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A First-Order Parametric Model of Steady-State Heat Transfer Through Layered Materials

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14. ABSTRACT

A parametric model of steady-state heat transfer through layered materials is examined. This model is formally based on a first-order approximation of the solution to advection-diffusion equation, which includes the effects of multiple layers with varying thermal diffusivities, interface effects (e.g., large changes in thermal properties), contact resistance, and the effects of singular heat-sinks that are represented by negative heat sources. This model provides parametric representations of temperature distributions within layered-material systems, which can be utilized for their design and optimization for layer-configuration, including heat-sink control of thermal transport. Results of prototype modeling of controlled heat transfer in layered-material systems are presented and validated, demonstrating general aspects of the parametric model for thermal analysis and simulation of heat-transfer control using layer configurations and embedded heat sinks.

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Table of Contents

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Introduction

The control of heat transfer through multilayer materials is significant for many applications, ranging from heat treatment of materials to thermal management of systems. Optimizing heat transfer through multilayer materials requires estimating the thermal response of layered composite materials. This can be achieved using parametric models that combine heat-transfer characteristics of a specified layered system and thermal material properties, enabling prediction of temperature fields. These models should be conveniently adaptable for estimating the thermal response of different types of layered materials. The present study presents a parametric model of heat transfer through a layered material system. The parametric model is motivated by welding processes [1,2], where workpiece temperatures are controlled by material composition and thermal contact to base plates. The control of heat transfer through multilayer materials can also utilize heat sinks. The general physical character of heat sinks is that their thermal diffusivities are substantially greater than those of the workpieces whose temperature fields are to be controlled [3,4]. Accordingly, thermal coupling of heat sinks to adjacent layers can be represented parametrically by phenomenological negative heat sources [5].

Parametric modeling of temperature fields follows the approach of inverse analysis [6], i.e., inverse thermal analysis [7,12], which is for estimating optimal parameter values for a given system specification. The parameter values (e.g., effective thermal diffusivities) should be achievable for realistic system design. For complex material systems, the determination of material response functions is well posed in terms of inverse analysis. The multiscale character of layered materials poses an inverse analysis problem, which follows from the realization that layered-material thermal properties are not the same as those of bulk materials. In addition, for layered materials there is typically influence on thermal transport due to advection at bounding surfaces of the layered system. Accordingly, parametrization should be extended to include effective diffusivities, which are based formally on replacing the advection-diffusion operator with an effective-diffusion operator. Physically, advection is not expected to manifest as influencing thermal transport locally within a layered-material system but rather as influencing thermal transport over the its entire length of the layered. Accordingly, the phenomenological influence of advection, which is associated with ambient environments at surface boundaries of a layered-material system, again poses a problem of inverse thermal analysis for determination of effective diffusivities. Finally, heat transfer across interfaces of layers making up a layered material system can be effectively singular with respect to heat-transfer trend characteristics because of thermal contact resistance and possible large differences of thermal diffusivity. Accordingly, with respect to parametric modeling, an interface may be represented by a layer having singular characteristics with respect to heat transfer. Heat transfer through a characteristically singular layer from adjacent layers of material will depend on the characteristic thermal coupling of layer interfaces, which again is a complex material property, not known a priori, and thus appropriately posed for inverse thermal analysis.

There exists different types of configurations for heat-sink coupling to a heated system. The parametric model considered here, for heat-sink coupling to a heated layered system, is as described schematically in Figure 1, where coupling occurs at edges of the system. This configuration poses a specific problem with respect to inverse thermal analysis.

This study presents a parametrric model for heat transfer through layered material systems, formally based on a first-order approximation of the solution to advection-diffusion equation [5], which includes the effects of multiple layers with varying thermal diffusivities, interface effects

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(e.g., large changes in thermal properties), contact resistance, and the effects of singular heat-sinks that are represented by negative heat sources. Organization of subject areas presented are as follows. First, a parametric model of temperature fields for layer controlled heat transfer in layered materials is presented. Second, prototype analyses using the parametric model are described. Finally, a discussion and conclusion are given.

Derivation of Parametric Model

Presented in this section is a parametric model for heat transfer through a thin layered material heated at one surface, with transfer of heat to an ambient environment at the other surface by means of radiation and convection, with the possibility of heat-sink channel cooling, at steady state. This model is based formal extensions and approximations of analyical solutions to the heat conduction equation [5].

The constructed model is based on a particular interpretation of the Rosenthal equation of a moving heat source in one dimension [2], i.e.,

$$T(z) = \frac{C_0}{|z - z_0|}\, e^{-\frac{V(z-z_0)}{\kappa}} \qquad Eq. 1$$

where V is the speed of the heat source of strength C_0 moving within a material whose thermal diffusivity is κ. One notes that the temperature field within a frame of reference fixed to a moving heat source, Eq. 1, is equivalent to the temperature field generated from a source consisting of both advection and diffusion, where V can be interpreted formally as an effective advection coefficient, which represents rates of energy input to the layered-material system. Next, one assumes that the layered-material thickness is thin, and thus the $(z\text{-}z_0)^{-1}$ dependence can be neglected and C_0 is assigned the value of the heated surface temperature T_H, i.e.,

$$T(z) = T_H e^{-\frac{V(z-z_0)}{\kappa}} \qquad Eq. 2$$

and that one adjusts the effective advection coefficient V such that $Pe = V/\kappa < 1$, which is sufficient for a linear approximation of Eq. 2, i.e.,

$$T_1(z) = T_H \left[1 - \frac{V(z - z_0)}{\kappa}\right] \qquad Eq. 3$$

where the Peclet number Pe, is a metric for relative contributions of advection and diffusion to a heat-transfer process [13], i.e., $Pe = \frac{\text{advective transport rate}}{\text{diffusive transport rate}}$. Next, the parameterization of Eq. 3 is extended by consideration of heat transfer to first order, i.e., that the heat flux Q_H is given by

$$Q_H = -k\frac{dT}{dz} \qquad Eq. 4$$

and integrating over interval (z_0, z), one obtains

$$T_2(z) = T_H - \frac{Q_H(z - z_0)}{k} \qquad Eq.5$$

It follows by comparison of Eqs. (3) and (5) that

$$Pe = \frac{V}{\kappa} = \frac{Q_H}{kT_H} \qquad Eq.6$$

Thus, a consistent parametric reformulation of Eq. 2, is

$$T(z) = T_H e^{-\frac{Q_H(z-z_0)}{T_H k}} \qquad Eq.7$$

which extended formally, based on the property of series thermal resistances is

$$T(z) = T_H\, e^{-\frac{Q_H}{T_H}\left(\frac{z-z_n}{k_n}+\sum_{k=0}^{n-1}\frac{z_{k+1}-z_k}{k_k}\right)} \qquad \text{Eq. 8}$$

Next, the parametric model Eq. 8 is extended by introduction of contact conductance at interfaces between layers, and an embedded heat-sink boundary at $z = z_c$, i.e.,

$$T(x,y,z) = T_H \exp\left[-\frac{Q_H}{T_H}\left(\frac{z-z_n}{k_n} + \sum_{k=0}^{n-1}\frac{z_{k+1}-z_k}{k_k}\right) + \sum_{j=1}^{N_j}\frac{u(z-z_j)}{h_{icj}}\right] - f_{hs}(x,y,z) \quad Eq.9$$

$$f_{hs}(x,y,z) = T_C(x,y)\exp\left[-\frac{Q_c}{T_{C0}}\left(\frac{z-z_{hs}}{k_m}\right)\right] \qquad Eq.10$$

The quantities h_{icj}, j = 1-N_j, are the interface contact conductances and $u(x)$ is the Heaviside unit step function. The function $T_C(x,y)$ is the 2D temperature field of the heat sink and Q_c specifies its strength of coupling to the layered system. Note that physically, for a given location (x,y), heat sink cooling is for all z. Our goal, however, is parametric modeling of heat transfer to the surface of the layered sytem, and therefore only $z > z_c$ is of interest.

Next, the function $T_C(x,y)$, whose model representation is assumed purely phenomenoological and parameters adjusted with respect to measurements, can given by functions of the general form

$$T_c(x,y) = T_{co} e^{-\frac{Q_{bx}(x-x_0)}{k_{hs}}} e^{-\frac{Q_{by}(y-y_0)}{k_{hs}}} \qquad \text{Eq. 11}$$

Referring to Eq. 9, the parametric model is defined in terms of two adjustable parameters, Q_H and Q_c, associated with heat transfer through the layer system. The adjustable parameters Q_{bx} and Q_{by}, are associted with heat-sink coupling to the layered system

Next, given a surface boundary at $z = z_s$, letting $T_S = T(x,y,z_s)$ defined by Eq. (9), for $z > z_s$,

$$T(x,y,z) = T_A + (T_s - T_A)e^{-\frac{Q_S}{T_S}\left(\frac{z-z_S}{k_A}\right)} \qquad \text{Eq. 12}$$

where

$$Q_s = h_c(T_s - T_A) + \varepsilon\sigma(T_s^4 - T_A^4) \qquad Eq. 13$$

Where h_c, ε, σ, and T_A are the convective heat-transfer coefficient, emissivity of the outer surface, Stefan-Boltzmann Constant ($5.6704\cdot10^{-8}$ $W\cdot m^{-2}\cdot K^{-4}$) and the ambient-environment temperature, respectively.

Next, referring to Eq. 9, one can note that heat-sink influence within a layered system can be approximated phenomenologically by an interface conductance. Accordingly, an alternate form of the parametric model defined by Eq. 9 is given by

$$T(x,y,z) = T_H \exp\left[-\frac{Q_H}{T_H}\left(\frac{z-z_n}{k_n} + \sum_{k=0}^{n-1}\frac{z_{k+1}-z_k}{k_k}\right) + \sum_{j=1}^{N_j}\frac{u(z-z_j)}{h_{icj}} + \frac{u(z-z_{hs})}{h_{hs}(x,y)}\right] \qquad Eq. 14$$

where location of the "heat-sink interface" is $z = z_{hs}$, and locations of layer-layer interfaces are at $z = z_j$. The phenomenological contact conductances h_{icj} and h_{hs} are adjustable parameters that are determined in principle according to experimental measurements, i.e., inverse thermal analysis.

Equations 9-14 define a parametric model for heat transfer through a layered material, from a hot body to a ambient environment external to the outer surface, with cooling due to an embedded heat sink. An interesting aspect of this model is that advection coefficients can be associated physically with both steady-state "putting in" (heat transfer from the hot body) and "pulling out" (convection and radiation at the outer surface) of energy. For this model the inclusion of contact resistance can be achieved formally by localized (essentially singular) variations of the diffusivity that "draw off heat."

Prototype Analyses

This section presents computational experiments describing parametric modeling of layer-configuration and heat-sink controlled thermal transport within a layered material system. The design of these experiments, using physically realistic thermal properties and phenomenologically adjustable parameters, was not to demonstrate optimal layer-configuration and heat-sink thermal control, but rather general characteristics of the parametric model for modeling and simulation of such control, as well as demonstrating feasibily of such control using multilayer and heat-sink materials. The computational experiments described in this section represent both prototype inverse thermal analyses and simulations, which are both the goal of parametric modeling. The temparature field of the basic layered system adopted for prototype analyses, to be modified in computational experiments that follow, is shown in Figure 2.

Our first prototype simulation is of heat-sink cooling of the layered system shown in Figure 2 using the parametric model defined by Eq. 9. Shown in Figure 3 is the heat-sink source function used for this simulation, as a function of position *z* within layered system, and position *x* relative to location of heat-bath coupling to system (see Figure 1). Shown in Figure 4 are results of this simulation.

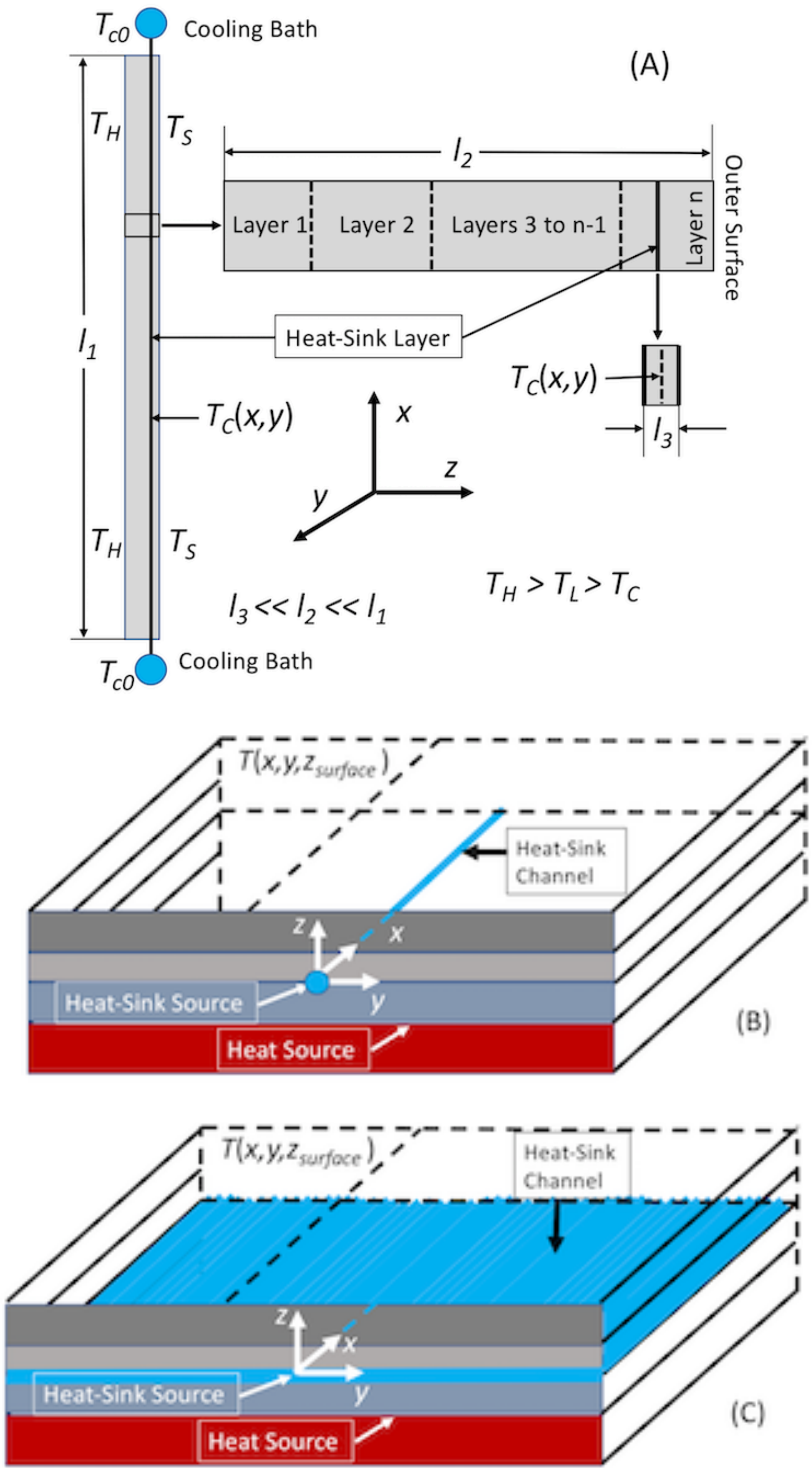


Figure 1. (A) Schematic representation of layer and heat-sink controlled heat transfer in layered materials, where T_H and T_S are temperatures of heated surface and surface at ambient atmosphere, respectively, and T_C(x,*y*) is temperature of heat sink at location (*x,y*), along the heat-sink layer relative to the cooling bath. (B) Schematic representation of cooling using embedded heat-sink channel. (C) Schematic representation of cooling using heat-sink layer.

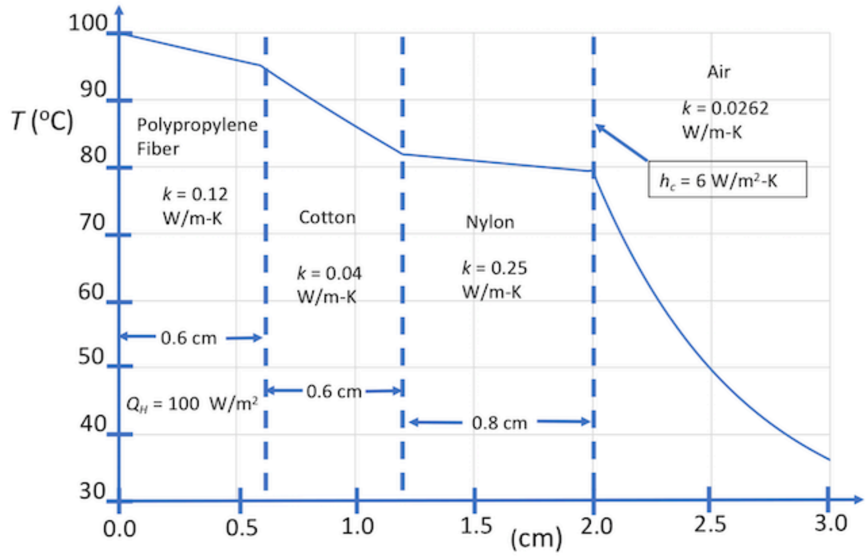


Figure 2. Temparature field of the basic layered system adopted for prototype analyses.

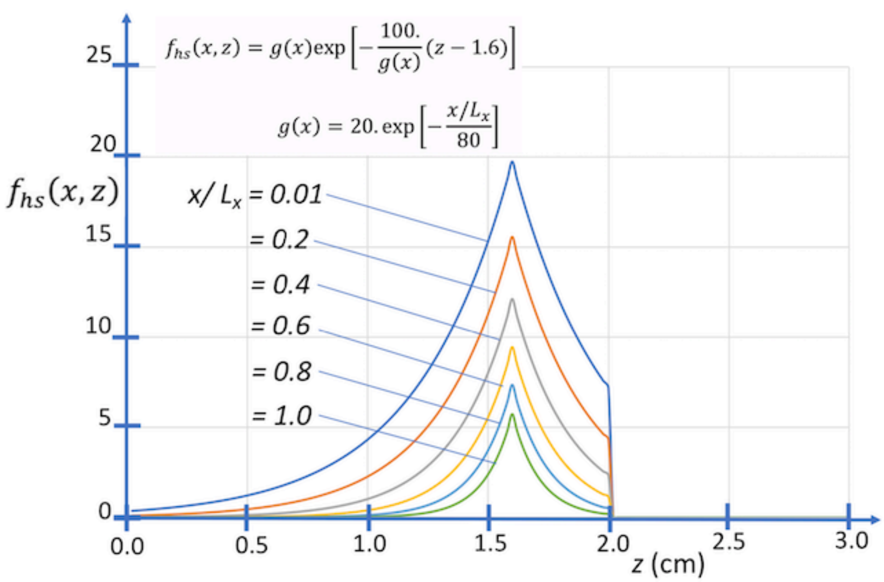
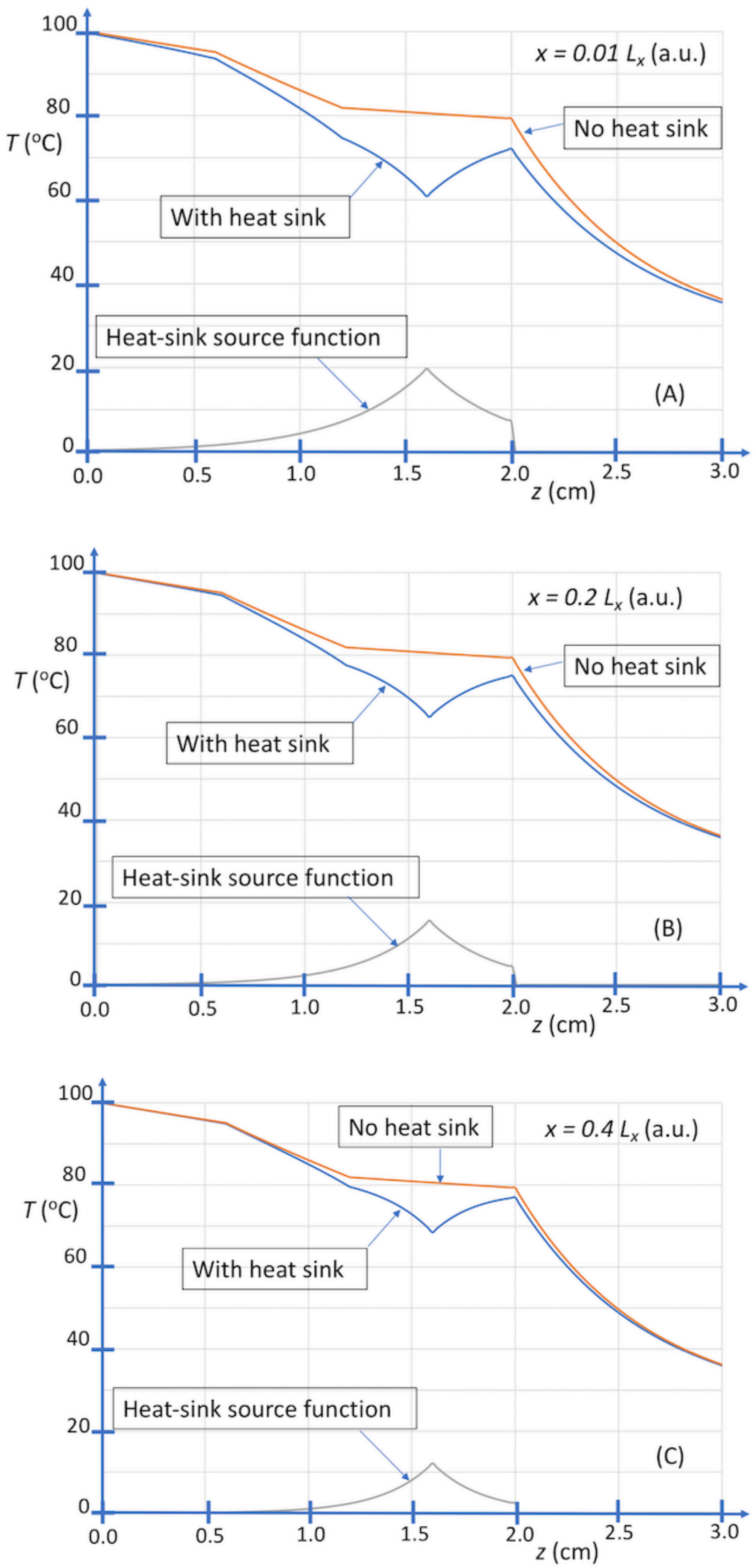


Figure 3. Phenomenological heat-sink source function, whose stength depends on penetration distance x within layered system.

Our second prototype simulation is of either contact conductance at interfaces between layers of the system or heat-sink cooling, using the parametric model defined by Eq. 14. Shown in Figure 5 are results of this simulation. For this simulatiion, values of the parameters h_{icj} (if interpreted as contact conductance) or h_{hs} (if interpreted as heat-sink cooling) were assigned arbitrarily.


x = 0.01 L_x (a.u.)
No heat sink
With heat sink
Heat-sink source function
(A)
T (°C)
z (cm)
x = 0.2 L_x (a.u.)
No heat sink
With heat sink
Heat-sink source function
(B)
T (°C)
z (cm)
No heat sink
x = 0.4 L_x (a.u.)
With heat sink
Heat-sink source function
(C)
T (°C)
z (cm)

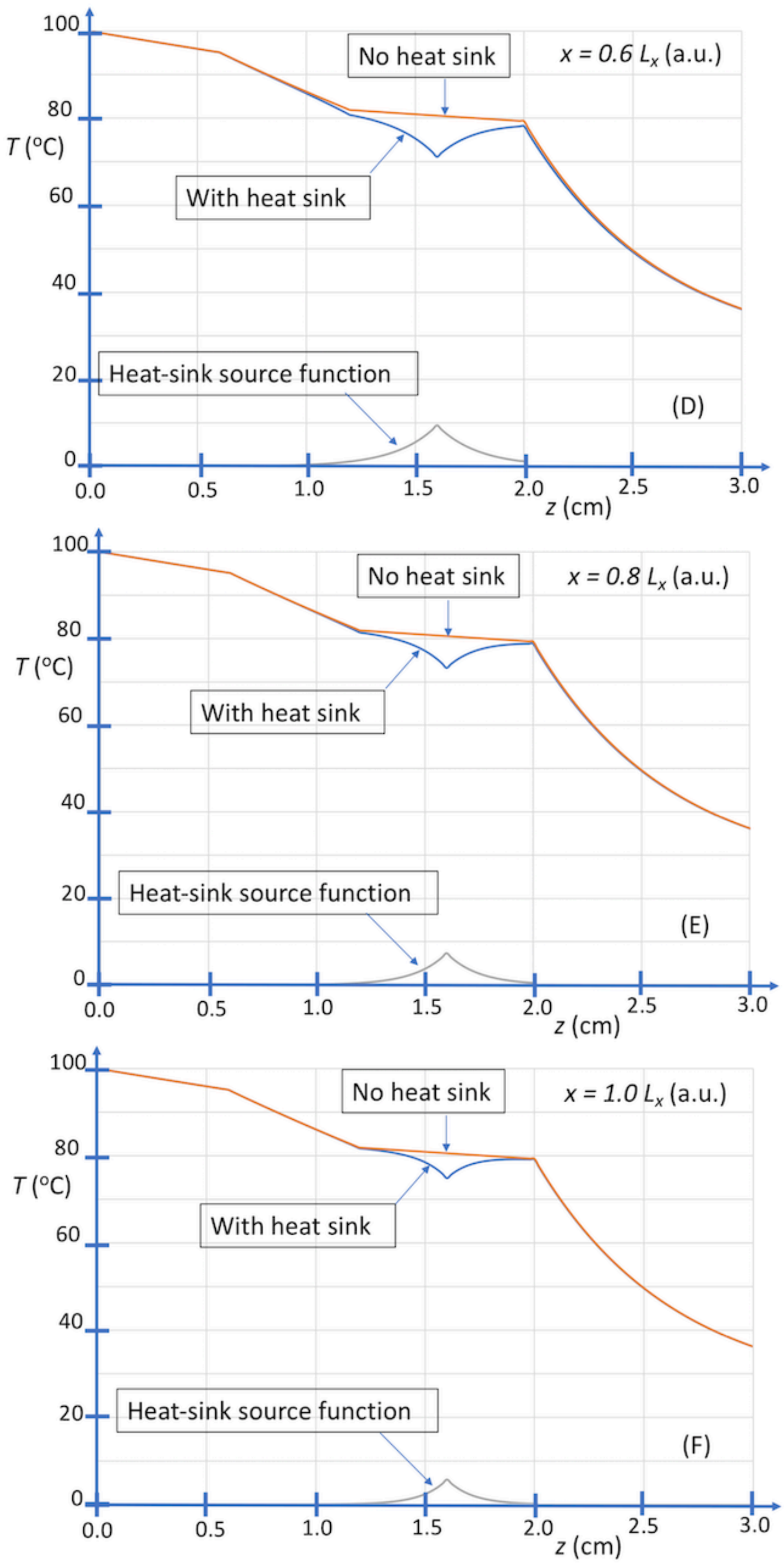


Figure 4. Simulation of heat-sink cooling of layered system shown in Figure 2 using the parametric model defined by Eq. 9, where $T_H = 100$ °C and $Q_H = 100$ W/m^2.

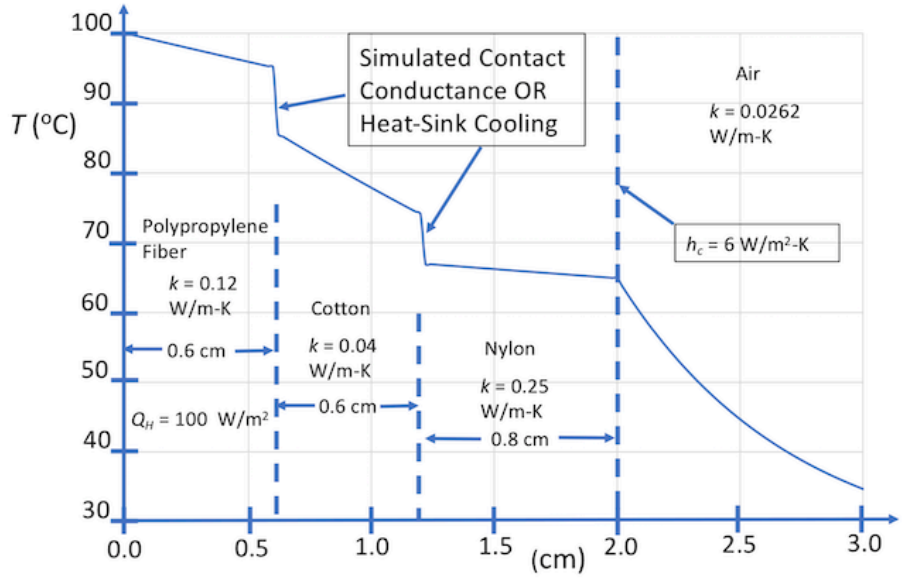
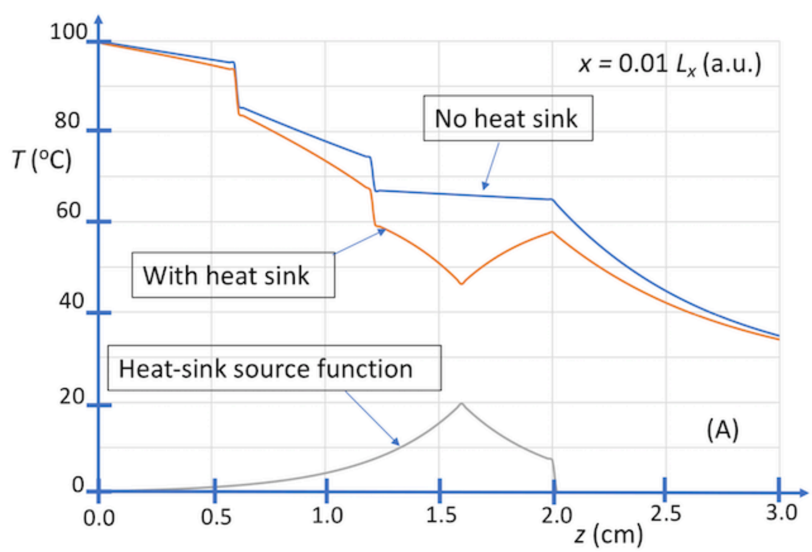
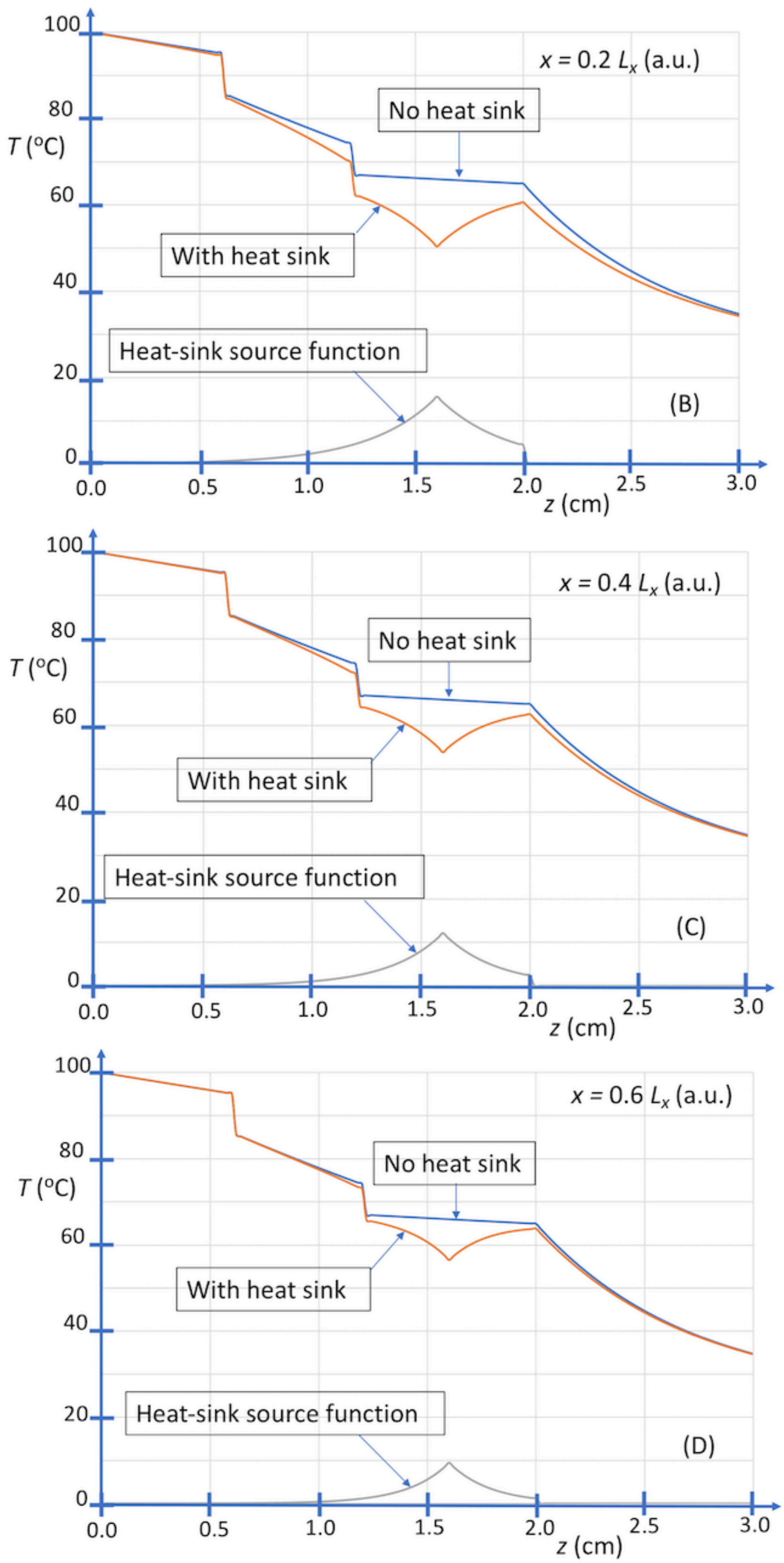


Figure 5. Simulation of contact conductance or heat-sink cooling of layered system shown in Figure 2 using the parametric model defined by Eq. 14, where $T_H = 100$ °C and $Q_H = 100$ W/m^2.

Our third prototype simulation is of heat-sink cooling of the layered system shown in Figure 5, where the presence of contant conductance is assumed, using the parametric model defined by Eq. 9, the heat-sink source function shown in Figure 3, and parameter values h_{icj} assigned arbitrarily. Shown in Figure 6 are results of this simulation.





100
80
60
40
20
0
T (°C)
x = 0.2 Lx (a.u.)
No heat sink
With heat sink
Heat-sink source function
(B)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
z (cm)
x = 0.4 Lx (a.u.)
No heat sink
With heat sink
Heat-sink source function
(C)
x = 0.6 Lx (a.u.)
No heat sink
With heat sink
Heat-sink source function
(D)

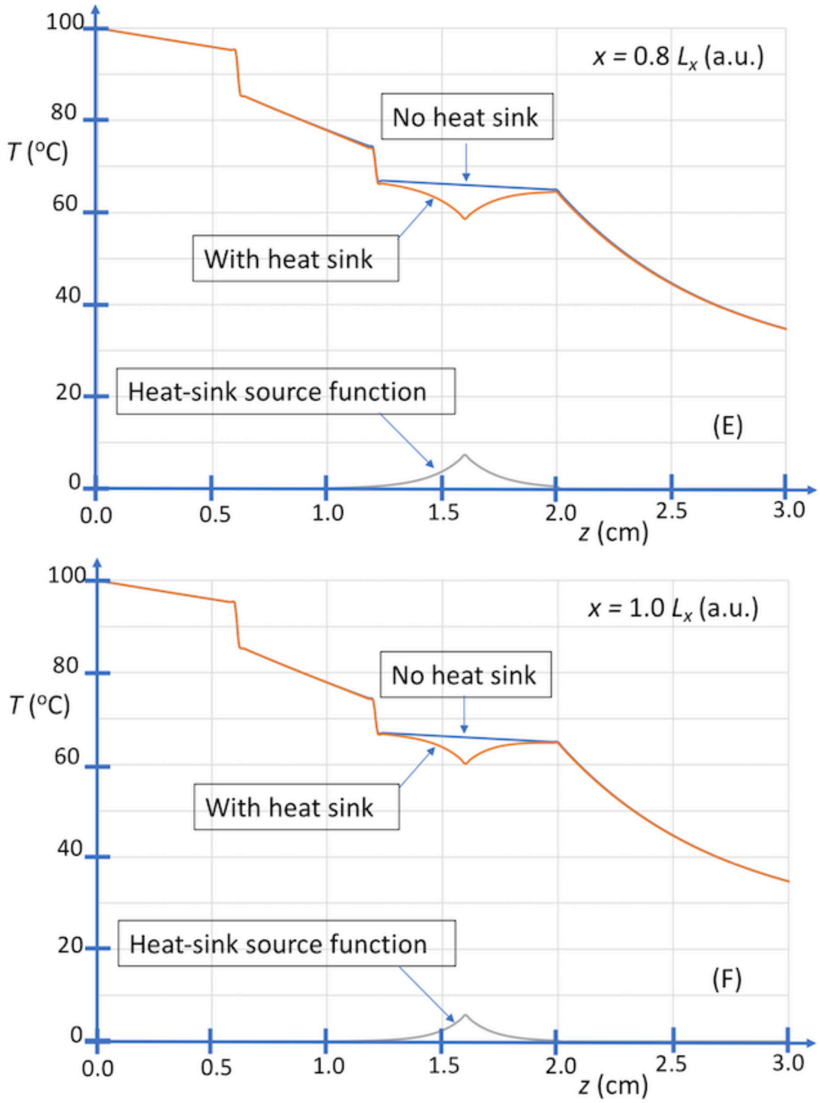


Figure 6. Simulation of heat-sink cooling of layered system shown in Figure 5, having contact conductance between layers, using the parametric model defined by Eq. 9, where $T_H = 100$ °C and $Q_H = 100$ W/m^2.

Discussion

The parametric model defined by Eqs. 9-14 are a formal adaption of the Rosenthal solution for simulation of heat-transfer in layerd materials which are coupled to heat sinks at edge boundaries. As described schematically in Figure 1, the model is of a three-dimensional system that is inherently multiscale in nature. The parametric model combines both energy-transport theory and phenomenological parameterization. Equation 9, whose construction is according to physical theory, includes the phenomenological source function $f_{hs}(x,y,z)$ representing multiscale coupling of embedded heat sinks, where cooling baths are located at edges of the system. This particular heat-sink configuration can be characterized by anisotropic heat diffusion along embedded heat-sink channels, having different geometries (as described schematically in Figure 1). Determination of anisotropic heat diffusion characteristics poses a problem for inverse thermal analysis. Equation 14, similarly, whose dominant ansatz is physical theory, includes the phenomenological contact conductance $h_{hs}(x,y)$, determined in principle according to experimental measurements, i.e., inverse thermal analysis. In the spirit of inverse analysis, the parametric forms of phenomenlogical components of the model are not unique, but adapted for convenience. For example, depending on the characteristic of anisotropic heat diffusion associated withh embedded heat sinks, perhaps Eq. 11 can be replaced by a product of parameterized Gaussian functions.

Conclusion

Determination of optimal process parameters for achieving a given target temperature field for heat transfer through a layered material using material-layer configurations and heat sinks poses a specific problem. The results of this study demonstrate use and general features of a parametric model, which can provide estimation of heat-transfer characteristics for layered materials coupled to heat sinks.

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