



AFRL-AFOSR-UK-TR-2021-0010

Phase-changing Morphing Metamaterial with Hieararchical Musco-Skeletal Structure

Lanzara, Giulia
UNIVERSITA' DEGLI STUDI ROMA 3
VIA VITO VOLTERRA 62
ROMA, ROMA, 146
ITA

05/14/2021
Final Technical Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
Air Force Office of Scientific Research
European Office of Aerospace Research and Development
Unit 4515 Box 14, APO AE 09421

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.					
1. REPORT DATE (DD-MM-YYYY) 14-05-2021		2. REPORT TYPE Final		3. DATES COVERED (From - To) 15 Mar 2017 - 14 Sep 2020	
4. TITLE AND SUBTITLE Phase-changing Morphing Metamaterial with Hieararchical Musco-Skeletal Structure				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER FA9550-17-1-0014	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Giulia Lanzara				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) UNIVERSITA' DEGLI STUDI ROMA 3 VIA VITO VOLTERRA 62 ROMA, ROMA 146 ITA				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) EOARD UNIT 4515 APO AE 09421-4515				10. SPONSOR/MONITOR'S ACRONYM(S) AFRL/AFOSR IOE	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-AFOSR-UK-TR-2021-0010	
12. DISTRIBUTION/AVAILABILITY STATEMENT A Distribution Unlimited: PB Public Release					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT Bio-inspired morphing air vehicles have the potential to provide superior system capabilities by performing multi-scale shape reconfigurations. Biological systems are often formed with materials/mechanisms that allow them to adapt their body shape at the micro- or macro-scale in response to continuously monitored environmental parameters or to achieve desired functions. This inspired the current work. This grant explored design and multi-physics modeling of a multi-stable muscular carbon fiber composite plate and a 3D muscular composite material to achieve macro, global shape change. It also developed self-activated muscular piezoelectric "pixels" material that achieved localized shape change and tuning through resonate frequency modulation. This grant also produced a prototype and concept demonstrator. Multiple papers were published and well-recieved conference talks were given.					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			DAVID GARNER
U	U	U	SAR	2	19b. TELEPHONE NUMBER (Include area code) 314-235-6420

Highly reconfigurable, multistable composite with tunable global/local morphing capability

Final Report

May 14, 2021

Sponsor: US Dept of Air Force
Sponsor Ref: FA9550-17-1-0014

Submitted to
Dr. David Garner
Program Manager
European Office of Aerospace Research and Development (EOARD)
email: david.garner.2@us.af.mil

By

Giulia Lanzara (PI)
Engineering Department
University of Rome, RomaTre

giulia.lanzara@uniroma3.it

Walter Lacarbonara (Co-PI)
Structural Engineering Department
Sapienza University of Rome

walter.lacarbonara@uniroma1.it

TABLE OF CONTENTS

INTRODUCTION

1 THE OVERALL PROJECT OBJECTIVE

2 MUSCULAR COMPOSITE FOR GLOBAL MORPHING

2.1 MATERIAL DESIGN CONCEPT AND OBJECTIVES

2.2 MUSCULAR CARBON FIBER COMPOSITE PLATE

2.2.1 Multi-physics modelling of the muscular composite plate

2.2.2 Experimental validation of the Muscular Composite plate

2.2.2.1 Fabrication and morphology characterization

2.2.2.2 Testing and mechanical characterization of the CF composite

2.2.2.3 Functionality of the Muscular Composite Response: Experiments vs Theory

Passive Response

Self-actuated muscular composite

Self-tuning: frequency and damping

Tuning snap-through energy

2.3 THREE-DIMENSIONAL (3D) MUSCULAR COMPOSITE

2.3.1 Towards enhanced morphing with higher load-bearing capability

2.3.1.1. Self-activated 3D muscular composite

2.3.1.2. Enhancing morphing while improving load-bearing capability

2.3.1.3 Effect of the composite curvature

2.3.2 Towards radical shape changes

2.4 PROTOTYPE OF A SMART PROSTHETIC LEG VIA A “SENSITIVE” MUSCULAR 3D COMPOSITE

2.4.1 Prosthetic design and manufacturing

2.4.2 Autonomous adaptation of the muscular composite vs. Walking Mode

2.4.2.1 The algorithm

2.4.2.2 Characterization of the heater through the Microcontroller

2.4.3 The functionality assessment

2.4.4. Final demonstration of the multi-functionality of the prosthesis

3 LOCAL MORPHING

3.1 LOCAL MORPHING OF COMPOSITE SKIN VIA SELF-ACTIVATED MUSCULAR COMPOSITE IN PIXELS

3.2 DYNAMIC MORPHING VIA PIEZOELECTRIC ACTUATION

3.2.1 Dynamic morphing of membranes with embedded PVDF wires

Nonlinear model and equations of motion
Analytic approach

3.2.2 Dynamic morphing via principal parametric resonances

3D ABAQUS simulations

3.2.3 Dynamic morphing of PVDF panels

The PVDF nonlinear model
Morphing activation regions
Frequency response curves of the morphing response
Validation via ABAQUS simulations
Pixelated PVDF panels
Summary

3.2.4 Dynamic morphing of panels with embedded PVDF wires

Nonlinear model

3.2.5 Piezoelectrically induced morphing shapes

Principal parametric resonance
Primary resonance induced morphing
Superharmonic resonance morphing
Subharmonic resonance morphing

3.3. EXPERIMENTAL VALIDATION OF STATIC AND DYNAMIC MORPHING VIA PIEZOELECTRIC ACTUATION

3.3.1 Electro-Mechanical Model in support of the experiments

Poling directions

Buckling Analysis

3.3.2 Static morphing

3.3.3 Dynamic morphing by exploiting parametric resonance

3.3.4 Local morphing of a pixelated coating by exploiting parametric resonance

INTRODUCTION

Bio-inspired morphing air vehicles have the potential to provide superior system capabilities by performing multi-scale shape reconfigurations. This represents today one of the greatest challenges in the aviation field. Biological systems are in fact often formed with materials/mechanisms that allow to adapt their body shape at the micro- or macro-scale in response to continuously monitored environmental parameters or to achieve desired functions. For instance, falcons [1] are able to change and control largely their wing size and shape macroscopically. A high aspect ratio of their wings allows them to loiter on-station using air currents until they detect a prey while the morphed strike configuration allows them to swoop down on a victim. The secret of birds wing in general relies on a special musco-skeletal *mechanism* where couples of criss-crossing muscles act in opposition to produce movement of the anchored skeletal structure. Nature has been able to deliver other formidable wing architectures like that of bats in which a *membrane* wing structure is tuned to maximize flight performance [2,3]. The uniqueness relies on a primary skeletal structure that interfaces an array of small and weak muscles (the plagiopatagiales) forming a mechanism, embedded and distributed all over the wing membrane (through its full thickness/area). These muscles have the role of modulating membrane stiffness to improve wings efficiency, to passively modify the membrane deformations, to actively maximize force generation as well as to control wing shape. A key advantage of such a membrane structure is that the resulting wing is much thinner and lighter than other structures, thus leading to low power consumption due to the reduced inertial cost of flapping. The remarkable beauty is that these functions are activated to modulate the membrane stiffness either in synchrony for a global response, or by producing gradients along the membrane resulting in an outstanding local feature. Small-scale morphing actions can also be found in feathered birds that are able to change feathers position accordingly to flight conditions and help to reduce drag for slow predatory land birds by using low aspect-ratio wings. Further scale reductions of local morphing can be found in other living creatures, like in dolphins [4], whose skin is corrugated with active micro-scaled dermal ridges that are able to drastically reduce drag in response to sensed fluid flow.

In other words, nature provides creatures with a multi-scale morphing capability that is activated as a reaction or an adaptation to sensed stimuli at the local (micro-scale) and global level (macro-scale).

These lessons from nature indicate that the major requirements for materials employed in smart morphing applications can be summarized as follows: (1) capability of sensing local parameters over large areas; (2) flexibility in locally or globally adapting to different shapes; (3) capability of morphing in multi-scale; (4) capability to reconfigure into a large number of shapes; (5) capability of modulating rigidity; (6) capability of carrying loads; (4) light weight (important, for example, for reducing fuel thus energy consumption during flights); (5) low manufacturing costs; (6) elimination of heavy, fault-sensitive control systems and, finally; (7) skin continuity.

It is not possible to list all the researchers' attempts to mimic these functions. The overall attempts focused on in-plane, out-of-plane wing morphing as well as in airfoil adjustment, the latter of which relates to changes of the wing profile (e.g. thickness) without meaningfully changing the wing camber. To name just a few examples, a great deal of work [5] has been performed since the beginning of the century entirely using mechanical actuations and lately, from 1979, in the US Air Force research laboratory (AFRL) and (NASA) Dryden Flight Research Center, with hydraulic actuations of discrete components. However, these approaches did not prove to be satisfactory due to the increases in weight, complexity and volume which are against aerodynamic benefits. Further studies by these two major agencies considered the use of flexible materials to reduce weight and improve aerodynamics and then, from 1990s the interest switched toward the development of adaptive structural morphing for biologically-inspired flight systems. From 2002 up to today's attempts, a major focus has been devoted to the development of passive, intrinsically adaptive or active smart materials that could facilitate the achievement of the above mentioned requirements. The general conceptual frame is that more electric materials or structures for aircrafts could replace the centralized aircraft hydraulic power systems with an electrically based power system allowing for improved reliability, maintainability, and

supportability as well as the potential for significant performance improvements in terms of weight, volume and system complexity [5,6].

The mostly investigated materials/structures for morphing applications are certainly [5,6,7]: elastomers, auxetic passive materials, multistable composites, segmented structures, shape-memory alloys (SMA) / shape-memory polymers (SMP) and piezoelectric ceramics / polymers. The beauty of elastomers is that their extreme deformability is ideal for morphing skins, but this comes at the expenses of a negligible load-bearing capacity. Auxetic passive materials are characterized by a negative Poisson's ratio which allows for synclastic shape changes as necessary in morphing. Moreover, the nature of these materials is such that they are characterized by high energy absorption capability, fracture toughness and resistance to indentation, but they are not able to provide autonomous morphing capability. Finally, as of today, auxetics have a macroscopic cellular structure which requires their coupling with other materials adding weight and complexity. Segmented structures can cause critical discontinuities on the surface which is detrimental in aerodynamics. Shape-memory alloys and shape-memory polymers, which can be programmed to switch from one configuration to the other, are very slow in reaction limiting their applicability. On the contrary, multistable composites [8,9], which are capable of switching from one stable configuration to another allowing global potential morphing, are definitely much faster although this comes with other limitations such as the lack of control of the transition speed, the onset of nonlinear vibrations upon snapping, and relatively small snap-through amplitudes. The latter, on one hand, entails the advantage of requiring low actuation power, but also the disadvantage of low load-bearing capability. Moreover, as of today SMP/SMA materials and multistable laminated composites are severely limited by the low number of achievable configuration shapes (up to three, at most), are not capable of providing global/local morphing and are not reconfigurable in new shapes besides the originally programmed ones. Dense arrays of piezoceramics may "in principle" overcome some of the above limitations, but not only the associated technologies are not yet mature [10-16], PZTs also require continuous power for operation, they exhibit very small deformation ranges with limitations for large-amplitude morphing.

Despite the above summarized important efforts in morphing wing and skin, most studies focused on large-scale morphing and not on local morphing. There are just few examples in which novel designs were proposed to modify the fluid-structure interaction at the local scale. Some of these approaches do not involve "morphing" since they are "passive" design strategies. The most famous of these passive strategies takes inspiration from the skin of high speed sharks [17-20] that comprises micro-scaled riblets, over critical skin areas, that are responsible for a reduction in friction. When using a surface with riblet structures, the turbulent momentum transfer, which is responsible for the skin friction, is in fact hampered. This strategy works for all turbulent flowing fluidic media and for this reason it was used to design novel airfoil wings with several different riblet geometries. For instance, researchers at NASA Langley Research Center tested riblet structured surfaces with a zigzag profile [17] which provided 10% shear stress reduction. Other attempts used lighter polymer laminated riblets [20]. To the best of our knowledge, effort towards the implementation of "active local dynamic morphing" to modify the fluid-structure interaction is really limited. Limited examples of dynamic morphing to control local flow behaviour can be found for turbines applications where an active interaction with a fluid is established. In this context PZT microactuators were used to control vortex induced vibrations in a two cylinder experiment. By inducing local vibration of the wall, 50-80% reduction of cylinder vibrational energy was observed [21]. The authors concluded that a better response can be achieved with actuators that are capable of providing higher amplitudes. Large-scale local morphing (involving macro-scale areas) was also achieved through a complex structural concept where a compliant cellular truss structure, with tendons used as active elements and with cells inter-connected through compliant joints, was investigated [22].

The above brief overview clearly highlights that the design of materials that can provide the listed key features (local, global and multi-scale morphing; reconfigurability into a large number of shapes; rigidity modulation; load-bearing; light weight, material continuity) still represents the greatest challenge in the morphing field.

References

- [1] J. E. Blondeau, and D. J. Pines (2004) Pneumatic Morphing Aspect Ratio Wing. In: *45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference*, 19-22 April, Palm Springs, CA, Paper No. AIAA2004-1808.
- [2] J. A. Cheney et al. (2014) Membrane muscle function in the compliant wings of bats. *Bioinspiration & Biomimetics* 9, 025007
- [3] J. M. Zook, presented at *Third European Bat Research Symposium*, Aberdeen, U.K., August 1985.
- [4] M. O. Kramer (1965) Hydrodynamics of the Dolphin, in *Advances in Hydrosience Vol. 2* (Ed: V. T. Chow), Academic Press, New York 1965, 111-130.
- [5] C. Thill, J. Etches, I. Bond, K. Potter, P. Weaver (2008) Morphing Skin, *The Aeronautical Journal*, 112, 1129, 117-139.
- [6] A.Y.N. Sofla, S.A. Meguid, K.T. Tan, W.K. Yeo (2010) Shape morphing of aircraft wing: Status and challenges, *Materials and Design*, 31, 1284-1292.
- [7] S. Özgün, Y. Yaman, M. Şahin, G. Seber, L. Ünlüsoy, E. Sakarya, T. İnsuyu (2010) Morphing air vehicle concepts *Proceedings of the International Workshop on Unmanned Vehicles -UVW2010*
- [8] M. Santer, S. Pellegrino (2008) Compliant Multistable Structural Elements, *International Journal of Solids and Structures*, Volume 45, Issue 24.
- [9] F. Mattioni, P.M. Weaver, K.D. Potter, M.I. Friswell (2008) Analysis of thermally induced multistable composites, *International Journal of Solids and Structures*, Volume 45, Issue 2, 15 January 2008, Pages 657-675.
- [10] G. Lanzara, N. Salowitz, Z. Guo, F.K. Chang (2010) A Spider-Web-Like Highly Expandable Sensor Network for Multifunctional Materials *Advanced Materials*, Volume 22, Issue 41, November 2, 2010, Pages 4643-4648.
- [11] G. Lanzara, J. Feng, F.-K. Chang (2010) Design of micro-scale highly expandable networks of polymer-based substrates for macro-scale applications, *Smart Materials and Structures*, 19, 045013.
- [12] N. Salowitz, Z. Guo, S.-J. Kim, Y.-H. Li, G. Lanzara, F.-K. Chang (2014) Micro-fabricated expandable sensor networks for intelligent sensing materials. *IEEE Sensors Journal*, 14, 7, 2014.
- [13] G. Lanzara, L. Zhang, P. Peumans, F.-K. Chang (2009) Embedded Nano/Micro Sensor Networks for Intelligent Structures. *NSF Engineering Research and Innovation Conference, Honolulu, Hawaii*, June 2009
- [14] G. Lanzara, N. Salowitz, Z. Guo, D. Chatterjee, K. Kim, P. Peumans and F.-K. Chang (2009) Multi functional sensor nodes in stretchable network for structural health monitoring. *7th International Workshop on Structural Health Monitoring*, Stanford, September 2009.
- [15] N. Salowitz, Z. Guo, Y.-H. Li, K. Kim, G. Lanzara, F.-K. Chang (2013) Bio-Inspired Stretchable Network Based Intelligent Composites, *Journal of Composite Materials*, 47(1), 97-105.
- [16] G. Lanzara and F.-K. Chang (2008) A Large Area Multifunctional Stretchable Network for Smart Structures. *The 15th International Symposium on: Smart Structures and Materials & Nondestructive Evaluation and Health Monitoring*, SPIE, San Diego, USA, March 2008
- [17] W. Hage. Zur Widerstandsverminderung von dreidimensionalen Riblet-Strukturen und anderen Oberflächen. PhD Thesis, TU Berlin (2004).
- [18] K. Oehlert, J. R. Seume, B. Wang, B. Denkena, W. Hage, K. Knobloch, R. Meyer, F. Siegel, A. Ostendorf, T. Vynnyk and E. Reithmeier (2007) Exploratory Experiments on Machined Riblets for 2-D Compressor Blades. In: *IMECE 2007 - 43457*.
- [19] D. W. Bechert, M. Bruse and W. Hage (2000) Experiments with three-dimensional riblets as an idealized model of shark skin. *Exp. Fluids*, 28, 403.
- [20] D. W. Bechert, M. Bruse, W. Hage and R. Meyer (2000). Fluid Mechanics of Biological Surfaces and their Technological Application. *Naturwissenschaften* 87, 157.
- [21] Bao-Qing Li, Yang Liu, Jia-Ru Chua (2007) Vortex-induced Vibration Control by Micro Actuator. *Journal of Mechanical Science and Technology* 21, 1408-1414.
- [22] S. Ramrakhiani, George A. Lesieutre, M. Frecker, and S. Bharti (2005) Aircraft Structural Morphing Using Tendon-Actuated Compliant Cellular Trusses Deepak, *JOURNAL OF AIRCRAFT* Vol. 42, No. 6, November-December 2005.

1. THE OVERALL PROJECT OBJECTIVE

The objective of the project is to engineer a bat-inspired morphing composite with particular focus on the following unique capabilities: dramatic global reconfigurability, passive local/global morphing, local dynamic morphing, orders of magnitude tunable rigidity and damping. This is enabled by: (a) a novel muscular composite design which assembles, in a strategic manner, carbon fibers (CF) in a thermosetting matrix; (b) by a pattern of piezoelectric wires and/or films that drive locally a polymer layer and/or the piezoelectric film itself; and (c) by studying a novel parametric resonance-induced morphing for ultra-fast multi-shape actuation. Figure 1 shows a schematic representation of the overall material design.

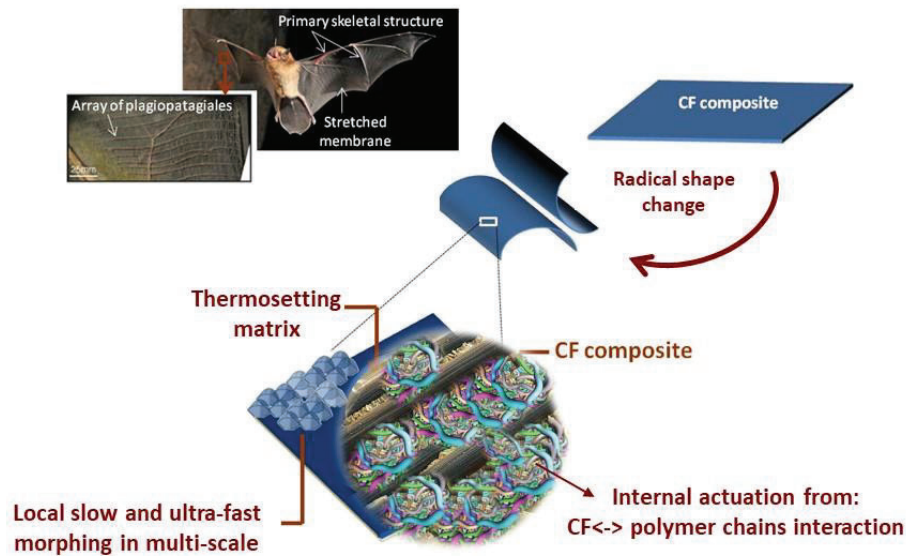


Figure 1: Schematic representation of the proposed concept design.

The above objectives were assessed with a theoretical, numerical and experimental approach. To highlight the major project objectives, the report is structured with the following key sections:

1. Design, realization, and characterization of the novel muscular carbon fiber composite for global morphing
2. Local morphing in multi-scale: static, quasi-static and dynamic with and without the exploitation of parametric resonance

Note. The reported experimental activities were performed at the Multifunctional Materials Laboratory, Engineering Department, University of RomaTre.

2 MUSCULAR COMPOSITE FOR GLOBAL MORPHING

The research activities are focused on the development of a carbon fiber composite that has the capability to morph, modify its rigidity and damping capability without the need of an external actuation. Those functions are indeed delivered without the risk of undergoing into material damage as would occur in standard carbon fiber composites which can only accept ultra-small strain levels. The material design concept, modeling, fabrication, testing and characterization for global as well as for local skin morphing, are discussed in the following sections.

2.1 MATERIAL DESIGN CONCEPT AND OBJECTIVES

Morphing of a carbon fiber composite plate is here achieved without the exploitation of a shape memory effect, of a bi-stability concept or of the integration of actuation devices and systems. Avoiding the usage of such materials/devices, allowed the development of a carbon fiber composite which provides simultaneously cyclic shape changes, large morphing excursions and tunable rigidity. Such properties are in contrast with SMP (which, as said, lack of cyclability and load-bearing capability) or with multistable composites (which additionally deliver just small excursions). The above design choices and morphing specs, let us define this innovative material as a “**Muscular composite**” in which *distributed internal muscles* are the result of a strategic combination of *internal mechanical stresses* and *temperature dependent material properties*. It is worth nothing that the muscular composite can morph “passively” as a result of environmental thermal effects, as well as through “self-actuation” by actively controlling the material temperature. Moreover, the active control of temperature, leads to the capability of tuning the structural stiffness as well as damping. This implies that the proposed approach can be adopted not only for morphing purposes but also to control structural frequencies and the material response to dynamic excitations.

The above mentioned material design concept is applied within this research to carbon fiber composite plates in order to make them morph from a bent to a flat configuration, as well as to composites with initial 3D configurations that morph toward a flat shape and viceversa. The study also tackles both cases of composites that are designed to provide orders-of-magnitude rigidity modulation as well as those that morph in large-scale without an excessive loss of rigidity. The material design concept is presented in this section for the case of a 2D carbon fiber composite plate that provides a wide range of rigidity modulation, while the discussion of 3D morphing composites at reduced rigidity modulation will be detailed in the following section.

2.2 MUSCULAR CARBON FIBER COMPOSITE PLATE

Multiple layers of 0°/90° carbon fiber mats are embedded into a thermosetting high temperature-sensitive resin characterized by orders of magnitude variations in stiffness with temperature.

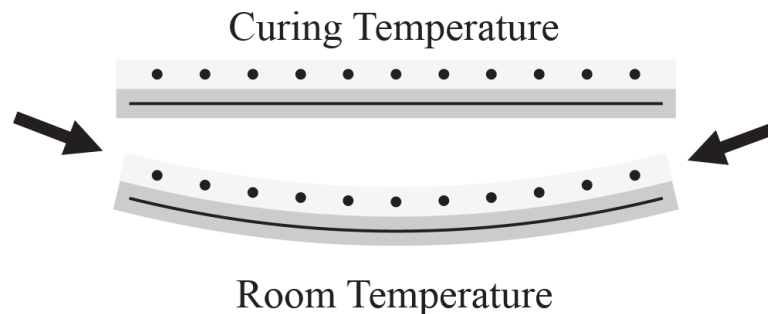


Figure 2: Possible shapes of the composite.

The $0^\circ/90^\circ$ layers start interacting and, due to the different coefficient of thermal expansion along the longitudinal and transverse direction, strong internal thermal stresses arise. This interaction is generated by the differential shrinkage in the two orthogonal directions arising in the cooling phase of the manufacturing process. Such stresses cause a thermal contraction in the direction denoted by the black arrows in Figure 2. Consequently, the composite is bended reaching a curved shape at room temperature.

By inducing thermal gradients into the material, it is possible to prescribe stiffness variations in agreement with the proposed material design. The beauty is that the matrix can be cyclically and gradually switched from a rigid to a soft phase **activating** or **de-activating internal stresses**, by simply controlling the matrix temperature.

A schematic example is reported in Figure 3 which clearly highlights the logic of the cyclic morphing of the composite plate. The schematic also highlights the case study that will be later discussed, in which the material comprises an internal heater to self-tune the matrix rigidity by exploiting Joule Effect.

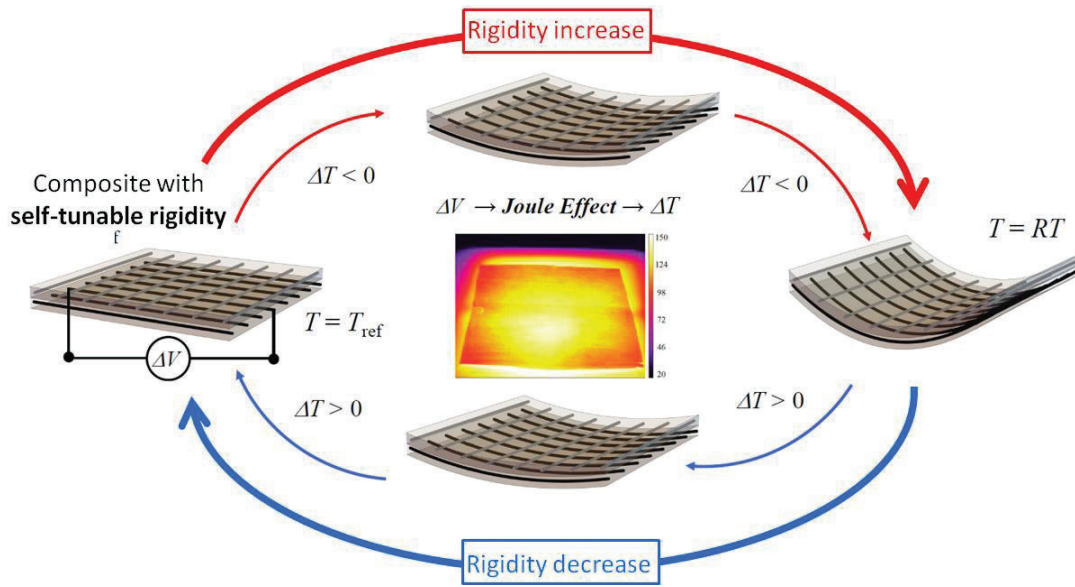


Figure 3: Concept design of a muscular composite plate with self-tunable rigidity

2.2.1 Multi-physics modelling of the muscular composite plate

A parametric model was developed in order to design the composite to be manufactured according to specific requirements. The uniqueness of this model relies on the fact that the composite consists of a specific lay-up of carbon fibers that are embedded in a hosting matrix whose rigidity is strongly influenced by temperature (for orders of magnitude). This is clearly deeply different from standard cases in which the matrix rigidity is not significantly influenced by temperature.

The implemented model is based on Hyer theory that allows to predict the room-temperature shape/curvature of cross-ply laminates depending on geometry, layup sequence and material parameters. In our case, however, the material parameters that are strongly affected by temperature, such as Young's modulus, are introduced in the model in order to accurately predict the composite behavior. The implemented model is then verified through a thorough experimental campaign.

Strain-Displacement Relationship

The coordinates x, y are the directions in the plane of the plate and the origin is set in the center of the plate. The horizontal displacements along x and y are u and v respectively, and u^0, v^0 and w^0 denote the displacements of the mid-plane, while the vertical displacement is denoted by w^0 . By assuming the Kirchhoff hypothesis valid and introducing the geometric nonlinearity according to the Von Karman theory, the strains in the mid-plane are

$$\begin{aligned}\epsilon_x^0 &= u_{,x}^0 + \frac{1}{2}(w_{,x}^0)^2 \\ \epsilon_y^0 &= u_{,y}^0 + \frac{1}{2}(w_{,y}^0)^2 \\ \gamma_{xy}^0 &= \frac{1}{2}(u_{,y}^0 + v_{,x}^0 + w_{,x}^0 w_{,y}^0)\end{aligned}, \quad (\text{A.1})$$

and the curvatures are

$$\kappa_x^0 = -w_{,xx}^0, \quad \kappa_y^0 = -w_{,yy}^0, \quad \kappa_{xy}^0 = -2w_{,xy}^0, \quad (\text{A.2})$$

where the subscript $(,k)$ denotes $\partial/\partial k$ with $k = (x, y, z)$. Strains and curvatures are collected in the vectors ϵ^0 and κ^0 , respectively. Thus, the general strain-displacement relationship is expressed as

$$\epsilon = \epsilon^0 + z \kappa^0. \quad (\text{A.3})$$

According to the expected displacement fields, observed experimentally, the trial function are drawn from [24] and can be defined as follows

$$\begin{aligned}u^0 &= xc_1 + xy^2 \left(c_3 - \frac{c_{11}^2}{8} \right) + \frac{x^2 y}{2} \left(c_4 - \frac{1}{2} c_{11} c_9 \right) + yc_{12} + \frac{c_{13}}{3} y^3 + \frac{x^3}{3} \left(c_2 - \frac{c_9^2}{2} \right), \\ v^0 &= \frac{xy^2}{2} \left(c_8 - \frac{1}{2} c_{10} c_{11} \right) + \frac{y^3}{3} \left(c_7 - \frac{c_{10}^2}{2} \right) + x^2 y \left(c_6 - \frac{c_{11}^2}{8} \right) + xc_{12} + \frac{c_{14}}{3} x^3 + yc_5 \\ w^0 &= \frac{1}{2} (y^2 c_{10} + xyc_{11} + x^2 c_9).\end{aligned}, \quad (\text{A.4})$$

These expressions were proposed to investigate the behavior of general oriented laminates, but can be successfully adopted to study cross-ply laminates.

Constitutive law for a plain-stress state

The composites laminate is a material that exhibits different behaviors along the fibers and transversely to them, thus it can be called transversely isotropic. The constitutive law can be expressed as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_y \end{Bmatrix} \Rightarrow \sigma = \bar{Q} \quad (\text{A.5})$$

where $\bar{Q}$ denotes the reduced stiffness matrix, which is symmetric and the independent components are:

$$\begin{aligned}\bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta, \quad \bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12}(\cos^4 + \sin^4) \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta, \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta, \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta, \quad \bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + \\ &\quad (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta + Q_{66} \cos^4 \theta\end{aligned}$$

(A.6)

where θ is the orientation of the fibers with respect to the x -axis and

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}. \quad (\text{A.7})$$

Contrary to common composites, in some cases the temperature dependence of the material has non-negligible effect on its properties. This means that it is necessary to introduce the constitutive parameters of the material as functions of temperature T . In particular, the Young's moduli, the shear modulus and the Poisson's ratios must be defined as $E_1 = E_1(T)$, $E_2 = E_2(T)$, $E_G = G_1(T)$, $\nu_{12} = \nu_{12}(T)$ and $\nu_{21} = \nu_{21}(T)$, respectively. Under this assumptions the reduced stiffness matrix, and thus the stress vector are function of the temperature, as follows

$$\bar{\mathbf{Q}} = \tilde{\mathbf{Q}}(T), \quad \boldsymbol{\sigma} = \tilde{\boldsymbol{\sigma}}(T) = \tilde{\mathbf{Q}}(T)\boldsymbol{\epsilon}. \quad (\text{A.8})$$

Appendix A.3. Thermal stresses

The effect of the thermal contraction arising at the end of the curing process results internal thermal stresses, which are defined as follows

$$\boldsymbol{\sigma}_T = \mathbf{Q}(T)\boldsymbol{\epsilon}_T \quad (\text{A.9})$$

where $\boldsymbol{\epsilon}_T = (a_x\Delta T, a_y\Delta T, a_{xy}\Delta T)^T$ denotes the vector listing the thermal deformations. The coefficients of thermal expansion are referred to the x, y directions, but they depend on the fiber orientation according to the following expressions:

$$a_x = a_1 \cos^2 \theta + a_2 \sin^2 \theta,$$

$$a_y = a_1 \sin^2 \theta + a_2 \cos^2 \theta, \quad (\text{A.10}) \quad a_{xy} = 2(a_1 - a_2)\cos\theta \sin\theta$$

where a_1, a_2 are the coefficients of thermal expansion in the longitudinal and transverse directions, respectively, referred to the fibers orientation.

Equilibrium equations

The total potential energy of the system including the effect of thermal expansion is defined as the sum of the elastic part and the inelastic part. The elastic deformation $\boldsymbol{\epsilon}_{el}$ and the elastic stresses $\boldsymbol{\sigma}_{el}$ must be defined. They are

$$\boldsymbol{\epsilon}_{el} = \boldsymbol{\epsilon}(T) - \boldsymbol{\epsilon}_T \quad \text{and} \quad \boldsymbol{\sigma}_{el} = \tilde{\mathbf{Q}}(T)\boldsymbol{\epsilon}_{el}. \quad (\text{A.11})$$

The total potential energy is thus defined as

$$W = \int_V \left(\frac{1}{2} \boldsymbol{\sigma}_{el}^T \boldsymbol{\epsilon}_{el} + \boldsymbol{\sigma}_T^T \boldsymbol{\epsilon}_T \right) dV. \quad (\text{A.12})$$

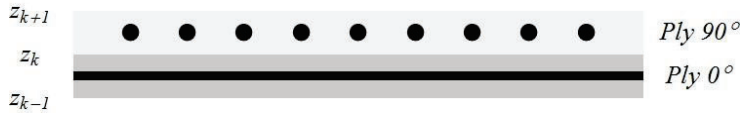


Figure 4. Schematic of a plate section to represent the plies superposition.

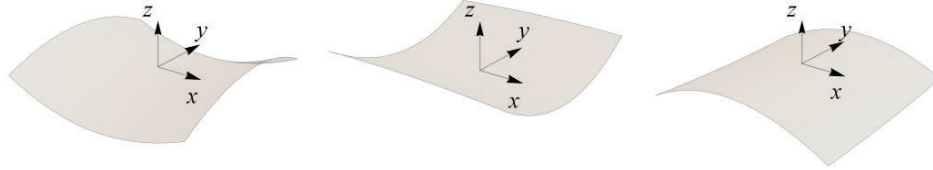
By considering that the composite is a superposition of several layers, as illustrated in Fig. 4, the volume integral can be written in explicit form as

$$W = \int_S \left[\sum_{k=1}^{N_{ply}} \int_{z_{k-1}}^{z_k} \left(\frac{1}{2} \boldsymbol{\sigma}_{el}^T \boldsymbol{\epsilon}_{el} + \boldsymbol{\sigma}_T^T \boldsymbol{\epsilon}_T \right) dS \right] dz \quad (\text{A.13})$$

where N_{ply} represents the number of plies that constitute the plate, while S denotes their surface. The equilibrium equations can be finally derived, according to the Rayleigh-Ritz principle of minimization of the total potential energy. This implies that the solutions of the problem are derived by searching zeroes of the first variation of W , i.e.

$$\frac{\partial W}{\partial \alpha_i} = \sum_i \frac{\partial c}{\partial \alpha_i} = 0 \quad (A.14)$$

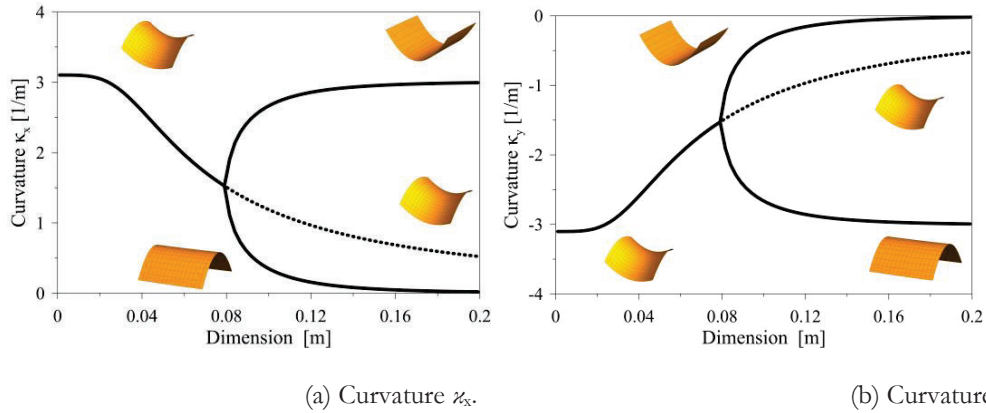
The latter is satisfied when every single term of the summation is 0 at the same time, thus they represent the equilibrium equations for the composite. Depending on the plate dimensions and the ply characteristics, the feasible room-temperature shapes that can be predicted by the model are three: a perfect saddle and two cylinder with the curvature axis in the opposite direction. Such shapes are represented in Fig. 5.



(a) Perfect saddle: $a = -b = 0$, $b > 0$ (b) Cylinder with $a < 0$, $b > 0$ (c) Cylinder with $a < 0$, $b < 0$. 1.6 m^{-1} .

Figure 5. Cross-ply laminates: room-temperature shapes.

According to the mechanical model it is possible to design the samples to get a determined room-temperature configuration, as illustrated in Fig. 6. In particular, by fixing the stack sequence, one can predict the room-temperature shape of the plate in function of its dimensions. Below a certain length the plate has only one shape, i.e. a stable saddle (black solid line), whereas for higher lengths the saddle becomes unstable (dashed line) and the two opposite cylindrical shapes arise (red and blue lines), with curvatures increasing with the dimension. These data are performed by inserting the *material properties that were experimentally tested for the fabricated composites* (as reported in the following sections).



κ_y .

Figure 6: The curvatures κ_x , κ_y are evaluated by varying the plate dimension.

2.2.2 Experimental validation of the Muscular Composite plate

2.2.2.1 Fabrication and morphology characterization

[0 , 90] carbon fiber reinforced composites are fabricated with unidirectional fibers tows (T300 from Torayca[®]) whose properties are given by the manufacturer as shown in the Table in

Figure 7. According to the provided values, even though the fibers diameter is $7\text{ }\mu\text{m}$, the overall thickness of the fiber mat reaches an average value of $100\text{ }\mu\text{m}$.

A Scanning Electron Microscope (SEM) is used to perform a preliminary assessment of the fibers quality and evaluate their actual configuration, in order to search for eventual imperfection or unwanted particles. As illustrated in Fig. 7, the SEM images allow to understand that not all the fibers are aligned in the same direction, there are smaller areas with overlaying fibers and other with lack of fibers, moreover it can be seen that some particles are already present within the fibers. This allows to expect a nonperfect composite, but of a quality that is compatible with the main focus of the present study. An epoxy resin based on bisphenol-A and the curing agent (4,4'-methylenedianiline) are mixed with a 5:1 ratio. The carbon fiber composite is manufactured by adopting the hand lay-up procedure followed by an oven curing process (1 hour at $80\text{ }^{\circ}\text{C}$, 1 hour at $120\text{ }^{\circ}\text{C}$ and two hours at $150\text{ }^{\circ}\text{C}$) with vacuum-bagging; the samples are subsequently trimmed to their final geometry.

A series of samples are thus fabricated in order to investigate different features of the composite. First, bare epoxy resin samples are prepared in a rectangular shape ($30.00 \times 5.00 \times 1.96\text{ mm}$) to investigate the mechanical properties of the matrix. Subsequently, a $[0_3, 90_3]$ cross-ply laminate are fabricated and trimmed to the dimensions $160 \times 160\text{ mm}$ with an average thickness of 0.8 mm , the obtained fiber volume ratio is $\varphi = 0.55$; this plate is characterized by a cylindrical room-temperature configuration and such sample is adopted to investigate the bistable response when the temperature is passively varied.

CarbonFibersProperties	
Tensile Strength[MPa]	3530
Tensile Modulus[MPa]	230
YieldStrain[%]	1.5
FiberDiameter[μm]	7.0

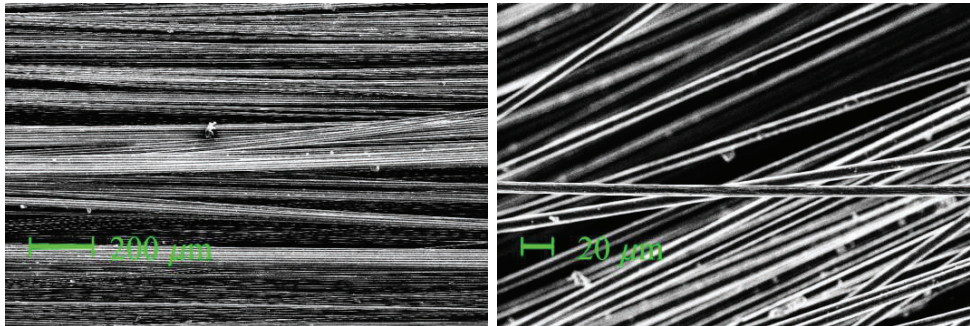


Figure 7: SEM images of the carbon fibers.

A smaller sample, $[0^{\circ}, 90^{\circ}]$ with dimensions $38 \times 38\text{ mm}$, is then produced to study the response of the material to applied loads together with indirectly prescribed temperature variations. Finally, a cross-ply laminate is produced via an ad-hoc treatment of the fibers tips to transform their tips in suitable electrodes. This step is the fundamental point that must be introduced in the manufacturing process in order to avoid that the epoxy matrix wraps the fibers providing unwanted electrical insulation. In particular, a conductive paste is applied onto the fiber tips with the double role of avoiding the insulation on one hand and, on the other hand, of allowing their connection with an aluminum foil for maximized electrical conductivity. By doing so, the resulting sample is a self-activated composite that can actively regulate its stiffness according to the selected applied voltage and prescribed working conditions.

An example of one of the fabricated muscular plates is reported in Figure 8.

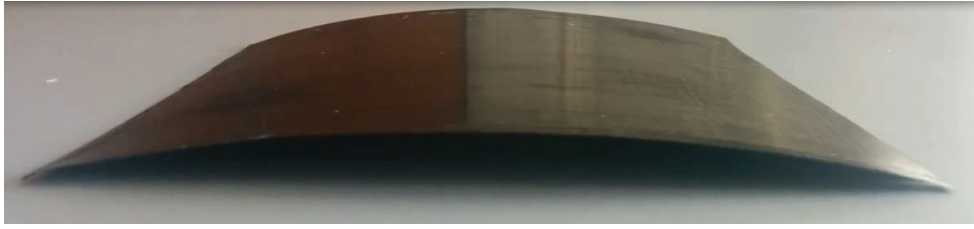


Figure 8: Muscular composite plate

The SEM images of the fabricated two layer sample, show similar particles to those observed on bare fibers (see Fig. 9), but the presence of voids is reasonably negligible. The overall quality of the manufactured laminates is then considered to be appropriate to investigate the complex functionality of the proposed material.

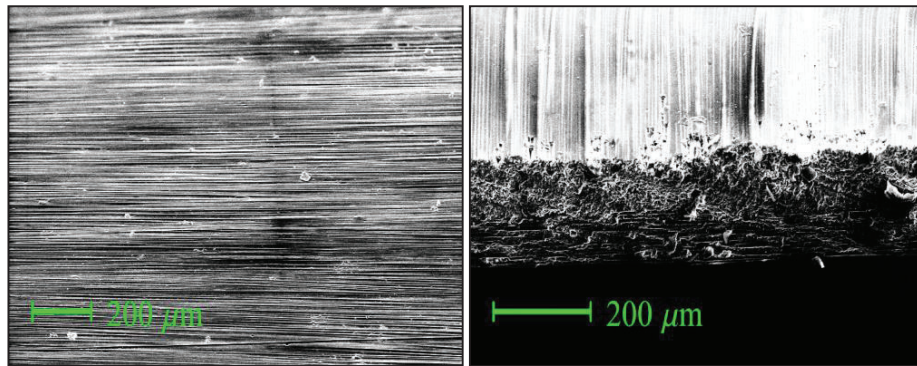


Figure 9: SEM image of the manufactured composite: top view and cross section

2.2.2.2 Testing and mechanical characterization of the CF composite

The mechanical properties of the manufactured composite are derived from the Voigt/Reuss model. It delivers the Young's moduli of a single layer laminate, under the assumption of perfectly longitudinally aligned fibers and perfect bonding between fibers and matrix. The model is also known as the rule of mixtures and its inverse, which are based on the knowledge of the fibers volume fraction and the Young's modulus of the fibers and the bare matrix. The modulus is retained as a constant (according to literature's common assumptions) while the second is evaluated experimentally. A rectangular sample ($30 \times 5 \times 1.96$ mm) of bare epoxy resin is thus tested in a single cantilever mode using a Dynamic Mechanical Analyzer (DMA) through a discrete temperature/strain sweep. At each temperature value a strain sweep is performed to derive the value of the storage and loss moduli. The results are represented in figure 10.

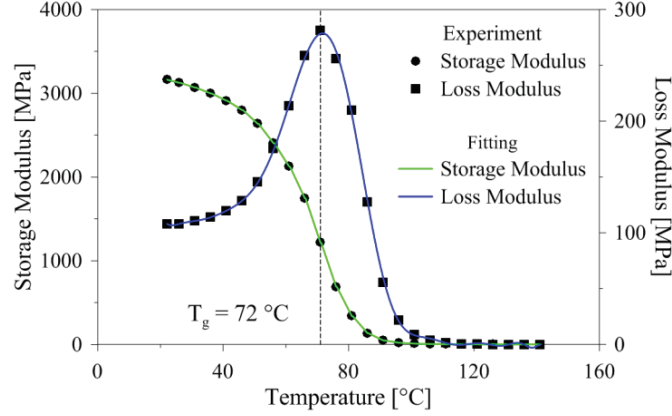


Figure 10: Adopted hosting matrix Young's and Loss Moduli identified experimentally.

The derived Young's modulus of the epoxy resin reveals the strong dependence of the material properties on temperature. Thanks to this feature, the composite is characterized by the high capability to change its stiffness when the temperature varies. This means that stiffness can be modulated by actively controlling the material temperature.

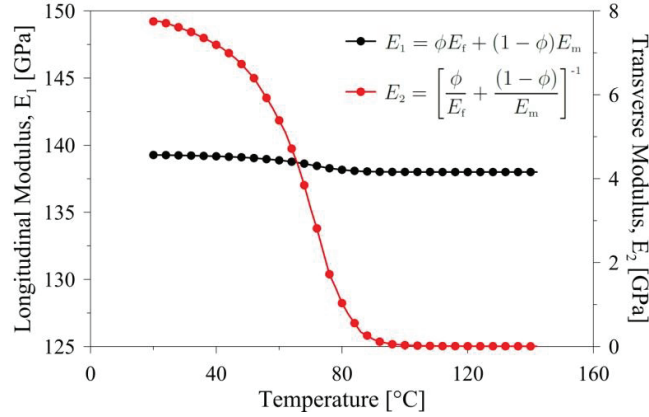


Figure 11: Derived longitudinal and transverse moduli of the composite.

According to these results, the longitudinal and transverse moduli of the composite are obtained by assuming an evaluated average fibers volume fraction of $\phi = 60\%$, that is in agreement with the literature values for composites fabricated with the same process. As expected, the Young's modulus in the longitudinal direction, E_1 , is not considerably affected by temperature variations. This is because in that direction the behavior is governed by the fibers, thus the modulus value mostly depends on the actual volume fraction. On the other hand, in the transverse direction the temperature strongly influences the actual value of the Young's modulus, E_2 , in particular, it resembles the behavior of the epoxy resin showing a sudden drop of the mechanical properties around $T = 80^\circ\text{C}$.

The remaining mechanical parameters are derived from literature, since they do not strongly influence the response of the composites here considered. The Poisson's ratios $\nu_{12} = 0.28$, $\nu_{21} = 0.064$ are drawn from the literature and retained as constant. The shear modulus G is derived according to $E_2/(2(1 + \nu_{21}))$, thus it is characterized by a trend in temperature similar to that of E_2 . The coefficients of thermal expansion are retained as constant in temperature and can be assumed as $\alpha_1 = -0.406 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$ in the longitudinal direction, which corresponds to the fibers coefficients, whereas $\alpha_2 = 25.6 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$ in the transverse direction which represents the coefficients of the epoxy resin.

The material properties were then adopted to implement the previously described model for design purposes.

2.2.2.3 Functionality of the Muscular Composite Response: Experiments vs Theory

Passive Response

A curved laminate is used to investigate the response of the material when it is exposed to a controlled temperature environment. This is done first by exposing the material to an external heating system in order to study the intrinsic material properties, independently by the performance of the heating system. Then, an internal heating system is implemented to create a material whose thermo-mechanical properties can be self-activated.

A laminate with a specific lay-up sequence was designed with the help of the implemented model in order to achieve a cylindrical shape with a specific known curvature when the matrix is rigid at room temperature. When the temperature increases the matrix decreases its rigidity and the corresponding curvature obviously decreases because internal stresses are gradually released. This process is reversible.

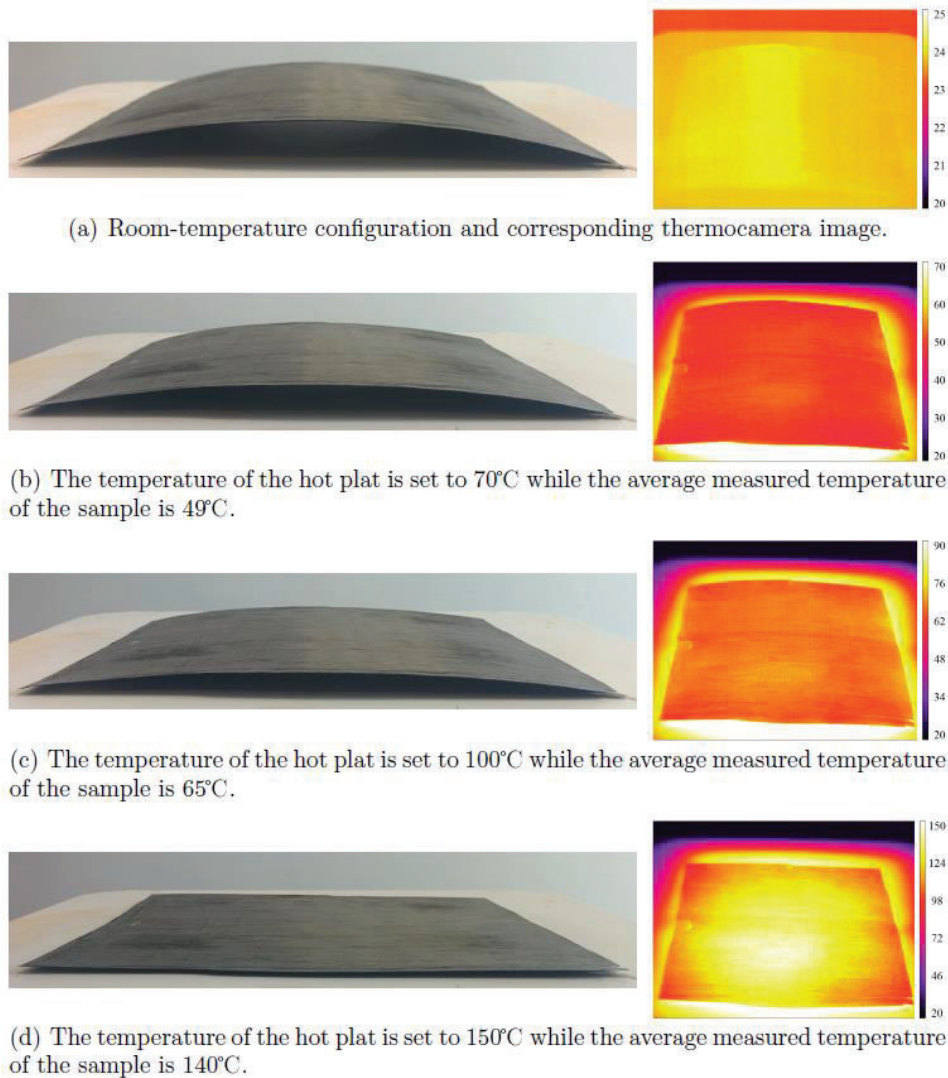


Figure 12: Setup on the hotplate to measure the curvature variations with temperature.

Preliminary tests are conducted by placing the sample on a hot plate in order to control the temperature according to a prescribed path. The **rigidity variation is monitored indirectly** by

constantly monitoring with a videocamera the displacement (and thus the curvature) of the center of the sample. While the actual temperature is monitored with the help of a thermal camera. Figure 12 illustrates the composite plate on the hot plate (left column) during the experiment for three cases: $T = 25, 75$ and 130°C , together with the corresponding thermocamera images (right column). The temperature contour colors are represented at the bottom of the figure and the range is $20^\circ\text{C} \leq T \leq 150^\circ\text{C}$. It is possible to appreciate the gradual rigidity losses by observing curvature variations starting from the room temperature (maximum curvature). The maximum rigidity loss is recorded when the plate reaches a perfectly flat configuration due to a total loss of the internal stresses.

By the identification of the midpoint displacement and the evaluation of the average temperature of the sample it is possible to represent the typical curvature/temperature trend. The collected data are represented in figure 13.

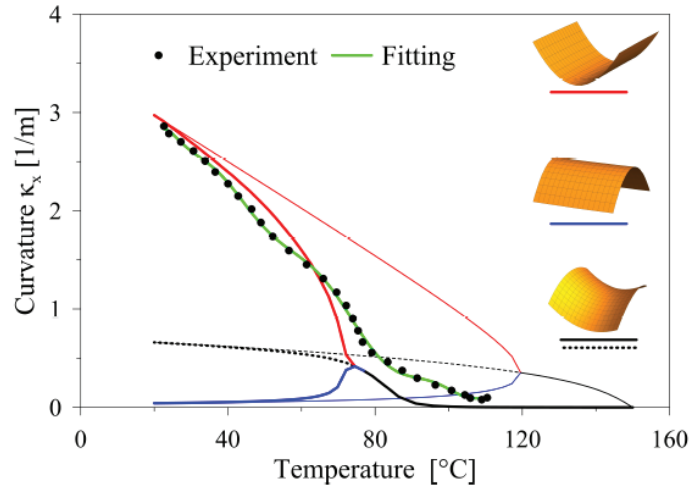


Figure 13: Curvature against Temperature.

The black dots denote the experimental results, while the green line represents the obtained fitting. As expected, the initial curvature of the sample measured at room temperature starts decreasing as temperature increases. When the temperature approaches the value $T = 70\text{--}80^\circ\text{C}$, the curvature experiences a more intense decrease, and this is due to the peculiar behavior of the adopted epoxy resin which allows to reach the flat shape very rapidly.

On the other hand, the black, red and blue lines represent the results obtained by the parametric model. According to the theory, the model allows to predict all the possible configurations of the structure, including the saddle, which is only theoretical at selected temperatures, while becomes the only one when a certain threshold is exceeded.

It is worth to note that the thin lines well predict the structure curvature only at room temperature while completely fails to resemble the real behavior at higher temperatures. Such lines represent the behavior when the dependency of the material parameters to temperature is neglected in the model. On the other hand, when such relation is implemented, the model well predicts the real behavior over the full range of temperature.

The hot plate test allowed to investigate the rigidity-change capability of the composite in terms of curvature. What is more relevant is the capability of the material to vary its response to an external excitation such as a prescribed displacement in order to quantify the rigidity modulation of the material itself. This is done by a special DMA test set-up that is shown next.

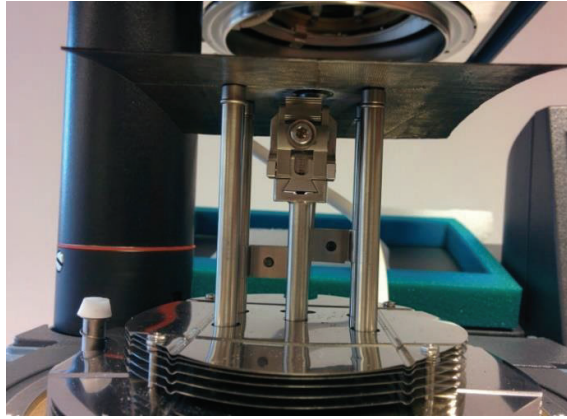


Figure 14: DMA setup.

The square sample is simply-supported on the four posts shown in figure 14, while the DMA driving shaft is rigidly connected to the plate center point. A displacement ramp is applied to the sample center point while the temperature is directly driven using the DMA furnace. The results in terms of force-displacement diagrams are illustrated in Fig. 15 where the displacement ramp is adjusted at each of the five selected temperature values ($25^{\circ}\text{C} \leq T \leq 70^{\circ}\text{C}$), in order to avoid in this case the instability of the composite.

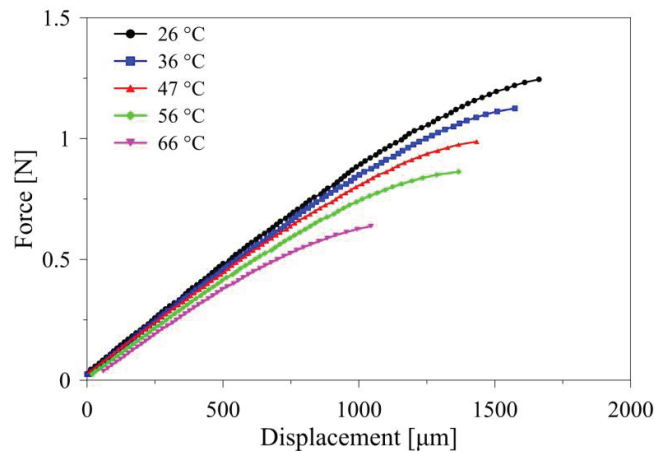


Figure 15: Force-displacement diagram at different temperatures (DMA).

It is clear that as the temperature increases the sample exhibits a moderate loss of the initial tangent stiffness even though the curvature, as well as, the load-bearing capability, are characterized by considerable variations. As before, the actual curvature of the sample is derived by the imposed displacement and, as illustrated in the next table a 31% variation is obtained when the temperature reached 66°C . Moreover, the maximum allowable load that prevents the material to snap into a new configuration, is evaluated and the obtained values clearly show the overall stiffness variations undergone by the plate: at 66°C **the maximum allowable load is almost 50% lower**. These results clearly highlight the **reduction in rigidity with increasing temperature**. These data are quantified in the table below.

Table: curvature and max load variations with temperature.

Temperature [$^{\circ}\text{C}$]	26	36	47	56	66
Curvature [$1/\text{m}$]	14.67	13.83	12.78	11.83	10.14

	Loss[%]	-6	-13	-19	-31
Stiffness [Nm]	1.25	1.13	0.99	0.86	0.64
	Loss[%]	-10	-21	-31	-49

In other words, as the temperature increases, the stiffness decreases, but also the shape of the structure is changing since the curvature is decreasing. In particular, the curvature experiences a maximum variation of 31%, above which the composite becomes almost flat, since it is approaching the glass transition temperature of the matrix and the residual thermal stresses are negligible. On the other hand, the linear stiffness, that is represented by the slope of the force-displacement curves, is considerably reducing.

As previously mentioned, the procedure can be cyclically repeated, thus the strategy can be suitably adopted to *tune the structural stiffness and adapt its response to external excitations*.

Self-actuated muscular composite

The self-actuation capability of the curved composite is achieved by using an internal heating system.

An initial study was performed to validate the capability of carbon fibers to work as embedded heaters. This is possible thanks to the Joule effect that is exploited when current is forced to pass in the fibers. Electrical current was applied by connecting the fibers tips to a power supply following the fabrication procedure previously described. The composite prototype is shown in the inset of Fig. 16, where the aluminum electrodes, that are glued to the carbon fibers tips, can be clearly identified.

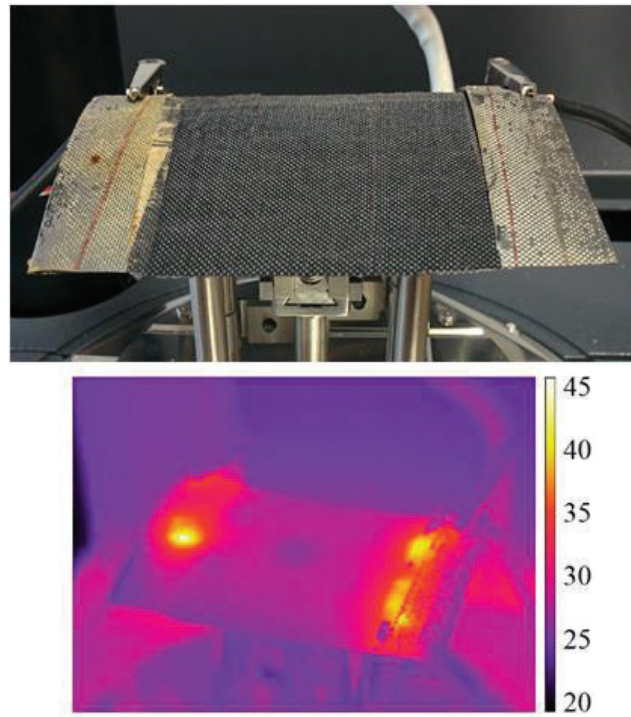


Figure 16: (Left) Self-Heating composite with fibers as internal heaters, mounted in a DMA with an external power supply; (Right) Obtained non-uniform heating.

By properly tuning the applied current, temperature gradients are achieved, and the composite changes its shape accordingly. The composite mechanical response while being self-

activated is studied with the help of the same unconventional set-up assembled in the DMA described before. A prescribed displacement field is applied to the composite while a power supply drives current into the sample at 0.5 A, 1.0 A and 1.5 A. The resulting force-displacement cycles are shown in the next figure.

When a current of 0.5 A is applied, the sample reaches an average temperature of about 27°C (here considered as room temperature). In this condition the sample exhibits the force-displacement cycle denoted by the black line and in correspondence of the largest prescribed displacement it reaches a maximum force of about 1.48 N. As expected, the cycle is characterized by an initial tangent stiffness that starts decreasing and approaches zero as the displacement increases towards the flat configuration. When the current is increased to 1.0 A, the average temperature of the sample reaches $T = 33^{\circ}\text{C}$ and the force-displacement cycle is denoted by the blue line. It can be noted that the initial tangent stiffness is lower and this becomes more evident in terms of the maximum load which experiences a considerable drop. Finally, when the current reaches 1.5 A the temperature set at $T = 40^{\circ}\text{C}$. The loss in initial stiffness is more evident as well as the drop in maximum measured load. Results are summarized in the next table together with the input value of current and the corresponding average temperature measured on the sample. The initial tangent stiffness is reported together with the maximum load for the three values of applied current. The effect of temperature is measured in terms of stiffness reduction that varies from an initial 12% to a final 27%. The corresponding maximum load reduction is 10% and 23%, respectively.

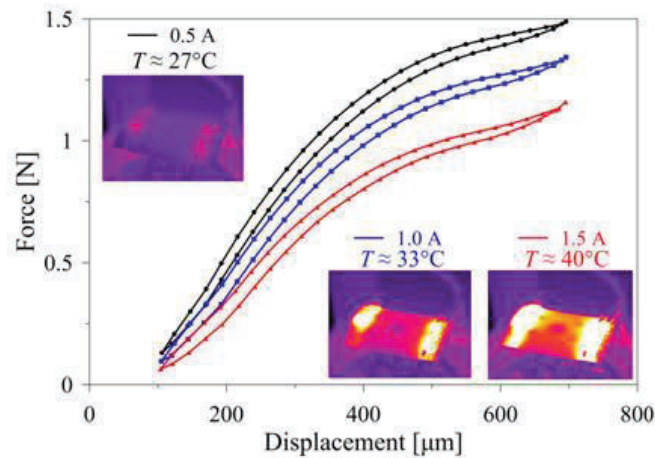


Figure 17: Force-displacement cycles at different temperatures (power supply).

The conducted experimental campaign is intended to investigate the capability of the manufactured composites to change shape according to prescribed input. The results showed that the initially curved plate can turn into a flat plate and then go back to the starting configuration following the assigned temperature path. Moreover, the ad-hoc treatment adopted to prepare the fibers to become suitable electrodes, lead to an active control of the temperature, though affected by a non-perfectly uniform temperature field.

Current [A]	0.5	1.0	1.5
Temperature [°C]	27	33	40
Stiffness [N/m]	4133	3639	3035
	Loss [%]	-12	-27
Max load [N]	1.48	1.33	1.13
	Loss [%]	-10	-23

The conducted experiments allowed to investigate the stiffness variations, but revealed that the proposed heating system was not able to uniformly heat the samples. Thus, further investigations were made on a different heating approach. This is achieved by integrating the composite with an embedded heating system whose performance is here reported. In particular, heat is again produced by exploiting the Joule effect in a resistor, that is here realized with a simple copper wire with a diameter of 0.1 mm embedded within the composite layer, as illustrated in Figure 18.

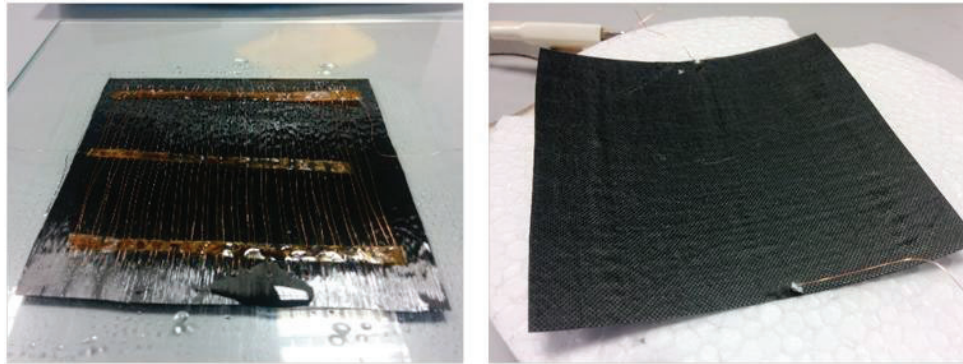
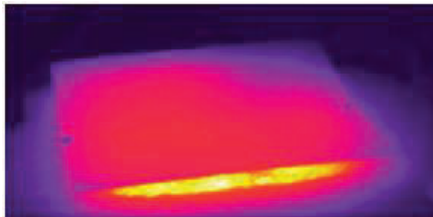


Figure 18: Composite with internal resistor during and after fabrication.

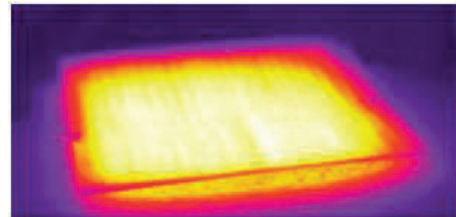
The wire is coiled between two adjacent layers of the composite during the wet lay-up phase of the manufacturing process. The wire ends are then left outside the composite and the curing process is completed. The obtained structure is shown in the right part of Figure 18. The overall length of the resistor is 4.1 m and the resulting resistance is $R=9.19 \, \Omega$ (copper resistivity $1.76 \times 10^{-8} \, \Omega \text{m}$). A power generator is used to induce current into the resistor and thus heat the material. In order to investigate the performance of the heating system, the obtained temperature is monitored by a thermo-camera up to a value of $T=150^\circ\text{C}$ that is reached with a current of 0.96 A (8.5 W).

$$T_{\max} = 30 \, ^\circ\text{C}$$

$$T_{\max} = 60 \, ^\circ\text{C}$$



$$T_{\max} = 100 \, ^\circ\text{C}$$



$$T_{\max} = 150 \, ^\circ\text{C}$$

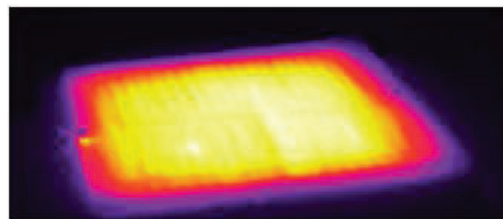
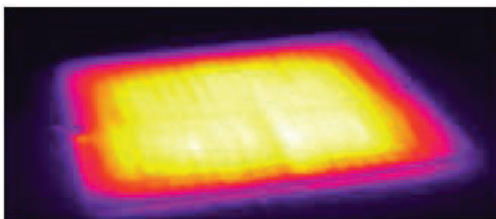


Figure 19: Thermal images of the heating system performance test.

The results are shown in Figure 19 where thermal images are reported together with the corresponding maximum temperature reached by the material as the applied current is increased.

The investigations clearly show that the designed heating system not only can accurately control the composite temperature, but also a uniform and well distributed heating is achieved. A quantified analysis was then performed by correlating the applied current to the resulting plate temperature.

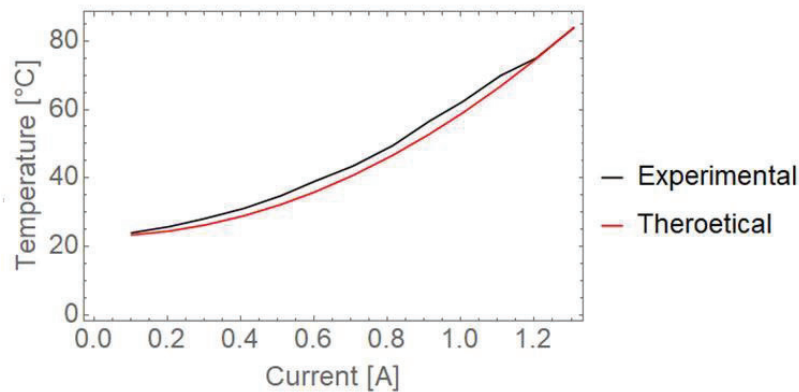


Figure 20: Validation of the heating capability of the embedded heating system.

In this case the current applied to the internal resistor is increased up to 1.3 A, while the resulting temperature is measured continuously via the thermal camera, the results are represented by the black line in figure 20. On the other hand, a simple theoretical model was adopted to design the heating system, thus allowing to predict the system performance. The expected temperature is represented by the red line and it is clear how the manufactured heating system well resembles the expected designed behavior. This implies that the model can well be adopted to design the active composite according to actual prescriptions.

The same type of internal heating system was then validated on a larger-in-scale sample and its muscular response is reported in the set of images below (Figure 21) captured during morphing. The images highlight very clearly the smooth self-actuation capability of the muscular composite and its cyclic response while morphing from a bent to a flat configuration and viceversa.

The proposed strategy represents a promising approach to develop a morphing material with variable curvature that can actively be controlled according to prescribed inputs. In other words, the plate is capable to switch gradually and cyclically from a curved shape to a planar shape, by simply applying voltage within the plate, making this design interesting for morphing applications.

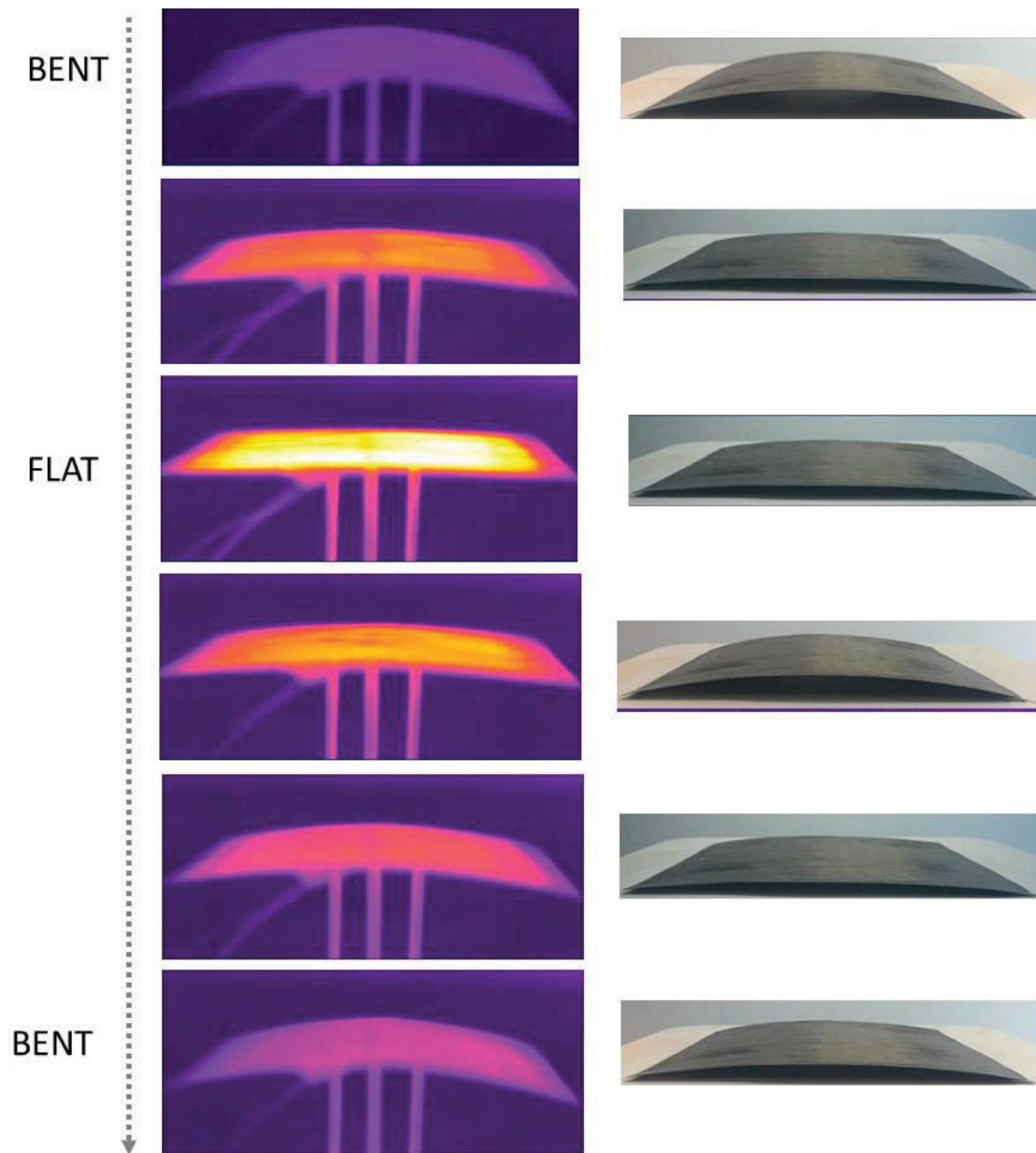


Figure 21: Snap-shots of the self-actuated muscular composite during morphing.

Self-tuning: frequency and damping

The results illustrated in the previous sections, show that the proposed approach can well be adopted to actively control the properties and the shape changing capability of the manufactured morphing material. However it is possible to observe that the fabricated composite plate, in its rigid state (room temperature), has the same lay-up of a bistable plate and can be snapped into a new stable configuration by simply applying an actuation load perpendicularly to the plate, as experimentally verified in Figure 22. A typical undesired phenomenon is that snapping induces undesired vibrations. A study was then performed within this project, to benefit of the unique features (rigidity modulation and heating capability) of the muscular composite material, to damp-out such vibrations.

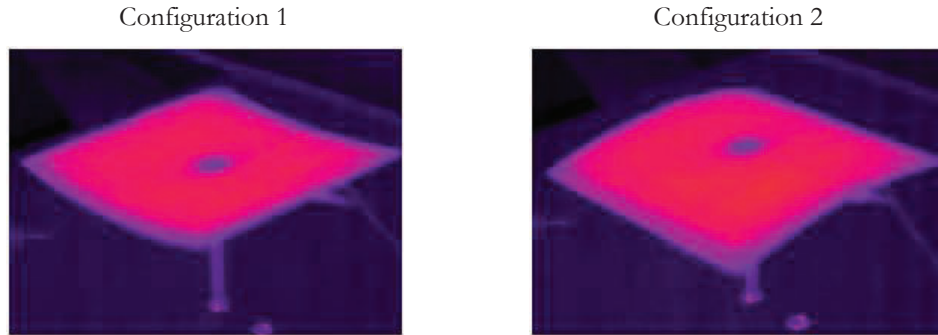


Figure 22: The two bistable configurations of the manufactured muscular composite at room temperature.

In this case study, the self-heating composite is tested by actively controlling the material temperature thanks to the previously described embedded heating system.

The sample is fully constrained in the center point and free vibration tests are performed. In particular, vibrations, thus the oscillations, are induced by making the material snapping from one configuration to the other as shown in figure 22. It is in fact well known that once the plate changes configuration it vibrates around the new stable configuration. This is usually an undesired phenomenon which can hardly be controlled with conventional approaches. Here these vibrations are monitored with the help of a laser vibrometer pointed at the free corner of the plate.

The response is obtained in terms of velocity which allows to determine the actual oscillation frequency as well as the material damping, as shown in figure 23. The laser was directly pointed onto the surface of the composite and no specific target was adopted, thus the acquired signal is affected by some noise. Despite this issue, it is clearly possible to identify the first vibration frequency, that, for the room temperature configuration, is around 50 Hz.

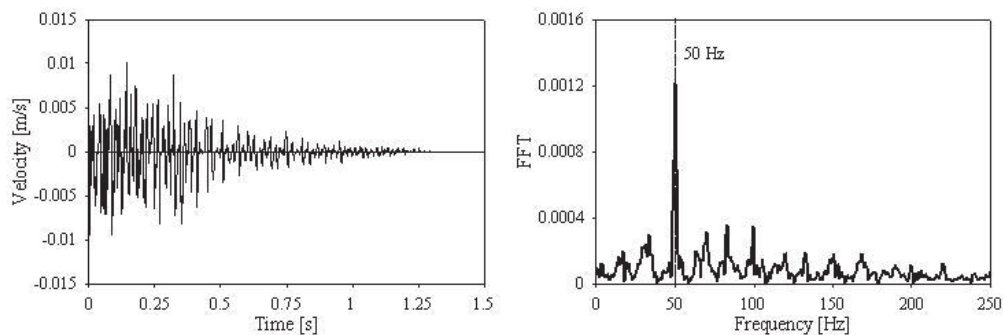


Figure 23: Snap-through induced oscillations in terms of velocity and corresponding frequency.

The time history signal allows also to evaluate the material damping that can be derived by fitting the absolute-value velocity time history and evaluating the logarithmic decrement that characterizes the decrease of the peaks.

The test is repeated at multiple input values of the current which lead to different temperature levels ranging from room temperature to 90°C. Selected time histories of the acquired signals are illustrated in Figure 24. At room temperature the oscillations induced by the snap-through are characterized by a higher amplitude and the lowest structural damping.

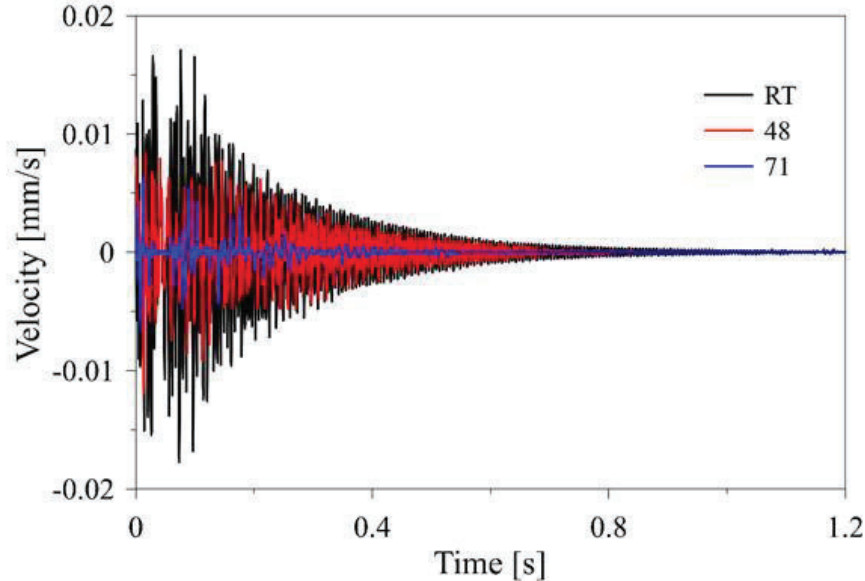


Figure 24: Oscillation of the plate after snapping.

On the other hand, as current (and thus temperature) increases, the maximum amplitude of oscillation decreases while damping increases. It is possible to appreciate that the proposed approach can control the energy necessary to induce the shape change in such bi-stable composites and also to reduce the oscillations taking place right after the instability.

From the time histories it is possible to evaluate numerically the corresponding oscillation frequency and equivalent damping. The derived frequencies and damping are reported as a function of the applied temperature, including the room temperature condition. These results are resumed in figure 25, in terms of frequency and identified damping as a function of temperature.

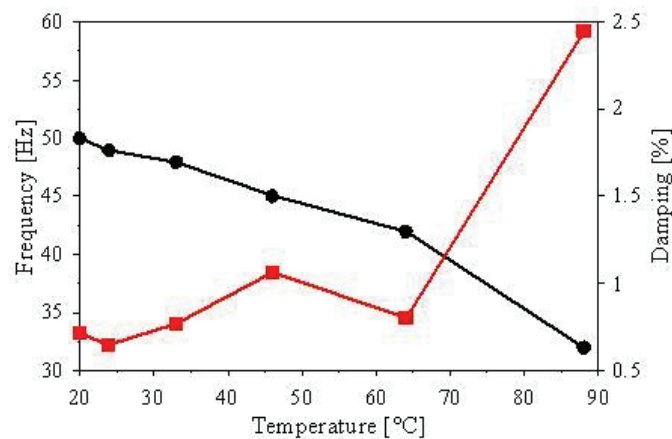


Figure 25: Identified frequency and damping at different temperatures.

The results show that the material properties experience a large variation: the frequency changes from 50 Hz to almost 30 Hz, with a maximum variation of 40%; on the other hand, the identified damping increases from less than 0.5% at room temperature to a maximum value of 2.5%.

This implies that by performing the shape change at higher temperatures it is possible to reduce the activation energy, but also that the oscillations arising from the snap-through are considerably reduced thanks to the higher damping.

Tuning snap-through energy

The same set-up described in the previous section is adopted to investigate the snap-through phenomenon that describes the dynamic instability taking place when the bi-stable composite is forced to switch from one configuration to another.

Such aspect is investigated during unstable phenomenon (snap-through) taking place when the material experiences the transition between its two stable configurations. In this case, the prescribed displacement is increased until the plate undergoes the snap-through. A further displacement increase is observed and allows to study the pre- and post-instability behavior of the material.

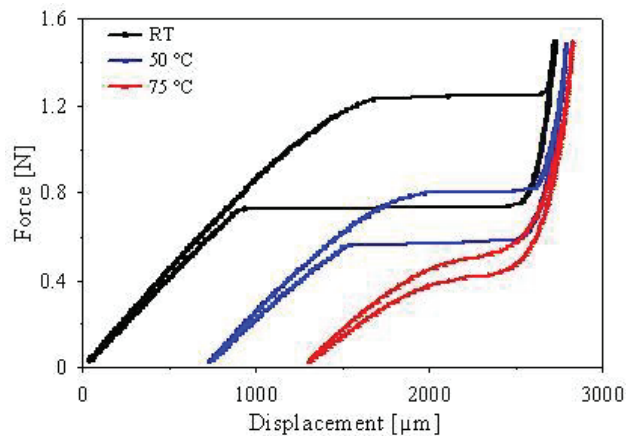


Figure 26: Force-Displacement paths with induced snap-through at different temperatures prescribed by DMA furnace.

The results are illustrated in figure 26: Force-Displacement paths with induced snap-through at different temperatures prescribed by DMA furnace where the response of the structure is reported in terms of force-displacement diagrams at the different selected temperatures. The black solid line represents the structure at room temperature and it is possible to observe the onset of the snap-through when the force approaches 1.2 N; then, a sudden increase of displacement is observed, while the force remains constant until the second configuration is reached. At that point, the material recovers stiffness and its value is much higher than that in the first configuration. This is mainly due to the fact that the material in this case is almost in a pure tensile state. When the versus of the imposed displacement is inverted the snap-through takes place at a much lower load level, thus a considerable hysteresis cycle is obtained.

By increasing temperature, the material loses curvature and this can be clearly identified by the initial displacement of the force displacement cycle (blue and red lines). When the temperature reaches 50°C (blue line), the critical load that activates the snap-through is around 0.8 N, but the overall behavior resembles the room-temperature cycle (black line).

On the other hand, when the temperature reaches 75°C (that is higher than the polymer T_g) the snap-through phenomenon disappears and the structure goes gradually from the first configuration to the second.

These results imply that by heating the material, the shape change can be obtained with different energy levels that can be actively tuned. Moreover, above a certain temperature

threshold, the change in configuration can be performed gradually without undesired (because uncontrollable) instabilities.

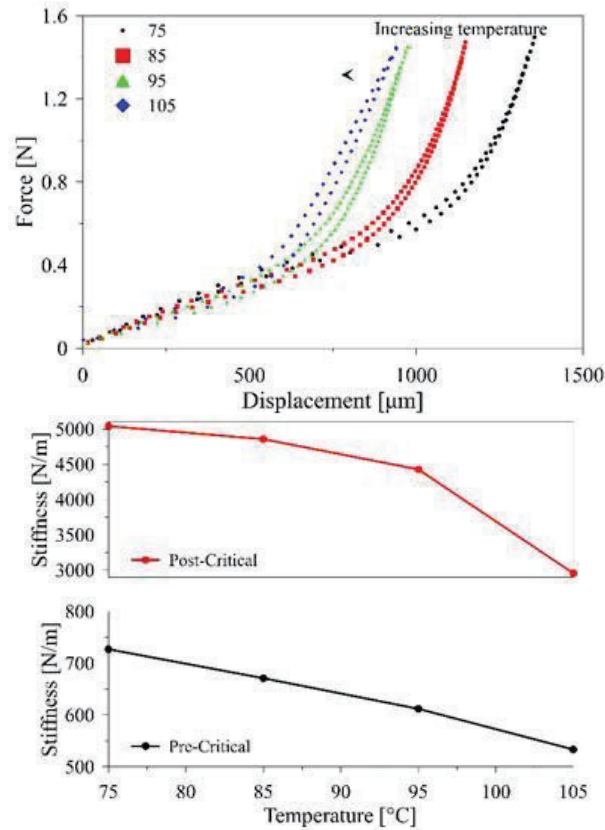


Figure 27: Cycles with increasing temperature (left); Stiffness variation as a function of temperature pre- and post-snapping.

The same test is pushed at higher temperatures and, as expected, the shape change is well recognizable from the results in terms of force-displacement cycles shown in Figure 27. It can be noticed that the instability does not take place, even though an evident stiffness change is observed at a certain deformation level. The stiffness is evaluated at the beginning of the cycle and also after the shape change. As obvious, the stiffness in the second configuration is much higher.

The snap-through phenomenon was not observed, but the approach shows that it is possible to obtain a configuration change without the instability and those unwanted uncontrolled oscillations. This becomes possible if a suitable temperature control system is embedded within the material.

2.3 THREE-DIMENSIONAL (3D) MUSCULAR COMPOSITE

The results reported in the previous sections highlighted the great potential of the muscular composite to provide morphing in a passive and self-actuated manner as well its damping and rigidity tuning capability. However, two key aspects were considered for further developments: (1) is it possible to morph in 3D and enhance morphing excursions to move toward “radical” shape changes? (2) what if a specific application requires large-in-scale morphing without excessive rigidity losses? (for instance, in cases in which it is necessary to carry loads while morphing). Studies were then performed to investigate the mentioned key challenges and allowed us to give some answers to the listed questions.

The above questions are quite broad and it is difficult to provide a material whose properties are satisfactory to all possible applications. To narrow the topic, we firstly ***focused on enhancing the shape changing capability, in 3D, while offering higher load-bearing capability***, and then we performed a preliminary study to ***move towards radical shape changes***.

2.3.1 Towards enhanced morphing with higher load-bearing capability

To enhance the initial curvature of the composite and to enhance the shape changing capability, a different manufacturing approach is investigated. The composite is manufactured with an initially curved cylindrical mold.

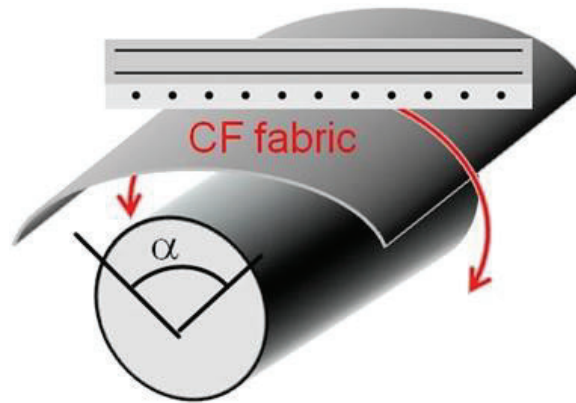


Figure 1: 3D design and manufacturing of the muscular composite.

The effect of the ***curved mold*** is coupled to the induction of ***additional internal stresses***. Our idea was to acquire enhanced morphing capability through the combination of these two features. This idea was initially investigated by considering different layup sequences as those reported in Figure 2.



Figure 2: Investigated composite lay-ups that were fabricated on a 3D mold.

By considering the top face in contact with the mold, the stacking sequence on the left revealed to lead to a composite with ***decreased*** curvature. This is due to effect of internal thermal stresses that in this case act in a direction that opposes the initial curvature of the mold. On the other hand, it was observed that this configuration reveals a ***bi-stable*** room temperature configuration, as represented in figure 3. Such configurations are both stable and the switching from one to another happens via the so-called snap-through phenomenon.

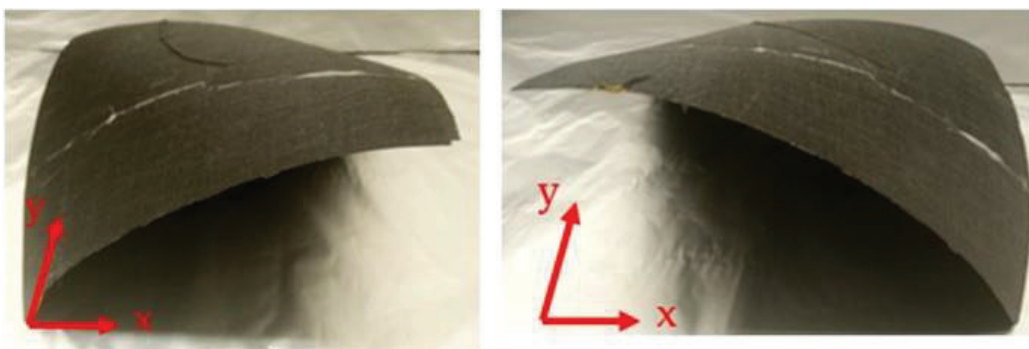


Figure 3: Bi-stable samples obtained through a curved mold.

The other configuration instead leads to a room temperature shape that is **mono-stable**, but the most interesting aspect is that *the obtained curvature is greater than that of the mold*, as depicted in Figure 4, in which the curvature enhancement of the manufactured composite with respect to the previous case, is qualitatively evident. A quantitative analysis proves the result, in fact, the radius of the sample after fabrication is of 4.16 cm which implies an initial curvature χ of 0.24 cm^{-1} , while that of the mold is of 7cm. Upon heating to 150°C , a 40.5% increase in curvature was recorded. The curvature increase $\Delta\chi$ of +71%, in just 50 sec., is due to the effect of the thermal stresses that in this case act in the same direction of the mold curvature, adding-up.

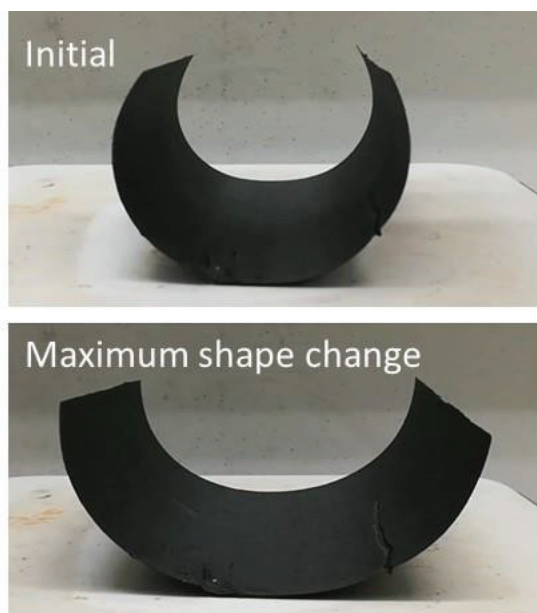


Figure 4: Morphing of the curved composite.

2.3.1.1. Self-activated 3D muscular composite

In order to realize a **self-shaping** curved composite, an alternative design of the embedded heating system was investigated first. Figure 5a shows the original fabrication approach of the heating system whose geometry was first pinned on a soft substrate, with sacrificial needles, so that the copper wire was layered around them. As discussed in the previous sections, this resulted being a successful approach which allowed to fabricate and characterize several morphing composites during the research activities. However, this heater design often turned out to be time consuming. Shortage of the composite samples was in fact often found and, of course this

was an unfixable problem since the samples were already fully cured with the resistor trapped between the composite layers. Electrical shortage was the result of a partial erosion of the copper wire coating in the areas in which the wire was folded around the needles with a 180° angle (see Figure 5a). The exposed copper wire was then getting in direct electrical contact with the surrounding carbon fibers that are electrically conductive.

To avoid this issue, an alternative copper pattern was investigated as shown in Figure 5 b.

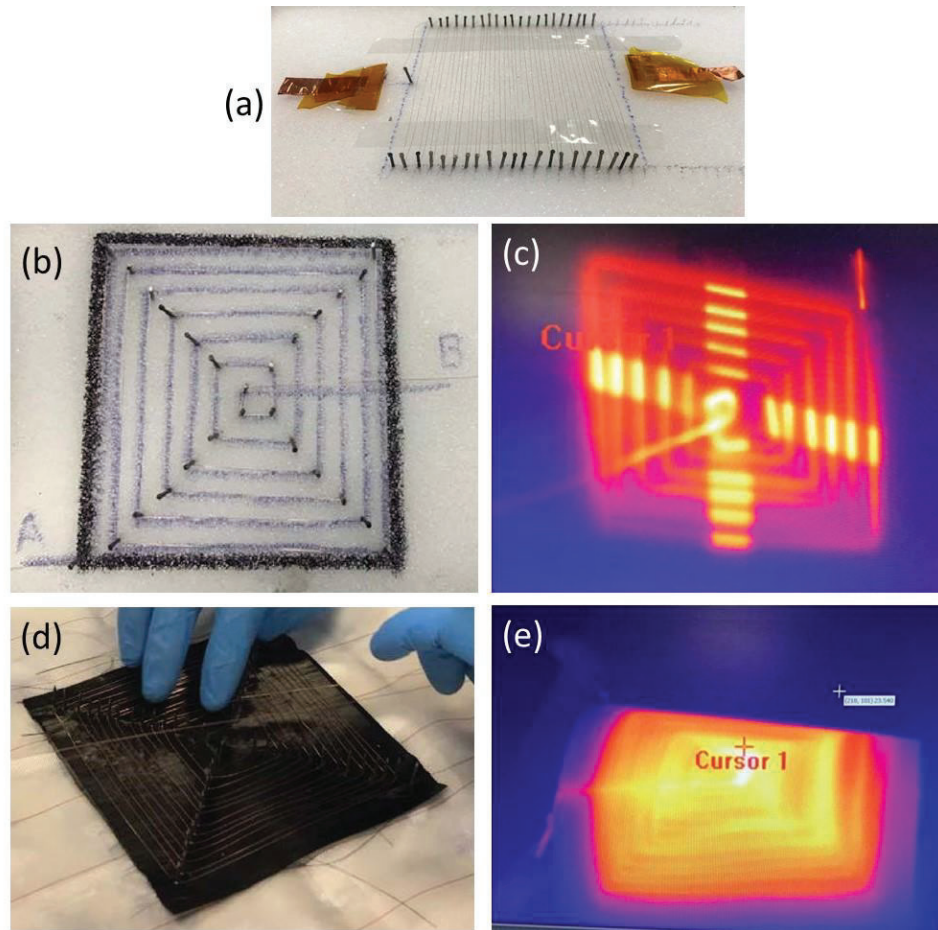


Figure 5: (a) Initial heating system manufacturing logic; (b) New resistor design with 90° folding angles; (c) Thermal camera image of the resistor in (b); (d) New resistor design implemented in a carbon fiber composite; (e) Thermal camera image of a curved composite with the embedded heating system (new heater design).

In particular, a concentric square pattern was implemented since it required a reduced folding angle of the wires around the pins (90° angle). Such reduced folding angle was expected to reduce the stress of the wire coating, avoiding its damage, thus reducing electrical shortage. The resulting resistor highlighted a good and uniform heating as reported in Figure 5c in which the single wires of the resistor could be recognized. A mesh with a higher wire density was then implemented directly within the layers of the composite during manufacturing (Figure 5d), resulting in an excellent and uniform composite heating (see Figure 5 e).

Initial testing of the 3D muscular composite with self-activating capability were performed with a system designed in our laboratory in collaboration with Prof. Matthew Snyder (from United States Air Force Academy). The system showed in Figure 6, consists of two rotational bearings in which two sides of the sample are inserted and constrained in the x, y, z directions, as well as in all rotations with the exception to the rotation around the rotational bearings. Such rotations are permitted without friction. A large cylindrical indenter and the load cell were mounted on a digital stage characterized by a displacement resolution in the nanometer scale. A linear displacement ramp was then applied and the corresponding load was monitored with the load cell mounted on the other side of the indenter. The sample temperature was controlled and induced through the internal heater powered with an external power supply and was monitored with a thermal camera focused on the sample during the full testing.

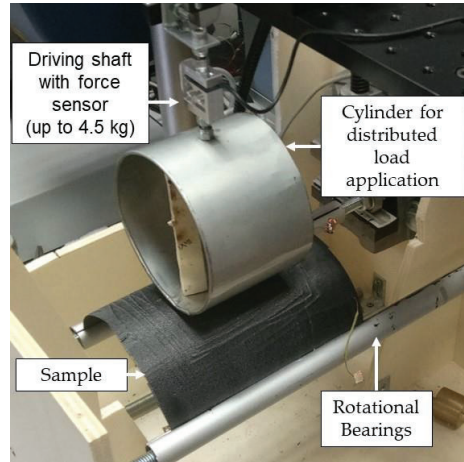


Figure 6: System designed to test 3D composites with large-in-scale size.

A bi-layer sample with an asymmetric lay-up was tested with the above described testing feature following a loading/unloading ramp at temperatures ranging from room temperature to 100°C.

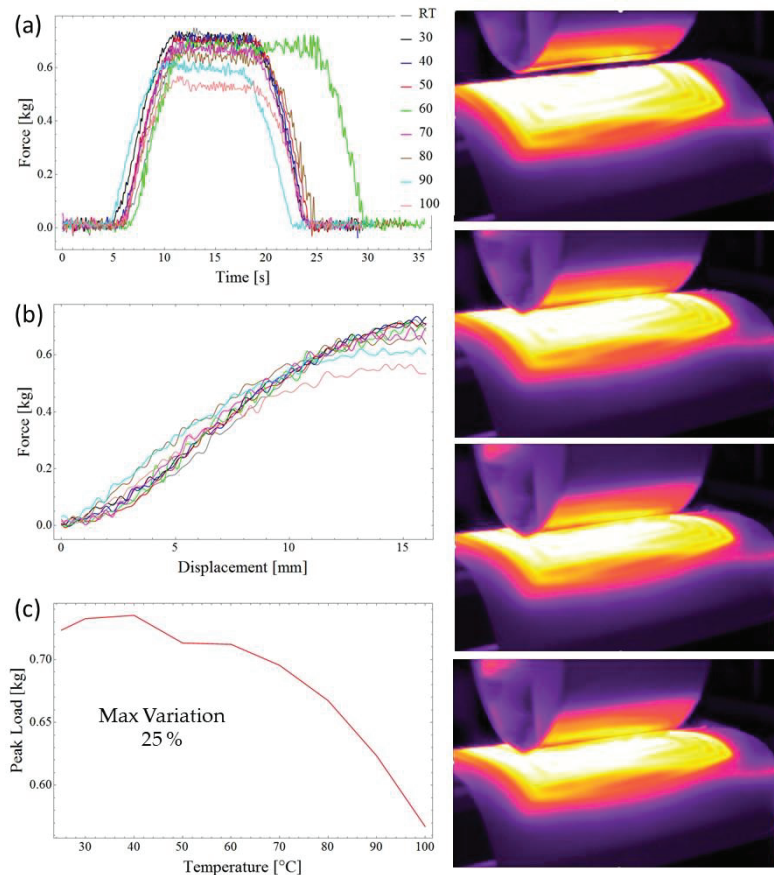


Figure 7: Mechanical response of a bi-layered curved composite with embedded heater: (a) force vs. time response of a full cycle at various temperatures; (b) force vs displacement at different temperatures

(loading) (c) peak load vs temperature of the loading phase at various temperatures; Right column: thermal images captured during the first loading phase.

The results of the curved composite are reported in Figure 7. The complete loading-unloading cycles for each of the tested temperatures, are shown in figure 7a, while the loading ramps in terms of Force vs. Displacement, are reported in Figure 7b and some thermal camera images of half cycle (loading ramp) are shown on the right column of the figure. These results highlight that the loaded composite has a slight viscoelastic response. A peak load is in fact reached at the end of the loading ramp just prior the rapid stabilization of the material to a constant load when the indenter was resting at the maximum applied displacement. Then the force is fully recovered when the system is brought to its initial configuration. The force vs displacement curve instead highlights a slight non-linear response of the material for the full loading range and such nonlinearities become more dominant with increasing temperature. Such non-linear response is related to the conjunction of geometric non-linearities, due to the particular shape of the sample that has a softening response, and, to some extent, to the material non linearities especially when loaded in large-scale (as in this case) and at high temperatures. The analyses of the peak load reached for each temperature, is reported in Figure 33c and shows that, as expected, the load-bearing capability decreases with increasing temperature for a total variation of 25% when temperature varies from room temperature to 100°C.

2.3.1.2. Enhancing morphing while improving load-bearing capability

The idea and thus the focus of this part of the work is to improve the morphing as well as the load-bearing capability of the curved composite during morphing, which in principle might seem to be contradictory. For this reason the role of internal thermal stresses and the optimal number of layers to maximize: -composite curvature, -morphing amplitude and -rigidity, were investigated in depth and the key results are here summarized.

The investigated concept is that, since the additional curvature observed in the previously investigated bi-layer composite, is mainly due to the internal stresses happening in the layer that is in contact with the mold, by increasing the number of layers on such side higher curvatures should ideally be reached and this should consequently also imply higher rigidity.

Three families of composites were fabricated with the rationale reported in Figure 8.

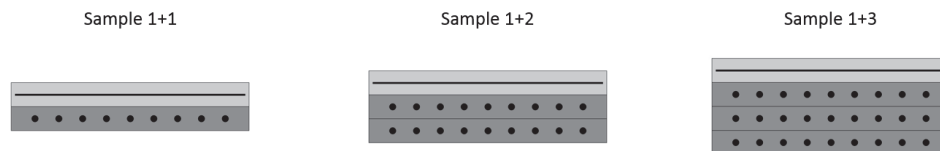


Figure 8: Lay-up of the curved composites investigated to maximize morphing and rigidity.

After fabricating the samples (Figure 9), it was observed that the room temperature configuration was characterized by an internal radius of

Sample	1+1	1+2	1+3
Radius [1/cm]	4.22	4.16	3.75

since the initial radius of the mold is 7 cm, the proposed approach nearly delivered samples with a doubled curvature with respect to the mold.

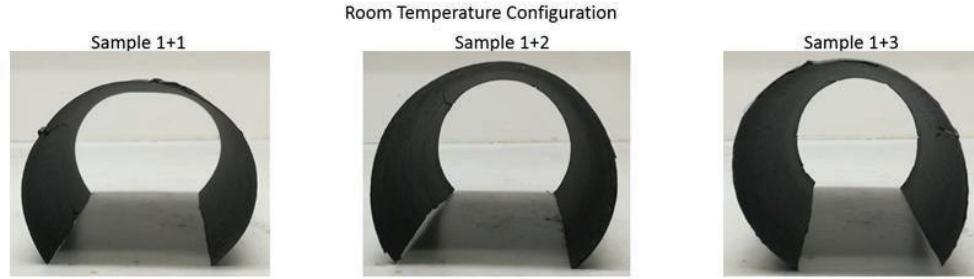


Figure 9: Room temperature configuration of the manufactured curved composites characterized by the layer sequences of figure 8.

The morphing capability of such composite is tested by evaluating the curvature at different temperatures ranging from room temperature (25°C) up to 150 °C, that is the curing temperature and thus corresponding to the mold curvature (total release of internal stresses). As shown in Figure 10 and comparing these images with Figure 9, the capability of the composite to change shape is evident. Moreover, since the reference curvature is fixed by the mold the shape change can be suitably tuned to arbitrary values, allowing dramatic shape changes. The most important aspect is that with this approach the structure with the highest morphing capability is also the one with the highest mechanical properties and the highest number of fiber plies. In particular, the highest curvature change is attained by the sample called 1+3, that can change curvature up to 93 %, while the first and second sample delivered a curvature variation of 69% and 71%, respectively. It is also observed that the time required to change shape is also related to the number of layers: the bi-layer (first sample) is the fastest (50 s), while the second and third samples experience morphing in 61sec and 84 sec, respectively. This latter is related to the higher thickness with increasing number of layers and can be tuned by optimizing the heating system.

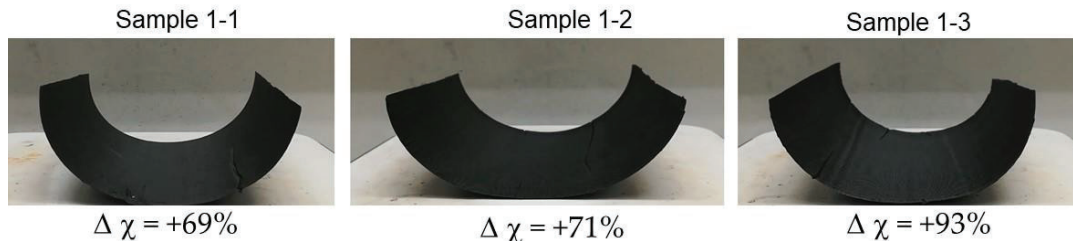


Figure 10: Maximum morphing response at 150°C of the composites characterized by the layer sequences of figure 8.

Since the best response was observed for the third sample, further deeper studies focus on this configuration. First, the design as well as the mechanical and morphing responses are deepened, and then the effect of the mold curvature is taken into account.

The layup configuration of the sample is shown in Figure 11 during and after manufacturing on a cylindrical mold of radius R_0 that is only partially covered by the carbon fibers. The result is an arch-type structure. As presented above, the outer layer has the fiber disposed transversely to the cylinder (denoted by the thick black line), while the inner layers are all disposed with fibers in the longitudinal direction (denoted by the dots). At high temperature (during curing) the sample has the same curvature of the tool plate (top part of Fig. 11), that is $\chi_0 = 1/R_0$. When the temperature decreases (room temperature) the internal thermal stresses induce the structure to bend and such action is designed to act in the same direction of the mold curvature, thus giving rise to a configuration with increased curvature (bottom part of Fig. 11). The temperature control of the material, allows to actively control the shape of the composite as well as its actual stiffness. This is the key aspect of this portion of the study: *analyze in depth the increased curvature*

due to the internal stresses, by adding fiber layers that increase bending. The single layer of fibers in the transverse direction plays the only role of structural reinforcement but does not take part in the morphing phase. On the other hand, the three layers disposed in the longitudinal direction represent not only structural reinforcement, but instead participate to the morphing process. As previously discussed, a cross-ply composite is fabricated having three layers in the longitudinal direction on the inner face plus one layer on the outer side with the fibers in the transverse direction. The latter layer has only a structural role, while **the three inner layers** not only act as structural part, but are **also act as**

actuators. This means that the thermal stresses allowing morphing are mainly concentrated in those layers. The sample is cured at high temperature (1 h at 80°C, 1 h at 120°C, 2 h at 150°C) and the adopted mold has, as earlier mentioned, a radius $R_0 = 7.0$ cm, an initial curvature $\chi_0 = 1/R_0 = 0.14$ cm⁻¹. At the end of curing, at room temperature, the sample attains an increased curvature considerably higher than that of the mold.

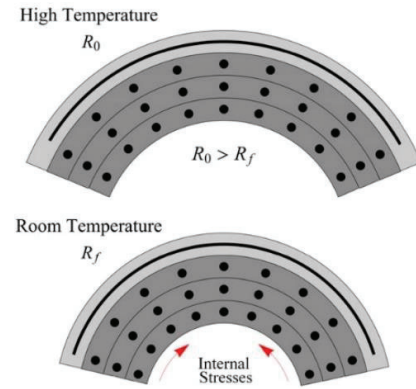


Figure 11: Sample design in cross section before and after manufacturing.

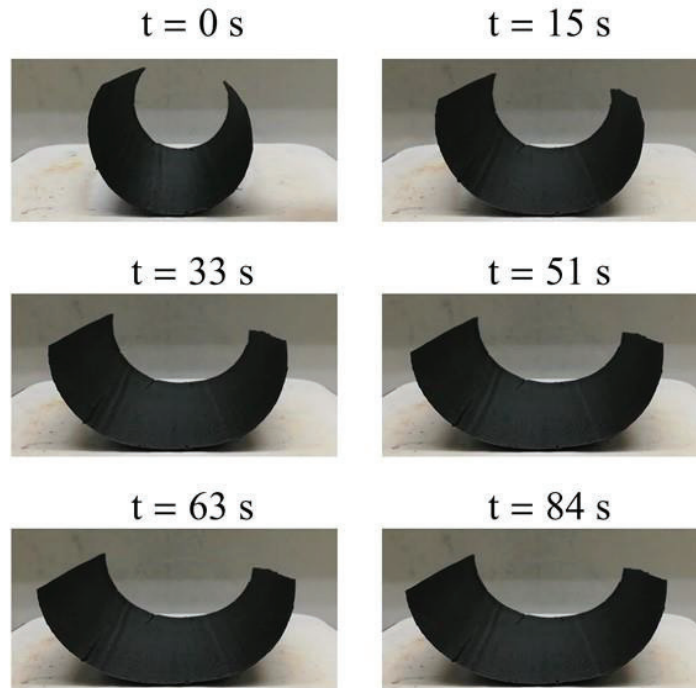


Figure 12: Snap-shots of the tested sample during morphing.

The measurement revealed a new radius of $R_f = 3.75$ cm, this means that the final curvature is equal to $\chi_f = 1/R_f = 0.27$ cm⁻¹, an increase with respect to the initial configuration up to $(0.27-0.14)/0.14 = 93\%$. On the other hand, when the sample is heated again the thermal stresses are released and the additional curvature is lost. This aspect was investigated by placing the sample on a hot plate at the curing temperature (i.e. 150 °C). When the internal stresses are not present, the sample reaches the less curved shape corresponding to the initial configuration equal to the tool plate curvature $\chi_0 = 1/R_0 = 0.14$ cm⁻¹. The test allowed to investigate the shape change of the composite when the temperature is varied and to evaluate the morphing capability. The outcome of the performed experiment demonstrates that the proposed approach is a very

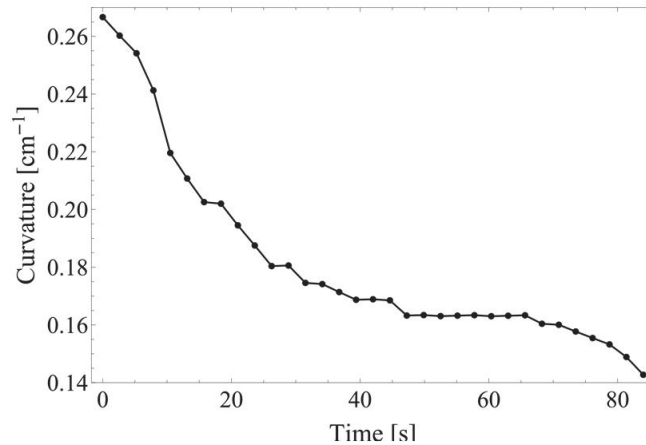


Figure 13: Monitored curvature vs time resulting from the DIC analysis

promising procedure to develop morphing structures and that the considerable shape change can be obtained with load bearing properties. Another important aspect in morphing characterization is the velocity at which the shape change takes place. This property is investigated by placing the sample on a hot plate at 150°C, the actual configuration is continuously monitored by a video camera so to measure the shape change over time. A representation of the performed experiment is given in Fig. 12, where an image of the sample is given at selected time instants to show the shape changing process. It can be seen how the sample changes shape gradually from the highest curvature configuration at room temperature to the lowest curvature configuration at 150 °C. By applying a digital image correlation algorithm to the captured video, it is possible to evaluate the actual curvature of the sample at each instant of time. At room temperature the sample displays the highest curvature, that continuously decreases up to 150 °C, as represented in Fig. 13, where the dots indicate the average curvature evaluated every 2.55 s. The lowest curvature, $\chi_0 = 0.14$ cm⁻¹, is attained after 84 s on the hot plate. It can be observed that, in this case, the heating process is not uniform since the sample is heated-up with an external heater, and thus the speed at which the shape change takes place is not optimized. The heated portion of the material is located in the center, while the regions around the sample edges reach the same temperature at different times. Of course this influences negatively the performance of the system.

The capability of the material to adapt its properties according to the prescribed temperature is then investigated from the mechanical point of view. In particular, the sample is subject to an external applied displacement and the corresponding force is measured continuously by the use of a load cell using the setup in Figure 14(a), where the sample is depicted during one of the performed mechanical tests. The sample is retained by an external support and the input displacement is prescribed by the driving shaft, while the corresponding force is measured by the load cell. The temperature is indirectly varied by the use of the hot plate placed below the sample, while the actual temperature of the material is monitored via a thermal camera. An example of the heating obtained in the material is given in Fig. 14, where the image captured by the thermal camera is represented when the sample reaches a temperature of approximately 55 °C. It can be noted that the hot plate temperature is much higher (around 70 °C) than that attained on the sample, but this is necessary since there is no direct contact between the two parts.

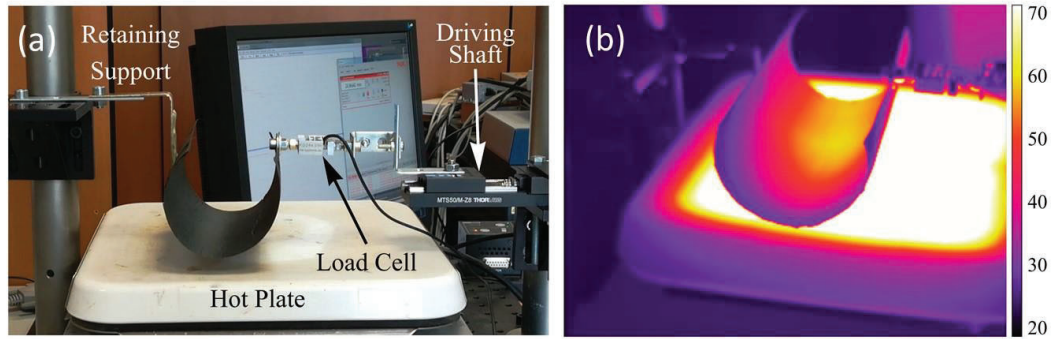


Figure 14: (a) Test setup; (b) thermal camera image during testing.

The use of the thermal camera allowed evaluation of the mismatch between the selected temperature on the hot plate and the average temperature on the composite. The sample is sufficiently uniformly heated except the regions at the edges that remains at a lower temperature, however this is an acceptable approximation for this specific phase of the study. The composite is tested by applying a prescribed displacement with a rate of 2.4 mm/s and the maximum stroke is set in the range $37 \div 45$ mm, that is adjusted as the temperature increases to match the initial configuration of the sample. The displacement ramp is applied at selected temperatures in order to measure the corresponding force exerted by the material and evaluate the variations of such force at each temperature.

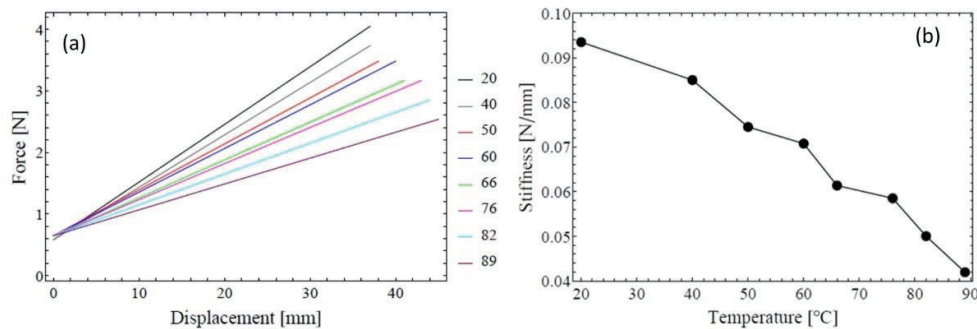


Figure 15: (a) Mechanical response at the different testing temperatures; (b) Variation of stiffness with temperature.

As illustrated in Fig. 15, the force-displacement diagram is reported at each selected temperature. It can be noted that, even though the response of the composite always displays a linear behavior, the average force measured by the load cell reveals to be considerably lower at higher temperatures. In particular, at room temperature the force measured at the maximum displacement is 4.04 N, while at the highest induced temperature the maximum measured force is 2.54 N. This demonstrates that not only the shape is changing but also the overall stiffness of the structure is changing accordingly, up to 37%. By deriving the linear tangent stiffness from the force-displacement diagrams it is possible to evaluate the actual stiffness of the material and its variation with temperature. The result of this analysis is illustrated in Fig. 15b, where the stiffness is calculated at each prescribed temperature. The trend of the variation reveals to be regular showing an almost linear decrease, starting at 0.094 N/mm and reaching the value of 0.042 N/mm with a maximum difference of 55%. The observed behavior confirms the great capability of the curved composite to not only change shape, but also to vary its mechanical response according to the prescribed temperature. Moreover, the large shape change demonstrated by the composite is associated to an interesting mechanical response that is not common at all in morphing materials.

2.3.1.3 Effect of the composite curvature

Further investigations were performed to understand the role of the mold curvature on the morphing capability and on the mechanical response of the curved composite. The study focuses again on the best lay-up among the ones above investigated, and in particular on the design that comprises an outer layer in which the fibers are disposed transversely to the mold and the inner layers that are disposed with fibers in the longitudinal direction. Three molds with decreasing radii, and so with increasing curvature, were adopted for the study giving rise to sample 1, sample 2 and sample 3 (see Fig. 16). The test setup is shown in Figure 17 (a). The sample was sitting on the hot-plate with the concave down and was loaded with the support of a flat indenter along the vertical direction. As in the other cases, the flat indenter and the load cell were directly connected with the driving shaft controlled by a digital microstage with a nanometer resolution. The applied time history is reported in Figure 17 through the example of the test of sample 3. A series of temperature steps lasting 75 sec each, were planned to increase temperature from room temperature to 100°C, as reported in Figure 17 (c). During each single temperature step the sample was loaded with 5 consecutive loading-unloading cycles in a displacement control mode.

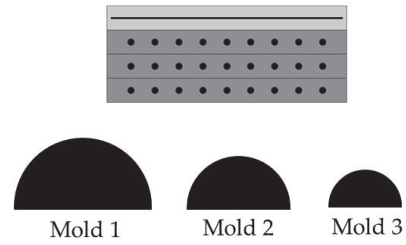


Figure 16: Design logic to study curvature effects.

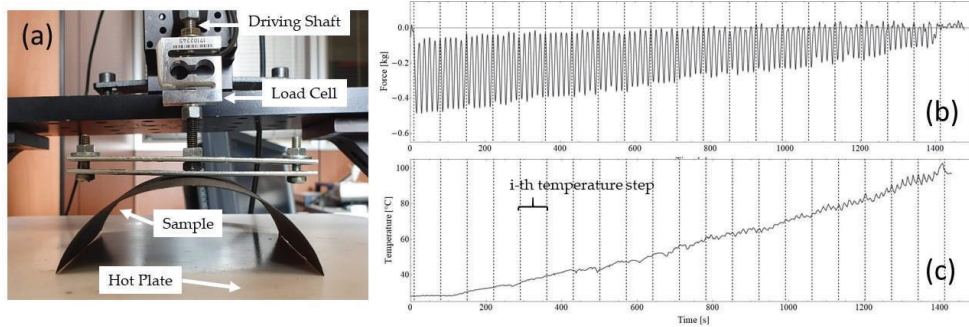


Figure 17: (a) test setup; (b) Force vs time response of sample 3; (c) Time history of temperature.

The corresponding forces were simultaneously detected with the load cell (Figure 17 (b)). The results are in line with the previous study. With increasing temperature the load-bearing capability of the material decreases. This can be observed from the decrease of cycles amplitude of the Force time history with increasing temperature but also in Figure 18 (a) that shows the comparison of three independent cycles recorded at three distinct temperature steps (always for sample 3). In the latter case a downward shift of the Force vs displacement cycle is evident with increasing temperature. This latter diagram also highlights an hysteretic response of the material which is mostly related to geometric nonlinearities.

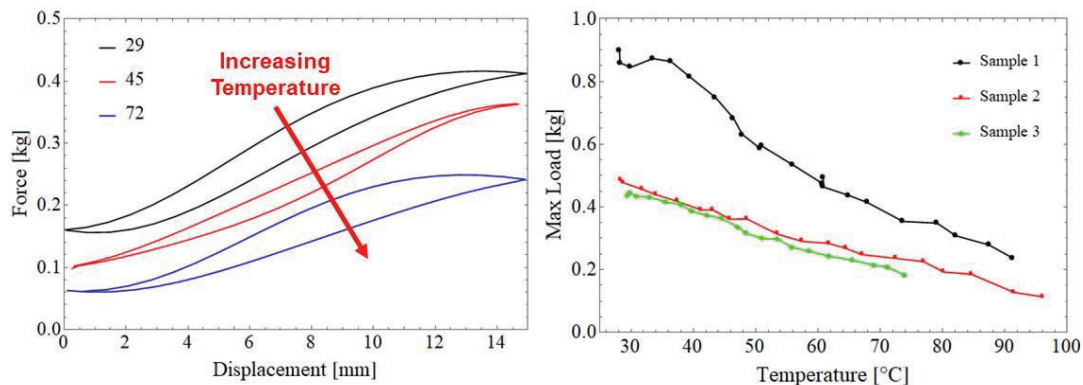


Figure 18: Comparison of 3 cycles at 29°C, 45°C and 72°C and comparison of the recorded maximum load for each temperature step, for the three tested curvatures.

The comparison of the three curvatures shows that the sample with lower curvature (sample 1) is characterized by a more pronounced decrease in load bearing capability with increasing temperature.

2.3.2 Towards radical shape changes

The material design discussed in the previous sections provided an interesting approach to achieve large-scale morphing in carbon fiber composites without the need of using external actuators and without damaging the composite material which is typically rigid in its nature. The fact that morphing in this design is all related to the cyclic acquisition/releases of internal stresses, has the beauty that it can be achieved in a simple manner and that the material can be switched from one configuration to the other “cyclically” over the life-time of the structure, but has the drawback that the “shapes” that the material can reach are fixed to the initial material design. For this reason the last part of the work on the composite, was devoted in understanding if it is possible to design a carbon fiber composite that can provide radical shape variations with the idea that, in the long term, such changes should be supported by additional muscular systems to be embedded in the material.

Preliminary tests were then performed on 3D composites which did not comprise internal stresses upon manufacturing. A four layered symmetric lay-up depicted in Figure 19 (a) with an embedded heating system as that discussed in Figure 5, was then selected for this scope. The picture in Figure 19 (b) shows one of the fabricated 4-layered samples which, in the symmetric lay-up case, assume the same radius as that of the mold (7 cm). The mechanical response of three separated tests that were performed at room temperature on this sample using the test setup described in Figure 6, is reported in Figure 19 (c). The result shows a nonlinear behavior which is well fitted with numerical simulations that also help in understanding that the initial change in the slope is due to a rigid motion taking place inside the support bearings, while the final loss of stiffness is mostly related to the loss of geometric stiffness, typical of arch-type structures. Such behavior is well captured by simulations when geometric nonlinearities are taken into account (dashed red line), the agreement between numerical results and experiments is more accurate when a rigid motion is prescribed to the model (green solid line). The linear model (red solid line) is also reported for comparison.

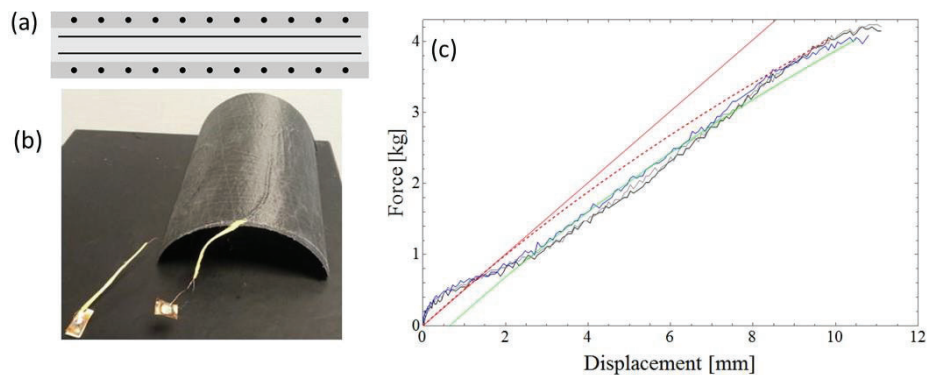


Figure 19: (a) Schematic of the composite lay-up; (b) Picture of one of the fabricated samples with the structure in (a); (c) Mechanical response of the composite under small deformations.

The test is repeated at different temperatures up to 100°C (Figure 20) applying a *large displacement of 25mm* to the indenter to simulate **radical shape changes**. It is observed that with increasing temperature the slope of the force vs time decreases which is related to the stiffness decrease depicted in figure 20 (b), while the force vs displacement trend follows a similar nonlinear trend described in Figure 19. The difference is that in this latter case the ultra-large applied deformations cause rigidity losses due to geometric nonlinearities typical of arch structures.

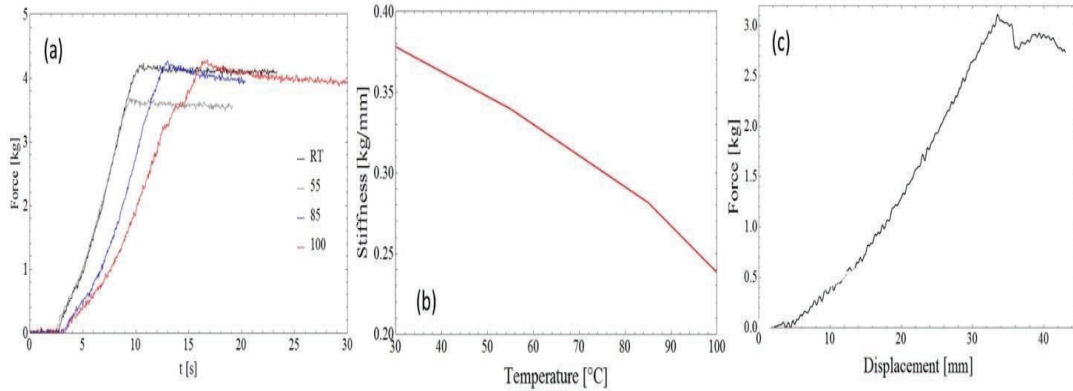


Figure 20: (a) Load-time history response at increasing temperature; (b) Stiffness variation of the sample with increasing temperature under radical shape changes; (c) Mechanical response of the composite under large deformations (radical variations).

Despite the quantitative analysis above described, it is probably more effective to show the snapshots of the induced displacements which, for this particular test, represent a **radical shape change** in the composite sample. The sample is in fact fully squeezed along the vertical direction, thus losing its original dome-like structure. This result suggests that a symmetric carbon fiber sample can be shaped into new configurations thanks to the usage of this particular thermosetting matrix. The snap-shots below (Figure 21) also highlight that once the load is removed, the heated sample recovers its initial shape.

These observations were useful for the following latter investigation. Instead than unloading the sample while being heated, as in this case, the sample is constrained in a radical change of shape while being cooled down, and then, once re-heated, it recovers its initial shape, in other words, it could be treated as a shape memory composite. The difference in this case is that this is done with a thermosetting matrix whose shape memory properties are not known in the literature, but have been investigated in our laboratory. Moreover, in this case study, the shape memory thermosetting composite can self-shape thanks to the integration of the embedded heating system.

The radical shape change of the composite and the recovery of the initial shape is reported in Figure 22. The composite is self-heated and loaded, under severe deformations, with the same procedure described above but it is frozen into this new configuration by cooling down the system before unloading. The new shape of the sample is radically different from the initial configuration and this is possible, without damaging the material, thanks to the matrix properties and to the embedded heating system. Once the sample is re-heated, it fully recovers its original configuration.

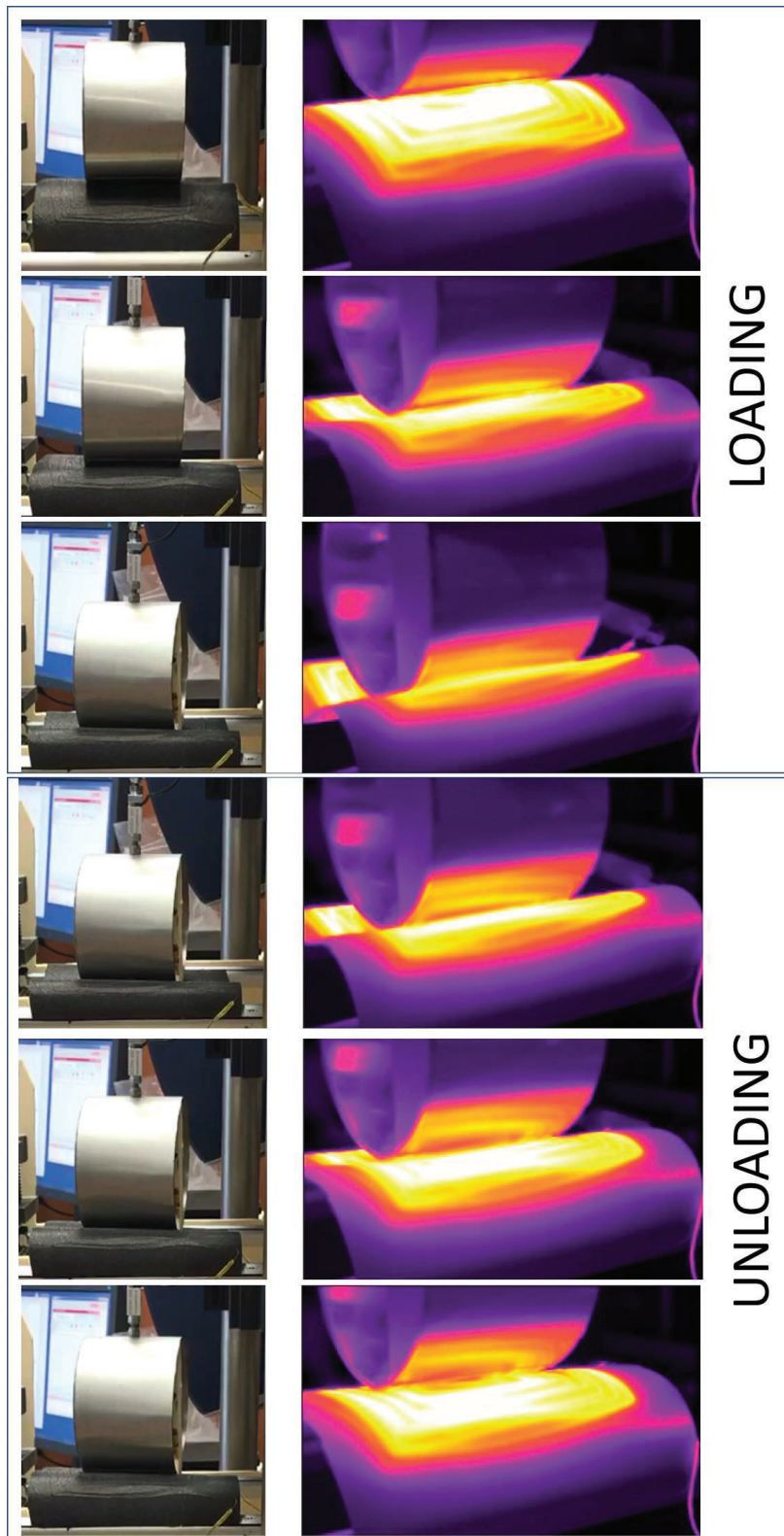


Figure 21: Snap-shots of a radical shape change of a symmetric layup while heated followed by an automatic and spontaneous recovery upon removal of the load at high temperature.

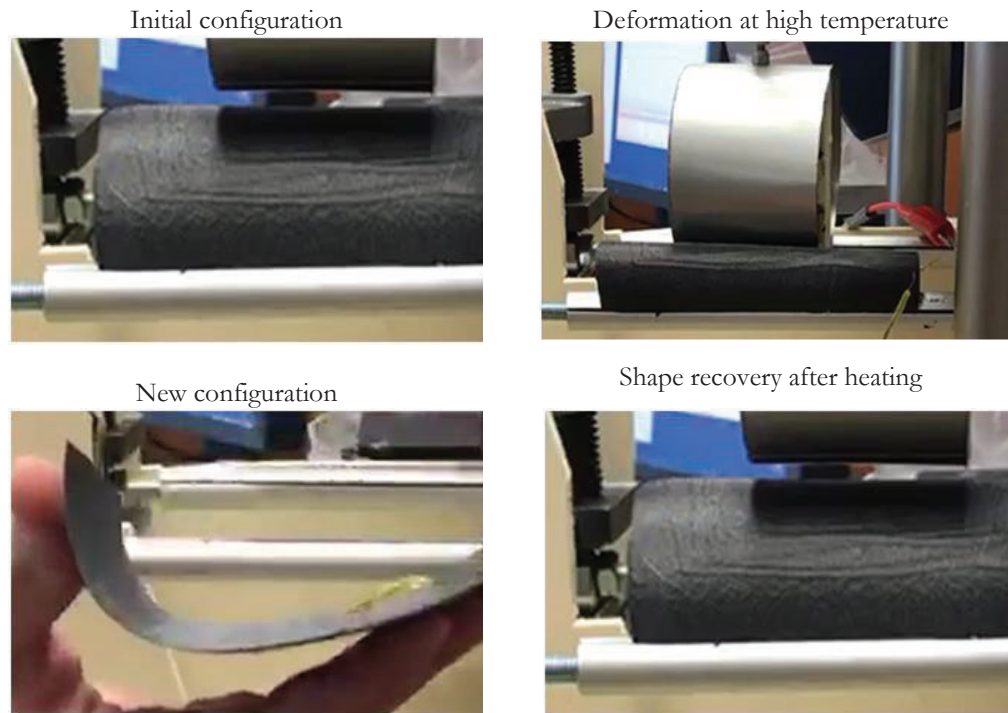


Figure 22: Testing to validate the frozen radical shape change of the composite.

2.4 PROTOTYPE OF A SMART PROSTHETIC LEG VIA A “SENSITIVE” MUSCULAR 3D COMPOSITE

This section illustrates a prototype that comprises a 3D muscular composite that embeds a number of independent carbon nanotube-based strain sensors, electrospun piezoelectric sensors and thermal heaters, to form an innovative transtibial prosthetic leg for a youth. The idea is that the leg should adapt its rigidity, locally or globally, as a function of the walking mode (walking and running) which is continuously monitored over time with the embedded sensors. In agreement with the previous study, the local and global rigidity tuning of the carbon fiber composite is allowed, respectively, by a local and global temperature modulation through the embedded heaters which can be actuated independently. Since the adopted epoxy has a considerable temperature-dependent behavior, it is possible to control its stiffness and thus to control the structural frequencies and damping. By coupling the sensing system with the control system, the structural properties are tuned to match prescribed working conditions, thus optimizing the performance of the proposed smart system. It is worth noting that in this study, the muscular shape change functionality was not implemented since it would have increased the design complexity, going beyond the scope of this project, with respect to fine-tuning shape changes with walking mode, patient weight and impulsive loads. Instead the interest in this case study was mostly related to validate the multiple-functionality of the material for a given shape. For this reason, the composite design that is adopted in this case study is a “symmetric lay-up”, as discussed in the following.

2.4.1 Prosthetic design and manufacturing

The design of the prosthetic leg had to keep in mind not only the project objectives but also some laboratory constraints. For instance, the ovens in the laboratory have a limited size, for this reason we decided to focus on a kid prosthetic that is located between the knee and the ankle. The selected transtibial prosthetic follows the classical shape of a racing prosthetic (Fig. 1a) with the following specs: 52 cm total length and 5 cm width. As mentioned, the idea is to locally tune the prosthetic rigidity as a function of the walking mode. The prosthesis should be more rigid while walking and less rigid while running. Such variations were thought to be concentrated out of the “foot” area, and in particular, in an area that has a length of 25 cm as shown in the schematic in figure (2a). This area was split in 5 distinct zones, each of which comprised a nanocomposite strain sensor, an electrospun piezoelectric sensor (Fig. (2c)) and a heating system (Fig. (2b)), all interconnected with an external controller for automatic:

sensing→processing→ actuation.

The devices were inserted within the carbon fibers lay-up in agreement with the schematic in Figure (1e), during the composite manufacturing. In particular, the sensors are located on top of the bottom layer where the higher bending deformations are expected, while the heaters are located in the middle of the composite in order to get the most uniform heating on both sides. Their thin and light nature does not strongly affect the global behavior of the structure. It is to be noted that all devices (CNT sensors, piezoelectric electrospun sensors and heaters) were fabricated in our laboratory with the thickness and size requested for this study.

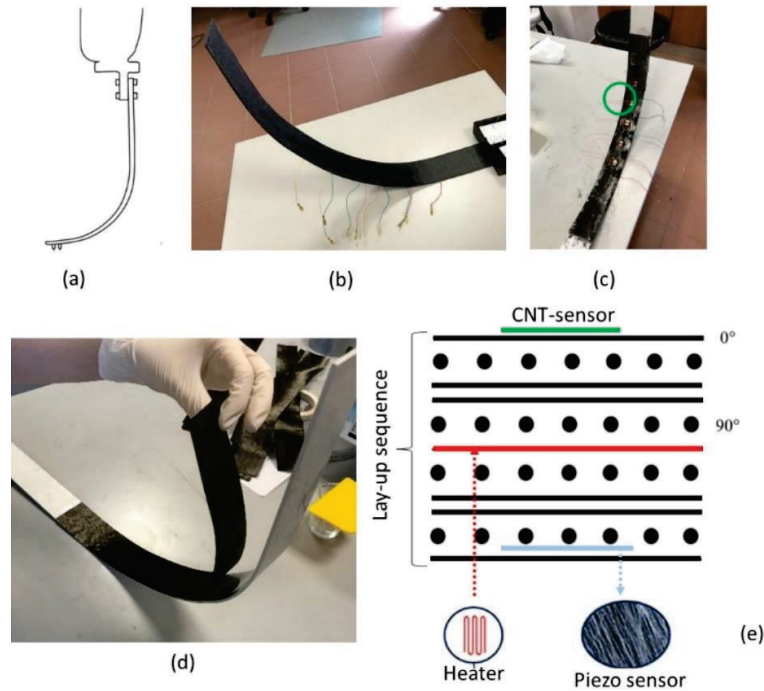


Figure 1. (a) Conventional racing prosthesis; (b) Carbon fiber prosthesis fabricated in our laboratory and equipped with an array of 5 heaters, an array of 5 PVDF sensors and of 5 nanocomposite sensors shown in (c); (d) fabrication step of the prosthesis on an Al mold; (e) Schematic of the lay-up of the composite and of the embedded devices.

The cross-ply plates are obtained by the use of unidirectional carbon fibers, whose main characteristics are the Young's modulus of 230 GPa and a diameter of $7\ \mu\text{m}$, with an overall thickness of the fiber tow of 0.1 mm. On the other hand, the adopted hosting matrix is the same thermosetting epoxy resin used for the previous study, whose Young's modulus is 3.6 GPa. The manufactured composite realized via the hand-layup process with a vacuum curing phase of 24 h is composed of 10 fiber layers with the following orientation $[0_3^{\circ}, 90_4^{\circ}, 0_3^{\circ}]$.

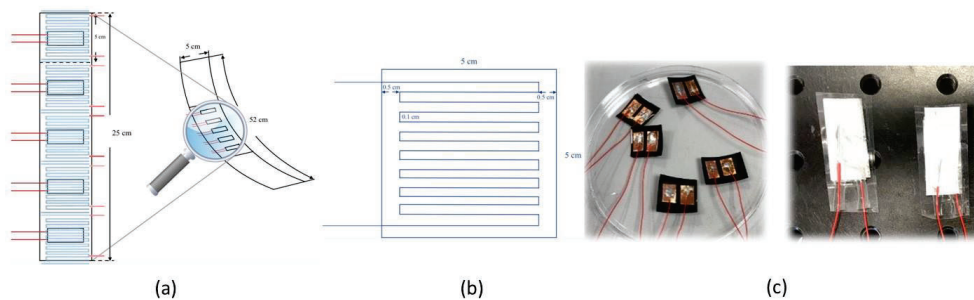


Figure 2: (a) schematic of the embedded sensors and heaters within the prosthetic; (b) schematic of the designed heaters; (c) pictures of the fabricated sensors.

The system of internal heaters, that can be independently activated, is also embedded in the composite in order to locally control the composite stiffness. In line with the above research activities, the heaters were designed so as to exploit Joule effect through a coiled 0.1 mm in diameter copper wire. By properly tuning the applied electrical current it is possible to govern the actual stiffness of the epoxy resin whose mechanical properties experience considerable

variations with temperature. The Young's modulus E of the epoxy resin under such curing conditions, was experimentally measured at different temperatures, as resumed in Tab. 1. The results show that the adopted epoxy resin can vary its modulus up to 15%.

TABLE 1. VARIATION WITH TEMPERATURE OF THE EPOXY YOUNG'S MODULUS.

[°C]	25	30	35	40	45	50
E [GPa]	3132	3077	3009	2926	2817	2662

Since the overall mechanical properties of the final composite are determined not only by the fibers, but also by the selected matrix, the behavior of the composite will experience similar variations.

The described system of sensors and heat actuators is governed by a micro-controller. The main idea is to monitor local composite strain deformations or external excitations, and consequently tune the stiffness of the composite in order to adapt its response to such detected input

2.4.2 Autonomous adaptation of the muscular composite vs. Walking Mode

2.4.2.1 The algorithm

The autonomous functionality of the muscular composite in this case study, consists on the capability of the system to acquire the electric signal of the sensors, then elaborate it to interpret the walking mode and consequently send the right amount of current to the corresponding heater. The heater is then heated-up to a specific temperature which is of course in agreement with the corresponding walking mode. The maximum value of current is the result of a simple model, described below, which takes into account the resistance of the wire, its length, the section of the prosthesis to be heated and the maximum temperature that must be reached. It is obvious that the walking mode is unknown to the system, it depends on the person wearing the prosthesis. For this reason two approaches were investigated to let the system understand the walking mode: the first one is to use the frequency of detection of the prosthetic strain, while the second is based on the study of the magnitude of the deformation. Such procedures were both implemented in a microcontroller that has been used to acquire the signals from the sensors, elaborate them to "understand" the walking mode and finally "control" an electronic component (a RELAY and a MOSFET) which is able to supply a current up to the required Ampere. These components were alimented with 4V and the reached temperature over the powered resistor was monitored with thermal analyses. To speed up the resistor heating, the system was activated with the maximum current for a given time and then it was switched to the current level of the target temperature.

As mentioned earlier, in the frequency strain monitoring approach, the frequency at which a deformation of the prosthesis is detected, is correlated with the type of walking mode. For instance, running implies a higher deformation frequency, while walking implies a lower frequency. In other words, the frequency of the signal detected by one of the PVDF

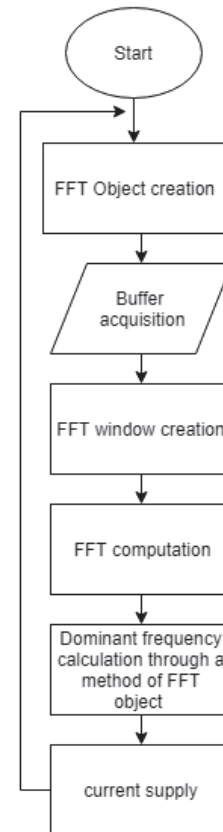


Figure 3. Block diagram of the frequency algorithm

electrospun sensors is used to modulate the local temperature of the prosthesis. Since the softer leg is desired for a running mode, the higher is the monitored strain frequency, the higher will be the required current within the resistor to reach a higher temperature. With this in mind, a maximum frequency was selected as design choice, by considering the maximum frequency at which a very fast person can run. In this way, a so called “duty cycle” (in percentage) was defined as the ratio between the monitored frequency (over time) and the maximum (set) frequency. The block diagram of the implemented algorithm is reported in the schematic in Figure 3. In brief: first an FFT object is created so a space in the memory is reserved for a class instance. The buffer is acquired with a length that depends on the duration and sampling frequency of the acquisition. This step is followed by the creation of the FFT computation window which can be of various types (hamming, Hanning, Gauss, Kaiser etc) which can affect the standard deviation. Finally, the FFT computation is performed to extract the frequency content of the signal in the buffer, thus calculating the value of the duty cycle and consequently the current that the MOSFET must supply.

In the deformation algorithm the idea is to evaluate directly the amplitude of the deformation and consequently decide the target temperature to which the prosthesis must be brought. Therefore the walking mode is correlated with the amplitude of deformation.

In this algorithm two important parameters had to be decided: the maximum deformation that the sensor can detect and the value of a threshold below which the signal is considered noise. The maximum deformation was calculated considering the maximum voltage value (from here, called *absolute maximum*) that the sensor can provide when it is subjected to a deformation. The second one, instead, was calculated directly amplifying by a factor of 2, the sensor noise. Another important design choice is the integration of a counter in the algorithm. The algorithm is in fact designed in such a way that current is provided only after three consecutive deformation measurements to exclude deformations that are not directly related to a walking mode (for instance a hit on the prosthesis). So, it has to be able to count the number of deformations detected which is done with a counter.

At the beginning, the algorithm initializes the counter to zero, then it checks if the system is providing current and, at the same time, if the user is inactive in a specific time interval. In case of inactivity the microcontroller stops providing current (the patient is at rest). Instead, if at least one of the two conditions are not true, then the microcontroller acquires the data from the sensor putting them into a buffer. The dimension of this buffer depends on the sampling frequency and on the interval of acquisition that, in this case, is of 1 second. The length of the buffer has been set considering two important aspects: avoid the analysis of multiple peaks of the same deformation and avoid to influence the system processing speed.

During the acquisition the algorithm also calculates the maximum value of deformation that is acquired in that buffer (*relative maximum*) and then, in the next step, it calculates the percentage of deformation in order to calculate the target temperature of the prosthesis. First, the relative maximum, is compared to the noise threshold and with the absolute maximum. If the value is not included in these two extremes a new buffer is acquired. If, instead, it is included the counter of the detected deformation is increased. Then the value of the counter is checked. If it is equal to one, the time instant in which the deformation is detected is saved. In the case, instead, that the number of detected deformation is two, the time instant of the deformation is saved but, after this, it is checked if the difference between this second instant and the first one is lower than a certain threshold chosen during the planning phase. If this is true a new buffer is acquired to see if there is a new deformation. If instead, the difference between the interval is higher than the threshold, this means that those two deformations are not correlated and so the counter is reset to zero.

The last case is the one in which the number of deformations is equal to three. Even in this case the time instant is saved and compared with the second one. As in the previous case, if the difference between the two times instant is higher than the threshold, the counter of the deformation is reset to zero. If, instead, it is lower than the threshold, to speed up the heating system, the heating is started with the maximum value of duty cycle for a given time, that is calculated inverting the 2° degree formula obtained from the characterization of the heater reported below, and, after this time, the value of duty cycle is switched at a value that maintains

the temperature at the calculated target temperature. To test this method two target temperatures are chosen and the corresponding trend of the system is reported in Figure 5.

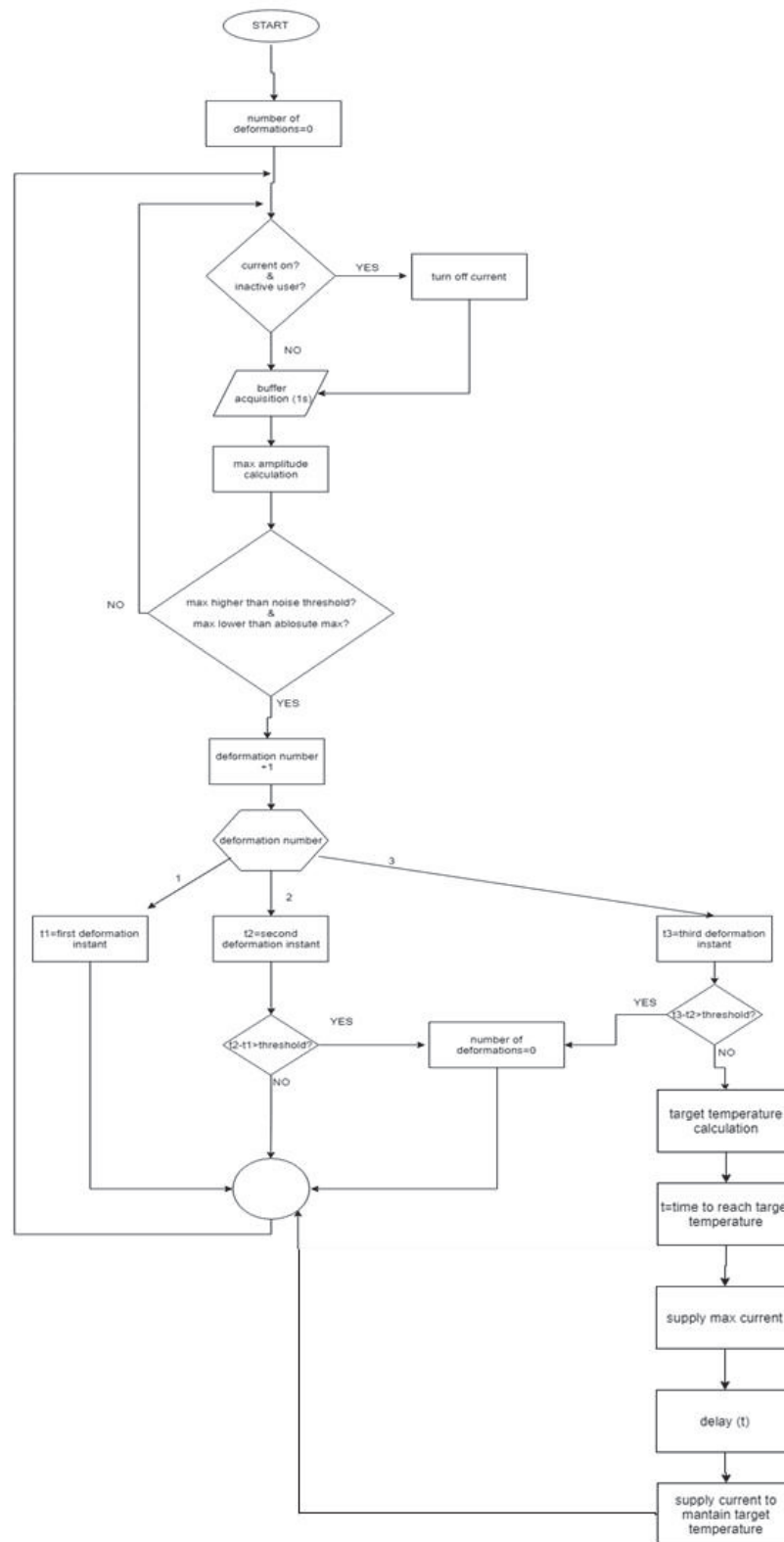


Figure 4. Block diagram of the deformation algorithm

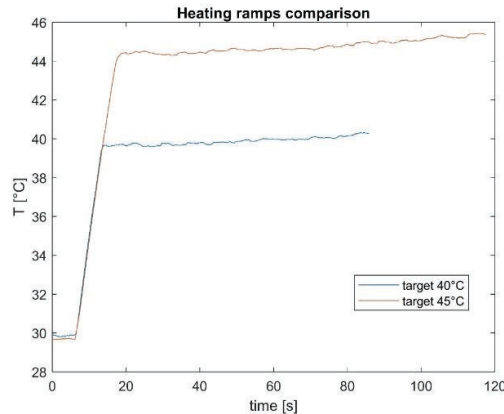


Figure 5: trend for different target temperatures.

The two ramps in Figure 5 start with the same inclination and then, once they reach their target temperature, they remain stable at that temperature. As previously mentioned, this is possible using before a duty cycle at 100% and then using the correct duty cycle for a given target temperature.

The algorithm that reflects the above concept in the case in which the difference between the two time instants is lower than the threshold, is the following. The algorithm calculates the target temperature that the prosthesis has to reach taking the inverted mathematical relation (discussed below) between the deformation and temperature. The value of duty cycle that is used in that formula is calculated as the ratio between the relative deformation and the absolute deformation. At this point the algorithm calculates the time that the system should use to reach the temperature just calculated using a value of duty cycle of 100%. Even here, to calculate this time, the previously obtained formula that connects the variation of time with the variation of temperature, is used. The time is calculated fixing the target temperature to that calculated in the first step and inverting the formula.

Now the maximum value of current is provided to the heater for an interval of time that equals the calculated time and, after this time, a value of current that is able to maintain the temperature of the prosthesis at the value of the target temperature, is also provided.

This last value of current is obtained by calculating the right value of duty cycle that is needed to provide the output. This value of duty cycle can be calculated from the relation between the duty cycle and saturation temperature which is obtained by the characterization of the heater.

2.4.2.2 Characterization of the heater through the Microcontroller

The characterization of the heater is important to associate each value of duty cycle to a value of saturation temperature and to understand what is the mathematical function that correlates these two quantities. The characterization gives also information about the heating speed in reaching the relative saturation temperature with a given value of duty cycle.

This characterization is important because over a certain temperature (50°C) the piezoelectric sensors depolarize. In addition, it is important to regulate the temperature of the prosthesis.

The instruments used to characterize the heater are the thermal camera and the power supply. Other parts of the setup are the controller, the circuit with the control device (relay or mosfet) and, obviously, the heater.

Initially the power supply used was a wall power cube that was able to provide maximum 12V and 2A, while the control device was the relay. The schematic of the circuit that has been used is schematically represented in Figure 6.

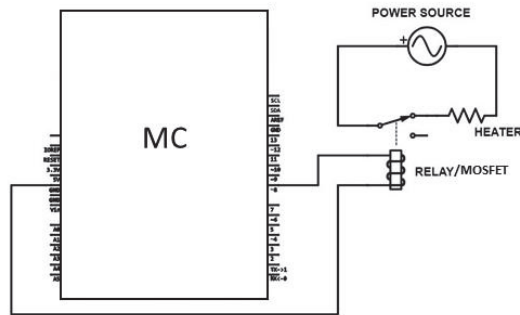


Figure 6: circuit scheme with relay and power cube

However, both power cube and relay gave some issues and were replaced with a bench power supply and with a MOSFET. By setting a 4V output and considering 2 Ohm for the resistance of the heater, the maximum current that could be supplied is 2A.

With this configuration it was possible to go to higher values of duty cycle with respect to the initial trials. The validation of the approach was tested by giving a known value of duty cycle to see the value of saturation temperature for that duty cycle. This approach was repeated for more values of duty cycle. The data acquired are reported in Figure 7b. The graph shows that with this configuration there is a large dynamic of duty cycles and this allows a fine regulation of the temperature for prosthetic heating. If the maximum temperature at which the prosthesis must be brought is 45°C (50°C is the limit for piezoelectric sensors), then for each duty cycle value, it is possible to detect the corresponding saturation temperature (Figure 7b).

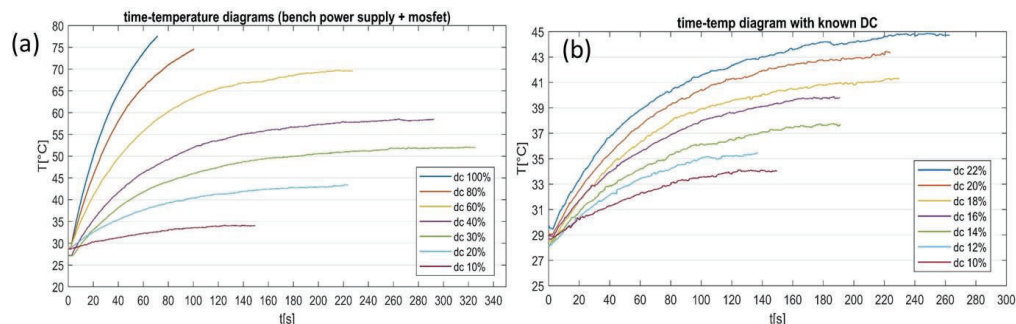


Figure 7: (a) time vs temperature (bench supply+mofset); (b) time vs. temperature with known duty cycle

It can be observed that the maximum temperature at which the prosthesis can be heated up, is reached at sufficiently low values of duty cycle.

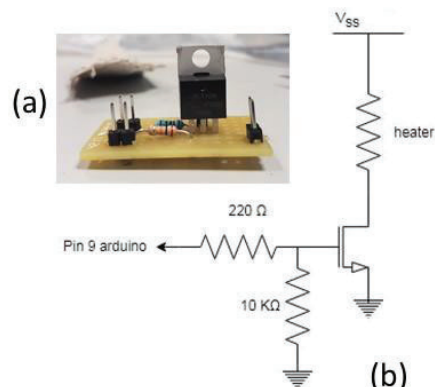


Figure 8: schematic and realized electrical circuit

The electrical schematic and the realized circuit of this configuration is represented in Figure 8 in which Vss is the power supply (4V), the heater is connectet to the gain, the source is connected to the ground and the gait is the control pin wich is connected to the microcontroller.

Using the data acquired from the characterization of the heater, a mathematical function that could describe those data was identified to be implicitly or explicitly used in the algorithm. In particular, the relationship between duty cycle and saturation temperature is calculated from the curve of Figure 7(b) and

is reported in Figure 9 (well interpolated with a linear trend).

This array of maximum temperatures has been put in relation with the values of duty cycle from which these maximums have been generated. The interpolation of the data shows a linear trend, $T = (0.362 * DC\%) + 27.77$ which can be observed in the graph in figure 9a.

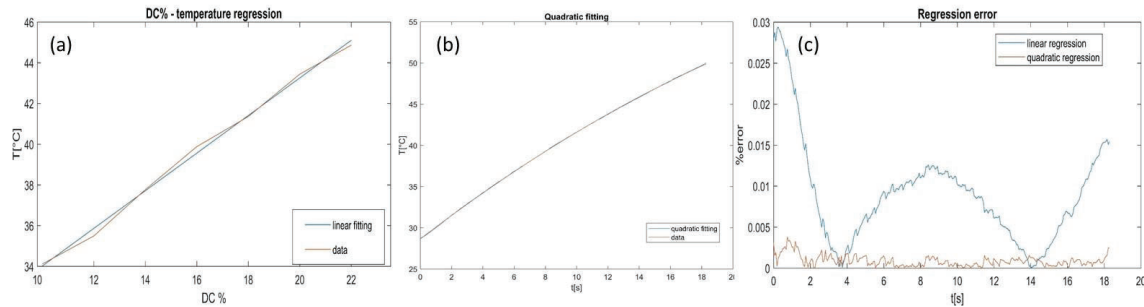


Figure 9. (a) fitting, (b) quadratic fitting of temperature vs. time; (c) comparison between linear and quadratic errors

For the relation between time and target temperature a duty cycle with a value of 100% was used to extrapolate the temperature-time trend below 50°C, which was well interpolated with a 2° degree function in figure 9b:

$$T = (-0.016 * t^2) + (1.455 * t) + 28.65$$

A fitting error E for the quadratic fitting was calculated and compared with that of a linear fitting, using the following formula:

$$E = \left| \frac{x_{fit} - x_{data}}{x_{data}} \right|$$

The quadratic fitting is confirmed to be reliable also from this prospective. Its error is in fact significantly lower (factor of 10) than that of a linear fitting and is constant over time.

2.4.3 The functionality assessment

The performed tests are mainly focused on the characterization of the system and the dialog between the embedded heating system and the sensors. The embedded heaters are composed of five copper wires (each having a length of 0.5 m) in order to provide a resistance of 1.1 Ω . The wire ends are connected to a power supply. Figure 10 shows the resulting temperature distribution of the composite, given by a single heating element, that is measured with a thermo-camera at two current values (600 mA (left) and 800 mA (right)). The blue color denotes the room temperature, while the red color represents 50°C which is also the maximum reachable temperature to avoid, as earlier mentioned, the piezoelectric sensors depolarization. Even though the temperature is affected by a heat concentration at the connection with the external electrodes, it is found to be uniformly distributed in the composite zone that corresponds to that heater.

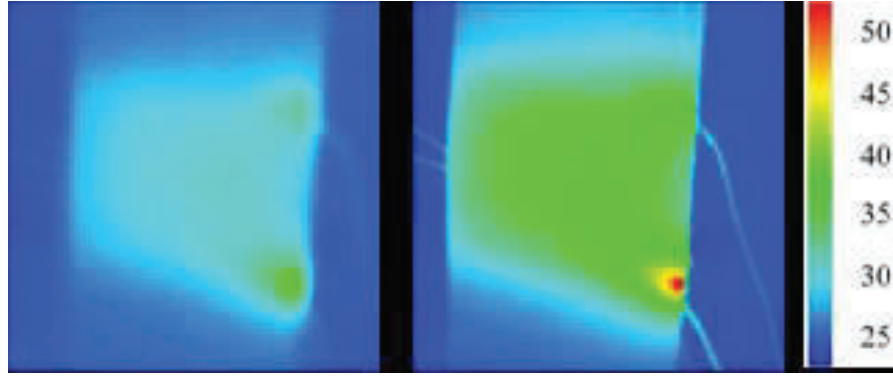


Figure 10: Thermal images of the heating system. The temperature scale is expressed in degrees Celsius.

Figure 11 shows the comparison between the average temperature measured across the composite when the current is increased from 0 to 800 mA (gray line) and the temperature derived theoretically (red line). As illustrated in fig. 11, the theoretical model is capable to predict very well the experimental data. The model is based on the knowledge of the electrical resistance of the heaters, which is defined by $R = \rho \frac{l}{S} 1.12 \Omega$ where $\rho = 1.76 \times 10^{-8} \Omega \text{m}$ is the copper resistivity, l is the wire length and S is its cross section area. The power developed by the wire with a current I is given by $P = I^2 R$ and the corresponding temperature variation is evaluated according to the following expression:

$$\Delta T = \frac{P}{c_{\text{add}} \cdot A_1}$$

where A_1 is the lateral surface of the heater and $c_{\text{add}} = 400 \text{ W}/(\text{m}^2 \text{K})$ is the adduction coefficient, that has been experimentally derived for the actual composite. This is important to facilitate the material design and its optimization, since driving the proper amount of current into the structure allows to control its temperature and thus its actual mechanical properties, which were proved to be strongly affected by temperature.

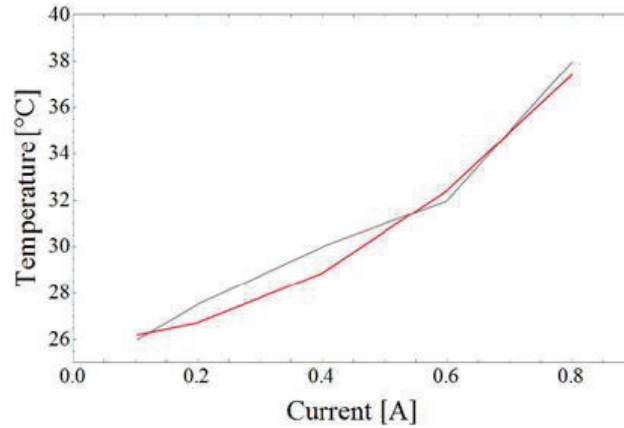


Figure 11. Theoretical (red) and experimental (gray) temperature vs. applied current.

Preliminary acquisitions are conducted to identify the sensors noise. As illustrated in Fig. 12, in fact all the sensors are affected by a peculiar noise that acts at a specific frequency, that is 50 Hz. Such noise is mostly due to the acquisition system that is powered by a 50 Hz voltage. Moreover, additional peaks can be observed in Fig. 12 at higher frequencies, but they are located in correspondence of odd 50 Hz multiples (150, 250, etc.) which means that they are due to the

same issue previously mentioned. This noise affecting the sensors can be reduced by adopting a suitable signal post-processing procedure, that consists in a band pass filter around 50 Hz. It would also be better to eliminate noise at higher frequencies, but since the structural modes happen at a lower range, it would be less significant. It was observed that the response given by the sensors is comparable in terms of noise and generated voltage, thus, though the experiments are always conducted simultaneously on all the sensors, the next results are illustrated only for sensor 3.

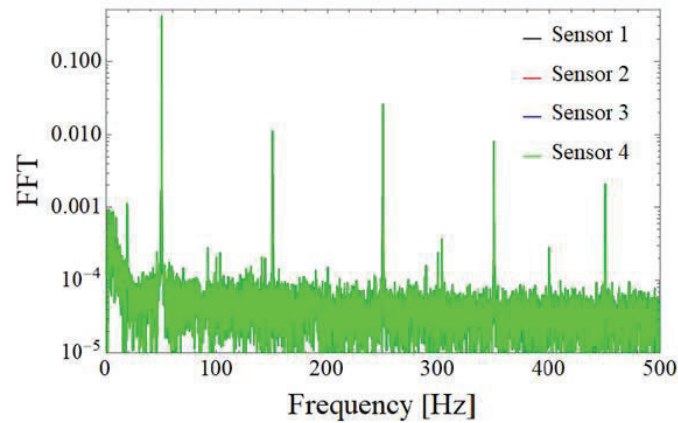


Figure 12: Acquired signal FFT affected by strong noise at 50Hz.

To this aim, the composite is subject to an impulsive excitation at the free end: the results in terms of time history and frequency (FFT) are shown in Fig. 13. The gray lines denote

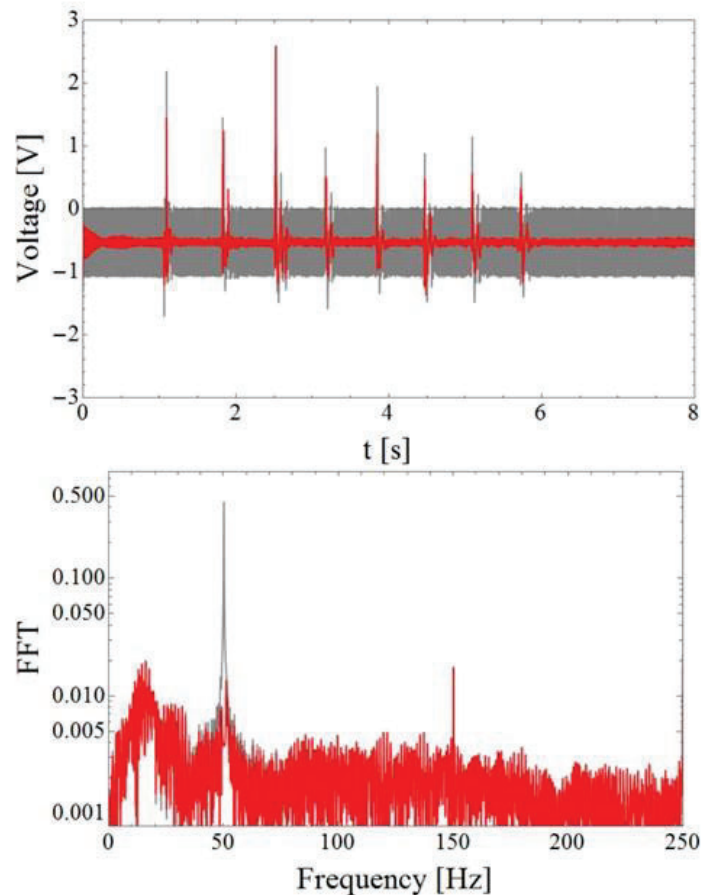


Figure 13: Time and frequency response.

the acquired signal which is affected by strong noise. The red lines represent the signal after the filtering. It is worth noting that the applied impulses are considerably strong and can be identified even without data post-processing.

2.4.4 Final demonstration of the multi-functionality of the prosthesis

The capability of the smart composite to change its stiffness is tested via the setup shown below. An impulsive load is applied to the material, the micro-controller is set to activate the heaters in order to tune the actual stiffness of the composite when the external load is detected by the embedded sensors. The response of the structure is monitored also externally via a laser scanning vibrometer and it is compared with that obtained by the internal nanocomposite sensors. It can be seen from the plots in figure 15 that even if the signal is not clean, it allows to detect the external load thus allowing the automatic activation the heating system. The signal quality is affected by the nanocomposite sensor used for this demonstration which is not ideal for dynamic monitoring, its best performance is for quasi-static measurements. The capability of the system to change stiffness is then evaluated by measuring the change in natural frequency derived from the acquired time histories. It was observed that the so-designed material can actively change its frequency up to a 39% variation, in agreement with the prosthesis design.

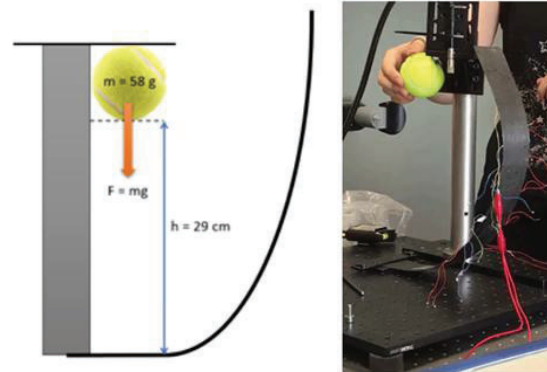


Figure 14: Test setup of the prosthetic leg.

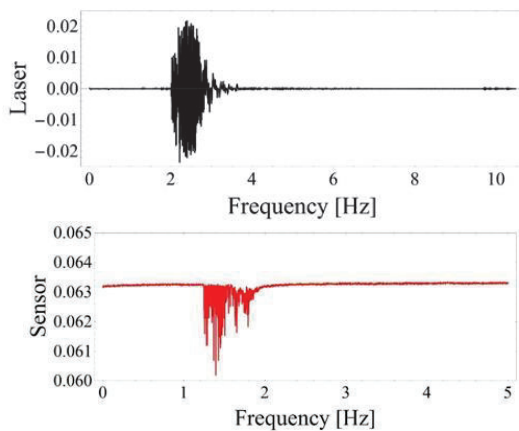


Figure 15: Experimental response of the prosthetic leg.

one prosthesis was in fact equipped with the array of sensors, while the other with the array of resistors. This was done just because of damages that occurred in the sensors embedded with the resistors, so, to save fabrication time, instead than refabricating the full system, a prosthesis with sensors only was fabricated for the final test.

The test successfully validated the full functionality of the prosthesis.

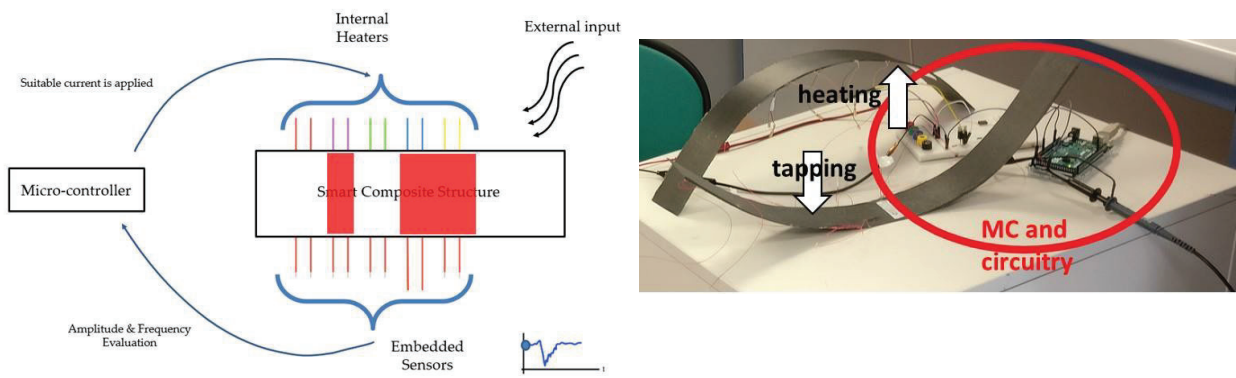


Figure 16: Left. Schematic of the functionality/testing. Right. The two prosthesis tested simultaneously in which the circuitry and MC can also be viewed.

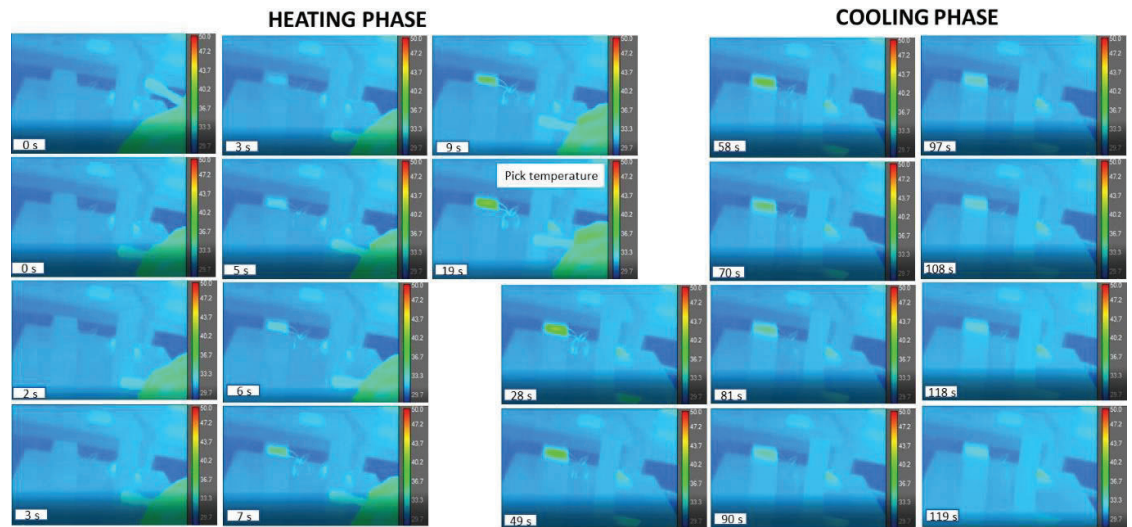


Figure 17: Snap-shots from the demonstration of the prosthesis multi-functionality automatically activated.

References

Arnaldo Casalotti and Giulia Lanzara, Matthew Snyder, Towards highly reconfigurable carbon fiber composite, Proceedings of the ASME 2019 Conference on Smart Materials, Adaptive Structures and Intelligent Systems, September 9–11, 2019, Louisville, KY

Casalotti A., Chinnam K.C., Lanzara G., SELF-ADAPTABLE CARBON FIBER COMPOSITE, Proceedings of the ASME 2018 Conference on Smart Materials, Adaptive Structures and Intelligent Systems, September 10–12, 2018, San Antonio, TX, USA, (SMASIS2018-8058);

Casalotti A., Lanzara G., Self-Actuated Morphing Composite with Tuneable Frequency and Damping, 6th International Conference on Integrity - Reliability - Failure (IRF), 22-26 July 2018;

Casalotti A., Lanzara G., Shape-Changing Carbon Fiber Composite with Tunable Frequency and Damping, The 18th International Conference on Experimental Mechanics (ICEM 2018) Proceedings 2018, 2(8) 465; 1-5 July 2018

Giulia Lanzara, Arnaldo Casalotti, «Mechanical Memory» in carbon fiber composites, Advanced Materials, In preparation

Arnaldo Casalotti, Giulia Lanzara, Enhanced damping and rigidity tuning in bi-stable composites, Materials and Design, In preparation

3 LOCAL MORPHING

Local morphing is studied to provide quasi-static as well as dynamic morphing. Figure 1 shows the logic of the investigated idea. A composite structure can be partitioned in an array of elements each of which can independently morph with respect to the other. Each element, which can be envisioned as a pixel, can be built with a size that is convenient for the specific application allowing local morphing in specific structural areas. Moreover, in this project we imagined that each skin pixel is further pixelled with a smart coating to provide dynamic morphing, at a smaller scale, which can be again tuned in frequency and space extension depending on the application. Within the project this idea was investigated to some extent. Focus was given to experimentally demonstrate local morphing with larger-in-scale pixels to allow morphing at ultra-low-frequencies, then dynamic morphing was investigated theoretically and experimentally via a unique piezoelectric pixeling as well as through the exploitation of parametric resonance. The work performed on this task is reported in the following subsections.

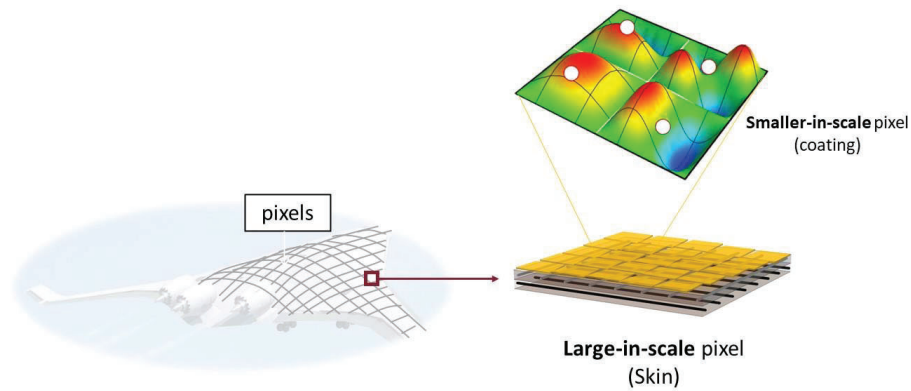


Figure 1: Concept of the proposed multi-scale local morphing.

2.1 LOCAL MORPHING OF COMPOSITE SKIN VIA SELF-ACTIVATED MUSCULAR COMPOSITE IN PIXELS

Preliminary tests are conducted on a morphing surface made of an assembly of the previously implemented muscular composite. A number of muscular composite samples are connected in series to form a linear array of pixels. The interconnection between each panel is performed via a soft inter-sample bond whose role was just that of giving structural continuity to the composite skin without however constraining the pixels degrees of freedom. Each pixel was fixed on the substrate just along two parallel sides. This was chosen to let the panel to freely bend and flatten. The described optimal configuration is the result of a series of tests and different boundary conditions that were investigated within the project. When the samples were rigidly interconnected, their morphing capability was altered with respect to the original morphing design.

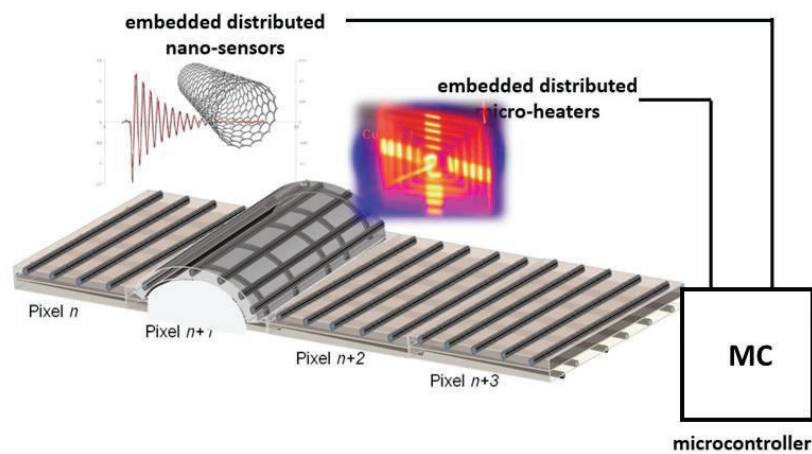


Figure 1: Schematic of the composite with autonomous local morphing capability.

A soft or rigid interconnection between the panels constraining just the two extreme edges of the series of samples, was again compromising the morphing result. It is obvious that the solution and boundary conditions chosen in this study represent just one of the many design solutions that could be implemented with further studies. In this project we were only interested in proving the concept that local morphing of the composite skin could be achieved automatically by pixeling the material and conceiving each of the formed pixels as an independent muscular composite panel. The automatic response was achieved with the same design logic presented for the prosthetic leg. Each composite pixel comprised embedded internal heaters (the discussed copper coil), embedded PVDF sensors for dynamic morphing as well as carbon nanotube nanocomposite samples for strain monitoring. All the devices were interconnected with a circuitry and a microcontroller that was programmed following the same block-diagram (deformation algorithm) discussed before. The internal sensors were used to monitor static or dynamic strain in each pixel, and the heaters were used to change rigidity and morph each single pixel. It is worth noting that the embedded PVDF sensors lost their sensing capability when the system was brought to very high temperatures (150°C). The system should have been tested at lower temperatures. So the demonstration below was performed by monitoring strain with the embedded nanocomposite strain sensors.

Such composites are assembled together thus forming a global surface that can change shape locally, in the sense that each pixel composing the skin can be activated independently from the others.

A multitude of tests were performed to proof the overall system functionality.

TEST 1. INDEPENDENT PIXELS ACTUATION AND MORPHING

The capability of the skin to change shape is tested by activating its pixels one at the time, the shape is monitored by a video camera, while the temperature induced in the material is measured over time with a thermal camera.

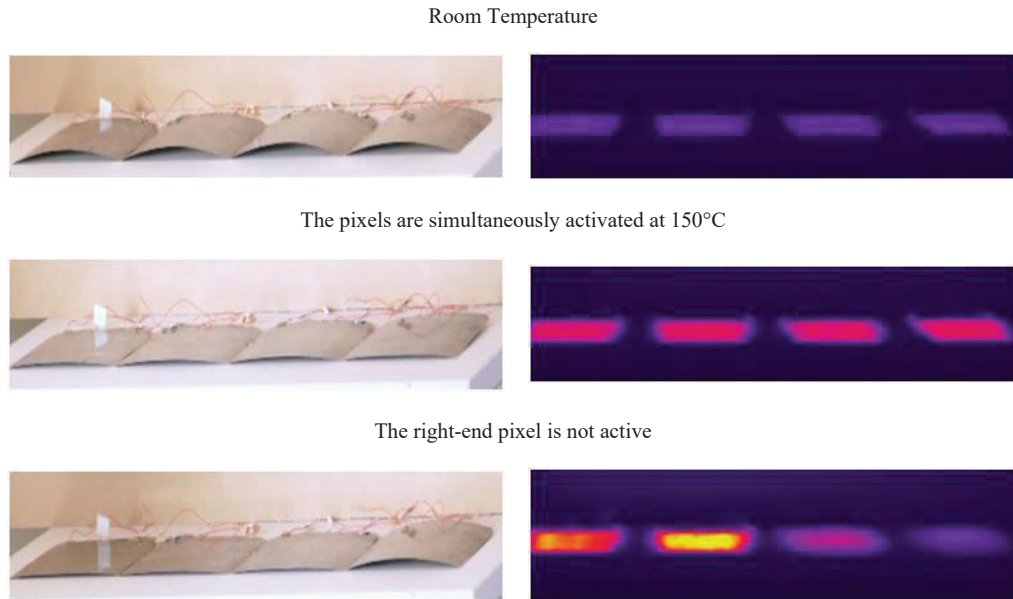


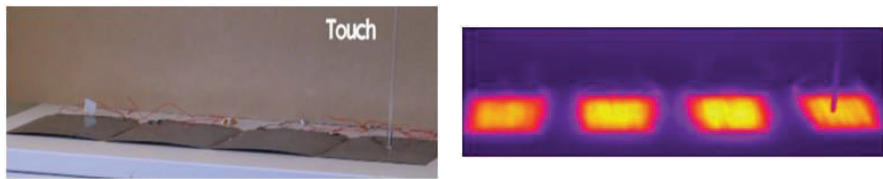
Figure 2: Snap-shots from the first test.

In particular, tests were performed with a smart skin composed of 4 independent pixels, being each pixel a muscular composite. At room temperature all pixels show a curved shape. Then all pixels are activated by heating them up to 150°C which allowed them to morph together to a flat configuration. Finally, just one of the pixels was let to bend going back to its initial local configuration.

TEST 2. INDEPENDENT PIXEL MORPHING BY RESPONDING TO A DETECTED STRAIN

This second test was planned to first actuate all pixels at the same time to flatten them, then one of the pixels was touched with an external bar, that pixel was able to detect strain with its embedded nanocomposite sensor and consequently was de-activated to go back to its initial curved reference configuration.

The right-end pixel detects an external input



All pixels are activated at 150°C

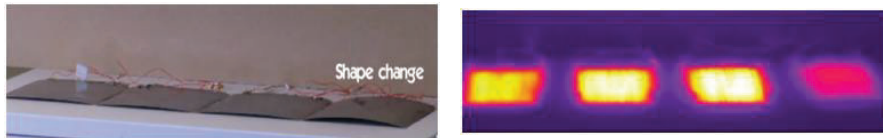


Figure 3: Snap-shots from the first test.

TEST 3. PIXELS MORPHING TO CONTROL OF A GLASS SPHERE

This morphing skin can also be used to interact with airflow (e.g. to modify its interaction with the structure for drag reduction) or, more in general, with bodies moving along the surface.

To show the feasibility, a simple example was implemented.

The interaction is simulated by governing the motion of a glass bead placed on top of the surface. As depicted in the snap-shots in Figure 4, the pixel locating the bead is turned off and starts gaining curvature. When a threshold curvature is reached the bead starts moving and it oscillates in-between two pixels until it stops right in the middle.

This is just a Pictionary example to prove the wide potential of the morphing skin.

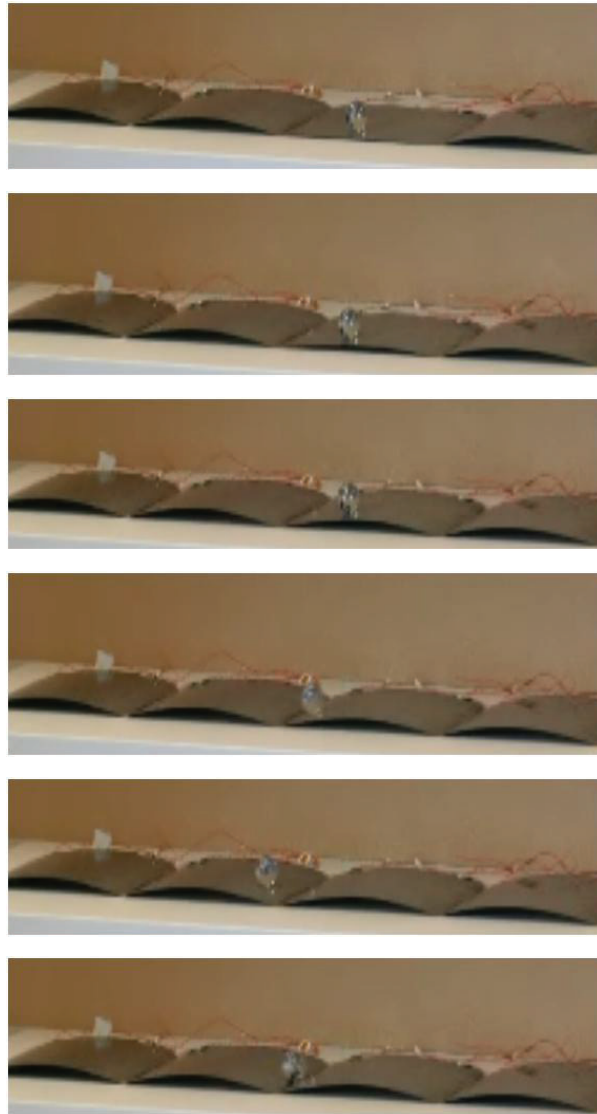


Figure 4: Snap-shots from the bead test.

3.2 DYNAMIC MORPHING VIA PIEZOELECTRIC ACTUATION

Introduction and state of the art. Dynamic morphing of surfaces is one of the rapidly emerging capabilities needed in various application fields ranging from aerospace to biomedical engineering. It is a kind of new paradigm aiming to realize self-actuating, active, smart structures such as skins, wings, biosurfaces that undergo shape changes when subject to external stimuli (pressure, velocity, temperature change, light, etc.). In aerospace, for example, the concept of morphing wing is likely to enhance performance and efficiency over a wider range of flight conditions in future aerostructures [1]. In the past, the goal of structural morphing capability was obtained at the expenses of high costs and energy consumption. Today, the challenge consists in the development of morphing structures and mechanisms capable of integrating efficiency, reliability, low energy consumptions and low complexity levels [13]. This requires innovative ways of engineering microsensors, actuation mechanisms and energy sources coordinated by microprocessors according to high-fidelity, robust models and control theories [14, 15]. Recent state-of-the-art-reviews about morphing of aircraft structures can be found in [16, 17, 18] where concepts, strategies and models are systematically illustrated.

Applications of the morphing concept in aeronautical engineering are devoted to flutter suppression achieved through targeted increase of the flutter speed in aircraft wings. To this end, a morphing strategy based on the axially moving rates of a cantilever plate (representing an aircraft wing) was proposed in [2] to suppress flutter. The upper limit of the morphing rate was determined by investigating the wing transient stability during the morphing motion.

To perform shape morphing of aerodynamic surfaces, several techniques can be adopted, ranging from miniaturized electro-actuated servomechanisms [3] to active smart materials [4], which change their shape when a current, or a magnetic field, passes through them [5], or to piezoelectric patches that can be attached onto the aerodynamic surfaces to prescribe strains which can induce curvature changes [6]. The latter is a technology that nowadays is becoming feasible due to the possibility of manufacturing piezoelectric parts of incredibly small size (i.e., in the range of microns) making use of new piezoelectric polymers [5, 7]. Piezoelectric fibers can be assembled into composite materials which may undergo large strains when a voltage (both static or dynamic) is applied to them, and different motions can be achieved by the piezoelectric action, such as elongation and bending [8, 9]. The latter relies on a characteristic also known as flexoelectricity. Applications of piezoelectricity and flexoelectricity are numerous, although these can be summarized into three main categories: energy harvesting, sensing, and actuation. A recent work [10] proposed a continuum-based parametric model of elastic beams including the two above mentioned effects to study the coupled electrodynamic response of bimorph cantilevers for energy harvesting and actuation. Moreover, it was shown that the effective electromechanical coupling of a particular piezoelectric material (i.e., the barium titanate) is significantly enhanced for nanoscale thickness levels. On the other hand, by applying a voltage to piezoelectric actuators attached to a structural element it is possible to induce large-amplitude nonlinear oscillations also in the case in which the actuators exhibit linear piezoelectricity with constitutive equations including electromechanical coupling [11]. Furthermore, in [12] it was also shown that the mechanical characteristics of distributed piezoelectric actuators in cylindrical shells can be optimized to improve the performance of multi-mode vibration control.

Shape morphing can be achieved at the macroscale for global shape changes (see Fig. 1a) or at much smaller scales for local shape changes thanks to the idea of pixelling the material in 2D arrays of subelements (see Fig. 1b). While the former is easier to achieve, the second (dynamic local surface morphing) has more complexity in terms of

assembling, manufacturing, electronic and operational costs.

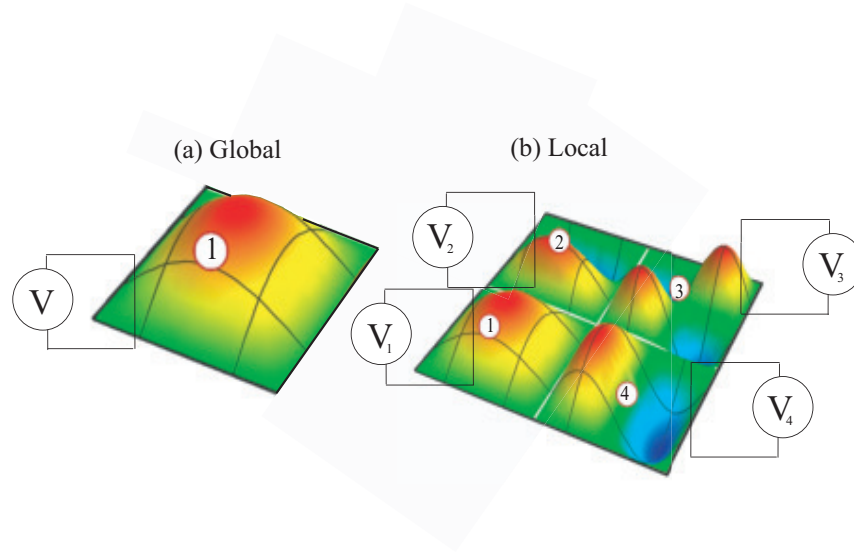


Figure 1: The concept of dynamic morphing according to two architectures: (a) global morphing and (b) local morphing in a pixelled membrane with embedded piezoelectric wires subject to a voltage made of a DC and an AC pulsating component.

Nowadays, piezoelectric materials represent the best candidates to obtain dynamic structural morphing in terms of reliability, control versatility (shapes and actuation frequencies), relative low energy consumptions and robustness. In 1969, Kawai found very high piezo activity in the polarized fluoropolymer, polyvinylidene fluoride (PVDF) [21]. Over the last few years, new copolymers of PVDF have been developed. These innovative materials have expanded the applications of piezoelectric polymer sensors. The continuous discovery and enhancement of the piezoelectric capabilities in classic as well as new materials [19] led many researchers to develop models for accurate simulations of the macro scale piezoelectric multiphysics response [20].

In this project we propose to exploit the parametric resonance, also known as dynamic buckling, as an effective nonlinear actuation phenomenon in PVDF-based materials to enhance the morphing capabilities in terms of control feasibility and variety of morphing shapes. For thin panel-like structures, an internal pulsating axial strain induced by the voltage applied to the PVDF wires or material under suitable conditions is such to induce a parametric resonance activating an out-of-plane morphing on selective mode shapes.

The first study reported in [22] dealt with the possibility to perform dynamic shape morphing exploiting the parametric resonances of a pre-tensioned membrane actuated by the in-plane strain field induced by a pulsating voltage (see Fig. 1). The PDE representing the mechanical problem was discretized by means of the Galerkin method and solved asymptotically via the method of multiple scales [23] thus obtaining the actuation gains in terms of the so-called Strutt diagrams.

On the other hand, the dynamic morphing concept via parametric resonance was fully explored and validated numerically taking into account a full PVDF panel polarized in the thickness direction as in Fig. 2a. The electrodes, where the voltage is assigned, coincide with the top and bottom surfaces. A suitable mechanism for tuning the stiffness and resonance frequencies of the piezoelectric beam was employed, namely, the prestress effected by a DC voltage gradient across the beam thickness. The developed analytical formulation aimed to evaluate the reliability of using PVDF panels for dynamic shape morphing arising out of parametric resonances. This task was explored investigating the

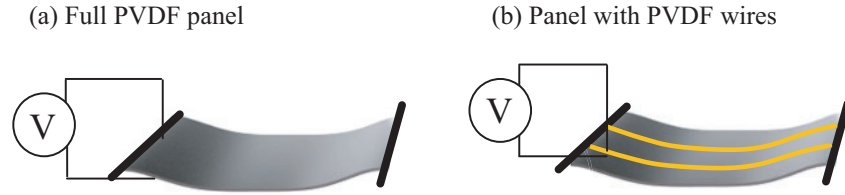


Figure 2: The concept of dynamic morphing: (a) full PVDF panel and (b) thin panel with embedded PVDF wires subject to a voltage made of a DC and a pulsating AC component.

possibility of actuating different mode shapes with meaningful amplitudes within feasible ranges of voltage and in a robust manner with respect to the pulsating frequency.

Differently from the case of a full PVDF panel, the case of a panel actuated by time-varying voltages in the embedded PVDF wires was thoroughly investigated in [24]. Besides the parametric resonance, a rich variety of other nonlinear resonance scenarios acting in the initially curved shapes were further explored for dynamic morphing including primary, superharmonic, subharmonic resonances. For panels with different geometric features ranging from ultrathin to thick geometries, corresponding to nonlinear softening, hardening and quasi-linear behaviors, parametric studies showed the effect of the cross-sectional size of the PVDF wires, the piezoelectric coupling coefficient, and the structural damping.

3.2.1 Dynamic morphing of membranes with embedded PVDF wires

Dynamic buckling of membranes achieved via parametric resonances is not a thoroughly investigated area of research. A geometrically fully nonlinear model of prestressed micromembranes was employed in [25] to explore the nonlinear range in which the high sensitivity of the nonlinear frequency with respect to the applied pressure was proposed for pressure sensing purposes. Nonlinearly viscoelastic cylindrical/spherical shells under pulsating pressures were studied in [26] via the method of multiple scales (MMS) to capture the dynamic buckling regions for breathing motions. In the context of dynamic morphing of membranes carried out within this project, the nonlinear partial differential equation of motion describing the membrane out-of-plane motion was discretized according to the Faedo-Galerkin procedure, and a fifth-order asymptotic expansion of the solution for the m th mode was performed. To prove the accuracy of the analytical results, a path following of the parametric response of the morphing membrane was performed. The Strutt diagrams with the actuation regions, where the parametric resonance morphing can be realized, were obtained for the lowest few modes. A study of the nonlinear morphing response was further carried out via time integration of the nonlinear equation of motion to demonstrate that dynamic morphing of membranes can be suitably driven and optimized for innovative designs of active surfaces in aerospace engineering applications.

Nonlinear model and equations of motion. A nonlinear model of pretensioned membranes subject to parametric excitation was employed. The source of the parametric excitation is provided by time-varying strains acting along a selected number of orthogonal active PVDF wires embedded in the membrane. As depicted in Fig.3, the reference square membrane configuration is assumed to be planar and coinciding with the equilibrium under the effect of a uniform prestress N_0 . Such prestress may be induced in the membrane by the DC component of the voltage applied to the wires. The chosen

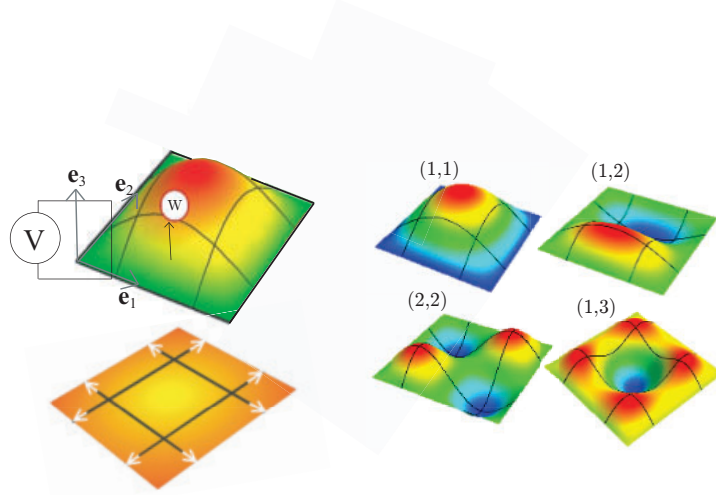


Figure 3: Membrane geometry with the embedded PVDF wires (left) and the lowest four modes (right).

fixed frame $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is such that $\mathbf{e}_3$ is normal to the reference configuration and the membrane midplane lies in the $\{\mathbf{e}_1, \mathbf{e}_2\}$ -plane. The position of each material point of the membrane is thus described by the space coordinates x_1 and x_2 along $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively, while $w(x_1, x_2, t)$ represents the transverse displacement, and t denotes time. In the case study here investigated, the thickness is $h = 0.2$ mm and the edge length is $l = 2$ cm. The membrane is assumed to be made of PEEK with Young modulus, Poisson ratio, and mass density set to $E = 3.6$ GPa, $\nu = 0.4$, and $\rho = 1320$ kg/m³, respectively.

The analytical model is cast in nondimensional form by adopting the membrane edge length l and the frequency $\omega_c = \sqrt{E/[(1-\nu^2)\rho l^2]}$ as characteristic length and frequency, respectively, for the nondimensionalization. Therefore, the space and time nondimensional coordinates become $\bar{x}_i = x_i/l$ ($i = 1, 2$) and $\bar{t} = \omega_c t$, respectively, while $\bar{N}_0 = N_0/[\rho h \omega_c^2 l^2]$ and $\alpha = E/[(1-\nu^2)\rho \omega_c^2 l^2]$ are the nondimensional prestress and elastic stiffness, respectively.

The parametric excitation is provided by the wires forces N_1 and N_2 in directions $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively, are given by

$$\bar{N}_1 = \alpha e_0 \cos \bar{\Omega} t \sum_{i=1}^{n_s} \delta(\bar{x}_2 - \bar{x}_{2,i}), \quad \bar{N}_2 = \alpha e_0 \cos \bar{\Omega} t \sum_{j=1}^{n_s} \delta(\bar{x}_1 - \bar{x}_{1,j}), \quad (1)$$

where $e_0 \cos \bar{\Omega} t$ is the pulsating strain in the piezoelectric wires due to the AC voltage component, n_s is the number of active wires, $\bar{x}_{1,j}$ and $\bar{x}_{2,i}$ indicate the position of the wires across the membrane. The number of wires and their position are chosen so as to provide optimal actuation for both symmetric and skew-symmetric membrane modes. In particular, here it is $n_s = 2$ and $\bar{x}_{1,1} = 1/4$, $\bar{x}_{1,2} = 3/4$, $\bar{x}_{2,1} = 1/4$, $\bar{x}_{2,2} = 3/4$, respectively.

By assuming a linear viscous force and enforcing the balance of linear momentum, the nondimensional equation of motion can be written as:

$$\begin{aligned} \frac{\partial^2 \bar{w}}{\partial \bar{t}^2} + \zeta \frac{\partial \bar{w}}{\partial \bar{t}} - \bar{N}_0 \nabla^2 \bar{w} - \frac{\alpha}{2} \left[\frac{\partial^2 \bar{w}}{\partial \bar{x}_1^2} \int_0^1 \left(\frac{\partial \bar{w}}{\partial \bar{x}_1} \right)^2 d\bar{x}_1 + \frac{\partial^2 \bar{w}}{\partial \bar{x}_2^2} \int_0^1 \left(\frac{\partial \bar{w}}{\partial \bar{x}_2} \right)^2 d\bar{x}_2 \right] \\ + \alpha e_0 \cos \bar{\Omega} t \sum_{i=1}^{n_s} \delta(\bar{x}_2 - \bar{x}_{2,i}) \frac{\partial^2 \bar{w}}{\partial \bar{x}_1^2} + \alpha e_0 \cos \bar{\Omega} t \sum_{j=1}^{n_s} \delta(\bar{x}_1 - \bar{x}_{1,j}) \frac{\partial^2 \bar{w}}{\partial \bar{x}_2^2} = 0, \end{aligned} \quad (2)$$

together with the boundary conditions $\bar{w}(0, \bar{x}_2, \bar{t}) = \bar{w}(1, \bar{x}_2, \bar{t}) = 0$, $\bar{w}(\bar{x}_1, 0, \bar{t}) = \bar{w}(\bar{x}_1, 1, \bar{t}) = 0$, $\partial_{\bar{x}_1}^2 \bar{w}(0, \bar{x}_2, \bar{t}) = 0 = \partial_{\bar{x}_1}^2 \bar{w}(1, \bar{x}_2, \bar{t})$, $\partial_{\bar{x}_2}^2 \bar{w}(\bar{x}_1, 0, \bar{t}) = 0 = \partial_{\bar{x}_2}^2 \bar{w}(\bar{x}_1, 1, \bar{t})$, and the initial conditions. In Eq.(2), $\partial \bar{t}$ and $\partial \bar{x}_i$ ($i = 1, 2$) indicate partial differentiation with respect to the nondimensional time and space coordinates, respectively, ∇^2 is the Laplace operator, and ζ is the damping ratio.

Analytical approach. The Faedo-Galerkin method was employed to reduce the partial differential equation of motion into an ODE governing the dynamics of each mode. According to the adopted discretization, the transverse displacement was expressed as $\bar{w}(\bar{x}_1, \bar{x}_2, \bar{t}) = 2q_{mn}(\bar{t}) \sin m\pi \bar{x}_1 \sin n\pi \bar{x}_2$, where $q_{mn}(\bar{t})$ is the (mn) th generalized coordinate, thus, the single-mode projection of Eq.(2) can be written as

$$\begin{aligned} \ddot{q}_{mn} + \zeta \dot{q}_{mn} + \bar{\omega}_{mn} q_{mn} + \frac{3}{4} \alpha \pi^4 (m^4 + n^4) q_{mn}^3 \\ + \alpha \pi^2 \left[-2(m^2 + n^2) + m^2 \left(\cos \frac{n\pi}{2} + \cos \frac{3n\pi}{2} \right) \right. \\ \left. + n^2 \left(\cos \frac{m\pi}{2} + \cos \frac{3m\pi}{2} \right) \right] e_0 q_{mn} \cos \bar{\Omega} \bar{t} = 0, \end{aligned} \quad (3)$$

where the overdot indicates differentiation with respect to the nondimensional time $\bar{t}$ and $\bar{\omega}_{mn} = \pi \sqrt{(m^2 + n^2)} \bar{\omega}_0$ is the linear nondimensional circular frequency of the (m, n) th mode.

The method of multiple scales was employed to perform an asymptotic analysis of Eq.(3) yielding the approximate solutions. The analytical derivation is reported in [22].

3.2.2 Dynamic morphing via principal parametric resonances

Before investigating the parametric resonance morphing, variation of the nonlinear frequency $\bar{\omega}_{mn}^{\text{NL}}$ of each mode with the modal amplitude (i.e., the so-called backbone curve) was investigated. The backbone curves highlight a hardening characteristic since each membrane nonlinear mode exhibits a frequency which increases with the oscillation amplitude. Then the asymptotic solution was studied for the lowest nine modes and the stability ascertained by monitoring the eigenvalues of the Jacobian of the bifurcation equations.

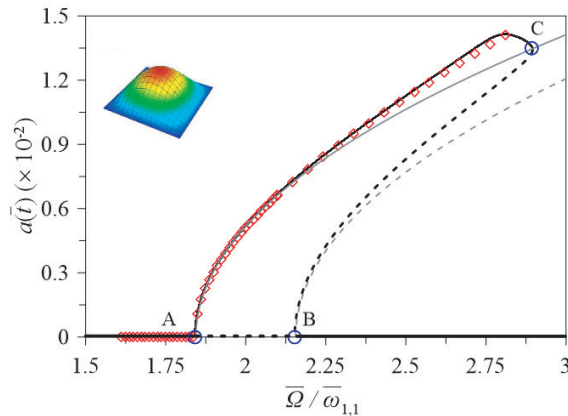


Figure 4: Frequency response curve of the lowest mode for $e_0 = 1.5 \times 10^{-4}$ and $\zeta = 0.2\%$: stable branch (solid line), unstable branch (dashed line), amplitudes of the stable parametric oscillations via numerical integration (red squares). The black (grey) lines indicate a fifth-order (third order) approximation.

The perturbation solution allows to obtain closed form expressions of the *transition curves* (i.e., bounding curves of the actuation regions) with the limitation given by the third- or fifth-order approximations. To assess the range of e_0 for which the fifth-order asymptotic expansion gives a suitable approximation of the stability curves, numerical continuation of the periodic solutions obtained from the nonlinear modal equation (3) was performed by means of a Matlab-based continuation toolbox called COCO. To delve more into the details of the numerical continuation procedure, starting from a parametric response calculated via time integration of the equation of motion of the (m, n) th mode, the path of the nonresonant dynamic response having $2\omega_{mn}$ frequency is followed to determine the period doubling (PD) bifurcations marking the onset of the parametric response possessing the frequency ω_{mn} . Once the PD bifurcation points are determined for a given value of e_0 , the continuation of the PD points is performed in a selected range of piezoelectric strain amplitudes e_0 . Figure 5 shows the actuation regions for the lowest two modes (i.e., $(m, n) = (1, 1)$ and $(m, n) = (1, 2)$) and the third-order (grey dashed-dotted-dotted lines) and fifth-order approximations (black dashed-dotted lines) are compared with the numerical continuation curves (black solid lines). The comparisons confirm that, up to moderately high values of the actuation strain, $e_0 < 1.4 \times 10^{-3}$, the predictions of the fifth-order perturbation treatment is in excellent agreement with the numerical results.

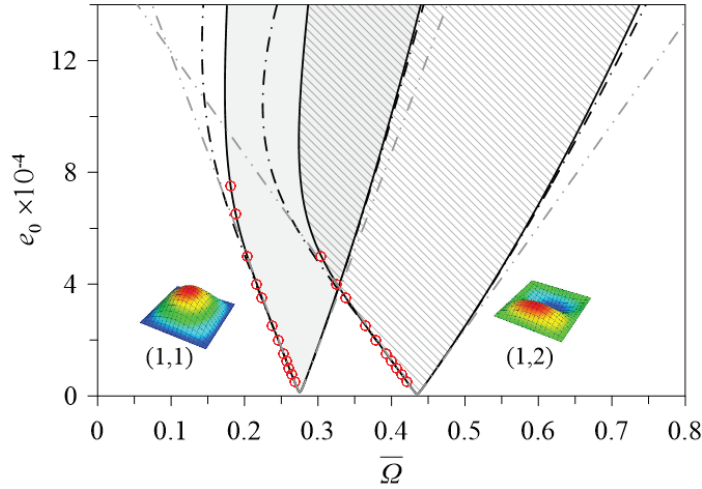


Figure 5: Actuation regions of the parametric resonance of the lowest two modes for $\zeta = 0.2\%$. Comparison between the asymptotic predictions via fifth-order (black dashed-dotted line) and third-order (grey dashed-dotted-dotted) solutions and the exact numerical continuation (black solid line).

Finally, from direct time integrations of the nonlinear equation of motion of modes $(1, 1)$ and $(1, 2)$, performed in the range of excitation frequency Ω near the parametric resonance and for selected values of the strain e_0 , the bifurcation points (i.e., where the stable response loses the $2(2\pi/\bar{\omega}_{mn})$ periodicity) were calculated and are reported in Fig. 5 as red circles to show the agreement with the stability curves obtained via numerical continuation.

Figure 6 shows the Strutt diagrams for the lowest six modes obtained via numerical continuation of the nonlinear modal equations. In particular, Fig. 6 allows to appreciate the bandwidth frequency where the actuation regions (shaded regions) overlap and multi-mode responses are expected to arise. In Fig. 9 the actuation regions are shown for the dimensional membrane for the lowest six modes where it is to be noted that the lowest activation voltage for the $(2, 2)$ -mode is 246 Volts.

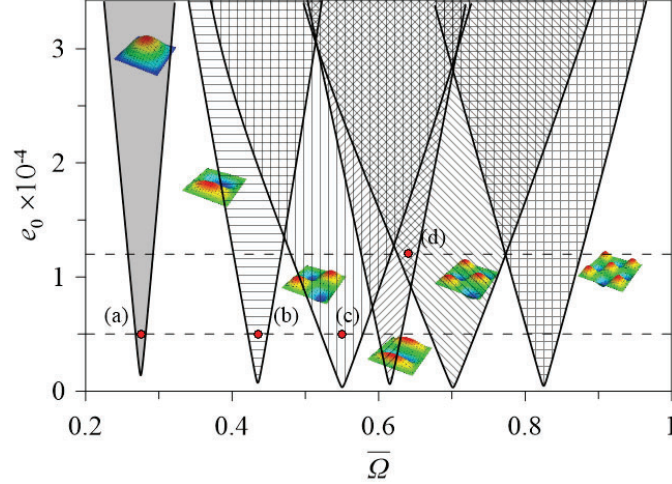


Figure 6: Nondimensional actuation regions (Strutt diagrams) for the lowest six modes.

To show the multi-mode response in the overlapped instability regions, a direct time integration of the equation of motion was performed by projecting Eq. (2) into the basis of trial functions given by the lowest nine membrane modes (i.e., all the modes investigated in the perturbation analysis, including the companion modes (2, 1), (3, 1), and (3, 2)). The results of the numerical simulations are shown in Fig. 9, where the time histories and the fast Fourier transforms (FFTs) were obtained upon varying the excitation frequency to seek parametric response involving single- or multi-mode oscillations. In particular, the membrane morphing through single-mode motion is shown in Fig. 9 (a), (b) and (c), while mixed-mode motions are shown Fig. 9(d).

3D ABAQUS simulations. A full 3D FE model was constructed in ABAQUS CAE [27]. The wires were meshed with C3D8E (piezoelectric) elements while the remaining parts with C3D8R (3D Solid) elements. The principal parametric resonance of the lowest mode was simulated in a Dynamic Implicit Step, in which the solid was subject to its own weight and the actuation force was provided by the wires. The presence of gravity supplies a suitable natural initial condition that perturbs the structure from its trivial solution. Electrical boundary conditions are imposed at the ends of each PZT wire (in the shape of a bar) in order to achieve a periodic electrical field through the length direction oscillating at a frequency nearly twice that of the first mode. The 3D FE model allows to investigate stress distributions within the solid and in particular at the interface between the wires and the plate. The maximum von Mises stress detected should not reach the ultimate tensile stress for PEEK. A critical issue could be the shear stress generated at the interface because of the combination of strain in $\mathbf{e}_3$ direction, due to out-of-plane oscillations, and in-plane strains, due to the actuation force. As expected, the elements with the highest stress concentration are the interface elements collocated across the modal peaks. The maximum von Mises stress is 11 MPa, while the maximum shear stress is 0.8 MPa, both below the limit stress.

A macro scale model of a 2×2 pixelled plate was also implemented in ABAQUS.

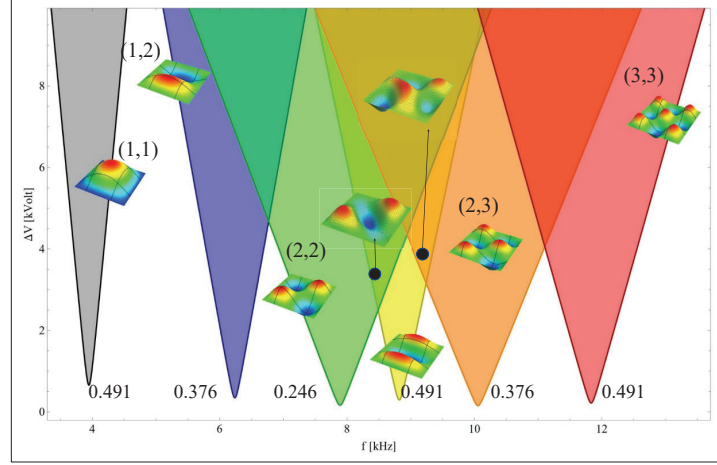


Figure 7: Dimensional actuation regions (Strutt diagrams) for the lowest six modes of a PEEK membrane with length $l = 2$ cm, thickness $h = 0.2$ mm.

Each pixel is the prestressed 3D membrane model, constrained to a rigid connection. In this way the modal properties of the membrane remain unchanged and each pixel can be actuated independently exciting different modes. The full 3D model was not further pursued for computational reasons. Indeed the number of nodes needed to mesh a 3D solid is much higher than the number of nodes used for a 2D membrane. Because for membrane elements in ABAQUS the electrical-mechanical coupling is not implemented the actuation was provided through the introduction of thermal temperature gradients along the lines representing the wires. The PEEK thermal expansion coefficient was set to: $\alpha_C = 2 \cdot 10^{-5}/C$. Different morphing shapes can be activated independently on each pixel by applying periodic temperature oscillations with suitable frequencies.

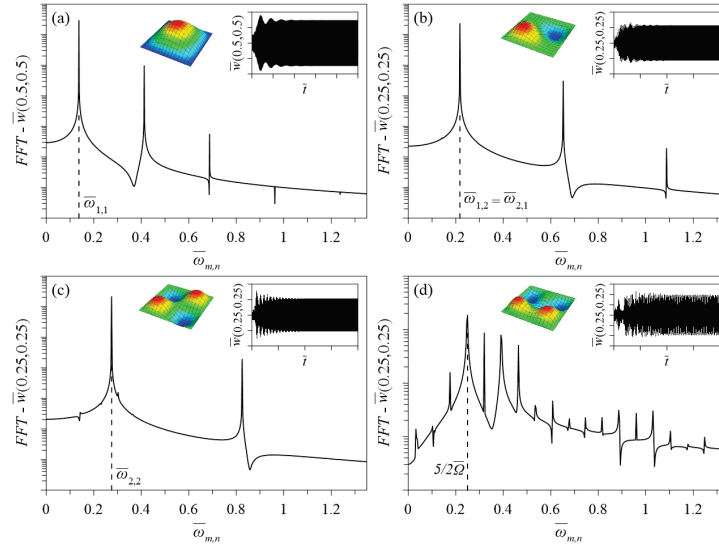


Figure 8: FFTs and time histories of the deflection at selected parametric resonance conditions corresponding to (a), (b), (c) and (d) of Fig. 6.

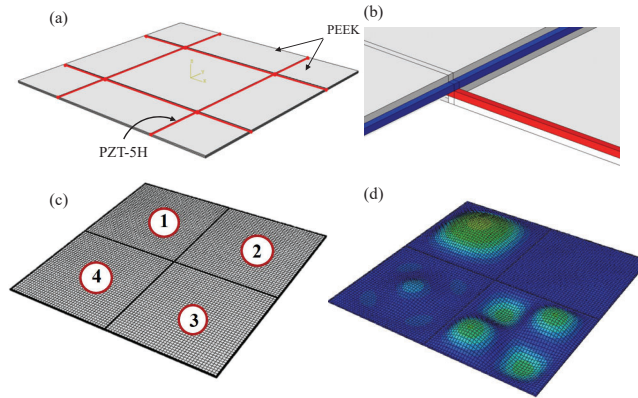


Figure 9: (a) Geometric model of the 2×2 pixelled PEEK membrane, (b) 3D zoom of the crossing between two piezoelectric wires, (c) FE mesh and (d) simulated morphing of the multi-pixelled membrane.

3.2.3 Dynamic morphing of PVDF panels

The PVDF nonlinear model. A nonlinear model of a PVDF sample was constructed making use of the nonlinear beam theory and the linear theory of 3D piezoelectricity. Although nonlinear piezoelectric effects have been documented in the literature, there is lack of sufficient experimental data to motivate a nonlinear constitutive piezo-electrical theory when moderately large strains are involved. The electric boundary conditions allow the introduction of a classical ansatz that consists in assuming the voltage constant across the beam span. The voltage is assumed to vary in the thickness direction with a parabolic law. These assumptions uncouple the two physics. Thus the electric problem can be solved in succession after the mechanical problem is tackled. The nonlinear piezoelectric beam model is grounded on the Euler-Bernoulli nonlinear beam formulation assuming moderately large displacements and rotations. The problem was specialized for clamped-clamped boundary conditions as often encountered in applications (see Fig. 10). The 3D linear constitutive law, in the reference configuration described by vector

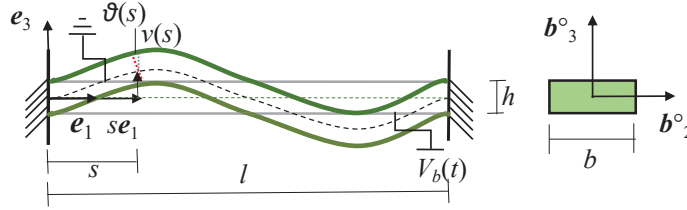


Figure 10: PVDF sample in the reference and deformed configurations with the cross-section deflection $v(s)$ and rotation $\theta(s)$.

$\mathbf{x} = se_1 + ye_2 + ze_3$ in the fixed frame (O, e_1, e_2, e_3) , can be expressed according to the Voigt notation in the following form:

$$\boldsymbol{\sigma} = \mathbf{L}(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{\text{PZT}}), \quad \boldsymbol{\epsilon}^{\text{PZT}} := \mathbf{D}\mathbf{h} \quad (4)$$

where $\boldsymbol{\sigma} = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{13}, \sigma_{23}, \sigma_{12}]^T$ and $\boldsymbol{\epsilon} = [\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, \epsilon_{13}, \epsilon_{23}, \epsilon_{12}]^T$ denote the stress and the strain vectors, respectively, $\mathbf{L}$ (6×6) indicates the matrix of elastic constants for a general anisotropic solid, $\boldsymbol{\epsilon}^{\text{PZT}}$ is the piezoelectric induced strain, $\mathbf{h}$ (3×1) is the electric field vector, and $\mathbf{D}$ (6×3) is the piezoelectric strain coefficient matrix. This 6×3 matrix for a general piezoelectric material polarized in the e_3 direction (i.e., thickness direction) exhibits nontrivial entries D_{3i} with $i = 1, 2, 3$ indicating the normal piezoelectric strains per unit electric field, and D_{42} and D_{51} which denote the transverse piezoelectric shear strains per unit electric field. For most piezoelectric materials - including PVDF co-polymer uni-axial or bi-axial film specimens - the coefficients D_{31} and D_{32} are negative while D_{33} is positive. The shear strain coefficients are negligible and set to zero as suggested in previous works [28].

The stress-charge form of the piezoelectric constitutive law (4) is obtained as

$$\boldsymbol{\sigma} = \mathbf{L}\boldsymbol{\epsilon} - \mathbf{E}\mathbf{h}, \quad \mathbf{E} = \mathbf{L}\mathbf{D} \quad (5)$$

where the coefficients of $\mathbf{E}$ have the units of C/m^2 .

The elastic matrix for a PVDF co-polymer bi-axial film of relatively small thickness is of orthotropic type. The two stiffer directions are those contained in the (e_1, e_2) -plane and the 6 Lamé constants $(k_p, k_f, \lambda_p, \lambda_f, \mu_p, \mu_f)$ can define the constitutive relationship. The resulting stress coefficient matrix contains three nontrivial elements, namely, E_{31} ,

E_{32} , and E_{33} .

Since the electric field vector $\mathbf{h}$ can be expressed as the gradient of the voltage $\mathbf{h} = -V_{\mathbf{x}}$ (here, $V_{\mathbf{x}} := \nabla V$ denotes the gradient vector), the stress-voltage relationship is written as

$$\boldsymbol{\sigma} = \mathbf{L} \boldsymbol{\epsilon} + \mathbf{E} V_{\mathbf{x}} \quad (6)$$

The electro-mechanical coupling requires the definition of the electric displacement vector

$$\mathbf{k} = \mathbf{E}^{\top} \boldsymbol{\epsilon} + \varepsilon_0 \boldsymbol{\varepsilon}_r \mathbf{h}, \quad (7)$$

where $\boldsymbol{\varepsilon}_r$ is the (diagonal) 3×3 vacuum relative permittivity matrix and the components of $\mathbf{k}$ are given by

$$k_1 = \varepsilon_0 \varepsilon_{r1} h_1, \quad k_2 = \varepsilon_0 \varepsilon_{r2} h_2, \quad k_3 = E_{31} \epsilon_{11} + E_{32} \epsilon_{22} + E_{33} \epsilon_{33} + \varepsilon_0 \varepsilon_{r3} h_3. \quad (8)$$

The terms ε_0 and ε_{ri} ($i = 1, 2, 3$) indicate the vacuum permittivity and the relative permittivity of the material in the $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ directions, respectively. The electric displacement vector has to obey the Gauss law for electric charge conservation:

$$\int_{\mathcal{B}} (\nabla \cdot \mathbf{k}) dV = 0. \quad (9)$$

The linear multiphysics problem is governed by the 3D equations of motion which read as

$$\frac{\partial \boldsymbol{\sigma}}{\partial \mathbf{x}} - c \frac{\partial \mathbf{u}}{\partial t} + \mathbf{b} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \quad (10)$$

together with the Gauss law, where $\mathbf{b}$ is the force per unit volume, ρ is the mass per unit volume, c is the viscous damping coefficient, $\mathbf{u} = [u_1, u_2, u_3]^{\top}$ is the displacement vector. The linear strain-displacement relationships, in engineering notation, are

$$\epsilon_{ij} = \frac{1}{2} [u_{i,j} + u_{j,i}], \quad i, j = 1, 2, 3. \quad (11)$$

where the subscripts 1, 2 and 3 in the left-hand side indicate differentiation with respect to the coordinates in the directions $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, respectively.

Substituting Eqs. (11), (5), (7) and (6) into (10) and (9), the coupled problem is formulated in the unknown displacement field $\mathbf{u}(s, y, z, t)$ and the electric potential $V(s, y, z, t)$.

The planar piezoelectric panel is modelled as a 1D beam derived from the 3D constitutive law assuming the Saint-Venant ansatz for the stress and strain states. The straight panel strip has a rectangular cross-section of width b and thickness h and the cross-section positions across the beam span l , in the reference configuration, are described by the vector $\mathbf{r}^0 = s \mathbf{e}_3$ with $s \in [0, l]$ spanning a base line taken as the beam centerline. The local frame, co-rotational with the beam cross sections, in the reference configuration is denoted by $(\mathbf{b}_1^0, \mathbf{b}_2^0, \mathbf{b}_3^0)$. The axis $\mathbf{b}_1^0$ is collinear with $\mathbf{e}_1$ while $\mathbf{b}_2^0$ and $\mathbf{b}_3^0$ are collinear with the principal axes of inertia of the beam cross sections.

The panel strip is assumed to deform in the $(\mathbf{e}_1, \mathbf{e}_3)$ -plane. Hence, the displacement of the baseline is denoted by $\mathbf{u}(s) = u(s) \mathbf{e}_1 + v(s) \mathbf{e}_3$. The flexural rotation of the beam cross sections assumed as rigid about $\mathbf{b}_2^0 = \mathbf{e}_2$ is denoted by $\theta(s)$. According to the Saint-Venant ansatz, the stresses $(\sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{12})$ and, consequently, the strains $(\gamma_{12}, \gamma_{23})$ are set to zero. On the other hand, the assumed rigidity of the cross sections together with the unshearability condition leads to $\epsilon_{22} = \epsilon_{33} = \gamma_{13} = 0$. The longitudinal strain follows from the cross-section internal rigidity constraint in the form

$$\epsilon_{11}(s, z, t) = \epsilon_0(s, t) - \mu(s, t)z, \quad (12)$$

where ϵ_o is the elongation of the centerline and μ is the bending curvature. The generalized strain-displacement relationships are $\epsilon_o = \sqrt{(1 + u')^2 + v'^2} - 1$ and $\mu = \theta'$ with $\theta = \arctan[v'/(1 + u')]$ (ensuing from the unshearability of the beam) and the prime denoting differentiation with respect to s . The beam displacement field $\mathbf{u}$ has components expressed in terms of the centerline displacement and cross-section rotation as

$$u(s, z, t) = u(s, t) - z \sin \theta(s, t), \quad (13)$$

$$v(s, z, t) = v(s, t) - z[1 - \cos \theta(s, t)], \quad (14)$$

while the cross-section out-of-plane (warping) displacement is assumed to be zero in consonance with the assumed cross-sectional rigidity. The deformation gradient $\mathbf{F} = \partial \mathbf{p} / \partial \mathbf{x}$ can be evaluated differentiating the actual position vector $\mathbf{p}(s, t) = \mathbf{x} + \mathbf{u}(s, t)$ with respect to the reference position vector $\mathbf{x}$.

As far as the electric field is concerned, the voltage V is assumed to vary only in the z direction according to the assumed electric boundary conditions. We consider the problem in which the top beam surface at $z = h/2$ is connected to ground, namely, and $V(h/2) = 0$. At the bottom surface, a non-null voltage is assigned, $V(-h/2) = V_b(t)$. Moreover, the voltage is assumed to vary according to the following quadratic law across the panel thickness:

$$V(z, t) = \Phi_o(t) + \Phi_1(t)z + \frac{1}{2}\Phi_2(t)z^2. \quad (15)$$

Imposing the above stated boundary conditions yields $\Phi_1(t) = -\frac{V_b(t)}{h}$.

The nonlinear electric displacement vector is

$$\hat{\mathbf{k}} = \det(\mathbf{F})\mathbf{F}^{-1}\mathbf{k} \quad (16)$$

where $\mathbf{k}$ is given by (7) [29]. According to the Saint-Venant and electric potential ansatz illustrated above, the only non-null component is $k_3 = E_{31}\epsilon_{11} + \epsilon_o\epsilon_{r3}h_3$. Equation (16) can be written according to the formula for inverting a square matrix

$$\hat{\mathbf{k}} = \mathbf{F}^a \mathbf{k}, \quad (17)$$

where $\mathbf{F}^a$ denotes the adjoint matrix of $\mathbf{F}$ whose elements are: $F^a_{11} = \cos \theta$, $F^a_{22} = \cos \theta(1 + \epsilon_o) - z\mu + v' \sin \theta$, $F^a_{33} = \epsilon_o - \mu z \cos \theta + 1$, $F^a_{13} = \sin \theta$, $F^a_{31} = \mu z \sin \theta - v'$, while the remaining components are zero. The geometric approximations $\cos \theta \approx 1$ and $\sin \theta \approx \theta$ are introduced. The nonlinear piezoelectric displacement vector exhibits two nontrivial components, $\hat{\mathbf{k}}^\top = [\hat{k}_1, 0, \hat{k}_3]$ whose expressions are $\hat{k}_1 = \theta k_3$ and $\hat{k}_3 = (\epsilon_o - \mu z + 1)k_3$.

The Gauss law that expresses the charge balance in the region occupied by the panel reference configuration reads

$$\int_B \nabla \cdot \hat{\mathbf{k}} dV = 0 \quad (18)$$

Solving (18) yields

$$\Phi_2 = \frac{E_{31} \int_0^l (\epsilon'_o \theta - \epsilon_o \mu - \mu) ds}{\epsilon_o \epsilon_{r3} \check{l}}. \quad (19)$$

Note that in the denominator, the actual deformed beam length $\check{l} = (l + \int_0^l \epsilon_o ds)$ appears. Finally, the constant Φ_0 is evaluated though the potential boundary condition on the top or bottom surfaces.

According to the relationships described above, the piezoelectric constitutive equation (5) becomes

$$\sigma_{11}(s, z, t) = E\epsilon_{11}(s, z, t) + E_{31} \frac{\partial V}{\partial z}, \quad (20)$$

where E denotes here the Young modulus. The axial force and the bending moment can be obtained by integration of the constitutive relationship for the longitudinal stress over the cross-sectional domain D ; that is,

$$N(s, t) = \int_D \sigma_{11}(s, z, t) dA = EA\epsilon_0(s, t) + E_{31}A\Phi_1(t), \quad (21)$$

$$M(s, t) = \int_D \sigma_{11}(s, z, t) z dA = EI\mu(s, t) + E_{31}I\Phi_2(t), \quad (22)$$

where $A = b \cdot h$ is the cross-sectional area and $I = b \cdot h^3/12$ is the moment of inertia about b_2^0 . The piezoelectric stress coefficient E_{31} obtained as $E_{31} = ED_{31}$.

The equations of motion for the nonlinear panel in the local co-rotational frame can be written as [30]

$$N' + \mu/\nu M' = \rho A(\ddot{u} \cos \theta + \ddot{v} \sin \theta), \quad (23)$$

$$\mu N - (M'/\nu)' = -\rho A(\ddot{u} \sin \theta - \ddot{v} \cos \theta), \quad (24)$$

where the overdot indicates differentiation with respect to time, $\nu = 1 + \varepsilon$ is the generalized beam stretch, and the shear force has been filtered out using the angular momentum balance upon neglecting the rotary inertia term. In the computation of the inertia force per unit length, the following approximations can be adopted: $\cos \theta = (1 + u')/\nu \approx 1$ $\sin \theta = v'/\nu \approx v'$.

The elongation of the beam centerline, here and henceforth, will be denoted by ε after dropping the subscript for ease of notation. We followed the same steps of Mettler's theory for axially constrained beams. The longitudinal inertia force (i.e., the right-hand side of (23)) can be neglected for the considered boundary and loading conditions. Moreover, dropping the higher-order curvature-induced nonlinear load-bearing term, $\mu/\nu M'$, in (23) yields $N'(s) = 0$. The tension is thus constant along the beam span and it can be expressed as its average value

$$N(s, t) \approx \bar{N}(t) = \frac{EA}{2l} \int_0^l v'^2 ds + E_{31}A\Phi_1. \quad (25)$$

Expanding ε up to second order terms, the expression for the longitudinal displacement, on account of the boundary conditions (i.e., $u(0) = u(l) = 0$), becomes

$$u(s, t) = \frac{s}{2l} \int_0^l v'^2 ds - \frac{1}{2} \int_0^s v'^2 ds. \quad (26)$$

The final equation of motion, expressing the balance of linear momentum in the current transverse direction, can be written as

$$\rho A(\ddot{v} - \ddot{u}v') + 2\rho A\zeta\omega\dot{v} + (M'/\nu)' - \mu\bar{N} = f(s, t) \quad (27)$$

where $\bar{N}$ and u are given by (26), ζ is the damping ratio, ω is the circular frequency of one of the bending modes. Moreover, the stretch and bending curvature are given by

$$\nu = \sqrt{(1 + u')^2 + (v')^2}, \quad \mu = [v''(1 + u') - u''v']/\nu^2. \quad (28)$$

The equation of motion can be nondimensionalized introducing the following dimensionless variables: $\tilde{s} = s/l$, $\tilde{z} = z/h$, $\tilde{v} = v/l$, $\tilde{V} = V/V_c$, $\tilde{t} = \omega_0 t$, $\tilde{\mu} = \mu l$ with $\omega_0 = \sqrt{EI/(\rho Al^4)}$ and V_c is a suitable characteristic voltage. The nondimensional equation of motion can be obtained substituting the nondimensional variables in Eq. (27)

and multiplying each term by $l^3/(EI)$. Thus, neglecting the axial inertia contribution, we obtain

$$\ddot{v} + 2\tilde{\omega}\zeta \dot{v} + (\tilde{M}'/\nu)' - \alpha\tilde{\mu}\tilde{N} = \tilde{f} \quad (29)$$

where $\alpha = Al^2/I$, $\tilde{\omega} = \omega/\omega_0$, and $\tilde{f}$ is the nondimensional force per unit length. Here the prime and overdot indicate differentiation with respect to the nondimensional space and time variables, respectively. Substituting the constitutive expressions for the tension and bending moment yields the following equation:

$$\ddot{v} + 2\tilde{\omega}\zeta \dot{v} + \{[\tilde{\mu}' + (\tilde{E}_{31}\frac{l}{h}\tilde{\Phi}_2)']/\nu\}' - \alpha\tilde{\mu}[\frac{1}{2} \int_0^1 \tilde{v}'^2 d\tilde{s} - \tilde{E}_{31}V_b(t)] = \tilde{f} \quad (30)$$

The term $(\alpha\tilde{E}_{31}V_b(t))\tilde{\mu}$ expresses the coupling between the piezoelectrically induced tension $\tilde{N}^{\text{PZT}} = \alpha\tilde{E}_{31}V_b(t)$ and the flexural curvature and provides the parametric resonance excitation mechanism once we set $V_b(t) = V_{\text{DC}} + V_{\text{AC}} \cos \Omega t$.

Morphing activation regions. Equation (30) is solved asymptotically via the method of multiple scales directly applied to the PDE and boundary conditions assuming $\Omega \approx 2\omega_j$ for the parametric resonance/dynamic buckling of the j th mode. The details to achieve the closed form solutions are reported in [31]. The frequency response curves (FRC) up to third order are expressed as

$$a_j = \pm \sqrt{2} \sqrt{-\frac{2(\Omega\omega_j - 2\omega_j^2)}{\Gamma_j} \pm \frac{\sqrt{\Gamma_j^2(\alpha^2\tilde{E}_{31}^2 V_{\text{AC}}^2 \kappa_j^2 - 16\zeta^2 \omega_j^4)}}{\Gamma_j^2}} \quad (31)$$

where ω_j , Γ_j and κ_j are the prestressed linear resonance frequency, the effective nonlinearity coefficient and the effective parametric resonance coefficient for the j th mode, respectively.

The transition curves bounding the regions of parametric resonances of the various modes in the plane (Ω, V_{AC}) can be found solving (31) with respect to V_{AC} and setting the amplitude a_j to zero. The resulting solutions up to third order are:

$$V_{\text{AC}} = \pm \frac{2\omega_j \sqrt{(\Omega - 2\omega_j)^2 + 4\zeta^2 \omega_j^2}}{\alpha\tilde{E}_{31}\kappa_j}. \quad (32)$$

The novelty of our approach is the higher order direct asymptotic treatment which has the advantage of expressing the parametric resonance motion at larger amplitude thanks to the higher order expansion of the frequency response function obtained in algebraic form as $f(\Omega, a_j) = 0$. This equation can be solved numerically using a pseudo-arclength scheme based on the Newton-Raphson algorithm whereby the forcing frequency Ω is employed as continuation parameter. The initial guess necessary to perform the continuation is provided by the third order solution given by Eq. (31) since the third and fifth order solutions are very close for small oscillation amplitudes. Finally, the transition curves to within fifth order are obtained letting $a_j = 0$ in $f(\Omega, a_j) = 0$ and solving with respect to V_{AC} .

The actuation regions in the plane pulsating frequency-voltage amplitude (i.e., Ω - V_{AC}) are bounded by the transition curves. The DC voltage component V_{DC} regulates the level of piezoelectric tensile stress and, consequently, the frequencies. The damping level, among other things, affects the pulsating voltage amplitude V_{AC} for which the parametric resonance/dynamic buckling is activated.

The following PVDF physical parameters were considered [19]: $b = 2$ mm, $l = 20$ mm, $h = 30$ μm , $E_{31} = -4.02 \times 10^{-2}$ C/m², $E = 3.011$ GPa. The transition curves for

the parametric resonances of the lowest three modes are shown in Fig. 11 for 3 different damping values and a DC voltage $V_{DC} = 5$ V. The prestressed resonance frequencies for the lowest 6 modes are 113.89 Hz, 295.56 Hz, 563.20 Hz, 918.27 Hz, 1361.56 Hz and 1892.67 Hz. The horizontal gray lines denote the threshold voltage, $V_{AC}=V_{DC}$, past which static buckling occurs and dynamic morphing cannot be achieved. Increasing the damping ratio, the parametric resonance regions shift up towards higher, yet feasible, voltages. The same trend occurs when higher modes are considered since their activation requires more energy. The trends of the critical voltages (i.e., the activation voltage V_{AC} for $\Omega = 2\omega_j$) for the lowest six modes versus the damping ratio are depicted in Fig. 12. The gray lines in Fig. 12 denote the damping ratios assumed in the computation of the activation regions in Fig. 11. The intersection points with the colored solid lines indicate the activation voltages associated to each mode when $\Omega = 2\omega_j$. The damping ratio plays an important role for selecting a suitable piezoelectric material for dynamic actuation towards morphing. In fact, the resonance regions of the higher modes are severely influenced by the damping ratio and thus the actual activation can be challenging. Of course, this is not the only constraint on such choice since it is also important that the selected material is sufficiently flexible.

The prestressing DC voltage V_{DC} is another important tuning parameter because: (i) it shifts the activation frequency towards higher frequencies due to the largely tunable beam stiffening effect caused by the piezoelastic axial stress; (ii) it limits the range of the pulsating voltage V_{AC} for which dynamic morphing can be achieved (see the gray horizontal lines in Fig. 11). Figure 13 shows the morphing activation regions for the lowest three modes with $\zeta = 0.3$ % and $V_{DC} = (0, 2.5$ V). Increasing the DC voltage V_{DC} , the activation regions are shifted up according to the increment of the linear frequencies ω_j associated to the stiffening effect [31]

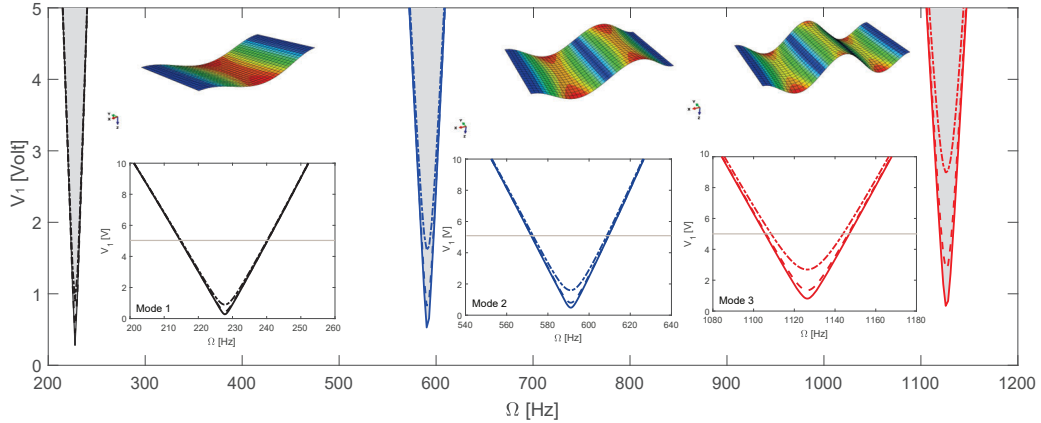


Figure 11: Activation regions for the lowest three modes and damping ratio set to 0.3 % (solid lines), 0.5 % (dashed lines), and 1 % (dashed-dotted lines) when the DC voltage is $V_{DC} = 5$ V; the horizontal gray lines denote the threshold voltage $V_{AC} = V_{DC}$.

Frequency response curves of the morphing response. The FRCs for the lowest mode of the PVDF thin panel up to the 5th and 3rd order were evaluated for the geometrical and physical parameters used in the previous section setting the damping ratio to $\zeta = 0.3$ %. The DC and AC voltages were set to $V_{DC} = V_{AC} = 50$ V. Note that the 5th order solution is necessary to correctly evaluate the actuation amplitude in the large amplitude range. Moreover, the solution indicates a frequency range between 500 Hz and 600 Hz (large frequency bandwidth) in which morphing is robust and can be

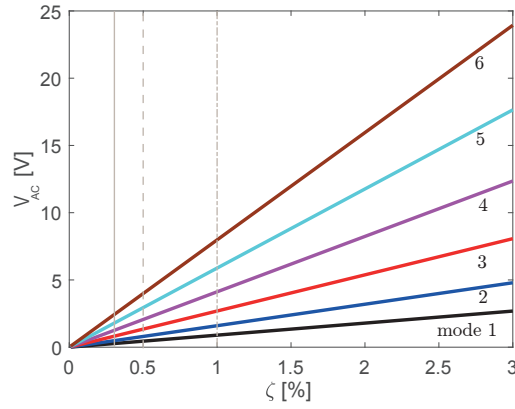


Figure 12: Activation voltage vs. damping ratio for the lowest six modes when $V_{DC} = 5V$. The gray vertical lines denote the damping ratios considered for the activation regions in Fig. 11.

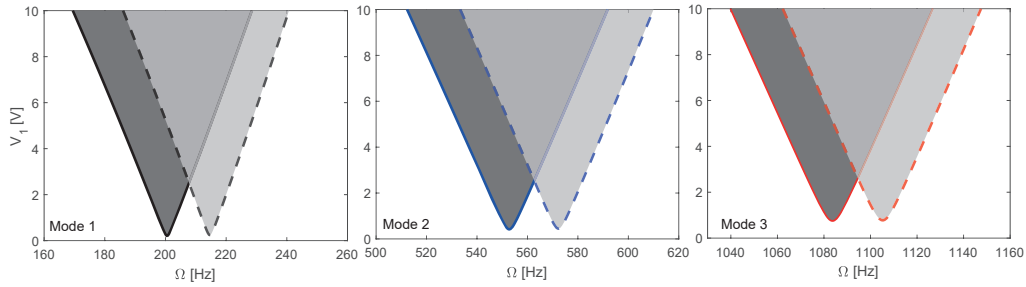


Figure 13: Effect of the DC voltage on the activation regions for the lowest three modes: $V_{DC} = 0$ V (solid lines) and $V_{DC} = 2.5$ V (dashed lines).

activated achieving maximum amplification around a mean value of about 60 mm.

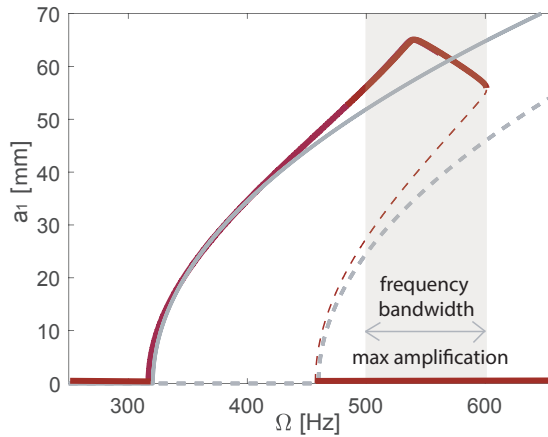


Figure 14: FRC for the lowest mode assuming $\zeta = 0.3$ % and $V_{DC} = V_{AC} = 50$ V where the magenta solid and dashed lines are the stable and unstable branches up to 5th order, respectively, and the gray solid and dashed lines indicate the stable and unstable branches up to 3th order, respectively; the shaded area denotes a frequency range in which morphing can be actuated with maximum amplification.

The morphing capabilities can be appreciated plotting the analytical solution which is given up to third order by

$$v_j(s, t) = a_j \phi_j(s) \cos(\omega_{Nj} t) + a_j \alpha \tilde{E}_{31} V_1 (q_j(s) + r_j(s)) \cos[(\Omega - \omega_{Nj}) t] \\ + a_j^3 \psi_j(s) \cos(\omega_{Nj} t) + a_j^3 \chi_j(s) \cos(3\omega_{Nj} t), \quad (33)$$

where the nonlinear oscillation frequency is expressed as the sum of the effective non-linearity coefficient multiplied by the squared amplitude and a linear frequency shift contribution [32]

$$\omega_{Nj} = \omega_j - \frac{a_j^2 \Gamma_j}{8\omega_j} - \frac{\alpha \tilde{E}_{31} \kappa_j V_{AC}}{4\omega_j}, \quad (34)$$

where the amplitude a_j can be substituted using the solution provided by Eq. (31). The functions χ_j and ψ_j denote nonlinear space functions correcting the j th mode shape and whose expressions are reported in [31]. Figure 15 shows the morphing shapes of the PVDF thin panel when subject to dynamic buckling ($\Omega = 2\omega_j$) of the lowest 4 modes (i.e. $j = 1, \dots, 4$) at $t = 0$ and for increasing values of the pulsating voltage V_{AC} with a step of 10 V. The lowest modes of the thin panel can be activated with significant amplitudes and relatively low voltage gradients. The amplitude and frequency content of the induced motion can be highly modulated by means of the AC voltage V_{AC} and frequency Ω according to Eqs. (31) and (33). Snapshots of the lowest beam morphing motion during an actuation cycle assuming $V_{DC} = 50$ V, $V_{AC} = 10$ V and $V_{DC} = V_{AC} = 50$ V are shown in Figs. 15(a) and (b), respectively. The level of the DC and AC voltages can be employed to obtain a flatter morphing shape at large oscillation amplitudes as shown in Fig. 15(b).

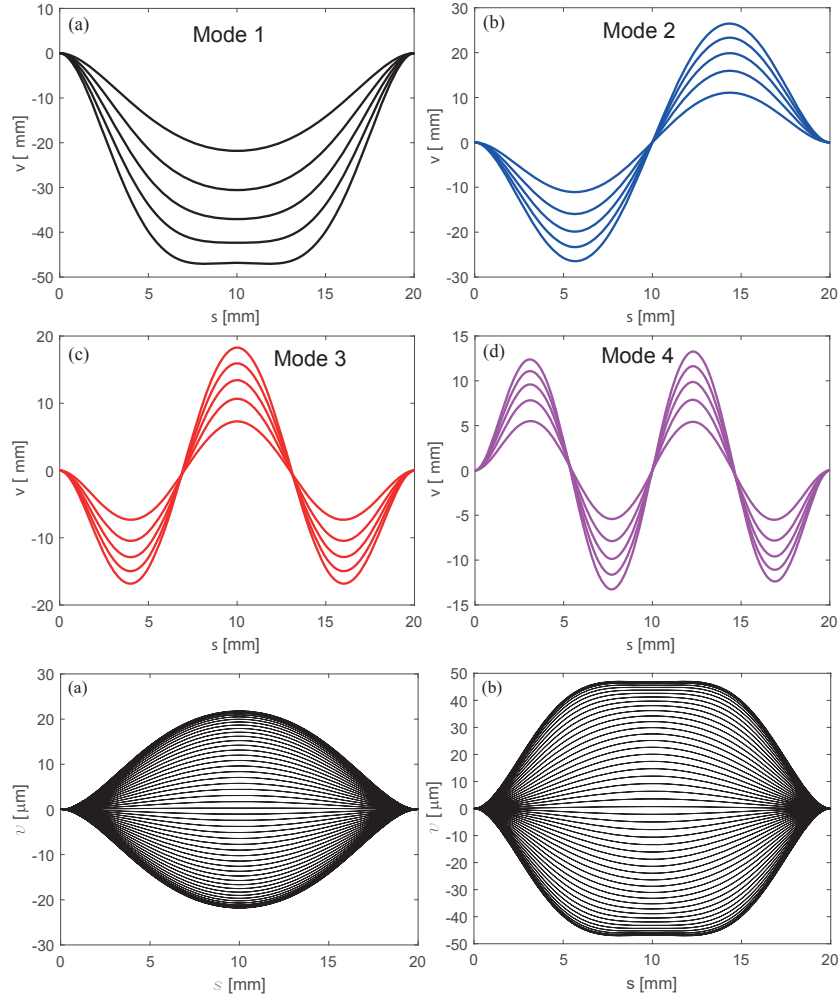


Figure 15: Morphing of the lowest 4 modes with DC voltage $V_{DC} = 50$ V and for 5 values of pulsating voltage V_{AC} from 10 to 50 V with a step of 10 V. Bottom: morphing configurations for the lowest mode during a morphing time cycle of 2π with $\Omega = 2\omega_1$ assuming (a) $V_{DC} = 50$ V, $V_{AC} = 10$ V and (b) $V_{DC} = V_{AC} = 50$ V.

Validation via ABAQUS simulations. The analytical results obtained via the asymptotic approach were compared with the predictions of nonlinear time integration analysis performed in ABAQUS. The studied PVDF thin paper is different from the previously investigated system and consists of a relatively thin PVDF film of length 2 cm, width 1 cm, and thickness $8 \mu\text{m}$. Here, the thickness-to-span ratio is $2.5 \cdot 10^{-3}$. The adopted geometry was inspired by the PVDF specimen which was experimentally investigated in another campaign within this project. The 3D solid model was discretized with 800 C3D20RE finite elements which are 20-node quadratic bricks. Note that this PVDF sample has an aspect ratio equal to 2 which is typical of rectangular plates. Therefore the results obtained by making use of the beam-like mechanical model suffer from this limitation. The aim of this study is to show that a full 3D numerical discretization of the piezo-mechanical problem yields the same kind of parametric resonance phenomenology instead of pursuing a quantitative comparison.

The mechanical problem comprises two loading steps: (i) application of a tensile stress to the whole film by assigning a DC voltage on the bottom surface; (ii) superposition of a pulsating voltage on the bottom surface in order to induce a parametric resonance of some selected modes. The transition curves for the parametric resonances were computed in closed form for a given DC voltage making use of the mechanical piezoelectric beam model. The analytical results for the transition curves were compared with those obtained via direct integration of the nonlinear equations of motion and charge conservation in ABAQUS according to a strategy that will be detailed next.

Before delving into the numerical simulations of the parametric resonance conditions, the elastic and piezoelectric properties of the analytical beam model were calibrated using as baseline the frequencies of the multiphysics FE beam model in the stress-free and prestressed conditions.

The elasticity of the PVDF film was defined using 5 parameters while the piezoelectric strain coefficients are 3. The material density is 1780 kg/m^3 , while the other parameters, in accordance with the guidelines in [28], are reported in Tabs. 1 and 2.

C_{11}	C_{22}	C_{33}	C_{12}	C_{13}	C_{23}	C_{44}	C_{55}	C_{66}
3.8	3.8	1.2	1.9	0.9	0.9	0.9	0.9	0.7

Table 1: Elastic constants of the PVDF material sample to be multiplied by $10^9 \frac{N}{m}$.

Piezoelectric constants $\times 10^{-11}$			Dielectric constants $\times 10^{-11}$		
$D_{31}[\frac{C}{N}]$	$D_{32}[10^{-11} \frac{C}{N}]$	$D_{33}[\frac{C}{N}]$	$\epsilon_{r1}[\frac{F}{m}]$	$\epsilon_{r2}[\frac{F}{m}]$	$\epsilon_{r3}[\frac{F}{m}]$
-1.358	-0.1482	3.38	6.5521	8.23439	6.72918

Table 2: Piezoelectric strain coefficients and permittivity constants of the PVDF material sample.

The through-the-thickness polarized film was fixed at the two ends of width equal to 1 cm. The Young modulus E and the piezoelectric constant E_{31} for the beam model were estimated from the 3D model. The Young modulus was set to $E = 2.53 \text{ GPa}$ considering that $C_{11} = \lambda + 2\mu$ and $C_{44} = \mu$. On the other hand, $E_{31} = ED_{31} = -3.44 \times 10^{-2} \text{ C/m}^2$. The longitudinal prestress in the beam was due to the DC voltage set to 5 V on the bottom surface while the top surface was grounded. Thus, in Eq.(32) the nondimensional DV voltage was set to $V_{DC} = 5/V_c$. The natural frequencies of the beam model evaluated based on the stated values of E and E_{31} did not match those of the

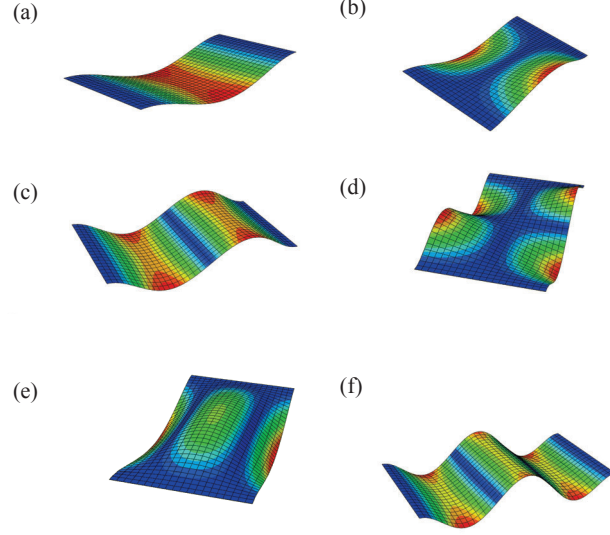


Figure 16: Lowest six mode shapes obtained with ABAQUS. The natural frequencies of the modes (a) through (f) are reported in Tab. 3.

FE model. This led to a calibration of E and E_{31} so as to match the frequencies and modes with and without prestress obtained using the FE model and the analytical formulation. With slight adjustments of the above mentioned parameters ($E = 3.011$ GPa and $E_{31} = -4.02 \times 10^{-2} \text{ C}/m^2$), the resonance frequencies of the analytical model perfectly matched those provided by ABAQUS (see Tab. 3). The lowest three analytical beam modes match the first, third and sixth lowest modes of the FE model in the stress-free and prestressed conditions. The mode shapes are shown in Fig. 16. Note that the second, fourth and fifth mode shapes obtained in ABAQUS are plate-like modes which cannot be described by the beam mechanical model.

mode type	ABAQUS		Analytical	
	stress-free	prestress	stress-free	prestress
BEAM	26.73	102.97	26.74	102.97
PLATE	41.45	189.02		
BEAM	73.70	210.79	73.70	210.69
PLATE	96.42	269.55		
PLATE	131.08	359.27		
BEAM	145.09	327.86	144.45	327.46

Table 3: Comparison of modal frequencies (in Hz) obtained via the analytical model and ABAQUS

Transition curves. We computed the transition curves of the lowest three modes of the prestressed beam obtained using Eq. (32) for comparison with the FE simulations. The dimensionless parameters were set to $\alpha = 7.5 \times 10^7$ and $\tilde{E}_{31} = 1.6716 \times 10^{-6}$, the reference voltage was set to $V_c = 1$ V. The damping ratio for the analytical beam model was adjusted to the equivalent value adopted in the ABAQUS simulations. The damping ratio identified in the numerical solution for the three lowest modes was about 3 %. These estimated values were thus adopted in the analytical model.

Figure (17) shows the transition curves obtained in the $(\Omega - V_{AC})$ plane (frequency-voltage). The solid black lines denote the transition curves obtained analytically with fifth order solutions for the lowest three modes. The gray vertical thin lines, dividing the resonance regions in two parts, are located at exactly twice the frequencies of the beam bending modes. The blue vertical lines are located at twice the frequencies of the lowest two plate modes which are not captured by the analytical model. The color filled circles denote different kinds of numerical solutions provided by the ABAQUS simulations. A time integration was performed for each frequency and the out-of-plane displacement of suitable points was monitored. The black filled circles represent nonresonant conditions according to the FE simulations (i.e., the solution decays thus entailing that the trivial equilibrium is stable) while the red filled circles indicate the onset of the parametric resonance of the investigated modes when a periodic response at the frequency of the excited mode was achieved. On the other hand, the blue filled circles represent the panel resonant responses which could be predicted only by the FE model.

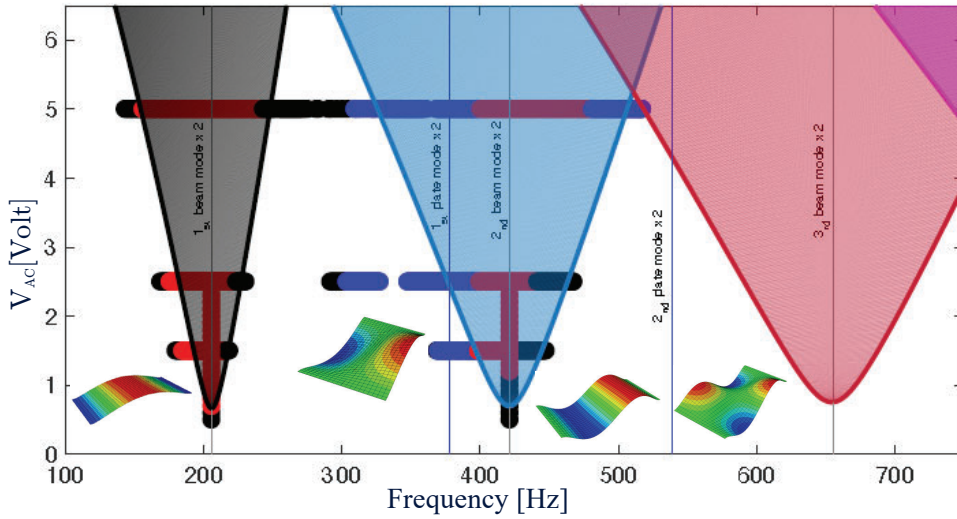


Figure 17: Activation regions for the PVDF sample where the black solid lines were obtained according to the fifth order solutions for the lowest three beam modes assuming $\zeta = 3 \%$ and $V_{DC} = 5$ V; the black and red circles denote nonresonant and resonant responses provided by ABAQUS while the blue circles indicate resonant responses for the plates modes; the gray and blue vertical thin lines denote twice the frequencies of the beam-like modes and twice the plate-like frequencies, respectively; the black dashed horizontal lines denote the amplitudes of the AC voltage V_{AC} for which the ABAQUS simulations were run, namely, (1.5, 2.5, 5) V.

The transition curves computed through the ABAQUS simulations are in accordance with the analytically obtained transition curves. In fact, the threshold points provided by ABAQUS are very close to the analytic predictions. The damping ratio is crucial for obtaining the correct critical/activation voltage V_{AC} at the forcing frequency equal to twice the resonance frequency. The closed form solution slightly underestimates the

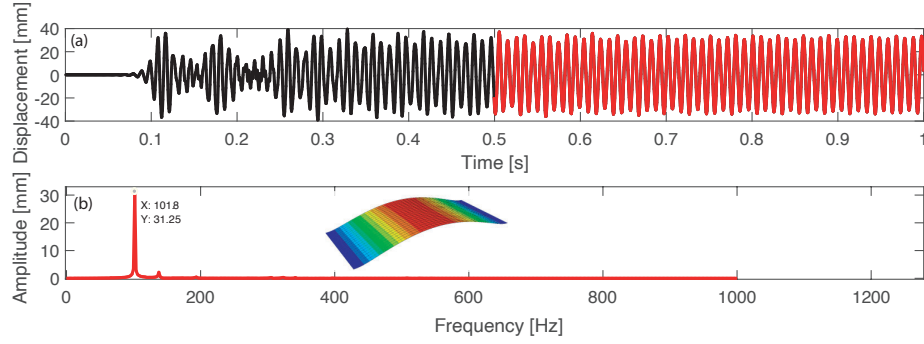


Figure 18: (a) Time history of the morphing response at the midspan and (b) associated FFT obtained for a pulsating voltage $V_{AC} = 5$ V and frequency $\Omega = 204.21$ Hz inducing the parametric resonance of the lowest beam-like mode; the black and red solid lines indicate the transient and stationary parts of the response provided by ABAQUS, respectively.

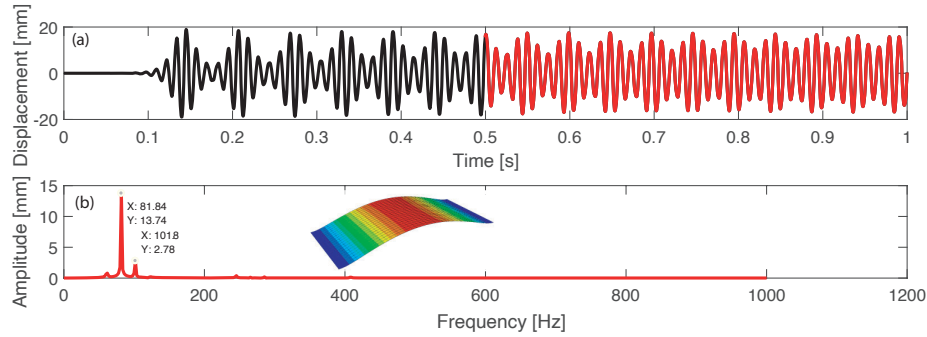


Figure 19: (a) Time history of the morphing response at the midspan and (b) associated FFT obtained for a pulsating voltage $V_{AC} = 5$ V and frequency $\Omega = 163.26$ Hz inducing the parametric resonance of the lowest beam mode; the black and red solid lines indicate the transient and stationary parts of the response provided by ABAQUS, respectively.

activation voltage with respect to ABAQUS. However, it is worth pointing out that the effective damping provided by ABAQUS in the computations may be different for different modes with respect to that introduced in the analytical model regardless of the considered mode. Thus, the small discrepancies about the activation voltage can be further reduced going much deeper into the issue of numerical damping introduced by the FE code.

The time histories of the FE responses and associated FFTs are shown in Fig. 18 through Fig. 21. The assumed control points are provided in the captions. Figures 18 and 19 show the morphing responses in the first beam mode while Fig. 20 shows a nonresonant response. Finally, Fig. 21 shows the morphing response in the lowest plate mode. More specifically, Fig. 19 shows the time history response obtained for a voltage amplitude $V_{AC} = 5$ V and pulsating frequency $\Omega = 163.26$ Hz, respectively. This point is located on the left border of the lowest mode resonance region of Fig 17. The frequency content of the signal exhibits two main frequencies which are the frequency of the lowest mode and the driving frequency demonstrating the incipient transition between parametric resonance and nonresonance.

Pixelled PVDF panels. 4×1 pixelled PVDF panels were also investigated analytically.

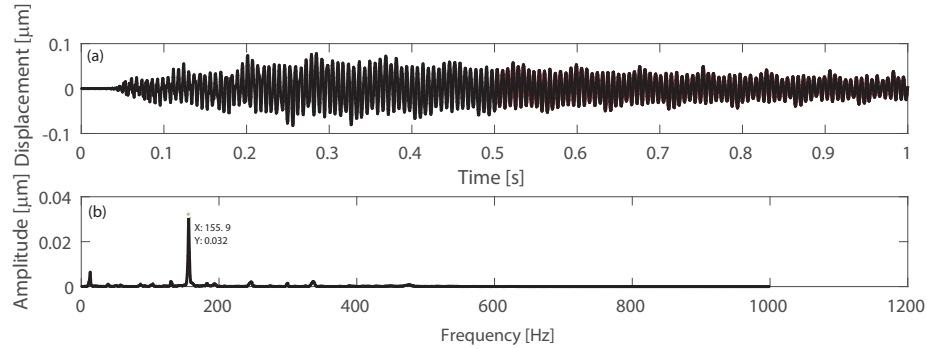


Figure 20: (a) Time history of the response at the midspan and (b) associated FFT obtained for a pulsating voltage $V_{AC} = 5$ V and frequency $\Omega = 143.24$ Hz in the ABAQUS model, for which the parametric resonance is not achieved.

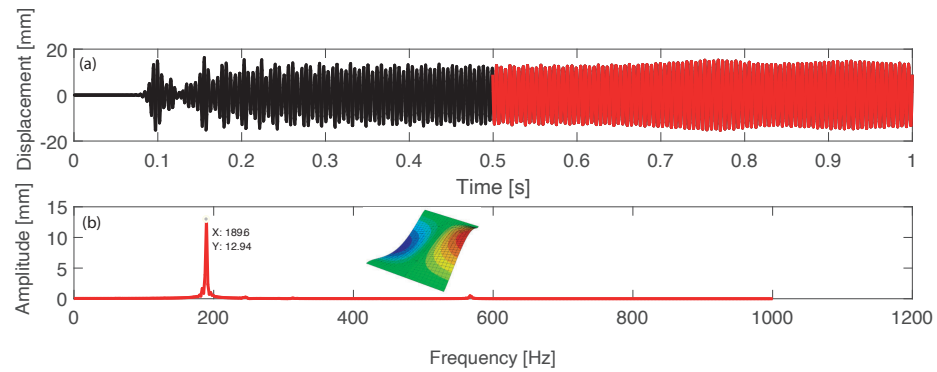


Figure 21: (a) Time history of the morphing response at the midspan point on the long beam edge and (b) associated FFT obtained for a pulsating voltage $V_{AC} = 5$ V and frequency $\Omega = 379.54$ Hz inducing the parametric resonance of the lowest plate mode; the black and red solid lines indicate the transient and stationary parts of the response provided by ABAQUS, respectively.

Each pixel is an independent panel which can be morphed according to the selected parametric resonance. Therefore, virtually any morphing shape can be achieved by the local morphing control of the j th pixel regulated by the two fundamental control gains $V_{DC,j}$ and $V_{AC,j}$ and the pulsating frequency Ω_j . In Fig. 22 the first instance shows the activation of mode 1 through 4 while a non-symmetric morphing shape and a symmetric shape are shown in part (b) and part (c), respectively.

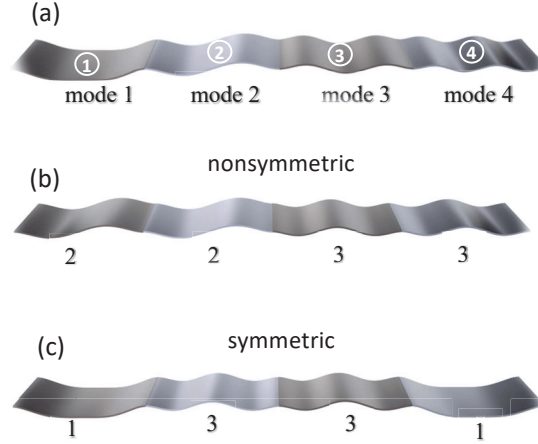


Figure 22: Different morphing modes: (a) 1-2-3-4 mode sequence, (b) skew-symmetric 2-2-3-3 mode sequence, and (c) symmetric 1-3-3-1 mode sequence.

Summary. The piezoelectric properties of the PVDF film can effectively induce via a DC voltage an initial tensile stress that makes the film stiffer and thus allows to tune the frequencies. A nonlinear piezoelectric 1D model was developed for the PVDF panel and the activation regions for the parametric resonance of various bending modes were computed employing an asymptotic method directly applied to the PDE and boundary conditions to overcome discretization issues. The obtained results were compared with those obtained with ABAQUS in which the piezoelectric system was discretized as a solid geometrically nonlinear problem. Different large-amplitude morphing modes could be activated varying the driving frequency and providing feasible voltage amplitudes. Moreover, an interesting outcome of the analytical study was the enhanced bandwidth of the parametric resonance response at larger excitation amplitudes whereby the region around the peak amplification shows some sort of flattening effect. These aspects demonstrate that PVDF films are optimal candidates for dynamic shape morphing driven by parametric resonances of selected modes.

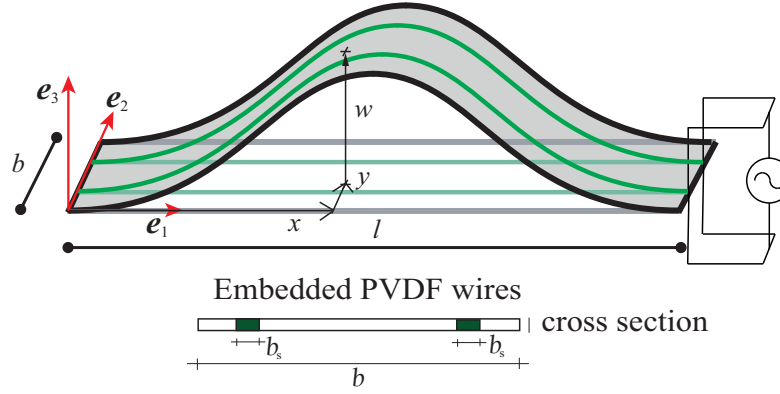


Figure 23: Panel reference (thick solid gray lines) and current (thick solid black lines and green lines) configurations.

3.2.4 Dynamic morphing of panels with embedded PVDF wires

We further investigated the feasibility of exciting large-amplitude oscillations of selected modes in ultrathin panels for dynamic morphing purposes by exploiting not only parametric resonances but also primary and secondary (i.e., sub- and super-harmonic) resonances. Such variety of resonances can be obtained thanks to the large static deflections achievable in ultrathin plates under their own weight which entail coupling of the piezoelectric axial force with the plate initial curvature generating a direct dynamic excitation term in addition to the parametric one.

Resorting on the initial study proposed in [33] on flexible thin plates, in the context of this project a full characterization of the dynamic morphing capability of thin elastic plates driven by embedded piezoelectric wires was pursued. Due to the pronounced thinness, the panel vibrates about a nonlinear equilibrium induced by its own weight. Parametric studies with respect to the wires cross-sectional size, the panel damping, and the piezoelectric coupling coefficient were carried out to identify the threshold voltages of the different resonance regions. The governing PDE was discretized according to the Faedo-Galerkin approach, and higher-order asymptotic expansions (i.e., up to seventh order) were tailored to study the morphing motions arising out of primary, superharmonic, subharmonic, and principal parametric resonances.

Nonlinear model. A nonlinear model of thin elastic panels actuated by piezoelectric wires was adopted to investigate their response induced by direct and parametric excitations, respectively. By introducing the in-plane coordinates x , along the span l , and y , along the width b , the boundary conditions were assumed to be stress-free along the two long edges at $y = 0$ and $y = b$, respectively, and clamped on the other two edges (i.e., at $x = 0$ and $x = l$). The parametric actuation was provided by two PVDF piezoelectric wires embedded in the panel thickness h along the x direction and subject to a time-varying elongation $\varepsilon(t)$ due to an oscillating (AC) voltage $V(t) = V \cos \Omega t$. The induced elongation can be expressed as $\varepsilon(t) = \varepsilon \cos \Omega t$, where $\varepsilon = dV/h$ and d is the piezoelectric coupling coefficient.

In the problems here investigated, the panel thickness was set to $h = 50 \times 10^{-6}$ m, its length $l = 51 \times 10^{-3}$ m, the width $b = 2 \times 10^{-3}$ m; the Young modulus was $E = 2.53$ GPa, the Poisson ratio $\nu = 0.41$, the mass density $\rho = 1.78 \times 10^3$ kg/m³, and the damping ratio ζ was set to 1% [34]. On the other hand, the two PVDF wires have a piezoelectric coupling coefficient $d_o = 1.358 \times 10^{-11}$ C/N (i.e., m/Volt), while the cross-sectional area is $A_s = b_s h$, being $0 < b_s \leq b/2$. The parametrized piezoelectric wires width used in this

study allows to vary the excitation amplitude and to find feasible voltage levels for the activation of the parametric resonance. The number of wires was set to $n_s = 2$, while $d = d_o$ and $b_s = b/50$ were chosen for the simulations.

Since the investigated panel is characterized by an aspect ratio $l/b = 25.5$ and, given the symmetry of the loading and the considered boundary conditions, it was assumed that the panel undergoes only cylindrical transverse motions, which are uniform along the y direction and depend only on the coordinate x and time t , respectively. Moreover, the geometric and mechanical features of the investigated panel allowed to consider an equivalent beam model to study both the equilibrium problem and the motion about it.

The axial force provided by the wires acts in the panel equilibrium under its own weight, therefore, it is worth separating the effects due to static and dynamic loads, respectively. To this end, the total plate displacement $w_T(x, t) = w_o(x) + w(x, t)$ was expressed as sum of the static deflection $w_o(x)$ and the dynamic incremental deflection $w(x, t)$. Similarly, the cross-sectional total rotations and total curvatures of the equivalent beam were expressed, in linearized form, as $w'_T(x, t) = w'_o(x) + w'(x, t)$ and $w''_T(x, t) = w''_o(x) + w''(x, t)$, respectively, where $w'_o(x)$ and $w''_o(x)$ are the static and incremental dynamic cross-sectional rotations, respectively, while $w'(x)$ and $w''(x)$ are the static and incremental dynamic bending curvatures, respectively.

As shown in Fig. 23, the reference stress-free configuration of the thin, rectangular panel was assumed to be planar and the overall mechanical problem was formulated in the fixed Cartesian frame $\{e_1, e_2, e_3\}$ having its origin in one of the panel corners. Moreover, the direction e_3 , normal to the reference configuration, corresponds to the vertical direction (i.e., the direction opposite to the gravity acceleration), while the plate mid-plane lies in the $\{e_1, e_2\}$ -plane. The position of each material point of the plate reference plane is thus given by the vector $\mathbf{x} = x \mathbf{e}_1 + y \mathbf{e}_2$.

Geometric nonlinearities arising from large oscillations are taken into account as in the Mettler theory [35] providing the equation of motion of the equivalent beam in the form

$$\int_0^b \left[\rho h \ddot{w}_T + c \dot{w}_T + D w_T''' - \frac{Eh}{2l} w_T'' \int_0^l (w_T')^2 dx - N(y, t) w_T'' + f_g \right] dy = 0, \quad (35)$$

together with the boundary conditions $w_T(0, t) = 0 = w_T(l, t)$ and $w'_T(0, t) = 0 = w'_T(l, t)$, and the natural initial conditions. In Eq. (35), the prime and the dot indicate differentiation with respect to x and t , respectively, while c is the damping coefficient, $D = Eh^3/12$ is the bending rigidity, and $f_g = \rho h g$ is the weight per unit area, respectively. Finally, $N(y, t)$ is the axial force provided by the PVDF wires expressed as

$$N(y, t) = N(t) \sum_{j=1}^{n_s} \delta(y - y_j), \quad (36)$$

where $\delta(\cdot)$ is the Dirac-delta function whose physical dimension, in this application, is the inverse of a length, while y_j is the coordinate along the panel width of the j th wire and n_s is the number of wires.

The PVDF-induced force $N(t)$ was here assumed to be in the form $N(t) = E A_s \varepsilon \cos \Omega t$ and to act in the equilibrium under the panel weight f_g . Such equilibrium is governed by the following integro-differential equation:

$$, D w_o''' - \frac{Eh}{2l} w_o'' \int_0^l (w_o')^2 dx + f_g = 0. \quad (37)$$

On the other hand, the statement of the balance of linear momentum per unit length of

the equivalent beam in the transverse direction can be written in incremental form as:

$$\begin{aligned} \rho h \ddot{w} + c \dot{w} + D w'''' - \frac{Eh}{2l} \left\{ w'' \int_0^l (w'_o)^2 dx + 2 w'' \int_0^l w'_o w' dx + w'' \int_0^l (w')^2 dx \right. \\ \left. + 2 w''_o \int_0^l w'_o w' dx + w''_o \int_0^l (w')^2 dx \right\} - \frac{n_s}{b} N(t) w'' - \frac{n_s}{b} N(t) w''_o = 0. \end{aligned} \quad (38)$$

The panel length l and the frequency $\omega_c = \sqrt{E h^2 / (12 \rho l^4)}$ were adopted as characteristic parameters so as to cast the equations in nondimensional form. Hence, the nondimensional space and time coordinates are $\bar{x} = x/l$ and $\bar{t} = \omega_c t$, respectively, while $\bar{w}_o = w_o/l$ and $\bar{w} = w/l$ are the nondimensional out-of-plane static and dynamic displacements, respectively. Here and henceforth, the bars will be dropped from the nondimensional variables for ease of notation.

The nondimensional form of the nonlinear equilibrium equation was cast as

$$w''''_o - \kappa w''_o \int_0^1 (w'_o)^2 dx + p = 0, \quad (39)$$

together with the nondimensional boundary conditions $w_o(0) = 0 = w_o(1)$ and $w'_o(0) = 0 = w'_o(1)$. The closed-form solution of the nonlinear equilibrium was found in the form:

$$\begin{aligned} w_o = \frac{p}{2(\kappa C)^{3/2}} \left[(x^2 - x) \sqrt{\kappa C} - \coth\left(\frac{1}{2}\sqrt{\kappa C}\right) \cosh(x\sqrt{\kappa C}) + \sinh(x\sqrt{\kappa C}) \right. \\ \left. + \coth\left(\frac{1}{2}\sqrt{\kappa C}\right) \right]. \end{aligned} \quad (40)$$

Moreover, $\kappa = 6(l/h)^2$ is the axial-to-flexural stiffness ratio (i.e., $\kappa = (Eh/2l)/(Eh^3/12l^3)$), while $p = f_g l^3/D$ is the nondimensional panel weight per unit reference length. By dividing both sides of Eq. (38) by $\rho h \omega_c^2 l$, the nondimensional form of the equation of motion was found to be

$$\begin{aligned} \ddot{w} + \bar{c} \dot{w} + w'''' - \kappa \left\{ w'' \int_0^1 (w'_o)^2 dx + 2 w'' \int_0^1 w'_o w' dx + w'' \int_0^1 (w')^2 dx \right. \\ \left. + 2 w''_o \int_0^1 w'_o w' dx + w''_o \int_0^1 (w')^2 dx \right\} - n_s \eta \bar{N} w'' - n_s \eta \bar{N} w''_o = 0, \end{aligned} \quad (41)$$

It is worth noting that in Eq. (41) the coefficient of w'''' is $(D/l^3)/(\rho h \omega_c^2 l) \equiv 1$, while $\eta = l/b$ is the panel aspect ratio, $\bar{c}$ is the damping coefficient, and $\bar{N} = \kappa_s \varepsilon \cos \Omega t$, where $\kappa_s = E A_s/(\rho h \omega_c^2 l^3)$ is the nondimensional PVDF wires axial stiffness.

A further nondimensional parameter is the slenderness $\lambda = l/h$ of the equivalent beam. In terms of λ , the beam nondimensional axial stiffness κ can be expressed as $\kappa = 6\lambda^2$, while the weight per unit length can be written in nondimensional form as $p = (12\rho h g/E)\lambda^3$. Thus, λ influences drastically the beam equilibrium and its dynamic behavior since it regulates the ratio between the beam axial stiffness and the flexural stiffness. Indeed, high values of λ entail a flexural stiffness much lower than the axial stiffness. In the limit, $\lambda \rightarrow \infty$, the beam behaves as a cable with vanishing flexural stiffness. Moreover, as λ increases, the nondimensional weight increases with the third power. According to the mechanical properties previously described, $\eta = 25.5$, while the nondimensional parameters of the equivalent beam were found to be expressed in terms of the slenderness λ as follows: $\kappa = 6\lambda^2$, $\kappa_s = 9.41176 \times 10^{-3} \lambda^2$, $p = 4.22396 \times 10^{-6} \lambda^2$. On the other hand, the amplitude ε of the harmonic elongation has the form $\varepsilon = 2.66275 \times 10^{-10} V \lambda$. The effect of λ on the morphing response was extensively studied in the range $\lambda \in [10, 1100]$, which corresponds to the physical range

spanned by very thick up to ultrathin panels. To study the modal characteristics of the beam subjected to the weight-induced prestress, Eq. (41) is linearized and the axially induced piezoelectric excitation $\bar{N}$ is neglected together with the dissipative force $\bar{c}\dot{w}$. The ensuing homogeneous PDE describing free vibrations of the prestressed, precurved, beam is the following [36]:

$$\ddot{w} + w'''' - \kappa \left[w'' \int_0^1 (w'_o)^2 dx + 2w''_o \int_0^1 w'_o w' dx \right] = 0, \quad (42)$$

where w_o is given by Eq. (40).

To obtain the reduced-order model of the beam dynamics, the incremental displacement $w(x, t)$ is approximated as $w(x, t) \approx \sum_{j=1}^{N_m} q_j(t) \phi_j(x)$, where $q_j(t)$ is the j th generalized coordinate and $\phi_j(x) = B_1 \sin \Lambda_j x + B_2 \cos \Lambda_j x + B_3 \sinh \Lambda_j x + B_4 \cosh \Lambda_j x$ is the j th mode shape of the clamped-clamped beam (i.e., normalized according to $\int_0^1 \phi_j^2 dx = 1$). The coefficients B_k ($k = 1, \dots, 4$), together with the j th nondimensional space frequency Λ_j , can be found for each mode shape by solving the eigenvalue problem (i.e., the homogeneous equation $\phi_j(x)'''' - \Lambda_j^4 \phi_j(x) = 0$) together with the boundary conditions $\phi_j(0) = 0 = \phi_j(1)$ and $\phi'_j(0) = 0 = \phi'_j(1)$. On the other hand, due to the adopted nondimensionalization, the nondimensional natural frequency ω_j of each mode shape could be straightforwardly calculated as $\omega_j = \Lambda_j^2$. By adopting N_m trial functions $\phi_j(x)$ as weight functions to minimize the residual resulting from the substitution of the approximate displacement into Eq. (41), by multiplying the residual by $\phi_j(x)$ and integrating over the domain $[0, 1]$, the system of N_m nonlinear ordinary differential equations of motion could be written in matrix form as

$$\mathbf{I} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + (\mathbf{K} + \mathbf{K}_o) \mathbf{q} + \mathbf{g}_o^{(2)} + \mathbf{g}^{(3)} + \mathbf{K}_s \mathbf{q} \varepsilon \cos \Omega t + \mathbf{f}_\varepsilon^o \varepsilon \cos \Omega t = \mathbf{o}. \quad (43)$$

In Eq. (43) the $(N_m \times N_m)$ matrix $\mathbf{K}_o = \mathbf{K}_o(w_o^2)$ and the $(N_m \times 1)$ nonlinear vector-valued operator $\mathbf{g}_o^{(2)} = \mathbf{g}_o^{(2)}(w_o, \mathbf{q}, \mathbf{q})$ are the linear and the quadratic geometric counterparts of the beam stiffness, respectively, due to the prestressed equilibrium w_o . Moreover, $\mathbf{q} = [q_1(t), \dots, q_{N_m}(t)]^\top$ is the vector collecting the N_m generalized coordinates, $\mathbf{K}$ is the $(N_m \times N_m)$ linear elastic stiffness matrix, $\mathbf{g}^{(3)} = \mathbf{g}^{(3)}(\mathbf{q}, \mathbf{q}, \mathbf{q})$ is the $(N_m \times 1)$ nonlinear vector-valued operator collecting the cubic terms of the beam (stretching-induced) restoring force, while $\mathbf{K}_s$ is the modal matrix of the dynamic stiffness provided by the PVDF wires and $\mathbf{f}_\varepsilon^o = \mathbf{f}_\varepsilon^o(w_o)$ represents the $(N_m \times 1)$ associated excitation vector. Finally, $\mathbf{o}$ indicates a $(N_m \times 1)$ vector of zeros. It is also worth noting that, due to the orthogonality and the normal form of the selected trial functions ϕ_j (i.e., $\int_0^1 \phi_j \phi_k dx = 0$ for $j \neq k$, and $\int_0^1 \phi_j^2 dx = 1$, respectively) in Eq. (43) the modal mass matrix coalesces to the identity matrix $\mathbf{I}$. Furthermore, $\mathbf{C} = 2\zeta\omega_j\mathbf{I}$, where ζ is the damping ratio.

The eigenvalue problem can be obtained from Eq. (42) dropping the nonlinear, damping and excitation terms yielding

$$\mathbf{I} \ddot{\mathbf{q}} + (\mathbf{K} + \mathbf{K}_o) \mathbf{q} = \mathbf{o}. \quad (44)$$

The j th mode shape ψ_j is sought as a linear combination of the N_m mode shapes of the stress-free beam, i.e., $\psi_j = \sum_{k=1}^{N_m} c_{jk} \phi_k$, where c_{jk} is the (jk) th coefficient (i.e., the k th element of the j th eigenfunction of Eq. (44)), while $\tilde{\omega}_j$ is the j th nondimensional frequency of the prestressed beam (i.e., the square root of the j th eigenvalue of Eq. (44)). Moreover, the same orthonormality properties for ψ_j hold as for ϕ_j . In Fig. 24 the lowest six mode shapes of eminently thin beams (i.e., $\lambda = 1020$) together with the nondimensional frequencies $\tilde{\omega}_j$ are depicted. Note that the nondimensional frequencies are higher than the frequency of the stress-free beam, and this is due to the hardening

effect provided by the equilibrium under tensile stresses.

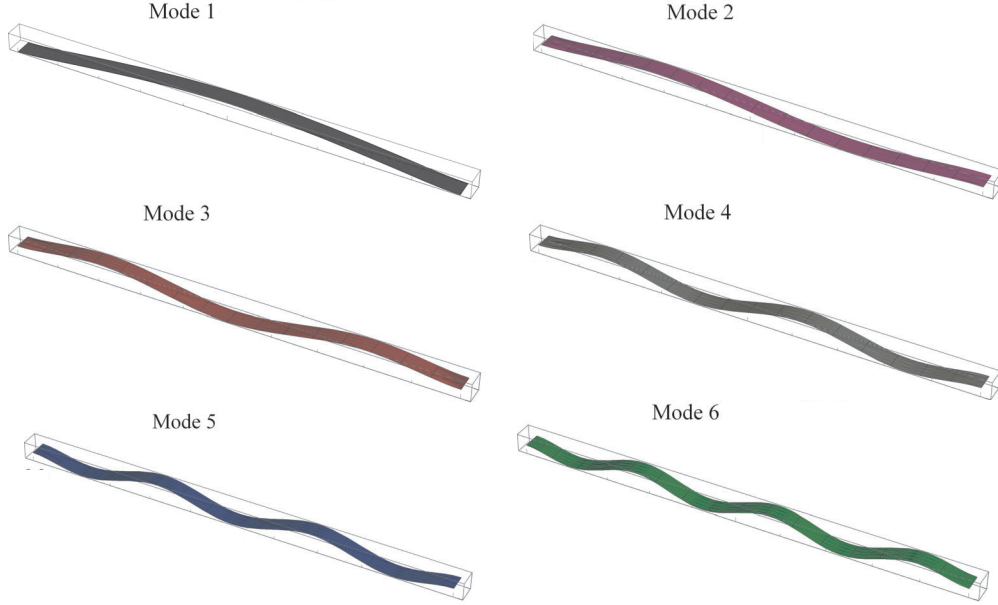


Figure 24: Mode shapes of the lowest six modes of the prestressed beam with $\lambda = 1020$.

The Faedo-Galerkin (FG) method was employed to perform a discretization of the equation of motion (41) yielding a reduced-order model (ROM). A convergence analysis was necessary to validate the accuracy of the ROM. To this end, the results in terms of natural frequencies calculated by adopting an increasing number of trial functions were compared with the results obtained by solving Eq. (41) by means of a nonlinear finite FE solver called Comsol Multiphysics [37].

In a single-mode discretization, we let $w(x, t) \approx \psi_j(x)\tilde{q}_j(t)$, where $\tilde{q}_j(t)$ is the j th generalized coordinate. The reduced-order model was obtained by retaining only one mode in Eq. (43); that is,

$$\ddot{\tilde{q}}_j + 2\zeta\tilde{\omega}_j\dot{\tilde{q}}_j + \left(\tilde{K}_j + \tilde{K}_{o,j}\right)\tilde{q}_j + \tilde{K}_{o,j}^{(2)}\tilde{q}_j^2 + \tilde{K}_j^{(3)}\tilde{q}_j^3 + \tilde{K}_{s,j}\varepsilon\tilde{q}_j\cos\Omega t + \tilde{f}_{\varepsilon,j}^o\varepsilon\cos\Omega t = 0, \quad (45)$$

where $\tilde{K}_j$ is the elastic component of the beam stiffness, $\tilde{\omega}_j = \sqrt{\tilde{K}_j + \tilde{K}_{o,j}}$ is the frequency, and $\tilde{K}_j^{(3)}$ is the coefficient of the stretching-induced cubic term. Moreover, $\tilde{K}_{o,j}$ and $\tilde{K}_{o,j}^{(2)}$ are the linear and quadratic geometric stiffness terms, respectively, while $\tilde{K}_{s,j}$ is the modal projection of the dynamically modulated stiffness given by the piezoelectric wires. Finally, $\tilde{f}_{\varepsilon,j}^o\varepsilon\cos\Omega t$ represents the modal projection of the direct forcing provided by the wires.

When the beam is thick, the effect of the initially bent equilibrium $w_o(x)$ on the nonlinear dynamic response becomes negligible and Eq. (43) gets simplified with $\mathbf{K}_o = \mathbf{O}$, $\mathbf{g}_o^{(2)} = \mathbf{O}$, and $\mathbf{f}_\varepsilon^o = \mathbf{O}$.

By setting $\varepsilon = 0$ in Eq. (45), it is possible to study asymptotically the nonlinearity of the modes of the beam and the influence of λ . It was found that the frequency of the j th mode can be expressed as:

$$\tilde{\omega}_j^{\text{nl}} = \tilde{\omega}_j + \tilde{\Gamma}_j a_j^2, \quad \tilde{\Gamma}_j = \frac{3}{8\tilde{\omega}_j} \tilde{K}_j^{(3)} - \frac{5}{12\tilde{\omega}_j^3} \left(\tilde{K}_{o,j}^{(2)}\right)^2, \quad (46)$$

where $\tilde{\Gamma}_j$ is the effective nonlinearity coefficient of the j th prestressed mode. When $\tilde{\Gamma}_j < 0$, the nonlinearity is softening, when $\tilde{\Gamma}_j > 0$, it is hardening. There exists a boundary between softening and hardening behaviors depending on λ (panel thinness). When the prestress induced by the beam weight is negligible, the effective nonlinearity coefficient is given by $\Gamma_j = 3K_j^{(3)}/(8\omega_j)$ upon setting to zero the quadratic contribution in Eq. (46) and the modes become all hardening.

Figure 25 shows the effective nonlinearity coefficient $\tilde{\Gamma}_j$ (solid lines) and Γ_j (dashed lines), respectively, for the lowest six modes of the prestressed and stress-free beam, respectively. The nonlinearity of the lowest symmetric mode in Fig. 25 (a) changes from softening to hardening when the panel becomes increasingly thicker, i.e., for decreasing values of λ , while the higher modes (i.e., $j \geq 2$) are hardening regardless of the panel thinness/slenderness (see Fig. 25 (b)).

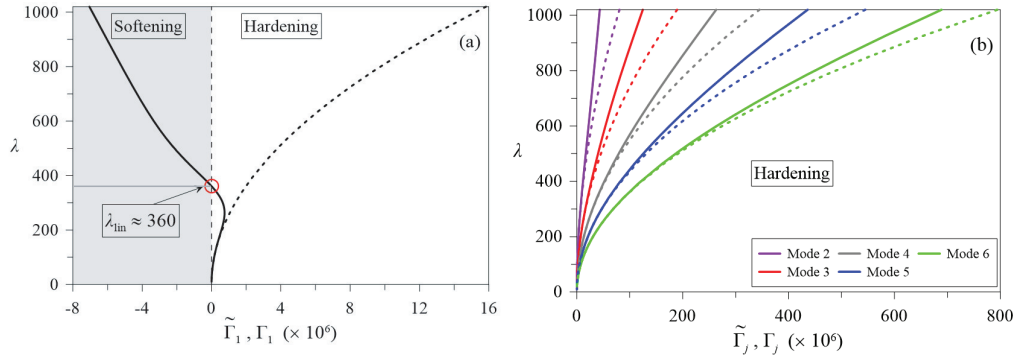


Figure 25: Variation of the effective nonlinearity coefficients of the lowest six modes with the slenderness for the straight (dashed lines) or initially bent panels (solid line).

3.2.5 Piezoelectrically induced morphing shapes

The dynamic excitation induced by the piezoelectric wires provides two types of loading conditions depending on whether the effects of the prestressed bent equilibrium are neglected or not. For very thin panels (i.e., $\lambda \geq 200$), the piezoelectrically induced excitation may cause direct resonances or principal parametric resonances, respectively. The direct forcing term in Eq. (41) is due to $n_s \eta \bar{N} w_o''$ whose single-mode projection in Eq. (45) is $\tilde{f}_{\varepsilon,j}^o \varepsilon \cos \Omega t$. Moreover, due to the presence of nonnegligible quadratic nonlinearities, direct resonances can be primary, superharmonic, or subharmonic, depending on whether $\Omega \approx \tilde{\omega}_j$ (primary resonance), $\Omega \approx \tilde{\omega}_j/n$ (superharmonic resonances), or $\Omega \approx n\tilde{\omega}_j$ (subharmonic resonances), respectively, being n an integer number greater than one (i.e., $n > 1$). On the other hand, when $\Omega \approx n\tilde{\omega}_j$, due to the presence of the term $\tilde{K}_{s,j} \varepsilon \cos \Omega t$ multiplied by the modal displacement $\tilde{q}_j$, principal parametric resonances of the j th mode can be excited.

For thick panels (i.e., $\lambda < 200$), the equilibrium state described by $w_o(x)$ becomes negligible and the term providing direct excitation disappears from the equation of motion (45) as λ goes to zero. Therefore, for these panels, principal parametric resonances only can be exploited for morphing. The above mentioned resonances are investigated via a higher order perturbation analysis.

Principal parametric resonance. Principal parametric resonances can be triggered when the harmonic excitation term multiplicative of the modal amplitude $\tilde{q}_j$ has frequency $\Omega \approx 2\tilde{\omega}_j$. The closed form expressions of the transition curves based on the third-order

asymptotic solution are given by

$$\varepsilon_{1,2} = \pm \frac{2\tilde{\omega}_j}{\tilde{K}_{s,j}} \sqrt{\sigma^2 + 4\zeta^2\tilde{\omega}^2}, \quad (47)$$

where, by setting $\sigma = 0$, the threshold elongation ε which triggers the parametric resonance is found :

$$\varepsilon_{1,2}^{cr} = \left| \frac{4\zeta\tilde{\omega}_j^2}{\tilde{K}_{s,j}} \right|. \quad (48)$$

The transition curves and the frequency response curves were also computed using fifth- and seventh-order expansions. The details are reported in [24]. We discuss the role of the approximation next. In Fig. 26, the activation regions of the parametric resonance obtained with third-, fifth-, and seventh-order asymptotic analysis are compared. Differences become appreciable above a certain actuation strain level.

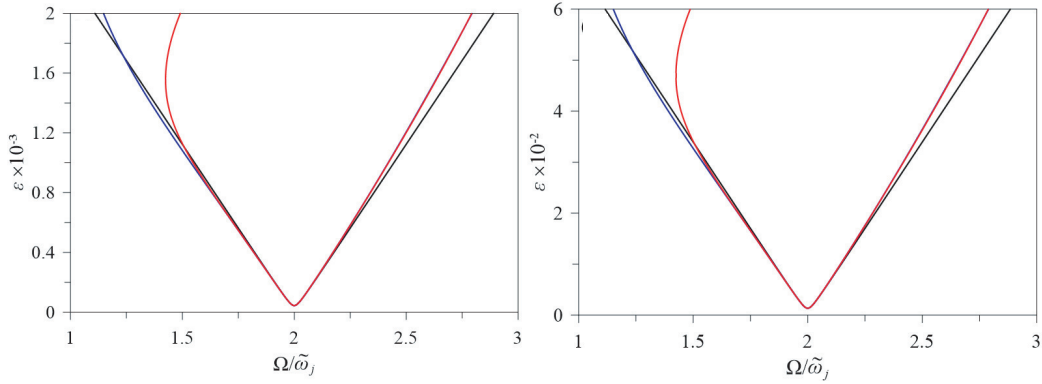


Figure 26: Activation regions for (a) $\lambda = 1020$ and (d) $\lambda = 50$. Black, blue, and red lines indicate the results of the third-, fifth-, and seventh-order asymptotic analysis, respectively.

Moreover, the frequency response curves in Figs. 27 computed according to various orders of approximation prove that for very slender panels, at sufficiently high amplitudes, only the seventh-order approximation yields somewhat qualitatively correct predictions. To achieve more accurate results, the asymptotic analysis should tackle a richer ROM model with more modes including in the discretization process.

Next the activation regions obtained with the lowest order approximation are discussed for the lowest six modes in Fig. 28. Note that high values of AC voltage are considered to show the trends of the activation regions. However, the critical values of the voltage at the onset of the parametric resonance shown in Fig. 29 prove that tens or hundreds of Volts can trigger the morphing response. It is interesting to note that, for very thin panels (i.e., $\lambda = 1020$) (see Fig. 29 (a)) the critical voltage V_{cr} activating the parametric resonance of the lowest mode is higher than that obtained for higher modes while the difference is milder for $\lambda = 500$ (see Fig. 29 (b)). This is consistent with the results revealing that the lowest symmetric mode is softening or hardening depending on whether λ is greater or smaller than $\lambda \approx 360$.

Figure 30 shows the critical voltage for $d = d_o$ upon varying the width of the two piezoelectric strips in the range $b/50 \leq b_s \leq b/2$ and damping ratio in the range $0.1\% \leq \zeta \leq 2.5\%$. We see that decreasing the width of the PZT wires, as expected more voltage is needed to trigger the parametric morphing, a trend which is emphasized for increasingly thicker panels.

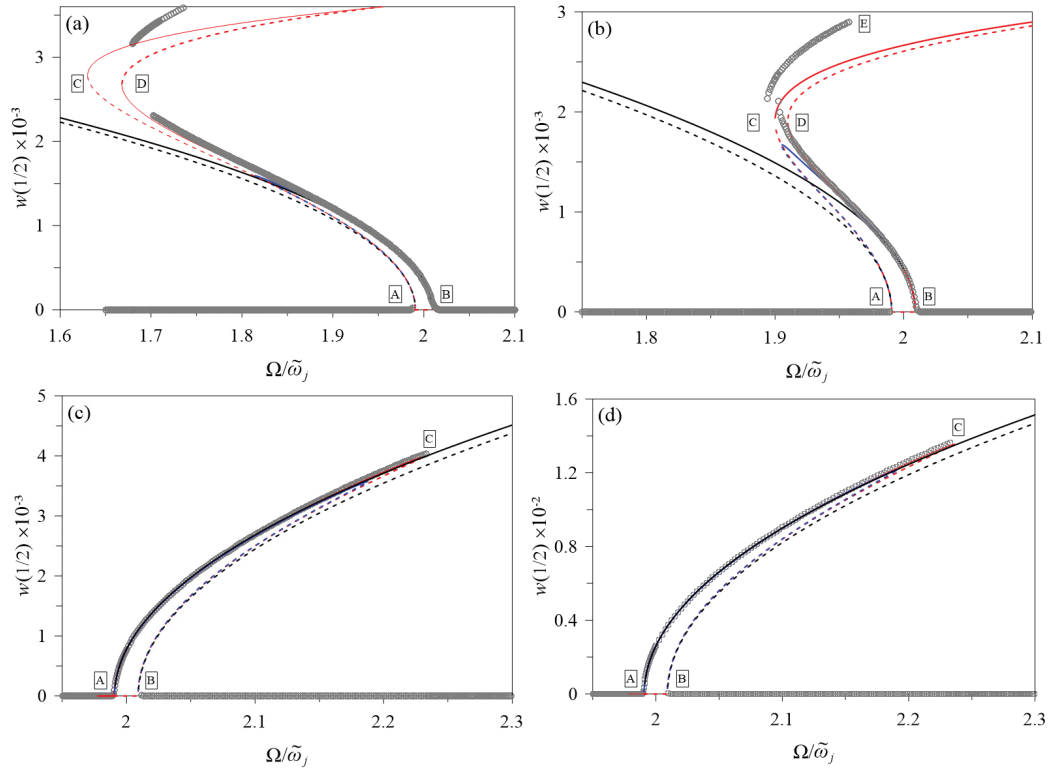


Figure 27: Principal parametric resonance frequency-response curves for (a) $\lambda = 1020$, $\varepsilon = 4.9 \times 10^{-5}$, (b) $\lambda = 500$, $\varepsilon = 4.1 \times 10^{-5}$, (c) $\lambda = 170$, $\varepsilon = 1.3 \times 10^{-4}$, and (d) $\lambda = 50$, $\varepsilon = 1.5 \times 10^{-3}$. Black, blue, and red lines indicate the results of the third-, fifth-, and seventh-order asymptotic analysis, respectively, while gray circles indicate the numerical time integration results with 10 mode shapes.

Primary resonance induced morphing. A primary resonance response is excited when the driving frequency is $\Omega \approx \tilde{\omega}_j$. The frequency-response curves of the lowest symmetric mode of the prestressed panel calculated for four selected values of λ are shown in Fig. 31. Numerical time integration analysis of the nonlinear equations of motion, obtained with 10 mode shapes ψ_j of the prestressed panel, shows that, for very thin panels (i.e., large λ), a soft-hardening behavior is obtained consistently with the results on the lowest mode nonlinearity (see gray circles in Fig. 31). For very thin panels, neither the third-order nor the fifth-order approximations can predict the soft-hardening behavior of the lowest mode whereas for moderately thin or thick panels the third order approximation suffices. Another observation is that, at the same voltage levels, the principal parametric resonance and the primary resonance give rise to comparable amplitudes of the morphing motions.

Superharmonic resonance morphing. Secondary resonances can be excited when the excitation frequency is an integer multiple or an integer fraction of the mode frequency $\tilde{\omega}_j$ (i.e., $\Omega \approx n \tilde{\omega}_j$ or $\Omega \approx \tilde{\omega}_j/n$ with $n > 1$). A superharmonic resonance of order 2 is excited when the frequency of the AC voltage is $2\Omega \approx \tilde{\omega}_j$.

The frequency response curves for the 1:2 superharmonic resonance morphing obtained for $V = 92$ Volt and $V = 375$ Volt are shown in Fig. 32 for $\lambda = (1020, 500)$, respectively. The chosen voltages are the same employed for the primary resonance morphing examples (see Fig. 31). However, the vibration amplitudes arising out of the superharmonic resonance are 30 times lower than the primary resonance scenario. The simulations

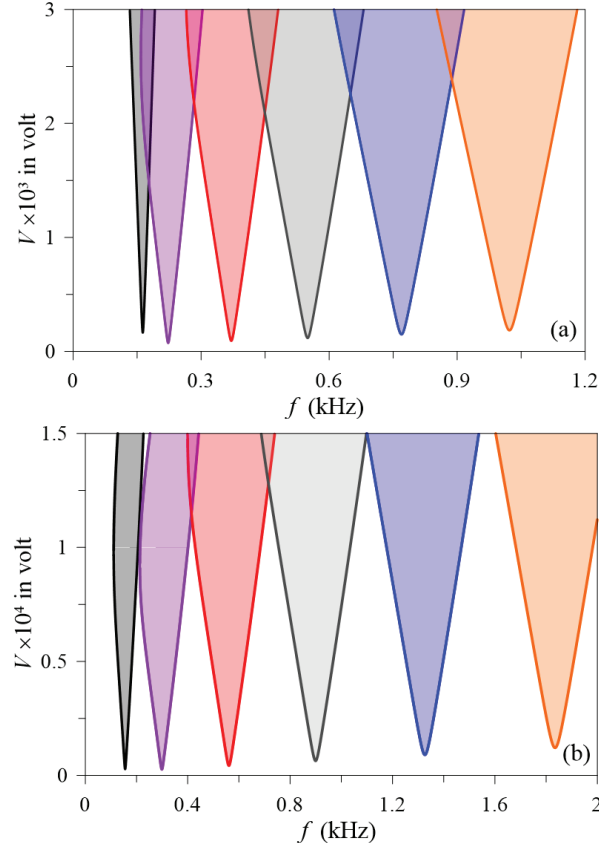


Figure 28: Activation regions of the principal parametric resonance of the lowest six modes for (a) $\lambda = 1020$ and (b) $\lambda = 500$.

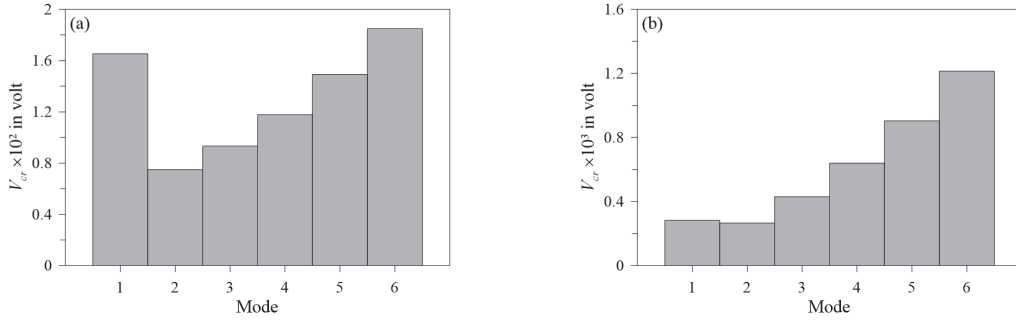


Figure 29: Critical voltage V_{cr} of the lowest six modes, assuming $d = d_o = 1.358 \times 10^{-11}$, $\zeta = 1\%$ and $b_s = b/50$ for (a) $\lambda = 1020$ and (b) $\lambda = 500$.

here performed took into account the default piezoelectric and mechanical parameters: $\zeta = 1\%$, $n_s = 2$, $b_s = b/50$ and $d = d_o = 1.358 \times 10^{-11}$ C/N. The voltages necessary to induce the superharmonic resonance can be reduced by varying any of the parameters above mentioned. The regions of multi-stability of the superharmonic resonance could also be found in closed form.

Figures 32 (a), (b) and (c) show the frequency-response curves of the superharmonic resonance for $\lambda = 1020$, $\varepsilon = (0.75, 1, 1.25) \times 10^{-4}$ and for $\lambda = 170$ and $\varepsilon = (1, 1.25, 1.5) \times 10^{-2}$, respectively. Here, solid and dashed lines indicate stable and unstable branches, respectively.

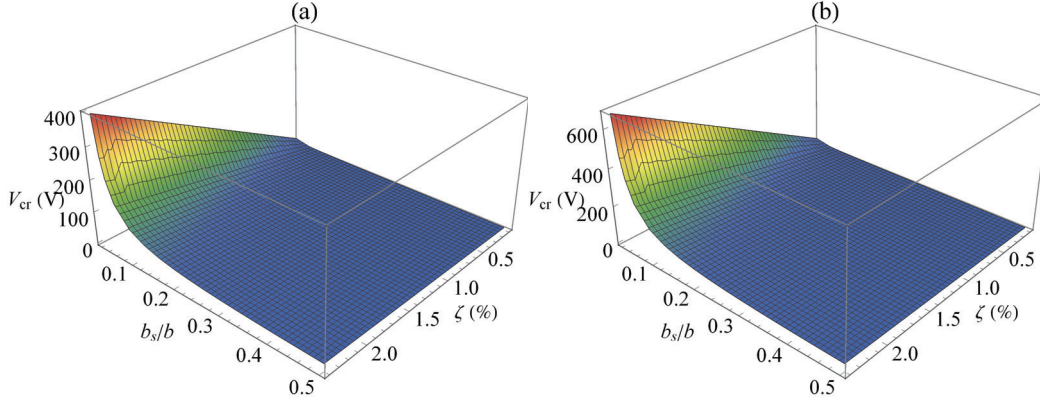


Figure 30: Critical voltage V_{cr} of the lowest mode, assuming $d = d_o = 1.358 \times 10^{-11}$ and varying ζ and b_s , for (a) $\lambda = 1020$ and (b) $\lambda = 500$.

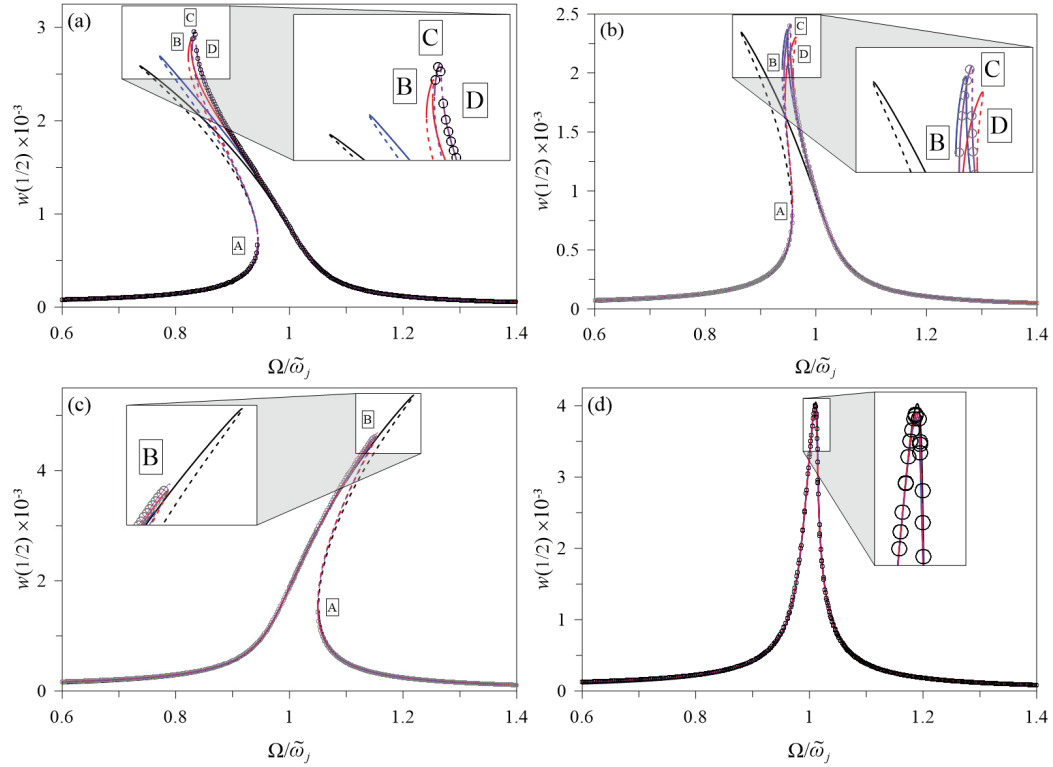


Figure 31: Frequency-response curves of the primary resonance of the lowest mode for (a) $\lambda = 1020$, $\varepsilon = 2.5 \times 10^{-5}$; (b) $\lambda = 500$, $\varepsilon = 5 \times 10^{-5}$; (c) $\lambda = 170$, $\varepsilon = 1 \times 10^{-3}$; (d) $\lambda = 50$, $\varepsilon = 1 \times 10^{-1}$. Black, blue, and red lines indicate the results of the third-, fifth-, and seventh-order asymptotic analysis, respectively, while gray circles indicate the results of the numerical time integrations of the reduced-order model with 10 modes, $n_s = 2$, $d = d_o$, and $b_s = b/50$.

Subharmonic resonance morphing. A subharmonic resonance of order one-third was investigated for morphing purposes with the AV voltage driving frequency $\Omega \approx 3\tilde{\omega}_j$. The frequency-response curves of the subharmonic resonance are shown in Fig. 33 for $\lambda = (1020, 500)$ and for selected values of the excitation amplitude ε . Moreover, Figure 33 also shows the bifurcation curves for different values of the damping ratio ζ .

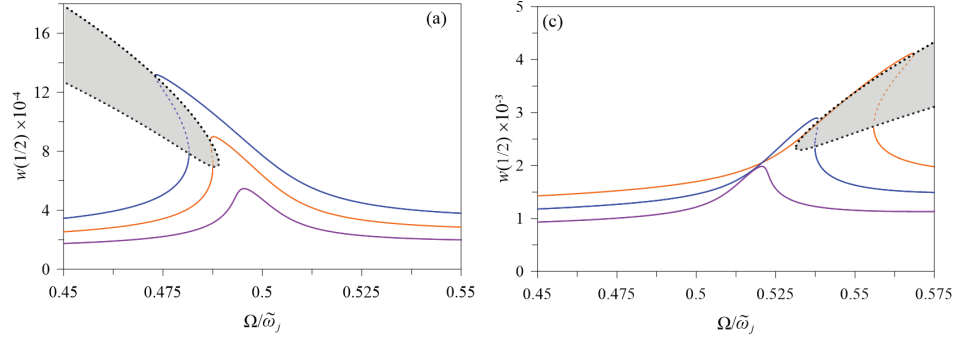


Figure 32: Frequency-response curves of the 2:1 superharmonic resonance of the lowest mode for (a) $\lambda = 1020$ and (c) $\lambda = 170$.

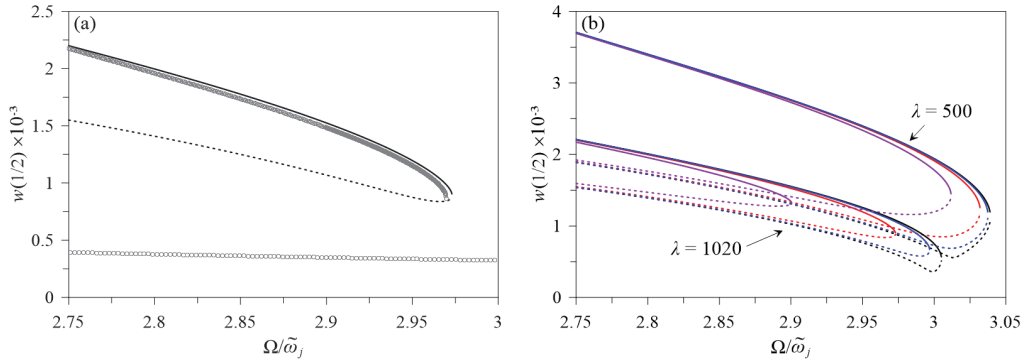


Figure 33: Frequency response curves of the one-third subharmonic resonance morphing for (a) $\lambda = 1020$ and $\zeta = 1\%$: results of the MS analysis (black lines) compared with the numerical time integration analysis (gray circles); and (b) for $\lambda = (1020, 500)$ and $\zeta = (0.1, 0.5, 1, 2)\%$. The solid lines indicate stable solutions, while dashed lines indicate unstable solutions.

To achieve large amplitude oscillations triggered by the one-third subharmonic resonance, higher voltages are necessary compared to the voltages driving parametric, primary, and superharmonic resonances. This is true when considering the mechanical parameters here adopted: $\zeta = 1\%$, $n_s = 2$, $b_s = b/50$ and $d = d_o = 1.358 \times 10^{-11}$ C/N. In particular, the curves shown in Fig. 33 are obtained for $\varepsilon = 1.25 \times 10^{-3}$ and $\varepsilon = 2.5 \times 10^{-3}$, respectively for $\lambda = (1020, 500)$ and $V = (4.6 \times 10^3, 18.7 \times 10^3)$ Volt, respectively. An optimization study spanning the panel design parameters within reasonable bounds may find more suitable ranges of voltages to trigger acceptable subharmonic resonance morphing. Yet, the obtained results suggest that the principal parametric resonances are the most powerful dynamic morphing mechanism.

Summary. Dynamic morphing of lightweight panels can be driven by embedded piezoelectric wires. This can be achieved by inducing various kinds of resonances some of which resort on the prestressed bent equilibrium due to the panel weight for very thin panels.

A full characterization of the large-amplitude nonlinear morphing motions born out of parametric resonances as well as direct resonances (i.e., primary, subharmonic and superharmonic) was obtained for the lowest modes. In particular, the activation regions for the frequencies and voltage amplitudes triggering the principal parametric resonances of various modes were determined analytically. The frequency response functions regulating

the morphing responses were calculated by adopting various asymptotic approximations. It was shown that higher-order approximations can attain an accurate description of the morphing motions as well as their range of multi-stability. This was also numerically confirmed via direct time integration of the panel equation of motion. Comprehensive parametric studies on the effect of the PVDF wires cross-sectional size, panel damping, and piezoelectric coupling coefficient, respectively, were carried out so as to identify the threshold voltages of the activation regions associated with the different resonances in order to achieve the desired shape morphing. The proven strong sensitivity of the activation voltage with the PVDF wires width and coupling coefficient entails that a design optimization for morphing applications can be achieved. The analyses performed for different panels ranging from ultra-thin to thick panels highlighted different nonlinear dynamic responses ranging from softening to hardening behaviors.

An optimization study searching for optimal panel design parameters values within reasonable bounds may lead to lower voltages to trigger meaningful morphing amplitudes under secondary resonances. The obtained results confirm that the principal parametric resonances, as suggested in the original research plan, are indeed the most powerful dynamic morphing mechanism that can be actuated by suitable voltages and, more importantly, this mechanism is robust in a reasonably large frequency bandwidth.

References

- [1] C. Thill, J. Etches, I. Bond, K. Potter, P. Weaver, Morphing skins, *Aeronautical Journal* 112 (2008) 117–139.
- [2] W. Li, D. Jin, Flutter suppression and stability analysis for a variable-span wing via morphing technology, *Journal of Sound and Vibration* 412 (2018) 410–423.
- [3] T. L. Grigorie, S. Khan, R. M. Botez, M. Mamou, Y. Mébarki, Design and experimental testing of a control system for a morphing wing model actuated with miniature BLDC motors, *Chinese Journal of Aeronautics* 33(4) (2020) 1272–1287.
- [4] J. Sun, Q. Guan, Y. Liu, J. Leng, Morphing aircraft based on smart materials and structures: A state-of-the-art review, *Journal of Intelligent Material Systems and Structures* 27(17) (2016) 1–24.
- [5] R. Bogue, Smart materials: a review of recent developments, *Assembly Automation* 32(1) (2012) 3–7.
- [6] O. Bilgen, M. I. Friswell, Piezoceramic composite actuators for a solid-state variable-camber wing, *Journal of Intelligent Material Systems and Structures* 25(7) (2014) 806–817.
- [7] I. Vessonen, Smart materials and structures: VTT Research Program 2000-2002, no. 225 in VTT Symposium, VTT Technical Research Centre of Finland, Espoo, 2003.
- [8] V. Guillot, A. T. Savadkoobi, C.-H. Lamarque, Analysis of a reduced-order nonlinear model of a multi-physics beam, *Nonlinear Dynamics* 97 (2) (2019) 1371–1401.
- [9] A. Givois, C. Giraud-Audine, J.-F. Deü, O. Thomas, Experimental analysis of nonlinear resonances in piezoelectric plates with geometric nonlinearities, *Nonlinear Dynamics* 102 (3) (2020) 1451–1462.
- [10] A. G. Moura, A. Erturk, Combined piezoelectric and flexoelectric effects in resonant dynamics of nanocantilevers, *Journal of Intelligent Material Systems and Structures* 24(3) (2018) 266–273.

- [11] J. Przybylski, G. Gasiorowski, Nonlinear vibrations of elastic beam with piezoelectric actuators, *Journal of Sound and Vibration* 437 (2018) 150–165.
- [12] K. M. Hu, H. Li, Multi-parameter optimization of piezoelectric actuators for multi-mode active vibration control of cylindrical shells, *Journal of Sound and Vibration* 426 (2018) 166–185.
- [13] S. Barbarino, O. Bilgen, R. M. Ajaj, M. I. Friswell, D. J. Inman, A review of morphing aircraft, *Journal of intelligent material systems and structures* 22 (9) (2011) 823–877.
- [14] I. Chopra, Review of state of art of smart structures and integrated systems, *AIAA journal* 40 (11) (2002) 2145–2187.
- [15] W. Lacarbonara, B. Carboni, Tensairity structure with shape-memory wire ropes, *uS Patent* 10,407,939 (Sep. 10 2019).
- [16] I. K. Kuder, A. F. Arrieta, W. E. Raither, P. Ermanni, Variable stiffness material and structural concepts for morphing applications, *Progress in Aerospace Sciences* 63 (2013) 33–55.
- [17] J. Sun, Q. Guan, Y. Liu, J. Leng, Morphing aircraft based on smart materials and structures: A state-of-the-art review, *Journal of Intelligent material systems and structures* 27 (17) (2016) 2289–2312.
- [18] D. Li, S. Zhao, A. Da Ronch, J. Xiang, J. Drofelnik, Y. Li, L. Zhang, Y. Wu, M. Kintscher, H. P. Monner, et al., A review of modelling and analysis of morphing wings, *Progress in Aerospace Sciences* 100 (2018) 46–62.
- [19] M. L. Thompson, On the material properties and constitutive equations of piezo-electric poly vinylidene fluoride (pvdf).
- [20] T. Ikeda, T. Ikeda, *Fundamentals of piezoelectricity*, Vol. 16, Oxford university press Oxford, 1990.
- [21] N. Murayama, K. Nakamura, H. Obara, M. Segawa, The strong piezoelectricity in polyvinylidene fluoride (pvdf), *Ultrasonics* 14 (1) (1976) 15–24.
- [22] A. Arena, F. Massimi, W. Lacarbonara, Dynamic morphing of actuated elastic membranes, in: *IUTAM Symposium on Exploiting Nonlinear Dynamics for Engineering Systems*, Springer, 2018, pp. 37–48.
- [23] A. H. Nayfeh, D. T. Mook, *Nonlinear oscillations*, John Wiley & Sons, 2008.
- [24] A. Arena, W. Lacarbonara, Piezoelectrically induced nonlinear resonances for dynamic morphing of lightweight panels, *Journal of Sound and Vibration* 498 (2021) 115951. doi:<https://doi.org/10.1016/j.jsv.2021.115951>.
- [25] A. Arena, W. Lacarbonara, Free vibration of micromembranes subject to prestress and pressure, in: *Proceedings of the ASME Design Engineering Technical Conference*, Vol. 4, Cham, 2015, boston, United States, 2–5 Aug 2015.
- [26] W. Lacarbonara, S. S. Antman, Parametric instabilities of the radial motions of non-linearly viscoelastic shells under pulsating pressures, *International Journal of Non-Linear Mechanics* 47 (2012) 461–472.
- [27] D. S. Simulia, *Abaqus 6.13 user’s manual*, Dassault Systems, Providence, RI.

- [28] P. F. Sensors, Technical manual, Measurement Specialties, Inc 25.
- [29] M. Pasquali, P. Gaudenzi, A nonlinear formulation of piezoelectric plates, *Journal of Intelligent Material Systems and Structures* 23 (15) (2012) 1713–1723.
- [30] W. Lacarbonara, *Nonlinear structural mechanics: theory, dynamical phenomena and modeling*, Springer Science & Business Media, 2013.
- [31] C. S. L. W. Carboni, B., Parametric resonances of nonlinear piezoelectric beams exploiting in-plane actuation, *Mechanical Systems and Signal Processing* in press.
- [32] W. Lacarbonara, C.-M. Chin, R. Soper, Open-loop nonlinear vibration control of shallow arches via perturbation approach, *J. Appl. Mech.* 69 (3) (2002) 325–334.
- [33] A. Arena, W. Lacarbonara, Dynamic morphing of elastic plates via principal parametric resonance, in: *Proceedings of the ASME Design Engineering Technical Conference*, Vol. 2, 2020.
- [34] A. Arena, M. Talò, M. P. Snyder, W. Lacarbonara, Enhancing flutter stability in nanocomposite thin panels by harnessing CNT/polymer dissipation, *Mechanics Research Communications* 104 (103495) (2020) 1–6.
- [35] E. Mettler, Stability and vibration problems of mechanical systems under harmonic excitation, in: *Dynamic Stability of Structures*, 1967, pp. 169–188.
- [36] W. Lacarbonara, A. H. Nayfeh, W. Kreider, Experimental validation of reduction methods for nonlinear vibrations of distributed-parameter systems: analysis of a buckled beam, *Nonlinear Dynamics* 17 (1998) 95–117.
- [37] COMSOL AB, *COMSOL Multiphysics User Guide and Model Library*, version 3.5a, Sweden (2008).

3.3. EXPERIMENTAL VALIDATION OF STATIC AND DYNAMIC MORPHING VIA PIEZOELECTRIC ACTUATION

The experimental validation of the local morphing capability of the designed skin coating, is presented in this section. Here the outer skin is considered to be coated with a piezoelectric film that is pixelated and that is used to provide static and dynamic morphing with or without the exploitation of parametric resonance. The study is completed with an experimental validation of the pixelated coating and its dynamic “local” actuation. The morphing capability of each single pixel is reached by combining in the design, the piezoelectric effect with mechanical instabilities (e.g. buckling). To this end, the pixels of the film are fully constrained so as to induce out-of-plane deformations when the film is piezoelectrically expanded. The degree of shape changes is controlled by controlling the voltage intensity. The pixels design for the experimental tests, and the validity of the experimental response are performed with the support of numerical simulations by investigating the coupled electro-mechanical problem and comparing the results in terms of critical voltage value and out-of-plane deformations obtained with different film thicknesses.

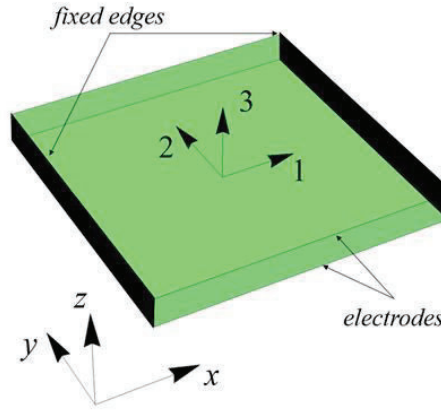


Figure 1: Schematic of the investigated structure with global and local reference frames.

The piezoelectric effect is adopted to develop out-of-plane deformation by exploiting the buckling phenomenon in a PVDF pixel. The study is conducted first by modeling the film, as a plate, via a finite element approach and its mechanical response is first evaluated. Preliminary considerations about poling orientation and geometric limitations are discussed in order to optimize the out-of-plane response of the film without exceeding in applied voltage, to prevent breakdown failure. The buckling phenomenon is then investigated by considering films with different thicknesses with particular focus on the onset of the critical condition as well as the films response in the post-buckling phase. Contrary to the literature, in this case, the axial force that induces the instability is provided by the film itself, thus no additional compliant layers are coupled to the system. Not only this implies a potentially simpler manufacturing process, but it also implies the capability of the film to self-activate. The work is carried out all the way up to the material pixeling for local morphing, static or dynamic, with or without parametric resonance.

3.3.1 Electro-Mechanical Model in support of the experiments

A square PVDF thin film with a length of $L=1$ mm, and a variable thickness in the range 5-20 μm is investigated. The plane of the plate is identified by the x, y axes, while the vertical direction is identified by the z -axis, as illustrated in Fig. 1. The coupled electro-mechanical problem is studied via a 3D finite-element approach by COMSOL Multiphysics. Typically, the constitutive equation of the 3D problem of a piezoelectric material is expressed in the so-called strain-charge form:

$$\begin{aligned}\mathbf{D} &= \mathbf{d}\boldsymbol{\sigma} + \epsilon_0 \boldsymbol{\epsilon}^r \mathbf{E}, \\ \boldsymbol{\epsilon} &= \mathbf{s}^E \boldsymbol{\sigma} + \mathbf{d}^\top \mathbf{E} \quad ,\end{aligned}\tag{1}$$

where $\mathbf{D}$ (3x1) is the electric displacement vector [C/m²], $\mathbf{d}$ (3x6) is the coupling matrix [C/N], $\epsilon_0=8.854 \cdot 10^{-12}$ is the permittivity constant [F/m], $\boldsymbol{\epsilon}^r$ (3x3) is the relative permittivity matrix and $\mathbf{E}$ represents the electric field [V/m].

The mechanical strains are collected in the vector $\boldsymbol{\epsilon}$ (6x1) and are related to the stress vector $\boldsymbol{\sigma}$ (6x1) [N/m²] via the compliance matrix $\mathbf{s}^E$ (6x1) that is expressed in [m²/N] and represents the inverse of the elasticity matrix $\mathbf{c}^E=(\mathbf{s}^E)^{-1}$.

Typically, the constitutive equations are expressed in the local reference frame, so that the stress and strain vectors appearing in Eqns. (1) are expressed as

$$\begin{aligned}\boldsymbol{\sigma} &= \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{31}, \sigma_{12}\}^\top, \\ \boldsymbol{\epsilon} &= \{\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, \epsilon_{23}, \epsilon_{31}, \epsilon_{12}\}^\top.\end{aligned}\tag{2}$$

According to this notation, the matrices containing the material electro-mechanical properties assume the following expressions:

$$\begin{aligned}\mathbf{s}^E &= \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & \cdot & \cdot & \cdot \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & \cdot & \cdot & \cdot \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{G_{12}} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \frac{1}{G_{23}} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{G_{31}} \end{bmatrix}, \\ \mathbf{d} &= \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & d_{15} & \cdot \\ \cdot & \cdot & \cdot & d_{24} & \cdot & \cdot \\ d_{31} & d_{32} & d_{33} & \cdot & \cdot & \cdot \end{bmatrix}, \\ \boldsymbol{\epsilon}^r &= \begin{bmatrix} \epsilon_{11}^r & \cdot & \cdot \\ \cdot & \epsilon_{22}^r & \cdot \\ \text{sym} & \cdot & \epsilon_{33}^r \end{bmatrix}.\end{aligned}\tag{3}$$

This mechanical model allows to properly study the piezoelectric effect and is here adopted to investigate the behavior of a poled PVDF thin film. According to the literature, poled PVDF exhibits strong orthotropic properties that are induced during the poling process, while the non-poled material is an isotropic polymer.

Table 1. Top: Young's and shear moduli [GPa]. Bottom: Poisson's ratios [-].

E_1	E_2	E_3	G_1	G_2	G_3
2.25	1.98	0.88	0.55	0.59	0.69

ν_{12}	ν_{23}	ν_{31}
0.22	0.27	0.27

The mechanical parameters adopted in the following numerical investigations are referred to an orthotropic material and the values are defined in Tab. 1, while the remaining Poisson's ratios are obtained by the following expressions:

$$\nu_{21} = \frac{E_2}{E_1} \nu_{12}, \quad \nu_{13} = \frac{E_1}{E_3} \nu_{31}, \quad \nu_{23} = \frac{E_2}{E_3} \nu_{32}. \quad (4)$$

On the other hand, the parameters governing the piezoelectric problem are given in Tab. 2. The reported values are drawn from the literature and resemble common poled PVDF characteristics: it can be noticed also a strong orthotropy not only in the mechanical properties but also in the piezoelectric properties.

Table 2. Top: Coupling matrix $\mathbf{d}^E$ [10^{-11} m/V]. Bottom: permittivity matrix ϵ^r [-].

$\mathbf{d}_{31}$	$\mathbf{d}_{32}$	$\mathbf{d}_{33}$	$\mathbf{d}_{15}$	$\mathbf{d}_{24}$
2.25	1.98	0.88	1.93	2.01

ϵ^r_{11}	ϵ^r_{22}	ϵ^r_{33}
6.8	8.4	7.3

In the next sections, the behavior of the film is investigated by exploiting its piezoelectricity to induce the out-of-plane buckling. Contrarily to traditional approaches where the axial force inducing the instability is provided by additional compliant mechanisms, in this study the compressive stress is directly provided by the film itself. First it is investigated the influence of the poling direction together with the induced orthotropy of the material. Subsequently, the buckling of the film is induced by applying voltage to the structure to study and understand how the different material parameters influence the overall behavior. This is useful to design and optimize the structural, as well as the piezoelectric properties.

Poling Directions

Preliminary simulations are conducted on the square plate with thickness $10 \mu\text{m}$ to understand the behavior of PVDF as the poling direction changes. The plate is mechanically constrained to suppress rigid motions while allowing the displacements induced by the applied voltage.

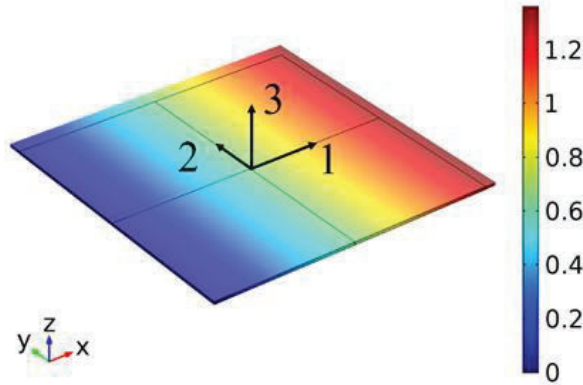


Figure 2: Deformed shape in configuration 1 where the local axes are aligned with the global axes, 1 kV is applied through the thickness of the film. Color legend refers to u [μm], the displacement along the x-axis.

Table 3. Comparing the effect of the poling directions.

	u [μm]	v [μm]	w [μm]
Condition 1	1.36	1.94	0.0297
Condition 2	0.0297	0.0136	1.94 10^{-4}
	ϵ_x [%]	ϵ_y [%]	ϵ_z [%]
Condition 1	1.36	1.94	0.0297
Condition 2	0.0297	0.0136	1.94 10^{-4}

A. Condition 1

The plate is poled in the most common configuration, i.e. the electrodes are placed on the top and bottom faces, thus the "strong" direction is oriented along the vertical axis. A 1000 V tension is applied to the bottom electrode (induced electric field is 1.0×10^8 V/m) and the deformed configuration is represented in Fig. 2.

B. Condition 2

The plate is poled in the in-plane direction, i.e. the electrodes are placed on the left and right faces, thus the "strong" direction is oriented along the x-axis. A 1000 V tension is applied to the right electrode (the induced electric field is 1.0×10^6 V/m) and the deformed configuration is represented in Fig.3.

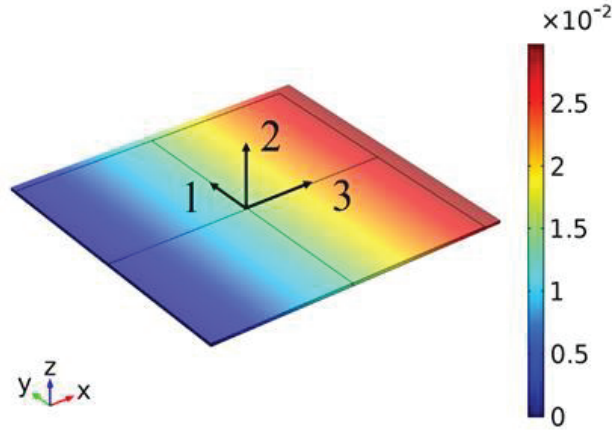


Figure 3: Deformed shape in configuration 2 where the local axes are aligned with the global axes, 1 kV is applied through the length of the film in the x-direction. Color legend refers to u [μm], the displacement along the x-axis.

Comparing the two obtained configurations, by adopting the same tension, a lower displacement is obtained in condition 2. In particular, by analyzing the values reported in Tab. 3, the first condition leads the highest displacement in the y direction, even though the higher deformation is achieved in the z direction. On the other hand, in condition 2, where the material is poled in order to maximize the response in the x-direction, the largest deformation is achieved in the x-direction, but the correspondent displacement is much lower than that obtained in condition 1.

This is due to the fact that, since the same voltage was applied in both conditions, the generated electric field is completely different in the two cases: in condition 1 the electric field acts through the thickness of the film ($10 \mu\text{m}$), while is acting in the x direction in condition 2 (1 mm). This implies that the resulting electric field in condition 2 is 99% smaller. This leads to a lower displacement in the desired direction, thus the best way to maximize the in-plane response is to pole the material in the z-direction and configure the electrodes as illustrated in Fig. 1.

Moreover, it can be noticed that, due to the strong orthotropy of poled PVDF, the highest in-plane piezoelectric coefficient is d_{32} . This implies that by applying voltage in the vertical direction, the highest deformation is induced in the local direction 2. Thus, to maximize the axial deformation

that might induce buckling it is of convenience to rotate the local axes around the z direction so to have the local axis 2 aligned with the x-direction and the local axis 1 aligned to -y.

This aspect is investigated in the next simulation, where the lateral displacement of the plate is restrained in the x-direction (as depicted in Fig. 1), in order to analyze the capability of the piezoelectric effect to induce stress. The results are shown in Fig. 4, where the stresses σ_{xx} along the x axis are reported in MPa.

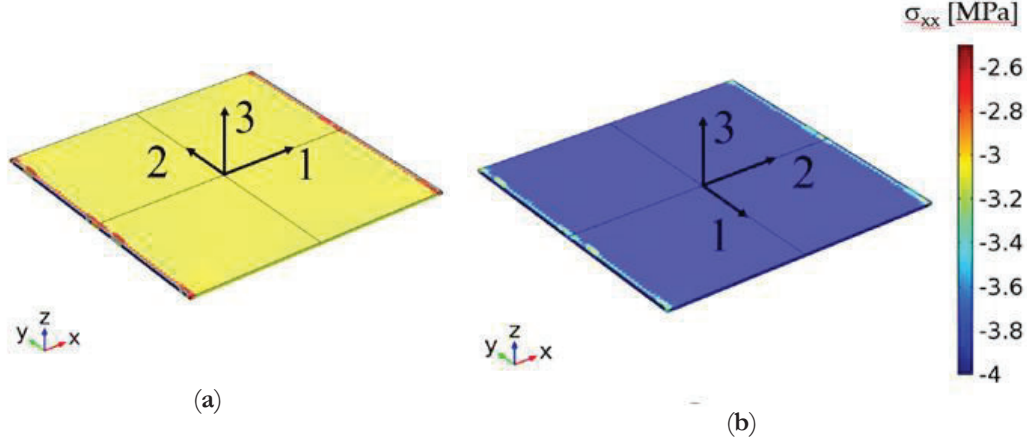


Figure 4: Stress along the x axis, σ_{xx} [MPa] when a voltage of 1000 V is applied. (a) the material axes are aligned with the global reference frame, the average compressive stress is 3.0 MPa; (b) the material axes are rotated, the average compressive stress is 3.8 MPa.

The induced compressive stress developed in the first configuration (left part) is around 3.0 MPa, while in the second configuration (right part) it increases up to 3.8 MPa. This analysis allows to understand how the strong orthotropy that characterizes the adopted material can be exploited to optimize the actuation effect: by properly constraining the morphing film it is possible to gain +30% more of compressive stress. It is thanks to this compressive state that it is possible to induce buckling and thus change shape to the structure.

Buckling Analysis

Once the axial displacement is constrained and voltage is applied, due to the piezoelectric effect, the plate tends to expand, and an axial stress arises. If there are no imperfection and the plate is perfectly flat the instability will not take place, but real structures are always characterized by the presence of imperfections. To induce the instability in the model it is necessary to introduce a perturbation: a small pressure (1.0×10^{-11} MPa) on the bottom face of the plate is applied to trigger buckling. As intended, such pressure induces a negligible out-of-plane displacement (0.012%) even though sufficient to be considered as a perturbation.

This is clearly shown in Fig. 5 where the obtained vertical displacement (w/L [%]) in center of the plate is represented as a function of the applied voltage. The simulations are conducted on 4 different thickness that are 5, 10, 15 and 20 μm , respectively.

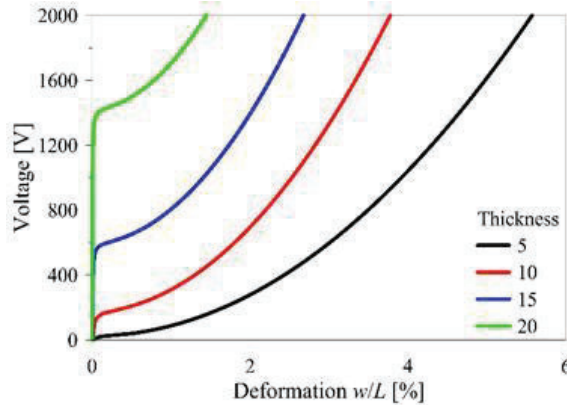


Figure 5: Applied voltage against the deformation in the center of the plate for the different considered thicknesses (5, 10, 15 and 20 μm).

As expected, all studied cases show that if the voltage is below a certain threshold value, a negligible vertical displacement is produced, whereas when it goes above the critical value, the plate undergoes a large vertical displacement up to 5.5% that corresponds to $w = 55 \mu\text{m}$ for the 5 μm -thick film.

Figure 6 shows the deformed shape at the highest voltage, 2000 V, for the different considered thicknesses. The results highlight the obtained out-of-plane displacement whose entity is mainly affected by the applied voltage. When the highest voltage is applied, the largest displacement is obtained by the thinner film, whose buckling is also activated at smaller voltages.

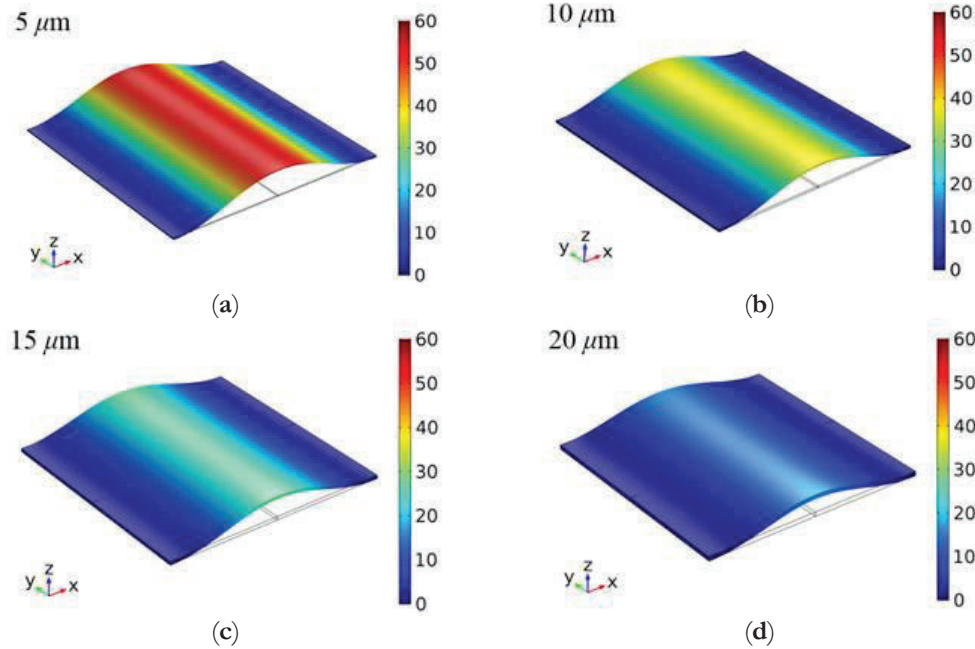


Figure 6: Deformed configuration at 2000 V for the different considered thicknesses. The color legend refers to the vertical displacement w [μm]. (a) 5 [μm]; (b) 10 [μm]; (c) 15 [μm]; (d) 20 [μm].

This observation has its negative counterpart: the film can be activated with small amount of energy and by doing this it can undergo large displacements, however its load-bearing capacity is limited. This is why thicker films are further considered.

On the other hand, it can also be noticed that by increasing the voltage above 2000 V, also the thicker films might reach larger displacements, but such high voltage will be closer to the breakdown voltage, that will cause an overall failure of the morphing system.

Such analysis allows to evaluate the critical value of tension above which the instability phenomenon takes place and the plate buckles. This can be done by evaluating the tangent to the w-V curve: Fig. 7 (left part) shows the behavior w-V curves for small deformations in order to highlight the triggering of buckling and the correspondent critical voltage for the different films. In particular, the critical voltage is obtained by considering the equivalent stiffness (right part of Fig. 7) that is represented by the local tangent of the w-V curve. More specifically, the voltage that causes buckling is identified as the local minimum of the equivalent stiffness, as shown in the right part of Fig. 7. The buckling voltages are reported in Tab. 4.

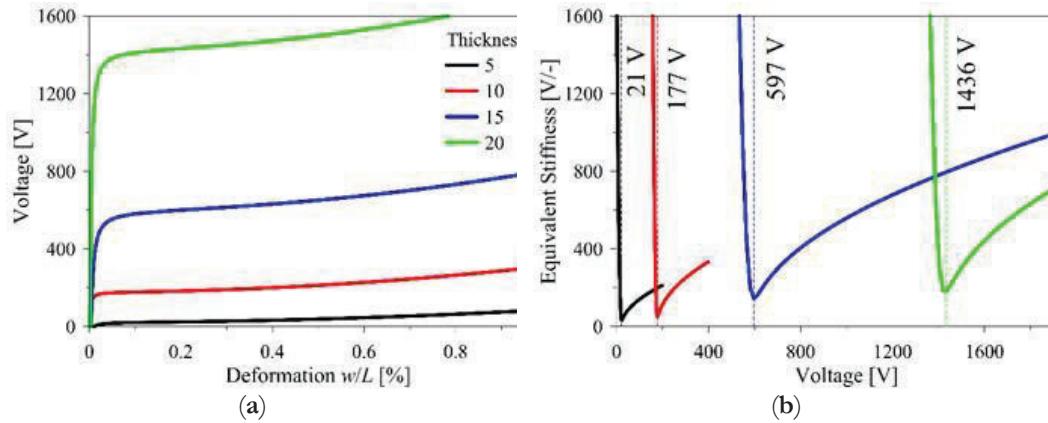


Figure 7: Identification of the critical tension that gives rise to the buckling phenomenon. (a) Zoom of the deformation-Tension graph to highlight the buckling onset; (b) equivalent stiffness (voltage/deformation) against correspondent voltage: the minimum of the curve identifies the critical voltage that corresponds to the buckling condition.

As expected, the critical voltage necessary to induce buckling increases considerably on thicker films: the 20 μm film is only 4 times thicker than the 5 μm film but its critical voltage is two orders of magnitude higher.

Table 4. Buckling critical voltage.

Thickness [μm]	5	10	15	20
Voltage [V]	21	177	597	1436

It is also of interest to investigate how stresses grow within the film when the voltage increases. Figure 8 shows the behavior of the axial stress σ_{xx} evaluated in the center of the film, in particular on the bottom face (solid lines) and on the top face (dashed lines). Before that the instability takes place, the film expands with increasing voltage, but since the axial displacement is restrained the film is subject to a uniform compressive state: both the bottom and the top face are characterized by a negative σ_{xx} . On the other hand, as the voltage reaches the critical value, the film experiences a sudden change of configuration switching to a bending state. In this case, since the deformation is in the direction of positive z, the bottom face results always compressed, while the top face switches in a tensile state.

The analysis is performed by considering the four thicknesses in order to compare the response of different membranes. It is worth noting that the thinner film (black line) experiences the instability at extremely low voltages and as voltage increases the developed axial stress increases rapidly. In particular, when the voltage is below the critical value of the thicker films the resulting stress in the 5 μm film is higher, i.e. at 1200 V the developed stress in the 20 μm film is $\sigma_{xx} = 2$ MPa, while in the 5 μm film it is $\sigma_{xx} = 5$ MPa.

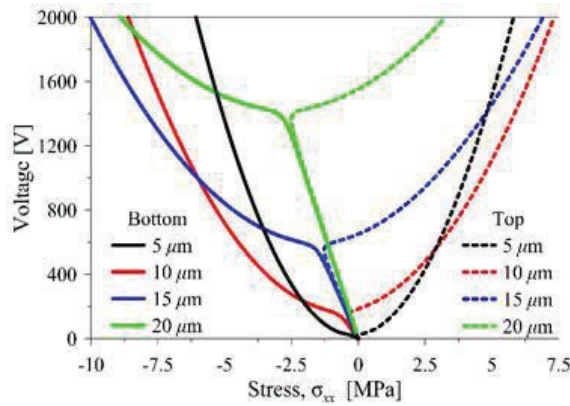


Figure 8: Axial stress σ_{xx} in the center of the film evaluated at the bottom face (solid lines) and at the top face (dashed lines).

Such study allows to understand the variation of stress within the structure before and after the instability takes place. It is thus possible to design a morphing structure that changes shape thanks to its piezoelectricity. The study allows to accurately tailor the shape change with a careful control of the induced stresses.

3.3.2 Static morphing

The numerical investigations discussed in the previous sections are devoted to predict the electro-mechanical behavior of the proposed morphing structure. The results are also validated through preliminary experimental campaign. To this aim, piezoelectric specimens are prepared with piezoelectric films of two different thicknesses, 10 and 40 μm , trimmed to an overall length of 6 mm and a width of 2 mm. The final dimensions were defined in order to have a free length of 4 mm plus 1 mm on each end to provide mechanical clamping as well as the required electrical connection, see Fig. 9. The specimens object of the experimental campaign are reduced to a beam-like structure instead of a square plate in order to simplify the mechanical behavior and better highlight the fundamental aspects of the proposed morphing material.

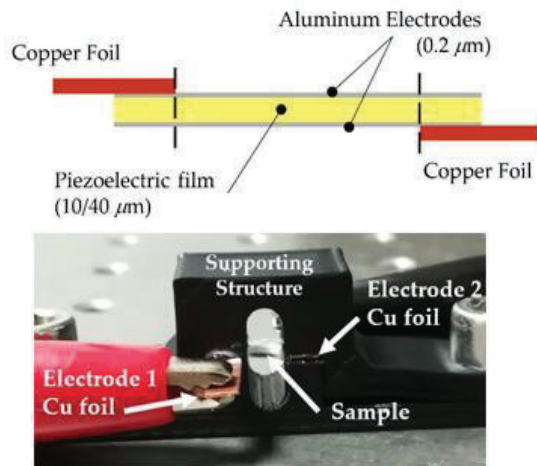


Figure 9: Schematic of the experimental set up (left) and picture of the actual testing rig (right).

As depicted in the right part of Fig. 9, an ad hoc supporting structure was designed and in-house manufactured. This revealed to be a key point of the conducted experiments, because it ensures very stable electro-mechanical boundary conditions which allow an accurate repeatability of the tests.

Besides the testing structure, the experiments were conducted by the use of a high voltage power supply to provide the required tension (up to 2 kV), while the corresponding displacements were acquired via a microscope camera that allowed to continuously monitor the specimen configuration. This implies that the measurement of the displacements are performed via a digital image correlation approach.

The experiments are conducted in order to induce the samples buckling under the application of increasing voltages. As expected, and predicted by the simulations, at low voltages the configuration of the tested samples does not change significantly, whereas when a specific threshold voltage is reached a dramatic configuration change is observed. Such behavior was captured during the test and the corresponding pictures are illustrated in Fig. 10.

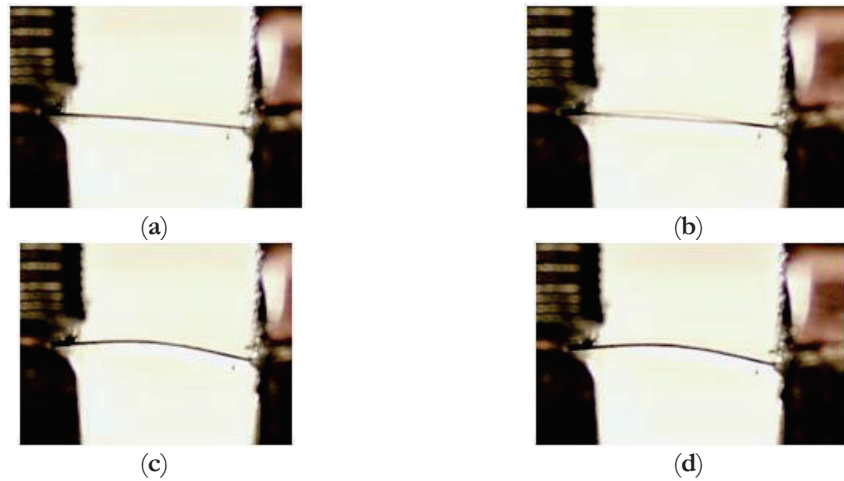


Figure 10: Deformed configuration of the sample with thickness 40 μm at selected voltage values. (a) 0 V; (b) 500 V; (c) 750 V; (d) 1000 V.

The pictures were taken at selected voltage values in the range 0-1 kV to capture the buckling of the sample, that in the case of 40 thickness takes place around 0.8-1.0 kV. As expected, at 0 V the sample is straight, while as the voltage increase small displacements are experienced, but when the voltage approaches the critical value a clear bent shape is attained.

Thanks to an image correlation algorithm it was possible to correlate the out-of-plane displacement and the input voltage. Results are shown in Fig. 11, where the measured displacements are reported against the corresponding applied voltages.

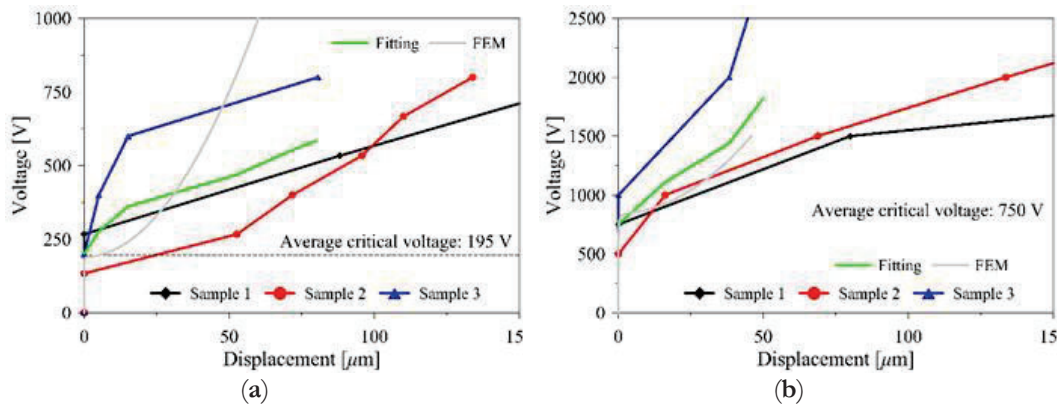


Figure 11: Experimental buckling diagram evaluated for families of samples with different thicknesses. (a) 10 μm ; (b) 40 μm .

The test was performed on 3 samples per each selected thickness, i.e. 10 and 40 μm , respectively. To better understand the behavior of the manufactured samples, the results are resumed by averaging the response of the three samples, and the fitting line is represented in Fig. 11 by the green lines. Moreover, the results of the numerical simulations are also represented in Fig. 11 by the gray lines. It can be noticed that the thinner samples attain the critical condition at lower voltages and the average value that induces buckling is 195 V, in agreement with the value predicted by the numerical simulations (see gray line in Fig. 11(a)). On the other hand, the results obtained for the thicker samples reveal a higher critical voltage, 750 V, such value is still in good agreement with the simulations. Besides the accuracy of the theoretical predictions, the experimental data are affected by a considerable large dispersion in the results. This is mainly due to manufacturing imperfections that lead to non-perfectly straight and flat samples, moreover the trimming technique caused non-negligible edge distortions. Larger dispersion was observed in the thinner samples, because those effects were more evident.

Although, the experiments well confirmed the capability of the material to change shape according to the applied input. The buckled configuration is characterized by large displacements, that can be actively governed.

3.3.3 Dynamic morphing by exploiting parametric resonance

The manufactured samples have been extensively investigated when D/C voltage is applied, thus in this section the results obtained with A/C voltage will be discussed. To predict the dynamic behavior of the material a reduced order model is adopted, and preliminary simulation are conducted to evaluate the possible resonance frequencies.

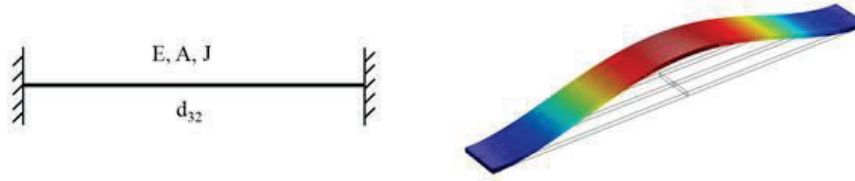


Figure 12: Schematic of the studied case.

The thin film is modeled according to the Mettler beam theory, where the transverse beam motion is studied taking into account the geometric nonlinearity arising from the axial stress. The length of the beam is l and both ends are clamped, as depicted in Fig. 12. The equation of motion, in moderate displacement regimes, can be written as

$$\ddot{w} + \zeta \dot{w} + w'''' - w'' \left[\chi \int_0^1 \frac{1}{2} (w')^2 dx - \chi \varepsilon_{pz} \right] = f \quad (1)$$

where ζ is the damping factor and $\chi = AE^2/J$ is the nondimensional axial stiffness ratio in which A and J are the cross section area and the moment of inertia. The piezoelectric effect is reduced to the only active strain component $\varepsilon_{pz} = d_{32}E_z$. The assumption is made by considering the film poled in the thickness direction and thus the electric field is applied in the vertical direction, i.e. $E_z = V/s$, where V is the applied voltage and s is the film thickness. The piezoelectric coefficient that acts in the axial direction is d_{32} .

The equation is solved by adopting the Galerkin approach, using a suitable number of modes. The response is compared with the solution obtained by a fully 3D nonlinear finite element model, to evaluate the good agreement with respect to the proposed formulation.

Preliminary simulations are conducted to evaluate the response in a static regime when the film is subject to a transverse load. The solution is compared with a full 3D nonlinear finite element model. The left part of Fig. 13 shows the result of the comparison between the two approaches in two different cases: only a gravitational load is applied (solid lines) and also a

voltage of 40 V is applied (dashed lines). In both cases it is evident how the parametric model well captures the behavior of the beam even when the piezoelectric effect is induced: the difference in terms of maximum displacement turns out to be less than 6%. It is thus observed the onset of the static buckling of the beam (right part of Fig. 13): for low voltages the structure shows a pseudo-linear behavior, although when the voltage overcomes a certain threshold, the equilibrium instability is observed. The response is then compared to that obtained by the FEM model, and a good agreement is observed for moderately large displacement amplitudes. The static analyses allow to foresee the good capability of the proposed method to develop self-activated morphing skins, although the actuation voltages are considerably high.

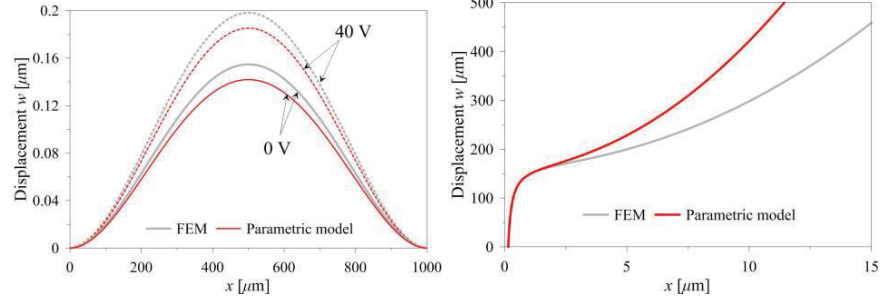


Figure 13: Left: Static deformation at constant voltage. Right: maximum displacement with increasing voltage.

The effect of a harmonic voltage is investigated next. This means that the term ε_{pz} appearing in Eqn. (1) is a function of time as: $\varepsilon_{pz} = d_{32}V_0 \cos(\Omega t)/s$. In this case, the applied voltage ($V_0 = 30$ V) is characterized by a frequency which is varied around the first natural frequency of the structure, in order to investigate possible resonance conditions.

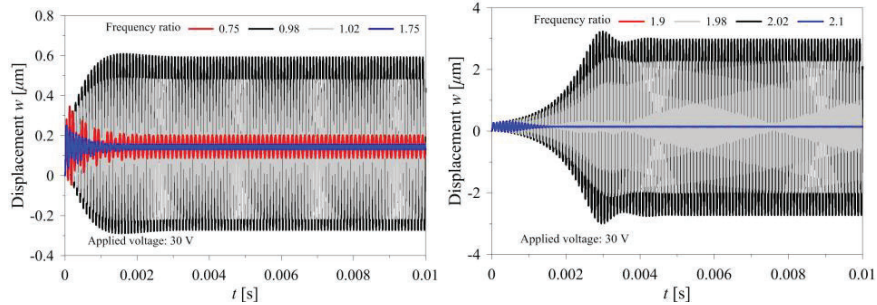


Figure 14: Response of the beam in primary resonance condition, $\Omega \approx \omega_1$ (left part) and under parametric resonance, $\Omega \approx 2\omega_1$ (right part).

The excitation given by the piezoelectric effect acting in the axial direction gives rise not only to a parametric resonance ($\Omega \approx 2\omega_1$), but also to a primary resonance ($\Omega \approx \omega_1$), as shown in Fig. 14. The latter condition is due to the presence of a non trivial equilibrium configuration (i.e., given by the self-weight).

This allows to drive the deformed shape of the film in dynamic regimes towards moderate amplitudes, when the actuation frequencies are close to ω_1 , or towards large amplitudes when the parametric resonance arises.

The same setup described before is adopted to test the sample in dynamic conditions, but since the signal generator can apply a maximum voltage of ± 10 V, only the $8 \mu\text{m}$ samples are considered in this test.

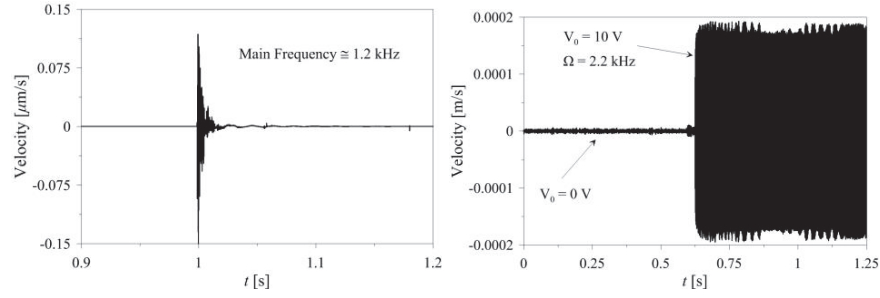


Figure 15: Left part: Free oscillation tests performed to identify the main frequency of the structure. Right part: dynamic buckling test at specific frequency (2.2 kHz).

Preliminary analyses are conducted to identify the natural frequencies of the structure in order to properly set the frequency range of the dynamic buckling tests. The structure is subject to an impulse and the free oscillations are acquired by the use of a laser scanner vibrometer: the main frequency turns out to be ≈ 1.2 kHz as illustrated in the left part of Fig. 15. The dynamic test will thus be focused in the range $[0, 3.0]$ kHz. The right part of Fig. 15 shows the result in terms of velocity when the signal generator is activated during the acquisition. The piezoelectric effect is clearly visible since the response of the structure switches to a strong harmonic oscillation state.

It is thus possible to investigate the behavior of the structure when the frequency of the applied voltage varies in the range previously identified. The results are shown in Fig. 16 where the maximum oscillation amplitude is reported against the corresponding applied frequency.

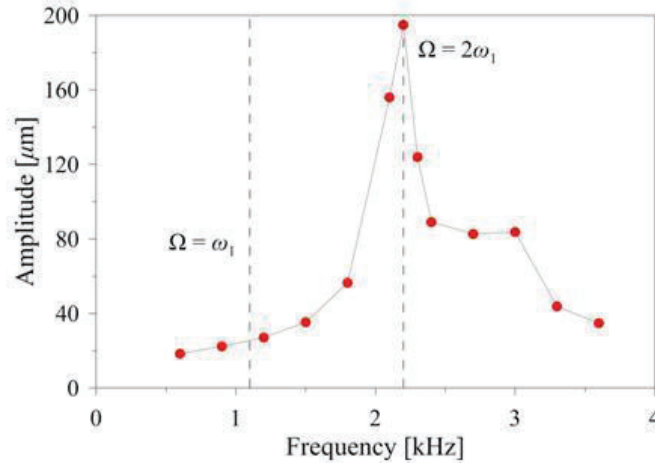


Figure 16: Maximum oscillation amplitude against the corresponding applied frequency.

As expected, the oscillation amplitude experiences a large increase when the applied frequency is close to $2\omega_1 \approx 2.1\div 2.3$ kHz: the system is subject to the parametric resonance. On the contrary, the direct resonance $\Omega = \omega_1$ was not observed in the experiments. This might be due to the fact that the real initial configuration of the sample is not only influenced by the self-weight as supposed in the simulations, but the actual initial deformed shape is mostly affected by fabrication imperfections which induce unpredictable deformations.

An improved setup is designed to evaluate the dynamic response with higher accuracy. The supporting system is first designed with CAD software and then 3D-printed. The resulting setup is shown below in Figure 17.

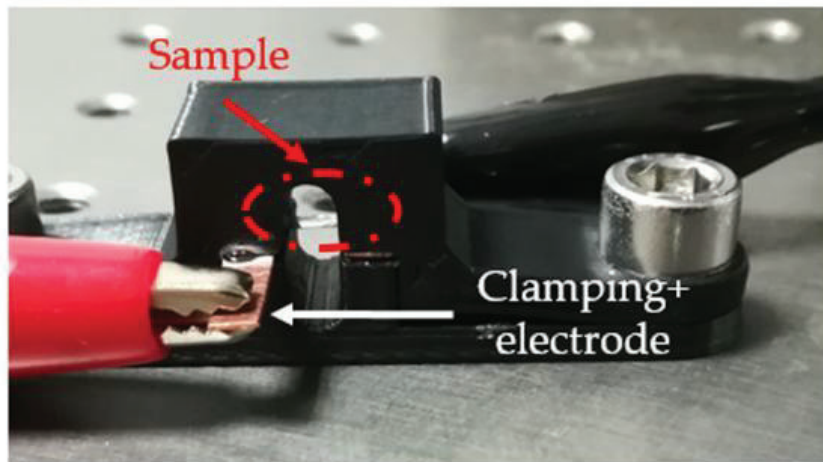


Figure 17: Improved setup.

Such setup allows to get better response signals making easier the post-processing phase.

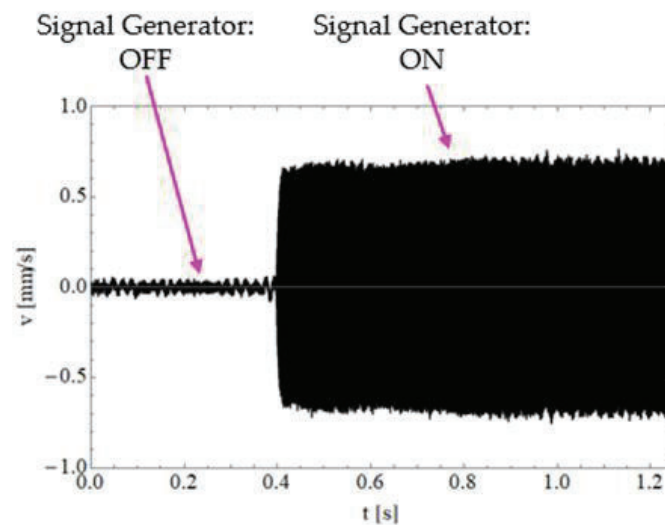


Figure 18: Experimental result.

The response of the structure is evaluated by using a A/C signal generator that allows to apply a 20 V (peak-to-peak) voltage with a continuously varying frequency ranging from 0.5 kHz up to 20 kHz. The buffer time for such frequency sweep is set up to 2 s. the approach allows to quickly and automatically detect the resonance frequencies of the system. As example a typical acquired response is shown in Figure 19.

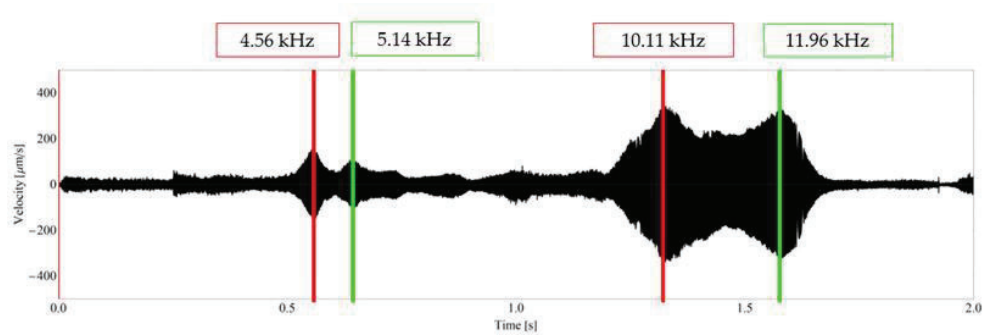


Figure 19: Time domain response.

Multiple resonance peaks can be observed, but the key aspect is that the morphing structure can be tuned to change its shape and vibrate at different frequencies as well as different amplitudes. This means that the approach can be adopted to design shape-changing structures that have a wide range of tunability in resonance frequencies as well as in amplitudes.

3.3.4 Local morphing of a pixelated coating by exploiting parametric resonance

After characterizing the single element, the morphing principle is extended to a uniform surface that has local “pixel” where each of them is identical to the single element described before.

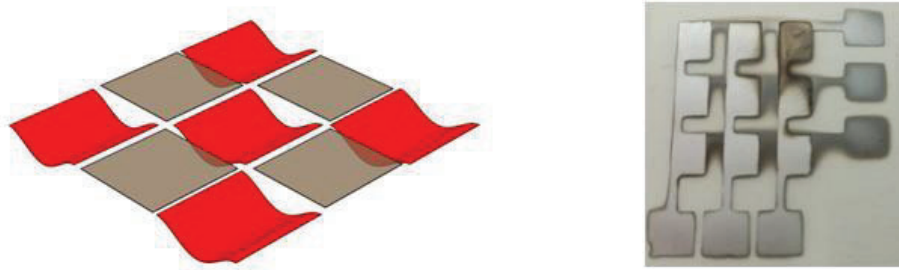


Figure 20: Fabricated sample and experimental morphing reported in terms of max amplitude of deformation per actuated pixels.

The uniform surface is made by a thin film of PVDF, where aluminum electrodes were deposited on its surface. The logic with which the electrodes have been designed is represented in figure 21.

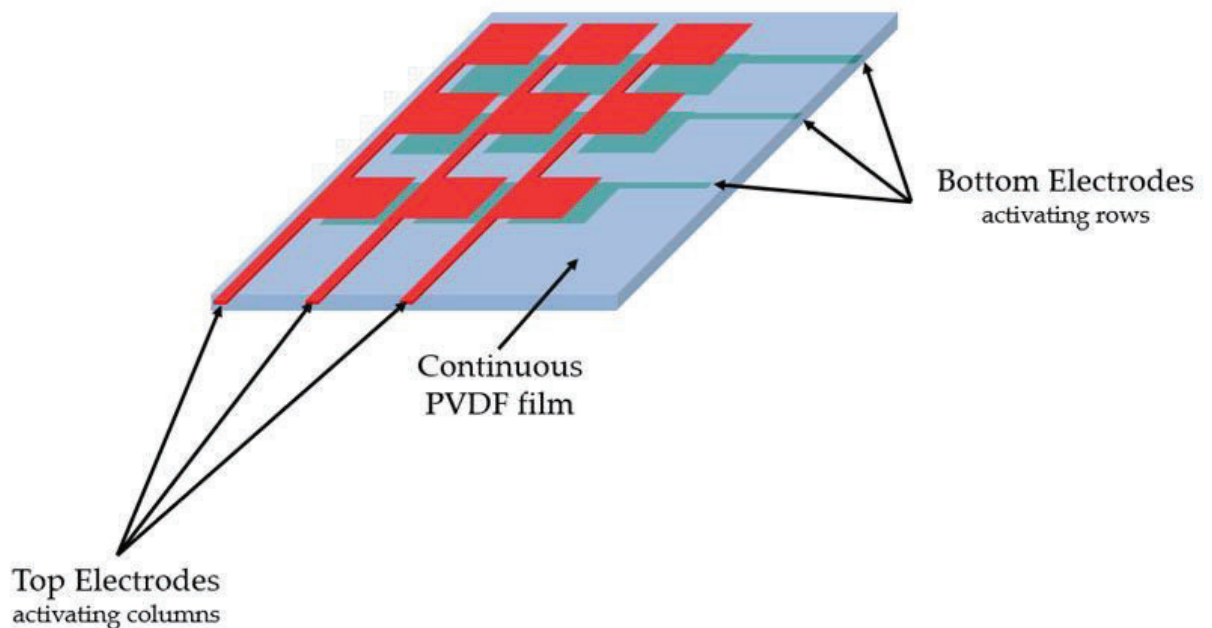


Figure 21: Logic of the sample functionalization to activate independently each pixel.

By selecting separately a single row (i) and a single column (j) it is possible to activate the single pixel (i,j). Such arrangement allows to reduce drastically the overall number of connecting wires, that is $i \times j$, while in a traditional arrangement will be $2 \times i \times j$.

The fabricated sample is then installed on a fixed support as depicted in Figure 22.

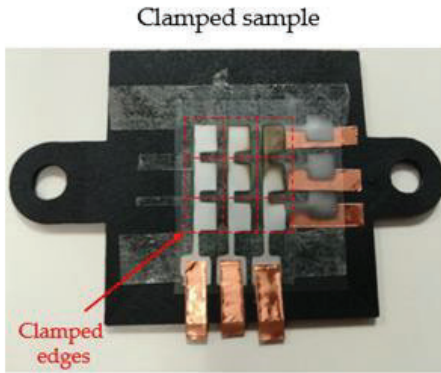


Figure 22: 3D printed and functionalized support for local testing of the sample.

Additional copper tape patches are adopted to allow a proper connection with external wires. Finally, the sample is activated by applying specific A/C voltage by the use of a signal generator. To identify the main frequencies of the system a frequency sweep is performed according to the following picture.

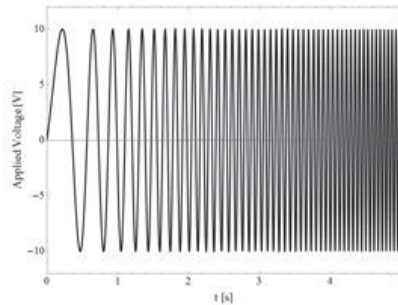


Figure 23: Frequency sweep.

A buffer of 5 s is adopted while the frequency ranges continuously from 0.5 kHz to 20 kHz. Such signal is applied subsequently at each pixel and the mechanical response of the morphing skin is acquired via a laser scanning vibrometer pointing subsequently on each pixel. An example of the acquired response in terms of time history is represented below.

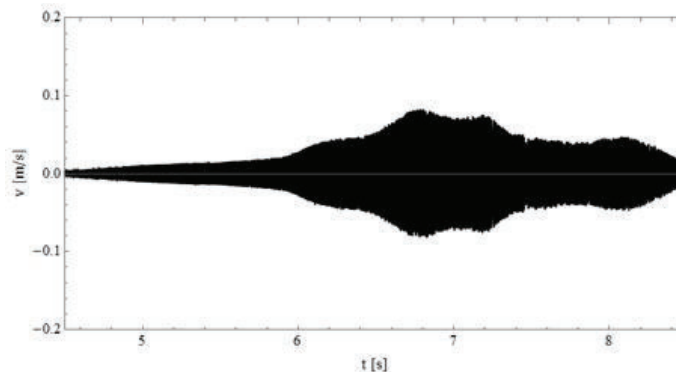


Figure 24: Acquired signal from the frequency sweep.

Such analysis allows to evaluate the main frequencies of the skin and the peak frequency is adopted to perform the characterization of the morphing skin. In particular, a single frequency is selected and the signal generated is applied continuously to a selected pixel, while the mechanical response is evaluated on all the pixels even if not directly activated.

Figure 25 shows an example of the acquired experimental data. The activated pixel is highlighted in red and the response amplitude obtained on each pixel is represented by the black bullet. It can be observed that the activated pixel is undergoing a displacement while the others are almost blocked.

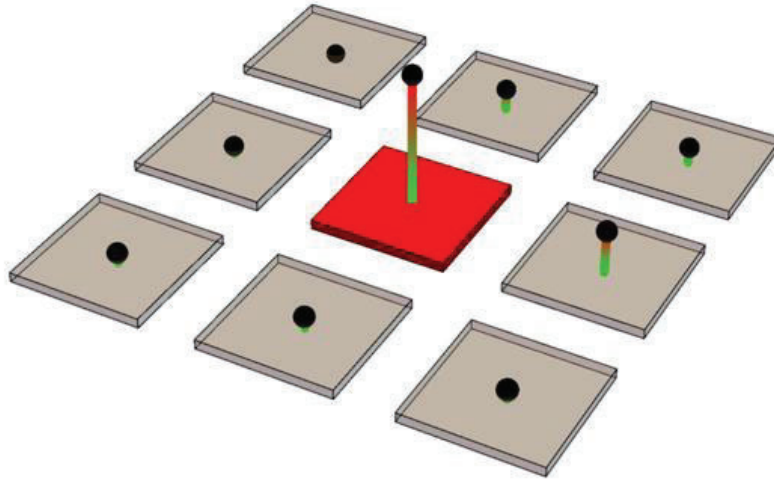


Figure 25: Experimental result of local morphing when the center pixel is activated.

Unfortunately, the response is not perfect for all tested pixels. When the pixels represented in figure 26 are activated, also other pixels are being activated to some extent. Since the pixels are mechanically independent, it is believed that such effect is due to migration of charges to adjacent electrodes. This issue can be solved easily by modifying slightly the fabrication process. Conceptually the proposed idea was demonstrated successfully in most pixels as shown for instance for the case in Figure 25.

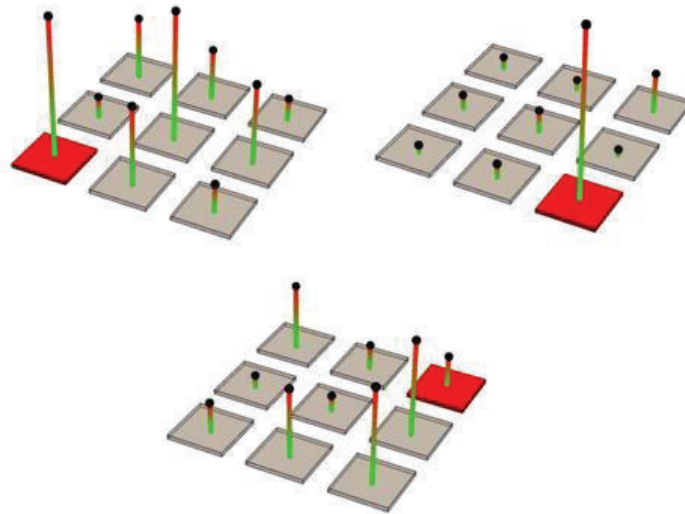


Figure 26: Experimental result of local morphing when the center pixel is activated.

REPORT DOCUMENTATION PAGE					Form Approved OMB No. 0704-0188	
The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to the Department of Defense, Executive Service Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.						
1. REPORT DATE (DD-MM-YYYY) 07-04-2021		2. REPORT TYPE Final			3. DATES COVERED (From - To) 15 Mar 2017 - 31 Dec 2020	
4. TITLE AND SUBTITLE Highly reconfigurable, multistable composite with tunable global/local morphing capability					5a. CONTRACT NUMBER	
					5b. GRANT NUMBER FA9550-17-1-0014	
					5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Prof Giulia Lanzara Prof Walter Lacarbonara					5d. PROJECT NUMBER	
					5e. TASK NUMBER	
					5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Rome, Roma Tre Rome, IT					8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) EOARD Unit 4515 APO AE 09421-4515					10. SPONSOR/MONITOR'S ACRONYM(S) AFRL/AFOSR/IOE (EOARD)	
					11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Distribution A; Approved for public release; distribution is unlimited						
13. SUPPLEMENTARY NOTES The subtitle erroneously says "Annual Report April 20, 2018." However, this is a copy/paste typo. The contents of this report are actually the final report covering the entire period of performance. The subtitle correction is not worth the effort to fix.						
14. ABSTRACT Bio-inspired morphing air vehicles have the potential to provide superior system capabilities by performing multi-scale shape reconfigurations. Biological systems are often formed with materials/mechanisms that allow them to adapt their body shape at the micro- or macro-scale in response to continuously monitored environmental parameters or to achieve desired functions. This inspired the current work. This grant explored design and multi-physics modeling of a multi-stable muscular carbon fiber composite plate and a 3D muscular composite material to achieve macro, global shape change. It also developed self-activated muscular piezoelectric "pixels" material that achieved localized shape change and tuning through resonate frequency modulation. This grant also produced a prototype and concept demonstrator. Multiple papers were published and well-recieved conference talks were given.						
15. SUBJECT TERMS musculo-skeletal; morphing; EOARD; IOE; metamaterials; multi-stable; composites						
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT SAR	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON David M. Garner	
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER (Include area code) +44 1895 616 420	

INSTRUCTIONS FOR COMPLETING SF 298

1. REPORT DATE. Full publication date, including day, month, if available. Must cite at least the year and be Year 2000 compliant, e.g. 30-06-1998; xx-06-1998; xx-xx-1998.

2. REPORT TYPE. State the type of report, such as final, technical, interim, memorandum, master's thesis, progress, quarterly, research, special, group study, etc.

3. DATES COVERED. Indicate the time during which the work was performed and the report was written, e.g., Jun 1997 - Jun 1998; 1-10 Jun 1996; May - Nov 1998; Nov 1998.

4. TITLE. Enter title and subtitle with volume number and part number, if applicable. On classified documents, enter the title classification in parentheses.

5a. CONTRACT NUMBER. Enter all contract numbers as they appear in the report, e.g. F33615-86-C-5169.

5b. GRANT NUMBER. Enter all grant numbers as they appear in the report, e.g. AFOSR-82-1234.

5c. PROGRAM ELEMENT NUMBER. Enter all program element numbers as they appear in the report, e.g. 61101A.

5d. PROJECT NUMBER. Enter all project numbers as they appear in the report, e.g. 1F665702D1257; ILIR.

5e. TASK NUMBER. Enter all task numbers as they appear in the report, e.g. 05; RF0330201; T4112.

5f. WORK UNIT NUMBER. Enter all work unit numbers as they appear in the report, e.g. 001; AFAPL30480105.

6. AUTHOR(S). Enter name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. The form of entry is the last name, first name, middle initial, and additional qualifiers separated by commas, e.g. Smith, Richard, J, Jr.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES). Self-explanatory.

8. PERFORMING ORGANIZATION REPORT NUMBER. Enter all unique alphanumeric report numbers assigned by the performing organization, e.g. BRL-1234; AFWL-TR-85-4017-Vol-21-PT-2.

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES). Enter the name and address of the organization(s) financially responsible for and monitoring the work.

10. SPONSOR/MONITOR'S ACRONYM(S). Enter, if available, e.g. BRL, ARDEC, NADC.

11. SPONSOR/MONITOR'S REPORT NUMBER(S). Enter report number as assigned by the sponsoring/monitoring agency, if available, e.g. BRL-TR-829; -215.

12. DISTRIBUTION/AVAILABILITY STATEMENT. Use agency-mandated availability statements to indicate the public availability or distribution limitations of the report. If additional limitations/ restrictions or special markings are indicated, follow agency authorization procedures, e.g. RD/FRD, PROPIN, ITAR, etc. Include copyright information.

13. SUPPLEMENTARY NOTES. Enter information not included elsewhere such as: prepared in cooperation with; translation of; report supersedes; old edition number, etc.

14. ABSTRACT. A brief (approximately 200 words) factual summary of the most significant information.

15. SUBJECT TERMS. Key words or phrases identifying major concepts in the report.

16. SECURITY CLASSIFICATION. Enter security classification in accordance with security classification regulations, e.g. U, C, S, etc. If this form contains classified information, stamp classification level on the top and bottom of this page.

17. LIMITATION OF ABSTRACT. This block must be completed to assign a distribution limitation to the abstract. Enter UU (Unclassified Unlimited) or SAR (Same as Report). An entry in this block is necessary if the abstract is to be limited.