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Inverse Scattering Problems in Time Dependent Random Media, Waveguides and Cavities

Borcea, Liliana
REGENTS OF THE UNIVERSITY OF MICHIGAN
503 THOMPSON ST
ANN ARBOR, MI,
US

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14. ABSTRACT
We introduce a direct, linear sampling approach to imaging in an acoustic waveguide with sound hard walls. The waveguide terminates at one end and has unknown geometry due to compactly supported wall deformations. The goal of imaging is to determine these deformations and to identify localized scatterers in the waveguide, using a remote array of sensors that emits time harmonic probing waves and records the echoes. We present a theoretical analysis of the imaging approach and illustmte its performance with numerical simulations.

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Final report for AFOSR Grant FA9550-18-1-0131
Inverse Scattering Problems in Time Dependent Random Media, Waveguides and Cavities

Liliana Borcea
Department of Mathematics
University of Michigan, Ann Arbor

1 Objectives

The objectives of our research are:

1. Quantify the effect of multiple scattering on wave propagation in complex environments. The complexity is due to small amplitude, uncertain variations of the wave speed and/or boundaries of the domain, and it is modeled as a random process. The goal of our work is to quantify how the randomness of the wave speed and of the boundary is mapped by the wave equation to the randomness of the wave field. In particular, we quantified the range scales on which the wave field becomes significantly different than its statistical mean i.e., the wave loses coherence. We also studied how power is transferred among the wave components due to scattering in the random medium and/or the random boundaries, and how the wave evolves at long range of propagation in the complex environment.
2. Mitigate the wave distortion due to scattering in complex environments in order to obtain robust and high resolution images using either active arrays of sensors or synthetic aperture radar. We considered imaging in open environments and in waveguides.

2 Accomplishments

We describe the results and publications grouped by research theme, and in chronological order:

2.1 Wave propagation and imaging in waveguides

• L. Borcea, F. Cakoni, S. Meng, A direct approach to imaging in a waveguide with perturbed geometry, Journal of Computational Physics, Vol. 392, 2019, p. 556-577.

Abstract *We introduce a direct, linear sampling approach to imaging in an acoustic waveguide with sound hard walls. The waveguide terminates at one end and has unknown geometry due to compactly supported wall deformations. The goal of imaging is to determine these deformations and to identify localized scatterers in the waveguide, using a remote array of sensors that emits time harmonic probing waves and records the echoes. We present a theoretical analysis of the imaging approach and illustrate its performance with numerical simulations.*

The motivation of this work is the problem of imaging in tunnels. The mathematical model of the tunnel is a waveguide with reflecting walls, which terminates at one end. The main challenge is that the waveguide has unknown geometry, modeled by compactly supported variations of the walls, which are to be determined in conjunction with imaging targets in the waveguide. The study is carried out in the context of imaging with active sensor arrays which probe the waveguide with time harmonic waves and record the returns i.e., the data. We show that it is possible to solve this imaging problem using the linear sampling approach. The detailed analysis is based on the modal wave decomposition in the reference waveguide with undeformed walls. The unknown wall deformations and the targets lead to mode coupling. This is a nonlinear effect which is taken fully into account in the paper, for both full and partial aperture arrays. The performance of the linear sampling approach is illustrated with numerical simulations carried out in two dimensions.

• L. Borcea, S. Meng, Factorization method versus migration imaging in a waveguide, Inverse Problems, 35, 2019, p. 124005 (33 pages).

Abstract *We present a comparative study of two qualitative imaging methods in an acoustic waveguide with sound hard walls. The waveguide terminates at one end and contains unknown obstacles of compact support, to be determined from data gathered by an array of sensors that probe the obstacles with waves and measure the scattered response. The first imaging method, known as the factorization method, is based on the factorization of the far field operator. It is designed to image at single frequency and estimates the support of the obstacles by a Picard range criterion. The second imaging method, known as migration, works either with one or multiple frequencies. It forms an image by backpropagating the measured scattered wave to the search points, using the Green's function in the empty waveguide. We study the connection between these methods with analysis and numerical simulations.*

This paper is also motivated by the application of imaging in tunnels. It introduces a comparative mathematical analysis between two qualitative active array imaging methods: the factorization method and reverse time migration. The factorization method, which is popular in the mathematics community, is based on a factorization of the far field operator that maps the incident wave to the scattered wave. This method was developed mostly for imaging in open environments, although there have been some recent applications to imaging in waveguides or cavities. The problem with the latter has been that due to technical difficulties, either unphysical illuminations or illuminations that can only be realized with full aperture arrays have been used. In this paper we show that the factorization method can be used in a terminated waveguide, for physical incident waves generated by sensors in an array that lies far from the obstacle, on the opposite side of the end wall. The factorization method, like its mathematical sibling, the linear sampling method, is designed for imaging at a single frequency. There is no clear way, at the moment, to combine multi-frequency data. The main result of our paper is to show that there is a connection between the factorization method and the migration approach, which is popular in the applied community. Explicitly, we show that the factorization method has a variational (optimization) formulation and explain that the migration imaging function can be understood as the optimization objective evaluated at a very good test function. This result is important because the migration method has a natural formulation for broadband signals. Therefore, it should be useful in guiding the development of qualitative methods like factorization or linear sampling for such signals. The analysis is complemented with detailed numerical simulations in both two and three dimensions.

• L. Borcea, J. Garnier, A ghost imaging modality in a random waveguide, Inverse Problems, 34, 2018, p. 124007 (33 pages).

Abstract *We study the imaging of a penetrable scatterer, aka target, in a waveguide with randomly perturbed boundary. The target is located between a partially coherent source which transmits the wave, and a detector which measures the spatially integrated energy flux of the wave. The imaging is impeded by random boundary scattering effects that accumulate as the wave propagates. We consider a very large distance (range) between the target and the detector, where the cumulative scattering is so strong that it distributes the energy evenly among the components (modes) of the wave. Conventional imaging is impossible in this equipartition regime. Nevertheless, we show that the target can be located with a ghost imaging modality. This forms an image using the cross-correlation of the measured energy flux, integrated over the aperture of the detector, with the time and space resolved energy in a reference waveguide, at the search range. We consider two reference*

waveguides: the waveguide with unperturbed boundary, in which we can calculate the energy flux, and the actual random waveguide, before the presence of the target, in which the energy flux should be measured. We analyze the ghost imaging modality from first principles and show that it can be efficient in a random waveguide geometry in which there is both strong modal dispersion and mode coupling induced by scattering, provided that the standard ghost imaging function is modifed and integrated over a suitable time offset window in order to compensate for dispersion and diffusion. The analysis quantifies the resolution of the image in terms of the source and detector aperture, the range offset between the source and the target, and the duration of the measurements.

This paper addresses imaging in a waveguide with random boundary, in a regime where the boundary scattering effects on the measured wave are extreme. By this we mean that:

1. The wave field is incoherent i.e., the statistical mean is essentially zero and therefore the wave consists of random fluctuations.
2. The energy becomes distributed uniformly over the propagating components of the wave (the so-called waveguide modes). This is known as the equipartition regime and it means practically that the wave has lost any direction information.

The first item means that it is impossible to use any coherent imaging method, like say matched field. The second item says that the situation is even worse: one cannot image even using incoherent methods e.g., based on transport theory. However, a new imaging modality, called "ghost imaging" in the optics community, can be adapted to allow imaging. The main result of the paper is to show how such imaging can be carried out and to quantify in detail the resolution of the resulting image. The ghost imaging modality is designed for a transmission setup, where the source and receiver (detector) are at opposite ends of the waveguide. The source emits incoherent (noise like) signals and the detector measures the net energy flux. The strong scattering over the long travel distance between the source and the detector makes the imaging basically independent of the detector aperture, meaning that good results can be obtained even with a very small (almost point-like) detector. However, the distance between the source and the target has a strong influence: If the target is close enough to the source, so the wave reaching it still has some coherence, then imaging can be done using cross-correlation of the net flux at the detector with the computed flux at the target. This computation is done in an ideal waveguide i.e., without the random boundary. Otherwise, if the target is further away, then imaging can only be done using prior calibration i.e., using measurements of the transmission matrix from the source plane to the target plane. This could be used for monitoring a certain region in the waveguide.

- **L. Borcea, J. Garnier, Wave propagation in randomly perturbed weakly coupled waveguides, SIAM Multiscale Modeling and Simulations, 18(1), 2020, p. 44-78.**

Abstract *We present an analysis of wave propagation in a two step-index, parallel waveguide system. The goal is to quantify the effect of scattering at randomly perturbed interfaces between the guiding layers of high index of refraction and the host medium. The analysis is based on the expansion of the solution of the wave equation in a complete set of guided, radiation and evanescent modes with amplitudes that are random fields, due to scattering. We obtain a detailed characterization of these amplitudes and thus quantify the transfer of power between the two waveguides in terms of their separation distance. The results show that, no matter how small the fluctuations of the interfaces are, they have significant effect at sufficiently large distance of propagation, which manifests in two ways: The first effect is well known and consists of power leakage from the guided modes to the radiation ones. The second effect consists of blurring of the periodic transfer of power*

between the waveguides and the eventual equipartition of power. Its quantification is the main practical result of the paper.

This study began with an investigation of a leaky waveguide with random boundary. We were interested in extending our work on random waveguides to the case of penetrable boundaries, so that we can understand how coupling to radiation modes affects the statistics of the propagating modes in the waveguide. In this paper we went beyond this question, as we addressed the problem of two waveguides, where not only we quantified the loss of energy via radiation through the boundary and due to the mode coupling induced by scattering at the random boundary, but we also studied the coupling between waveguides. This could be of interest in the design of optical circuits. The results show that scattering at the random boundary has two effects: The first one, which is well known, is the loss of power to the radiative modes. The second one, which is not found in the literature, consists of a blurring of the deterministic coupling effects (which are known to consist of a periodic power transfer between the two waveguides). In particular, we show that at long enough distance, the balance of power reaches a steady state, corresponding to equipartition between the two waveguides, no matter what the initial excitation was.

- **L. Borcea, J. Garnier, K. Solna, Onset of energy equipartition among surface and body waves, Proc. R. Soc. A, 2021, 477: 20200775 (28 pages).**

Abstract *We derive a radiative transfer equation that accounts for coupling from surface waves to body waves and the other way around. The model is the acoustic wave equation in a two-dimensional waveguide with reflecting boundary. The waveguide has a thin, weakly randomly heterogeneous layer near the top surface, and a thick homogeneous layer beneath it. There are two types of modes that propagate along the axis of the waveguide: those that are almost trapped in the thin layer, and thus model surface waves, and those that penetrate deep in the waveguide, and thus model body waves. The remaining modes are evanescent waves. We introduce a mathematical theory of mode coupling induced by scattering in the thin layer, and derive a radiative transfer equation which quantifies the mean mode power exchange. We study the solution of this equation in the asymptotic limit of infinite width of the waveguide. The main result is a quantification of the rate of convergence of the mean mode powers toward equipartition.*

The fundamental question of wave propagation in random media is how scattering redistributes power among the wave components, commonly called modes. In regimes of forward scattering, this is described by a radiative transfer equation, which is an evolution (in range) equation for the mean mode powers. The radiative transfer equation is relatively well understood in open environments. There are also studies (late 1990's and early 2000's) that consider a random boundary (i.e., a Robin boundary condition with random impedance). These studies establish effective boundary conditions for the radiative transfer equation and account for the loss of power via radiation. However, there is an outstanding question concerned with the power exchange (back and forth) between surface and body waves that is not understood at all. This question has been explored in a phenomenological way by geophysicists, based on new data which shows that coda (i.e., the long tailed part of signals) emerging from a random earth appears at first glance to correspond to a regime of energy equipartition, but in fact the power of the modes still evolves. The motivation of our study was to quantify the multiscale evolution of the wave towards equipartition. Our experience with waveguides allowed us to make progress towards this goal. By using a waveguide that contains (1) a thin layer filled with a random medium, which leads to modes that are trapped there and thus mimic surface waves, and (2) a thick layer that supports propagating modes that mimic body waves, we obtained a specific quantification of back and forth power exchange between the modes and a proof of the multiscale nature of the evolution towards equipartition.

2.2 High resolution imaging

- **L. Borcea and I. Kocyigit, A Multiple Measurement Vector Approach to Synthetic Aperture Radar Imaging, SIAM Journal on Imaging Sciences 11 (1), 2018, p. 770-801.**

Abstract *We study a multiple measurement vector (MMV) approach to synthetic aperture radar (SAR) imaging of scenes with direction-dependent reflectivity and with polarization diverse measurements. The unknown reflectivity is represented by a matrix with row support corresponding to the location of the scatterers in the scene and columns corresponding to measurements gathered from different subapertures or different polarization of the waves. The MMV methodology is used to estimate the reflectivity matrix by inverting in an appropriate sense the linear system of equations that models the SAR data. We introduce a resolution analysis of imaging with MMV, which takes into account the sparsity of the imaging scene, the separation of the scatterers, and the diversity of the measurements. The results of the analysis are illustrated with some numerical simulations.*

The goal of this paper is to derive a resolution theory of SAR imaging of direction dependent reflectivities, using the multiple measurement vector (MMV) optimization approach. We also take into consideration polarization diverse measurements. The theory quantifies the support of the reconstructions by taking into account the target separation and analyzes in detail the conditions under which the MMV approach is useful. This involves studying carefully proper segmentations of the aperture in overlapping sub-apertures, taking advantage of the different angle views of the reflectivity, and how data is fused across apertures. The resolution theory is complemented by extensive numerical simulations.

- **L. Borcea, J. Garnier, High-resolution interferometric synthetic aperture imaging in scattering media, SIAM Imaging sciences, 13(1), 2020, p. 291-316.**

Abstract *The goal of synthetic aperture imaging is to estimate the reflectivity of a remote region of interest by processing data gathered with a moving sensor which emits periodically a signal and records the backscattered wave. We introduce and analyze a high-resolution interferometric method for synthetic aperture imaging through an unknown scattering medium which distorts the wave. The method builds on the coherent interferometric (CINT) approach which uses empirical cross-correlations of the measurements to mitigate the distortion, at the expense of a loss of resolution of the image. The new method shows that, while mitigating the wave distortion, it is possible to obtain a robust and sharp estimate of the modulus of the Fourier transform of the reflectivity function. A high-resolution image can then be obtained by a phase retrieval algorithm.*

This paper addresses the challenging problem of synthetic aperture radar (SAR) imaging through the turbulent atmosphere. It is relevant for high frequency (10GHz or higher) SAR, where the interaction of the waves with the atmosphere becomes strong at distances of tens or hundreds of km. The imaging method introduced in this paper is based on cross-correlations of the measurements made by the antenna, that is to say, it uses the transport theory for energy density (Wigner transform) propagation in the random medium. The established such method known as coherent interferometry (CINT) was developed by the PI in collaboration with George Papanicolaou and Chrysoula Tsogka more than ten years ago. Other notable contributions that followed are due to Josselin Garnier and Knut Solna. CINT accounts for the loss of coherence and statistical decorrelation of the components of the wave due to scattering in random media. It images by back-propagating point by point in the imaging region the cross-correlations of the measurements calculated over carefully chosen time and sensor offset windows. The windowing has a statistical smoothing (stabilization) effect, which is necessary for robust imaging, but comes at the expense of resolution. In this paper we show, after a careful study of the cross-correlations of the measurements, that they actually contain information for much better imaging. This information is lost in CINT because of the backpropagation which is based on a linear approach that is adequate for a single point target or for well separated point targets. We introduce a better way of back-propagating the cross-correlations, to a pair of points in the imaging region, instead of a single point. The result is a more complicated imaging function, which allows estimation of differences of distances between points in the targets with no loss of resolution due to the random medium. One way to get an image out of this imaging function is to calculate the Wigner transform of the two point imaging function. This allows the estimation of the modulus of the Fourier transform of the unknown reflectivity function and reduces the imaging to a phase retrieval problem. We show with analysis and numerical simulations that while the two point imaging function is very robust both to atmosphere and noise effects, the phase retrieval part is affected by noise. We will pursue other avenues to get rid of the phase retrieval step in order to get a more robust and efficient way of obtaining high resolution images in random media.

2.3 Wave propagation in random open environments and applications

- **L. Borcea, J. Garnier, K. Solna, Wave propagation and imaging in moving random media, SIAM Multiscale Modeling & Simulation 17 (1), 2019, 31-67.**

Abstract *We present a study of sound wave propagation in a time dependent random medium and an application to imaging. The medium is modeled by small temporal and spatial random fluctuations in the wave speed and density, and it moves due to an ambient flow. We develop a transport theory for the energy density of the waves, in a forward scattering regime, within a cone (beam) of propagation with small opening angle. We apply the transport theory to the inverse problem of estimating a stationary wave source from measurements at a remote array of receivers. The estimation requires knowledge of the mean velocity of the ambient flow and the second-order statistics of the random medium. If these are not known, we show how they may be estimated from additional measurements gathered at the array, using a few known sources. We also show how the transport theory can be used to estimate the mean velocity of the medium. If the array has large aperture and the scattering in the random medium is strong, this estimate does not depend on the knowledge of the statistics of the random medium.*

In this paper we developed, from first principles, a rigorous theory of sonar imaging in time dependent random media. The time dependence is due to a flow e.g., wind, modeled by a random flow velocity. The equation of conservation of mass and the constitutive relation that defines the sound speed give that the mass density and sound speed must also have random, time dependent fluctuations, which are statistically correlated to the fluctuations of the flow velocity. The wave equation is not the usual one, but one that is obtained from the fluid dynamics equations linearized about the flow. Our analysis reveals that the time variation in the medium has two beneficial effects for imaging: First, it leads to self-averaging (statistically stable) quantities (like cross-correlations of the wave field measured by an array of sensors) that involve time integrals over longer durations than the time scale of variation in the medium. This is important for having high fidelity images that do not depend on the particular realization of the random medium. Second, it leads to a broadening of the bandwidth, which allows for improved range estimation. The paper gives a detailed analysis of imaging and estimation of the mean flow velocity, which makes use of these effects.

- **L. Borcea, J. Garnier, K. Solna, Multimode communication through the turbulent atmosphere, JOSA A, 37(5), 2020, p. 720–730.**

Abstract *A central question in free-space optical communications is how to improve the transfer of information between a transmitter and receiver. The capacity of the communication channel can be increased by multiplexing of independent modes using either: (1) the MIMO (Multiple-Input-Multiple-Output) approach, where the communication is done with modes obtained from the singular value decomposition of the transfer matrix from the transmitter array to the receiver array, or (2) the OAM (Orbital Angular Momentum) approach, which uses vortex beams that carry angular momenta. In both cases, the number of usable modes is limited by the finite aperture of the transmitter and receiver, and the effect of the turbulent atmosphere. The goal of this paper is twofold: First, we show that the MIMO and OAM multiplexing schemes are closely related. Specifically, in the case of circular apertures, the leading singular vectors of the transfer matrix which are useful for communication are essentially the same as the commonly used Laguerre-Gauss vortex beams, provided these have a special radius that depends on the wavelength, the distance from the transmitter to the receiver and the ratio of the radii of their apertures. Second, we characterize the effect of atmospheric turbulence on the communication modes using the phase screen method put in the mathematical framework of beam propagation in random media.*

In this work we built upon our experience on beam propagation in random media to study an optimal scheme for communication between an emitter array and a receiver array. There is plenty of literature on this topic concerned with the use of orbital angular momentum (OAM) beams for increasing the capacity of the communication channel. However, most of the literature ignores the fact that the arrays have finite aperture and therefore does not study carefully its effect on the communication scheme. We show that the optimal way of approaching this problem is by measuring the transfer matrix between the receiver and transmitter array. This is especially important in strong turbulence. If the turbulence is not too strong, then one can get away with working with a model transfer matrix, corresponding to the propagation in a reference (homogeneous) medium. The SVD of this matrix then leads to an optimal communication scheme, that interestingly can be related to communication with OAM (Laguerre-Gauss) beams in the case of circular apertures.

• L. Borcea, Imaging with waves in random media, Notices of the AMS, vol 66(11), 2019, pp. 1800-1812.

This paper was written by invitation from the Notices of the AMS, to introduce to the mathematical community the field of imaging with waves in random media. It describes the challenges of the field and the advances made by the PI and collaborators on imaging with cross-correlations for both passive and active arrays, for probing the medium with controlled sources and for imaging with ambient noise sources. The reviewed developments are described in the context of applications, such as imaging satellites or energetic space debris using powerful ground based antennas and receivers on aircraft flying above the turbulent atmosphere.

2.4 Other research

• L. Borcea, B. Riviere, Y. Wang, Nonoverlapping domain decomposition method with preconditioner from asymptotic analysis of steady flow in high contrast media, International Journal of Computer Mathematics, Published Online Dec 30, 2020, (18pp).
https://doi.org/10.1080/00207160.2020.1870681

Abstract *We present a nonoverlapping domain decomposition method for steady flow in high contrast heterogeneous media modeled by an elliptic equation with coefficients that have very large amplitude variations* *on a small spatial scale. The linear system of equations resulting from matching the solution trace and the fluxes through the boundary of the subdomains is ill-conditioned, especially for fine meshes needed to capture the rapid variations of the solution. Our main contribution is to show with analysis and numerical simulations how to use an asymptotic approximation of the Dirichlet to Neumann (DtN) map of the sub-domain problems to obtain a preconditioner for an efficient domain decomposition algorithm.*

This study is concerned with developing an efficient numerical method for computing flow in high contrast media. We consider the so-called conductivity problem that models d.c. electrical flow or steady flow in porous media. The governing equation for the field $u(x)$ is of the form $\nabla\cdot[\sigma(x)\nabla u(x)]$ in some domain Ω, with some appropriate boundary conditions. The main challenge is that $\sigma(x)$ can take very large (almost infinite) values in Ω and in addition, it fluctuates at small scale. Think of a composite material with highly conducting inclusions packed closely in Ω. The field $u(x)$ undergoes rapid transitions in Ω and thus it is challenging to compute. The paper shows how to use asymptotic analysis in order to obtain an accurate approximation of the Dirichlet to Neumann map, that can then be used to devise an efficient domain decomposition approach to computing $u(x)$.

The following two papers are results of S. Meng, obtained while he was a postdoc at University of Michigan, supported on this grant:

- **C Lackner, S Meng, P Monk, Determination of electromagnetic Bloch variety in a medium with frequency-dependent coefficients, Journal of Computational and Applied Mathematics, vol. 358, 2019, pp. 359-373.**

Abstract *We provide a functional framework and a numerical algorithm to compute the Bloch variety for Maxwell's equations when the electric permittivity is frequency dependent. We incorporate the idea of a mixed formulation for Maxwell's equations to obtain a quadratic eigenvalue for the wave-vector in terms of the frequency. We reformulate this problem as a larger linear eigenvalue problem and prove that this results in the need to compute eigenvalues of a compact operator. Using finite elements, we provide preliminary numerical examples of the scheme for both frequency independent and frequency dependent permittivity.*

Wave propagation in periodic media has numerous applications in science, engineering, and technology. The dispersion relation or, more generally, the Bloch variety, represents the relationship between a possibly complex-valued wave vector and a possibly complex- valued frequency. The computation of electromagnetic Bloch variety for frequency-dependent dielectric photonic materials is motivated by novel applications in optical metamaterials and dispersive photonic crystals. This paper incorporates the idea of a mixed formulation for Maxwell's equations to obtain a quadratic eigenvalue problem for the wave vector as a function of frequency. Finite elements are used to obtain a numerical scheme for photonic materials with both frequency-independent and frequency-dependent permittivity. Numerical examples illustrate the performance of the algorithm and the convergence rate.

- **BB Guzina, S Meng, O Oudghiri-Idrissi, A rational framework for dynamic homogenization at finite wavelengths and frequencies, Proceedings of the Royal Society A, vol. 475, issue 2223, 2019, p. 20180547.**

Abstract *In this study, we establish an inclusive paradigm for the homogenization of scalar wave motion in periodic media (including the source term) at finite frequencies and wavenumbers spanning the first Brillouin zone. We take the eigenvalue problem for the unit cell of periodicity as a point of departure, and we consider the projection of germane Bloch wave function onto a suitable eigenfunction as descriptor of effective wave motion. For generality the finite wavenumber, finite frequency(FW-FF) homogenization is pursued in $\mathbb{R}^d$ via* *second-order asymptotic expansion about the apexes of "wavenumber quadrants" comprising the first Brillouin zone, at frequencies near given (acoustic or optical) dispersion branch. We also consider the junctures of dispersion branches and "dense" clusters thereof, where the asymptotic analysis reveals several distinct regimes driven by the parity and symmetries of the germane eigenfunction basis. In the case of junctures, one of these asymptotic regimes is shown to describe the so-called Dirac points, that are relevant to the phenomenon of topological insulation. On the other hand, the effective model for nearby solution branches is found to invariably entail a Dirac-like system of equations that describes the interacting dispersion surfaces as "blunted cones". For all cases considered, the effective description turns out to admit the same general framework, with differences largely being limited to (i) the eigenfunction basis, (ii) the reference cell of medium periodicity, and (iii) the wavenumber-frequency scaling law underpinning the asymptotic expansion. We illustrate the analytical developments by several examples, including the Green's function near the edge of a band gap and clusters of nearby dispersion surfaces.*

Recently, given the potential to illustrate dynamic properties and to retrieve microscopic information about periodic media, dynamic models that transcend the quasi-static limit have attracted much attention. Furthermore, recent advances in meta-materials have motivated the study on finite or high frequency homogenization. This paper integrates the Bloch-wave approach in Willis' dynamic homogenization, with the asymptotic expansion underpinning the multiple-scales description, to establish a dynamic (second-order) description of the scalar wave equation driven by a body force at finite frequencies and finite wave numbers. Numerical examples illustrate that the model is capable to approximate the dynamic wave field and to better approximate the dispersion relation.

3 Honors and other scientific activities:

- **Honors:**
  1. Editor in chief of the SIAM Multiscale Modeling and Simulations journal.
  2. Elected to the SIAM Board of Trustees for a 3 year term, starting January 1, 2021.
  3. Peter Field Collegiate Chair University of Michigan (this is the renewal of the chair, for another 5 years), 2020.
  4. SIAM Fellow, class of 2018.
  5. Distinguished Women in Mathematics lecture, University of Texas at Austin, February 5, 2018.
- **Editorial boards:**
  1. Editor in chief: SIAM Journal on Multiscale Modeling and Simulations
  2. Communications of the American Mathematical Society
  3. SIAM Journal on Uncertainty Quantification
  4. Journal of Computational Physics
  5. Inverse Problems
  6. Inverse Problems and Imaging
- **Scientific boards**

1. Elected to the SIAM Board of Trustees, for a 3 year term, starting January 1, 2021.
2. Scientific advisory board, ICERM Institute at Brown University, 2018-2021.
3. Scientific Advisory Board of the Johann Radon Institute for Computational and Applied Mathematics, Linz Austria, 5 year term starting 2018.

- **Plenary lectures and colloquia** (a few plenary talks were postponed for 2022 due to COVID)

  1. *Untangling the nonlinearity in inverse scattering using data-driven reduced order models*, Distinguished Women in Mathematics lecture, University of Texas, Austin, February 5, 2018.
  2. *Pulse reflection in a random waveguide with a turning point*, Analysis Seminar, Department of Mathematics at University of Texas, Austin, February 6, 2018.
  3. *Imaging a laser beam from the off-axis scattered intensity*, Inverse Problems in the ALPS II, Obergurgl University Center, Austria, March 21-23, 2018.
  4. *Transport of power in a random waveguide with a turning point*, Conference on Transport and Localization in random media: theory and applications, Columbia University, New York, May 1-3, 2018.
  5. *Transformation of active array data to its Born approximation via truncated ROM's*, Symposium on Waves, Model Reduction and Imaging, TU Delft, Netherlands, July 3, 2018.
  6. *Reduced order model for active array data processing in inverse scattering*, Conference on Mathematics of Wave Phenomena, Karlsruhe Institute of Technology, Germany, July 23-27, 2018.
  7. *Reduced order model for active array data processing in inverse scattering*, Department of Computational Mathematics Colloquium, Michigan State University, Lansing, September 10, 2018.
  8. *Untangling the nonlinearity in inverse scattering using data-driven reduced order models*, Colloquium at University College of London, UK, October 5, 2018.
  9. *Reduced order model for active array data processing in inverse scattering*, Clements Scientific Computing Seminar, Southern Methodist University, Dallas, November 29, 2018.
  10. *Reduced order model for active array data processing in inverse scattering*, Applied Mathematics Colloquium, University of Arizona, Tucson, November 30, 2018.
  11. *Reduced order model approach for inverse scattering*, Applied Mathematics Colloquium, Columbia University, March 5, 2019.
  12. *Reduced order model approach for inverse scattering*, Applied Mathematics Colloquium, Department of Statistics, University of Chicago, March 7, 2019.
  13. *Quantitative inverse scattering via reduced order modeling*, Hong Kong Inverse Problems, Imaging and PDEs conference, Institute of Advanced Studies, HKUST, Hong Kong, May 20-24, 2019.
  14. *Quantitative inverse scattering via reduced order modeling*, Women In Analysis conference, Banff International Research Station (BIRS), Canada, June 9 - 14, 2019.
  15. *Reduced order modeling approach to inversion for parabolic partial differential equations*, Ecole Polytechnique applied mathematics seminar, June 19, 2019.
  16. *Quantitative inverse scattering via reduced order modeling*, Oberwolfach Workshop Computational Multiscale Methods 28 July - 3 August 2019.

17. *Wave propagation and imaging in moving random media*, Inverse kinetic theory conference, University of Wisconsin Madison, Oct 25-27, 2019.
18. *Mathematical and Computational Aspects of Imaging with Waves*, Peter Field Collegiate Chair lecture, University of Michigan, February 6, 2020.
19. *Mathematical and computational aspects of wave imaging*, SCEE 2020 International Conference, plenary lecture, Eindhoven, Netherlands, Feb 16-20, 2020.
20. *Mathematical and computational aspects of wave imaging*, UC Irvine seminar, March 2, 2020.
21. *Mathematical and computational aspects of wave imaging*, Caltech seminar, March 3, 2020.
22. *Reduced order modeling for inverse problems*, International Zoom Inverse Problems seminar, August 20, 2020, https://www.youtube.com/watch?v=KZzD3XjIFU0

- **Organizer of conferences**

  1. Banff International Research Station in 2019, Canada Workshop on Women In Analysis - A Research Collaboration Conference for Women June 9-14, 2019. This runs a bunch of themes. With Fioralba Cakoni we organized the part on inverse problems.
  2. Oberwolfach Workshop in December 6-12, 2020 on Computational Inverse Problems for Partial Differential Equations was awarded. Workshop ID: 2050. Co-organizers are Thorsten Hohage, Götingen, Germany and Barbara Kaltenbacher, Klagenfurt, Austria.
  3. SIAM Annual Meeting, 2021. Organizing committee.