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Technical Report ARMET-TR-19016

ANALYTICALLY DERIVED FORM FUNCTIONS FOR ROLLED BALL PROPELLANT GRAIN GEOMETRY

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March 2021

U.S. ARMY COMBAT CAPABILITIES DEVELOPMENT COMMAND ARMAMENTS CENTER

Munitions Engineering Technology Center

Picatinny Arsenal, New Jersey

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-01-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (*From - To*)
March 2021	Final	

4. TITLE AND SUBTITLE	
Analytically Derived Form Functions for Rolled Ball Propellant Grain Geometry	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHORS	5d. PROJECT NUMBER
Christopher Houthuysen and Carlton Adam	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
U.S. Army DEVCOM AC, METC Energetics and Warheads Directorate (FCDD-ACM-EP) Picatinny Arsenal, NJ 07806-5000	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
U.S. Army DEVCOM AC, ESIC Knowledge & Process Management Office (FCDD-ACE-K) Picatinny Arsenal, NJ 07806-5000	11. SPONSOR/MONITOR'S REPORT NUMBER(S) Technical Report ARMET-TR-19016

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

This report documents the derivation of the form functions for a rolled ball geometry. The form functions were derived assuming that the rolled ball grains are uniformly ignited and burn in accordance to Piobert's law. The resulting expressions were validated with use of a practical example and the three dimensional (3D) modeling software, PTC Creo Parametric.

15. SUBJECT TERMS

Form functions   Piobert's law   Interior ballistics   Interior Ballistics of High Velocity Guns

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Christopher Houthuysen
U	U	U	SAR	17	19b. TELEPHONE NUMBER (Include area code) (973) 724-4551

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

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INTRODUCTION

Solid propellants are used in conventional ballistic systems to convert their stored chemical energy into kinetic energy of the projectile via combustion. Since the burning of solid propellants is a surface phenomenon, the rate at which gas is produced is highly dependent on the exposed surface area of the propellant grains per reference 1. In order to accurately model the gas evolution in a given weapon system, geometric form functions must be developed to compute the available volume and surface area of the solid propellant grains as functions of depth burned. For this paper, the general form functions for a rolled ball geometry are derived based on first principle mathematics.

DISCUSSION

Reference 2 discusses the physical geometry of the rolled ball grain shape; however, the resulting form functions are not provided and are not formally documented anywhere else to the authors' knowledge. In order to rectify this discrepancy, the governing geometry is reiterated in this paper, and the derivation is provided in the proceeding discussion. Essentially, the rolled ball geometry is obtained by flattening a sphere such that its cross-sectional area is that of a stadium. In this configuration, the diameter of the edge curve is the same length as the distance between the two flat sides. A simple diagram of this is shown in figure 1.

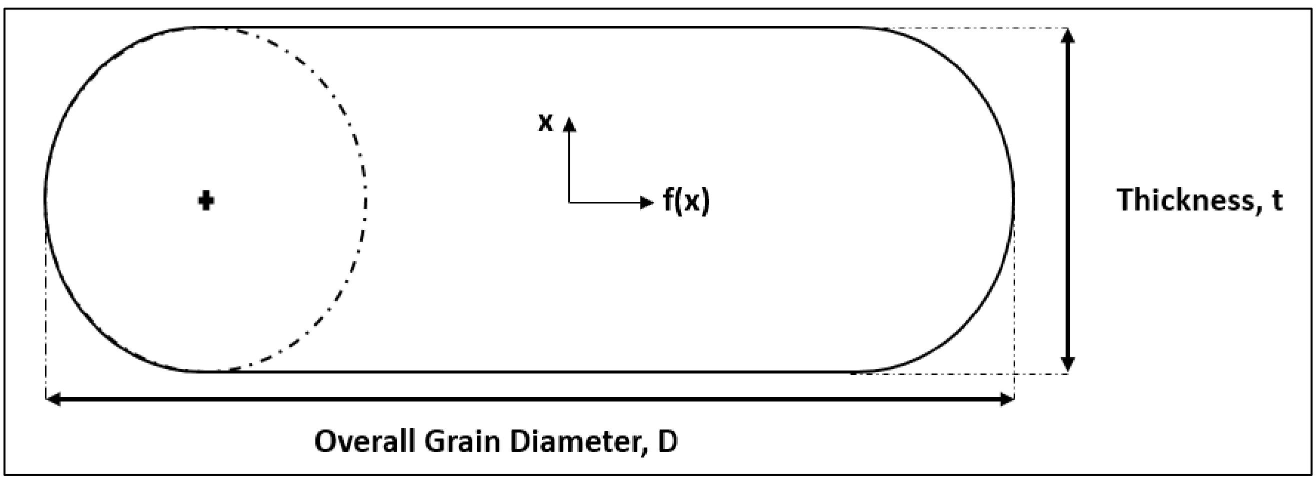

Figure 1
Cross-sectional representation of the rolled ball geometry

Based on figure 1, it can be seen that this geometry is symmetric on a quadrant basis with respect to a Cartesian plane fixed at the center of the cross section. In order to simplify the nature of this problem, a quarter symmetric model is used for the derivation and is shown in figure 2.

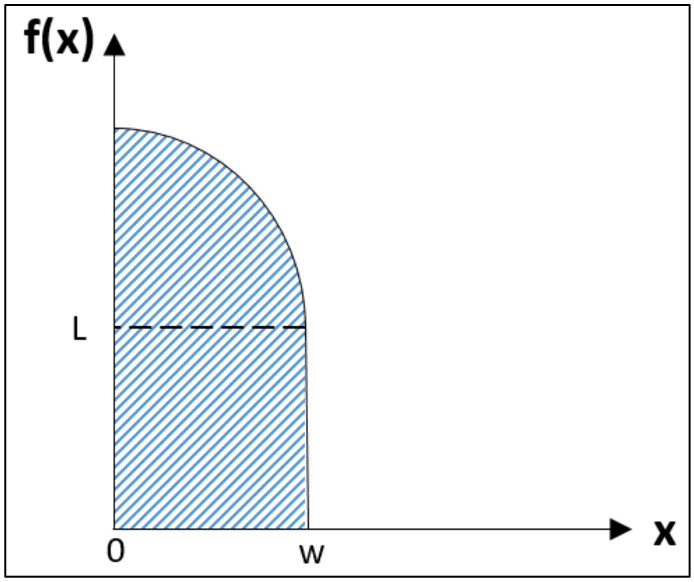

Figure 2
Quarter symmetric model of the rolled ball geometry

Upon inspection of figure 2, one will note that there is a “w” parameter displayed on the abscissa and an “L” parameter displayed on the ordinate of the axes shown. The significance of each parameter is discussed as follows: the “w” parameter represents simply half the thickness of the rolled ball geometry and the “L” parameter represents the length at which the flat side becomes tangent with the quarter circle. As an aside, note that the overall grain diameter shown in figure 1 is simply equivalent to the thickness of the grain plus twice the value of “L”. If the overall diameter and thickness of the grain are known, “L” can be easily determined; however, if the dimensions of the rolled ball geometry are known in terms of the original diameter of the spherical grain (prior to being flattened) and its corresponding reduction in web, “L” must be determined via more involved calculations. In either case, the resulting form functions are equivalent.

Computing the Initial Volume of the Rolled Ball Geometry

Based on the quarter symmetric model shown in figure 2, half of the rolled ball’s volume can be obtained by revolving its cross section 2π radians around the x-axis. This calculation can be performed via use of the disc method from integral calculus. The general equation for the disc method was obtained via reference 3 and is provided in equation 1:

$$\forall = \pi \int_a^b (f(x))^2 dx \tag{1}$$

From equation 1, $f(x)$ corresponds to the vertical position of a quarter circle of radius “w” whose center is offset by a vertical distance of “L”. The resulting lower and upper limits of integration are 0 and “w”, respectively. Again noting that the volume of the solid of revolution is only half of the rolled ball geometry, equation 1 is turned into equation 2:

$$\forall_{half} = \pi \int_0^w \left(L + \sqrt{w^2 - x^2}\right)^2 dx \tag{2}$$

After performing the integration and inserting the limits of integration, the aforementioned volume is determined. The full volume of the rolled ball geometry is then twice the previously mentioned quantity and is shown in equation 3. Note that the limit of the arctangent of a function as it approaches infinity is simply $\pi/2$. For simplicity, this term will be rewritten as such in later equations.

$$\forall = 2\pi\left(wL^2 + \frac{2w^3}{3} + Lw^2\tan^{-1}(\infty)\right) = 2\forall_{half} \tag{3}$$

If the dimensions of the grain are known only in terms of the original spherical geometry (prior to being flattened) and the value of "L" is not readily available, it can be determined by equating the volume of the rolled ball grain to that of the spherical grain. This is shown in equation 4, where "r" corresponds to the radius of the original spherical grain.

$$\frac{4\pi r^3}{3} = 2\pi wL^2 + \frac{4\pi w^3}{3} + \pi^2 Lw^2 \tag{4}$$

Upon further rearrangement, one will note that equation 4 takes the form of a quadratic equation. By using the quadratic formula, "L" is determined as shown in equation 5. Note that the negative root is not included since it is just an artifact of the mathematics and has no physical meaning for this geometry.

$$L = \frac{-\mathrm{B} + \sqrt{B^2 - 4AC}}{2A} \tag{5}$$

Where,

$$\mathrm{A} = 2\pi w$$
$$\mathrm{B} = \pi^2 w^2$$
$$C = \frac{4\pi}{3}(w^3 - r^3)$$

Computing the Initial Surface Area of the Rolled Ball Geometry

The total surface area of the rolled ball geometry comprised of two specific regions: one being the resulting area of the surface of revolution and the other being the surface area of the end caps (i.e., the circular tops of the flat sides of radius "L"). The general equation for the surface of revolution was obtained via reference 3 and is provided in equation 6:

$$S_{rev} = 2\pi\int_a^b \left(f(x)\sqrt{1 + (f'(x))^2}\right)dx \tag{6}$$

The same parameters defined previously in the volume computation are inserted into equation 6. The resulting surface of revolution of half of the rolled ball geometry is then obtained as shown in equation 7:

$$S_{rev,half} = 2\pi\int_0^w \left(\left(L + \sqrt{w^2 - x^2}\right)\left(\sqrt{1 + \left(\frac{-x}{\sqrt{w^2 - x^2}}\right)^2}\right)\right)dx \tag{7}$$

After performing the integration and inserting the limits of integration, the surface of revolution of half of the rolled ball geometry is determined. The full area of the surface of revolution is then twice the aforementioned quantity and is provided in equation 8.

$$S_{rev,full} = 4\pi\left(\frac{\pi Lw}{2} + w^2\right) = 2S_{rev,half} \tag{8}$$

As discussed previously, the surface area of the end cap regions must also be considered. Fortunately, this is simply the area of a circle of radius "L". By noting that there are two end caps, the total surface area of the rolled ball geometry is obtained by adding all of the computed surface areas together. The result is provided in equation 9.

$$A_s = 2\pi^2 Lw + 4\pi w^2 + 2\pi L^2 \tag{9}$$

Computing the Volume and Surface Area as Functions of Depth Burned

As of yet, the governing equations for the initial surface area and volume of the rolled ball geometry have been determined. However, these equations must be transformed into functions of depth burned. Prior to developing these expressions, it is first important to note that the following derivations assume that the grain is uniformly ignited and burns according to Piobert's law (i.e., the burning surface of the grain regresses layer-by-layer in the direction normal to the solid surface). The principle of Piobert's law is shown in figure 3 for this geometry, where the red lines indicate the inward normal vectors to the surface.

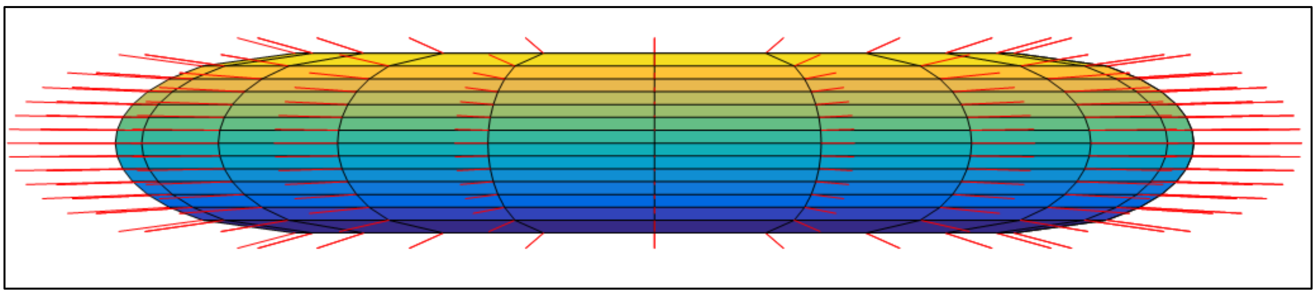

Figure 3
Visualization of Piobert's law for the rolled ball geometry

Based on the geometry of the rolled ball grain, the aforementioned equations can be easily modified to account for dimensional regression by replacing "w" with the difference between "w" and the burn depth, "u". The resulting form functions for volume and surface area as functions of depth burned are shown in equations 10 and 11, respectively. Note that the volume and surface area of this grain are zero when web burnout occurs. This happens when the depth burned is equivalent to half the thickness of the rolled ball (i.e., the "w" parameter).

$$\forall(u) = \begin{cases} 2\pi(w-u)L^2 + \frac{4\pi(w-u)^3}{3} + \pi^2 L(w-u)^2, & u < w \\ 0, & u \geq w \end{cases} \tag{10}$$

$$A_s(u) = \begin{cases} 2\pi^2 L(w-u) + 4\pi(w-u)^2 + 2\pi L^2, & u < w \\ 0, & u \geq w \end{cases} \tag{11}$$

With the form functions developed previously, their use is validated per the practical example provided in the following section.

Validation of the Model with a Practical Example

Problem Statement

Consider a spherical grain with a diameter of 0.5 mm, the sphere is then flattened into a rolled ball geometry with a 35% reduction in the web. By modeling the rolled ball geometry with the quarter symmetry approach defined previously, determine (1) the position at which the flat side is tangent with the circular arc (i.e., the “L” parameter), (2) the initial volume and surface area of the resulting geometry, and (3) the volume and surface area profiles of the grain until web burnout.

Part One

In order to solve this problem, the dimensions of the rolled ball grain must be determined. By noting that the web of the original spherical grain is reduced by 35%, this means that the overall thickness of the grain, “t” as defined in figure 1, is 0.325 mm [i.e., $t = 0.5mm \cdot (1 - 0.35)$]. The thickness of the quarter symmetric section is then half this thickness and is computed to be 0.1625 mm. Now that all of the necessary dimensions have been defined, equation 5 can be used to determine “L,” as shown in equation 12.

$$L = \frac{-\mathrm{B}+\sqrt{B^2-4AC}}{2A} = 0.1229\ mm \quad (12)$$

Part Two

With the “L” parameter determined, the initial volume and surface area of the rolled ball grain can be computed. First, the volume of the grain is computed via equation 3 as shown in equation 13:

$$\forall = 2\pi w L^2 + \frac{4\pi w^3}{3} + \pi^2 L w^2 = 0.0654\ mm^3 \quad (13)$$

As a cross check, this volume is validated against the original volume of the sphere (i.e., $\forall_{sphere} = \frac{4\pi(0.25mm)^3}{3}$). Since the volumes are in agreement, the calculation of “L” is correct. The initial surface area of the rolled ball geometry is then computed per equation 9 as in equation 14:

$$A_s = 2\pi^2 L w + 4\pi w^2 + 2\pi L^2 = 0.8212\ mm^2 \quad (14)$$

In order to validate the results, the initial volume and surface area of this geometry were then compared against the values produced by the three-dimensional (3D) modeling software, PTC Creo Parametric, as shown in figure 4.

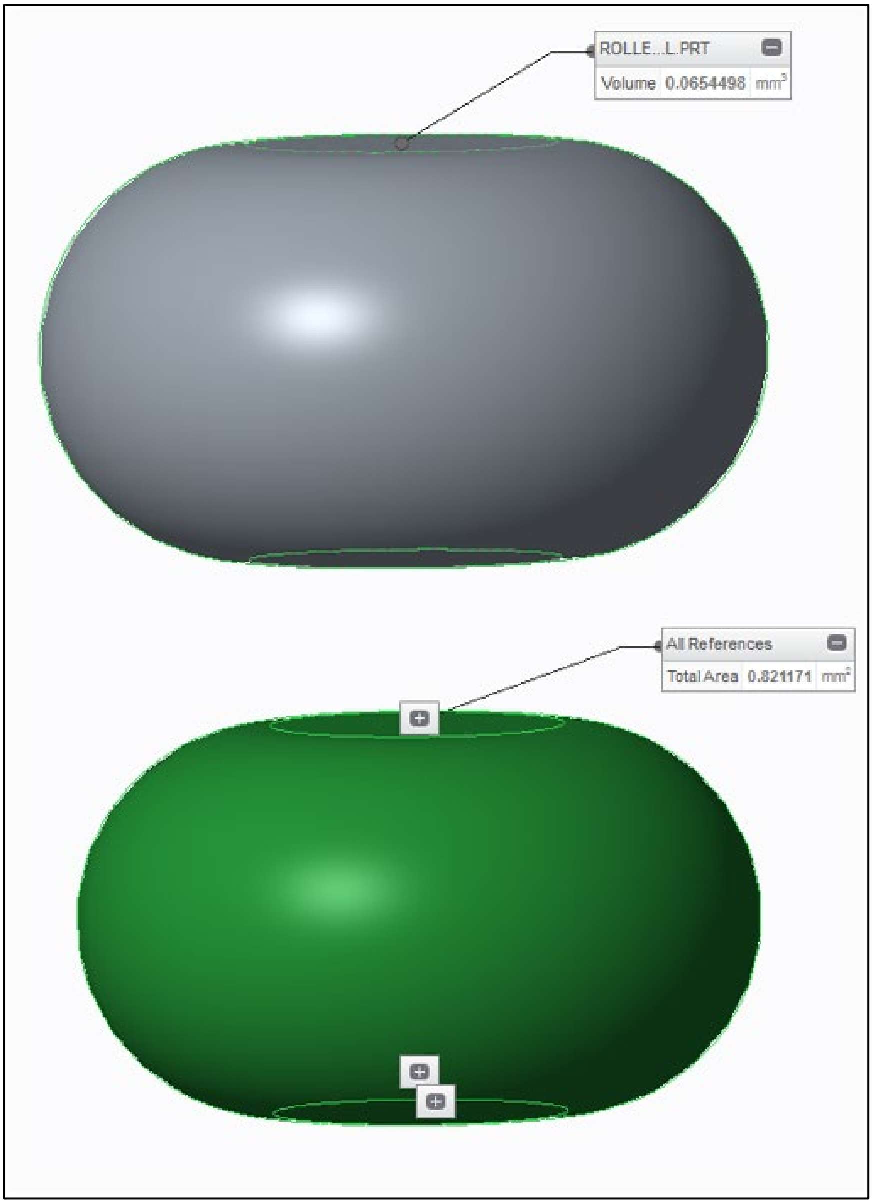

Figure 4
Initial geometry validation with Creo

Part Three

With use of equations 10 and 11, the profiles for the volume and surface area of the grain during deflagration are obtained as shown in figure 5. To validate the results, sensitivity studies were carried out with Creo to verify that the form functions are correctly computing the volume and surface area of the grain as it regresses. These sensitivity studies were conducted at explicit burn depths of 0.06, 0.12, and 0.16 mm. Per the previous equations, the volumes and surface areas at the aforementioned burn depths were computed to be 0.02699 mm^3 and 0.4578 mm^2, 0.0065 mm^3 and 0.2208 mm^2, and 2.4509e-04 mm^3 and 0.1011 mm^2, respectively. As shown in figures 5 through 8, the results produced by Creo are in perfect agreement with the analytical form functions.

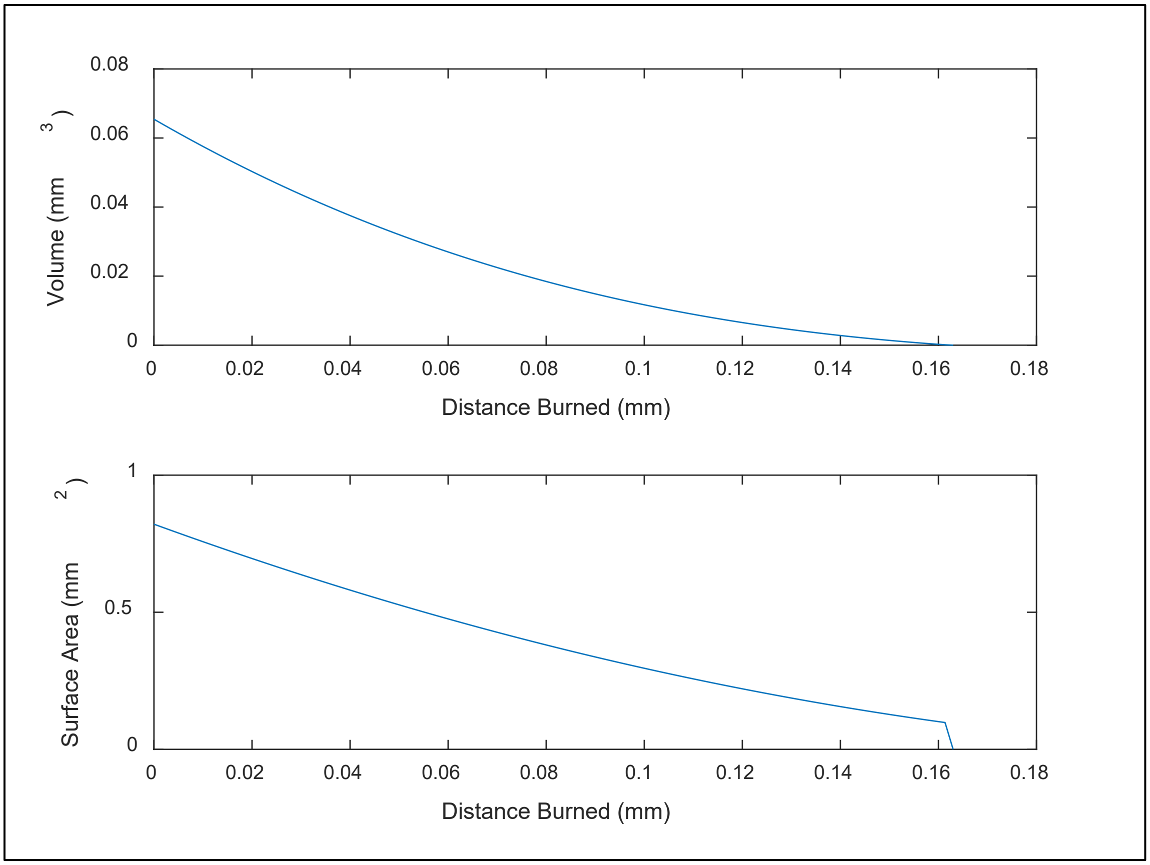

Figure 5
Volume and surface area plots as functions of depth burned for validation case

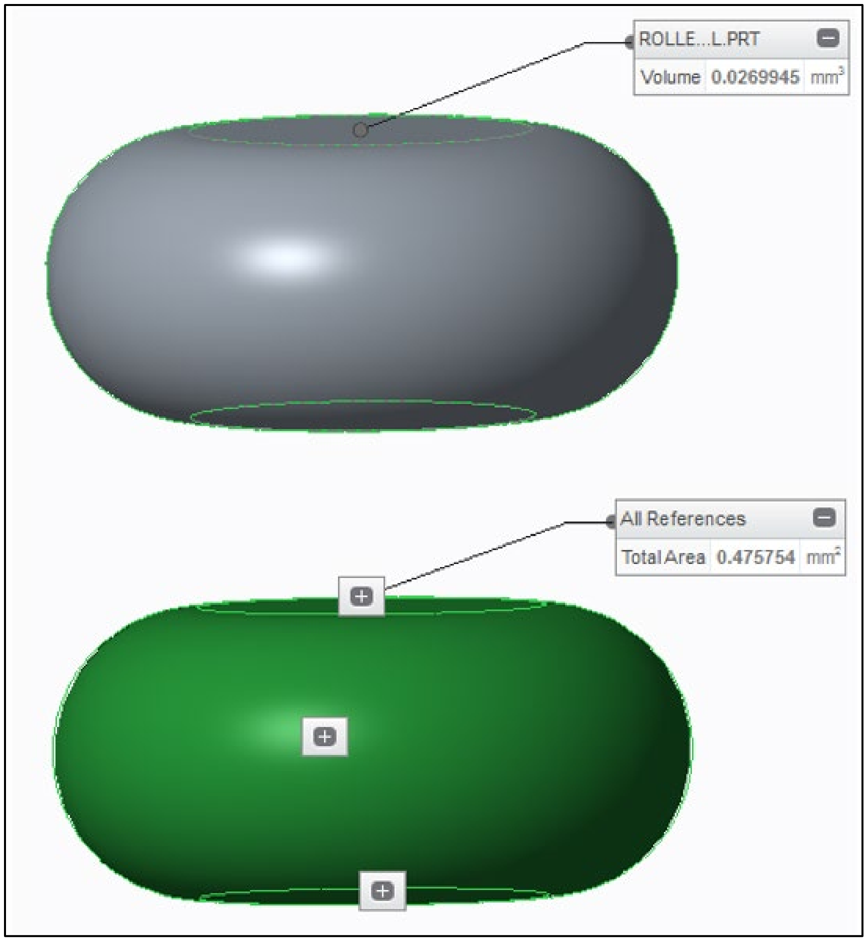

Figure 6
Volume and surface area of validation case at a burn depth of 0.06 mm

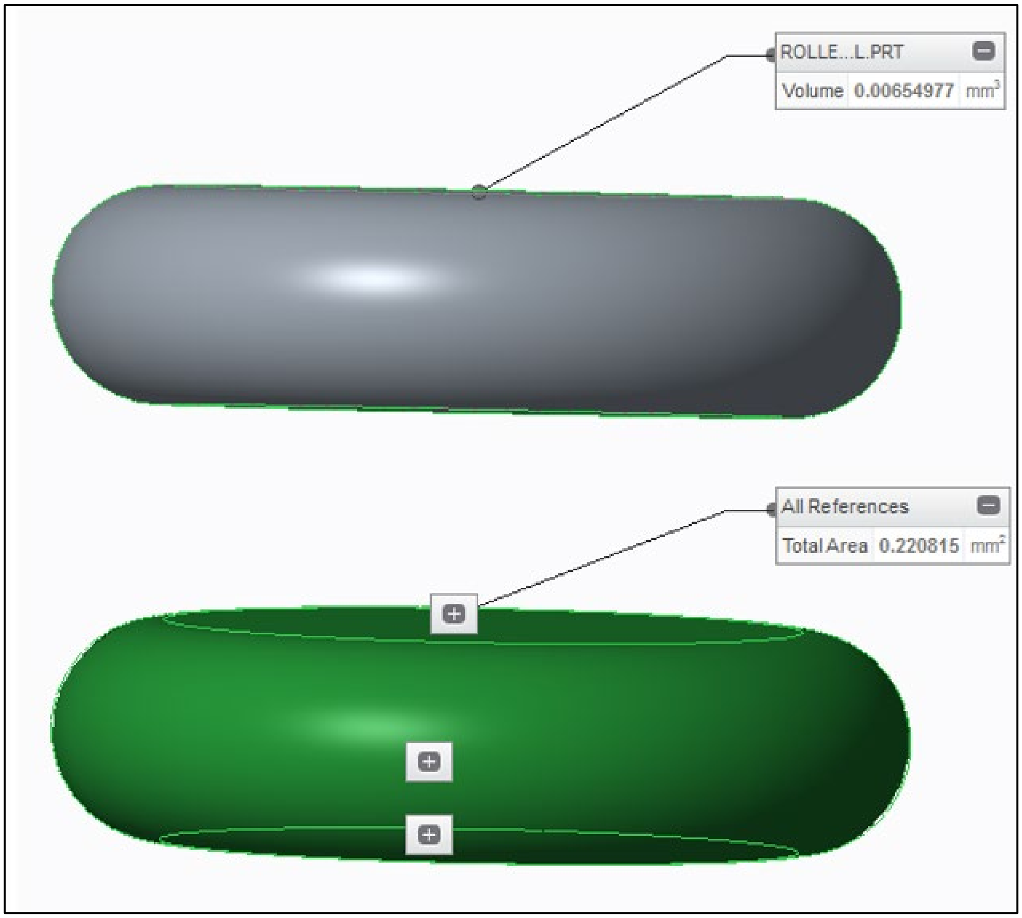

Figure 7
Volume and surface area of validation case at a burn depth of 0.12 mm

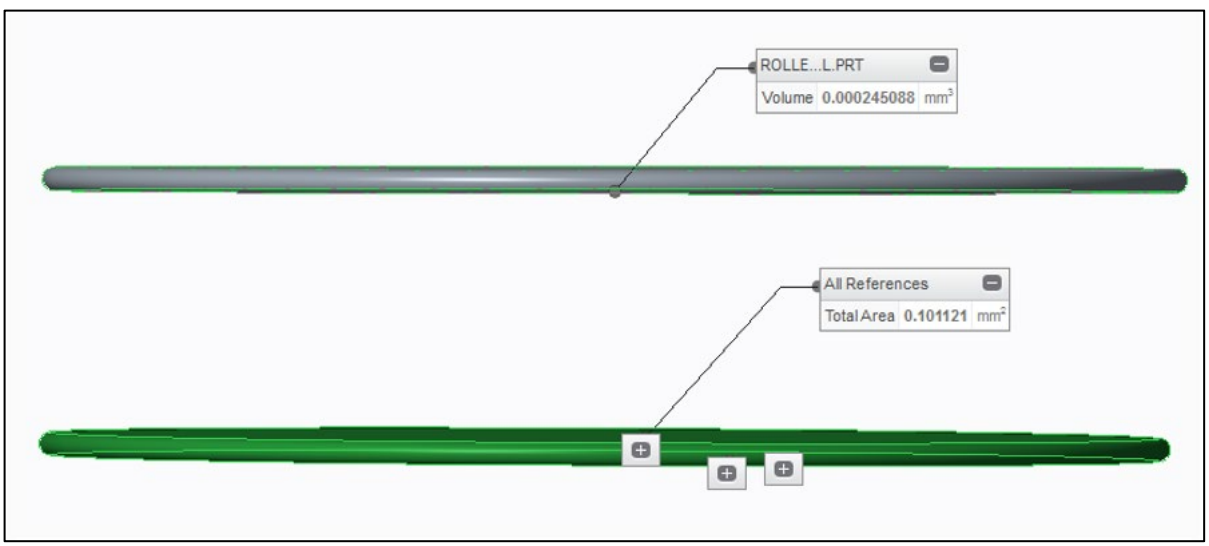

Figure 8
Volume and surface area of validation case at a burn depth of 0.16 mm

Furthermore, this practical example illustrates that the analytically derived form functions accurately compute the volume and surface area of the rolled ball grain geometry during deflagration.

CONCLUSIONS

Expressions for the volume and surface area of a rolled ball geometry have been developed with use of integral calculus. These expressions have been modified accordingly to account for dynamic grain regression. The enclosed results demonstrate that the analytically derived form functions correctly compute the effective volume and surface area of the grain as it burns uniformly and in accordance with Piobert's law. Furthermore, these expressions can be used to predict the rate of gas evolution in interior ballistic models.

REFERENCES

1 Carlucci, D. E. and Jacobsen, S. S., “Ballistics: Theory and Design of Guns and Ammunition,” CRC Press, Boca Raton, FL, August 2013.

2 Oberle, W. F., “Vivacity Calculation, Rolled Ball, and Rosette Grain Form Function and Editing Capability Update to the Closed-Chamber Data Analysis Program BRLCB,” Technical Report ARL-MR-61, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, April 1993.

3 Sharma, A. K., “Application of Integral Calculus,” Discovery Publishing Pvt.Ltd, New Delhi, India, January 2012.

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