Quantum Illumination Radar Feasibility

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Executive Summary

Primary Result

Quantum radar does not have the potential for long range standoff sensing (>10 km) at radio frequencies (<100 GHz)

Key Findings

- System requirements needed to realize quantum advantage limit the operating range
  - Low signal and strong background required for a quantum advantage
- Will take one second of integration to probe a single range/elevation/azimuth bin at 1 km using W band (92.5 GHz center frequency), much longer for lower frequency bands
- Quantum radar with 100 km range is not feasible
  - RF band operation requires 3 year integration time
  - Optical C-Band sensing possible (100 ms integration), but requires an unnaturally high background to achieve advantage over similar classical system

Contents

Quantum Radar Definition

- **Quantum Illumination (QI) Radar is a type of radar in which**
  1. **Two quantum-entangled beams (jointly Gaussian-states*) are generated at the radar**
  2. **One beam (the “signal”) is sent to the target, reflects, and partially returns to the radar**
  3. **The second beam (the “idler”) is kept at the radar**
  4. **A joint measurement is made using the returning signal and idler beams**

- **Quantum Illumination theory gives a theoretical bounds for the performance of *any* system that meets the above criteria**
  - **This includes all quantum radar systems feasible in the near future**

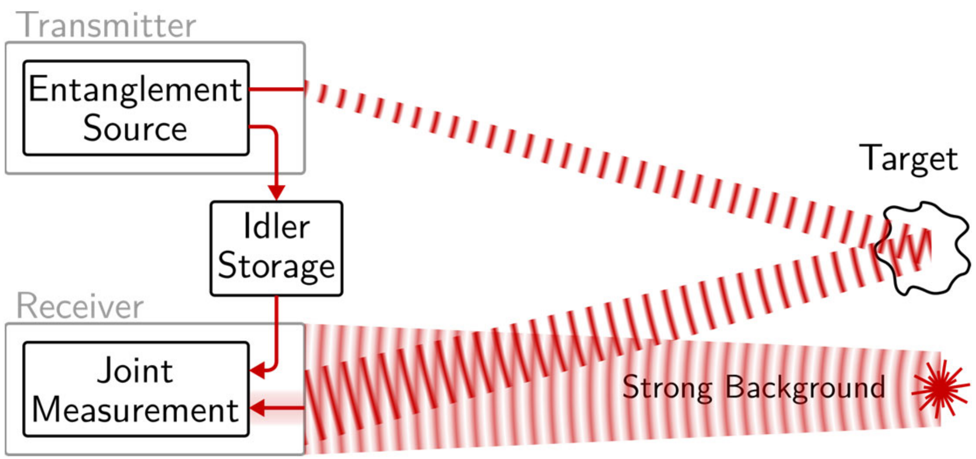
- **For clarity, we will refer to a Quantum Illumination Radar as a Quantum Radar (QR) throughout this presentation**

*Jointly Gaussian states encompass thermal radiation, lasers, classical radar emissions, and the fields created by spontaneous parametric downconversion used in entanglement experiments

Quantum Radar Advantage

Quantum Radar has potential 6 dB sensitivity advantage

Quantum Radar Framework



Increased sensitivity useful for LPI/LPD sensing, or when transmitter is energy-constrained

- **Range improved by:**
  - **Larger source flux, integration time, bandwidth, target cross section**
  - **Smaller background flux, wavelength**

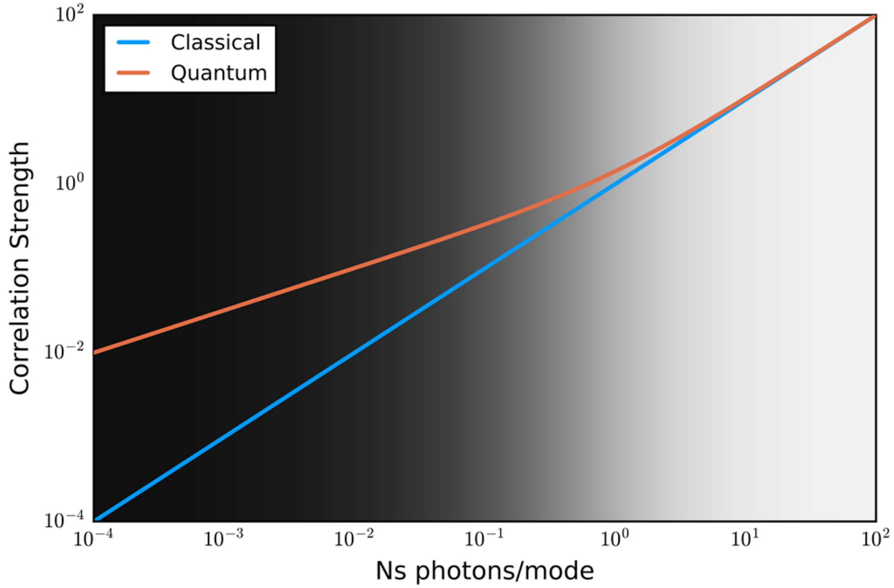
Classical and Quantum Radar performs better with stronger signal, weaker background

But…

Quantum Radar only outperforms classical radar under weak signal, strong background conditions

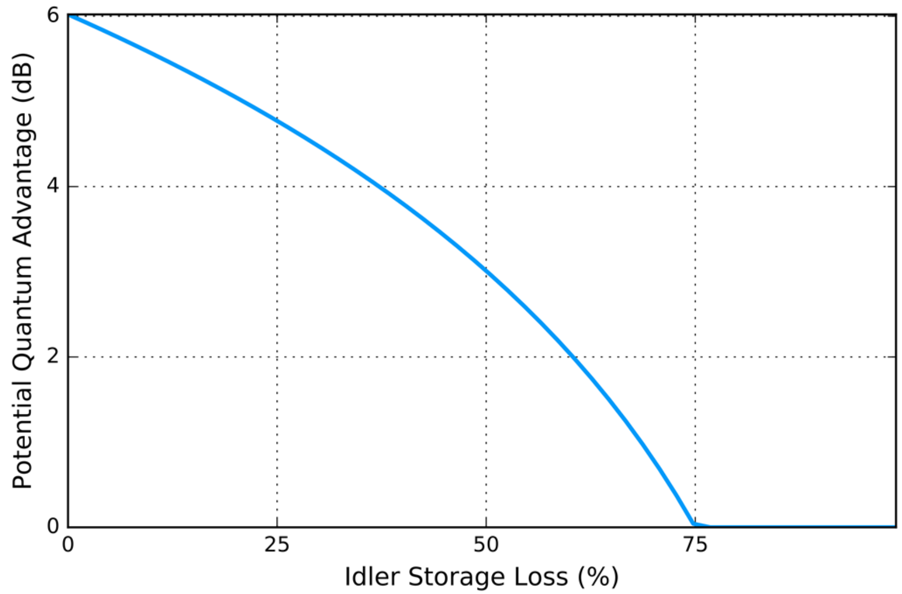
Quantum Parameter Consideration

(1) Signal strength (N_S) cannot grow without bound – quantum correlation advantage only for low-flux



(2) QR useful only when background much stronger than the signal, $N_B >> N_S$

(3) Quantum advantage only for low-loss idler storage



(4) QR can only measure one range bin at a time, so integration times are important

QR advantage only for low flux (N_S <<1), high background ($N_B >>N_S$), and low-loss idler storage

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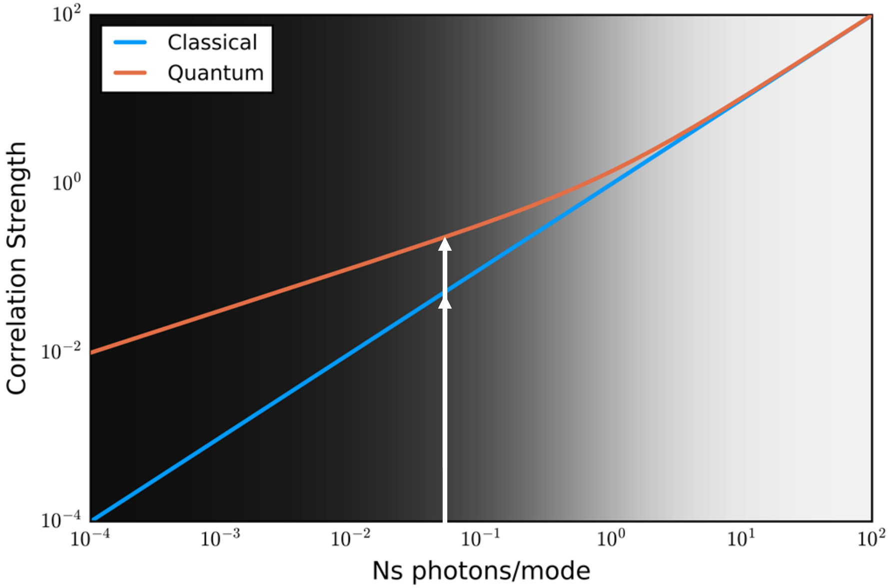
Quantum Parameter Consideration

(1) Signal strength (N_S) cannot grow without bound – quantum correlation advantage only for low-flux



(2) QR useful only when background much stronger than the signal, $N_B >> N_S$

(3) Quantum advantage only for low-loss idler storage

(1) In expanded study assume N_S=0.05 photons/mode

- Signal strength balances high flux with maintaining quantum advantage
  - Correlation is 95% of quantum asymptote
  - Higher signal quickly loses quantum advantage (for N_S=0.5, correlation is 66% of quantum asymptote)

(2) In expanded study maintain N_B>1 photon/mode

- Preserves N_B>>N_S for chosen N_S value

Quantum Parameter Consideration

(1) Signal strength (N_S) cannot grow without bound – quantum correlation advantage only for low flux

(3) In expanded study assume loss-less idler storage

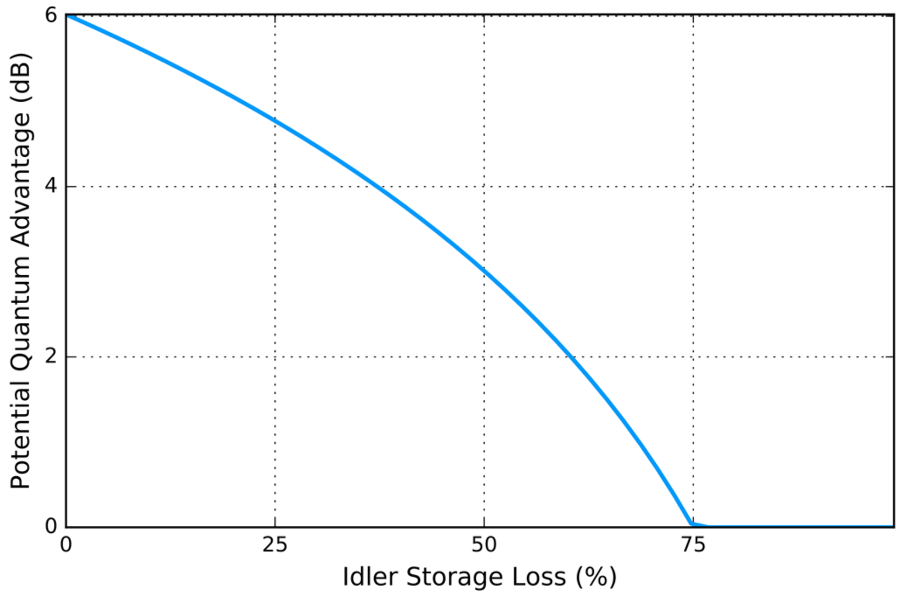
- Achieves maximum performance
- No current technology enables loss-less idler beam storage
- Optical fiber spool is most practical storage system

(4) In expanded study consider performance for sensing a single range/ elevation/azimuth/polarization bin

- To scan across multiple depths and rotations, a separate measurement is needed for each bin

(2) ... ger than the signal, $N_B >> N_S$

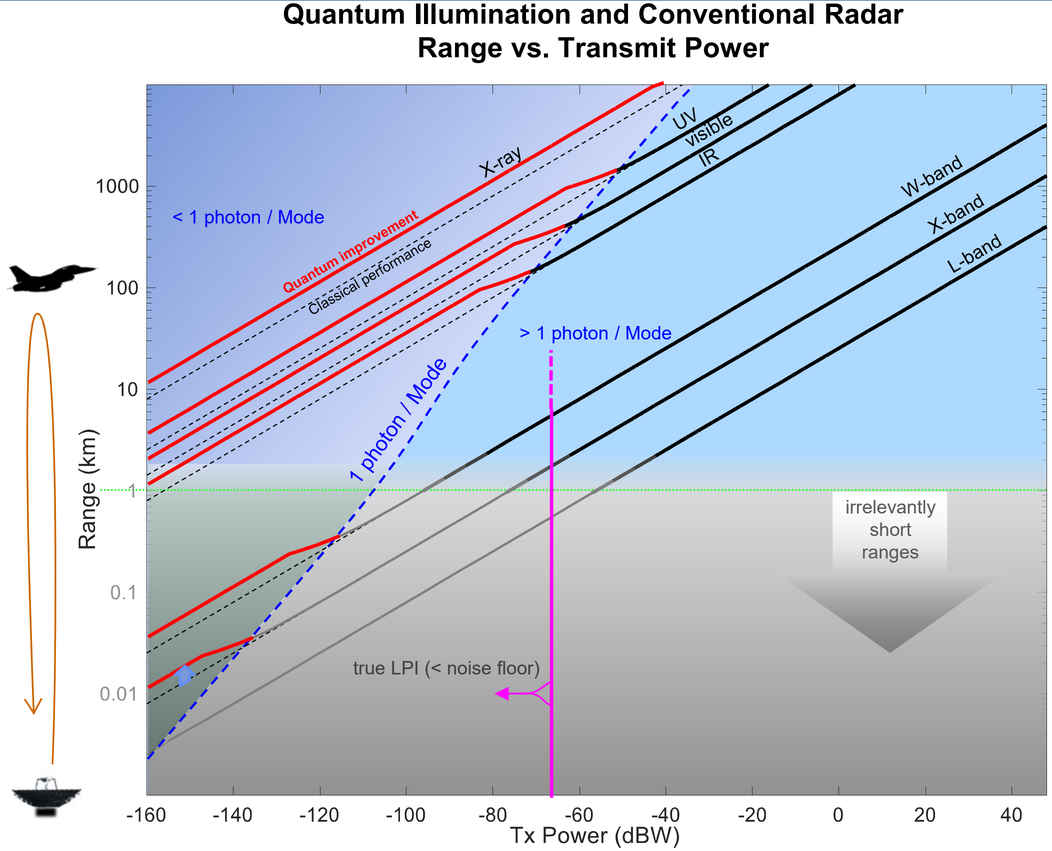
(3) Quantum advantage only for low-loss idler storage



(4) QR can only measure one range bin at a time, so integration times are important

Baseline Quantum / Classical Radar Comparison

- Radar compared in the L, X, and W radio bands, and optical C band (telecom)
- First-order analysis considered achievable range for following parameters:
  - 10^{-7} Error Probability
  - 1ms Pulse
  - Single Pulse
  - 1 m Aperture
  - 0 dBsm Cross Section
  - 40% BW at RF
  - 1% BW at optical
  - No losses



In baseline system range enhancement achieved for RF bands only for impractically short ranges, some utility at optical bands

Analysis and Parameter Expansion

Goal: Expand quantum radar baseline assumptions to realize enhanced range at RF bands

Analysis and Path Forward

- Time-bandwidth product (number of detected modes) and beam divergence are intrinsically limiting factors
  - Severe impact at radar frequencies, less impactful for optical
- Baseline analysis focused on a particular system, extend analysis to find feasible operating points
  - Address parameter assumptions
  - Look at alternate metrics
  - Consider alternate frameworks
  - Set source strength to N_S=0.05
    - Largest value that safely maintains quantum advantage

Baseline Parameter Expansion

1. Background noise temperature of 290 K
   - *Use more appropriate temperatures for inclination angle and wavelength*
2. Target cross-section of 0 dBsm (1 m^2)
   - *Identify relevant target sizes*
3. Fixed time-bandwidth product
   - *Look at longer integration times*
4. Error probability of 10^{-7}
   - *Explore error probability range*
5. Metric was range for fixed error probability
   - *Use receiver operating characteristic (ROC)*

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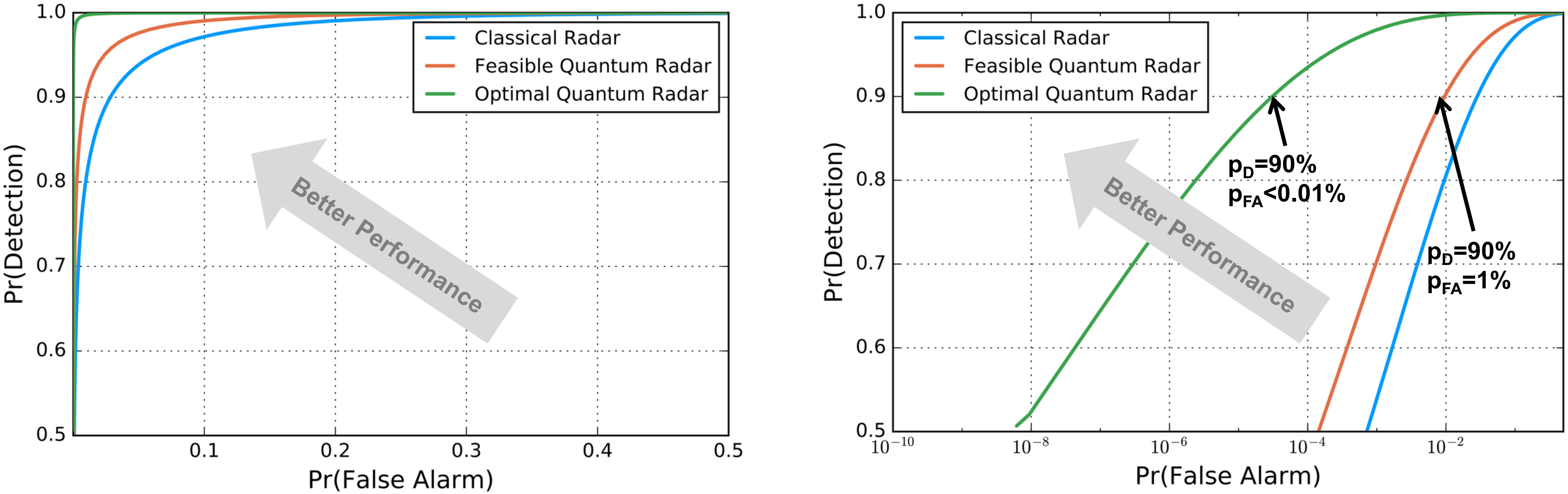
ROC Curves at 1 km

ROC for W-band, 1 km standoff, 1 second integration time

Other parameters same as used in slide titled "4. Error Probability"

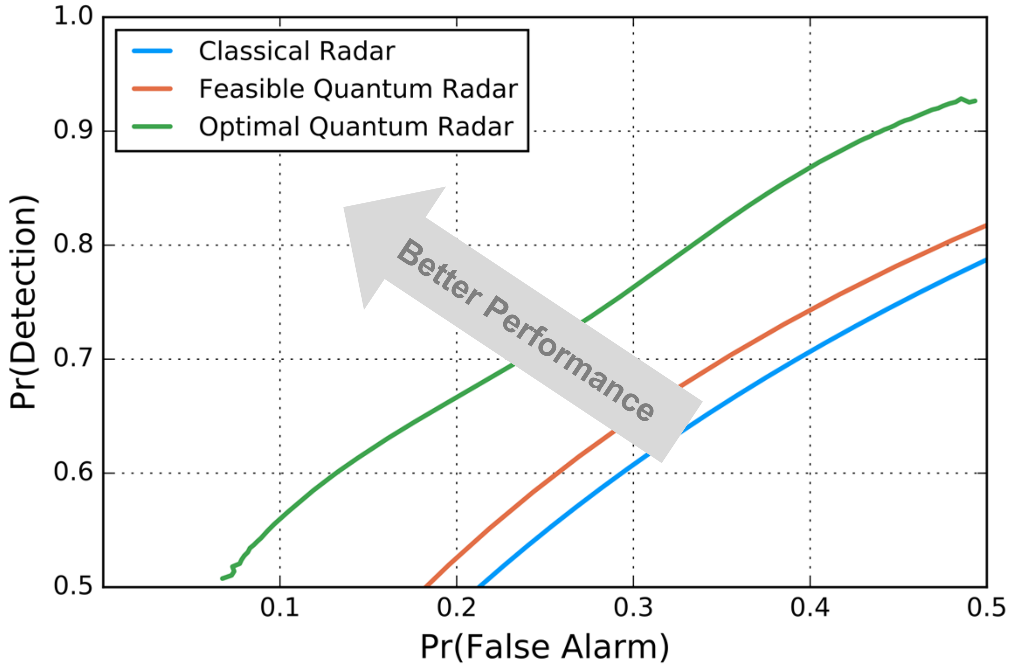


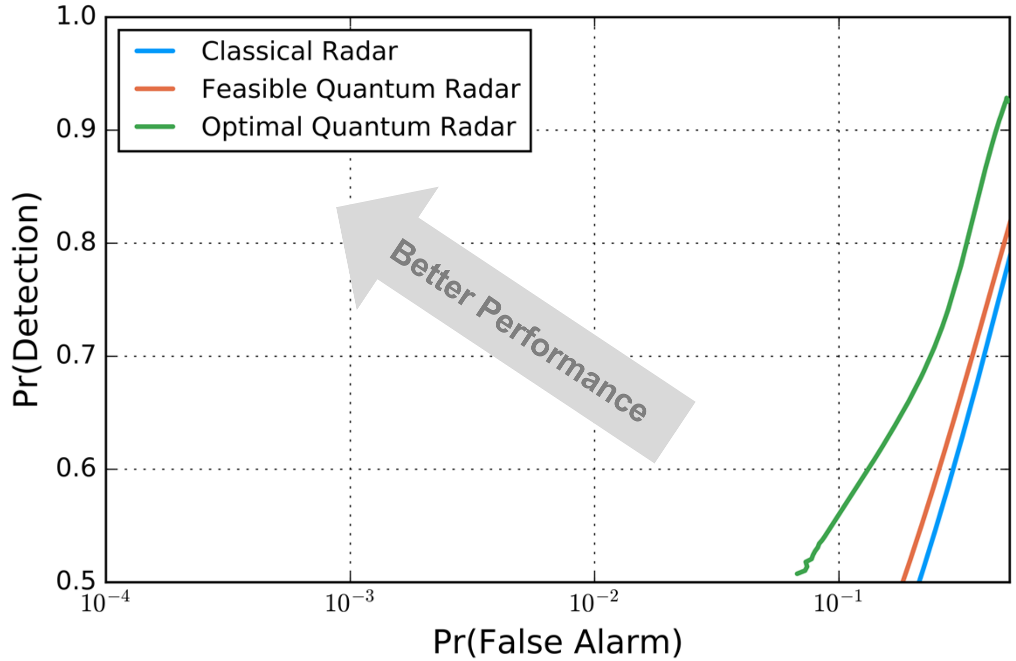
Optimal quantum radar curve has better scaling with p_{FA}, can achieve low p_{FA} with modest p_D

***Optimal QR curve only valid in "good" performance region where p_{FA} < 0.5 and p_D > 0.5**

ROC Curves at 2 km

ROC for W-band, 2 km standoff, 1 second integration time
Other parameters same as used in slide titled "4. Error Probability"





No useful operating points at 2 km (and beyond), .e.g., cannot have usefully low false alarm (p_{FA}<0.1) and high detection (p_D>0.9)

Quantum Radar at 100 km?

What is needed to make a quantum radar operate at 100 km standoff distance?

1. **Increase integration time per range-bin**
   - **Target and environment must stay constant during integration**
   - **Range (L) depends on integration time (T) as**

$$L \propto T^{1/4}$$

2. **Move to a higher frequency (e.g., optical C-band)**
   - **Higher bandwidth means more energy for same photons/mode (i.e., more transmit power)**
   - **Shorter wavelength means less geometric propagation loss (i.e., narrower beam)**
   - **Range (L) depends on bandwidth (W) and wavelength (λ) as**

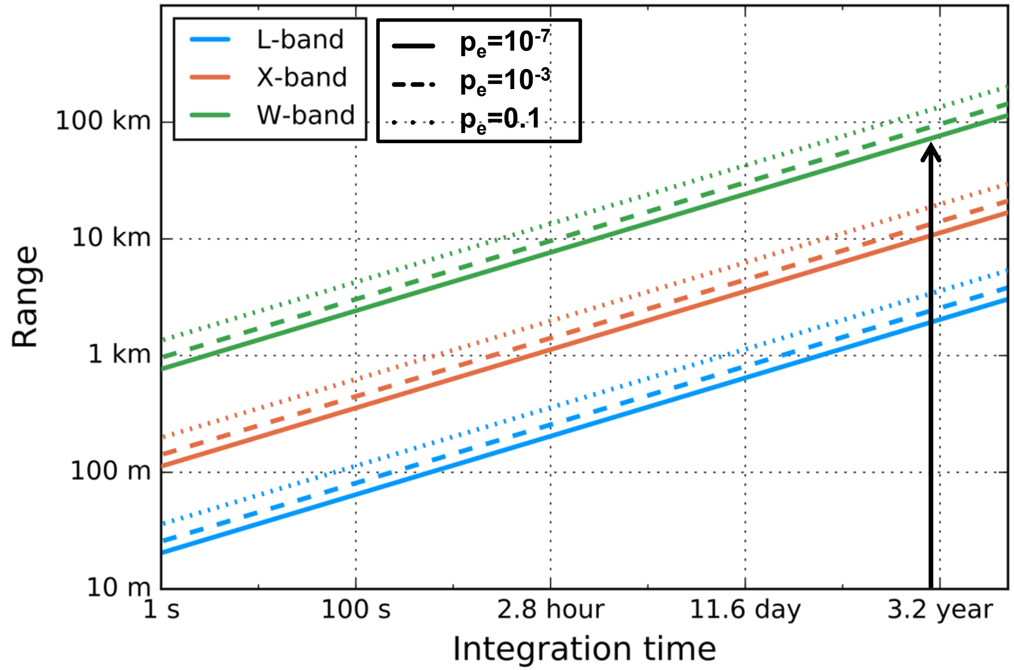
$$L \propto \left(\frac{W}{\lambda^2}\right)^{1/4}$$

100 km Radar Assumptions	
Aperture Diameter	1 m
Target Cross Section	10 m^2
Fractional Bandwidth	40%
Sky Temperatures	7, 15, 50 (L, X, & W band, vertical)
Photons / Mode (Ns)	0.05

1. 100 km Analysis – Integrate Longer

- **Range (L) depends weakly on integration time (T),** $L \propto T^{1/4}$

- **Target and environmental conditions need to remain unchanged over integration**



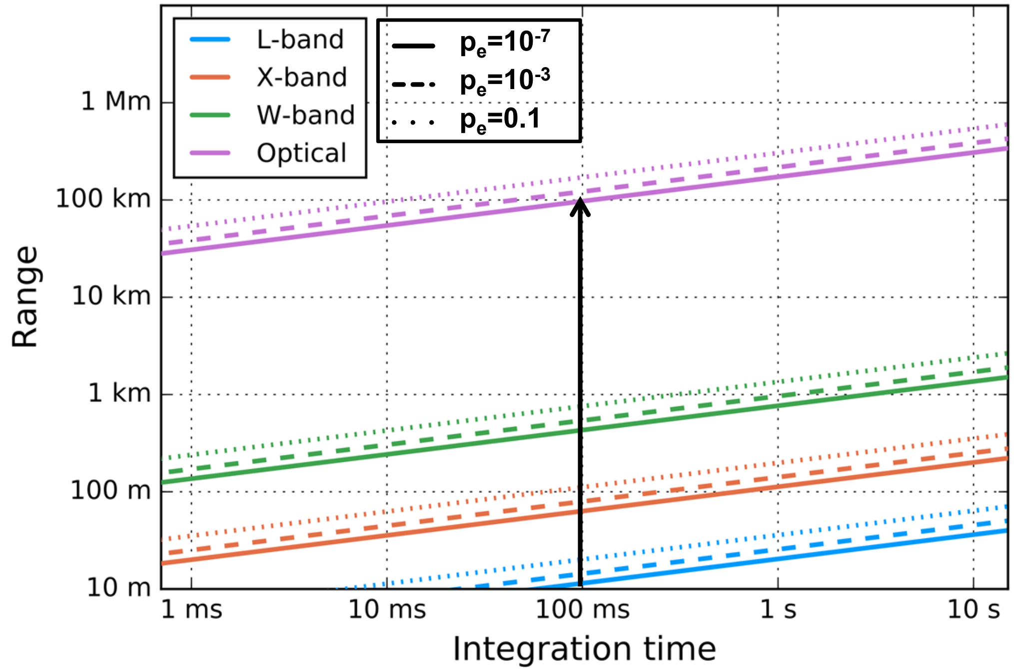
All plots use 100 km radar assumptions from previous slide

Need to integrate for 3 years to sense a single range-bin at 100 km

2. 100 km Analysis – Move to Optical Wavelengths

- **Optical telecom band has much shorter wavelength, greater bandwidth**
- **Different background noise, atmospheric absorption, and scattering at optical wavelengths may invalidate direct comparison to RF**

Radar Type	Center Wavelength	Fractional Bandwidth	Bandwidth
L-band	0.2 m	40%	600 MHz
X-band	0.03 m	40%	4 GHz
W-band	3.24 mm	40%	35 GHz
Optical	***1550 nm***	***1%***	***1.9 THz***



All plots use 100 km radar assumptions

An ideal optical quantum radar system can operate at 100 km with 100 ms integration per bin, but range enhancement may require unnaturally high background

Optical Quantum Radar in Practice

- Terrestrial applications generally see low natural background (<< 1 photon/mode)
  - Low background violates quantum enhancement assumptions, no expected advantage over classical radar
- Signal and idler path must be length-matched for quantum measurement
  - Radar needs to make a separate measurement for each range-bin
  - Idler must be stored for the time-of-flight of signal photons
- The idler (light kept at radar) cannot currently be stored losslessly
  - Best method to store in fiber, 0.2 dB/km loss
  - Theoretical advantage already lost at 30 km (6 dB SNR reduction)
- Laboratory experiment* achieved a 0.8 dB improvement over classical bound
  - Noise was artificially injected

*Zhang, Zheshen, Sara Mouradian, Franco N. C. Wong, and Jeffrey H. Shapiro. “Entanglement-Enhanced Sensing in a Lossy and Noisy Environment.” Physical Review Letters 114, no. 11 (March 20, 2015).

Summary

- **Quantum Illumination Radar at radio frequencies does *not* have the potential for long range standoff sensing (>10 km) at radio frequencies (<100 GHz)**
  - **Will take one second of integration to probe a single range/elevation/azimuth bin at 1 km**

- **Quantum radar with 100 km range is not feasible**
  - **RF band operation requires 3 year integration time**
  - **Optical C-Band operation requires unnaturally high background to achieve advantage over similar classical system**

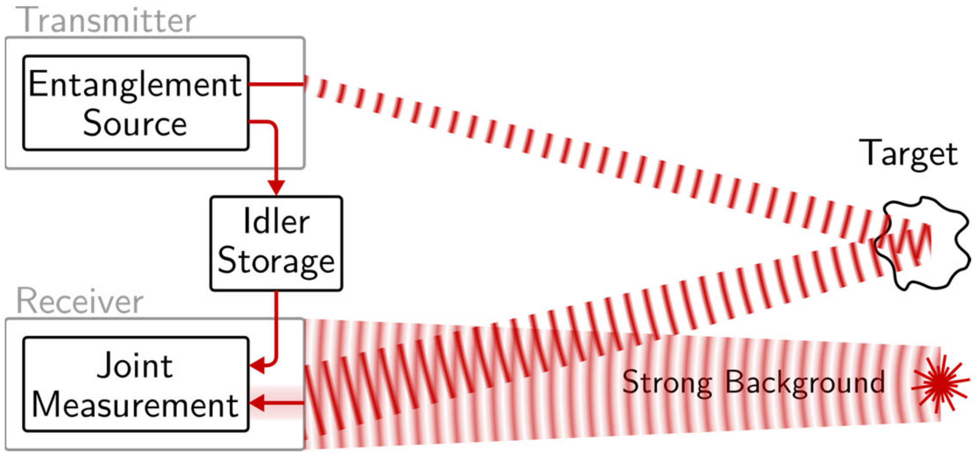
- **Analysis of receiver operating characteristic shows there are no useful trade-offs in error types at system operating points beyond 2 km (W-band, 1 sec integration)**

Quantum Radar Advantage

Quantum Radar has potential 6 dB sensitivity advantage

- **Quantum Radar probability of error exponent (SNR) is $\propto MN_S\kappa/N_B$**
- **Best classical system error exponent (SNR) is $\propto MN_S\kappa/4N_B$**
  - ***Factor of 4 in denominator is the 6 dB reduction in SNR***
  - **M is the number of modes (time-bandwidth product)**
  - **N_S is the transmitted photons/mode**
  - **κ is the round-trip transmission efficiency**
  - **N_B is the background photons/mode**

Quantum Radar Framework



Increased sensitivity useful for LPI/LPD sensing, or when transmitter is energy-constrained

Radar Parameter Considerations

- **Radar Equation:** $N_R = N_S \frac{G_T}{4\pi L^2} \frac{\sigma_T A_R}{4\pi L^2} \epsilon$
  - **N_R = # return photons**
  - **N_S = # source (transmitted) photons**
  - **G_T = transmitter directional gain**
  - **σ_T = target cross section**
  - **L = distance to target**
  - **ε = system efficiency**
  - **Round trip efficiency is** $\kappa = \frac{G_T}{4\pi L^2} \frac{\sigma_T A_R}{4\pi L^2} \epsilon$
- **Detection error probability in QR is**

$$p_e = \exp(-M N_S \kappa / N_B)/2$$

  - **N_B = # background photons**
  - **M = T W= time-bandwidth product**

- **Detection range for given error is then**

$$L = d \left(\frac{T W N_S}{N_B} \frac{\sigma_T \epsilon}{-4^3 \lambda^2 \log(2p_e)} \right)^{1/4}$$

- **Range improved by:**
  - **Larger source flux, integration time, bandwidth, target cross section**
  - **Smaller background flux, wavelength**

Classical and Quantum Radars perform better with stronger signal, weaker background

Radar Parameter Considerations

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- **Range improved by:**
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  - Smaller background flux, wavelength

Classical and Quantum Radars perform better with stronger signal, weaker background

But…

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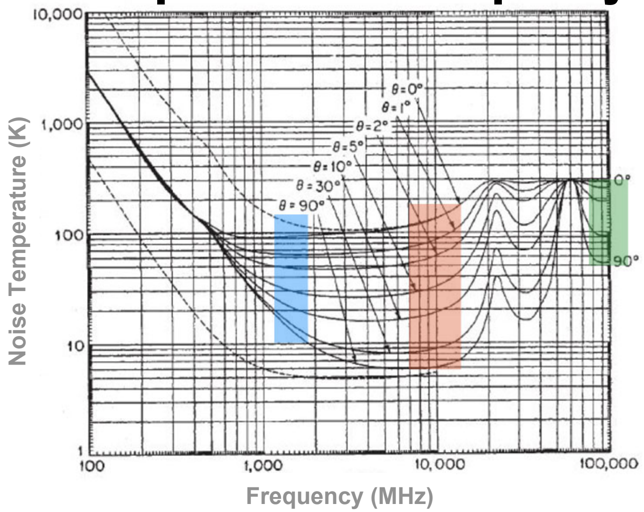
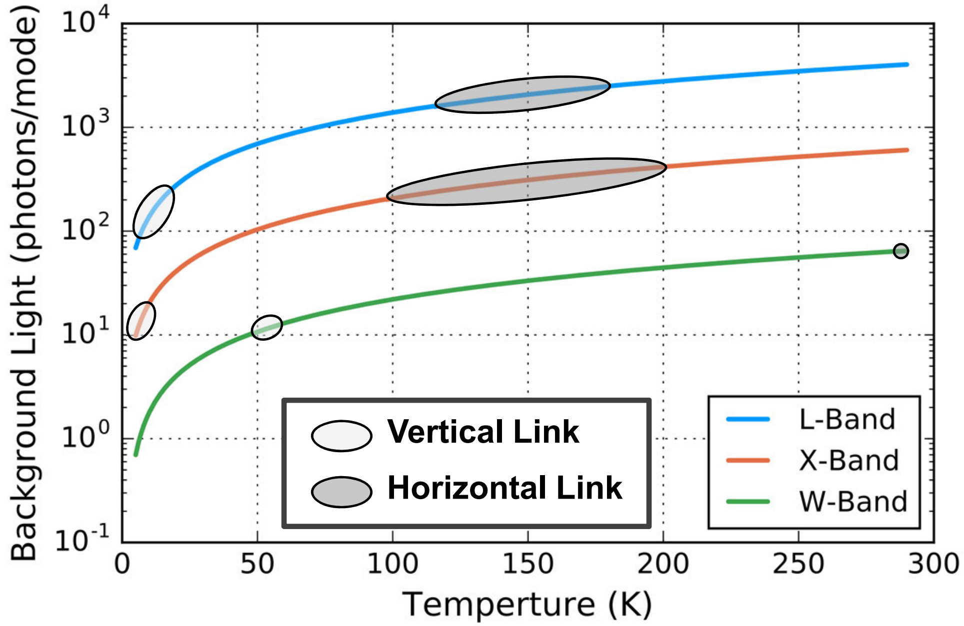
1. Sky Temperature by Frequency

Base Assumption: 290 K sky temperature

Background Noise Temperature vs. Frequency



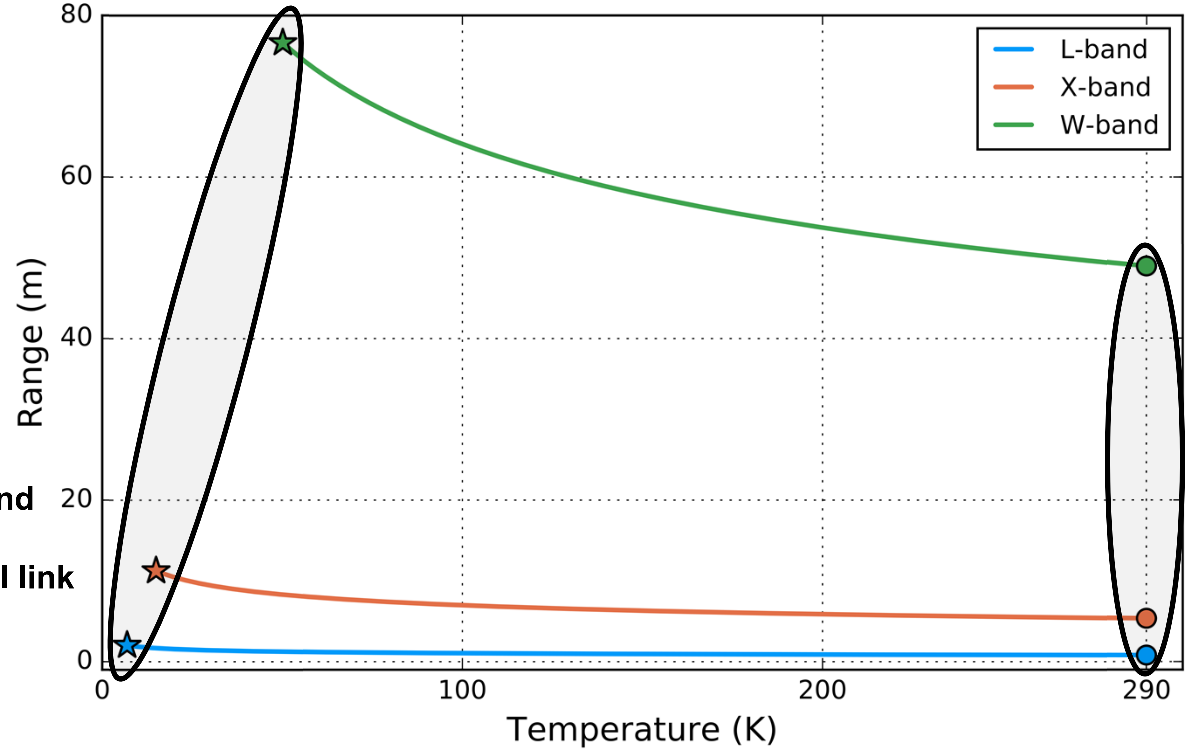


Background Noise Temperature (K)			
Link Type	L Band	X Band	W Band
Vertical	7	15	50
Horizontal	150	150	290

Practical system would see 1 – 2 order of magnitude reduction in background radiation compared to 290 K baseline

1. Sky Temperature by Frequency

Range inversely related to background photons per mode, $L \propto N_B^{-1/4}$



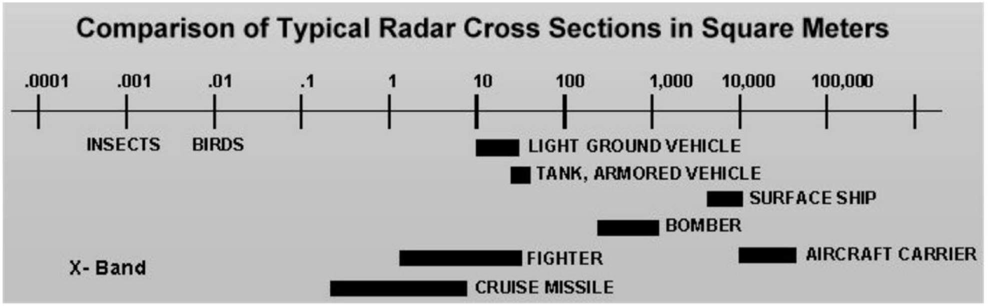
Reduced background temperatures, best estimate for vertical link

Baseline with 290 K background temperature

Range enhancement due to background reduction, but sensing range still limited to 10s of meters

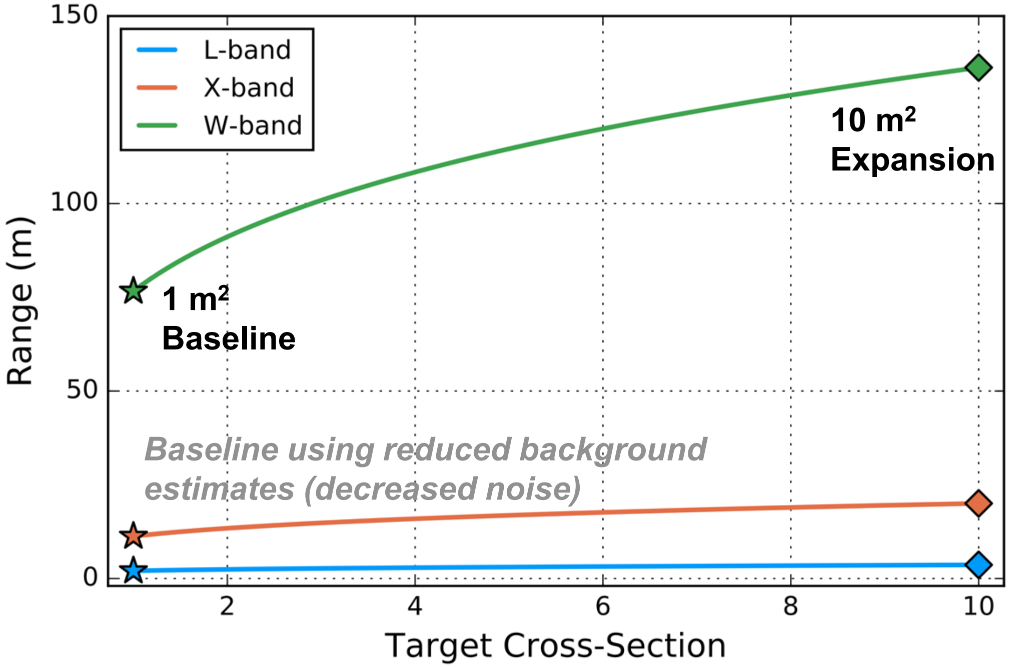
2. Target Cross-Section

Target Cross Section



- In X-band, 1 m^2 cross-section on low end of expected targets.
- Evaluate system up to 10 m^2 target

Range related to target cross-section as $L \propto \sigma_T^{1/4}$



Range enhancement due to cross-section increase, but sensing range still limited to 10s-100s of meters

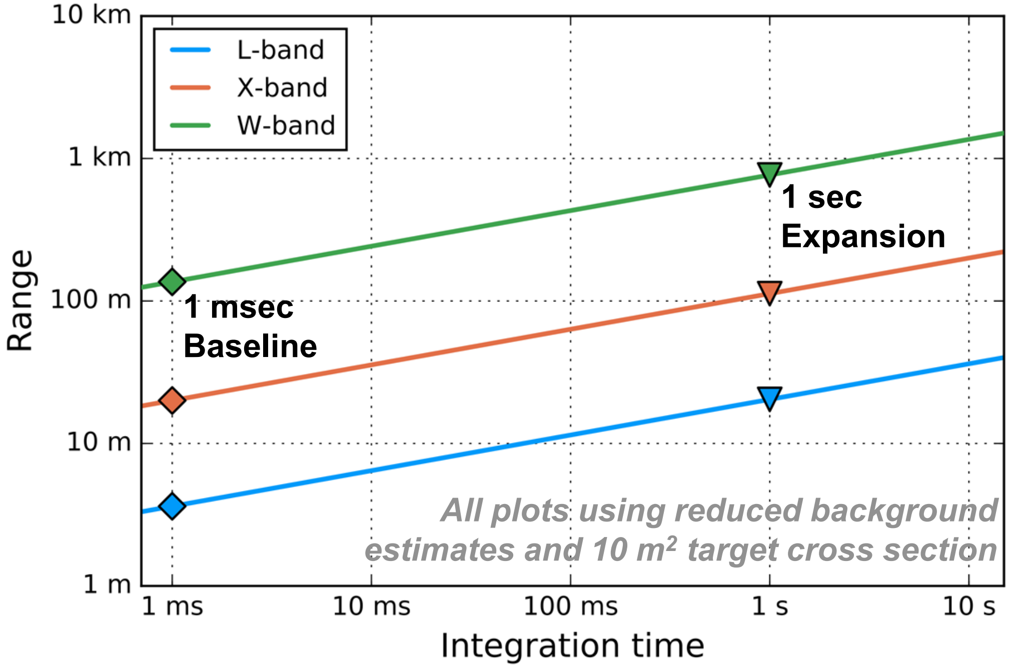
3. Time-Bandwidth Product

- **Look at L, X, and W bands at 40% fractional bandwidth**

Radar Type	Center Wavelength	Fractional Bandwidth	Bandwidth
L-band	0.2 m	40%	600 MHz
X-band	0.03 m	40%	4 GHz
W-band	3.24 mm	40%	35 GHz

- **40% fractional bandwidth near practical and fundamental limit**
- **Increasing integration time only feasible path to increase time-bandwidth product**

Range related to integration time as $L \propto T^{1/4}$



Range enhancement due to integration time increase, but limited to ~750 m for ≤1 sec integration time per range-bin (in W band)

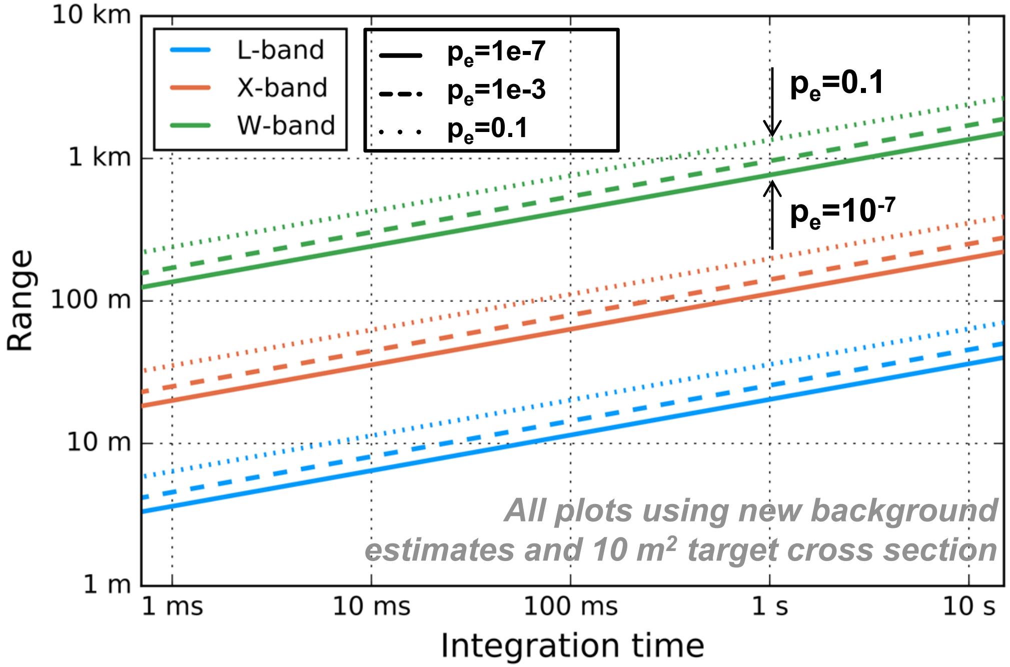
4. Error Probability

- **Baseline system has very low error probability of $p_e = 10^{-7}$**

- **Increased error probability achieved with lower SNR, which enables extended radar range**

- **Parameter expansion considers error probabilities up to $p_e = 0.1$ (10% chance of error)**

Range related to error probability as $L \propto (\log(2p_e))^{-1/4}$



Range enhancement due to error increase, but limited to ~1.35 km for $p_e \leq 0.1$ (in W band with 1 sec integration time per range-bin)

5. ROC Curve Analysis

Quantum Radar cannot achieve useful ranges (>10 km) at radio frequencies
For practical integration times (<1 ms) and **error probabilities (<0.1)**

- **Baseline probability-of-error analysis treats false alarm errors and target miss errors as equally important (assumes equal likelihood of target being present and absent)**
- **Radars are traditionally analyzed in terms of receiver operating characteristic (ROC)**
  - **Decision threshold varied to trace tradeoff between probabilities of detection and false alarm**
- **Expand analysis to investigate ROC curve for quantum radar to see if any advantage vs. a classical system**

Classical Coherent Sensor	QR with SPDC + OPA	QR Hypothesized Bound
$p_{\mathrm{D}}^{(coh)} \approx Q\left(Q^{-1}\left(p_{\mathrm{FA}}\right) - \sqrt{2}\sqrt{\frac{M\kappa N_S}{N_B}}\right)$	$p_{\mathrm{D}}^{(opa)} \approx Q\left(Q^{-1}\left(p_{\mathrm{FA}}\right) - 2\sqrt{\frac{M\kappa N_S}{N_B}}\right)$	**No closed form**
Same scaling with p_{FA}, QR has 3 dB better scaling with SNR (same advantage seen in p_e analysis)		