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NAVAL SURFACE WARFARE CENTER**

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NSWCDD/TR-20/167

**DERIVATION OF OPTIMUM DETECTOR FOR
RANGE MIGRATING TARGETS IN THE
PRESENCE OF CLUTTER**

BY TERRY L. FOREMAN, PH.D.

ELECTROMAGNETIC AND SENSOR SYSTEMS DEPARTMENT

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14. ABSTRACT This report describes the signal processing to detect range migrating targets in the presence of clutter. This has been an open problem. The signal processing derived here is based on the matched filter and Neyman-Pearson detector. The detector structure optimally processes fast time and slow time jointly. This provides the ability to differentiate the range migrating target and the clutter. This processing structure also allows one to change the transmitted radar signal on a pulse-to-pulse basis and still cancel clutter. The clutter can be stationary or range migrating. The target motion model is very general. It is only required to know the target range change as a function of time and its first derivative. In order to form this processing, it is necessary to provide estimates of the clutter parameters to the detector. This report shows that reasonable estimation accuracy provides good detector performance.					
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FOREWORD

This report is the product of a Naval Innovative Science and Engineering (NISE) project funded by Naval Surface Warfare Center, Dahlgren Division's (NSWCDD's) Chief Technology Office (Code 00T). The emergence of hypervelocity threats has greatly motivated this work. This report documents signal processing algorithms developed to solve the problem of detecting range migrating targets in clutter.

This document has been reviewed by Carrie Gonzalez, Head, Precision Sensing Branch (Code B43), and Chuck Campbell, Head, Advanced Sensor Systems Division (Code B40).

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GLOSSARY

A/D	analog-to-digital converter
AWGN	additive white Gaussian noise
Correlator+MTD	correlator and moving target detector
CPI	coherent processing interval
FFT	fast Fourier transform
GRP	ground reference point
IPP	inter-pulse period
LNA	low noise amplifier
MTD	moving target detection
N-P	Neyman-Pearson
NSWCDD	Naval Surface Warfare Center, Dahlgren Division
Pfa	probability of false alarm
PRI	pulse repetition interval
SIR	signal-to-interference ratio
SNR	signal-to-noise ratio

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1.0 PROBLEM STATEMENT

There is currently no method to allow a radar to suppress clutter when targets (or clutter) move across range cells during the span of a coherent processing interval (CPI) (see [1] p. 452 and [2] pp. 510-511), because compensating for range walk (or range migration) decorrelates the clutter. Clutter cancellation depends on the correlation properties of clutter. Hence, decorrelating the clutter destroys the radar's ability to cancel clutter. This problem can be visualized using the following simple illustrations. Start with the case, illustrated in Figure 1, where the target motion over the course of the coherent processing interval (CPI) is small compared to the range resolution of the radar.

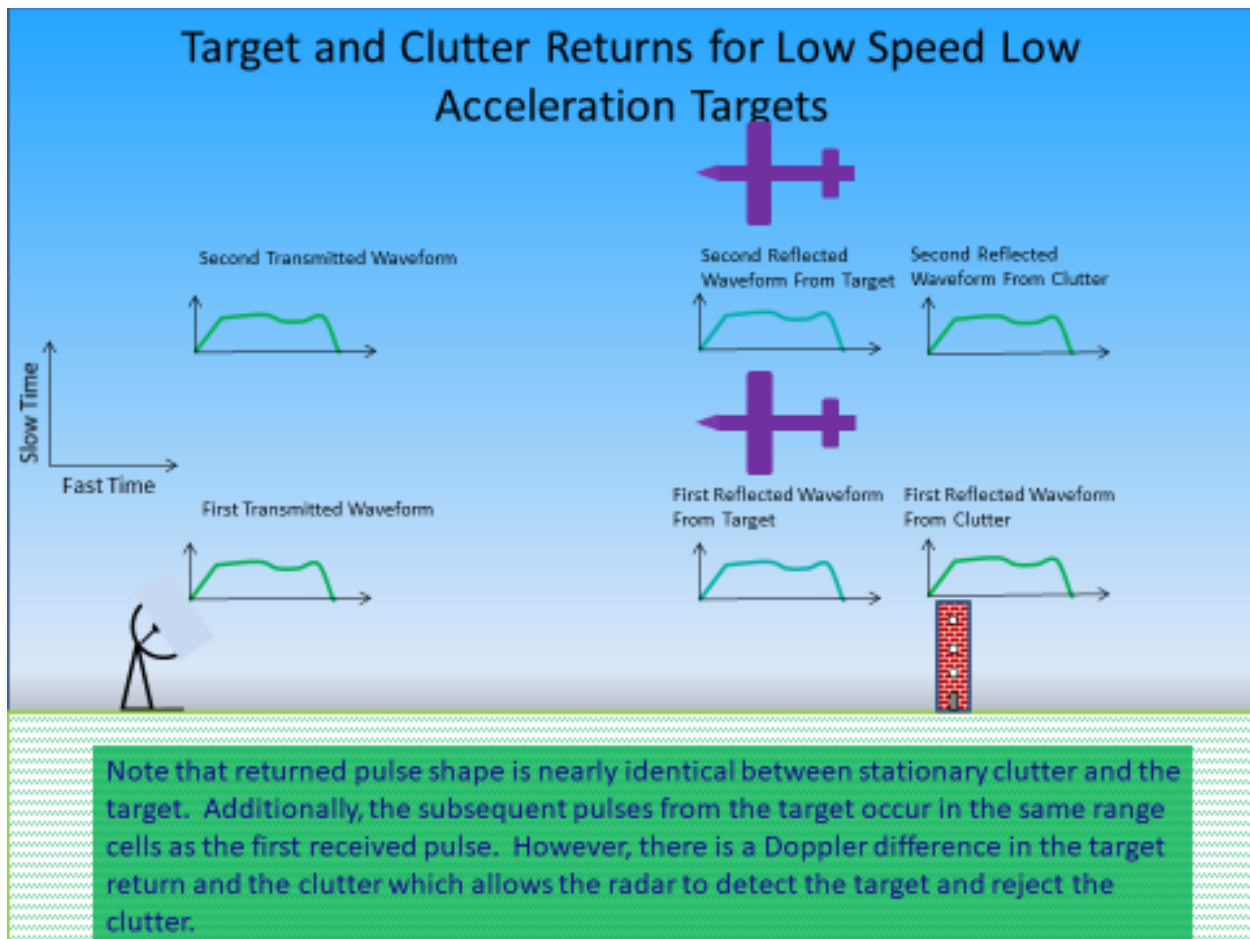


Figure 1. Target and clutter returns for low speed, low acceleration targets

In Figure 1, fast time/distance is represented by the horizontal dimension. The vertical dimension represents slow time (i.e., time between radar pulses). Figure 1 shows a radar transmitting two pulses. In the radar's field of view is an airplane (that is moving), representing the target, and a building, representing clutter (which is not moving). The target motion is slow enough that it appears not to change range from the time of one pulse transmission to the next.

Figure 2 illustrates the case where the target is flying fast enough that the radar observes it in different range cells from one pulse to the next. That is, the range change from the time of the first transmitted pulse to the second transmitted pulse is observable. Note that the Doppler processing integrates energy pulse to pulse. In other words, it adds up energy for a given range cell across slow time. Therefore, in the case of Figure 2 the target energy appears to leak between range cells; thus, it is lost. Therefore, if range migration is not compensated, there is a signal loss on the target.

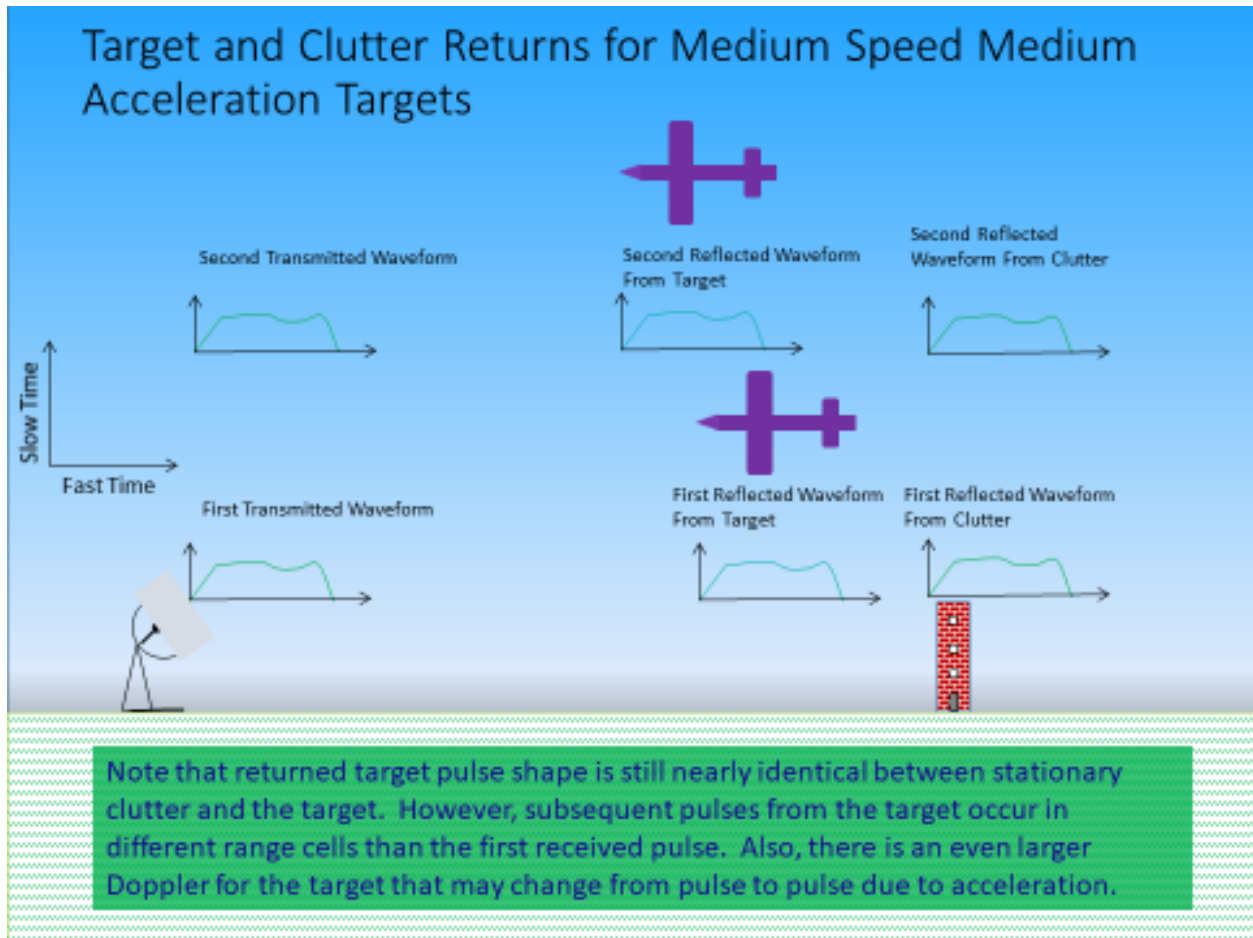


Figure 2. Target and clutter where target causes range migration, but pulse distortion can be ignored

Figure 3 illustrates the case of targets traveling so fast that, in addition to range migration, the reflected pulse shape is also distorted. This is caused by the observable motion over the course of a pulse width. Pulse distortion due to target motion is discussed in [1]. In this work, the narrow band assumption is used.

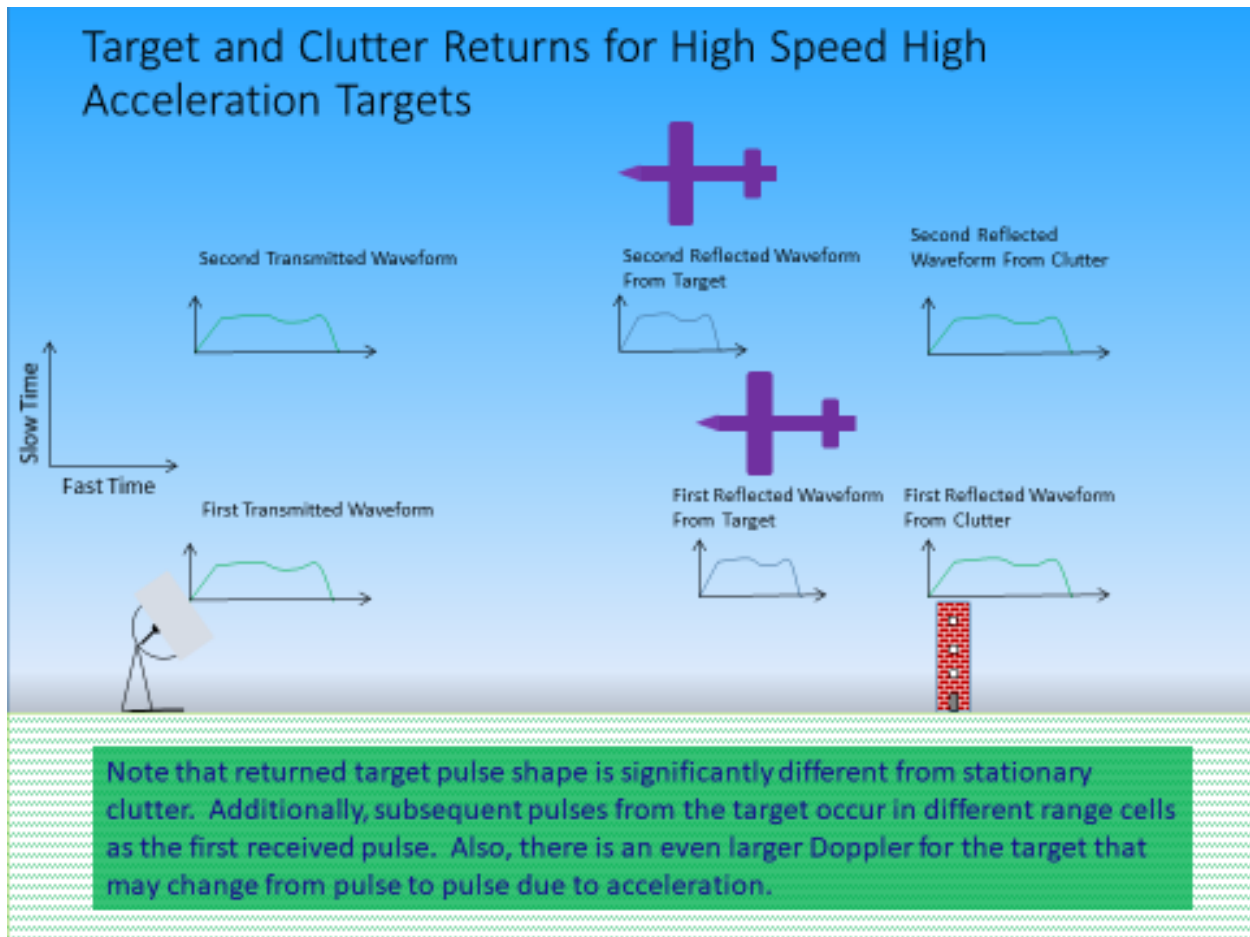


Figure 3. Target and clutter returns when target motion causes range migration and pulse distortion

There are techniques to compensate for range migration and pulse distortion. However, performing this compensation on the received data decorrelates radar clutter. This is illustrated in Figure 4 in which the target returns are aligned, and consequently, the clutter returns are misaligned causing serious degradation in clutter cancellation.

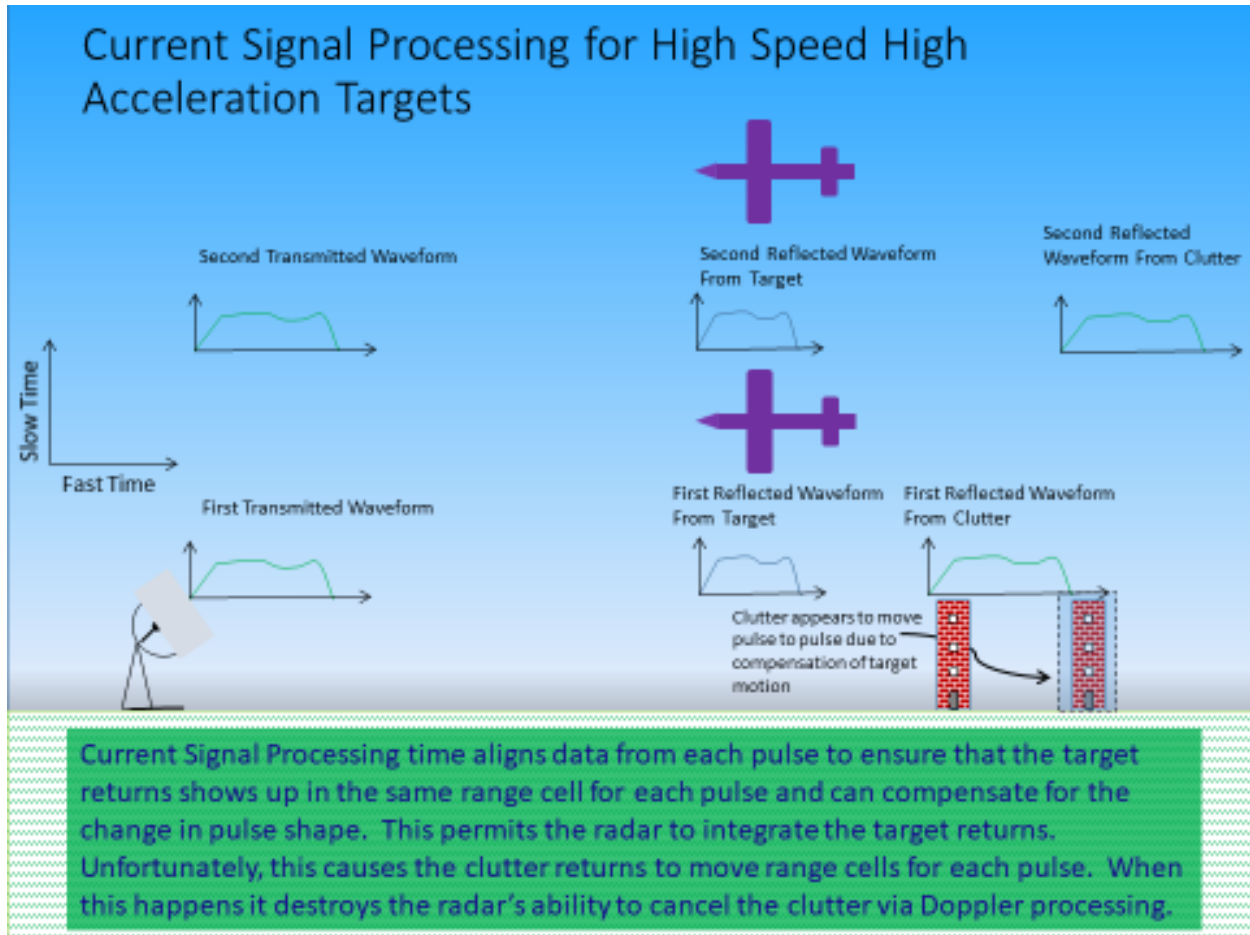


Figure 4. Range migration compensation and its impact on clutter

2.0 RADAR EQUIPMENT

The radar hardware is diagrammed in Figure 5. The radar transmitter generates and amplifies the signal that is radiated through the antenna. The signal then propagates to the target and then is reflected from the target and is received by the antenna. The low noise amplifier (LNA) amplifies the signal. Following the amplification, the signal is synchronously down-converted to baseband, producing the complex in-phase (real) and quadrature (imaginary) signals that are available to be processed in order to detect the target. Following the down-conversion, the analog signals are digitized at the Nyquist rate. The output of the analog-to-digital (A/D) converters is the data that is available to make a detection decision.

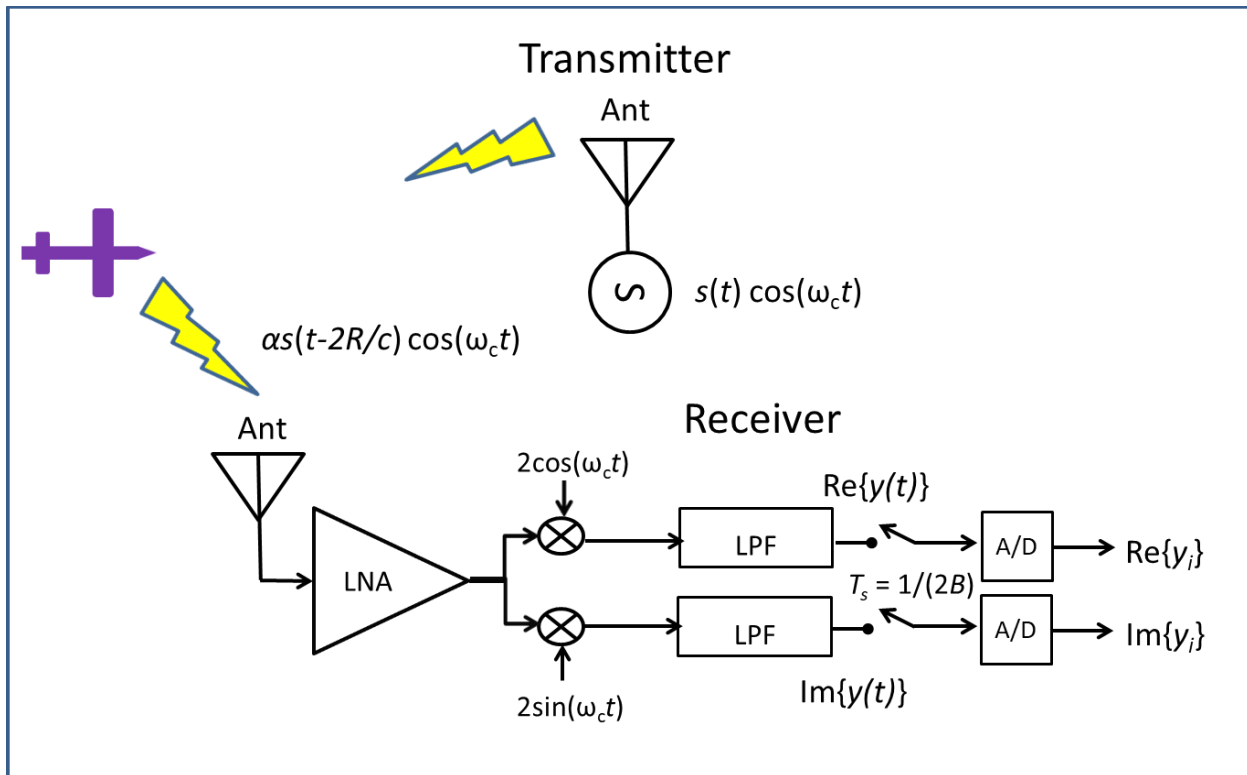


Figure 5. Radar hardware diagram

The radar transmits and receives multiple coherent pulses in order to make a detection decision. The receiver, in Figure 5, is gated on during the listen time for each pulse. The radar maintains timing such that it can time and phase-align the fast and slow time data. Note that fast time data is the receiver output due to the echoes produced by one pulse. Slow time indexes the subsequent fast time periods. Slow time and fast time are explained in [3], p 502.

3.0 DETECTORS FOR TARGETS IN CLUTTER

3.1. Non-Range Migrating Targets

The first detector developed will be for a slow-moving target (i.e., range migration can be ignored). This detector is optimal in that it maximizes the probability of detection for a specified probability of false alarm subject to the assumptions invoked in the text. The derivation will follow the approach in [4]. It will jointly process the fast time and slow time information to improve detectability as described in [4]. The input of this detector is the complex baseband data as shown in Figure 5.

3.1.1. Single-Pulse Detector

First presented is the case for a constant velocity target with a range rate and CPI time that satisfies

$$\dot{R}(t)MT_i \ll \frac{cT_s}{2},$$

where $R(t)$ is the target range as a function of time, $\dot{R}(t)$ is the target range rate as a function of time, M is the number pulses, T_i is the time between pulses, T_s is the sample time, and c is the speed of light. Further, this analysis will be at complex baseband. The radar transmits signal $s(t)$ in the direction of the target. The response of the receiver to this reflected signal is

$$y_t(t) = s(t) * \alpha h_t(t - \frac{2R(t_0)}{c}), \quad (1)$$

where α is the complex amplitude of the target, $h_t(t)$ is the impulse response of the target, and t_0 is the start of the CPI. For point targets, $h_t(t)$ is the Dirac delta function $\delta(t)$. Since the radar receiver produces data sampled by an analog-to-digital (A/D) converter, the convolution in (1) can be approximated by a discrete time convolution as follows:

$$y_t(kT_s) = \alpha \sum_{i=0}^{N-1} s(iT_s) h_t((k+1-i)T_s - \frac{2R(t_0)}{c}), \quad (2)$$

where T_s is the sample time interval and N is the total number of time samples that contain the transmitted pulse. The operation in (2) can be written in matrix notation as

$$\mathbf{y}_t = \alpha \tilde{\mathbf{S}} \mathbf{h}_t \quad (3)$$

where $\mathbf{y}_t$ is a column vector of size $(P+N-1)$ whose elements are the discrete time samples of $y_t(iT_s)$, and $\mathbf{h}_t$ is a column vector of size $(P+2(N-1))$ whose elements are the time samples of $h_t(iT_s - 2R/c)$ in reverse range order, and

$$\tilde{\mathbf{S}} = \begin{bmatrix} 0 & \cdots & 0 & \mathbf{s}^t \\ 0 & \cdots & \mathbf{s}^t & 0 \\ & \ddots & & \\ \mathbf{s}^t & 0 & \cdots & 0 \end{bmatrix} \quad (4)$$

which is a rectangle matrix with a size of $(P+N-1) \times (P+2(N-1))$. The matrix $\tilde{\mathbf{S}}$ is sometimes called a convolution matrix. Note that NT_s is the pulse width, $(P+N-1)$ is the length of the received data (i.e., number of samples received during the inter-pulse period (IPP)), and $(P+2(N-1))$ is the number of range cells that influence the IPP. Note that, for a point target, all elements of $\mathbf{h}_t$ are zero except the element that corresponds to the range cell containing the target. This element will be one.

The target model in (3) is adequate for stationary targets or targets whose Doppler can be ignored. The target Doppler manifests itself in the receive data as a phase change from sample to sample. The approach here is that used in [5] eq. (5). This can be accounted for by modifying the target model as follows:

$$\mathbf{y}_t = \alpha \tilde{\mathbf{S}}_d(m) \mathbf{h}_t, \quad (5)$$

where m is the pulse number that becomes relevant in the next section,

$$\tilde{\mathbf{S}}_d(m) = \begin{bmatrix} 0 & \cdots & s_1 & s_2 e^{j\phi(m,1)} & \cdots & s_N e^{j\phi(m,N-1)} \\ & & & \vdots & & \\ s_1 & s_2 e^{j\phi(m,1)} & \cdots & s_N e^{j\phi(m,N-1)} & \cdots & 0 \end{bmatrix}, \quad (6)$$

s_i are the elements of $\mathbf{s}$, and

$$\phi(m, n) = \frac{4\pi(R(mT_i + nT_s) - R(mT_i))}{\lambda}. \quad (7)$$

In (5) the target's position and impulse response is represented in $\mathbf{h}_t$, and the target's Doppler is represented in $\tilde{\mathbf{S}}_d(m)$. Note that, if the target is of constant velocity, then ϕ will only be a function n .

In a similar manner the receiver response to the clutter is

$$y_c(t) = s(t) * c(t) \quad (8)$$

where $c(t)$ is the fast time complex random process due to clutter backscatter. Similarly, the sampled response to clutter can be written as

$$\mathbf{y}_c = \tilde{\mathbf{S}} \mathbf{c} \quad (9)$$

where the clutter vector, $\mathbf{c}$, is a column vector of size $(P+2(N-1))$ whose elements are the complex clutter impulse response in reverse range (i.e., fast time) order. The receiver response to noise can be written as

$$\mathbf{y}_n = \mathbf{n} \quad (10)$$

where $\mathbf{y}_n$ and $\mathbf{n}$ are column vectors of length $(P+N-1)$. The noise is assumed to be zero mean additive white Gaussian noise (AWGN). The combined receiver response then is

$$\mathbf{y} = \mathbf{y}_t + \mathbf{y}_c + \mathbf{y}_n = \alpha \tilde{\mathbf{S}}_d \mathbf{h}_t + \tilde{\mathbf{S}} \mathbf{c} + \mathbf{n}. \quad (11)$$

The interference response of the receiver is

$$\mathbf{y}_I = \mathbf{y}_c + \mathbf{y}_n = \tilde{\mathbf{S}} \mathbf{c} + \mathbf{n}. \quad (12)$$

The correlation matrix for the interference process can be determined as

$$\mathbf{R}_I = E\{\mathbf{y}_I \mathbf{y}_I^H\} = E\{(\tilde{\mathbf{S}} \mathbf{c} + \mathbf{n})(\tilde{\mathbf{S}} \mathbf{c} + \mathbf{n})^H\}, \quad (13)$$

where the superscript H indicates a conjugate transpose.

By assumption, both $\mathbf{c}$ and $\mathbf{n}$ are zero mean and independent processes, which gives

$E\{\tilde{\mathbf{S}} \mathbf{c} \mathbf{n}^H\} = E\{\mathbf{n} \mathbf{c}^H \tilde{\mathbf{S}}^H\} = 0$. Additionally, $\mathbf{n}$ is AWGN, which gives $E\{\mathbf{n} \mathbf{n}^H\} = \sigma_n^2 \mathbf{I}$, where σ_n^2 is the noise variance (i.e., power) and $\mathbf{I}$ is a $(P+N-1) \times (P+N-1)$ identity matrix. Next, we assume that the clutter voltages are uncorrelated cell to cell. The clutter cell-to-cell uncorrelated assumption results from the clutter voltage from each cell having a random initial phase. This phase is uniformly distributed 0 to 2π and will be independent from cell to cell. This gives

$$\mathbf{R}_c = E\{\mathbf{c} \mathbf{c}^H\} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & \sigma_{P+2(N-1)}^2 \end{bmatrix}, \quad (14)$$

where σ_i^2 is the clutter variance (power) at range cell i . Having (14) now allows us to write the interference correlation matrix as

$$\mathbf{R}_I = \mathbf{R}_{y_c} + \mathbf{R}_{y_n} = \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H + \sigma_n^2 \mathbf{I}. \quad (15)$$

Clutter seen by a moving radar is addressed in a subsequent section.

The assumption that clutter is a compound Gaussian process is now invoked. This assumption has been verified in [6]. The compound Gaussian model states that the clutter voltage at any range cell i at time t is defined as

$$c_i(t) = \sigma_i g_i(t), \quad (16)$$

where σ_i is a random variable that is equal to the variance (power) at range cell i , and $g_i(t)$ is a unity variance complex Gaussian process that accounts for the pulse-to-pulse variance of the clutter complex amplitude. The correlation function of the clutter is

$$R_g^i(\tau) = E\{g_i(t+\tau)g_i^*(t)\}, \quad (17)$$

which defines the clutter spectral characteristics. (The autocorrelation of a random process and its power spectral density form a Fourier transform pair.) The process σ_i is often called the

texture, and $g_i(t)$ is called the speckle. This model accounts for the significant changes of clutter amplitude from range cell to range cell as well as the Doppler spectrum properties of the clutter. The clutter Doppler spectrum then is the Fourier transform of the time-correlation function.

If the clutter is compound Gaussian and the clutter variance is known, then the resulting clutter distribution is Gaussian. In other words, if σ_i^2 is known (through online measurement and estimation or through clutter modeling), then $\mathbf{R}_c$ in (14) is known. This in turn means the interference process as perceived by the radar receiver in $\mathbf{y}_I$ is a Gaussian random process whose correlation matrix is determined in (15) above. If the interference process is Gaussian, then it was shown in [4], eq. (38), that the Neyman-Pearson (N-P) detector can be formed by

$$\begin{array}{c} H_1 \\ \left| \mathbf{h}_t^H \tilde{\mathbf{S}}_d^H \mathbf{R}_I^{-1} \mathbf{y} \right| > \eta \\ H_0 \end{array} \quad (18)$$

where the value of η is chosen to achieve the desired probability of false alarm (Pfa).

3.1.2. Multiple-Pulse Detector

The receiver response to the target is determined first. Under the slow-moving target assumption, the target response is identical from pulse to pulse except for the phase change that is imparted due to the target's motion from pulse to pulse. Thus the receiver response to the target can be represented as stacked vector

$$\mathbf{Y}_t = \alpha \begin{bmatrix} u_0 \tilde{\mathbf{S}}_d(0) \mathbf{h}_t \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_d(M-1) \mathbf{h}_t \end{bmatrix}, \quad (19)$$

where u_i accounts for the phase change from pulse to pulse and is computed as

$$u_i = \exp(j4\pi \frac{R(iT_i)}{\lambda}), \quad (20)$$

and λ is the wavelength and T_i is the pulse repetition interval (PRI). The phase change is proportional to the range change as a function of time and is generally ascribed to the Doppler effect. For the case of a non-accelerating target (i.e., $\tilde{\mathbf{S}}_d$ is independent of pulse number), the Kronecker product $\otimes$ (see [7], pp. 406-407) allows (19) to be written compactly as

$$\mathbf{Y}_t = \mathbf{u} \otimes \tilde{\mathbf{S}}_d \mathbf{h}_t, \quad (21)$$

where $\mathbf{u}$ is a column vector whose elements are defined in (20). The size of $\mathbf{u}$ is $M \times 1$, where M is the number of pulses. Therefore, the size of $\mathbf{Y}_t$ is $M(N+P-1) \times 1$.

Next, the pulse-to-pulse clutter is determined. Using the compound Gaussian model, the time correlation function is designated $R_g(\tau)$ defined in (14). If we assume that the clutter over the

whole range extent is the same type (e.g., sea clutter), then it is not unrealistic to expect that $R_g(\tau)$ is the same for every range cell. This means that, for every range cell i , there is a random draw of the random variable σ_i determining its variance (power) and a random draw of the stationary random process $g(t)$ that is zero mean, unity variance complex Gaussian that is correlated in time according to $R_g(\tau)$. Note that the random and independent initial phase of the clutter voltage is uncorrelated from range cell to range cell as previously discussed. Also, it will be assumed that the complex Gaussian random processes from one range cell to the next are uncorrelated. Under these assumptions, we can define $\mathbf{c}(t)$ as the column vector of the clutter amplitudes at slow time t (noting that $\mathbf{c}(t)$ is a complex random process). Now write the stacked vector representing the response of the receiver due to clutter as

$$\mathbf{Y}_c = \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{c}(0) \\ \tilde{\mathbf{S}}\mathbf{c}((i-1)T_i) \\ \vdots \\ \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i) \end{bmatrix}. \quad (22)$$

The clutter correlation matrix for the stacked vector $\mathbf{Y}_c$ is determined as

$$\mathbf{R}_{Y_c} = \mathbf{E}\{\mathbf{Y}_c \mathbf{Y}_c^H\} = \mathbf{E}\left\{ \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{c}(0)\mathbf{c}^H(0)\tilde{\mathbf{S}}^H & \cdots & \tilde{\mathbf{S}}\mathbf{c}(0)\mathbf{c}^H((M-1)T_i)\tilde{\mathbf{S}}^H \\ & \ddots & \\ \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i)\mathbf{c}^H(0)\tilde{\mathbf{S}}^H & \cdots & \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i)\mathbf{c}^H((M-1)T_i)\tilde{\mathbf{S}}^H \end{bmatrix} \right\}. \quad (23)$$

This can be rewritten as

$$\mathbf{R}_{Y_c} = \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{E}\{\mathbf{c}(0)\mathbf{c}^H(0)\}\tilde{\mathbf{S}}^H & \cdots & \tilde{\mathbf{S}}\mathbf{E}\{\mathbf{c}(0)\mathbf{c}^H((M-1)T_i)\}\tilde{\mathbf{S}}^H \\ & \ddots & \\ \tilde{\mathbf{S}}\mathbf{E}\{\mathbf{c}((M-1)T_i)\mathbf{c}^H(0)\}\tilde{\mathbf{S}}^H & \cdots & \tilde{\mathbf{S}}\mathbf{E}\{\mathbf{c}((M-1)T_i)\mathbf{c}^H((M-1)T_i)\}\tilde{\mathbf{S}}^H \end{bmatrix}. \quad (24)$$

Each expectation in (21) can be represented as

$$\mathbf{E}\{\mathbf{c}(jT_i)\mathbf{c}^H(kT_i)\} = \begin{bmatrix} \sigma_1^2 R_g^1((j-k)T_i) & & 0 \\ & \ddots & \\ 0 & & \sigma_{P+2(N-1)}^2 R_g^{P+2(N-1)}((j-k)T_i) \end{bmatrix}. \quad (25)$$

The result in (25) is obtained by applying (14) and invoking the assumption that the clutter is zero mean and uncorrelated cell to cell. Under the assumption that the clutter has the same Doppler spectrum in each cell, it can be further simplified to (21). In that case, define

$$\rho_{j,k} = R_g((j-k)T_i). \quad (26)$$

This allows (21) to be written as

$$\mathbf{R}_{Y_c} = \begin{bmatrix} \rho_{1,1}\tilde{\mathbf{S}}\mathbf{R}_c\tilde{\mathbf{S}}^H & & \rho_{1,M}\tilde{\mathbf{S}}\mathbf{R}_c\tilde{\mathbf{S}}^H \\ & \ddots & \\ \rho_{M,1}\tilde{\mathbf{S}}\mathbf{R}_c\tilde{\mathbf{S}}^H & & \rho_{M,M}\tilde{\mathbf{S}}\mathbf{R}_c\tilde{\mathbf{S}}^H \end{bmatrix} = \mathbf{M}_c \otimes \tilde{\mathbf{S}}\mathbf{R}_c\tilde{\mathbf{S}}^H, \quad (27)$$

where

$$\mathbf{M}_c = \begin{bmatrix} \rho_{1,1} & & \rho_{1,M} \\ & \ddots & \\ \rho_{M,1} & & \rho_{M,M} \end{bmatrix}. \quad (28)$$

Remember that $\mathbf{R}_c$ is diagonal since clutter is uncorrelated range cell to range cell. Note also that $\mathbf{M}_c$ is size $M \times M$. The complete interference correlation matrix can be written as

$$\mathbf{R}_I = \mathbf{R}_{y_c} + \sigma_n^2 \mathbf{I} \quad (29)$$

where $\mathbf{R}_{y_c}$ is defined in (27) and (28). Note that $\mathbf{R}_I$, $\mathbf{R}_{y_c}$, and $\mathbf{I}$ are size

$M(P+N-1) \times M(P+N-1)$. Under the assumption that the Doppler spectrum is the same for all range cells, the correlation matrix for the interference process can be written as

$$\mathbf{R}_I = \mathbf{M}_c \otimes \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H + \sigma_n^2 \mathbf{I}. \quad (30)$$

With the signal model and interference model established, the N-P detector for the h_t can be written as

$$\left[\begin{array}{c} u_0 \tilde{\mathbf{S}}_d(0) \mathbf{h}_t \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_d(M-1) \mathbf{h}_t \end{array} \right]^H \mathbf{R}_I^{-1} \mathbf{Y} \left[\begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array} \right] \eta \quad (31)$$

where η is the detection threshold, which is set to the specified probability of false alarm.

Next, the case of non-homogeneous clutter is considered. This situation will arise when there are clutter boundaries, such as the boundary between land and sea. This situation can be handled by defining the diagonal matrix $\boldsymbol{\rho}_{j,k}$ of size $(P+2(N-1)) \times (P+2(N-1))$. The diagonal elements are determined as

$$\rho_{j,k(i,i)} = R_g^i((j-k)T_i), \quad (32)$$

where $R_g^i(t)$ is defined in (17) and i indicates the range cell number. With this, (25) can be rewritten as

$$E\{\mathbf{c}(jT_i)\mathbf{c}^H(kT_i)\} = \boldsymbol{\rho}_{j,k} \mathbf{R}_c. \quad (33)$$

With (33), the interference correlation matrix can be rewritten as

$$\mathbf{R}_I = \begin{bmatrix} \tilde{\mathbf{S}} \boldsymbol{\rho}_{1,1} \mathbf{R}_c \tilde{\mathbf{S}}^H & & \tilde{\mathbf{S}} \boldsymbol{\rho}_{1,M} \mathbf{R}_c \tilde{\mathbf{S}}^H \\ & \ddots & \\ \tilde{\mathbf{S}} \boldsymbol{\rho}_{M,1} \mathbf{R}_c \tilde{\mathbf{S}}^H & & \tilde{\mathbf{S}} \boldsymbol{\rho}_{M,M} \mathbf{R}_c \tilde{\mathbf{S}}^H \end{bmatrix} + \sigma_n^2 \mathbf{I}. \quad (34)$$

3.2. Detector for Range Migrating Target in Stationary Clutter

For this case, the target is moving so fast that its movement pulse to pulse relative to the sample interval is such that the inequality, $\dot{R}(t)MT_i \ll \frac{cT_s}{2}$, is not satisfied. That is, the target will move a significant fraction of a range cell from one pulse to the next. This will require some

form of range migration compensation such that the target returns can be integrated up. The target model will first address the range migration issue from pulse to pulse and then the effect of pulse distortion, which is caused by range migration within a pulse receive time. Finally, the Doppler effect will be included with the range migration effect.

Now the target model will be restated as

$$y_t(t) = s(t) * \alpha h_t(t - \frac{2R(t)}{c}), \quad (35)$$

where the fact that the target range changes with time is shown explicitly. In this case, the range change from pulse to pulse must be accounted for in the detector.

As before, (32) will be approximated by the discrete time convolution indicated by

$$y_t(kT_s) = \alpha \sum_{i=0}^{N-1} s(iT_s) h_t((k+1-i)T_s - \frac{2R(t)}{c}), \quad (36)$$

where, again, the time dependence of $R(t)$ is indicated. This time dependence has the effect of moving the target some significant fraction (or whole) sample times, T_s . While (33) is accurate, it is not in a convenient form. To address this, the target impulse response is redefined as

$$h_{t,R(\Delta T)}(t) = h_t(t - \frac{2R(\Delta T)}{c}), \quad (37)$$

where ΔT is the time since the beginning of the CPI. Thus, (37) accounts for the range of the target and its change over the course of the CPI. Next, we will show how to determine $h_{t,R(\Delta T)}(t)$ in terms of $h_{t,R(0)}(t)$. Remembering that $h_{t,R(0)}(t)$ is effectively discrete time sampled, we can use (3.64) from [8] to account for the sampling time offset induced by the range change of $R(\Delta T)$. This will account for fractional shifts in the target position (as well as whole sample shifts).

Thus, we have

$$h_{t,R(\Delta T)}(kT_s) = \sum_l h_{t,R(0)}(lT_s) \text{sinc}((kT_s - \Delta T_s - lT_s) / T_s), \quad (38)$$

where $\Delta T_s = 2(R(\Delta T) - R(0)) / c$ and $\text{sinc}(x) = \sin(\pi x) / (\pi x)$. Note that (38) can be interpreted as the discrete time convolution of $h_{t,R(0)}(t)$ with $\text{sinc}(t)$. Now, to determine $y_t(kT_s)$ for subsequent pulses we can write

$$y_t(kT_s) = \alpha \sum_{i=0}^{N-1} s(iT_s) h_{t,R(\Delta T)}((k+1-i)T_s), \quad (39)$$

where $\Delta T = (m-1)T_i$ and m = pulse number.

Next, we apply (38) to give

$$y_t(kT_s) = \alpha \sum_{i=0}^{N-1} s(iT_s) \sum_l h_{t,R(0)}(lT_s) \text{sinc}(((k+1-i)T_s - \Delta T_s - lT_s) / T_s), \quad (40)$$

where $\Delta T_s = 2(R((m-1)T_i) - R(0)) / c$. Equation (40) can be rewritten to change the order of convolution to give

$$y_t(kT_s) = \alpha \sum_l h_{t,R(0)}(lT_s) \sum_{i=0}^{N-1} s(iT_s) \text{sinc}(((k+1-i)T_s - \Delta T_s - lT_s)/T_s). \quad (41)$$

The target model as stated in (38) allows expression of the target model in a manner similar to (5) as

$$\mathbf{y}_t = \alpha \tilde{\mathbf{R}}_m(i) \tilde{\mathbf{S}}_d \mathbf{h}_{t,R(0)}, \quad (42)$$

where $h_{t,R(0)} = h_t$, $\tilde{\mathbf{S}}_d$ is defined in (6) and $\tilde{\mathbf{R}}_m(m)$ is the range migration matrix that convolves the sinc() function whose elements are determined as

$$\tilde{\mathbf{R}}_m(m)_{(k,l)} = \text{sinc}(((k-l)T_s - \Delta T_s)/T_s), \quad (43)$$

where $\Delta T_s = 2(R(mT_i + lT_s) - R(0))/c$, and $m \in \{0, \dots, M-1\}$. ΔT_s is a function of l , the sample number, as well as the pulse number, m . Including lT_s accounts for the target motion over the time of the pulse width. Making note that $\mathbf{h}_{t,R(0)}$ is a column vector of size $(P+2(N-1)) \times 1$, $\tilde{\mathbf{S}}_d$ is a rectangular matrix of size $(P+N-1) \times (P+2(N-1))$ and $\tilde{\mathbf{R}}_m(m)$ is a square matrix of size $(P+N-1) \times (P+N-1)$.

Using (41) we can write the complete (for all pulses) target model as

$$\mathbf{Y}_t = \alpha \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \mathbf{h}_{t,R(0)} \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \mathbf{h}_{t,R(0)} \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \mathbf{h}_{t,R(0)} \end{bmatrix}. \quad (44)$$

The detector for fast targets then will be

$$\left| \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \mathbf{h}_{t,R(0)} \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \mathbf{h}_{t,R(0)} \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \mathbf{h}_{t,R(0)} \end{bmatrix}^H \mathbf{R}_I^{-1} \mathbf{Y} \right| \begin{matrix} H_1 \\ > \\ < \\ H_0 \end{matrix} \eta, \quad (45)$$

where $\mathbf{R}_I$ is defined in (29).

4.0 PERFORMANCE EQUATIONS

4.1. Correlator and Moving Target Detector

The processing often done in multiple pulse radars is correlation (i.e., matched filter) in fast time and moving target detection (MTD) in slow time to integrate returns over slow time. Note that MTD is equivalent to pulse Doppler processing. This processing is optimal for targets immersed in AWGN.

4.1.1. Non-Range Migrating Target

To simplify this development, the target acceleration will be ignored (targets are assumed to be constant velocity). Including target acceleration is straightforward as shown in previous sections.

On a per pulse basis, the output of the correlator at range cell k given the receive data vector $\mathbf{y}$ is

$$x_{corr} = \boldsymbol{\delta}_k^H \tilde{\mathbf{S}}_d^H \mathbf{y}, \quad (46)$$

noting that x_{corr} is a scalar. The multiplication by $\tilde{\mathbf{S}}_d^H$ performs the operation of correlation on $\mathbf{y}$ with the transmitted signal $\mathbf{s}$. The multiplication by $\boldsymbol{\delta}_k^H$ sifts out the response of the correlator to a target at range cell k . The output of the MTD Doppler filter matched to the target Doppler is

$$\mathbf{z}_{corr+MTD} = \mathbf{u}^H \begin{bmatrix} x_{corr_1} \\ \vdots \\ x_{corr_M} \end{bmatrix} \quad (47)$$

where x_{corr_i} is the output of the correlator from the i 'th pulse received. The correlator response to a point target for the i 'th pulse is

$$x_{corr_i} = \boldsymbol{\delta}_k^H \tilde{\mathbf{S}}_d^H \mathbf{y}_{T_i} = u_i \alpha \boldsymbol{\delta}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \boldsymbol{\delta}_k. \quad (48)$$

Using (47), the received power due to the target is

$$\left| \mathbf{z}_{corr+MTD_T} \right|^2 = |\alpha|^2 \left| \mathbf{u}^H \begin{bmatrix} u_0 \boldsymbol{\delta}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \boldsymbol{\delta}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \boldsymbol{\delta}_k \end{bmatrix} \right|^2. \quad (49)$$

On a per pulse basis, the response of the correlator to the interference is

$$x_{corr_{i_i}} = \boldsymbol{\delta}_k^H \tilde{\mathbf{S}}_d^H (\tilde{\mathbf{S}} \mathbf{c}((i-1)T_i) + \mathbf{n}). \quad (50)$$

Based on (47), the output of the Correlator + MTD to the interference is

$$\mathbf{z}_{corr+MTD_I} = \mathbf{u}^H ((\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{c}(0) + \mathbf{n} \\ \vdots \\ \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i) + \mathbf{n} \end{bmatrix}), \quad (51)$$

where the multiplication by $(\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H)$ is understood to be done on every element of the matrix, where each element is in the form of $\tilde{\mathbf{S}}\mathbf{c}((m-1)T_i) + \mathbf{n}$. The power of the interference in the output of Correlator + MTD is

$$\mathbb{E}\{|\mathbf{z}_{corr+MTD_I}|^2\} = \mathbb{E}\{\mathbf{u}^H ((\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{c}(0) + n \\ \vdots \\ \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i) + n \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{c}(0) + n \\ \vdots \\ \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i) + n \end{bmatrix}^H (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k)) \mathbf{u}\}, \quad (52)$$

which can be factored as

$$\mathbb{E}\{|\mathbf{z}_{corr+MTD_I}|^2\} = \mathbf{u}^H ((\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbb{E}\{ \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{c}(0) + n \\ \vdots \\ \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i) + n \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{S}}\mathbf{c}(0) + n \\ \vdots \\ \tilde{\mathbf{S}}\mathbf{c}((M-1)T_i) + n \end{bmatrix}^H \}) (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k)) \mathbf{u}. \quad (53)$$

By applying (23) through (29), we have

$$|\mathbf{z}_{corr+MTD_I}|^2 = \mathbf{u}^H ((\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k)) \mathbf{u}. \quad (54)$$

If the clutter has the same Doppler spectrum for all range cells, then (30) can be applied giving

$$|\mathbf{z}_{corr+MTD_I}|^2 = \mathbf{u}^H ((\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) (\mathbf{M}_c \otimes \tilde{\mathbf{S}}\mathbf{R}_c \tilde{\mathbf{S}}^H + \sigma_n^2 \mathbf{I}) (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k)) \mathbf{u}. \quad (55)$$

Equation (55) can be rewritten as

$$|\mathbf{z}_{corr+MTD_I}|^2 = \mathbf{u}^H ((\mathbf{M}_c \otimes (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}\mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k)) + \sigma_n^2 \mathbf{I} \otimes \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k) \mathbf{u}, \quad (56)$$

where $\mathbf{I}$ is the identity matrix of size $M \times M$.

Now applying (49) and (56), the SIR for the Correlator + MTD can be determined as

$$SIR_{corr+MTD} = \frac{|\alpha|^2 \mathbf{u}^H \begin{bmatrix} u_0 \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k \\ \vdots \\ u_{M-1} \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k \end{bmatrix}}{\mathbf{u}^H ((\mathbf{M}_c \otimes (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}\mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k)) + \sigma_n^2 \mathbf{I} \otimes \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k) \mathbf{u}}. \quad (57)$$

4.1.2. Range Migrating Target

Next, the Correlator + MTD is evaluated for performance against fast-moving targets. That is, targets that are subject to range migration. First is the SIR for the Correlator + MTD when the detector is not compensating for the range migration of the targets. Under this assumption, the target model is

$$\mathbf{y}_{T_i} = \alpha u_i \tilde{\mathbf{R}}_m(m) \tilde{\mathbf{S}}_d \delta_k. \quad (58)$$

Based on this, the output of the Correlator + MTD for a point target is

$$\mathbf{z}_{corr+MTD} = \alpha \mathbf{u}^H \begin{bmatrix} u_0 \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d \delta_k \\ \vdots \\ u_{M-1} \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d \delta_k \end{bmatrix}. \quad (59)$$

This, in turn, gives the power output of the detector due to the target as

$$|\mathbf{z}_{corr+MTD}|^2 = |\alpha|^2 \left| \mathbf{u}^H \begin{bmatrix} u_0 \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d \delta_k \\ \vdots \\ u_{M-1} \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d \delta_k \end{bmatrix} \right|^2. \quad (60)$$

Since there is no range migration compensation, the interference power output of the Correlator + MTD detector is the same as for the slow target case above. Therefore, the SIR for this case is

$$SIR_{corr+MTD} = \frac{|\alpha|^2 \left| \mathbf{u}^H \begin{bmatrix} \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d \delta_k \\ \vdots \\ \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d \delta_k \end{bmatrix} \right|^2}{\mathbf{u}^H ((\mathbf{M}_c \otimes (\delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \mathbf{R}_c \tilde{\mathbf{S}}_d \tilde{\mathbf{S}}_d \delta_k)) + \sigma_n^2 \mathbf{I} \otimes \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{S}}_d \delta_k) \mathbf{u}}. \quad (61)$$

It is expected that, as range migration comes more into play, the Correlator + MTD is suboptimal, therefore degrading the output SIR. This is what (61) allows us to calculate. However, it is straightforward to modify the Correlator + MTD to account for range migration. Under this condition, the Correlator + MTD retains its optimality when used in AWGN alone. In order to determine its performance, the SIR for this case will be derived. The output of the correlator will be

$$corr_i = \delta_k^H \tilde{\mathbf{S}}_d^H \alpha u_i \tilde{\mathbf{R}}'_m(m) \tilde{\mathbf{R}}_m(m) \tilde{\mathbf{S}}_d \delta_k \quad (62)$$

where $\tilde{\mathbf{R}}'_m(m)$ is defined the same as $\tilde{\mathbf{R}}_m(m)$ except $\Delta T_s = 2(R(0) - R(mT_i + kT_s))/c$. Thus, multiplication by $\tilde{\mathbf{R}}'_m(m)$ compensates for the target range migration producing $\tilde{\mathbf{R}}'_m(m) \tilde{\mathbf{R}}_m(m) \approx \mathbf{I}$. The output of the Correlator + MTD due to the target will be the same as in the slow target case under the assumption that the range migration has been perfectly compensated. The output of the detector due to interference will change, since the range migration compensation will impact the clutter. Thus the output of the detector due to interference is

$$\mathbf{z}_{corr+MTD_i} = \mathbf{u}^H \left(\begin{bmatrix} \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}}_d \delta_k \\ \vdots \\ \delta_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}}_d \delta_k \end{bmatrix} + \mathbf{n} \right). \quad (63)$$

This can be rewritten as

$$\mathbf{z}_{corr+MTD_i} = \mathbf{u}^H (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \left(\begin{bmatrix} \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{c}(0) \\ \vdots \\ \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{c}((M-1)T_i) \end{bmatrix} + \mathbf{n} \right), \quad (64)$$

where it is understood that $(\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H)$ multiplies each element in the vector where each element is in the form of $\tilde{\mathbf{R}}'_m(i-1) \tilde{\mathbf{S}} \mathbf{c}((i-1)T_i)$. From this, the output power due to interference will be

$$\begin{aligned} \mathbb{E}\{|\mathbf{z}_{corr+MTD_i}|^2\} &= \mathbf{u}^H (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbb{E}\left\{ \begin{bmatrix} \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{c}(0) \\ \vdots \\ \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{c}((M-1)T_i) \end{bmatrix} + \mathbf{n} \right\} \cdots \\ &\quad \left(\begin{bmatrix} \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{c}(0) \\ \vdots \\ \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{c}((M-1)T_i) \end{bmatrix} + \mathbf{n} \right)^H \} (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k) \mathbf{u}. \end{aligned} \quad (65)$$

Next, to perform the manipulations to get (65) in a more convenient form, we will simplify notations as follows, $\mathbf{c}'(i) = \mathbf{c}(iT_i)$, $0 \leq i \leq M-1$, and defining $\mathbf{R}_{zc}$ as

$$\mathbf{R}_{zc} = \mathbb{E}\left\{ \begin{bmatrix} \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{c}'(0) \\ \vdots \\ \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{c}'(M-1) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{c}'(0) \\ \vdots \\ \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{c}'(M-1) \end{bmatrix}^H \right\}. \quad (66)$$

Based on this the interference, power can be written as

$$\mathbb{E}\{|\mathbf{z}_{corr+MTD_i}|^2\} = \mathbf{u}^H (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) (\mathbf{R}_{zc} + \sigma_n^2 \mathbf{I}) (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k) \mathbf{u}. \quad (67)$$

Equation (66) can be factored as

$$\mathbf{R}_{zc} = \begin{bmatrix} \mathbf{a}_{1,1} & & \mathbf{a}_{1,2} \\ & \ddots & \\ \mathbf{a}_{2,1} & & \mathbf{a}_{2,2} \end{bmatrix}, \quad (68)$$

where

$$\begin{aligned} \mathbf{a}_{1,1} &= \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbb{E}\{\mathbf{c}'(0) \mathbf{c}'^H(0)\} \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(0), \\ \mathbf{a}_{2,1} &= \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbb{E}\{\mathbf{c}'(M-1) \mathbf{c}'^H(0)\} \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(0), \\ \mathbf{a}_{1,2} &= \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbb{E}\{\mathbf{c}'(0) \mathbf{c}'^H(M-1)\} \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(M-1), \\ \mathbf{a}_{2,2} &= \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbb{E}\{\mathbf{c}'(M-1) \mathbf{c}'^H(M-1)\} \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(M-1), \end{aligned}$$

and (25) can be used to determine $\mathbb{E}\{\mathbf{c}'(j) \mathbf{c}'^H(k)\}$. Under the assumption that all range cells have the same Doppler spectrum, $\mathbf{R}_{zc}$ can be simplified to

$$\mathbf{R}_{zc} = \begin{bmatrix} \rho_{1,1} \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(0) & & \rho_{1,M} \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(M-1) \\ & \ddots & \\ \rho_{M,1} \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(0) & & \rho_{M,M} \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m'^H(M-1) \end{bmatrix}. \quad (69)$$

Using (54), (67), and (69), the SIR for the Correlator + MTD matched to the fast target can be written as

$$SIR_{corr+MTD} = \frac{|\alpha|^2 \mathbf{A}_1}{\mathbf{u}^H (\mathbf{A}_2 + \sigma_n^2 \mathbf{I} \otimes \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k) \mathbf{u}}. \quad (70)$$

where

$$\mathbf{A}_1 = \left| \mathbf{u}^H \begin{bmatrix} u_1 \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k \\ \vdots \\ u_{M-1} \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k \end{bmatrix} \right|^2, \text{ and}$$

$$\mathbf{A}_2 = \begin{bmatrix} \rho_{1,1} \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m^H(0) \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k & \rho_{1,M} \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(0) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m^H(M-1) \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k \\ \vdots & \vdots \\ \rho_{M,1} \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m^H(0) \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k & \rho_{M,M} \tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H \tilde{\mathbf{R}}'_m(M-1) \tilde{\mathbf{S}} \mathbf{R}_c \tilde{\mathbf{S}}^H \tilde{\mathbf{R}}_m^H(M-1) \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k \end{bmatrix}.$$

4.2. Optimum Detector Performance

4.2.1. SIR for Optimum Detector for Slow-Moving Target

The detector for the slow-moving (no range migration) target is defined in (31). This detector is an optimum detector for the assumptions defined, including no range migration of the target and that the interference matrix is known a priori. In this section, the output SIR of the detector is determined. This will allow this detector to be compared to other detectors. It will also allow the assessment of performance when the parameters of the signal and interference processes are not exactly known. In this development, the target is assumed to be a point target. Under this assumption, the test statistic from the detector is

$$z_{Slow} = \mathbf{u}^H \otimes (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} \mathbf{Y}. \quad (71)$$

From the target model (21), the received data due to a point target is

$$\mathbf{Y}_T = \alpha \mathbf{u} \otimes \tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k. \quad (72)$$

The signal power level at the output of the detector is

$$|z_{Slow_T}|^2 = |\alpha|^2 \left| \mathbf{u}^H \otimes (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} \mathbf{u} \otimes (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k) \right|^2. \quad (73)$$

The output of the detector due to interference is

$$z_{Slow_I} = \mathbf{u} \otimes (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} (\mathbf{Y}_c + \mathbf{Y}_n). \quad (74)$$

The power from the interference is determined as

$$E\{z_{Slow_I} z_{Slow_I}^*\} = E\{\mathbf{u}^H \otimes (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} (\mathbf{Y}_c + \mathbf{Y}_n) (\mathbf{Y}_c + \mathbf{Y}_n)^H \mathbf{R}_I^{-1} \mathbf{u} \otimes (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k)\}, \quad (75)$$

which can be factored into

$$E\{z_{Slow_I} z_{Slow_I}^*\} = \mathbf{u}^H \otimes (\tilde{\boldsymbol{\delta}}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} E\{(\mathbf{Y}_c + \mathbf{Y}_n)(\mathbf{Y}_c + \mathbf{Y}_n)^*\} \mathbf{R}_I^{-1} \mathbf{u} \otimes (\tilde{\mathbf{S}}_d \tilde{\boldsymbol{\delta}}_k). \quad (76)$$

Recognizing that $\mathbf{R}_I = E\{(\mathbf{Y}_c + \mathbf{Y}_n)(\mathbf{Y}_c + \mathbf{Y}_n)^*\}$ allows simplifying (59) to

$$E\{z_{Slow_I} z_{Slow_I}^*\} = \mathbf{u}^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} \mathbf{u} \otimes (\tilde{\delta}_k \tilde{\mathbf{S}}_d). \quad (77)$$

This gives the SIR for the slow detector as

$$SIR_{Slow} = \frac{|\alpha|^2 \left| \mathbf{u}^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} \mathbf{u} \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k) \right|^2}{\mathbf{u}^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} \mathbf{u} \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k)}, \quad (78)$$

which can be simplified to

$$SIR_{Slow} = |\alpha|^2 \mathbf{u}^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_I^{-1} \mathbf{u} \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k). \quad (79)$$

Next, the SIR for a mismatched detector will be determined. To begin the target pulse-to-pulse phase, the change vector used by the detector is designated $\mathbf{u}_D$, while the detector's interference correlation matrix is

$$\mathbf{R}_{I_D} = \mathbf{M}_{c_D} \otimes \tilde{\mathbf{S}} \mathbf{R}_{c_D} \tilde{\mathbf{S}}^H + \sigma_{n_D}^2 \mathbf{I}, \quad (80)$$

where the subscript D denotes the detector's assumed parameter. In a similar manner, the target's actual pulse-to-pulse phase change vector is designated $\mathbf{u}_A$, and the actual interference correlation matrix is

$$\mathbf{R}_{I_A} = \mathbf{M}_{c_A} \otimes \tilde{\mathbf{S}} \mathbf{R}_{c_A} \tilde{\mathbf{S}}^H + \sigma_{n_A}^2 \mathbf{I}. \quad (81)$$

The subscript A denotes the actual value of the parameter. Based on (71), the detector test statistic becomes

$$z_{Slow} = \mathbf{u}_D^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) (\mathbf{M}_{c_D} \otimes \tilde{\mathbf{S}} \mathbf{R}_{c_D} \tilde{\mathbf{S}}^H + \sigma_{n_D}^2 \mathbf{I})^{-1} \mathbf{Y} = \mathbf{u}_D^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_{I_D}^{-1} \mathbf{Y}. \quad (82)$$

The input to the detector from the target is

$$\mathbf{Y}_T = \alpha \mathbf{u}_A \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k). \quad (83)$$

Applying this to (71) gives the mismatched detector's response to the target as

$$z_{Slow_T} = \mathbf{u}_D^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_{I_D}^{-1} \alpha \mathbf{u}_A \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k). \quad (84)$$

The power due to the target in the mismatched detector is

$$\left| z_{Slow_T} \right|^2 = |\alpha|^2 \left| \mathbf{u}_D^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_{I_D}^{-1} \mathbf{u}_A \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k) \right|^2. \quad (85)$$

In a similar way, the interference induced in the mismatched slow detector can be shown to be

$$E\{|z_{Slow_I}|^2\} = \mathbf{u}_D^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_{I_D}^{-1} \mathbf{R}_{I_A} \mathbf{R}_{I_D}^{-1} \mathbf{u}_D \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k). \quad (86)$$

This gives the SIR for the mismatched detector as

$$SIR_{Slow_{Mismatched}} = \frac{|\alpha|^2 \left| \mathbf{u}_D^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_{I_D}^{-1} \mathbf{u}_A \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k) \right|^2}{\mathbf{u}_D^H \otimes (\tilde{\delta}_k^H \tilde{\mathbf{S}}_d^H) \mathbf{R}_{I_D}^{-1} \mathbf{R}_{I_A} \mathbf{R}_{I_D}^{-1} \mathbf{u}_D \otimes (\tilde{\mathbf{S}}_d \tilde{\delta}_k)}. \quad (87)$$

4.2.2. SIR for Optimum Detector for Fast-Moving Targets

The detector is presented in (42). In this section, the target will be assumed to be a point target. The test statistic is computed as

$$\mathbf{z}_{Fast} = \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_I^{-1} \mathbf{Y}. \quad (88)$$

From the target model (41), the power output of the detector due to the target is

$$|z_{Fast_T}|^2 = |\alpha|^2 \left[\begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix} \right]. \quad (89)$$

The output of the detector due to the interference is

$$\mathbf{z}_{Fast_I} = \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_I^{-1} \mathbf{Y}_I. \quad (90)$$

The power of the interference is found by taking the expectation of $|z_{Fast_I}|^2$ giving

$$|z_{Fast_I}|^2 = \mathbb{E} \left\{ \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_I^{-1} \mathbf{Y}_I \mathbf{Y}_I^H \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix} \right\}. \quad (91)$$

This can be factored to produce

$$|z_{Fast_I}|^2 = \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_I^{-1} \mathbb{E} \{ \mathbf{Y}_I \mathbf{Y}_I^H \} \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}, \quad (92)$$

which can be simplified to

$$|z_{Fast_I}|^2 = \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}, \quad (93)$$

Combining (72) and (75) allows computing the SIR for the fast detector as

$$SIR_{Fast} = |\alpha|^2 \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_1 \tilde{\mathbf{R}}_m(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}. \quad (94)$$

Next, the SIR for a mismatched detector will be determined. To begin the target pulse-to-pulse phase, the change vector used by the detector is designated $\mathbf{u}_D$, $\tilde{\mathbf{R}}_{m_D}$ is the target range migration matrix used by the detector, and the detector's interference correlation matrix is $\mathbf{R}_{I_D}$ as defined in (63) where the subscript D denotes the detector's assumed parameter. In a similar manner, the target's actual pulse-to-pulse phase change vector is designated $\mathbf{u}_A$, $\tilde{\mathbf{R}}_{m_A}$ is the actual target range migration matrix, and the actual interference correlation matrix is $\mathbf{R}_{I_A}$ as defined in (81). The subscript A indicates the actual value.

The power output of the mismatched filter due to the target is

$$|z_{Fast_T}|^2 = |\alpha|^2 \begin{bmatrix} u_{D_0} \tilde{\mathbf{R}}_{m_D}(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_{D_1} \tilde{\mathbf{R}}_{m_D}(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{R}}_{m_D}(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_{I_D}^{-1} \begin{bmatrix} u_{A_0} \tilde{\mathbf{R}}_{m_A}(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_{A_1} \tilde{\mathbf{R}}_{m_A}(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{A_{M-1}} \tilde{\mathbf{R}}_{m_A}(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}. \quad (95)$$

To determine the mismatched filter interference power, (92) can be rewritten as

$$|z_{Fast_I}|^2 = \begin{bmatrix} u_{D_0} \tilde{\mathbf{R}}_{m_D}(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_{D_1} \tilde{\mathbf{R}}_{m_D}(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{R}}_{m_D}(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_{I_D}^{-1} \mathbb{E}\{\mathbf{Y}_I \mathbf{Y}_I^H\} \mathbf{R}_{I_D}^{-1} \begin{bmatrix} u_{D_0} \tilde{\mathbf{R}}_{m_D}(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_{D_1} \tilde{\mathbf{R}}_{m_D}(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{R}}_{m_D}(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}, \quad (96)$$

which can be simplified to

$$|z_{Fast_I}|^2 = \begin{bmatrix} u_{D_0} \tilde{\mathbf{R}}_{m_D}(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_{D_1} \tilde{\mathbf{R}}_{m_D}(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{R}}_{m_D}(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}^H \mathbf{R}_{I_D}^{-1} \mathbf{R}_{I_A} \mathbf{R}_{I_D}^{-1} \begin{bmatrix} u_{A_0} \tilde{\mathbf{R}}_{m_A}(0) \tilde{\mathbf{S}}_d(0) \delta_k \\ u_{A_1} \tilde{\mathbf{R}}_{m_A}(1) \tilde{\mathbf{S}}_d(1) \delta_k \\ \vdots \\ u_{A_{M-1}} \tilde{\mathbf{R}}_{m_A}(M-1) \tilde{\mathbf{S}}_d(M-1) \delta_k \end{bmatrix}. \quad (97)$$

The SIR for the fast mismatched filter is obtained by dividing (78) by (80) to produce

$$\begin{aligned}
 SIR_{FastMismatched} = & \frac{|\alpha|^2 \left[\begin{array}{c} u_{D_0} \tilde{\mathbf{R}}_{m_D} (0) \tilde{\mathbf{S}}_d (0) \delta_k \\ u_{D_1} \tilde{\mathbf{R}}_{m_D} (1) \tilde{\mathbf{S}}_d (1) \delta_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{R}}_{m_D} (M-1) \tilde{\mathbf{S}}_d (M-1) \delta_k \end{array} \right]^H \mathbf{R}_{I_D}^{-1} \left[\begin{array}{c} u_{A_0} \tilde{\mathbf{R}}_{m_A} (0) \tilde{\mathbf{S}}_d (0) \delta_k \\ u_{A_1} \tilde{\mathbf{R}}_{m_A} (1) \tilde{\mathbf{S}}_d (1) \delta_k \\ \vdots \\ u_{A_{M-1}} \tilde{\mathbf{R}}_{m_A} (M-1) \tilde{\mathbf{S}}_d (M-1) \delta_k \end{array} \right]^2}{\left[\begin{array}{c} u_{D_0} \tilde{\mathbf{R}}_{m_D} (0) \tilde{\mathbf{S}}_d (0) \delta_k \\ u_{D_1} \tilde{\mathbf{R}}_{m_D} (1) \tilde{\mathbf{S}}_d (1) \delta_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{R}}_{m_D} (M-1) \tilde{\mathbf{S}}_d (M-1) \delta_k \end{array} \right]^H \mathbf{R}_{I_D}^{-1} \mathbf{R}_{I_A} \mathbf{R}_{I_D}^{-1} \left[\begin{array}{c} u_{D_0} \tilde{\mathbf{R}}_{m_D} (0) \tilde{\mathbf{S}}_d (0) \delta_k \\ u_{D_1} \tilde{\mathbf{R}}_{m_D} (1) \tilde{\mathbf{S}}_d (1) \delta_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{R}}_{m_D} (M-1) \tilde{\mathbf{S}}_d (M-1) \delta_k \end{array} \right]}. \quad (98)
 \end{aligned}$$

5.0 OPTIMUM DETECTORS FOR MULTIPLE NON-IDENTICAL PULSES FOR TARGETS IN CLUTTER

This section describes the optimum detector for the case in which multiple pulses are transmitted coherently; it is desired to maximize detection probability for a given probability of false alarm in the presence of clutter. Normally, radars transmit identical pulses; however, recent papers ([9] and [10]) have shown how the pulses may be different and subject to constraints. This approach removes these constraints while producing an optimum detector (subject to the previously imposed assumptions.)

5.1. Optimum Detector

In this development, for simplicity of notation, we will assume that range migration can be ignored (i.e., $\dot{R}(t)MT_i \ll \frac{cT_s}{2}$), that the target acceleration can be ignored, and the target is a point target. Note that including range migration, target acceleration, and finite extent into this formulation is straightforward using the previously developed approach.

Each pulse transmitted is $\mathbf{s}_i$, where $\mathbf{s}_i$ is a vector containing the baseband samples of the i 'th pulse that is transmitted. This produces the respective signal convolution matrices as

$$\tilde{\mathbf{S}}_i = \begin{bmatrix} 0 & \cdots & 0 & \mathbf{s}_i^t \\ 0 & \cdots & \mathbf{s}_i^t & 0 \\ & \ddots & & \\ \mathbf{s}_i^t & 0 & \cdots & 0 \end{bmatrix}. \quad (99)$$

The received data due to a point target is then

$$\mathbf{Y}_T = \alpha \begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}. \quad (100)$$

In a similar manner, the received data due to clutter will be

$$\mathbf{Y}_c = \begin{bmatrix} \tilde{\mathbf{S}}_1 \mathbf{c}(0) \\ \tilde{\mathbf{S}}_2 \mathbf{c}(T_i) \\ \vdots \\ \tilde{\mathbf{S}}_M \mathbf{c}((M-1)T_i) \end{bmatrix}. \quad (101)$$

From this, the clutter correlation can be determined as

$$\mathbf{R}_{Y_c} = E\left\{ \begin{bmatrix} \tilde{\mathbf{S}}_1 \mathbf{c}(0) \\ \tilde{\mathbf{S}}_2 \mathbf{c}(T_i) \\ \vdots \\ \tilde{\mathbf{S}}_M \mathbf{c}((M-1)T_i) \end{bmatrix} \begin{bmatrix} \mathbf{c}^H(0) \tilde{\mathbf{S}}_1^H & \mathbf{c}^H(T_i) \tilde{\mathbf{S}}_2^H & \cdots & \mathbf{c}^H((M-1)T_i) \tilde{\mathbf{S}}_M^H \end{bmatrix} \right\}. \quad (102)$$

Under the assumption that the clutter has the same Doppler spectrum, the clutter correlation matrix can be written as

$$\mathbf{R}_{y_c} = \begin{bmatrix} \rho_{1,1} \tilde{\mathbf{S}}_1 \mathbf{R}_c \tilde{\mathbf{S}}_1^H & & \rho_{1,M} \tilde{\mathbf{S}}_1 \mathbf{R}_c \tilde{\mathbf{S}}_M^H \\ & \ddots & \\ \rho_{M,1} \tilde{\mathbf{S}}_M \mathbf{R}_c \tilde{\mathbf{S}}_1^H & & \rho_{M,M} \tilde{\mathbf{S}}_M \mathbf{R}_c \tilde{\mathbf{S}}_M^H \end{bmatrix}. \quad (103)$$

Clearly, the noise correlation matrix is unchanged. Therefore, it is straightforward to write the optimum detector for non-identical pulses as

$$\left[\begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}^H \mathbf{R}_I^{-1} \mathbf{Y} \right] \begin{matrix} H_1 \\ > \\ < \\ H_0 \end{matrix} \eta, \quad (104)$$

where the interference matrix is

$$\mathbf{R}_I = \begin{bmatrix} \rho_{1,1} \tilde{\mathbf{S}}_1 \mathbf{R}_c \tilde{\mathbf{S}}_1^H & & \rho_{1,M} \tilde{\mathbf{S}}_1 \mathbf{R}_c \tilde{\mathbf{S}}_M^H \\ & \ddots & \\ \rho_{M,1} \tilde{\mathbf{S}}_M \mathbf{R}_c \tilde{\mathbf{S}}_1^H & & \rho_{M,M} \tilde{\mathbf{S}}_M \mathbf{R}_c \tilde{\mathbf{S}}_M^H \end{bmatrix} + \sigma_n^2 \mathbf{I}. \quad (105)$$

5.2. SIR for Non-Identical-Pulse Detector

The power output of the detector due to the signal is

$$|z_T|^2 = |\alpha|^2 \left[\begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}^H \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix} \right]^2. \quad (106)$$

The power output of the of the detector due to the interference is

$$|z_I|^2 = \left[\begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}^H \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix} \right]. \quad (107)$$

This produces the SIR for the non-identical-pulse detector as

$$SIR_{NIP} = \begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}^H \mathbf{R}_I^{-1} \begin{bmatrix} u_0 \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_1 \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{M-1} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix} \quad (108)$$

Next, the mismatched detector SIR will be determined. The design pulse-to-pulse phase change vector is $\mathbf{u}_D$; the actual pulse-to-pulse phase change vector is $\mathbf{u}_A$. The design interference correlation matrix is

$$\mathbf{R}_{I_D} = \begin{bmatrix} \rho_{D,1,1} \tilde{\mathbf{S}}_1 \mathbf{R}_{c_D} \tilde{\mathbf{S}}_1^H & & \rho_{D,1,M} \tilde{\mathbf{S}}_1 \mathbf{R}_{c_D} \tilde{\mathbf{S}}_M^H \\ & \ddots & \\ \rho_{D,M,1} \tilde{\mathbf{S}}_M \mathbf{R}_{c_D} \tilde{\mathbf{S}}_1^H & & \rho_{D,M,M} \tilde{\mathbf{S}}_M \mathbf{R}_{c_D} \tilde{\mathbf{S}}_M^H \end{bmatrix} + \sigma_{n_D} \mathbf{I}. \quad (109)$$

Similarly, the actual interference correlation matrix is

$$\mathbf{R}_{I_A} = \begin{bmatrix} \rho_{A,1,1} \tilde{\mathbf{S}}_1 \mathbf{R}_{c_A} \tilde{\mathbf{S}}_1^H & & \rho_{A,1,M} \tilde{\mathbf{S}}_1 \mathbf{R}_{c_A} \tilde{\mathbf{S}}_M^H \\ & \ddots & \\ \rho_{A,M,1} \tilde{\mathbf{S}}_M \mathbf{R}_{c_A} \tilde{\mathbf{S}}_1^H & & \rho_{A,M,M} \tilde{\mathbf{S}}_M \mathbf{R}_{c_A} \tilde{\mathbf{S}}_M^H \end{bmatrix} + \sigma_{n_A} \mathbf{I}. \quad (110)$$

From this, the SIR for the mismatched NIP detector can be determined as

$$SIR_{NIP_{mismatched}} = \frac{|\alpha|^2 \begin{bmatrix} u_{D_0} \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_{D_1} \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}^H \mathbf{R}_{I_D}^{-1} \begin{bmatrix} u_{A_0} \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_{A_1} \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{A_{M-1}} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}}{\begin{bmatrix} u_{D_0} \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_{D_1} \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}^H \mathbf{R}_{I_D}^{-1} \mathbf{R}_{I_A} \mathbf{R}_{I_D}^{-1} \begin{bmatrix} u_{D_0} \tilde{\mathbf{S}}_{1,d} \boldsymbol{\delta}_k \\ u_{D_1} \tilde{\mathbf{S}}_{2,d} \boldsymbol{\delta}_k \\ \vdots \\ u_{D_{M-1}} \tilde{\mathbf{S}}_{M,d} \boldsymbol{\delta}_k \end{bmatrix}}. \quad (111)$$

6.0 CLUTTER MOTION

In this section, the impact of radar motion will be incorporated into the detector. In the preceding developments, the motion of the clutter due was ignored. For clutter motion that will not cause range migration, the induced clutter motion can be handled a number of documented ways, such as in [2]. This section will deal with range migrating clutter as well as clutter that may have different mean Dopplers in each range cell.

6.1. Range Migrating Clutter

Radars mounted on platforms traveling at high speeds (such as aircraft or satellites) will produce range migration effects on the received clutter signal as well as a Doppler offset due the radar motion. To begin, the mean Doppler for ground reflections is

$$f_d = \frac{2v_r}{\lambda} \cos(\theta_{cone}) = \frac{2v_r}{\lambda} \cos(\theta_{az}) \cos(\theta_{el}), \quad (112)$$

where v_r is the radar velocity, θ_{cone} is the cone angle, θ_{az} is the azimuth angle, and θ_{el} is the elevation angle as described in [3], pp. 839-840. The angles are relative to the radar velocity vector and a specific location on the ground.

To illustrate how range migration compensation can be applied, the simple case of the radar traveling north at a constant velocity of v_r at a constant altitude over a flat earth will be used. The technique developed however, is not limited to the simple geometry; it is used to simplify the illustrations. The coordinate system used is as follows. The radar height above the ground is γ_{Z_R} , the radar east/west position γ_{X_R} is a constant zero, and the radar north/south position is $\gamma_{Y_R} = v_r t$, indicating that at time zero (the beginning of the CPI) the radar's position is $\gamma_{X_R} = 0, \gamma_{Y_R} = 0, \gamma_{Z_R} = \text{constant}$.

The spatial relationship of the radar relative to the stationary ground clutter is illustrated in Figure 6. This figure is in plan view (i.e., looking down) and hides the altitude and elevation aspect of the problem for simplicity. In this figure, the minimum and maximum range of the radar is indicated. The ranges are determined by the turning on and off of the radar receiver on a per pulse basis. The beamwidth of the radar antenna (as projected to the ground) is shown as well. These limits define the region of the ground that produce discernable clutter that will interfere with target detection. Also indicated in Figure 6 is the radar resolution cell projected on the earth's surface. This is the spatial region that produces clutter for receive sample y . This is the same region as indicated in [11], p. 405, Figure 7.1. At the center of this region is a ground reference point (GRP).

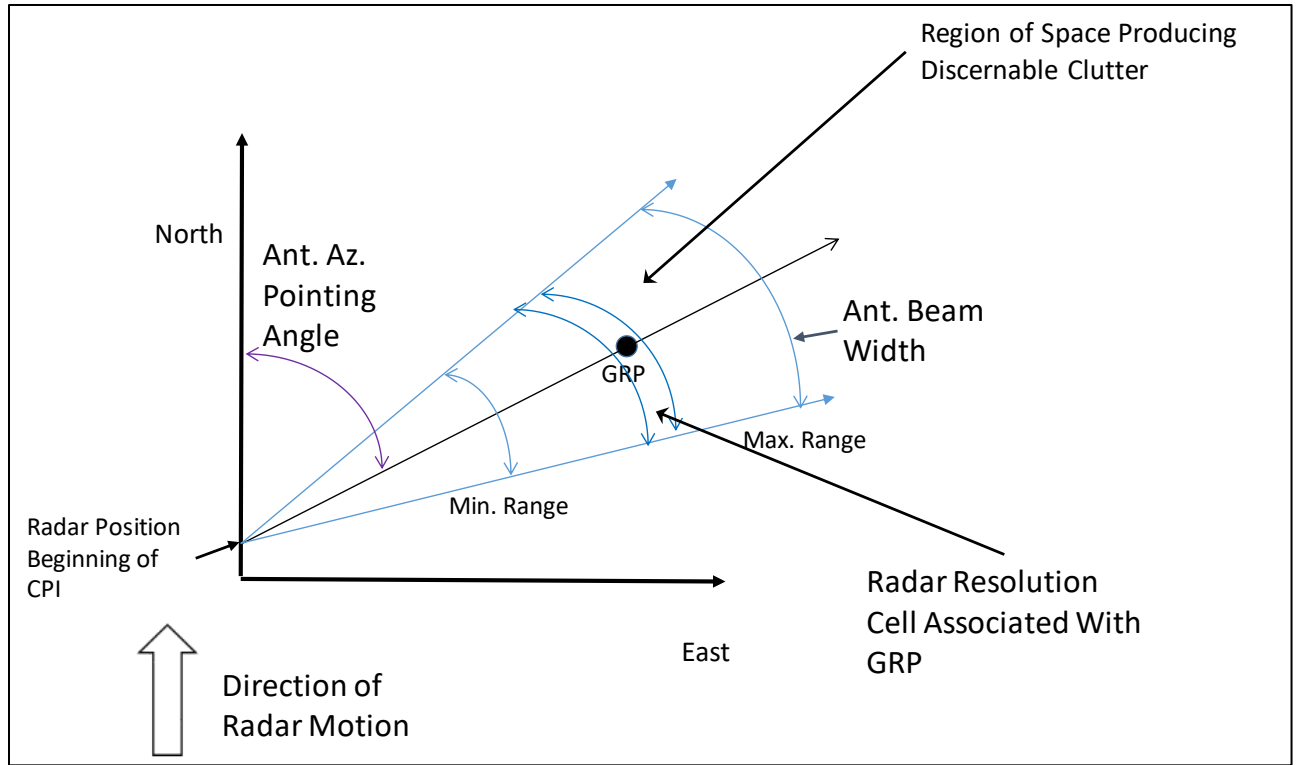


Figure 6. Plan view of radar position relative to antenna beam and clutter

The GRP provides the reference point to determine mean Doppler frequency (112) and range migration of the clutter. Consequently, each range resolution cell has an associated GRP as illustrated in Figure 7. From each GRP in each cell, the mean Doppler and corresponding range change can be calculated. There are $P + 2(N - 1)$ GRPs corresponding to the $P + 2(N - 1)$ elements of the vector $\mathbf{c}$ previously defined.

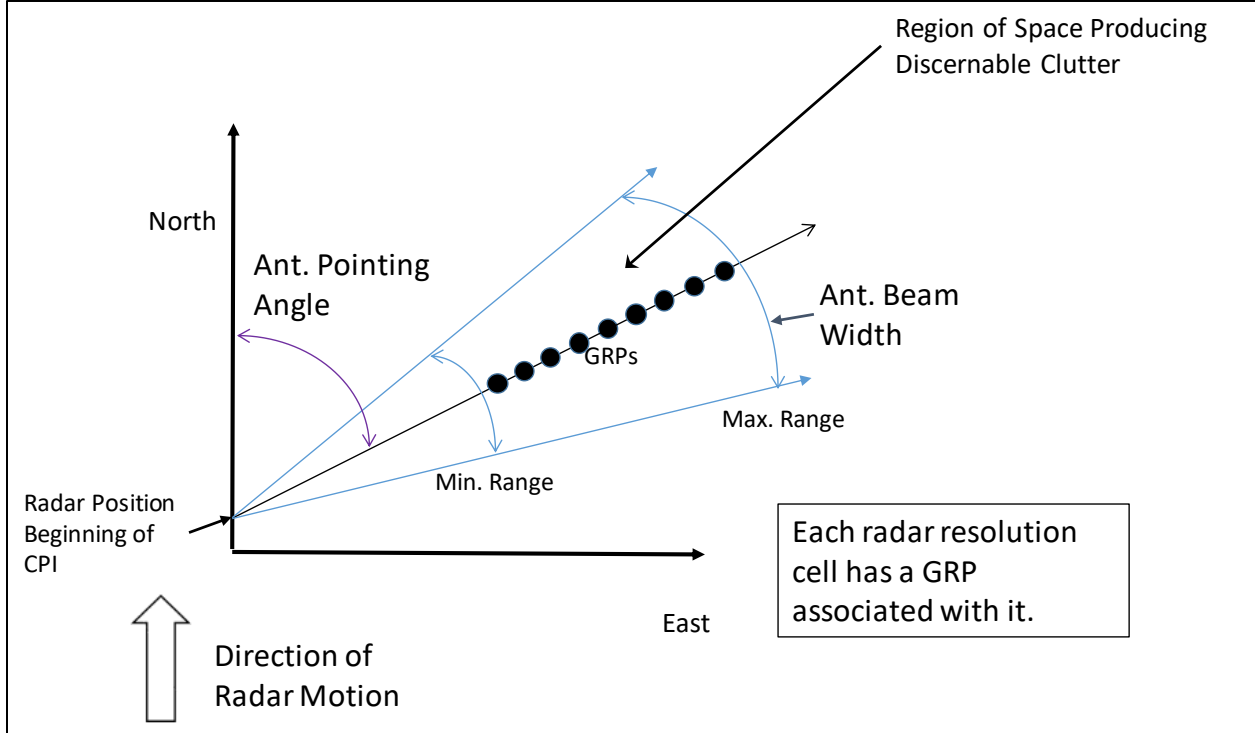


Figure 7. Plan view showing radar position relative to antenna main beam and ground reference points

As the radar moves through the time period of the CPI, the range to the GRP changes as well as its angle. This is illustrated in Figure 8. The range to a given GRP is

$$R_{GRP}(t) = \sqrt{\gamma_{X_{GRP}}^2 + (\gamma_{Y_{GRP}} - v_r t)^2 + \gamma_{Z_{GRP}}^2}. \quad (113)$$

The azimuth angle is

$$\theta_{az}(t) = \sin^{-1}\left(\frac{\gamma_{X_{GRP}}}{\sqrt{(\gamma_{Y_{GRP}} - v_r t)^2 + \gamma_{X_{GRP}}^2}}\right) \quad (114)$$

and the elevation angle is

$$\theta_{el} = \sin^{-1}\left(\frac{\gamma_{Z_{GRP}}}{\sqrt{(\gamma_{Y_{GRP}} - v_r t)^2 + \gamma_{X_{GRP}}^2}}\right), \quad (115)$$

where $\gamma_{X_{GRP}}$, $\gamma_{Y_{GRP}}$, and $\gamma_{Z_{GRP}}$ are the coordinates of the GRP. The cone angle of the GRP is

$$\theta_{cone}(t) = \cos^{-1}(\cos(\theta_{az}(t))\cos(\theta_{el}(t))). \quad (116)$$

The range rate is

$$\dot{R}_{GRP}(t) = \frac{-\gamma_{Y_{GRP}} v_r + v_r^2 t^2}{\sqrt{\gamma_{X_{GRP}}^2 + (\gamma_{Y_{GRP}} - v_r t)^2 + \gamma_{Z_{GRP}}^2}}. \quad (117)$$

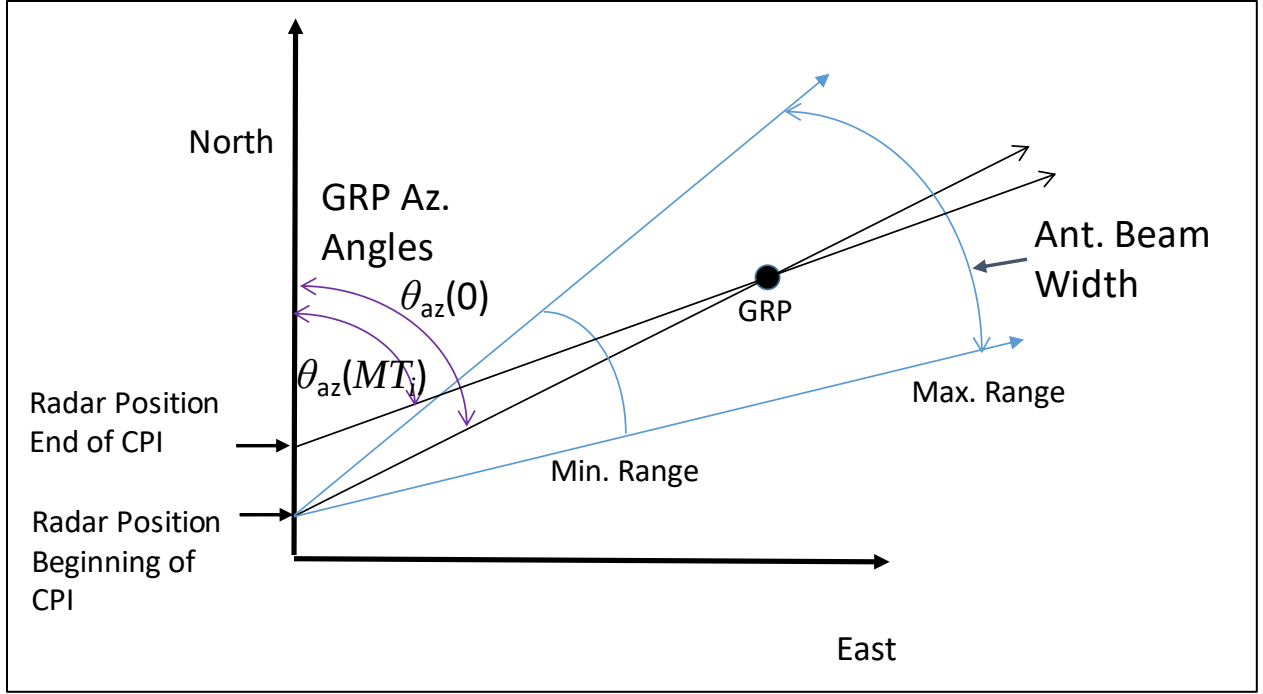


Figure 8. Plan view showing radar position and impact of radar motion on azimuth angle of the ground reference point

Next, it is convenient to rewrite $R_{GRP}(t)$ as a function of the initial range of the GRP and the initial angles. Starting with

$$R_{GRP}^2(t) = \gamma_{X_{GRP}}^2 + (\gamma_{Y_{GRP}} - v_r t)^2 + \gamma_{Z_R}^2 = R_{GRP}^2(0) + v_r^2 t^2 - 2\gamma_{Y_{GRP}} v_r t \quad (118)$$

and then substituting

$$\gamma_{Y_{GRP}} = R_{GRP}(0) \cos(\theta_{el}) \sin(\theta_{az}) \quad (119)$$

produces

$$R_{GRP}(t) = \sqrt{R_{GRP}^2(0) + v_r^2 t^2 - 2R_{GRP}(0) \cos(\theta_{el}) \sin(\theta_{az}) v_r t}. \quad (120)$$

As stated earlier, the range extent of the clutter is determined by the radar receiver switching time relative to the beginning of the transmit pulse time. If the receiver turn-on delay time (for each receive interval) is T_{rx_delay} , then the range of the closest GRP is $\frac{cT_{rx_delay}}{2}$. From this, the range of each GRP is determined as

$$R_{GRP}^i(0) = \frac{(T_{rx_delay} + (i-1)T_s)c}{2} \quad (121)$$

where there are $P+2(N-1)$ RGPs and $i \in 1..P+(N-1)$. Since the radar is in motion and the GRPs are stationary, the GRP ranges change over the course of the CPI. The range as a function of time for each RGP can be written as

$$R_{GRP}^i(t) = \sqrt{\frac{c}{2}(T_{rx_delay} + (i-1)T_s)^2 + v_r^2 t^2 - 2(\frac{c}{2}(T_{rx_delay} + (i-1)T_s)) \cos(\theta_{el}) \sin(\theta_{az})}. \quad (122)$$

Note that $\dot{R}_{GRP}^i(t)$ will also vary with i . The amount of change will depend on the geometry of the situation. In many cases the range migration for clutter will be substantially the same for each RGP. This is stated mathematically as

$$\left\| R_{GRP}^i(MT_i) - R_{GRP}^i(0) - \left[R_{GRP}^j(MT_i) - R_{GRP}^j(0) \right] \right\| \ll \frac{c}{2} T_s, \forall i, j. \quad (123)$$

In the next section, the detector is developed under the assumption that clutter range migration is the same for all range resolution cells.

6.2. Uniform Range Migration Detector

In this section, the detector that accommodates target range migration and clutter range migration will be developed. This detector is optimal subject to the previously stated assumptions. Further, in this section, it will be assumed that the range migration is the same for each range cell (i.e., (123) is satisfied). The next section will handle the general case of clutter range migration being different for each range cell.

Because the range rate for each GRP can be different, f_d will be different for the clutter that arises from each resolution cell. This is accommodated by reformulating the signal convolution matrix in a manner similar to $\tilde{\mathbf{S}}_d(m)$, which was formulated to account for the target Doppler.

Thus, the signal convolution matrix for the clutter $\tilde{\mathbf{S}}_c(m)$ is defined as

$$\tilde{\mathbf{S}}_c(m) = \begin{bmatrix} 0 & & s_1 e^{j\Delta_N(m,1)} & \dots & s_N e^{j\Delta_1(m,N)} \\ & \ddots & & & \\ s_1 e^{j\Delta_{P+2(N-1)}(m,1)} & \dots & s_N e^{j\Delta_{P+2(N-1)-(N-1)}(m,N)} & \dots & 0 \end{bmatrix} \quad (124)$$

where $\Delta_i(m, n)$ is the phase change of the clutter at GRP_i from the beginning of the CPI to the current pulse m and sample n and is calculated as

$$\Delta_i(m, n) = \frac{4\pi}{\lambda} (R_{GRP}^i(mT_i + (n-1)T_s) - R_{GRP}^i(0)). \quad (125)$$

Under the assumption that range migration is the same for each range cell, there is a single range migration matrix. Therefore, the range migration matrix is defined in (43), with ΔT_s calculated as

$$\Delta T_s = \frac{2}{c} (R_{GRP}(mT_i + lT_s) - R_{GRP}(0)). \quad (126)$$

The receive data due to clutter is

$$\mathbf{Y}_c = \begin{bmatrix} \tilde{\mathbf{R}}_m(0) \hat{\mathbf{S}}_c(0) \mathbf{c}(0) \\ \tilde{\mathbf{R}}_m(M-1) \hat{\mathbf{S}}_c(M-1) \mathbf{c}(M-1) \end{bmatrix}. \quad (127)$$

The clutter correlation matrix is determined as

$$\mathbf{R}_{Y_c} = E\{\mathbf{Y}_c \mathbf{Y}_c^H\} = \left\{ \begin{bmatrix} \tilde{\mathbf{R}}_m(0) \hat{\mathbf{S}}_c(0) \mathbf{c}(0) \\ \tilde{\mathbf{R}}_m(M-1) \hat{\mathbf{S}}_c(M-1) \mathbf{c}(M-1) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{R}}_m(0) \hat{\mathbf{S}}_c(0) \mathbf{c}(0) \\ \tilde{\mathbf{R}}_m(M-1) \hat{\mathbf{S}}_c(M-1) \mathbf{c}(M-1) \end{bmatrix}^H \right\} \quad (128)$$

which can be written as

$$\mathbf{R}_{Y_c} = \begin{bmatrix} \mathbf{a}_{1,1} & & \mathbf{a}_{1,2} \\ & \ddots & \\ \mathbf{a}_{2,1} & & \mathbf{a}_{2,2} \end{bmatrix}, \quad (129)$$

where

$$\mathbf{a}_{1,1} = \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_c(0) \mathbf{p}_{1,1} \mathbf{R}_c \tilde{\mathbf{S}}_c^H(0) \tilde{\mathbf{R}}_m^H(0),$$

$$\mathbf{a}_{1,2} = \tilde{\mathbf{R}}_m(0) \tilde{\mathbf{S}}_c(0) \mathbf{p}_{1,M} \mathbf{R}_c \tilde{\mathbf{S}}_c^H(M-1) \tilde{\mathbf{R}}_m^H(M-1),$$

$$\mathbf{a}_{2,1} = \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_c(M-1) \mathbf{p}_{M,1} \mathbf{R}_c \tilde{\mathbf{S}}_c^H(0) \tilde{\mathbf{R}}_m^H(0),$$

and

$$\mathbf{a}_{2,2} = \tilde{\mathbf{R}}_m(M-1) \tilde{\mathbf{S}}_c(M-1) \mathbf{p}_{M,M} \mathbf{R}_c \tilde{\mathbf{S}}_c^H(M-1) \tilde{\mathbf{R}}_m^H(M-1).$$

This is the clutter portion of $\mathbf{R}_I$ in (34). Substituting (129) into (34) allows using (45) as the detector to detect a range migrating target in the presence of range migrating clutter.

6.3. Non-Uniform Range Migration Detector

Under some geometries, the range migration of clutter may be significantly different for each range cell. Under this condition, the range migration matrix is indeed different for each range cell, and the range migration matrix is defined as

$$\tilde{\mathbf{R}}_m(m, i)_{(k,l)} = \text{sinc}(((k-l)T_s - \Delta T_s) / T_s), \quad (130)$$

where i indicates the range cell for which the range migration matrix is applicable and

$$\Delta T_s = \frac{2}{c} (R_{GRP}^i(mT_i + lT_s) - R_{GRP}^i(0)). \quad (131)$$

Next, the column vector $\mathbf{c}(t; i)$ is defined as a vector of length of $P+2(N-1)$ whose elements are all zero except the i 'th element. This element is equal to the i 'th element of $\mathbf{c}(t)$. Thus,

$$\mathbf{c}(t) = \sum_{i=1}^{P+2(N-1)} \mathbf{c}(t; i). \quad (132)$$

Using these definitions, the receive data $\mathbf{Y}_c$ due to the clutter for pulse m is

$$\mathbf{y}_c(m) = \sum_{i=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(m, i) \tilde{\mathbf{S}}_c(m) \mathbf{c}(t; i), \quad (133)$$

where $\tilde{\mathbf{S}}_c(m)$ has been previously defined. The full vector of received data due to clutter is

$$\mathbf{Y}_c = \begin{bmatrix} \mathbf{y}_c(0) \\ \vdots \\ \mathbf{y}_c(M-1) \end{bmatrix}. \quad (134)$$

Thus the correlation matrix of the clutter in the received data $\mathbf{Y}$ is

$$\mathbf{R}_{Y_c} = E \left\{ \tilde{\mathbf{Y}}_c \tilde{\mathbf{Y}}_c^H \right\} = \begin{bmatrix} E \left\{ \mathbf{y}_c(0) \mathbf{y}_c^H(0) \right\} & E \left\{ \mathbf{y}_c(0) \mathbf{y}_c^H(M-1) \right\} \\ & \ddots \\ E \left\{ \mathbf{y}_c(M-1) \mathbf{y}_c^H(0) \right\} & E \left\{ \mathbf{y}_c(M-1) \mathbf{y}_c^H(M-1) \right\} \end{bmatrix}. \quad (135)$$

Note that

$$E \left\{ \mathbf{y}_c(j) \mathbf{y}_c^H(k) \right\} = E \left\{ \begin{bmatrix} \sum_{i=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(j, i) \tilde{\mathbf{S}}_c(j) \mathbf{c}((j)T_i; i) \\ \dots \left(\sum_{l=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(k, l) \tilde{\mathbf{S}}_c(k) \mathbf{c}((k)T_l; l) \right)^H \end{bmatrix} \right\} \quad (136)$$

which can be factored into

$$E \left\{ \mathbf{y}_c(j) \mathbf{y}_c^H(k) \right\} = \sum_{i=1}^{P+2(N-1)} \sum_{l=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(j, i) \tilde{\mathbf{S}}_c(j) E \left\{ \mathbf{c}((j)T_i; i) \mathbf{c}^H((k)T_l; l) \right\} \tilde{\mathbf{S}}_c^H(k) \tilde{\mathbf{R}}_m^H(k, l). \quad (137)$$

Next, it is noted that

$$E \left\{ \mathbf{c}(jT_i; i) \mathbf{c}^H(kT_l; l) \right\} = [\mathbf{0}] \quad (138)$$

for $i \neq l$. This is due to the previously stated assumption that clutter is a zero mean process and is uncorrelated between range cells. On the other hand, if $i = l$ then

$$E \left\{ \mathbf{c}(jT; i) \mathbf{c}^H(kT; i) \right\} = \mathbf{p}_{j,k} \begin{bmatrix} 0 & \dots & 0 \\ & \ddots & \\ & & \sigma_i^2 & \\ & & & \ddots & \\ 0 & \dots & 0 \end{bmatrix}. \quad (139)$$

where $\mathbf{p}_{j,k}$ is defined in (33). That is, (139) is a matrix of all zeros except for the i 'th diagonal value, which is equal to the variance of the clutter at GRP_i multiplied by the j, k 'th time correlation coefficient for that range cell. Next, $\mathbf{R}_c(i)$ is defined as a matrix with all elements equal to zero except for the (i, i) 'th element, which is set to the (i, i) 'th element of $\mathbf{R}_c$, namely σ_i^2 . Note that

$$\mathbf{R}_c = \sum_{i=1}^{P+2(N-1)} \mathbf{R}_c(i). \quad (140)$$

With this, (137) can be written as

$$E \left\{ \mathbf{y}_c(j) \mathbf{y}_c^H(k) \right\} = \sum_{i=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(j, i) \tilde{\mathbf{S}}_c(j) \mathbf{p}_{j,k} \mathbf{R}_c(i) \tilde{\mathbf{S}}_c^H(k) \tilde{\mathbf{R}}_m^H(k, i) \quad (141)$$

Applying (141) to (135) produces

$$\mathbf{R}_{Y_c} = \begin{bmatrix} \mathbf{a}_{1,1} & & \mathbf{a}_{1,2} \\ & \ddots & \\ \mathbf{a}_{2,1} & & \mathbf{a}_{2,2} \end{bmatrix} \quad (142)$$

where

$$\mathbf{a}_{1,1} = \sum_{i=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(0,i) \tilde{\mathbf{S}}_c(0) \mathbf{p}_{j,k} \mathbf{R}_c(i) \tilde{\mathbf{S}}_c^H(0) \tilde{\mathbf{R}}_m^H(0,i),$$

$$\mathbf{a}_{1,2} = \sum_{i=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(0,i) \tilde{\mathbf{S}}_c(0) \mathbf{p}_{j,k} \mathbf{R}_c(i) \tilde{\mathbf{S}}_c^H(M-1) \tilde{\mathbf{R}}_m^H(M-1,i),$$

$$\mathbf{a}_{2,1} = \sum_{i=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(M-1,i) \tilde{\mathbf{S}}_c(M-1) \mathbf{p}_{j,k} \mathbf{R}_c(i) \tilde{\mathbf{S}}_c^H(0) \tilde{\mathbf{R}}_m^H(0,i),$$

and

$$\mathbf{a}_{2,1} = \sum_{i=1}^{P+2(N-1)} \tilde{\mathbf{R}}_m(M-1,i) \tilde{\mathbf{S}}_c(M-1) \mathbf{p}_{j,k} \mathbf{R}_c(i) \tilde{\mathbf{S}}_c^H(M-1) \tilde{\mathbf{R}}_m^H(M-1,i).$$

This is the clutter portion of $\mathbf{R}_l$ in (34).

6.4. Estimation of Clutter Mean Amplitude

There are a number of different methods in estimating mean clutter power σ_i^2 . Note that σ_i^2 must be estimated for each range cell i . One method is to use a clutter model such as the Littoral Clutter Model [12,13]. This model has been validated against a number of data sets and shown to estimate generally the clutter to noise to within a standard deviation of 10 dB (see [14]). Another method is to use the radar to make clutter measurements online. This, generally, is called clutter mapping; see [3], pp. 665-667.

6.5. Estimating Clutter Spectrum

As with amplitude, there are a number of different ways to estimate the clutter spectrum or slow time correlation of clutter ($\mathbf{M}_c$ and $\mathbf{p}_{j,k}$). As with clutter amplitude, the radar can estimate the clutter slow time correlation properties on-line. A common method to do that is used in weather radar and is called pulse pair processing; see [3], pp. 668-673.

For radars mounted on fast-moving vehicles, the motion of the radar often dominates the clutter spectrum. Under this condition, [14] provides a convenient way to calculate the Doppler time correlation properties of ground clutter. The motion of the antenna relative to the clutter is illustrated in Figure 8. The motion of the antenna beam across the ground modulates the clutter in addition to the Doppler changes based on angle; see (112).

To illustrate how to apply [14], the clutter correlation function $R_g^i(\tau)$ from (17) is used. To begin, the clutter observed in the receiver for a given range resolution cell centered at the i 'th GRP can be represented as

$$c_i(t) = \int \zeta_t(\theta) G_{ant}(\theta - \theta_{p_az}, \theta_{el} - \theta_{p_el}) d\theta, \quad (143)$$

where $c_i(t)$ is the clutter voltage present at the receiver for the range cell of interest, t is the slow time, $\zeta_t(\theta)$ is the incremental clutter response as a function of azimuth angle, $G_{ant}(\beta_{az}, \beta_{el})$ is the two-way (i.e., transmit and receive) antenna voltage gain as a function of azimuth angle β_{az} and elevation angle, β_{el} , θ_{p_az} is the azimuth pointing angle of the antenna, and θ_{p_el} is the elevation

pointing angle of the antenna. The integration in (143) sums up the back scatter from the ground according to the antenna gain. The variable of integration is azimuth (the value of changes of gain over elevation is assumed to be negligible). The angles are referenced at the beginning of the CPI.

The clutter voltage response for some time, τ , after the beginning of the CPI is

$$c_i(t + \tau) = \int \zeta_{t+\tau}(\theta) G_{ant}(\theta - \theta_{p_az} - \Delta\theta_{az}(\tau), \theta_{el} - \theta_{p_el}) d\theta, \quad (144)$$

where

$$\zeta_{t+\tau}(\theta) = \zeta_t(\theta) \exp(j2\pi\tau(f_d(\theta - \Delta\theta_{az}(\tau), \theta_{p_el}) - f_d(\theta_{p_az}, \theta_{p_el}))), \quad (145)$$

and $\Delta\theta_{az}(\tau)$ is the apparent azimuth change of the GRP over the course of the CPI as illustrated in Figure 8 and computed as

$$\Delta\theta_{az}(\tau) = \theta_{az}(\tau) - \theta_{az}(0), \quad (146)$$

where $\theta_{az}(t)$ is defined in (114) and the Doppler frequency f_d is defined in (112). The reason that the term $f_d(\theta_{p_az}, \theta_{p_el})$ is subtracted in (145) is that the mean Doppler is accounted for in (125) in the signal convolution matrix $\tilde{S}_c$. Thus, it must be removed in (145).

The time correlation of the clutter for the i 'th resolution cell associated with the i 'th GRP is

$$R_g^i(\tau) = E\{c_i(t)c_i^*(t + \tau)\}. \quad (147)$$

This can be written as

$$R_{gi}(\tau) = E\left\{\int \int \zeta_t(\alpha) \zeta_{t+\tau}^*(\beta) G_{ant}(\alpha - \theta_{p_az}, \theta_{el} - \theta_{p_el}) \cdots G_{ant}^*(\beta - \theta_{p_az} - \Delta\theta_{az}(\tau), \theta_{el} - \theta_{p_el}) d\alpha d\beta\right\}, \quad (148)$$

where α and β are dummy variables of integration.

The order of integration and expectation can be changed to give

$$R_g^i(\tau) = \int \int E\{\zeta_t(\alpha) \zeta_{t+\tau}^*(\beta)\} G_{ant}(\alpha - \theta_{p_az}, \theta_{el} - \theta_{p_el}) \cdots G_{ant}^*(\beta - \theta_{p_az} - \Delta\theta_{az}(\tau), \theta_{el} - \theta_{p_el}) d\alpha d\beta. \quad (149)$$

Next, the expectation can be rewritten as

$$E\{\zeta_t(\alpha) \zeta_{t+\tau}^*(\beta)\} = E\{\zeta_t(\alpha) \zeta_t^*(\beta)\} \cdots \exp(j2\pi\tau(f_d(\beta - \Delta\theta_{az}(\tau), \theta_{p_el}) - f_d(\theta_{p_az}, \theta_{p_el}))). \quad (150)$$

The assumptions used in [13] are applied here; namely

$$E\{\zeta_t(\alpha)\} = 0, \forall \alpha \quad (151)$$

and

$$E\{\zeta_t(\alpha) \zeta_t^*(\beta)\} = \begin{cases} 1, & \alpha = \beta \\ 0, & \alpha \neq \beta \end{cases}. \quad (152)$$

The assumption in (151) is arrived at by assuming the phase angle of the clutter voltage is uniformly distributed 0 to 2π for every location. There are two assumptions in (152). First, the power is unity at all locations. (This is done for convenience since the clutter power is captured in σ_i 's.) The second result comes from the assumption that the clutter voltage phase is independent from angle to angle.

Applying (150), (151), and (152) to (149) produces the following for the time correlation function:

$$R_g^i(\tau) = \int G_{ant}(\theta - \theta_{p_az}, \theta_{el} - \theta_{p_el}) G_{ant}^*(\theta - \theta_{p_az} - \Delta\theta_{az}(\tau), \theta_{el} - \theta_{p_el}) \cdots \exp(-j2\pi\tau(f_d(\theta - \Delta\theta_{az}(\tau), \theta_{p_el}) - f_d(\theta_{p_az}, \theta_{p_el}))) d\theta. \quad (153)$$

7.0 SUMMARY AND RESULTS

This study set out to develop signal processing that could detect range migrating targets in the presence of clutter. This has been an open problem. Equation (45) is the signal processor that was derived for this problem. In support of this effort, a very general target motion model was developed. (There was no requirement for constant velocity or constant acceleration of the target.) The signal processor was synthesized by deriving an optimum detector that jointly processes fast time and slow time data. This optimum detector solves the problem of maximizing the probability of detection of a target subject to range and pulse distortion in the presence of clutter.

In order simplify the development, the optimum detector (jointly processing fast time and slow time data) was first developed for non-range migrating targets. This was summarized in equation (31). This detector is a whitening matched filter. As such, it maximizes the SIR. This detector also maximizes the probability of detection for a given probability of false alarm (i.e., it is a Neyman-Pearson test). Based on the framework developed, the detector for the range migrating targets was derived by modifying the target model to include range migration and pulse distortion. Thus, the signal processor that detects range migrating, pulse-distorting targets is also a whitening matched filter (45).

7.1. Theoretical Performance

Since this signal processor is a linear processor, direct performance comparisons with other linear processors can be made by comparing the output SIR for each processor. The standard linear processor for radar is the Correlator+MTD. This processor can be shown to be optimal in AWGN. It is convenient to implement as it is often implemented with Fast Fourier Transforms (FFT). To demonstrate this comparison, Figure 9 plots the SIR versus ambiguous Doppler frequency for the optimum detector and the Correlator+MTD for a target in AWGN only (i.e., no clutter). In Figure 9, it can be seen that the SIRs for both detectors are the same. This is because the optimum detector reverts to the Correlator+MTD when there is no clutter.

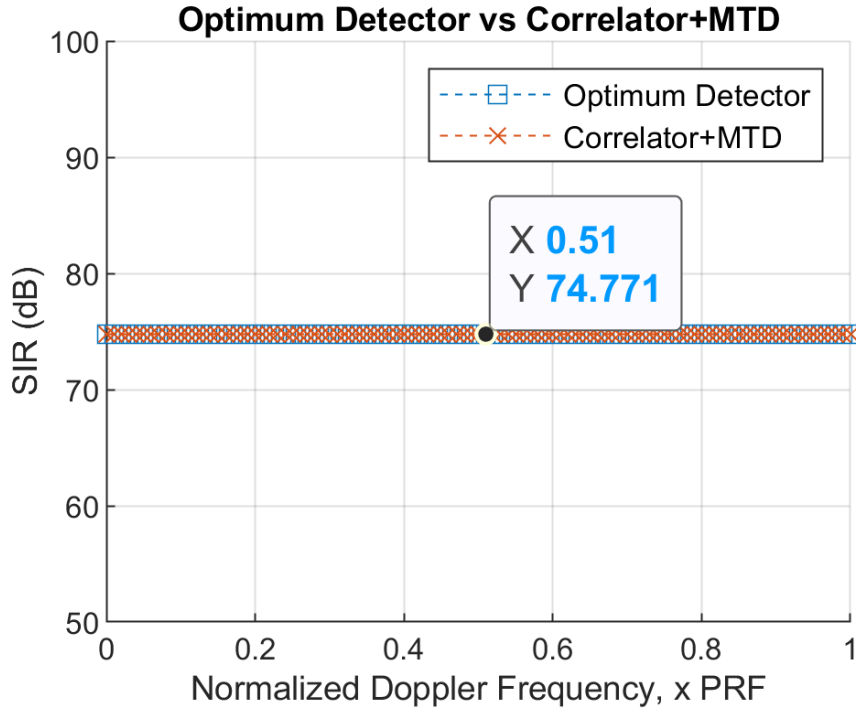


Figure 9. Signal-to-interference ratio for the optimum detector and the Correlator+MTD for a slow-moving (i.e., no range migration) target without clutter

Note that Table 1 gives the radar parameters used in the calculation of Figure 9 and subsequent figures in this section. The data input to the detector is complex data sampled at 80 MHz to satisfy the Nyquist sampling theorem. The waveform consists of 30 samples duration ($N = 30$), and the ten pulses give $M = 10$.

Table 1. Radar Parameters

Parameter	Value
Operating Frequency	15.728 GHz
Instantaneous Bandwidth	40 MHz
Number of Pulses	10
Number of Chips	15
PRF	1 kHz

For the case of a target in clutter, Figure 10 plots the SIR versus ambiguous Doppler frequency for the optimum detector and the Correlator+MTD for a target not subject to range migration. (The detector for this target as presented in (31).) For slow-moving targets (i.e., negligible range migration), there is small gain in performance (~ 2 dB) in the range of .35 PRF to .65 PRF normalized Doppler. The performance of the optimum detector, closer to zero Doppler, is due to the fact that it has perfect knowledge of the clutter spectrum as opposed to the Correlator+MTD. Note that this implementation of the Correlator+MTD in Figure 10 uses Blackman weighting.

The performance gain, of the optimum detector over Correlator+MTD near the center of the Doppler passband, is due to the loss of the Blackman weighting and the gain of the optimum detector over the correlator (see [4], Table 2).

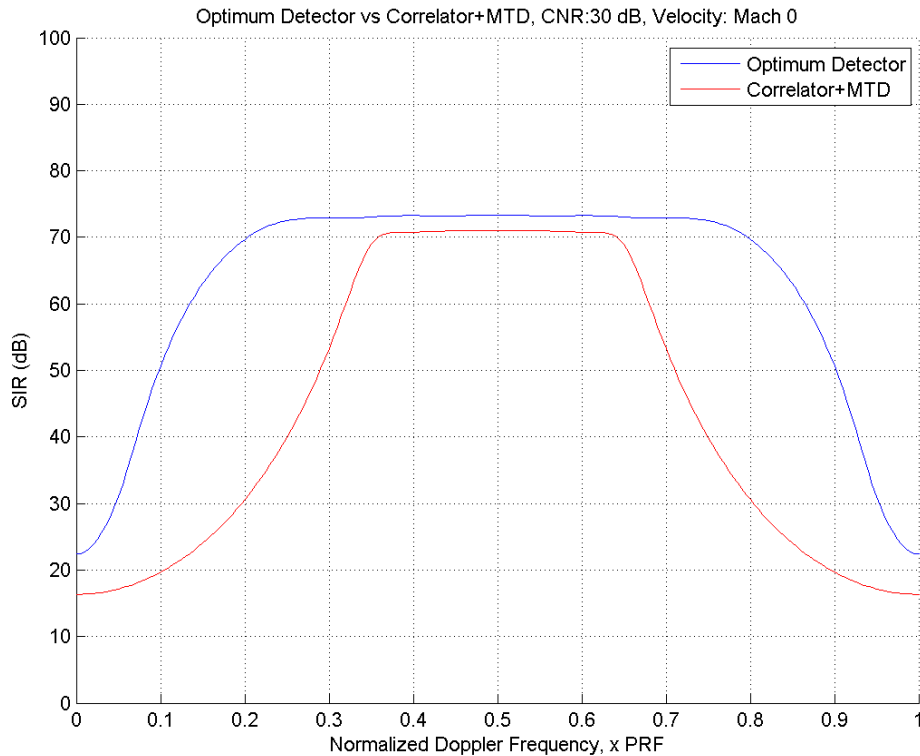


Figure 10. Signal-to-interference ratio for the optimum detector and the Correlator+MTD for a slow-moving (i.e., no range migration) target

Figure 11 illustrates the impact of range migration in the presence of clutter on the optimum detector developed here and on the Correlator+MTD. As can be seen in Figure 11, the Correlator+MTD suffers a substantial loss in the presence of clutter. On the other hand, the optimum detector keeps most of its performance for ambiguous Doppler of $PRF/2$ while gaining substantial performance for target Dopplers close to the clutter Doppler. The performance gain at target Doppler close to the clutter Doppler is due to the detector focusing on the target while rejecting the clutter. The optimum detector is able to achieve its performance because it jointly processes fast time and slow time data.

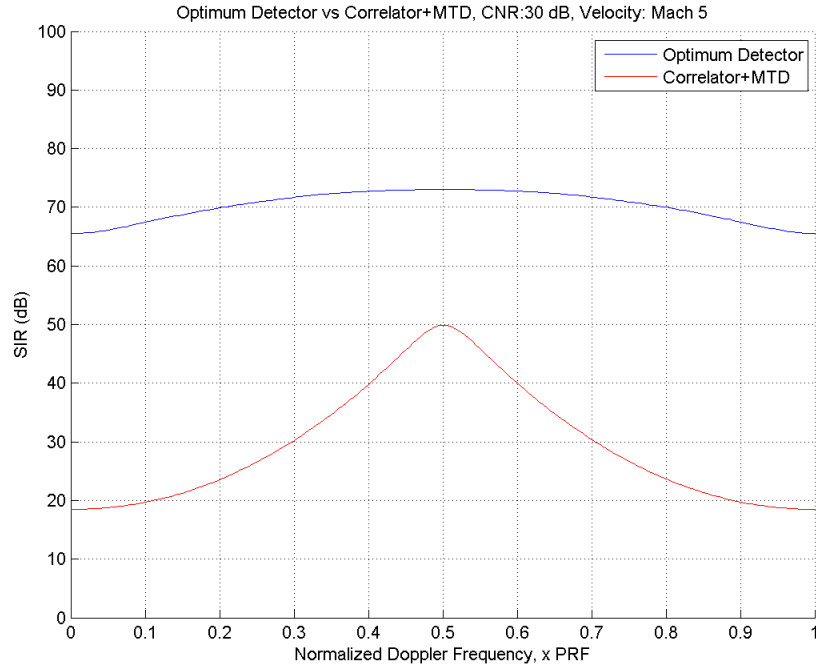


Figure 11. Signal-to-interference ratio for the optimum detector and the Correlator+MTD for a Mach 5 (i.e., significant range migration) target

The first attempt to jointly process fast time and slow time was [15]. That paper showed that, for an optimum detector, fast time and slow time could not be processed separately but needed to be processed simultaneously. This approach was more fully disclosed in [4].

7.2. Practical Performance

The current work expanded the optimum detector to include range migration of the target and the clutter. In order to apply the whitening matched filter in (31) and (45), the assumption had to be made that the clutter power and time correlation were known for each range resolution cell. Advances in clutter modeling ([12]-[14]) allow the clutter parameters to be estimated. This is in addition to online estimation techniques ([3], pp. 665-667). Also, there are methods to estimate the clutter spectrum, such as in [15] and online techniques such as in [3], pp. 668-673.

In order to evaluate the degradation of the optimum detector in the presence of estimation errors, performance equations were derived to calculate the output SIR under the condition of errors in clutter parameters (eq. (98)). This equation allows one to calculate the SIR when clutter parameters have been incorrectly determined. By taking the ratio of the (87) and (79) or (94) and (98), one can calculate the loss induced by clutter parameter estimation errors.

First, to compare this detection strategy, consider the case of a non-range migrating target. In this case, Figure 10 shows that the optimum detector has nearly a 2-dB advantage over the Correlator+MTD. This occurred because practical implementations of the Correlator+MTD

often use weighting to control Doppler sidelobe response and because it is not the optimum detector in clutter. Both of these features cause the Correlator+MTD to suffer a loss compared to the optimum detector.

In order to establish a meaningful assessment of the performance of the optimum detector with clutter parameter estimation errors, the following situation was modeled. First, it was specified that the actual clutter-to-noise ratio for every range sample is 30 dB. Next, the error in estimating the clutter-to-noise ratio is modeled as a normally distributed error with a standard deviation of 10 dB. That is, the error in decibels is normally distributed with a standard deviation of 10 dB and zero mean. (Reference [14] shows that it is possible to model clutter amplitude to this accuracy). This means the linear values of σ_i^2 have a lognormal error added for every range sample. Similarly, the Doppler spectrum is specified to conform to a textbook model of sea clutter [2]; namely, the spectrum is a Gaussian shape with the sigma determined by the sea state. Further, it is modeled that the sea state is estimated with a Gaussian error. This Gaussian error has a standard deviation $\frac{1}{2}$ sea state. Thus, every element of $\mathbf{p}_{j,k}$ is impacted by the sea state estimation error.

Figure 12 plots the losses incurred by optimum detector for a non-range migrating target when there are clutter parameter errors. This figure plots the results of 10 trials for each value of sea state. The bold line represents the mean loss value for the 10 trials. The average gain line represents the loss averaged over the whole ambiguous Doppler space. The peak gain line represents the loss for targets whose ambiguous Doppler is at the center of the Doppler pass band. The zero bin gain is for targets whose ambiguous Doppler is zero. As can be seen, the loss for all the plots, for all sea states (except zero), is 2 dB or less. The conclusion here, under reasonable assumptions of clutter parameter estimation error, is that detection performance will be on par with a Correlator+MTD. This assertion is justified since the optimum detector can have a roughly 2-dB advantage over a practical implementation of the Correlator+MTD (see Figure 10).

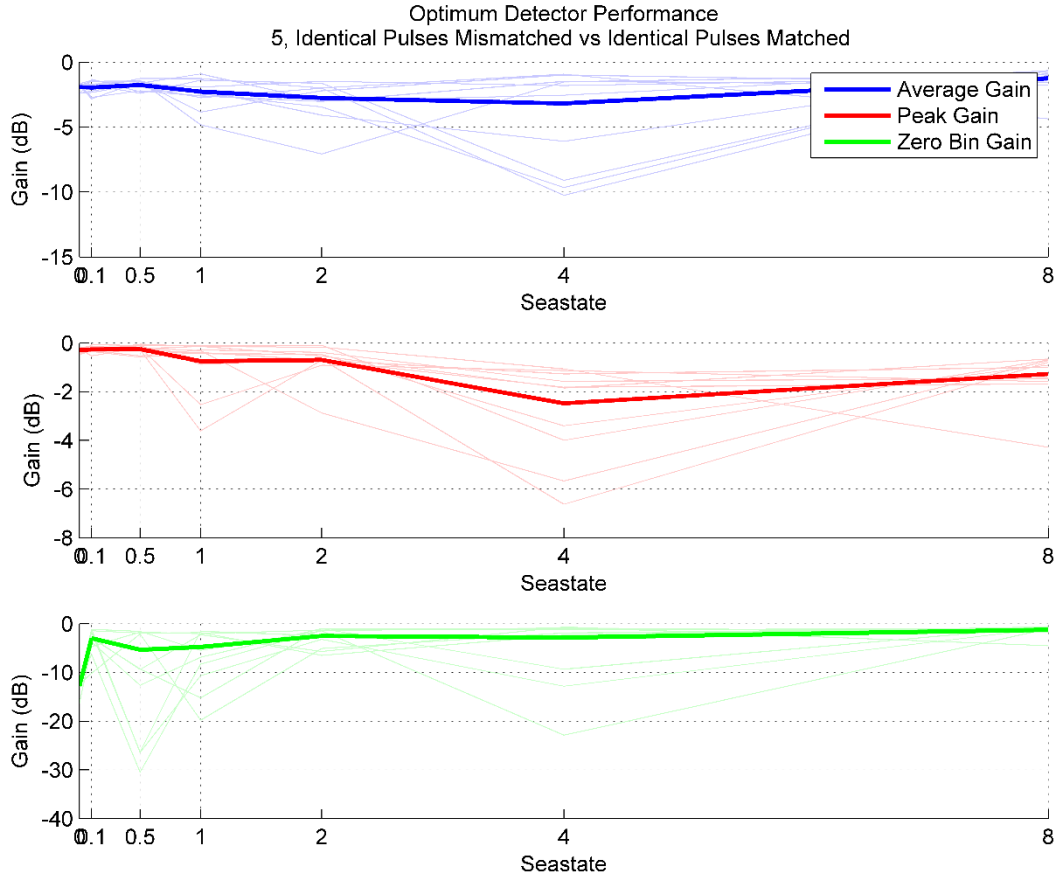


Figure 12. Comparison of matched and mismatched detectors versus sea state, non-range migrating target, 30-dB clutter-to-noise ratio

Next, to demonstrate that the optimum detector is robust to clutter estimation errors for different clutter amplitudes, Figure 13 plots detector loss versus clutter-to-noise level. Figure 13 plots the loss of the optimum detector with clutter parameter errors as a function of the true clutter-to-noise ratio. For this case, the actual sea state is fixed to 1 with a $\frac{1}{2}$ sea state standard deviation error. As can be seen in Figure 13, the mean loss over the trials is less than 2 dB.

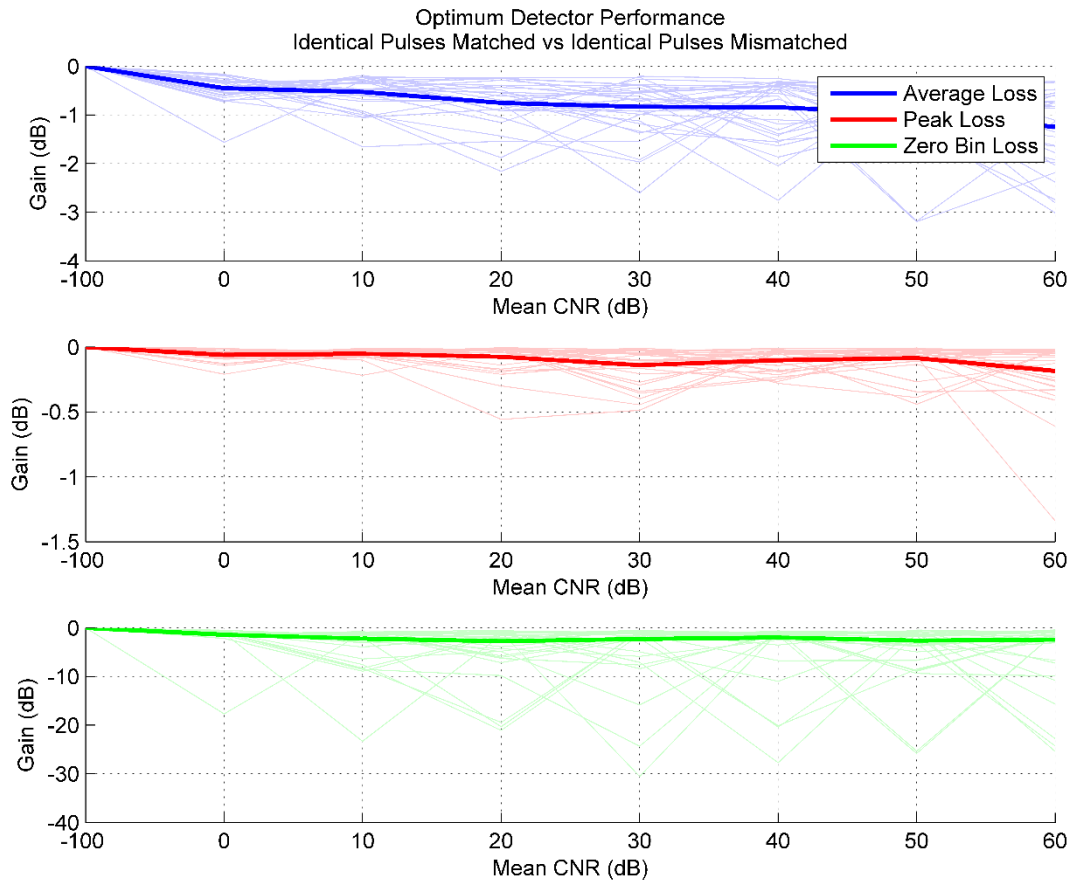


Figure 13. Comparison of matched and mismatched detectors versus clutter-to-noise ratio, non-range migrating target (*sea state = 1*)

Now the performance of the optimum detector for range migrating, pulse distortion targets under clutter parameter estimation errors is shown in Figure 14. In Figure 14, the loss due to estimation error is plotted versus sea state for a nominal target range rate of Mach 5. The true clutter-to-noise ratio is 30 dB (10-dB standard deviation estimation error). Here, the losses are somewhat larger; however, they are still very small compared to the gain in performance of the optimum detector compared to the Correlator+MTD (see Figure 11).

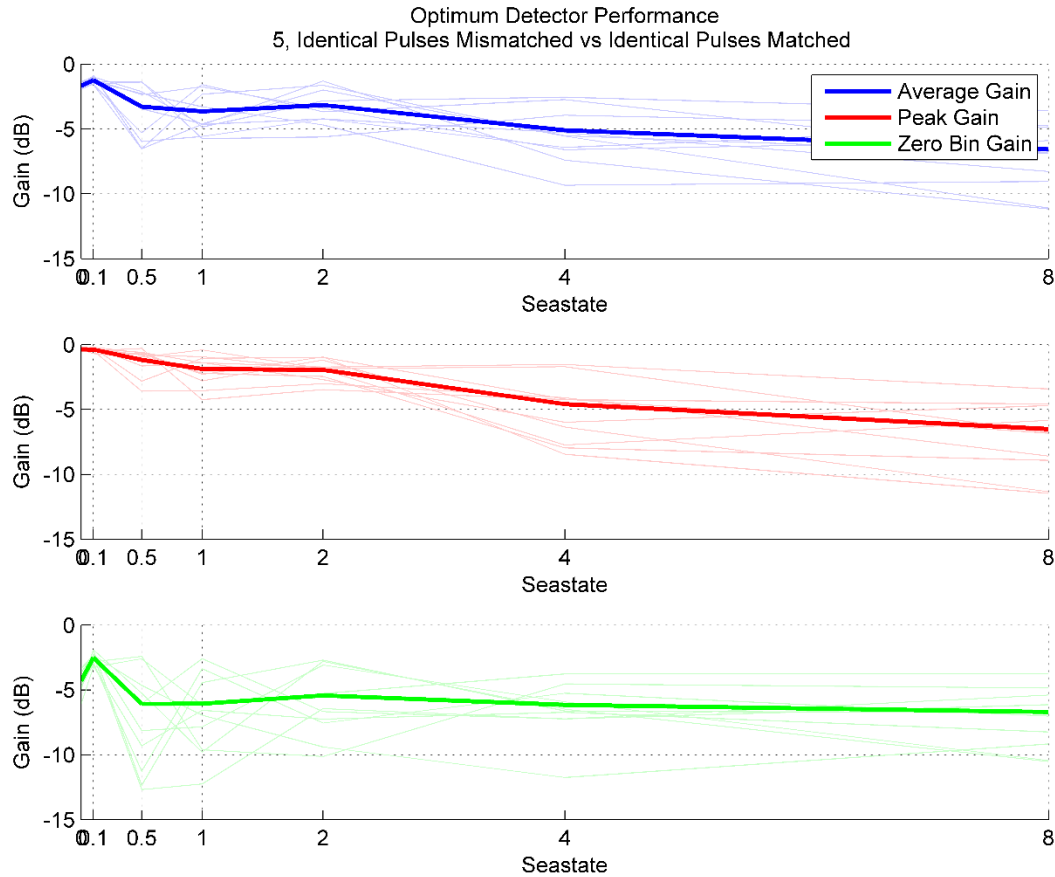


Figure 14. Comparison of matched and mismatched detectors versus sea state, Mach 5 target, moderate clutter-to-noise ratio of 30 dB

Next, Figure 15 plots the loss due to clutter parameter error versus nominal target range rate. For this case, the actual sea state is fixed to 1 with a $\frac{1}{2}$ sea state standard deviation error. In Figure 15, it can be seen that there can be 5 dB or more loss. However, this is balanced against the substantial gain over the optimum detector compared to the Correlator+MTD (see Figure 11). The conclusion again is that this signal processing approach is robust to errors in clutter parameter estimation.

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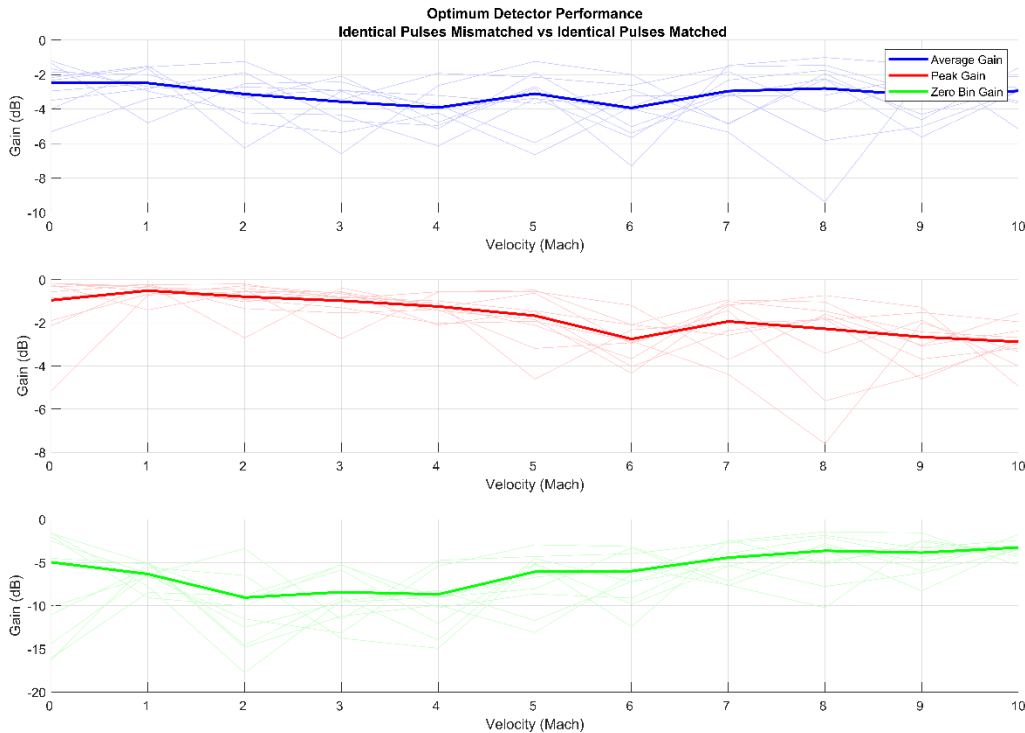


Figure 15. Comparison of matched and mismatched detectors versus nominal target range rate, clutter-to-noise ratio of 30 dB, sea state 1

Signal processing to address range migration targets can also detect targets in clutter when the transmitted signal changes pulse to pulse. Other approaches to allow the radar to change signals ([9] and [10]) place constraints on the transmitted signal. They require the range time sidelobes to be similar to limit the sidelobe clutter modulation that will not be canceled by the radar's Doppler processor. The signal processing developed here allows for any waveform modulation change from pulse to pulse. The detector for this situation was described by (104) and (105). The performance of the non-identical-pulse detector in the presence of clutter parameter errors is illustrated in Figure 16. For this case, the actual sea state is fixed to 1 with a $\frac{1}{2}$ sea state standard deviation error. In this figure, the loss is less than 2 dB. This loss is against an optimum detector with perfect knowledge of the clutter parameters using identical pulses.

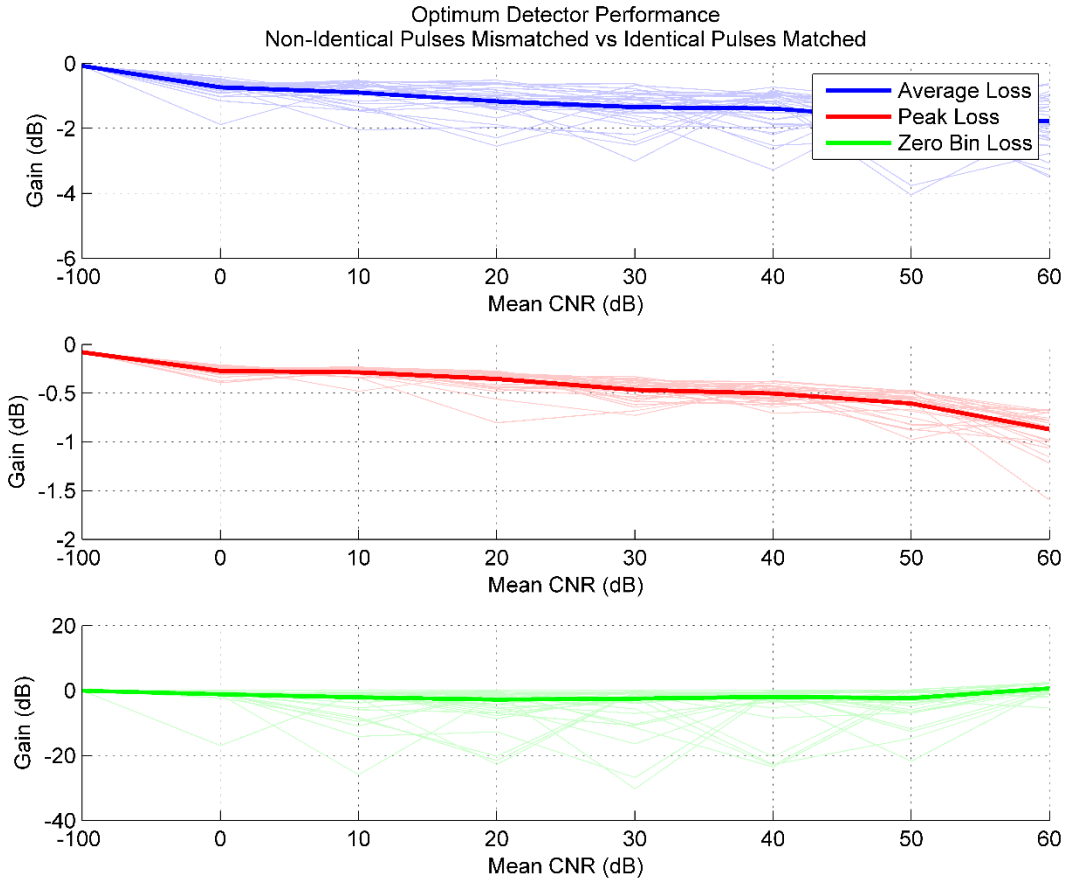


Figure 16. Comparison of mismatched detector using non-identical pulses versus matched detector using identical pulses as a function of mean clutter-to-noise ratio, slow speed target, constant sea state = 1

The final detectors developed here are for situations of range migrating clutter. This situation arises when the radar is on an aircraft or spacecraft looking down to the ground. The ground clutter appears to be moving to the radar and may experience range migration. This could be in addition to target range migration and might result in different amounts of range migration for the target and the clutter, depending on the radar and target velocity vectors. For situations of uniform clutter range migration, the clutter correlation matrix is determined by (120). Substituting (129) into (34) allows using (45) as the detector to detect a range migrating target in the presence of range migrating clutter. For the case of clutter range migration that varies with range, (142) was developed to compute the clutter correlation matrix. Substituting (142) into (34) allows using (45) as the detector to detect a range migrating target in the presence of range migrating clutter.

Finally, this work addressed calculating the clutter spectrum for radars in motion. For fast-moving radars, the clutter spectrum is dominated by the radar motion. Under this condition, [15] provides a convenient way to calculate the clutter time correlation parameters. The final development in this work showed how to apply [15] to airborne and spaceborne radars.

8.0 SYMBOLS USED

Table 2. Symbols Used in This Document

Symbol	Definition	Source of Information
α	Complex target voltage amplitude	This is an unknown parameter determined by the target size and precise range.
c	Speed of light	
$c(t)$	Complex clutter voltage	Clutter voltage is a random process whose amplitude cannot be predetermined, but its statistics can be estimated.
$\mathbf{c}(t)$	Column vector of discrete time sampled complex clutter voltages at slow time t	Clutter voltage is a random process whose amplitude cannot be predetermined but, its statistics can be estimated.
$\mathbf{c}(t;i)$	Vector of the same size as $\mathbf{c}(t)$; value of all elements are zero, except the i 'th element, which is equal to $\mathbf{c}(t)_{(i)}$	Clutter voltage is a random process whose amplitude cannot be predetermined but, its statistics can be estimated.
$\delta(t)$	Dirac delta function	
δ_k	Vector representation of the Kronecker delta function. It is a column vector with all elements equal to zero except element k , which is set to one.	The radar determines the value of k , which establishes the range cell under test (i.e., the range cell where a detection decision is being made).
$\Delta\theta_{az}(t)$	Change in azimuth angle over the course of the CPI	This value is determined by the geometry and speed of the radar.
e	Mathematical constant = 2.71828...	
$\exp(x)$	e^x	
$E\{\}$	Expectation operator	
f_d	Doppler frequency	Determined by the target range rate relative to the radar
$g(t)$	Unity variance complex Gaussian random variable	From assumption. See reference [6].
$G_{ant}(\theta, \varphi)$	Two-way antenna voltage gain as a function of azimuth (θ) and elevation (φ) angles	The antenna gain is determined by the design of the radar antenna. This is usually measured in an antenna range.
GRP	Ground reference point. Center of the resolution cell.	See Figure 6.
$\gamma_{X_{GRP}}$	X position (east/west) of the GRP, assuming flat earth	Provided by aircraft navigation system.
$\gamma_{Y_{GRP}}$	Y position (north/south) of the GRP, assuming flat earth	Provided by aircraft navigation system.
$\gamma_{Z_{GRP}}$	Z position (altitude) of the GRP, assuming flat earth	Provided by aircraft navigation system.

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γ_{x_R}	X position (east/west) of the radar, assuming flat earth	Provided by aircraft navigation system
γ_{y_R}	Y position (north/south) of the radar, assuming flat earth	Provided by aircraft navigation system
γ_{z_R}	Z position (altitude) of the radar, assuming flat earth	Provided by aircraft navigation system
$h_t(t)$	Target impulse response	This function is assumed known by hypothesis. The most common assumed version of $h(t)$ is that of the Dirac delta function $\delta(t)$, which is a point target.
$h_{t,R(\Delta T)}(t)$	Target impulse response taking into account the range change over the course of the coherent processing interval	This function can be calculated based on the hypothesis of the target impulse response function and the target assumed range versus time function ($R(t)$).
$\mathbf{h}_t$	Column vector of discrete time sampled target impulse function	This vector is assumed known by hypothesis. The most common assumed version of $\mathbf{h}_t$ is that of a vector with all elements equal to zero except the element that corresponds to the target range, which is a point target.
$\mathbf{h}_{t,R(0)}$	Column vector of discrete time sampled target impulse function valid at the beginning of the coherent processing interval	This vector is assumed known by hypothesis. The most common assumed version of $\mathbf{h}_t$ is that of a vector with all elements equal to zero except the element that corresponds to the target range corresponding to $R(0)$, which is a point target.
H_0	Hypothesis that the target is not present	
H_1	Hypothesis that the target is present	
η	Detection threshold	The detection threshold is set to guarantee a maximum probability of false alarm. Its value is determined from the radar receiver noise power σ_n^2 .
i	Integer index	
$\mathbf{I}$	Identity matrix, i.e., a matrix with ones on the diagonal and all other elements equal to zero	
j	Square root of -1	
k	Integer index	
l	Integer index	
λ	Wavelength of radar signal	This parameter is determined by the radar's hardware design.
m	Integer indexing the pulse number	
M	Number of pulses in the Coherent Processing Interval	This parameter is determined by the radar's hardware design.

$\mathbf{M}_c$	Time correlation matrix of the clutter	This matrix can be estimated by measurements of the radar or by clutter models. See equation (28).
$\mathbf{M}_{C_A}$	The time correlation matrix of the clutter that actually exists	This is used in calculating performance of the radar when the detector is mismatched.
$\mathbf{M}_{C_D}$	The time correlation matrix of the clutter to which the detector is designed as opposed to the clutter time correlation that the radar actually sees	This is used in calculating performance of the radar when the detector is mismatched.
$\mathbf{n}$	Vector of additive white Gaussian noise (AWGN).	The statistical properties of $\mathbf{n}$ are generally determined by the radar front-end low noise amplifier. The AWGN is justified if the data passes through an ideal filter and is sampled at the Nyquist rate.
N	Number of samples of support for the radar pulse	This parameter is determined by the radar's hardware design.
P	This parameter determines the number of samples in the receive interval for each pulse.	This parameter is determined by the radar's hardware design.
$\phi(m,n)$	Target phase angle as a function of pulse number and sample number	This value is determined by the target motion model and the radar's wavelength. It captures phase change over the pulse time.
$R(t)$	Target range as a function of time. $R(0)$ is the target range as at the beginning at coherent processing interval.	This function is assumed known by hypothesis. Note that targets with unknown dynamics can be detected with a bank of detectors with different $R(t)$ functions similar to what pulse Doppler radars do with banks of Doppler filters.
$\mathbf{R}_c$	Spatial correlation matrix for the clutter	See equation (14).
$\mathbf{R}_{C_A}$	The spatial correlation matrix for the clutter that the radar actually sees	This is used in calculating performance of the radar when the detector is mismatched.
$\mathbf{R}_{C_D}$	The spatial correlation matrix for the clutter for which the detector is designed as opposed to what the radar actually sees	This is used in calculating performance of the radar when the detector is mismatched.
$\mathbf{R}_c(i)$	Square matrix the same size as $\mathbf{R}_c$ with all elements equal to zero except the i,i 'th element, which is equal to the i,i 'th element of $\mathbf{R}_c$	See equation (140).
$R_g^i(\tau)$	Time correlation function of clutter at range cell i . Note that, under certain conditions, this	This function can be estimated by measurements in the radar, by clutter models, or by knowledge of the radar's motion relative to clutter and knowledge of the

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	function is the same for all values of i .	radar's antenna pattern (see U.S. Patent 9,857,455 B2)
$R_{GRP}(t)$	Range to the ground reference point (i.e., center of resolution cell)	This is determined by radar parameters and geometry. See Figure 6 and equation (120).
$R_{GRP}^i(t)$	Range to the i 'th ground reference point	This is determined by radar parameters and geometry. See Figure 7 and equation (122).
$\mathbf{R}_I$	The correlation matrix for the interference processes that include clutter and noise	See equation (13).
$\mathbf{R}_{I_A}$	The correlation matrix for the interference processes that include clutter and noise that actually are presented to the radar	This is used in calculating performance of the radar when the detector is mismatched.
$\mathbf{R}_{I_A}$	The correlation matrix for the interference processes that include clutter and noise for which the detector is designed as opposed to the actual correlation matrix	This is used in calculating performance of the radar when the detector is mismatched.
$\tilde{\mathbf{R}}_m(i)$	Range migration matrix. Multiplication by this matrix convolves the sinc function to account for range movement during the coherent processing interval for the i 'th pulse.	The range migration matrix is calculated based on the radar design parameters and knowledge of $R(t)$ (which is assumed by hypothesis).
$\tilde{\mathbf{R}}_m'(i)$	Range migration matrix. Multiplication by this matrix convolves the sinc function to account for range movement during the coherent processing interval for the i 'th pulse.	This matrix moves the target in the opposite direction. Note that $\tilde{\mathbf{R}}_m'(i)\tilde{\mathbf{R}}_m(i) \approx \mathbf{I}$.
$\mathbf{R}_{y_c}$	Correlation matrix of $\mathbf{Y}_c$	See equation (27).
$s(t)$	Baseband representation of transmitted pulse	The transmitted pulse is determined by the radar hardware design.
$\mathbf{s}$	Vector whose elements are the sampled values of $s(t)$	The transmitted pulse is determined by the radar hardware design.
$\tilde{\mathbf{s}}$	Signal convolution matrix. Multiplication by $\tilde{\mathbf{s}}$ performs the operation of convolution.	Determined by $\mathbf{s}$, N , and P , which are all determined by the radar hardware design
$\tilde{\mathbf{S}}_m$	Signal convolution matrix. Multiplication by $\tilde{\mathbf{S}}_m$ performs the operation of convolution where the waveform may be different pulse to pulse.	Determined by $\mathbf{s}_m$, N , and P , which are all determined by the radar hardware design

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$\tilde{\mathbf{S}}_d(m)$	Signal convolution matrix that includes the target Doppler for the m 'th pulse	Determined by $\mathbf{s}$, N , and P , which are all determined by the radar hardware design and knowledge of $R(t)$
$\tilde{\mathbf{S}}_{m,d}(m)$	Signal convolution matrix that includes the target Doppler for the m 'th pulse where the waveform may be different pulse to pulse	Determined by $\mathbf{s}_m$, N , and P , which are all determined by the radar hardware design and knowledge of $R(t)$.
$\tilde{\mathbf{S}}_c(m)$	Signal convolution matrix that has the includes the clutter Doppler for the m 'th pulse.	Determined by $\mathbf{s}$, N , and P , which are all determined by the radar hardware design and knowledge of $R_{GRF_i}(t)$
σ_i^2	Clutter power for range cell i	The clutter power can be estimated by measurements from the radar or from clutter models.
σ_n^2	Receiver noise power	The receiver noise power is a radar design parameter.
$\sigma_{n_A}^2$	Receiver noise power that actually exists	This is used in calculating performance of the radar when the detector is mismatched.
$\sigma_{n_D}^2$	Receiver noise power for which the detector is designed as opposed to the noise power that actually exists.	This is used in calculating performance of the radar when the detector is mismatched.
t	Time variable	
T_i	Time between transmission of radar pulses	This parameter is determined by the radar hardware design.
T_{rx_delay}	Time delay from the beginning of the radar transmitted pulse and the first receive sample recorded	This parameter is determined by the radar hardware design.
T_s	Time between digital samples of the received signal.	This parameter is determined by the radar hardware design.
$\square_i$	Clutter mean power at range cell i	This parameter is determined by the clutter scene presented to the radar. It can be estimated by online measurements from the radar or by a clutter model.
u_i	Complex value corresponding to the phase of the target return due to target motion for pulse i	See equation (20).
$\mathbf{u}$	Vector of u_i 's	See equation (20).
$\mathbf{u}_A$	Actual vector of phase per pulse due to target motion	This is used in calculating performance of the radar when the detector is mismatched.
$\mathbf{u}_D$	Design vector of phase per pulse that the detector uses as opposed to the actual vector	This is used in calculating performance of the radar when the detector is mismatched.
v_r	Radar velocity	
x_{corr}	Complex output of a correlator for the range cell under test	

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$\mathbf{y}$	Vector of time sampled receive data for one inter-pulse period	Output radar data illustrated in Figure 5
$\mathbf{y}_c$	Vector of time sampled receive data for one inter-pulse period due to the target	Output radar data illustrated in Figure 5
$\mathbf{y}_I$	Vector of time sampled receive data for one inter-pulse period due to the interference process (clutter and noise)	Output radar data illustrated in Figure 5
$\mathbf{y}_n$	Vector of time sampled receive data for one inter-pulse period due to noise.	Output radar data illustrated in Figure 5
$\mathbf{y}_t$	Vector of time sampled receive data for one inter-pulse period due to the target	Output radar data illustrated in Figure 5
$\mathbf{Y}$	Vector of time sampled receive data for all inter-pulse periods. This vector is a vertically stacked version of all the $\mathbf{y}$'s.	Output radar data illustrated in Figure 5
$\mathbf{Y}_c$	Vector of time sampled receive data for all inter-pulse periods due to clutter. This vector is a vertically stacked version of all the $\mathbf{y}_c$'s.	Output radar data illustrated in Figure 5
$\mathbf{Y}_I$	Vector of time sampled receive data for all inter-pulse periods due to interference processes (clutter and noise). This vector is a vertically stacked version of all the $\mathbf{y}_I$'s.	Output radar data illustrated in Figure 5.
$\mathbf{Y}_n$	Vector of time sampled receive data for all inter-pulse periods due to noise. This vector is a vertically stacked version of all the $\mathbf{y}_n$'s.	Output radar data illustrated in Figure 5
$\mathbf{Y}_t$	Vector of time sampled receive data for all inter-pulse periods due to the target. This vector is a vertically stacked version of all the $\mathbf{y}_t$'s.	Output radar data illustrated in Figure 5
$Z_{corr+MTD}$	Complex voltage output of correlation (in fast time) and Doppler processing (slow time) for the range cell under test. See (47).	Output of standard radar processing before detection decision. The detection decision is usually made on the absolute value of $Z_{corr+MTD}$
$Z_{corr+MTD_T}$	Complex voltage output of correlation (in fast time) and	

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	Doppler processing (slow time) for the range cell under test due to the target. See (49).	
$\rho_{j,k}$	Time correlation coefficient of clutter between the j 'th and k 'th pulses	This information can be estimated by measurements in the radar, by clutter models, or by knowledge of the radar's motion relative to clutter and knowledge of the radar's antenna pattern (see U.S. Patent 9,857,455 B2).
$\mathbf{p}_{j,k}$	Diagonal matrix of clutter correlation coefficients. The row/column indicates the range; j and k indicate this is the correlation coefficient between the j 'th and k 'th pulses.	This information can be estimated by measurements in the radar, by clutter models, or by knowledge of the radar's motion relative to clutter and knowledge of the radar's antenna pattern (see U.S. Patent 9,857,455 B2).
θ_{az}	Azimuth angle between the radar velocity vector and ground reference point	This is determined by the direction of travel of the radar, the antenna look angle.
θ_{cone}	Cone angle between the radar velocity vector and ground reference point	See equation (116).
θ_{el}	Elevation angle between the radar velocity vector and ground reference point	This is determined by the direction of travel of the radar, the antenna look angle.
θ_{p_az}	Azimuth pointing angle of the antenna	This angle may be controlled by the radar (if the antenna is mechanically scanned or is phased array). Otherwise, it is determined by antenna design and installation.
θ_{p_el}	Elevation pointing angle of the antenna	This angle may be controlled by the radar (if the antenna is mechanically scanned or is phased array). Otherwise, it is determined by antenna design and installation.
$\zeta_t(\theta)$	Random variable that represents clutter backscatter voltage as a function of time t and azimuth angle θ . The radar observes the clutter backscatter once it has been integrated over the antenna beam in azimuth as shown in (143).	

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