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14. ABSTRACT
Krakovsk•a et. al. suggest that convergent cross mapping (CCM) performance was among the `least successful' of the methods tested to detect causality due to a preponderance of false-positive results when detecting unidirectional causality. This conclusion stems from the method's detection of bidirectional causation when the description of the known state space dynamics is defined with a unidirectional link between coupled subsystems. The authors admit that this is not contrary to the theory of CCM in that it only claims to produce high cross mapping skill when the link exists, not low skill when no link exists. Though this point is true, this observation misses a critical note on the impact of generalized synchronization in the theory of CCM which motivated the development of extended CCM in subsequent work. Furthermore, because the issue of misidentified bidirectionality has been cited as a concern for the application of CCM in several recent publications and is used to justify caution against the use of CCM on real-world data, the issue warrants further discussion.

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Comment on "Comparison of six methods for the detection of causality in a bivariate time series"*

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Abstract

Krakovská *et. al.* suggest that convergent cross mapping (CCM) performance was among the 'least successful' of the methods tested to detect causality due to a preponderance of false-positive results when detecting unidirectional causality. This conclusion stems from the method's detection of bidirectional causation when the description of the known state space dynamics is defined with a unidirectional link between coupled subsystems. The authors admit that this is not contrary to the theory of CCM in that it only claims to produce high cross mapping skill when the link exists, not low skill when no link exists. Though this point is true, this observation misses a critical note on the impact of *generalized synchronization* in the theory of CCM which motivated the development of extended CCM in subsequent work. Furthermore, because the issue of misidentified bidirectionality has been cited as a concern for the application of CCM in several recent publications and is used to justify caution against the use of CCM on real-world data, the issue warrants further discussion.

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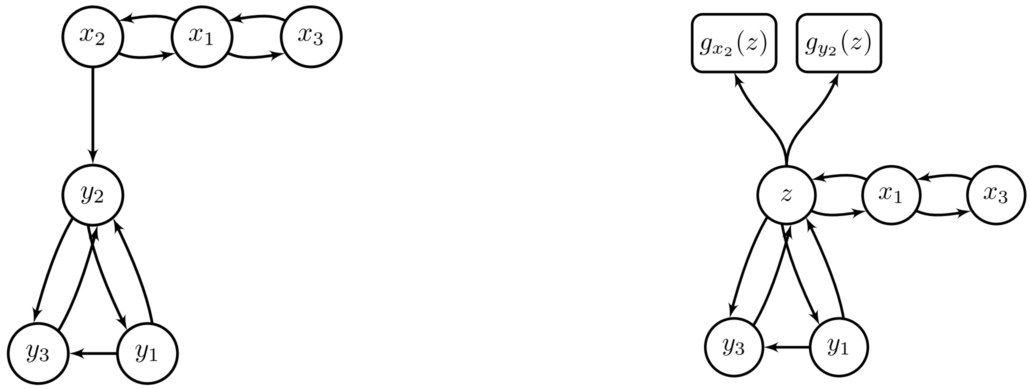
In Reference [1], Krakovská *et. al.* suggest that the convergent cross mapping (CCM) method fails to properly identify the direction of causality in unidirectionally coupled systems due to the appearance of 'false-positives' that imply bidirectionality in the causation. This assessment is based on the known unidirectional form of coupling of subsystems in the differential equations studied. However, the case of high bidirectional cross mapping performance is not a failure for the method to distinguish the direction of causality in as much as it implies that *general synchronization* (GS) as defined in Reference [2] has occurred in more than a colloquial sense. It is further worth noting that the motivation for the development of the extended convergent cross mapping method of Reference [3] was in part to handle the impact of synchrony in the system and that, though the cited Reference [4], cautions against the use of CCM on real-world data, this reference applied the full extended CCM and came to that conclusion for a much different reason. In that Reference, the issue was not so much the emergence of synchrony as it was a caution against applying CCM in essentially observational contexts where the assumption of time-invariant dynamics is not justifiable such as in ecology and epidemiology. This is certainly not the issue for the well defined ordinary differential equation systems studied in Reference [1].

The theorem defining *generalized synchronization* from Reference [2] is highly applicable in the context of CCM. Adopting the notation of this reference, a unidirectionally coupled system defined in terms of states $\mathbf{x}$ and $\mathbf{y}$ denote the *drive* and *response* subsystems described as shown in Equation 1.

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) \\ \dot{\mathbf{y}} &= \mathbf{g}(\mathbf{y}, \mathbf{h}(\mathbf{x})) \end{aligned} \tag{1}$$

Here, the drive system is an autonomous, time-invariant, differential equation and the response is coupled to the drive through some, potentially nonlinear, transformation, $\mathbf{h}(\mathbf{x})$. This system possess the property of general synchronization so long as there exists a transformation, $\mathbf{H}$, and manifold M such that $\mathbf{y} = \mathbf{H}(\mathbf{x})$ for all initial conditions within the basin of attraction of M. A key consequence of the theorem is that GS implies that the behavior of $\mathbf{y}$ can be predicted based on $\mathbf{x}$ and map $\mathbf{H}$ only. Furthermore, if $\mathbf{H}$ is invertible, $\mathbf{x}$ can be predicted from $\mathbf{y}$.

The high skill convergent bidirectional mappings constructed in CCM from observation



(a) Original Rössler→Lorenz System (b) Synchronized Rössler→Lorenz System

FIG. 1: Collapse of interaction graph for the Rössler system driving the Lorenz system due to synchronization. Circles denote independent system states while boxes denote auxiliary nonlinear observables.

$g_A(t)$ to observation $g_B(t)$ and vice versa are exactly such a maps, $\mathbf{H_{ccm}}$ and $\mathbf{H_{ccm}^{-1}}$, where the time delay vector coordinates, $\mathbf{g_A}$ and $\mathbf{g_B}$, of each observation's time series play the role of the unknown state variables in GS through Takens' diffeomorphism. In fact, whenever the CCM correlations converge to unity in either direction, it implies that observations g_A and g_B no longer depend on independent states of the system. This is a direct result of the Definition 1 of "state" provided in Reference [5].

Definition 1 *The state of a dynamic system is the smallest set of variables (called state variables) such that knowledge of these variables at $t = t_0$ together with input for $t \geq t_0$, completely determine the behavior of the system for any time $t \geq t_0$.*

For example, a perfectly reconstructed, $g_B(t)$, is simply an additional nonlinear observation of the time-delayed state, $\mathbf{g}_A$, through the converged map, $g_B = \mathbf{H_{CCM}}(\mathbf{g}_A)$, if the map $\mathbf{H_{ccm}}$ converges as $t \to \infty$. As long as the system remains invariant, observation g_B contains no additional information beyond what is already contained in g_A and map $\mathbf{H_{ccm}}$.

When both directions have converged, this mapping is invertible. This means that the once the dynamics have collapsed onto the lower dimensional invariant manifold of the attractor, the number of independent internal states has actually decreased. This implies that the interaction graphs for the coupled Lorez-Rössler system discussed in Reference [1] has effectively collapsed as shown in Figure 1.

This is what has occurred beyond a coupling factor, C, of approximately 2.8 as depicted in Figure 14 of that work. The result does not mean that CCM has failed to identify the correct direction of causality because the system has attained GS and the distinguishable states, x_2 and y_2, have become two different observations of the same synchronized state, z. In fact, the coupling of the Lorenz and Rössler systems is exactly the same system pair considered as a models for GS in Reference [2].

It is also worth noting that the set of variables $\{x_1, x_3, y_1, y_3, z\}$ may not actually represent the minimal "state" of the coupled, synchronized system, as the dimension of the attractor may still be less than five. This would imply additional nonlinear redundancy in the set, but further exploration of this topic in the context of the multi-view embedding of Reference [6] that inspired the spatiotemporal data fusion efforts of Reference [7] is beyond the scope of this comment.

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