

DEVELOPMENT OF A METHOD FOR COMPUTING WAVE-IMPACT EQUIVALENT STATIC ACCELERATIONS FOR USE IN SMALL PLANING CRAFT HULL DESIGN

FINAL



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14. ABSTRACT This report presents an example of how engineering rationale could be applied to create a method for using high-speed craft acceleration data to develop an impact load factor, also known as an equivalent static acceleration, used in procedures for small high-speed craft hull design. The theory and rationale used to develop the equations are presented and existing data is used to illustrate correlation results.					
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EXECUTIVE SUMMARY

The rigid body vertical acceleration response of a planing craft to a wave impact in head seas can be represented as a half-sine pulse of amplitude A_{peak} and duration $T/2$. This simple representation is important because it characterizes the vertical change in momentum of the craft caused by the impulsive load of the wave impact. The use of an equivalent static load to substitute for an impulsive load is demonstrated as long as the duration of the impulsive load is greater than the natural period (i.e., inverse of the natural frequency) of the hull structure plate or stiffener (or combination). The following observations support this conclusion.

A square acceleration pulse and a half-sine pulse both deliver the same impulse as long as the amplitude ratio of the square pulse to the half-sine pulse is equal to $2/\pi$. This is satisfied when the amplitude of the square pulse is equal to the average acceleration of the half-sine pulse.

Both the $2/\pi$ square pulse and the half-sine pulse result in the same relative displacement across a single-degree-of-freedom (SDOF) spring. The SDOF spring-mass system is a simple model used to investigate the maximum bending strain in a plate-stiffener structure. This suggests that both pulses will induce the same maximum structural bending strain.

Pulses with different durations do not affect the maximum SDOF spring relative displacements, as long as the duration of the pulse is greater than the natural period of the spring. The concept of an equivalent static acceleration is therefore applicable for wave impacts with typical durations greater than roughly 100 milliseconds for structural frequencies of interest (20 Hz to 80 Hz).

Acceleration data recorded during high speed trials of planing craft may be used to compute the equivalent static acceleration of the maximum peak rigid body acceleration (caused by the peak wave impact load). For the example data presented herein, the equivalent static acceleration was within one percent of the computed $A_{1/10}$ value.

RECOMMENDATION

The use of the $A_{1/10}$ acceleration value in equivalent static design procedures is not new. Some, but not all, researchers and designers have espoused its use for more than 50 years. The interesting aspect of the results presented in this report is therefore not that new acceleration amplitudes are suggested for planing craft hull design, but rather a rational approach and supporting theory based on rigid body mechanics and structural dynamics is presented that supports use of $A_{1/10}$ for hull design. The equations and methods presented herein should be considered for use in developing design tools for quantifying equivalent static accelerations for planing craft hull design.

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1 ADMINISTRATIVE INFORMATION

The contents of this report were approved for Distribution Statement A: approved for public release; distribution is unlimited, by the Congressional and Public Affairs Branch, Naval Surface Warfare Center Carderock Division on 9 March 2012, Request ID #NSWCCD-44. The work was originally presented at The Third Chesapeake Powerboat Symposium, Annapolis, MD, 14-15 June 2012 entitled: Development of a Method for Computing Wave-Impact Equivalent Static Accelerations for Use in Planing Craft Hull Design. The paper was selected as Best Paper, but a proceeding of the symposium was not published. The paper is therefore presented herein to document the work. Contract support was provided under contract number FD03.T083.G00618.4, project order 2332190817.

2 INTRODUCTION

Figure 1 illustrates the exhilarating experience of riding high-speed planing craft in waves. Riders experience motions in six degrees of freedom including heave, surge, and sway plus the rotations about the three principle axes. The challenge for the developers of craft hull design methods was to analyze these complicated motions to understand the fundamental cause and effect relationships between wave impact loads and craft response motions. They then had to synthesize lessons learned with simplifying assumptions to make tractable an otherwise seemingly unsolvable problem: what pressure values to use in design calculations to minimize the risk of structural damage. This report summarizes developments that are generally applicable to planing craft that weigh up to approximately 46,000 pounds with overall lengths 55 feet or less. Typical average planing speeds for these craft can vary from 25 knots to 40 knots or more in head seas characterized by significant wave heights typically in the range of 2.0 feet to 5.5 feet.



Figure 1. High Speed Planing Craft and Wave Impacts

3 DESIGN METHOD BACKGROUND

The current computational method for design evaluation of hull structure for high-speed planing craft evolved as a result of multiple sponsorships by the Office of Naval Research (ONR) under contract with the David Taylor Model Basin and later the David W. Taylor Naval Ship Research and Development Center. Since the focus of this report is on the development and use of acceleration values in design calculations, the evolution of the details of structural design stress calculations will not be summarized in this report.

The original work by Jasper in 1949 (reference 1) developed an approach where the complicated rigid body dynamics of a planing craft responding to complex wave impacts was simplified using

empirical factors for estimating an equivalent static pressure. The equivalent static pressure was defined as the static pressure that would produce a material strain in a structural element equal to the maximum strain produced by the dynamic pressure acting during the wave impact. A single degree of freedom model was employed to gain a fundamental understanding of the dynamic responses of craft structural elements to wave slam shock loads. Transverse and longitudinal load distribution factors determined from experimental data were defined to average the dynamic pressures over the width and length of the boat.

The design approach was modified by Heller and Jasper in 1960 (reference 2) to include a design acceleration curve that plotted the variation of the amplitude of the maximum vertical acceleration on a craft from bow to stern. It was explained that “the maximum linear (rigid body) acceleration of the center of gravity is to be expected at the instant at which the total pressure force on the boat is a maximum”. The acceleration plot could be determined from full-scale testing, scale model testing, or estimated based on impact theory. A linear plot of rigid body acceleration values from bow to stern was suggested for craft similar to the one reported in reference 1, including 11g at the bow, 4.7g at the longitudinal center of gravity (LCG), and about -2g at the stern. These acceleration values were then multiplied by transverse and longitudinal averaging factors, and mass and length terms to provide an equivalent static design pressure.

In 1969 Roper (reference 3) developed a relationship between the maximum acceleration during hull impact in waves and the maximum dynamic lift and maximum buoyant lift occurring during impacts. The recommended approach computed a load factor derived from the maximum recorded acceleration of a model minus 1g (to account for the equilibrium weight of the model). Combining this value with the pitch angular acceleration allowed calculation of a load factor at any position along the length of the craft. The extensive computational process showed good correlation with average accelerations recorded during scale model tests reported by Chey in 1963 (reference 4).

In 1978 Allen and Jones (reference 5) simplified the arduous experimental process of measuring and analyzing dynamic pressure time histories with a computational procedure that included a design reference area ratio along with transverse and longitudinal distribution factors. The vertical rigid body acceleration (defined as the impact load factor) at the craft LCG was also included in the equations. This was defined as that portion of the acceleration at the center of gravity due to impact forces as opposed to buoyancy forces. It could be determined from model test data, or previous operational experience, or it could be computed using an equation developed by Savitsky and Brown two-years earlier (reference 6) based on scale-model data reported by Fridsma (references 7 and 8). For those situations where little information was available, the authors provided a plot of Maximum Amplitude Vertical Acceleration versus Craft Displacement based on scant available full-scale trials data. The broad range of craft included 2.5 long-ton to 100 long-ton planing hulls, heavier hydro-foils, surface effect ships, air cushion vehicles, and displacement hulls. For planing hulls, the suggested range of LCG vertical acceleration values for design (i.e., impact load factor) was about 3.0g to 5.0g for craft weighing from 150 long-tons to 2.5 long-tons, respectively.

4 CLASSIFICATION SOCIETY DESIGN

The American Bureau of Shipping adopted lessons learned from the development history cited above and published its High Speed Naval Craft Guide with equations for high speed craft hull design (reference 9). This report will focus only on the accelerations used to predict bottom slamming pressure.

In the absence of model test data, theoretical computations, or service experience for a given design, the guide presents equation (1) to estimate the average of the 1/100th highest peak vertical accelerations at

the LCG of the craft. This value is then substituted into an equation along with other parameters to compute the bottom design pressure for wave impact loads.

$$\eta_{cg} = 0.0016 \left[\frac{12 h_{1/3}}{B_w} + 1.0 \right] \tau \left[50 - \beta_{cg} \right] \frac{V^2 B_w^2}{\Delta} \quad \text{Equation (1)}$$

Where:

τ = running trim (degrees) at speed V

$h_{1/3}$ = significant wave height (feet)

B_w = maximum waterline beam (feet)

β_{cg} = deadrise at LCG (degrees)

V = design speed (knots)

Δ = displacement at design waterline (pounds)

The running trim is generally not less than 4 degrees for craft less than 165 feet in length, and the deadrise is between 10 degrees and 30 degrees. The guide further specifies that the design acceleration, n_{cg} , is generally not to be taken greater than a design limit value computed using the following equation:

$$\eta_{cg \text{ Limit}} = 1.39 + 0.463 \frac{V_{design}}{\sqrt{L}} \quad \text{Equation (2)}$$

Additional guidance is provided for upper bound and lower bound design limit values. For example, n_{cg} should be no greater than 6g (for craft less than 40 feet traveling at speeds greater than about 60 knots), or no greater than 7g for search and rescue craft. It states that lower limit values are typically not less than 1g for craft lengths less than 79 feet, and not less than 2g for craft lengths less than 39 feet.

Figure 2 presents $A_{1/100}$ values computed using equation (1) for the following craft characteristics: 40 foot length, 8.5 foot beam, 4 degree trim, 18 degree deadrise, and a weight of 18,000 pounds. Values are plotted for significant wave heights from 2 feet to 4 feet. The red curve (solid diamond symbols) is the design limit accelerations computed using equation (2). At 35 knots in 3-foot seas, equation (1) predicts a value of $A_{1/100}$ equal to approximately 5.2g, but equation (2) limits the design value for this speed to about 4g. The design accelerations for this craft for speeds from 20 to 50 knots are therefore no larger than 2.8g to 5g depending upon the craft design speed. This range corresponds almost exactly with the impact load factor 3g to 5g range suggested by Allen and Jones in 1978. Figure 3 shows how the design limit values from equation (2) decrease with increasing craft length (from 32 feet to 60 feet).

5 STATIC DESIGN PRESSURE

Equation (3) is recommended by ABS for computing the static design pressure in pounds per square inch at any section along the length of the craft. The F_D term is a novel area ratio factor that averages peak dynamic pressures over a participating area of the bottom of the craft, and F_V is a longitudinal factor to account for variations in vertical acceleration fore or aft of the LCG. L_w is the craft waterline length in feet.

$$P_{bxx} = \frac{0.069\Delta}{L_w B_w} F_D F_V [1 + \eta_{cg}]$$

Equation (3)

The form of the equation illustrates the classical relationship for pressure being equal to mass times acceleration divided by an area term, plus the simplifying adjustment terms F_D and F_V . The equivalent static design pressure is a function of the equivalent static acceleration.

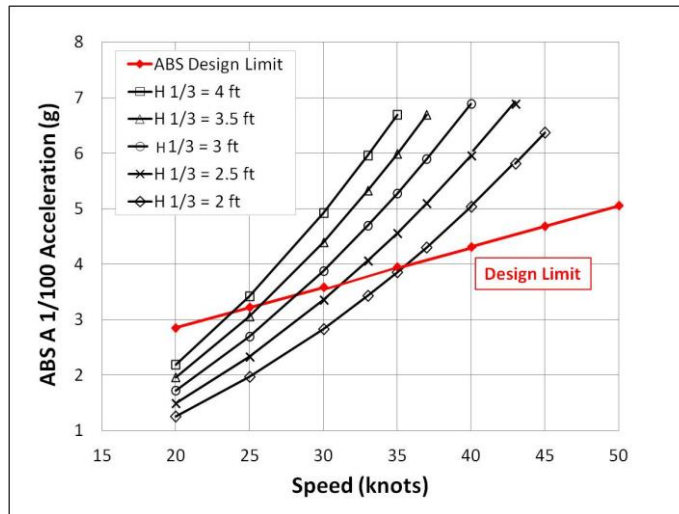


Figure 2. ABS $A_{1/100}$ Values and Design Limit Values

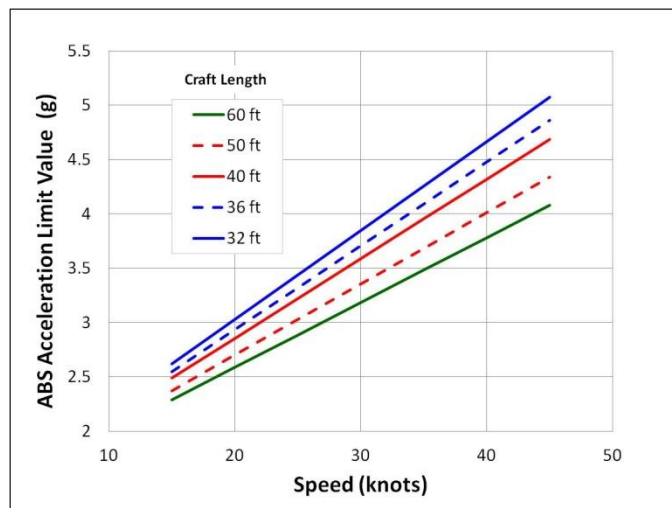


Figure 3. ABS Design Limit Acceleration Values

6 THE ELUSIVE ACCELERATION TERM

The fundamental approach established by Jasper and developed further by other researchers is the foundation for current high-speed craft design guides. An equivalent static pressure is determined from an equation that is a function of a load area ratio, a transverse factor, a longitudinal factor, the craft weight and design details, and an equivalent static acceleration. Equation (1) implies that the $A_{1/100}$ value is an important parameter for hull design, but only if it is less than the limit value computed by equation (2). Equation (2) establishes the upper bound of the equivalent static acceleration used in the ABS pressure equations for design.

The naval craft design community adopted the practice of using the $A_{1/10}$ statistic for hull design and quantifying crew endurance levels (references 10 to 15). Equation (4) was introduced by Hoggard and Jones in 1978 for estimating $A_{1/10}$, where FNV is the volumetric Froude number. Full-scale trials data was used in a least squares data fitting process to develop the equation. It was reported that the probability of error was +/- 36 percent with a 95 percent confidence level (reference 15).

$$n_{1/10cg} = 7.0 \left(\frac{H_{1/3}}{b} \right) \left(1 + \frac{\tau}{2} \right)^{0.25} (FNV^*)^{1.0} \left(\frac{L}{b} \right)^{-1.25}$$

Equation (4)

The choice of an equivalent static acceleration value has been an enigma for more than thirty years. Allen and Jones understood the elusive nature of the acceleration term. It was reported in reference 5 that the most difficult and controversial input required in the suggested (design) procedure was the impact load factor (i.e., the acceleration term). As recently as 2001 references 16 and 17 reported, “the single most pressing problem in the structural design of planing boats is to refine the method for selecting vertical acceleration...at the present state of the art (circa 2001) we have very little confidence that, for a given boat at a given speed in a given sea state, the calculated accelerations are anywhere near the actual acceleration”. Is $A_{1/100}$ the appropriate statistic to use in design, or is it $A_{1/10}$? Within the naval design community the use of $A_{1/10}$ for hull design has been a recommended practice, but it has been suggested that there was “no intrinsic reason” (reference 17) for its use other than the fact that it seemed to best represent what observers on craft felt during the ride experience.

7 VERTICAL DYNAMIC LOADING

The collective accomplishments achieved over a roughly 40-year period demonstrate that important hull design parameters were defined based on systematic testing, and increasing sea state severities were defined and craft responses in various headings were analyzed (see references 4, 7, 8, 12, 15, 18, and 19). The consistent lesson learned from scale-model testing and full-scale trials was that the largest amplitude bottom loads and craft response motions occur in the vertical direction for head sea conditions. This result reduced the focus to only a few degrees of freedom in terms of craft response, but as illustrated by equation (5), the vertical dynamic loading sequence and the vertical craft response was still complicated. In this equation the X and Y coordinates are oriented in transverse and longitudinal directions. During a wave impact sequence of events, the bottom impact pressure distribution, the area over which the pressure acts, and the vertical acceleration response all vary in X and Y coordinates and in time.

$$Vertical\ Static\ Load = \iiint \frac{[(mass)(Vert\ Accel(x, y, t))]}{Load\ Area(x, y, t)} dx dy dt \quad \text{Equation (5)}$$

The simplifying assumptions developed by Heller, Jasper, Allen, and Jones equate to an equation of the form shown by equation (6).

$$Static\ Design\ Pressure = \left(\frac{Mass}{Area} \right) (F_D F_V) (Vertical\ Accel_{eg}) \quad \text{Equation (6)}$$

In these simplifications the variation of the dynamic bottom pressure in space and time is averaged into the effective force acting over an area of the craft bottom governed by the area ratio term, F_D , and the variation of the acceleration along the length of the craft is accounted for by F_V . The remaining issue is how to transition a dynamic acceleration (historically characterized by $A_{1/N}$ statistics) into an equivalent static acceleration.

8 RECENT DEVELOPMENTS IN CRAFT MOTION MECHANICS

ONR initiated a research and development project at NSWC Carderock, Combatant Craft Division in 2005 to understand why acceleration values documented in historical test reports from different agencies could not be used in craft comparative analyses. This situation was a result of the complex nature of collecting, processing, and analyzing acceleration data, as well as the subjectivity that existed at various stages of data processing. There were no standard data processing methods. Reference 20 discusses the reasons why vertical accelerations are an important quantity to the naval engineering community. It also provides an explanation for the complexities related to this issue, summarizes recommended data acquisition best practices, and supplies the rationale for developing a standard approach.

What followed was a systematic series of developments, also funded by ONR and independent in-house research, that produced a recommended procedure to standardize the computation of average of the $1/n^{\text{th}}$ highest accelerations ($A_{1/n}$ accelerations) recorded during trials of high-speed craft (reference 21). This effort evolved under further ONR sponsorship into a pursuit to understand craft motion mechanics and wave-slam phenomenology, defined as the investigation of the phenomena associated with individual wave-slam events. It is an approach to analyzing wave slams to better understand the cause-and-effect physical relationships between impact loading and craft response motions. These motions are of interest because a broader awareness and a better understanding of loading phenomena and response mechanics could lead to improvements in model-scale to full-scale comparative evaluations, computer simulation validation, or design applications that address multiple factors associated with seaworthiness, including hull design loads, component ruggedness, and crew or passenger comfort and safety.

9 EQUIVALENT STATIC ACCELERATION

The static equivalent of the dynamic load acting on the bottom of a craft hull is the static pressure which produces the same maximum strain in a structural element as the dynamic load (reference 1). Two assumptions will be used to transition this definition to an equivalent static acceleration.

9.1 ASSUMPTION 1: HALF-SINE ACCELERATION PULSE

The dynamic vertical rigid body acceleration at the LCG caused by a single wave impact can be represented by a half-sine pulse.

As shown in Figure 4, data analyses of individual wave shapes from numerous trials have led to the adoption of the simple half-sine pulse to characterize the amplitude and duration of a craft’s acceleration response to wave impact forces (references 22 to 24). The acceleration amplitude and the duration of the pulse are important elements of the change in craft vertical velocity. The change in craft momentum (i.e., change in velocity of craft and entrained water) due to the impact is directly proportional to the impulse applied to the craft and the damage potential of a wave impact (references 25 to 28).

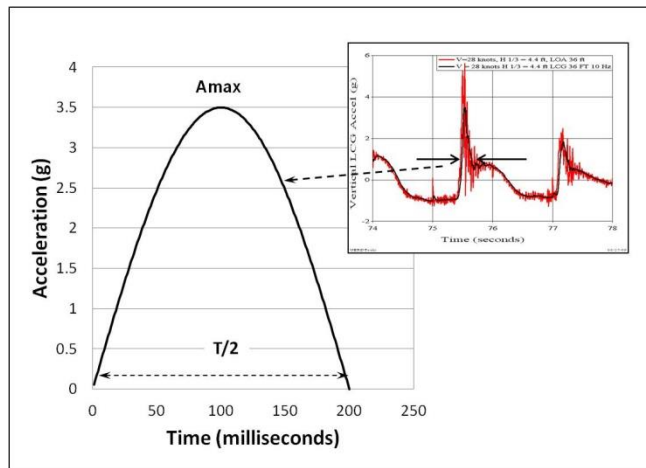


Figure 4. Half-sine Pulse Representation of a Single Wave Slam Impact

Assumption 1 is expressed mathematically by equation (7), where the pulse duration is $T/2$, and the peak acceleration of each wave slam pulse is A_{peak} .

$$A(t) = A_{peak} \sin\left(\frac{2\pi}{T} t\right) \quad \text{Equation (7)}$$

9.2 ASSUMPTION 2: HALF-SINE PRESSURE PULSE

This assumption includes the classical rigid body assumptions used by references 2 and 5, and adds the concept that the effective pressure acting over an area below the LCG has the same half-sine pulse shape as the half-sine rigid body acceleration response that it produces. This assumption is represented mathematically by equation (8).

$$P_{eff\ dynamic}(t) = \frac{mass}{area_{eff}} A_{peak} \sin\left(\frac{2\pi}{T} t\right) \quad \text{Equation (8)}$$

9.3 ASSUMPTION 3: AVERAGE PRESSURE PULSE

The effective design load in the vertical direction is the average bottom pressure over a spatial area, and it is also an average pressure over the duration ($T/2$) of the dynamic half-sine pressure pulse. This assumption follows the trend of the other simplifying assumptions that average the pressure amplitudes over an effective bottom area, and adds the assumption that, if the dynamic pressure over time has a half-sine shape, then the average pressure acting during this period is given by equation (9).

$$\bar{P}_{Static\ Design} = \frac{mass}{area_{eff}} \bar{A}(t) = \left(\frac{mass}{area_{eff}} \right) \frac{\bar{V}}{T/2} = \frac{A_{peak}}{T/2} \int_0^{T/2} \sin\left(\frac{2\pi t}{T}\right) dt \quad \text{Equation (9)}$$

Where:

\bar{V} = change in velocity during wave impact

$\bar{A}(t)$ = average acceleration during a wave impact

The important element of this assumption is the introduction of the simple average rigid body acceleration of the craft due to a single wave slam impact, $\bar{A}(t)$. The average acceleration acting from time zero to time $T/2$ therefore is a rectangular (or square) pulse of constant amplitude given by equation (10).

$$\bar{A}(t) = \frac{A_{peak}}{T/2} \int_0^{T/2} \sin\left(\frac{2\pi t}{T}\right) dt = \frac{2}{\pi} A_{peak} \quad \text{Equation (10)}$$

Figure 5 demonstrates that the average acceleration over time $T/2$ yields the same change in rigid body velocity, and the same absolute rigid body displacement at time $T/2$ as the half-sine acceleration pulse. Both pulses deliver identical impulses to a system. The example half-sine acceleration shown has peak amplitude 5g and 200 millisecond (msec) duration. Equation (10) yields an average acceleration of 3.18g. The resulting change in velocity for both pulse shapes is about 20.5 feet per second (fps), and the absolute displacement at 200 msec for both is 24.8 inches.

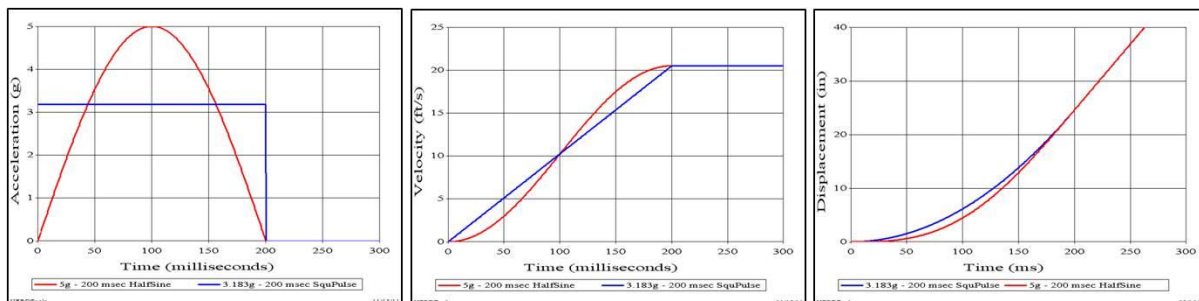


Figure 5. Half-sine and Square Pulse Accelerations, Velocities, and Displacements

10 STRAIN RESPONSE

Based on assumption 3, the dynamic pressure acting on the bottom of a craft during the wave impact period can be represented by a constant average bottom pressure that is directly proportional to the constant average acceleration response. The next step in understanding how a structure responds to this constant pressure pulse compared to the half-sine pulse can be pursued using the single-degree-of-freedom model introduced by Jasper in reference 1 using input acceleration pulses that are proportional to the input pressure pulses.

Figure 6 shows a simple representation of bottom plating between two support stiffeners in a craft that are subjected to rigid body accelerations simultaneously in the vertical direction. On the right side of the figure is an analogous single-degree-of-freedom (SDOF) mathematical model of the plate bending stiffness (k), damping coefficient δ , and the distributed mass of the plate. The natural frequency of the spring-mass system is governed by the stiffness of the spring (k) and the mass of the system.

As a first approximation, symmetry will be assumed where the plate and stiffeners are bounded on all sides by similar plate-stiffener elements, and half the pressure load will be averaged and applied as an input load at the supports. The input loads are proportional to input accelerations, either a half-sine pulse or a square acceleration pulse.

The model is not exact, but it is very useful for understanding parametric variations in a simple structure that can be modeled as a single degree of freedom system. In the SDOF mathematical model, the maximum relative displacement across the spring-damper is directly proportional to the maximum bending strain at the center of the plate. The SDOF model shown in Figure 6 can therefore be used to obtain a first order estimate of the maximum strain in the plate caused by a half-sine acceleration base input motion compared to the maximum strain caused by a square acceleration pulse.

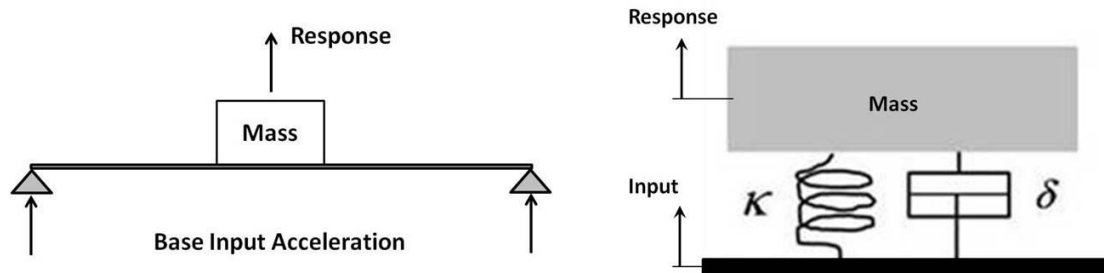


Figure 6. Single-Degree-of-Freedom Representation of a Plate and Stiffeners

Results of calculations will be presented in the next section that compare the maximum relative displacement (i.e., strain) across the spring caused by a half-sine acceleration input to that of a square acceleration pulse.

11 SHOCK RESPONSE SPECTRUM

A shock response spectrum (SRS) is a plot of the maximum response of a single-degree-of freedom system for a given input load as a function of system natural frequency and damping characteristics. The mathematics involved in the solution of the governing differential equations of motion may be found in reference 29. The parameters of interest in a SDOF model are typically the maximum absolute acceleration of the mass, the velocity of the system, or the maximum relative displacement across the spring. In this paper, the relative displacement across the spring is of primary interest, so the

acceleration and velocity parameters will not be presented. When a base input acceleration is applied to the SDOF, the mass will oscillate and damp out over a period of time governed by the damping coefficient. The SRS is a plot of the maximum relative displacement across the spring (obtained from a calculated time history response) as a function of system natural frequency.

Figure 7 shows results of an SRS calculation and compares the predicted maximum relative displacement across the spring caused by a square pulse (blue curve with diamond symbols) to that of a half-sine pulse (solid red curve). The plot shows how the maximum relative displacement (Y-axis in inches) varies with increasing system natural frequency (X-axis in Hz). For the calculations, it was assumed that the structural plate (i.e., the spring) is characterized by a damping coefficient that is nine percent of the critical damping value (reference 1). The red curve is the predicted maximum relative displacement for a 5g – 200 msec half-sine acceleration base input, and the blue curve (diamond symbols) is for a 3.18g – 200 msec square acceleration base input. The amplitude of the square pulse (3.18g) is the average value of the 5g half-sine pulse derived from equation (10).

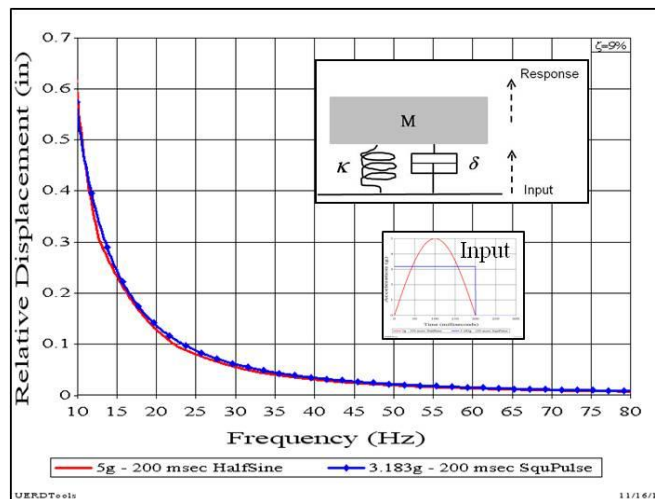


Figure 7. Relative Displacement SRS for Half-Sine and Square Pulses

The two curves are almost identical for all system natural frequencies, except for very small differences in the 12 Hz to 14 Hz range and the 20 Hz to 25 Hz range. Essentially the two curves are the same, indicating that a square pulse with amplitude given by equation (10) will result in approximately the same spring relative displacement (i.e., strain) for a broad range of system natural frequencies from 10 Hz to 80 Hz. In other words, the square pulse is equivalent to the half-sine pulse of equal duration (i.e., equal strain) as long as the amplitude of the square pulse is related to the peak acceleration of the half-sine pulse by equation (10).

The same equivalency can be shown to exist regardless of the amplitude of the half-sine pulse. The 5g value was selected here as an example of a realistic value representative of the higher end of rigid body maximum peak accelerations caused by individual wave slams found in full scale data for smaller craft.

The significance of this equivalency is related to the dynamic pulse of a wave impact (assumed half-sine shape) and its ability to be replaced by a constant square pulse of equal duration. The square pulse is a constant amplitude equivalent pulse that yields an equivalent maximum strain in the structure across a broad range of structural natural frequencies of interest. This is very close to Jasper’s original definition of an equivalent static acceleration that is proportional to an equivalent static load.

12 FREQUENCIES OF INTEREST

Accelerometer data recorded during high speed trials can be used to investigate the range of typical structural response frequencies. Analyses of Fourier spectra derived from numerous accelerometer records (most often positioned on deck plates above stiffeners) found the following ranges of structural frequencies associated with forced vibrations: Primary structural vibrations on stiffened decks are typically in the range of 22 Hz to 44 Hz. Secondary vibrations are typically in the range 44 Hz to 85 Hz. The categories “primary” and “secondary” used here mean the spectral amplitudes from wave impacts for the 22 Hz to 44 Hz are typically larger than the spectral amplitudes for the 44 Hz to 85Hz range. For hull structures the range of interest is therefore on the order of 20 Hz to 80 Hz. The general trend in trials data is that spectral amplitudes decrease as frequency increases. Figure 8 shows the vertical acceleration response of a craft to several wave impacts (from Figure 4) and illustrates the high frequency oscillations (grey unfiltered record) in the structure that damp out rapidly after each impact. In the figure the local structure vibration accelerations oscillate as much as +/- 1g to 2g around the estimated rigid body acceleration (black line, see reference 30) immediately after wave impact, but the vibration oscillation damps out in less than 500 msec. The frequencies of the oscillations are in the 24Hz to 26Hz range.

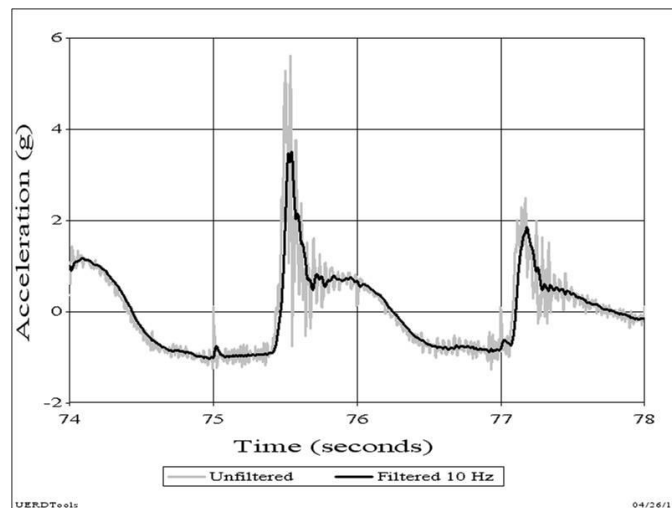


Figure 8. High Frequency Oscillations Excited by Wave Impacts

13 STATIC EQUIVALENCY

Typically the word static elicits the thought of a constant phenomenon with infinite duration, but in structural dynamics, where a maximum strain value is of interest, a load with short duration can have the same effect as a load whose duration is much longer. Figure 9 shows SRS for two square acceleration pulses of equal 3.5g amplitude with nine percent critical damping assumed for the spring natural frequency. The red curve was computed for a 200 msec pulse and the black curve (with triangle symbol) was computed for a 1000 msec pulse. Since the SRS are the same, they are equivalent pulses in terms of equal maximum strain in the spring element.

This concept can be extended to longer and longer time periods where the computed results are the same. As the duration of the pulse increases, the maximum strain responses in the spring do not change. As long as the maximum strain occurs within the duration of the acceleration pulse the SRS are the same

(i.e., the maximum strains are the same). This is illustrated further in Figure 10 where square acceleration pulses with durations of 50, 100, 200, and 1800 msec duration are shown to all have the same relative displacement SRS. Each pulse of increasing duration results in the same maximum strain in the spring because the fast response time of the spring (i.e., its frequency of response) causes the maximum strain to be achieved within the duration of the pulse. If a structural element has a response frequency of 20 Hz, its maximum strain will occur within 50 msec, for 30 Hz within 33 msec, for 40 Hz within 25 msec, and so on.

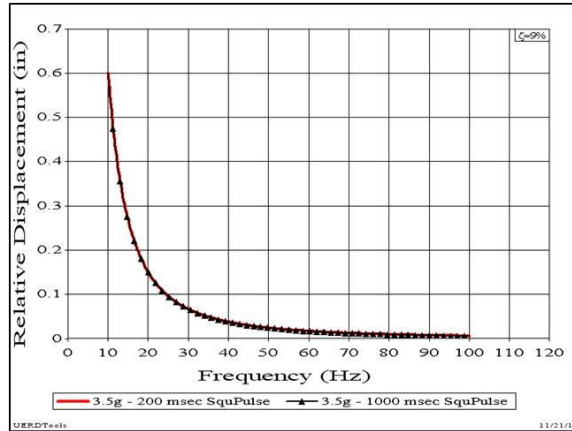


Figure 9. Displacement SRS for 200 Msec and 1000 Msec 3.5g Square Pulses

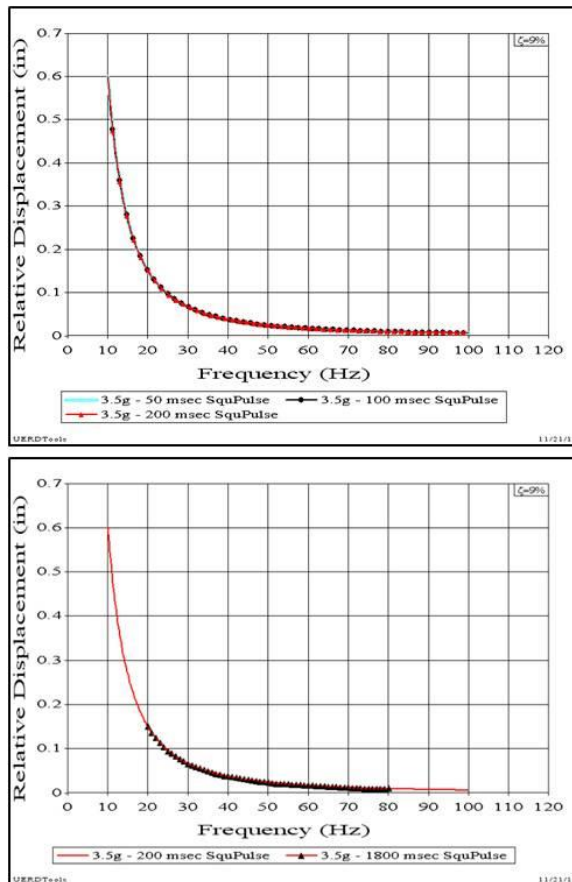


Figure 10. Displacement SRS for 50, 100, 200, and 1800 Msec 3.5g Square Pulses

In the limit, as the square pulse duration becomes large, the maximum relative displacements (i.e., strain) for all natural frequencies of interest remain constant. The square acceleration pulse is a constant amplitude load that yields the same strain as a load of very long duration (i.e., approaching static), and it produces the same strain as the dynamic half-sine acceleration pulse. Therefore, the relatively long duration average acceleration square pulse may be considered equivalent static acceleration pulses. The concept of static equivalency can be used for high speed planing craft because structural frequencies are typically greater than 20 Hz and have corresponding periodic cycle times that are very short compared to the relatively long wave impact durations. Full scale data indicates wave slam pulses are typically in the 100 msec to 400 msec range. From equation (10), it follows that the equivalent static acceleration (ESG) of a dynamic (e.g., half-sine) acceleration pulse is given by equation (11).

$$\text{Equivalent Static Acceleration (ESG)} = \bar{A} = \frac{2}{\pi} A_{peak} \quad \text{Equation (11)}$$

14 EXAMPLE FULL-SCALE DATA

Figure 11 shows the largest to smallest peak accelerations recorded during trials of a planing craft traveling at an average speed of 28 knots in seas characterized by a significant wave height of approximately 3.7 feet. The craft was approximately 39 feet in length and weighed less than 18,000 pounds. Each of the approximately 155 wave slam peaks greater than the RMS acceleration can be characterized as half-sine acceleration pulses of amplitude A_{peak} and duration $T_i/2$, where the i -subscript indicates wave slam number from 1 to N . The equivalent static acceleration of the largest peak acceleration (A_{max}) is:

$$ESG = \frac{2}{\pi} (5.6472 g) = 3.5951 g \quad \text{Equation (11)}$$

The acceleration data shown in Figure 11 was recorded during full-scale trials and processed using the standardized data analysis procedure for estimating craft rigid body accelerations (reference 30). This process is recommended for computing the average of the $1/n^{\text{th}}$ highest accelerations when analyzing accelerometer data recorded during trials of manned or unmanned small boats and craft. The methodology is based on analysis practices that evolved as a set of best-practices for achieving repeatability when $A_{1/N}$ calculations are performed for different data sets or by different analysts. The results of the statistical calculations are provided in Table 1. For this relatively small data set ($N=155$), the maximum peak acceleration is set equal to the average of the highest $1/100^{\text{th}}$ peak accelerations for illustration purposes.

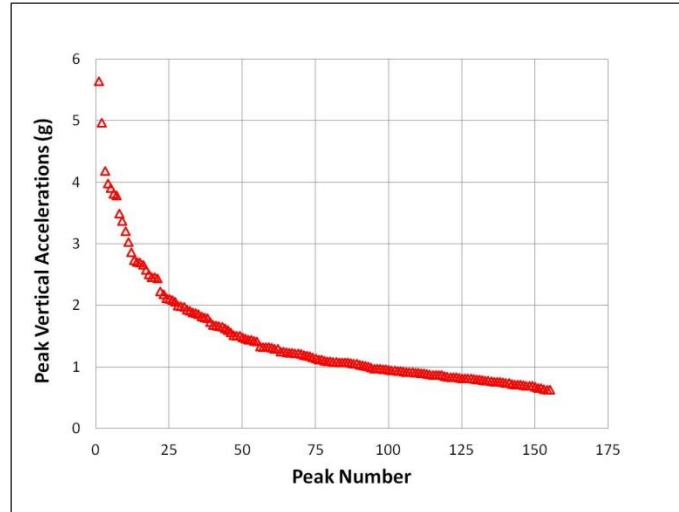


Figure 11. Wave Slam Peak Accelerations from Full-Scale Data

Table 1. Computed Acceleration Statistics for Example Data

Statistic	Acceleration (g)
$A_{1/100}$	5.6472
$A_{1/10}$	3.6326
$A_{1/3}$	2.4338
A_{avg}	1.4557
RMS	0.6302

15 EQUIVALENT STATIC ACCELERATION AND $A_{1/10}$

Reference (8) introduced the concept of using ratios of $A_{1/N}$ values for data sets with exponential cumulative distributions. The ratio is computed using equation (12).

$$\frac{A_{1/M}}{A_{1/N}} = \frac{1 + \ln M}{1 + \ln N} \quad \text{Equation (12)}$$

This ratio is useful for demonstrating an interesting relationship between the equivalent static acceleration given by equation (11) and the average of the 1/10th highest peak acceleration values. Table 2 compares acceleration ratios computed using equation (12) with acceleration ratios computed using average accelerations listed in Table 1.

Table 2. Computed Acceleration Ratios

Acceleration Ratio	Exponential Distribution	Example Data
$A_{1/100} / A_{1/10}$	1.6972	1.5545
$A_{1/10} / A_{1/3}$	1.5736	1.4926
$A_{1/10} / A_{1/100}$	0.5892	0.6432
ESG / $A_{1/10}$	1.0804	0.9896

The most interesting observation from Table 2 applicable to the equivalent static acceleration involves the ratios for A_{\max} to $A_{1/100}$ and $A_{1/100}$ to $A_{1/10}$. In equation (11), if the peak acceleration is defined to be the maximum peak acceleration in a data set (i.e., a conservative design approach), a ratio R can be defined as follows:

$$R = \frac{ESG}{A_{1/10}} = \frac{ESG}{A_{\max}} \left(\frac{A_{\max}}{A_{1/100}} \right) \left(\frac{A_{1/100}}{A_{1/10}} \right) = \frac{2}{\pi} \left(\frac{A_{\max}}{A_{1/100}} \right) \left(\frac{A_{1/100}}{A_{1/10}} \right) \quad \text{Equation (13)}$$

For the exponential distribution assumption, the ratio of A_{\max} to $A_{1/100}$ is 1.0, and the resulting ratio of ESG to $A_{1/10}$ from equation (13) is 1.08. In other words, the equivalent static acceleration is on the order of eight percent larger than the $A_{1/10}$ value.

For the example data, the ESG to $A_{1/10}$ ratio is 0.989 (i.e., within about one percent of the $A_{1/10}$ value). For A_{\max} to $A_{1/100}$ ratios up to 1.15, the data would suggest the ESG to $A_{1/10}$ ratio could be as large as $(0.6366)(1.15)(1.5545) = 1.138$, or within about 14 percent or less of $A_{1/10}$.

This is an important result because it means when the largest wave slam peak acceleration is extracted from a data set, the equivalent static acceleration of that largest peak value will be within approximately 8 percent to 14 percent of the $A_{1/10}$ value. This demonstrates the $A_{1/10}$ value is more appropriate for hull design applications than the $A_{1/100}$ value.

16 COMPARISON OF PREDICTED AND RECORDED ACCELERATIONS

The following calculations were performed for the 39 foot craft associated with the data shown in Figure 11. It is assumed that the craft's beam is 8.5 feet, the deadrise is 18 degrees, and the trim is 4 degrees. Table 3 compares predicted values of the ABS $A_{1/100}$ value from equation (1) with the data presented in Table 1. The recorded $A_{1/100}$ value (5.64g) is approximately 1.6g larger than (i.e., 39 percent larger than) the predicted value (for the given speed and significant wave height).

Equation (2) yields an ABS static design limit value of 3.44g. The equivalent static acceleration computed from the data (3.59g) using equation (13) is within 4 percent of the ABS design limit value. Both of these values fall within the 3g to 5g range of static accelerations suggested by Allen and Jones in reference (5).

Table 3. Comparison of Predicted and Recorded Acceleration Values

Parameter	Data	ABS
$A_{1/100}$	5.64 g	4.05 g
Static Design Value	3.59 g (1)	3.44 g (2)
Note (1)	Eq. (13) Equivalent Static Value	
Note (2)	Eq. (2) ABS Design Limit Value	

The full scale data listed in Table 3 is shown graphically in Figure 12 with trend curves for predicted ABS $A_{1/100}$ values from equation (1) and the ABS static design limit values from equation (2). The $A_{1/100}$ trend lines correspond to significant wave heights from 2 to 4 feet.

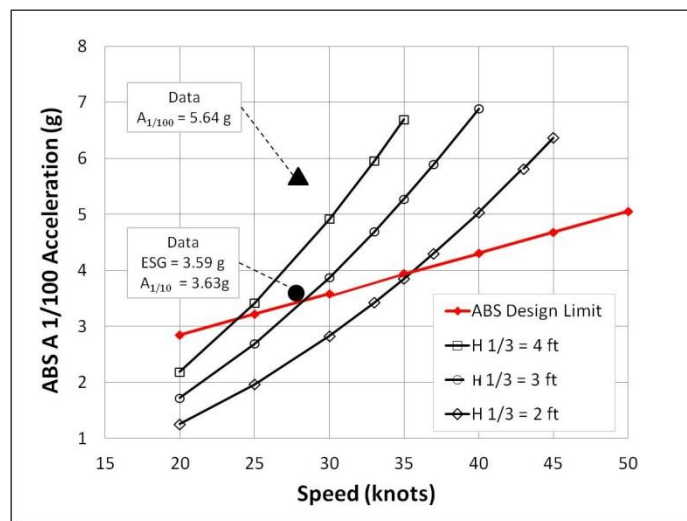


Figure 12. Comparison of ABS Accelerations with Example Data

17 OBSERVATIONS AND CONCLUSIONS

The rigid body vertical acceleration response of a planing craft to a wave impact in head seas can be represented as a half-sine pulse of amplitude A_{peak} and duration $T/2$. This simple representation is important because it characterizes the vertical change in momentum of the craft caused by the impulsive load of the wave impact. The use of an equivalent static load to substitute for an impulsive load is demonstrated as long as the duration of the impulsive load is greater than the natural period (i.e., inverse of the natural frequency) of the hull structure plate or stiffener (or combination). The following observations support this conclusion.

Figure 5 shows that a square acceleration pulse and a half-sine pulse both deliver the same impulse as long as the amplitude ratio is equal to $2/\pi$, from equation (10). This is satisfied when the amplitude of the square pulse is equal to the average acceleration of the half-sine pulse.

Figure 7 shows that both the $2/\pi$ square pulse and the half-sine pulse result in the same relative displacement across a SDOF spring. The SDOF spring-mass system is a simple model that can be used to investigate the maximum bending strain in a plate-stiffener structure. This suggests that both pulses will induce the same maximum structural bending strain.

Figure 9 and Figure 10 show that pulses with different durations do not affect the maximum relative displacements, as long as the duration of the pulse is greater than the natural period of the spring. The concept of an equivalent static acceleration is therefore applicable for wave impacts with typical durations greater than roughly 100 milliseconds for structural frequencies of interest (20 Hz to 80 Hz).

Equation (13) shows that data recorded during high speed trials of planing craft may be used to compute the equivalent static acceleration of the maximum peak rigid body acceleration (caused by the peak wave impact load). For the example data presented in Figure 11, the equivalent static acceleration was within one percent of the computed $A_{1/10}$ value.

The amplitudes of $A_{1/100}$ accelerations recorded during full-scale trials are typically on the order of 50 percent to 70 percent larger than $A_{1/10}$ values for craft weighing up to 38,000 pounds. For typical ratios of A_{\max} to $A_{1/100}$ up to 1.15, the amplitude of the equivalent static acceleration could be on the order of 14 percent more than the average of the 1/10th highest peak accelerations. The $A_{1/10}$ value is therefore more appropriate than the $A_{1/100}$ value for planing hull design applications that use equivalent static accelerations.

While the results presented herein are preliminary, experience with other individual data sets show the trends are similar, and values of ESG computed from data vary from 2.3g to 4.5g for craft weighing 14,000 pounds to 18,000 pounds. Additional research must be performed for larger weight craft to evaluate potential scale limitations associated with this approach.

The conclusion in this paper to focus on the use of the $A_{1/10}$ acceleration value in equivalent static design procedures is not new. Some, but not all, researchers and designers have espoused its use for more than 50 years. The interesting aspect of the results presented in this paper is therefore not that new acceleration amplitudes are suggested for planing craft hull design, but rather a rational approach and supporting theory based on rigid body mechanics and structural dynamics is presented that supports use of $A_{1/10}$ for hull design. The equations and methods presented herein should therefore be considered for use as design tools for quantifying equivalent static accelerations for planing craft hull design.

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