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A Category-Theoretic Approach to Agent Interaction: Information, Communication, Planning, and Learning

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This report, submitted to Doug Riecken at the Air Force Office of Scientific Research, summarizes my group's scientific progress on the research sponsored by
AFOSR grant FA9550-19-1-0113, titled A Category-Theoretic Approach to Agent Interaction: Information, Communication, Planning, and Learning.
Category theory is a theory of relationships and composition. As such it has
become a gateway to pure math, providing rigorous connections between algebra
and number theory, logic and programming, geometry and topology, and probability
and measure. My goal during this grant was to show that category theory is also
broadly applicable outside of mathematics, from data and knowledge, to information
and communication.
Below I will briefly summarize the work that this grant has enabled my team—
consisting of myself, Brendan Fong, Rémy Tuyeras, and

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A Category-Theoretic Approach to Agent Interaction: Information, Communication, Planning, and Learning

David I. Spivak

March 22, 2021

This report, submitted to Doug Riecken at the Air Force Office of Scientific Research, summarizes my group's scientific progress on the research sponsored by AFOSR grant FA9550-19-1-0113, titled *A Category-Theoretic Approach to Agent Interaction: Information, Communication, Planning, and Learning*.

Category theory is a theory of relationships and composition. As such it has become a gateway to pure math, providing rigorous connections between algebra and number theory, logic and programming, geometry and topology, and probability and measure. My goal during this grant was to show that category theory is also broadly applicable outside of mathematics, from data and knowledge, to information and communication.

Below I will briefly summarize the work that this grant has enabled my team—consisting of myself, Brendan Fong, Rémy Tuyeras, and Paolo Perrone—to complete over the past two years.

1 Summary of scientific progress

Graphical logic

A now classic query was posed by Tim Berners Lee: he wanted the internet to be able to answer questions like "tell me all the cardiologists in my area who take my insurance and are free when I am next week". This sort of question can be answered by a database, but there is a large barrier to entry in terms of learning SQL.

We have developed a formal theory of regular logic, in which such questions can be asked visually rather than via text. That is, we formulated a rigorous graphical syntax that has exactly the expressive power of regular logic. We also implemented it in a working graphical user interface prototype.

Behavioral Mereology

Mereology is the study of parts and wholes. Generally, these are understood materially: the leg is part of a table, or the floor is part of a room. But we are generally interested in things for how they behave—and how the behavior of one system affects that of another—rather than in terms of their material substance itself.

To that end, we developed a behavioral approach to mereology. Rather than A being materially part of B, we defined A to be *behaviorally* a part of B when B's behavior determines A's behavior, that is, when there is a surjection from B's set of possible actions to A's set. We defined modal operators that generalize the usual "alethic" notions of *necessity* and *possibility*, whereby constraints can be passed around between parts of a system.

Black boxing

One of the most important abilities that people have is the ability to summarize a complex situation, putting all the complexity in a *black box*. Given a bunch of transistors, we black box them to form a logic gate; we then black box a configuration of logic gates to form an adder circuit; we black box these to form a computer, etc. Our group worked on a general-purpose method for creating black-boxing functors.

Genetics

Genome-wide association studies (GWAS) remain critical to understanding the fundamental aspects of our biology. Our group developed new computational tools for aligning genome snippets and producing phylogenetic trees. This work was picked up by Manolis Kellis' lab at MIT.

Deep learning

Our group analyzed the structure of deep learning, namely that of interacting neurons—which we call learners—each of which parameterizes a function that can be updated by training data. We proved that the neurons and populations of neurons both can be considered in a uniform way, as objects in a symmetric monoidal category; this creates a kind of fractal-like view on deep learning, where each learner can be composed as a system of lower-level learners. With this description, we established a general form for learning algorithms that greatly generalizes standard deep learning approaches.

2 Publications and presentations

Below I will list some publications and presentations with which I have been involved—either as author or as postdoc supervisor (postdoc name in **bold**)—that were produced during the performance period of this AFOSR grant. Some are about the above topics, others are about tangential 'spin-off' topics.

2.1 Books

- Fong, B.; Milewski, B.; **Spivak, D.I.** (2020) *Categories for Programmers*. (*to appear*).
- Fong, B.; **Spivak, D.I.** (2018) *An Invitation to Applied Category Theory: Seven Sketches in Compositionality*. Cambridge University Press.

2.2 Journal papers, book chapters, conferences, and preprints

- **Fong, B.**; Myers, D.J.; **Spivak, D.I.** (2020) "Behavioral Mereology". *Proceedings of the 3rd Annual International Applied Category Theory Conference 2020*. Available online: http://arxiv.org/abs/1811.00420.
- **Fong, B.**; Sarazola, M. (2020) "A recipe for black box functors". *Theory and Applications of Categories* Vol. 35, No. 26, pp. 979 – 1011.
- **Fong, B.**; **Spivak, D.I.** (2019) "Regular and relational categories: Revisiting 'Cartesian bicategories I' ". Available online: http://arxiv.org/abs/1909.00069.
- **Fong, B.**; **Spivak, D.I.** (2019) "Supplying bells and whistles in symmetric monoidal categories". Available online: http://arxiv.org/abs/1908.02633.
- **Spivak, D.I.** (2019) "Generalized Lens Categories via Functors $C^{\mathrm{op}} \to \mathsf{Cat}$". Available online: https://arxiv.org/abs/1908.02202.
- **Fong, B.**; Johnson, M. (2019) "Lenses and Learners". Available online: https://arxiv.org/abs/1903.03671.
- **Fong, B.**; **Spivak, D.I.** (2018) "Graphical Regular Logic". Available online: http://arxiv.org/abs/1812.05765.
- **Tuyéras, R.** (2018) "Category Theory for Genetics I: mutations and sequence alignments". *Theory and Applications of Categories* Vol. 33, No. 40, pp 1269-1317.
- **Tuyéras, R.** (2018) "Category theory for genetics II: genotype, phenotype and haplotype". Available online: https://arxiv.org/abs/1805.07004.

2.3 Invited Presentations

Over the course of this performance period, I spoke about this work in the following seminars and conferences:

- Algebraic and Combinatorial Perspectives in the Mathematical Sciences, 2020/10/09
- ETH Zurich, Special event seminar, 2019/09/25
- Category theory and Logic seminar, McGill University, 2019/09/10
- Google X, 6-hour mini-course on category theory, 2019/09/05
- Compose conference, Keynote presentation, 2019/06/25
- Kensho AI Lab Academic Talk Series, 2019/02/27
- Carnegie Mellon University, Department of Philosophy, 2019/02/15