories to compare measurements with the USRD
- Disseminating the standard throughout the U.S.

To disseminate the standard, the USRD provides underwater acoustic measurement and calibration services to government laboratories, industry, and academia and builds, maintains, and leases calibrated acoustic transducers to the U.S. and international customers. These responsibilities establish a link called measurement traceability between any measurement of the underwater acoustic pascal made in the U.S. and the international standard of the unit.

The result of any measurement is approximate and is incomplete, or in some cases misleading, without a statement of uncertainty. Uncertainty statements are particularly important for USRD measurement results, as they apply to all equipment in the U.S. traceable to the international standard underwater acoustic pascal. The USRD provides uncertainty statements for accredited measurements as part of its Quality Management System (QMS) in accordance with International Organization for Standardization (ISO) 17025 (reference 1) and accredited by the National Voluntary Laboratory Accreditation Program (NVLAP) (reference 2). The USRD develops new uncertainty statements for any proposed additions to its scope of accreditation and works continuously to reduce uncertainty in accredited measurements using new or improved methods and equipment.

This report documents the USRD measurement uncertainty for the underwater acoustic pascal over a frequency range of 3 Hz to 1.6 kHz, temperature range of - 3° C to 40° C and for absolute hydrostatic pressures between 350 kPa and 13.9 MPa, simulating oceanic conditions. It begins by summarizing relevant guidance for estimating and propagating uncertainty. Next, it presents two measurement methods employed by the USRD, a primary method using a coupler

reciprocity technique and a secondary method using a comparison technique (reference 3). A brief description of the facilities and relevant measurement theory precede the uncertainty analysis for each method. All methods described here are performed in the USRD Low-Frequency Facility (LOFAC). Uncertainties in measurements at frequencies higher than 1.6 kHz are documented in references 4 and 5.

2. UNCERTAINTY ANALYSIS METHOD OVERVIEW

Uncertainties in this report are calculated according to International Bureau of Weights and Measures (BIPM) recommendations (reference 6) summarized in reference 7, along with statistical procedures that describe the propagation of uncertainty (reference 8).

Uncertainty is grouped into two types based on evaluation method: Type A and Type B. Type A uncertainty is evaluated by statistical methods. It arises from a random effect. Type B uncertainty is evaluated by any other means. It arises from a systematic effect. This distinction can be ambiguous, unless one constrains the classification to a particular measurement process (reference 7), in this case, the two hydrophone sensitivity measurements presented in this report. For example, in the primary measurement method, pressure uncertainty has both Type A and Type B components. The pressure in the test chamber can vary randomly by ± 70 kPa during a measurement, a Type A uncertainty. The effect of this uncertainty on the measured sensitivity, combined with other random effects, can be estimated by analyzing variation in repeated measurements. The accuracy of the pressure gage used to check the chamber pressure before and after the measurement, is coincidentally ± 70 kPa as well, a Type B uncertainty. The gage reading, wherever it lies within that range, skews all sensitivity measurements in the same way and so is invisible to analysis of repeated measurements. Instead, the effect on uncertainty can be estimated by incorporating the manufacturer's accuracy specification into the uncertainty in the sensitivity measurement. Type B uncertainty estimates can be based on scientific judgement using all relevant information available, including previous measurement data, experience of the behavior of materials and instruments, manufacturer's specifications, or calibration data for references used.

To combine Type A and Type B uncertainties, they must be converted to statistical quantities defined as standard uncertainties, u . Since Type A components are calculated statistically, the standard deviation of repeated measurement results, s , and the standard uncertainty are one in the same, i.e., $u_A = s$. Type B components must be converted to standard uncertainties, u_B , through various methods summarized in reference 7. The method used in this report treats manufacturer accuracy specifications for test equipment, for example; $a\%$ accuracy, as the boundary of a rectangular distribution where the true value is equally likely to occur anywhere within the reported range $\pm a\%$. It could be that the distribution is triangular or Gaussian, but without knowledge of the equipment manufacturer's process, a rectangular distribution is the conservative choice. In this case, the standard uncertainty is given by,

$$u_B = a/\sqrt{3} . \tag{1}$$

Once the Type A and B components are converted to statistical standard uncertainties, and assuming they are all independent, they can be combined into a single combined standard uncertainty u_c , by the law of propagation of uncertainty (reference 7). When the covariance between the standard uncertainty components is negligible, the law simplifies to,

$$u_c = \sqrt{\sum u_A^2 + \sum u_B^2}, \quad (2)$$

referred to as the root-sum-of-squares (RSS). The RSS always provides a smaller combined uncertainty than a sum. To appreciate the usage of RSS over a sum, imagine a particular measurement where one component may be at the low end of its uncertainty range while another may be high. These inaccuracies would cancel out in the combination. Therefore, a sum of uncertainties overestimates all but the single most extreme combination. In this report, the fractional uncertainty in hydrophone sensitivity, $\delta M_H/M_H$, the uncertainty in the sensitivity divided by the mean sensitivity, is calculated according to equation (2),

$$\frac{\delta M_H}{M_H} = \sqrt{\left(\frac{\delta M_H}{M_H}\right)_A^2 + \left(\frac{\delta M_H}{M_H}\right)_B^2}. \quad (3)$$

Here, the Type A and B uncertainties are independent. For example, the accuracy of the voltmeter used to measure the hydrophone voltage (Type B uncertainty) is independent of random pressure variation in the test chamber (Type A uncertainty). In cases where individual uncertainties are not independent, no partial cancellation occurs between uncertainty components, and a sum is used to estimate uncertainty instead of an RSS. For example, the uncertainty in the voltages measured from two different transducers with the same voltmeter are not independent.

In many practical situations, including this report, a measurement result, y , and combined standard uncertainty define an interval $y \pm u_c(y)$ in which the true value of the measurand, Y , is estimated to lie with a level of confidence of 68%. This statement is true when the individual distributions of u_A and u_B are approximately normal or when the conditions of the central limit theorem are met, the measurement result y is calculated from a significant number of other quantities with well-defined distributions, normal or otherwise. When it is important to define an interval in which the measurand is believed to lie with a level of confidence greater than 68%, an expanded uncertainty is used. In an expanded uncertainty, u_c is multiplied by a coverage factor, k , to cover a larger interval with a level of confidence of 95%. When the normal distribution applies and the uncertainty in u_c is small, $k = 2$. In one case in this report, the uncertainty in u_c is not considered small because it is estimated with too few independent measurements to use $k = 2$. In this case, the effective degrees of freedom, ν_{eff} , determines an appropriate coverage factor. The number of repeated measurements is used to estimate ν_{eff} using the Welch-Satterthwaite formula (reference 7). For this measurement however, where y is the sample mean and $u_c(y)$ is the estimated standard deviation of the mean;

$$\nu_{eff} = n - 1, \quad (4)$$

where n is the number of independent measurements. The appropriate coverage factor is the t -factor, $k = t_p(v_{eff})$, from the t -distribution for level of confidence p and effective degrees of freedom, v_{eff} . Tables specify t -factors from the t -distribution listed by degrees of freedom and level of confidence (reference 7, table B.1). For lower degrees of freedom, t_p , and therefore k , is greater.

The hydrophone sensitivities described in this report are calculated from a number of independent measurements. Uncertainties in those measurements must be propagated into the sensitivity result. Reference 8 describes the following procedures for propagation. The uncertainty in a quantity q that is a function of one or more variables, $q(x, \dots, z)$, is;

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \delta x\right)^2 + \dots + \left(\frac{\partial q}{\partial z} \delta z\right)^2}, \quad (5)$$

where $\delta x, \dots, \delta z$ are the uncertainties in the respective individual variables. Two simplifications derived from equation (5) find use in this report. First, if q is defined solely by products and quotients,

$$q = \frac{x \times \dots \times z}{u \times \dots \times w}, \quad (6)$$

then the fractional uncertainty in q , $\delta q/q$, is defined in terms of the fractional uncertainties in the individual variables, and

$$\frac{\delta q}{q} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \dots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \dots + \left(\frac{\delta w}{w}\right)^2}. \quad (7)$$

Equation (52), and parts of equation (11) follow this structure. Second, if q is calculated by a variable raised to a power, $q = y^n$, the fractional uncertainty $\delta q/q$ due to fractional uncertainty $\delta y/y$, is,

$$\frac{\delta q}{q} = n \frac{\delta y}{y}. \quad (8)$$

Since many measurements used to calculate sensitivity in this report fall under a square root, their fractional uncertainty contribution is multiplied by 1/2. Fractional uncertainties, like those in equations (7) and (8), are practical because the accuracy of equipment or previously published measurement results are often reported as percentages. When the more general relationship in equation (5) is used in this report, the uncertainties are converted to fractional uncertainties after taking partial derivatives.

3. PRIMARY CALIBRATION OF A HYDROPHONE

This hydrophone calibration relates the hydrophone voltage output to the underwater acoustic pressure in pascal at the hydrophone's sensitive element. This relationship is called the receiving voltage sensitivity (RVS), or just the sensitivity. Primary methods for measuring sensitivity require measurements of voltage, current, length, mass, and time, or the electrical impedances that are combinations of those parameters. No receiving sensitivity or transmitting response of any previously calibrated reference (i.e., underwater acoustic transducer) can take part in a primary calibration. Instead, the primary method discussed in this report relies on the reciprocity principle. In contrast, a secondary method includes any calibration that uses a reference transducer previously calibrated using a primary or secondary method (reference 9).

The USRD uses the coupler reciprocity method (reference 3) for the primary calibration of Type H48 hydrophones from 3 Hz to 1.6 kHz. A coupler is a closed pressure vessel, shown schematically in figure 3-1, designed for a single reference hydrophone, H, reciprocal transducer, Y, and projector, X. In the USRD implementation, the projector, X, is also reciprocal, and like Y, it can act as a projector and a receiver, allowing for a check of the reciprocity condition in the coupler. In contrast to reference 3, transducer X will be referred to as a reciprocal transducer instead of a projector in this report. Together, the chamber and three transducers form a closed volume as small as practicable. The USRD chamber diameter is 4.5 cm and the total length is 8.7 cm.

The coupler is filled under vacuum with de-gassed castor oil, a dielectric fluid that acoustically couples the three transducers. Primary calibrations for the sensitivity of the reference hydrophone are performed under simulated ocean environments from - 3° C to 40° C and hydrostatic pressures up to 13.7 MPa or a 1,365 m ocean depth. The measurement system consists of a preamplifier for conditioning received voltage signals from the reciprocal transducers, a function generator and amplifier to drive the reciprocal transducers, a standard capacitor and differential amplifier for converting the small drive currents to measureable voltages, and multimeters for measuring those voltages and the voltage from the hydrophone under test.

3.1 MEASUREMENT THEORY

Coupler reciprocity measurements, indicated in figure 3-1, are taken in two steps (reference 3). With reciprocal transducer X driving, the current i_x and voltages e_{xh} and e_{xy} are measured. Then with reciprocal transducer Y driving, the current i_y and voltage e_{yh} are measured. In this implementation there is an extra voltage measurement, e_{yx} , measured while the reciprocal transducer Y is driving that is not required by the method, but can be used to check the reciprocity condition in the coupler.

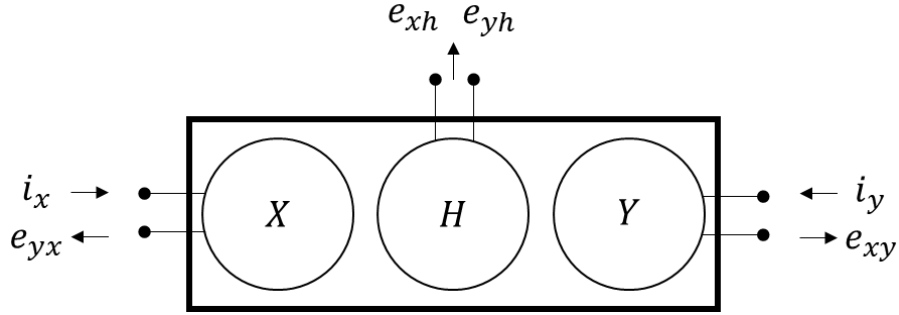


Figure 3-1. Coupler Schematic with a Projector P, a Reciprocal Transducer T, and a Hydrophone H to be Calibrated

The hydrophone sensitivity, M_H , is;

$$M_H^2 = J \frac{|Z_{xh}| |Z_{yh}|}{\sqrt{|Z_{xy}| |Z_{yx}|}}, \quad (9)$$

where J is the coupler reciprocity parameter, and Z are the electrical transfer impedances* between reciprocal transducer X , hydrophone H , and reciprocal transducer Y . An electrical transfer impedance, Z_{AB} is defined as the ratio;

$$Z_{AB} = \frac{e_{AB}}{i_A}, \quad (10)$$

where e_{AB} is the voltage measured at transducer B with transducer A driving and i_A is the drive current to transducer A. The voltage, current, and electrical transfer impedance are complex-valued, as indicated by bold font. In this implementation, only the moduli are used, although complex-valued calibration is possible (reference 10). From this point on, all indicated voltages and currents are real numbers, the moduli of complex quantities.

Equation (9) is in the same form as reference 3, except using the geometric mean of Z_{yx} and Z_{xy} because of the extra voltage, e_{xy} , recorded in the USRD coupler to perform a reciprocity check. If the reciprocity condition in the coupler holds true, Z_{xy} and Z_{yx} should be equal. This condition does not guarantee reciprocity, but it is a safe assumption (reference 9). In reality, these two transfer impedances are not exactly the same, so instead of choosing one or the other with which to calculate sensitivity, the geometric mean of the two is used. Unlike the arithmetic mean, the geometric mean of ratios and the ratio of geometric means are equal, allowing the voltages and currents in the electrical transfer impedances to be rearranged. Substituting voltages and currents for electrical transfer impedances as defined in equation (10), then, for the currents, substituting $i = e\omega C_p$, where i is a current through a standard capacitance C_p at an angular frequency ω , and e is the voltage measured across C_p , one gets;

* In many USRD reports, this quantity is referred to as the acoustical transfer impedance. Here the term electrical transfer impedance is used to conform to the recently released version of IEC-60565 (reference 3).

$$M_H = \sqrt{\frac{e_{xh}e_{yh}}{\omega C_p \sqrt{e_{xy}e_{xc} \times e_{yx}e_{yc}}}} J, \quad (11)$$

where e_{xh} and e_{yh} are the open circuit voltages measured at the hydrophone when reciprocal transducer X and Y are driving, respectively. e_{yx} is the open circuit voltage across the X transducer when transducer Y is driving. e_{xy} is the open circuit voltage across transducer Y when transducer X is driving. e_{yc} is the voltage across the standard capacitance when transducer Y is driving. e_{xc} is the voltage across the standard capacitance when transducer X is driving.

The reciprocity parameter (reference 11), J , is,

$$J = \omega C_t = \omega \left(\frac{V}{\rho c^2} + C_B \right), \quad (12)$$

where C_t is the total acoustic compliance of the coupler, consisting of the fill fluid compliance, $\frac{V}{\rho c^2}$, and boundary compliance, C_B (references 12 and 13). The fill fluid compliance is defined by V , the fluid volume, ρ , the fluid density, and c , the speed of sound in the fluid.

The equations describing hydrophone sensitivity in the coupler are correct under the following assumptions.

- The sound field is uniform throughout the cavity. Therefore, the coupler cavity must be smaller than the smallest wavelength of sound in the fill fluid. For a three-in-a-row configuration like figure 3-1, the length of the cavity must be less than 1/10 of the shortest wavelength of sound measured. If this condition is not met, the method is considered invalid (reference 3). Field nonuniformity in cavities near or longer than the 1/10 wavelength criterion can be calculated and corrected, however, to reduce this type of error (reference 13). Wavelength correction factors, calculated as in reference 10, are applied to the hydrophone (F_H) and receiving reciprocal transducer voltages (F_T) as shown in equation (13).

$$M_H = \sqrt{\frac{F_H e_{xh} F_H e_{yh}}{\omega C_p \sqrt{F_T e_{xy} e_{xc} \times F_T e_{yx} e_{yc}}}} J \quad (13)$$

- At least one transducer must be reciprocal. The reciprocity check performed by comparing Z_{xy} and Z_{yx} in a test setup where both transducers are reciprocal can verify the reciprocity condition. To avoid a breakdown of the reciprocity condition in otherwise reciprocal transducers, they must be operated well below their first resonance.

Non-linearity and cross-talk can also introduce errors in the measurement. Linearity of the devices can be checked by verifying electrical transfer impedances and does not change

while driving the transducers from the lowest to the highest levels used in the test. Cross-talk is eliminated at the USRD coupler by placing grounded metal screens between transducers.

3.2 TYPE A UNCERTAINTY ANALYSIS

The Type A measurement uncertainty was estimated from a statistical analysis on repeated, independent sensitivity measurements on H48 serial numbers 1, 2, 3, and 4, taken at the reciprocity coupler between February 2018 and October 2019.

The mean sensitivity of each H48 differs and varies with temperature and pressure, so the sensitivity variance across the four hydrophones at all environments does not represent Type A uncertainty. Sample variances of measurements from individual H48's at individual environmental conditions must be combined another way. At a given frequency, these sample variances should estimate the same population variance regardless of their mean values, making them good candidates for pooling into a single variance estimate (reference 14). For a single H48, if environments are numbered $i = 1, \dots, k$, the pooled variance s_p^2 across all environments at a particular frequency is given by;

$$s_p^2 = \frac{\sum_{i=1}^m (n_i - 1) s_i^2}{\sum_{i=1}^m (n_i - 1)} = \frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2 + \dots + (n_m - 1) s_m^2}{n_1 + n_2 + \dots + n_m - m}, \quad (14)$$

where n_i is the number of independent measurements and s_i^2 is the sample variance for each environment i . In this report, $n_i = 2$ or 3 independent measurements at each frequency and $m = 30$ to cover all environments. Similarly, the mean sensitivity of each H48 is different, but the variances should estimate the same population variance. The variances, s_p^2 , are pooled again across device serial numbers 1–4. The end result is a single pooled variance representing the Type A uncertainty in sensitivity at each frequency. With n_i between 2 and 3 for each s_p^2 , the number of independent measurements in the variances pooled across device serial numbers is between 7 and 10*, fewer effective degrees of freedom than that required for a 95% coverage factor $k=2$, as discussed in section 2. From table B.1 of reference 7 (or appendix G of reference 6), a coverage factor $k=2.52$ should be used for six effective degrees of freedom.

The standard deviation calculated from these pooled variances are shown for the H48 population in table 3-1 and figure 3-2. Environmental interference produced noise in the measurement between 50–60 Hz for H48 SN 2 and 3, which was reduced in the test setup for later measurements in SN 1 and 4.

* Environmental changes do not require draining the coupler fluid or removing the hydrophone under test. Therefore, a complete calibration, measured at all 30 test environments, counts as just one independent measurement. For example: three complete calibrations on H48 serial number two between 2018 and 2019 comprise 90 sensitivity measurements at each frequency but just three independent measurements ($n_i = 3$ for all i).

Table 3-1. Fractional Standard Uncertainties for Coupler Reciprocity Measurements

Calibration Frequency (kHz)	Pooled Sample Size (N)	Type A Pooled Std. Deviation (%)
3.15	10	0.91
4.00	10	0.88
5.00	7	0.85
6.30	7	0.82
8.00	10	0.80
10.0	7	0.81
12.5	7	0.77
16.0	10	0.76
20.0	7	0.72
25.0	7	0.77
31.5	10	0.68
40.0	7	0.72
50.0	7	1.06
63.0	10	0.62
80.0	7	0.68
100	7	0.61
125	10	0.58
160	7	0.55
200	7	0.52
250	10	0.53
315	7	0.49
400	7	0.48
500	10	0.49
630	7	0.45
800	7	0.48
1000	10	0.46
1250	7	0.55
1600	7	0.68
2000	10	0.82

Figure 3-2 compares the fractional standard uncertainties for H48 serial numbers one through four to the pooled fractional standard deviation.

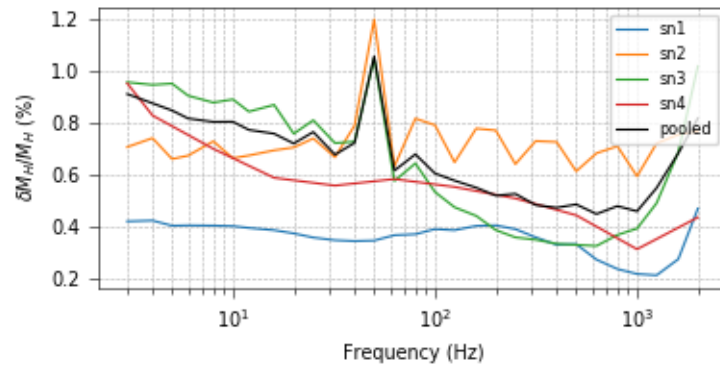


Figure 3-2. Fractional Standard Deviations for Each Type H48 Hydrophone Compared to the Pooled Fractional Standard Deviation

3.3 TYPE B UNCERTAINTY ANALYSIS

Type B uncertainties are calculated through means other than statistical analysis on repeated measurements, as described in section 2. The Type B uncertainty in the coupler is a combination of uncertainty in the compliance of the working fluid, the compliance of the boundaries, and the measurement system electronics. Table 3-2 lists the Type B uncertainties, providing an outline for the uncertainty terms described in more detail in the following sections of this report. These uncertainties are treated as constant over frequency.

Table 3-2. Type B Uncertainties

Section	Symbol	Description	Standard Uncertainty (%)
3.3.1.1	$2\delta c/c$	Speed of Sound in Castor Oil	1.31*
3.3.1.2	$\delta\rho/\rho$	Density of Castor Oil	0.70
3.3.1.3	$\delta V/V$	Fluid Volume in Coupler	0.72
3.3.2.1	$\delta C_w/C_t$	Compliance of Coupler Walls	0.47
3.3.2.2	$\delta C_T/C_t$	Compliance of Transducers	0.80
3.3.3.1	$\delta C_P/C_P$	Standard Capacitor Capacitance	0.06
3.3.3.2	$\delta G_d/G_d$	Differential Amplifier Gain	0.64
3.3.3.3	$\delta R_d/R_d$	Differential Amplifier Common Mode Rejection Ratio (CMRR)	0.92
3.3.3.4	$\delta G_p/G_p$	Preamplifier Gain	0.58
3.3.3.5	$\sqrt{6}\delta e/e$	Digital Multimeter	0.54
3.3.4	$(\delta M_H/M_H)_\lambda$	Wavelength Correction	≤ 0.58
3.3.5	$(\delta M_H/M_H)_{bdf}$	Frequency Dependent Compliance Variation	2.33
		Other	0.10

*The effect of sound speed uncertainty on sensitivity uncertainty is doubled

Equation (11), with the reciprocity parameter expanded in equation (12), define the relationship between the measurements and sensitivity as well as the relationship between the

uncertainty in the measurements and the uncertainty in the sensitivity. Applying the rules in equations (7) and (8) to (13), the Type B uncertainty in sensitivity is,

$$\begin{aligned} \left(\frac{\delta M_H}{M_H}\right)_{B1} = & \frac{1}{2} \left(\left(\frac{\delta F_H}{F_H}\right)^2 + \left(\frac{\delta e_{xh}}{e_{xh}}\right)^2 + \left(\frac{\delta F_H}{F_H}\right)^2 + \left(\frac{\delta e_{yh}}{e_{yh}}\right)^2 + \left(\frac{\delta C_p}{C_p}\right)^2 + \left(\frac{1}{2} \frac{\delta F_T}{F_T}\right)^2 + \right. \\ & \left. \left(\frac{1}{2} \frac{\delta e_{xy}}{e_{xy}}\right)^2 + \left(\frac{1}{2} \frac{\delta e_{xc}}{e_{xc}}\right)^2 + \left(\frac{1}{2} \frac{\delta F_T}{F_T}\right)^2 + \left(\frac{1}{2} \frac{\delta e_{yx}}{e_{yx}}\right)^2 + \left(\frac{1}{2} \frac{\delta e_{yc}}{e_{yc}}\right)^2 + \left(\frac{\delta C_t}{C_t}\right)^2 \right)^{\frac{1}{2}}. \end{aligned} \quad (15)$$

Grouping voltage and capacitance terms into a single term containing all uncertainty in measurement electronics, $\left(\frac{\delta M_H}{M_H}\right)_{el}$, and grouping wavelength correction terms into a single term, $\left(\frac{\delta M_H}{M_H}\right)_\lambda$, equation (15) becomes,

$$\left(\frac{\delta M_H}{M_H}\right)_{B1} = \frac{1}{2} \sqrt{\left(\frac{\delta M_H}{M_H}\right)_{el}^2 + \left(\frac{\delta M_H}{M_H}\right)_\lambda^2 + \left(\frac{\delta C_t}{C_t}\right)^2}. \quad (16)$$

Note that the measurement electronics term, $\left(\frac{\delta M_H}{M_H}\right)_{el}$, is modified later in equation (38).

An additional term, $\left(\frac{\delta M_H}{M_H}\right)_{bdf}$, containing uncertainty due to a design feature in the coupler chamber that conflicts with the requirements of reference 3, and is unrelated to any of the measurements in equation (13), is added to $\left(\frac{\delta M_H}{M_H}\right)_{B1}$ in an RSS to get,

$$\left(\frac{\delta M_H}{M_H}\right)_B = \sqrt{\frac{1}{4} \left(\left(\frac{\delta M_H}{M_H}\right)_{el}^2 + \left(\frac{\delta M_H}{M_H}\right)_\lambda^2 + \left(\frac{\delta C_t}{C_t}\right)^2 \right) + \left(\frac{\delta M_H}{M_H}\right)_{bdf}^2}. \quad (17)$$

From equation (12), the total system compliance is $C_t = \left(\frac{V}{\rho c^2} + C_B\right)$. Applying the relationship in equation (5) to the expression for C_t , one gets,

$$\delta C_t = \sqrt{\left(\frac{1}{\rho c^2} \delta V\right)^2 + \left(-\frac{V}{\rho^2 c^2} \delta \rho\right)^2 + \left(-\frac{2V}{\rho c^3} \delta c\right)^2 + \delta C_B^2}. \quad (18)$$

To transform equation (18) to a more useful expression of fractional uncertainty, divide by the compliance of the working fluid alone, which equates to the total system compliance C_t while neglecting the contribution of the boundary compliance C_B which is just over one order of magnitude smaller than C_t ;

$$\frac{\delta C_t}{C_t} = \frac{\sqrt{\left(\frac{1}{\rho c^2} \delta V\right)^2 + \left(-\frac{V}{\rho^2 c^2} \delta \rho\right)^2 + \left(-\frac{2V}{\rho c^3} \delta c\right)^2 + \delta C_B^2}}{\sqrt{\left(\frac{V}{\rho c^2}\right)^2}}, \quad (19)$$

the omission of which allows simplification to,

$$\frac{\delta C_t}{C_t} = \sqrt{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta \rho}{\rho}\right)^2 + \left(2 \frac{\delta c}{c}\right)^2 + \left(\frac{\delta C_B}{\frac{V}{\rho c^2}}\right)^2}. \quad (20)$$

Now the uncertainties in volume, density, and speed of sound can be represented as percentages. Since the boundary compliance C_B is an order of magnitude smaller than the fluid compliance, omitting it from the total compliance used to normalize the fractional uncertainty component in equation (20) results in a more conservative (though negligibly so) estimate. Grouping the volume, density, and sound speed uncertainty into a single term, $\left(\frac{\delta M_H}{M_H}\right)_{fl}$, and renaming the boundary uncertainty, $\left(\frac{\delta M_H}{M_H}\right)_{bd}$, one gets,

$$\frac{\delta C_t}{C_t} = \sqrt{\left(\frac{\delta M_H}{M_H}\right)_{fl}^2 + \left(\frac{\delta M_H}{M_H}\right)_{bd}^2}. \quad (21)$$

The final expression for Type B uncertainty is the combination of equations (17) and (21),

$$\left(\frac{\delta M_H}{M_H}\right)_B = \sqrt{\frac{1}{4} \left(\left(\frac{\delta M_H}{M_H}\right)_{el}^2 + \left(\frac{\delta M_H}{M_H}\right)_{\lambda}^2 + \left(\frac{\delta M_H}{M_H}\right)_{fl}^2 + \left(\frac{\delta M_H}{M_H}\right)_{bd}^2 \right) + \left(\frac{\delta M_H}{M_H}\right)_{bdf}^2}. \quad (22)$$

To review, $\left(\frac{\delta M_H}{M_H}\right)_{fl}$ is the uncertainty in hydrophone sensitivity due to uncertainty in the working fluid compliance, $\left(\frac{\delta M_H}{M_H}\right)_{bd}$ is the uncertainty in hydrophone sensitivity due to uncertainty in boundary compliances, $\left(\frac{\delta M_H}{M_H}\right)_{\lambda}$ is the uncertainty in hydrophone sensitivity due to uncertainty in the wavelength correction, and $\left(\frac{\delta M_H}{M_H}\right)_{el}$ is the uncertainty in hydrophone sensitivity due to uncertainty in the measurement system electronics. These four terms are groups containing the uncertainty from all terms in equation (11), described in detail in sections 3.3.1,

3.3.2, and 3.3.3. The final term, $\left(\frac{\delta M_H}{M_H}\right)_{bdf}$, added separately, represents uncertainty in the hydrophone sensitivity due to a frequency dependent boundary compliance. This uncertainty arises from a non-optimal design feature in the coupler measurement setup and is treated differently than the other three terms, as explained in section 3.3 5. This term is added as an RSS because it is independent of the other three terms.

3.3.1 Compliance of the Working Fluid

The largest contributor to total system compliance is the fluid (castor oil) that fills the coupler cavity and acoustically couples the transducers. The fluid also transfers heat from the temperature-controlled coupler cavity walls and pressure through the system plumbing from the pressure ram. The fluid compliance can be calculated from the fluid volume, V , the fluid density, ρ , and speed of sound in the fluid, c , as shown on the right side of equation (12), $\frac{V}{\rho c^2}$.

The effect on the uncertainty in sensitivity from uncertainty in those individual terms, $\left(\frac{\delta M_H}{M_H}\right)_{fl}$ can be calculated according to rules of propagation of uncertainty for products, quotients, and powers from equations (7) and (8) as,

$$\left(\frac{\delta M_H}{M_H}\right)_{fl}^2 = \left(2 \frac{\delta c}{c}\right)^2 + \left(\frac{\delta \rho}{\rho}\right)^2 + \left(\frac{\delta V}{V}\right)^2. \quad (23)$$

The following subsections explain each term in equation (23).

3.3.1.1 Speed of Sound in Castor Oil (c). Timme (reference 15) measured the speed of sound in castor oil in a coupler using a time of flight technique. These measurements were accurate to $\pm 1.1\%$ based on the ability of Timme's equipment to measure pressure, temperature, and time, as well as the repeatability of his test setup. The coupler processor interpolates directly from Timme's data, but the regression he used to fit the data,

$$c(T, p) = 1570[a_0 + a_1T + a_2T^2 + a_3Tp + a_4p + a_5p^2], \quad (24)$$

with coefficients specified in table 3-3, is still useful to estimate the extra contribution of pressure and temperature uncertainty on the sound speed uncertainty.*

* There is a mistake in equation 3 of reference 15 that is corrected in equation (24).

Table 3-3. Constants for the Empirical Relationship of Sound Speed in Castor Oil with Temperature and Pressure

Constant	Value
a_0	1
a_1	-2.15×10^{-3}
a_2	4.0×10^{-6}
a_3	2.5×10^{-6}
a_4	2.22×10^{-3}
a_5	-3.0×10^{-6}

Uncertainty in the temperature and pressure measurements add to the sound speed uncertainty. From manufacturer specifications, the uncertainty in the temperature reading of the Industronics AB-RTD100-188-U-S6-2 probe is $\delta T = 0.7^\circ\text{C}$. The uncertainty in the pressure reading of the Ashcroft A2YAM0242C610000#G is $\delta p = 70 \text{ kPa}$ (10 psi).

Equation (25) is the sum of the partial derivatives of equation (24) with respect to temperature and pressure. This expression relates the uncertainty in temperature and pressure measurements to uncertainty in sound speed.

$$\delta c(T, p) = 1570[(a_1 + a_2T + a_3p)\delta T + (a_3T + a_4 + 2a_5p)\delta p] \quad (25)$$

Calculating the δT and δp driven sound speed uncertainties separately over the environmental range of H48 calibrations (-3°C to 40°C , 345 kPa to 13790 kPa) and picking the worst case result for each gives a temperature driven uncertainty of 0.15% and pressure driven uncertainty of 0.02%. Since any change in temperature causes a corresponding change in pressure that is not compensated during the measurement, the temperature and pressure components are not independent and add directly as in equation (25) instead of the RSS sum typically used, for a total of 0.17%. Although a pressure change does not also cause a temperature change, it is more conservative to treat them as dependent. The total sound speed uncertainty is then given by;

$$\frac{\delta c}{c} = \frac{\sqrt{\left(\frac{\delta c}{c}\right)_{meas}^2 + \left(\frac{\delta c}{c}\right)_{tp}^2}}{\sqrt{3}} = 0.66\% , \quad (26)$$

where $\left(\frac{\delta c}{c}\right)_{meas}$ is the 1.1% uncertainty in Timme's measurement setup and $\left(\frac{\delta c}{c}\right)_{tp}^2$ is the 0.17% worst case error due to uncertainty in temperature and pressure. Each of these terms are treated as boundaries that form a rectangular error distribution and are, therefore, divided by $\sqrt{3}$ to convert to standard uncertainty as described in section 2. The effect of sound speed uncertainty on compliance uncertainty is doubled, or 1.32%.

3.3.1.2 Castor Oil Density (ρ). Stallard's measurements of castor oil density were precise, but questions about the consistency of castor oil density between batches and manufacturers likely caused differences between his measurements, the international critical tables, and other researchers (reference 16). The variation in these data suggest that the castor oil density is not known to be better than 0.7%; therefore, $\frac{\delta\rho}{\rho} = 0.7\%$. This variation is not divided by $\sqrt{3}$ to convert to a standard uncertainty, because unlike a manufacturer specification, there is no guarantee that the variation in the observed results are the boundaries of the true variation. To be conservative, the standard uncertainty is assumed to encompass all observed variations.

3.3.1.3 Castor Oil Volume (V). Three factors contribute to the volume uncertainty; (1) the measurement uncertainty in the coupler fluid volume, (2) the change in the chamber volume with pressure, and (3) the change in the chamber volume with temperature. The latter two factors are small, and instead of correcting for them during processing, they are treated as uncertainties.

Zalesak (reference 13) measured the castor oil volume in the coupler as $1.247 \times 10^{-4} \text{ m}^3$ with an estimated uncertainty of $\pm 1\%$ by injecting alcohol into the coupler cavity with a calibrated digital pipette. This procedure is repeated when changes to the test setup affect coupler chamber volume.

Using the chamber compliance calculated in section 3.3.2.1, the change in volume induced by a change in hydrostatic pressure is just the compliance multiplied by the pressure change. At the maximum test pressure of 13,790 kPa, the coupler chamber volume has increased approximately 10^{-8} m^3 , about 0.01% of the H48 chamber volume and is small enough to neglect.

The change in the length of steel relates approximately to a change in temperature through the linear coefficient of thermal expansion α_L ,

$$\frac{\Delta L}{L} = \alpha_L \Delta T, \quad (27)$$

where L is length and T is temperature. The linear coefficient of thermal expansion for 304 stainless steel is $17.3 \mu\text{m}/\text{m}^\circ\text{C}$. Assuming all H48 coupler dimensions apply at 20°C , ΔT is about 20°C since the calibration temperature range is -3°C to 40°C . That ΔT gives a length change of 0.03%. The uncertainty in cylinder volume is,

$$\frac{\delta V}{V} = \pi * \left(2 \frac{\delta r}{r} + \frac{\delta L}{L} \right). \quad (28)$$

Assuming a cylindrical coupler interior and an increase or decrease in cylinder length and circumference of 0.03%, the volume varies $\pm 0.1\%$ over the temperature range.

The 1% uncertainty in the volume measurement is treated as a rectangular error distribution and therefore, is divided by $\sqrt{3}$ before being added directly to the 0.1% the uncorrected change in volume over the range of test temperatures.

3.3.2 Compliance of fluid boundaries

The second term in the coupler system compliance of equation (12), the boundary compliance C_B , accounts for the compliance of all fluid boundaries, including the coupler wall compliance, C_W , the compliance of the transducers in the coupler, C_T , and the compliance of the high pressure tube feeding the coupler chamber, C_H . These boundary compliances can be treated as capacitances in parallel or in the analogous mechanical case, as springs in series. The compliance correction, C_B , is the sum of the individual compliances and can be expressed as:

$$C_B = C_W + C_T + C_H . \quad (29)$$

The fractional uncertainty in the hydrophone sensitivity due to the uncertainty in the boundary compliance is,

$$\left(\frac{\delta M_H}{M_H} \right)_{bd}^2 = \left(\frac{\delta C_W}{C_t} \right)^2 + \left(\frac{\delta C_T}{C_t} \right)^2 + \left(\frac{\delta C_H}{C_t} \right)^2 . \quad (30)$$

The boundary compliance uncertainties are made fractional by dividing each by the total system compliance. Unlike the simplification used in equations (19) and (20), the total system compliance is used in the denominator of these individual terms. Leaving out the contribution by C_B provides no benefits for simplifying equation (30). The total compliance, C_t , is the sum of the boundary compliance and fluid compliance, C_F .

$$C_t = C_F + C_B . \quad (31)$$

3.3.2.1 Compliance of Coupler Walls. Black and Siders estimated the compliance of the coupler walls with a finite element analysis (reference 17) checked by Zalesak's analytical calculation (reference 13). Difficulty in accurately modeling the boundary conditions of the ACME threaded coupler sealing plugs controlled the uncertainty in the finite element analysis. They analyzed a range of possible boundary conditions for the threads to estimate the wall compliance and uncertainty to be $C_W = 2.7 \times 10^{-15} \text{ m}^3/\text{Pa} \pm 10\%$, i.e. $\delta C_W = 3 \times 10^{-16} \text{ m}^3/\text{Pa}$.

3.3.2.2 Compliance of Transducers. The change in outer radius of the transducer ceramic with pressure can be calculated analytically from the formula for a thick-walled sphere (reference 18),

$$\Delta a = -\frac{qa}{E} \left(\frac{(1-\nu)(b^3 + 2a^3)}{2(a^3 - b^3)} - \nu \right), \quad (32)$$

where a is the outer radius, b is the inner radius, E is Young's modulus, and ν is Poisson's ratio. For Navy Type I lead zirconate-titanate, Young's modulus is $6.45 \times 10^{10} \text{ N/m}^2$ and Poisson's ratio is 0.343. Setting $q = 1 \text{ Pa}$, (32) gives the change in radius per Pa. To calculate the transducer compliance C_{sphere} , or change in volume per Pa, calculate the change in volume of the sphere due to the radius change calculated in equation (32).

$$C_{\text{sphere}} = \frac{\Delta V}{\Delta p} = \frac{4}{3}\pi((a + \Delta a)^2 - a^2). \quad (33)$$

Table 3-4 presents the relevant dimensions of the hydrophone and reciprocal transducers.

Table 3-4. Transducer Parameters (reference 13)

Parameter	Symbol	Value (H48)	Value (Reciprocal)	Units
Outer radius	a	12.7	6.35	10-3 m
Inner radius	b	9.53	5.18	10-3 m
Young's modulus	E	64.5	64.5	GPa
Poisson's ratio	ν	.343	.343	--

The reciprocal transducer compliance, C_r , is $7.40 \times 10^{-17} \text{ m}^3/\text{Pa}$ and the H48 transducer compliance, C_h , is $4.12 \times 10^{-16} \text{ m}^3/\text{Pa}$. With two reciprocal transducers and one H48 transducer, the total compliance, C_T , is $5.6 \times 10^{-16} \text{ m}^3/\text{Pa}$.

Uncertainty in the dimensions of the spheres is assumed to be 5% and most piezoelectric material suppliers quote $\pm 20\%$ on their material properties. The uncertainty from the material properties and dimensions on the compliance can be estimated conservatively at just over 100% since the inner and outer radii are cubed in equation (32), and Δa is cubed again to obtain the change in volume per unit pressure.

$$\frac{\delta \Delta a}{\Delta a} = \sqrt{\left(3 \frac{\delta b}{b}\right)^2 + \left(3 \frac{\delta a}{a}\right)^2 + \left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta E}{E}\right)^2 + \left(\frac{\delta \nu}{\nu}\right)^2} \quad (34)$$

$$\frac{\delta C_{\text{sphere}}}{C_{\text{sphere}}} = 3 * \frac{\delta \Delta a}{\Delta a} = 1.1 \quad (35)$$

Now the total uncertainty in the transducer compliance is,

$$\delta C_T = \sqrt{2 \left(\frac{\delta C_{\text{sphere}}}{C_{\text{sphere}}} * C_r\right)^2 + \left(\frac{\delta C_{\text{sphere}}}{C_{\text{sphere}}} * C_h\right)^2} = 4.6 \times 10^{-16} \text{ m}^3/\text{Pa}. \quad (36)$$

The transducer ceramic is two orders of magnitude stiffer than the coupler fluid and an order of magnitude stiffer than the coupler walls. Therefore, 100% uncertainty in the transducer compliance contributes only 1% uncertainty to the total system compliance.

3.3.2.3 Compliance of High-Pressure Tube. A 75 mm (3-in.) high-pressure tube feeds fluid in and out of the bottom of the coupler and separates the coupler from the system plumbing. From Roark's formulas for a thick-walled cylindrical shell with no end caps under uniform internal radial pressure (reference 18), the deformation of the inner diameter b with pressure is,

$$\Delta b = \frac{qb}{E} \left(\frac{a^2 + b^2}{a^2 - b^2} + \nu \right). \quad (37)$$

The High-Pressure Equipment company tubing, 20-LM6 316SS, has properties in table 3-5.

Table 3-5. High Pressure 3/8-in. OD 316SS, 3-in. Long Tube Parameters

Parameter	Symbol	Value	Units
Outer radius	a	4.7625	10-3 m
Inner radius	b	2.5781	10-3 m
Tube length	h	76.2	10-3 m
Young's modulus	E	190-205	GPa
Poisson's ratio	ν	.265-275	--

Note: In table 3-5, the (Material properties from <https://www.azom.com/properties.aspx?ArticleID=863>).

From equation (37), the change in inner diameter with unit pressure ($q = 1 \text{ Pa}$) is 2.74×10^{-14} , assuming the average value of all ranges in Table 3-5.

To find the compliance, calculate the change in volume associated with a unit change in pressure.

$$C_H = \frac{\Delta V}{\Delta p} = \pi h((b + \Delta b)^2 - b^2) = 3.4 * 10^{-17} \text{ m}^3/\text{Pa}.$$

Since this correction is so small, it is left out of the boundary compliance, C_B and contributes a negligible amount to uncertainty. As a fraction of the worst-case total system compliance, this error is only 0.06%.

3.3.2.4 Boundary Compliance Summary. Summing the individual boundary compliance contributions from equation (29), the boundary compliance is $C_B = 3.2 \times 10^{-15} \text{ m}^3/\text{Pa}$. From equations (30) and (31) the uncertainty $\left(\frac{\delta M_H}{M_H}\right)_{bd}$ is 0.9%. The boundary compliance uncertainty due to the transducers is 0.8%. The boundary uncertainty due to the coupler walls is 0.4%.

3.3.3 Measurement System Electronics

The uncertainties in the measurement system electronics group shown in equations (15) and (16) can be combined with one important modification, as shown in equation (38),

$$\left(\frac{\delta M_H}{M_H}\right)_{el}^2 = \left(\frac{\delta C_p}{C_p}\right)^2 + \left(\frac{1}{2} \frac{\delta e_{xc}}{e_{xc}} + \frac{1}{2} \frac{\delta e_{yc}}{e_{yc}}\right)^2 + \left(\frac{1}{2} \frac{\delta e_{xy}}{e_{xy}} + \frac{1}{2} \frac{\delta e_{yx}}{e_{yx}}\right)^2 + \left(\frac{\delta e_{xh}}{e_{xh}} + \frac{\delta e_{yh}}{e_{yh}}\right)^2. \quad (38)$$

In this modification, pairs of capacitors, receiving reciprocal transducer, and hydrophone voltage measurement uncertainties add directly rather than RSS, because their uncertainties are not independent, having been measured with the same instruments. If these quantities were ratios and not products in equation (13), their contribution would cancel. These pairs of voltages are shown at the top of figure 3-3.

The uncertainty in the voltage measurements arise from uncertainty in the signal conditioning equipment and multimeters. A block diagram of the measurement setup is shown in figure 3-3. Each voltage measurement is subject to a different chain of uncertainties. The hydrophone voltage measurements, e_{xh} and e_{yh} , contain uncertainty from the multimeter used to make the measurement, $\frac{\delta e}{e}$. The receiving reciprocal transducer measurements, e_{xy} and e_{yx} , contain uncertainty from the multimeter, $\frac{\delta e}{e}$, and preamplifier, $\frac{\delta G_p}{G_p}$. The standard capacitor measurements, e_{cx} and e_{cy} , contain uncertainty from the multimeter, $\frac{\delta e}{e}$, differential amplifier gain, $\frac{\delta G_d}{G_d}$, and differential amplifier common mode rejection ratio, $\frac{\delta R_d}{R_d}$.

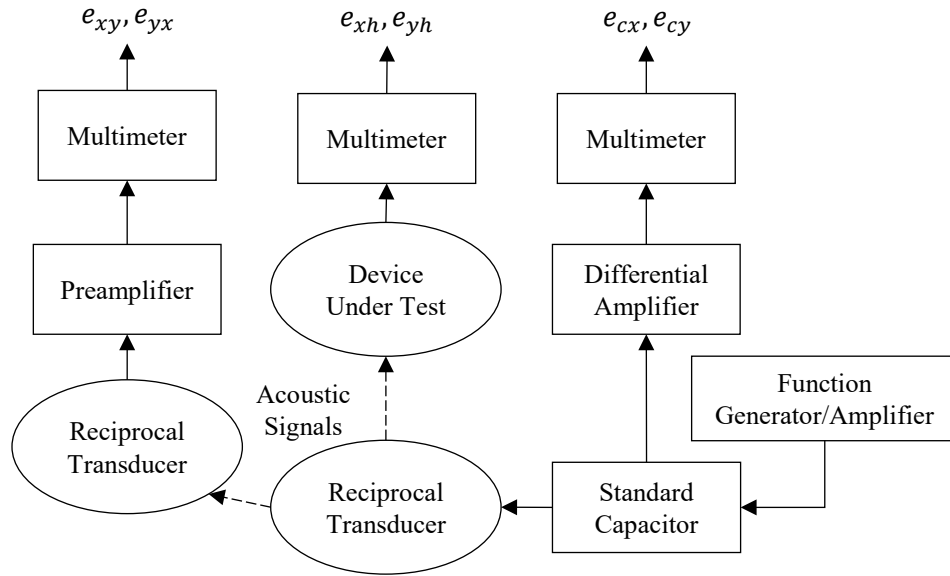


Figure 3-3. Voltage Measurement Block Diagram

Substituting the component uncertainties from the measurement chains in figure 3-3 and using the RSS, one gets the expressions in equations (39), (40), and (41) for the voltage uncertainties,

$$\frac{\delta e_{xc}}{e_{xc}} = \frac{\delta e_{yc}}{e_{yc}} = \sqrt{\left(\frac{\delta G_d}{G_d}\right)^2 + \left(\frac{\delta R_d}{R_d}\right)^2 + \left(\frac{\delta e}{e}\right)^2}, \quad (39)$$

$$\frac{\delta e_{xy}}{e_{xy}} = \frac{\delta e_{yx}}{e_{yx}} = \sqrt{\left(\frac{\delta G_p}{G_p}\right)^2 + \left(\frac{\delta e}{e}\right)^2}, \text{ and} \quad (40)$$

$$\frac{\delta e_{xh}}{e_{xh}} = \frac{\delta e_{yh}}{e_{yh}} = \frac{\delta e}{e}. \quad (41)$$

Substituting the expressions for voltage uncertainties from equations (39), (40), and (41) into equation (38) shows the relationship between $\left(\frac{\delta M_H}{M_H}\right)_{el}^2$ and the uncertainties in the test equipment,

$$\left(\frac{\delta M_H}{M_H}\right)_{el}^2 = \left(\frac{\delta C_p}{C_p}\right)^2 + \left(\frac{\delta G_d}{G_d}\right)^2 + \left(\frac{\delta R_d}{R_d}\right)^2 + \left(\frac{\delta G_p}{G_p}\right)^2 + \left(\sqrt{6} \frac{\delta e}{e}\right)^2. \quad (42)$$

The uncertainty components in equation (42) are defined in the following subsections. Each of these components, taken from manufacturer equipment specifications, are divided by $\sqrt{3}$ to convert to standard uncertainties, as shown in equation (1).

3.3.3.1 Standard Capacitor Capacitance $\left(\frac{\delta C_p}{C_p}\right)$. AC current to the projecting transducer passes through a General Radio type 1409-Y standard capacitor, which provides a standard impedance relating that current to a voltage. While its capacitance is specified to $\pm 0.05\%$, it is verified yearly using an HP 4192A impedance analyzer with capacitance uncertainty described in sheet 13 of (reference 19). From that table, with a Y range of $\sim 1S$, the uncertainty is given by,

$$\left(\frac{\delta C_p}{C_p}\right)^2 = \left(\left[0.12 + \frac{19}{f}\right]\% + \frac{3 \text{ counts}}{C}\right)^2. \quad (43)$$

While the capacitance is measured at many frequencies to ensure it is not frequency dependent, to achieve the lowest uncertainty, the capacitance value used was measured at $f = 200 \text{ Hz}$, the lowest value in the measured range. With a measured capacitance of $1 \mu F$ and $3 \text{ counts} = 0.1 \text{ pF}$ at that range, the uncertainty is 0.25% . The contribution to the voltage uncertainty is $\frac{0.25\%}{\sqrt{3}} = 0.12\%$.

3.3.3.2 Differential Amplifier Gain $\left(\frac{\delta G_d}{G_d}\right)$. A Burr-Brown 3562JG differential amplifier conditions the voltage signal across the standard capacitor. It is calibrated using an HP3562A dynamic signal analyzer, which has a gain accuracy of 1.1% per the HP 3562A datasheet. This uncertainty will contribute $\frac{1.1\%}{\sqrt{3}} = 0.7\%$ to voltage uncertainty.

3.3.3.3 Differential Amplifier Common Mode Rejection $\left(\frac{\delta R_d}{R_d}\right)$. The Burr-Brown 3562JG differential amplifier has a common mode rejection ratio of 83 dB . With a transducer capacitance of 4 nF and the standard capacitor of $1 \mu F$, the desired signal is 47 dB below the

common mode signal giving a measurement accuracy of $\pm 1.6\%$ (reference 13). For example, with a sinusoidal drive signal of amplitude 100 volts, the difference between the standard capacitor terminals is about 0.45 volts peak, while the common voltage is about 100 volts peak. After the 100 volt signal is attenuated 83 dB, the fractional error between this signal and the differential signal is 1.6%. This factor contributes $\frac{1.6\%}{\sqrt{3}} = 0.92\%$ to voltage uncertainty.

3.3.3.4 Preamplifier Gain $\left(\frac{\delta G_p}{G_p}\right)$. An Ithaco/DL Instruments model 1201 low-noise voltage preamplifier is used to condition the voltage signal received from the reciprocal transducer. According to the manufacturer specification, the gain accuracy on the preamplifier is 1% (reference 20), which contributes $\frac{1\%}{\sqrt{3}} = 0.6\%$ uncertainty to the hydrophone sensitivity.

3.3.3.5 Multimeter Voltage Measurement Accuracy $\left(\sqrt{6} \frac{\delta e}{e}\right)$. Keithley 2000 multimeters are used to measure the voltage from the hydrophone, receiving transducer by way of the model 1201 preamplifier, and standard capacitor by way of the Burr-Brown 3562JG. From the Keithley manufacturer specification, these voltages are measured with 0.38% accuracy (reference 21), contributing $\frac{0.38\% \sqrt{6}}{\sqrt{3}} = 0.5\%$ uncertainty to the hydrophone sensitivity.

3.3.4 Wavelength Correction

As the frequency of the sound in the coupler increases, the wavelength decreases. Even if the smallest wavelength is greater than 1/10 of the largest coupler dimension as required in reference 3, wavelength effects can introduce error into the sensitivity measurement. Assuming sound in the coupler behaves as a one-dimensional standing wave with a projector at one end and a closed end at the other, discussed in section 4, the wavelength correction factor used by the USRD is derived in references 10 and 13. This factor multiplies the measured voltages as shown in equation (13) and is given by:

$$F = \frac{\sin(kL)}{kL \cos(kd) \cos(kx)}, \quad (44)$$

where L is the length of the coupler, k is the wave number, and d is the offset distance between the projector and the nearest coupler wall. This factor reduces the peak amplitude at a distance x away from the projector to the amplitude if the pressure were the same everywhere in the coupler. To modify the hydrophone voltage, $x = L/2$ and

$$F_H = \frac{2\sin(kL/2)}{kL \cos(kd)}, \quad (45)$$

to modify the reciprocal transducer voltage, $x = d$, and

$$F_T = \frac{\sin(kL)}{kL \cos^2(kd)}. \quad (46)$$

Zalesak (reference 13) estimated the uncertainty in F to be ± 0.1 dB for a 1.0 dB correction at 2,000 Hz, or approximately a 10% multiplicative uncertainty, analyzing repeated measurements of the sensitivity of three field hydrophones calibrated in a coupler larger than one discussed in this report. For the receiving reciprocal transducer in the smaller coupler discussed here, the 10% multiplicative uncertainty corresponds to a 0.7% additive uncertainty or ± 0.06 dB at 2,000 Hz, decreasing as frequency decreases. At the hydrophone position, the additive uncertainty is just 0.01% at 2,000 Hz, since the hydrophone is closer to the projector.

From equation (15) the total uncertainty in sensitivity due to the wavelength corrections is,

$$\left(\frac{\delta M_H}{M_H}\right)_\lambda^2 = \left(\sqrt{2} \frac{\delta F_H}{F_H}\right)^2 + \left(\frac{1}{\sqrt{2}} \frac{\delta F_T}{F_T}\right)^2, \quad (47)$$

or 0.58% at 2,000 Hz.

3.3.5 Frequency Dependent Boundary Compliance

In coupler measurement theory, the total system compliance is treated as a constant in equation (12). For this assertion to be true there must not be elastomeric material exposed to the acoustic field (reference 3), as these types of material exhibit a frequency dependent bulk modulus (reference 22) and therefore, a frequency dependent compliance. This requirement complicates the way gasket materials would typically seal a pressure cavity. Elastomeric O-rings and transducer mounts have been used in the USRD coupler since the 1970s. The butyl rubber boot that covers the H48 sensitive element also violates the requirement. It was not until 2018 that the frequency dependent compliance due to this material was rigorously investigated in calibration results traceable to this primary measurement. Historical data show that the primary calibration was not performed at the lower pressures where the frequency dependence is most pronounced.

When frequency dependent sensitivity curves from low-pressure primary measurements were used in secondary calibrations at LOFAC in 2018, the systematic error became apparent and was documented in Corrective or Preventative Action Request (CPAR)-0032 (reference 23). During a subsequent investigation (reference 24), elastomers were removed from the acoustic field one-by-one, until only the O-rings remained. Once the O-rings were removed, the frequency dependence of the H48 sensitivity was greatly reduced.

Figure 3-4 shows several H48 sensitivity curves measured in the coupler at 40° C and 345 kPa, the environments where the effect of frequency dependent compliance on the sensitivity is greatest. In figure 3-4, the green and purple curves represent sensitivity with O-rings in place. These sensitivities should be independent of frequency over the range shown with the exception of preamplifier roll off below 10 Hz. Instead, the sensitivity between 10 Hz and 1,000 Hz varies by more than 1 dB. With aluminum washers in place of O-rings, the same hydrophone produced the orange and red sensitivity curves. The frequency dependent variation in sensitivity was reduced to about 0.2 dB over the same frequency range, similar to the frequency dependence of the higher pressure measurements when the dependence is at a minimum. In order to eliminate

elastomers in the coupler, one additional small O-ring must be removed that seals castor oil out of the H48 preamplifier cavity. This modification, as well as the elimination of the coupler chamber O-rings, requires a redesign of the system.

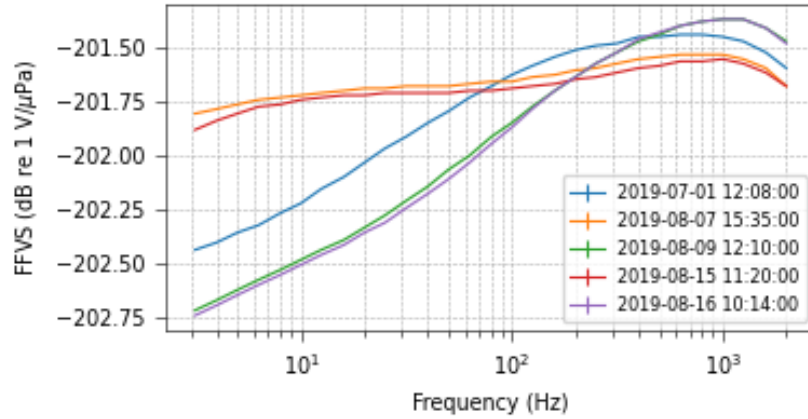


Figure 3-4. Sensitivity Curves for H48 SN1 Measured at 40° C and 345 kPa with and without Elastomeric O-Rings in the Coupler

The USRD will redesign the coupler and primary reference hydrophone to eliminate the elastomers and the frequency dependent system compliance. The new design will use metal seals and fiberglass transducer mounts to eliminate elastomers from the acoustic field. Until the new design is implemented, a single sensitivity value from the primary measurement replaces the sensitivity curve: the mean of the sensitivities measured between 500 Hz and 1,000 Hz where the frequency dependence is at a minimum. Preamplifier roll-off is measured separately and is added to the mean sensitivity to account for its effect on the hydrophone response below 10 Hz. Additionally, the uncertainty in the sensitivity is expanded by a factor, $\left(\frac{\delta M_H}{M_H}\right)_{bdf}$, which is set equal to 0.2 dB or 2.3%, the remaining frequency dependence in the sensitivity curve measured in reference 24 after removing as much elastomeric material as practical. If the frequency dependence of the sensitivity measurement is eliminated in the redesign of the coupler, the $\left(\frac{\delta M_H}{M_H}\right)_{bdf}$ factor in the Type B uncertainty can be removed.

3.3.6 Summary of Type B Uncertainties

A comparison of all Type B uncertainties shows which inputs have the greatest effect on the uncertainty in sensitivity. The relative effects of the component uncertainties in table 3-2 are not apparent by examining their individual sizes because the RSS exaggerates the impact of larger errors relative to a sum. Furthermore, as shown in equations (30) and (42), the sound speed uncertainty is doubled because it is squared in equation (23), and the multimeter uncertainty is multiplied by $\sqrt{6}$ because uncertainties in voltage pairs in equation (38) are not

independent. Figure 3-5 compares the relative contributions of the squared sources of uncertainty with appropriate multipliers as a fraction of the sum of all squared Type B errors.

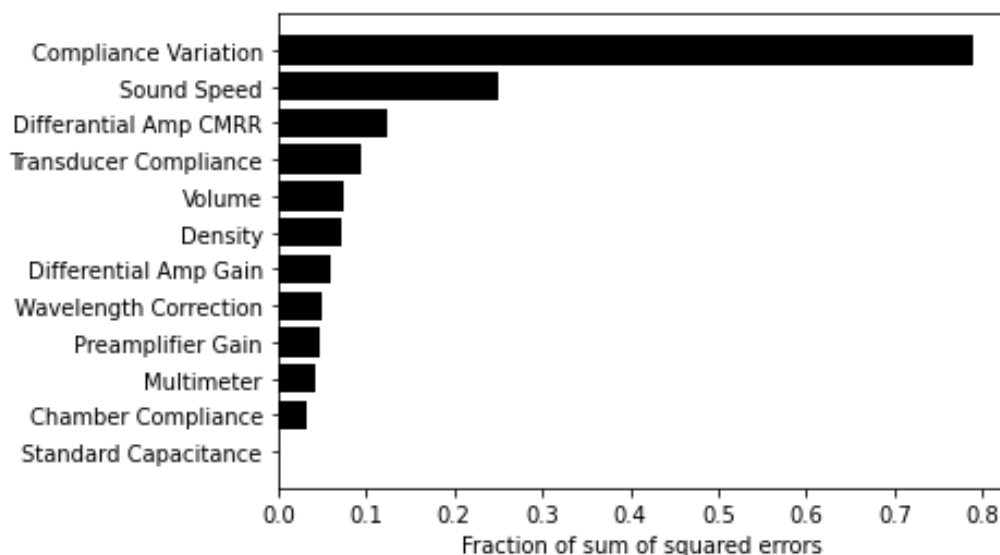


Figure 3-5. Comparison of the Sources of Type B Measurement Uncertainty in the H48 Coupler

The uncertainty due to the frequency dependent compliance in the coupler is, by far, the largest contributor to uncertainty in the primary sensitivity measurement. Sound speed uncertainty has the next largest effect followed by similarly sized sources. To reduce overall measurement uncertainty, the compliance variation must be eliminated and sound speed uncertainty reduced before other efforts will prove helpful.

3.4 COMBINED UNCERTAINTY

The uncertainty in the primary hydrophone sensitivity measurement is shown in table 3-6. The individual components of the Type A and B error subtotals used to estimate these uncertainties are discussed in sections 3.2 and 3.3. They are combined as shown in equation (3). As explained in sections 2 and 3.2, a coverage factor $k=2.52$ is used to calculate expanded uncertainty from standard uncertainty for the Type A component and for the Type B component, $k=2$. To illustrate the large effect of the contribution of the frequency dependent compliance variation, the right most two columns show the standard and expanded uncertainty terms calculated neglecting $\left(\frac{\delta M_H}{M_H}\right)_{fbd}$.

Table 3-6. Combined Uncertainty for Coupler Reciprocity Calibration

Calibration Frequency (kHz)	Uncertainty Components			Combined Uncertainty		Combined Uncertainty without $\left(\frac{\delta M_H}{M_H}\right)_{fbd}$	
	Type A (%)	Type B (%)	Type B without $\left(\frac{\delta M_H}{M_H}\right)_{fbd}$ (%)	Standard (68%) (%)	Expanded (95%) (dB)	Standard (68%) (%)	Expanded (95%) (dB)
3.15	0.91	2.61	1.17	2.76	0.48	1.49	0.28
4.00	0.88	2.61	1.17	2.75	0.48	1.47	0.28
5.00	0.85	2.61	1.17	2.74	0.48	1.45	0.27
6.30	0.82	2.61	1.17	2.73	0.47	1.43	0.27
8.00	0.80	2.61	1.17	2.73	0.47	1.42	0.27
10.0	0.81	2.61	1.17	2.73	0.47	1.42	0.27
12.5	0.77	2.61	1.17	2.72	0.47	1.41	0.26
16.0	0.76	2.61	1.17	2.72	0.47	1.40	0.26
20.0	0.72	2.61	1.17	2.71	0.47	1.38	0.25
25.0	0.77	2.61	1.17	2.72	0.47	1.40	0.26
31.5	0.68	2.61	1.17	2.70	0.46	1.36	0.25
40.0	0.72	2.61	1.17	2.71	0.47	1.38	0.25
50.0	1.06	2.61	1.17	2.81	0.49	1.58	0.30
63.0	0.62	2.61	1.17	2.68	0.46	1.33	0.24
80.0	0.68	2.61	1.17	2.70	0.46	1.36	0.25
100	0.61	2.61	1.17	2.68	0.46	1.32	0.24
125	0.58	2.61	1.17	2.67	0.46	1.31	0.24
160	0.55	2.61	1.17	2.67	0.46	1.30	0.23
200	0.52	2.61	1.17	2.66	0.46	1.29	0.23
250	0.53	2.61	1.17	2.66	0.46	1.29	0.23
315	0.49	2.61	1.17	2.65	0.45	1.27	0.23
400	0.48	2.61	1.17	2.65	0.45	1.27	0.23
500	0.49	2.61	1.17	2.65	0.45	1.27	0.23
630	0.45	2.61	1.17	2.65	0.45	1.26	0.22
800	0.48	2.61	1.18	2.65	0.45	1.27	0.23
1000	0.46	2.61	1.18	2.65	0.45	1.26	0.23
1250	0.55	2.61	1.18	2.67	0.46	1.30	0.23
1600	0.68	2.62	1.19	2.70	0.47	1.37	0.25
2000	0.82	2.62	1.21	2.75	0.48	1.46	0.27

4. SECONDARY CALIBRATION OF A HYDROPHONE

The LOFAC standing wave tube, referred to as system K, is a rigid-walled tube 13.9 cm in diameter and 183 cm in length. Unlike the coupler used in the primary measurement, it can accommodate a variety of hydrophones, provided:

- Their sensitive element is much less than a quarter wavelength at the highest calibration frequency*
- They are not operating near resonance (reference 25)
- They are much less compliant than the acoustic medium

Since this measurement is a secondary calibration, the output of the hydrophone under test is compared to a reference hydrophone, the USRD H48 and calibrated in the primary measurement described in section 3.

The system is filled with a water-glycol mixture that is degassed under vacuum. The chamber operates at pressures up to 13.79 MPa, temperatures from - 3° C to 40° C, and frequencies from 3 Hz to 1,600 Hz. The lower frequency limit is set by the electronics used to calibrate the H48 reference at the reciprocity coupler (section 3). A signal analyzer drives the projector through an amplifier and the analyzer's receive channels to compare the signals from the hydrophone under test, H , and the H48 reference, H_r . Figure 4-1 is a diagram of the setup.

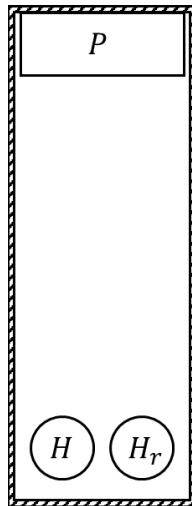


Figure 4-1. Standing Wave Schematic with a Projector, P , and Reference Hydrophone, R , and the Transducer Under Test, T

The sound field in the tube can be modeled as a one-dimensional standing wave. The boundary condition at the bottom wall is that of a rigid, closed tube end. There is a velocity node (point of zero velocity) and a pressure anti-node (acoustic pressure is at a maximum), as shown for several wavelengths in figure 4-2. The standing wave field is a superposition of the traveling wave generated by the projector and the wave reflected by the bottom boundary traveling in the opposite direction. At a pressure anti-node, the measured standing wave pressure amplitude is twice that of the individual traveling waves and the pressure gradient is zero. This pressure doubling provides the greatest sound pressure level at the hydrophone for a given drive current to

* In some circumstances, a special test setup allows for calibration of sensitive elements that are nearly as long as a quarter wavelength.

the projector. The zero or small pressure gradient means that the pressure amplitude at the hydrophone and reference hydrophone will be the same even for small differences in the relative heights of their acoustic centers.

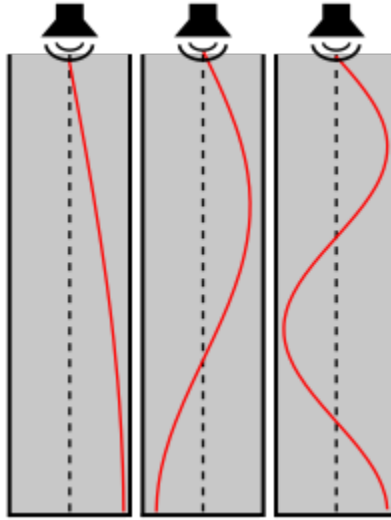


Figure 4-2. Diagram of a Pressure in an Acoustic Wave in a Tube with a Closed End²⁶

Note: In figure 4-2, the wavelength pictured is associated with the fundamental, third harmonic, and fifth harmonic resonance frequencies, from left to right.

The hydrophone being calibrated and reference hydrophone must both be positioned at the bottom wall to achieve the best results. As frequency increases, the standing wave pattern transitions from one longitudinal mode to the next (moving left to right in figure 4-2). Pressure nodes form at the projector surface and propagate down the tube, antinodes moving along with them. The wavelengths of the longitudinal modes are,

$$\lambda = \frac{4L}{n} \text{ for } n = 1, 3, 5, \text{ and} \quad (48)$$

where $L = 183$ cm, and n is odd because only odd harmonics are supported under these boundary conditions. The sound field sweeps through the first seven harmonics in the measured frequency range of 3 Hz to 1.6 kHz. The bottom wall boundary is the only position in the tube that the measurement benefits from pressure doubling and a near zero gradient throughout that frequency range.

The one-dimensional, longitudinal standing wave assumption breaks down as frequency increases. When the frequency is high enough to support the first radial mode in the tube, the standing wave field is two-dimensional. To maintain the validity of the one-dimensional longitudinal wave assumption, an upper frequency limit is set so the radius of the tube, a , is small compared to an acoustic wavelength, meaning $ka < 1$ where k is the acoustic wavenumber. To provide context for that criterion, the wavelength of the first radial mode in the tube is,

$$\lambda = \frac{2\pi a}{1.841}, \quad (49)$$

(reference 27), or 47.4 cm (for $a = 13.9$ cm) corresponding to a frequency of 3.3 kHz for a sound speed of 1,600 m/s, a lower bound for the environmental conditions in system K. Rewriting the $ka < 1$ criterion in terms of wavelength,

$$\lambda > 2\pi a, \quad (50)$$

with a corresponding frequency, and

$$f < \frac{c}{2\pi a}, \quad (51)$$

or 1.7 kHz. The criterion in equation (51) sets the upper frequency limit about one octave below that of the first radial mode, the frequency at which the radial acoustic wave can propagate without exponential attenuation. Below 3.2 kHz evanescent radial modes can form, so the extra octave frequency margin is important to ensure the sound field is the same at the hydrophone and reference hydrophone locations. The effect of evanescent radial modes can be seen in the Type A uncertainty analysis in section 4.2, where measurements at frequencies higher than 1.6 kHz show much greater random variation.

4.1 MEASUREMENT THEORY

Secondary comparison calibrations are performed in the standing wave tube with an H48 reference hydrophone next to the hydrophone under test (reference 3). The sensitivity of the unknown hydrophone, M_H , is

$$M_H = M_R \frac{e_H}{e_R} \quad (52)$$

where M_R is the sensitivity of the reference hydrophone, measured with coupler reciprocity, and e_H and e_R are the open circuit voltages of the unknown and reference hydrophones, respectively.

4.2 TYPE A UNCERTAINTY ANALYSIS

The Type A uncertainty for comparison calibrations is estimated using the standard deviation of repeated sensitivity measurements, as described in section 2 of this report, on four different hydrophones: H48 serial number 4, DT-369 serial number 1043, F42D serial number 217, and H56 serial number 125. The Type H56 hydrophone included 20 dB gain applied to its output, resulting in an effective sensitivity of - 144 dB re 1 V/ μ Pa, so that the Type A sensitivity could be evaluated near the upper limit of the accredited range. This cross section of hydrophones represents a variety of sizes, constructions and sensitivities. Between nine and eighteen measurements were taken for each hydrophone.

The sensitivity measurements from three of the four hydrophones in this Type A analysis are shown in figure 4-3 alongside their Type A standard uncertainties. The measured sensitivities range from -230 dB re 1 V/ μ Pa to -166 dB re 1 V/ μ Pa. The uncertainty in the results are generally within 4% across the frequency range valid in system K. The measurements continue above 1.6 kHz, where evanescent radial modes disrupt the one-dimensional longitudinal wave assumption and random variation increases sharply. Between 1.6 kHz and 2 kHz, the standard uncertainty doubles.

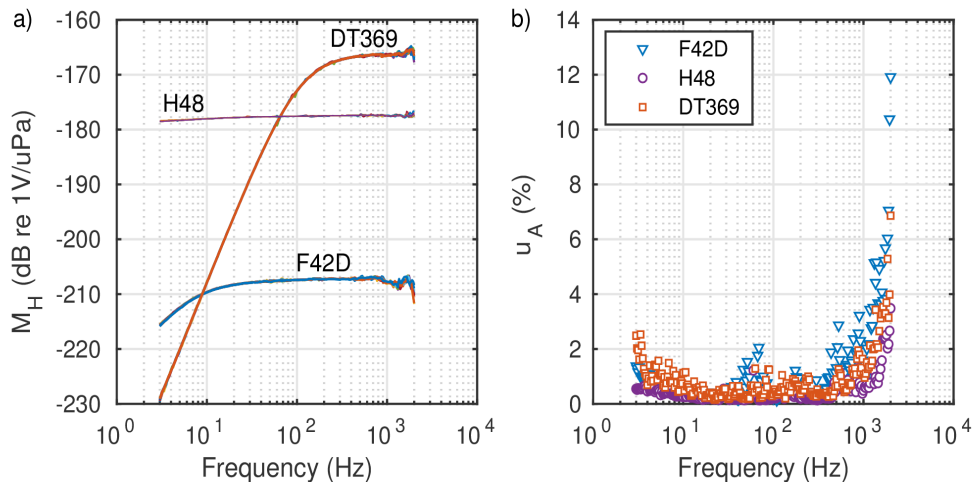


Figure 4-3. Results for Repeated Calibration of Three Different Hydrophone Types

Note: In figure 4-3, panel a (left) shows the measured sensitivities M_H . Panel b (right) shows the Type A uncertainty component u_A .

Since the sensitivities vary between hydrophones, the individual standard deviations were pooled, as described in equation (14), to estimate the Type A uncertainty for comparison calibrations. Table 4-1 presents the Type A uncertainty results for the individual hydrophones and the pooled Type A measurement uncertainty of the facility.

Table 4-1. Comparison Calibration – Type A Uncertainty

Frequency (Hz)	Device Type A Standard Uncertainty				Pooled Std. Deviation (%)
	H48	DT-369	F42D	H56 (+20dB)	
3.00	0.56	1.96	1.16	0.41	1.08
4.00	0.46	0.69	0.81	0.40	0.59
5.00	0.49	1.37	0.78	0.46	0.78
6.30	0.42	0.60	0.54	0.34	0.47
8.00	0.30	0.59	0.65	0.28	0.45
10.0	0.26	1.13	0.40	0.25	0.56
12.5	0.30	0.31	0.45	0.22	0.33
16.0	0.21	0.30	0.38	0.22	0.28
20.0	0.27	0.23	0.29	0.19	0.25
25.0	0.15	0.50	0.32	0.19	0.29
31.5	0.22	0.18	0.57	0.17	0.32
40.0	0.17	0.57	0.19	0.18	0.29
50.0	0.24	0.32	0.86	0.16	0.45
63.0	0.48	1.29	3.91	0.20	1.90
80.0	0.16	0.48	0.41	0.18	0.31
100	0.21	0.37	0.22	0.18	0.24
125	0.55	1.05	0.29	0.31	0.60
160	0.34	0.63	0.65	0.62	0.54
200	0.19	0.38	0.41	0.45	0.34
250	0.20	0.89	0.36	0.64	0.53
315	0.17	0.39	0.40	0.60	0.38
400	0.22	0.30	0.94	0.36	0.50
500	0.49	0.89	1.31	0.42	0.80
630	0.56	0.50	1.17	0.53	0.72
800	0.83	1.29	1.23	0.64	0.99
1000	0.49	1.06	1.22	0.54	0.82
1250	1.52	1.57	2.85	1.12	1.82
1600	1.58	3.26	4.07	0.72	2.56

4.3 TYPE B UNCERTAINTY ANALYSIS

The largest component of Type B uncertainty in the comparison calibration is the uncertainty in the sensitivity of the primary reference hydrophone, the H48. This uncertainty is just the combined uncertainty of the coupler reciprocity measurement shown in table 3-6.

Three other components contribute to the Type B uncertainty:

- The uncertainty in transfer function measured by the HP 3562A signal analyzer to compare the signals received from the H48 and the device under test.
- The uncertainty in the gain of the Ithaco/DL 1201 preamplifier used to condition signals from the device under test.

- At frequencies of 800 Hz and above, some variation in the acoustic field in the radial direction due to an evanescent radial mode in the tube.

As discussed in sections 3.3.3.2 and 3.3.3.4, the HP 3562A contribution to Type B uncertainty is 0.7% and the Ithaco/DL 1201 contribution is 0.6%. At frequencies of 800 Hz and above, evanescent radial modes contribute 1% uncertainty. An additional 0.5% uncertainty captures sources of error not accounted for elsewhere. Table 4-2 shows these components and the combined Type B standard uncertainty across frequency.

Table 4-2. Type B Uncertainty for the Comparison Calibration

Calibration Frequency (kHz)	Reference Measuring Hydrophone (%)	HP 3562A (%)	Ithaco Preamplifier (%)	Evanescent Radial Mode (%)	Other (%)	Type B Std. Uncertainty (%)
3.00	2.76	0.7	0.6		0.5	2.96
4.00	2.75	0.7	0.6		0.5	2.95
5.00	2.74	0.7	0.6		0.5	2.94
6.30	2.73	0.7	0.6		0.5	2.93
8.00	2.73	0.7	0.6		0.5	2.92
10.0	2.73	0.7	0.6		0.5	2.92
12.5	2.72	0.7	0.6		0.5	2.92
16.0	2.72	0.7	0.6		0.5	2.91
20.0	2.71	0.7	0.6		0.5	2.90
25.0	2.72	0.7	0.6		0.5	2.91
31.5	2.70	0.7	0.6		0.5	2.89
40.0	2.71	0.7	0.6		0.5	2.90
50.0	2.81	0.7	0.6		0.5	3.00
63.0	2.68	0.7	0.6		0.5	2.88
80.0	2.70	0.7	0.6		0.5	2.89
100	2.68	0.7	0.6		0.5	2.88
125	2.67	0.7	0.6		0.5	2.87
160	2.67	0.7	0.6		0.5	2.87
200	2.66	0.7	0.6		0.5	2.86
250	2.66	0.7	0.6		0.5	2.86
315	2.65	0.7	0.6		0.5	2.85
400	2.65	0.7	0.6		0.5	2.85
500	2.65	0.7	0.6		0.5	2.85
630	2.65	0.7	0.6		0.5	2.85
800	2.65	0.7	0.6	1.0	0.5	3.02
1000	2.65	0.7	0.6	1.0	0.5	3.02
1250	2.67	0.7	0.6	1.0	0.5	3.04
1600	2.70	0.7	0.6	1.0	0.5	3.07

4.4 COMBINED UNCERTAINTY

The uncertainties in the comparison calibration analysis are summarized in table 4-3. In the combined expanded uncertainty, a coverage factor of $k = 2$ multiplies all Type A and B combined standard uncertainties, for reasons discussed in section 2. The individual components of the Type A and B error subtotals used to estimate these uncertainties are discussed in sections 4.2 and 4.3 combined as shown in equation (3). The rightmost two columns show the same combined uncertainties, except neglecting the component of the primary reference uncertainty, $\left(\frac{\delta M_H}{M_H}\right)_{fbd}$, to illustrate the large effect that term has on the overall uncertainty.

Table 4-3. Combined Uncertainty for Comparison Calibration

Calibration Frequency (kHz)	Uncertainty Components		Combined Uncertainty		Combined Uncertainty without $\left(\frac{\delta M_H}{M_H}\right)_{fbd}$	
	Type A (%)	Type B (%)	Standard (68%) (%)	Expanded (95%) (dB)	Standard (68%) (%)	Expanded (95%) (dB)
3.00	1.08	2.96	3.14	0.5	1.91	0.3
4.00	0.59	2.94	3.00	0.5	1.68	0.3
5.00	0.78	2.94	3.04	0.5	1.76	0.3
6.30	0.47	2.93	2.97	0.5	1.64	0.3
8.00	0.45	2.92	2.96	0.5	1.64	0.3
10.0	0.56	2.92	2.98	0.5	1.67	0.3
12.5	0.33	2.92	2.93	0.5	1.61	0.3
16.0	0.28	2.91	2.92	0.5	1.60	0.3
20.0	0.25	2.90	2.91	0.5	1.59	0.3
25.0	0.29	2.91	2.93	0.5	1.60	0.3
31.5	0.32	2.89	2.91	0.5	1.61	0.3
40.0	0.29	2.90	2.92	0.5	1.60	0.3
50.0	0.45	3.00	3.04	0.5	1.64	0.3
63.0	1.90	2.88	3.45	0.6	2.47	0.4
80.0	0.31	2.89	2.91	0.5	1.60	0.3
100	0.24	2.88	2.89	0.5	1.59	0.3
125	0.60	2.87	2.93	0.5	1.69	0.3
160	0.54	2.86	2.92	0.5	1.66	0.3
200	0.34	2.86	2.88	0.5	1.61	0.3
250	0.53	2.86	2.91	0.5	1.66	0.3
315	0.38	2.85	2.88	0.5	1.62	0.3
400	0.50	2.85	2.89	0.5	1.65	0.3
500	0.80	2.85	2.96	0.5	1.77	0.3
630	0.72	2.85	2.94	0.5	1.73	0.3
800	0.99	3.02	3.18	0.5	2.11	0.4
1000	0.82	3.02	3.13	0.5	2.04	0.3
1250	1.82	3.03	3.54	0.6	2.61	0.4
1600	2.56	3.06	3.99	0.7	3.17	0.5

5. CONCLUSION

This report describes the uncertainty in hydrophone sensitivity measurements over the frequency range 3 Hz to 1,600 Hz at the USRD. Type A and Type B uncertainty analyses are performed for the random and systematic effects, respectively, that lead to uncertainty in the measurements. These types of uncertainties are combined to estimate the uncertainty for the primary and secondary hydrophone sensitivity measurements as recommended by the BIPM in reference 6. The controlling uncertainties of the USRD sensitivity measurement in this frequency range are derived from the frequency dependent compliance in the coupler and the speed of sound in castor oil, both of which affect uncertainty by way of the primary measurement. Other factors contribute less to the combined uncertainty, like measurement system electronics and evanescent radial modes in the standing wave tube.

Future work should focus on reducing the largest sources of uncertainty. An effort currently underway to redesign the coupler chamber and primary reference hydrophone will address the frequency dependent compliance problem. A system to measure the speed of sound in castor oil with less uncertainty than reference 15 would also make a meaningful reduction the USRD measurement uncertainty in this frequency range.

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