

# Using Polar Codes for Low Overhead Error Correction Encoding in Underwater Acoustics

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Polar coding is a linear block coding method notable for its channel capacity-achieving performance with low-complexity encoding and decoding algorithms. Polar codes are ideal within a presumed channel description, here an additive white Gaussian noise (AWGN) was assumed. This paper will demonstrate the application of polar codes to reduce bit errors for underwater acoustic communications by applying them to frequency shift keying (FSK) modulation. Compared with traditional rate-1/2 convolutional error correction encoding, polar was shown to have lower probability of bit error and had less overhead – another critical concern for very short transmission frames.

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## **EXECUTIVE SUMMARY**

Polar coding is a linear block coding method notable for its channel capacity-achieving performance with low-complexity encoding and decoding algorithms. Polar codes are ideal within a presumed channel description; here an additive white Gaussian noise (AWGN) was assumed. This paper will demonstrate the application of polar codes to reduce bit errors for underwater acoustic communications by applying them to frequency shift keying (FSK) modulation. Compared with traditional rate-1/2 convolutional error correction encoding, polar was shown to have lower probability of bit error and had less overhead – another critical concern for very short transmission frames.

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# USING POLAR CODES FOR LOW OVERHEAD ERROR CORRECTION ENCODING IN UNDERWATER ACOUSTICS

## 1. INTRODUCTION

Underwater acoustic communication present unique challenges compared with its terrestrial radio frequency counterparts. An acoustic channel often suffers from noise, multipath, reverberation, attenuation, and fading, as well as spatial and temporal variability. In order to mitigate the bit errors caused by these effects, strong channel coding together with sophisticated equalization are often utilized [1]. Due to limited bandwidth, acoustic communications often can support only low data rates, especially at long ranges. This, combined with the short coherence time of underwater acoustic channels, leads to the need for short transmission packets. Fortunately, practical underwater systems like instruments, gliders, and unmanned underwater vehicles mostly need short command-and-control or sensing messages in order to stay connected with each other.

Low-complexity and high-efficiency channel coding is desirable for underwater acoustic communications. Code complexity affects power consumption, memory, and latency – practical concerns especially for Size Weight and Power (SWaP)-limited devices like at-sea acoustic modems. Channel coding methods broadly fall under two categories: block and convolutional codes. Block codes work on a fixed block number of bits, for example, the Reed-Solomon codes found in hard drives. Convolutional codes work on arbitrary number of bits over a sliding window, for example, Viterbi codes. Research combining block and convolutional codes has led to such techniques as turbo codes (a hybrid of the aforementioned methods), the first method that approached the Shannon capacity limit with relatively moderate complexity. The application of turbo coding to underwater acoustics has shown promising results [2]. Because of their asymptotic nature, iteratively decodable codes developed for long frames tend to suffer from coding rate and performance loss when applied to short frames [3,4]. Conversely, using long frames for short data blocks also results in wasted bandwidth and increased latency. Popular low-density parity-check coding only becomes advantageous for frames over 1 kb. For this reason, tail-biting convolutional codes for very short frames has been demonstrated successfully in the ocean and has shown that full tail-biting codes can be used in applications where high bandwidth efficiency is required [5].

This paper instead will consider the use of polar codes for short-frame underwater communications. Polar codes have supplanted turbo codes in 5G enhanced mobile broadband control channels and are also the first and only explicitly proven capacity-achieving, yet practical, channel coding method [6]. An at-sea demonstration of the effectiveness of polar codes for underwater acoustic communications was performed and encoded FSK transmissions were made. Comparisons against convolutional encoding will show that it is an improved technique to reduce error in very short 256-bit frames for a modem with an unencoded bit rate of 160 bps.

## 2. THEORY

### 2.1 Polar Code

Polar is a type of block code that involves two key operations: channel combining and channel splitting [7]. Channel combining maps carefully selected combinations of bits to specific channels (similar to orthogonal frequency-division multiplexing symbol sub-blocks). Channel splitting implements a transformation operation (like a discrete Fourier transform) on the combination of bits into time domain vectors. Symmetric to the encoder, the decoder estimates the time domain bits *via* a successive-cancellation decoding technique (comparable to spectral domain estimation). As a total, this method converts the block and the distortion of the channel into polarized bit streams that are characterized as being a good or bad representation of the ideal channel. It has been shown theoretically that as the size of the block increases, the number of good representations approaches Shannon capacity. However, polar codes are quite effective even when the size of the block is small. Despite its exceptional performance, the algorithm complexity is far less than comparable hybrid codes, especially at the receiver.

Polar codes can reach Shannon channel capacity for its specific design channel. Several design channels such as binary symmetric and binary erasure have been developed, but, for the purpose of underwater acoustics, the AWGN is the most analogous [8]. Fast and efficient implementations have been developed and are freely available [9]. The AWGN channel requires an assumption for the SNR where  $SNR = 10 \log_{10} \left( \frac{E_b}{N_0} \right) \in (-\infty, +\infty)$  and where  $E_b/N_0$  is the signal energy per bit over noise power in the same spectral range. Practically, SNR is not typically known ahead of time and must be assumed. Figure 1 depicts the theoretical probability of bit error in additive white Gaussian noise (AWGN). Polar encoding (“X”) was coded for an ideal AWGN channel with a design SNR of 0 dB. For comparison, a convolutional encoder (“O”) is shown. If the assumption that the ocean can be represented by an AWGN channel holds true, polar coding is expected to outperform convolutional encoding while reducing overhead (unencoded Bits: 128, encoded bits Polar: 256 vs. Convolutional: 272). Note that for a binary choice in noise without

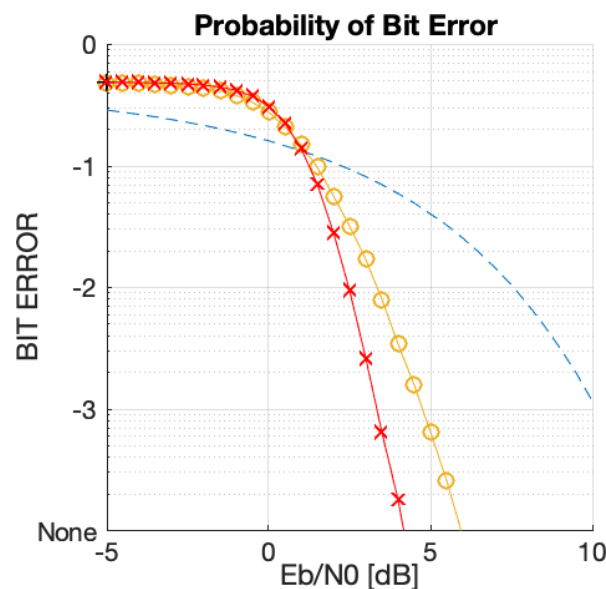


Fig. 12—Probability of bit error in additive white Gaussian noise. Unencoded (“dashed”) is the expected BE of the raw reception. Polar coding (“X”) is expected to outperform convolutional (“O”) encoding (8-bit trellis 752,561) and to take fewer bits to do it (unencoded Bits: 128, encoded bits Polar: 256 vs. Convolutional: 272).

coding, we expect the error to follow as binary phase shift keying (not binary frequency shift keying), *i.e.*,

$$P_{BE} = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_B}{2N_0}} \right), \quad (1)$$

which the dashed line depicting raw bit errors (BE) obeys.

### 2.1.1 Noncoherent Frequency Shift Keying

Frequency shift keying (FSK) is a well-known and robust method of communication, popular in underwater acoustics for its reliability and simplicity [10,11]. FSK modulates the frequency of the carrier signal so as to carry information. For digital baseband signals, it represents data through discrete variations in the frequency of a carrier signal. In FSK, the instantaneous frequency (or tone) of a constant-amplitude carrier signal is shifted between two (for BFSK) or more (for  $M$ -ary FSK) values by the baseband digital message at the beginning of each symbol duration (chip interval),  $T_S$ . In this paper, only BFSK transmissions will be discussed. FSK is not particularly susceptible to additive white Gaussian noise (AWGN) because noise affects the carrier's amplitude and not its frequency. Since FSK transmissions are unimodular (constant amplitude), power-efficient class-C non-linear power amplifiers can be used in the modem.

Noncoherent FSK demodulation is less bandwidth-efficient than coherent FSK, but requires, at most, only an additional 1 dB more ( $E_B/N_0$ ) than that for coherent FSK for the same probability of bit error of  $< 10^{-4}$  [12]. For the same symbol duration, noncoherent FSK requires more bandwidth than coherent FSK. In practical terms, the noncoherent receive system is easier to build because it doesn't require a reference signal with the additional benefit of being Doppler tolerant. The theoretical probability of bit error in the presence of AWGN can be written expressed as

$$P_{BE} = \frac{1}{2} \exp \left( -\frac{E_B}{2N_0} \right), \quad (2)$$

for BFSK transmissions. For binary transmissions, the energy per symbol is the same as the energy per bit. That is,  $E_C = E_b \log_2 M$ , where  $M$  is 2.

## 3. EXPERIMENT

The SACEX experimental area is shown in Fig. 2. The area of interest for the acoustic communications portion of the experiment was located in a relatively flat sea bottom with a median depth of 94 m. The bottom consists of a few meters of mud over a deep layer of sand as was profiled with a chirp sonar. Acoustic measurements were made via a commercially available Loggerhead Instruments Snap data logger moored at about 65-m depth with a sampling rate of 96 kS/s. A consistent downward-refracting profile was present with the M2 internal tide depth located near 25 m.

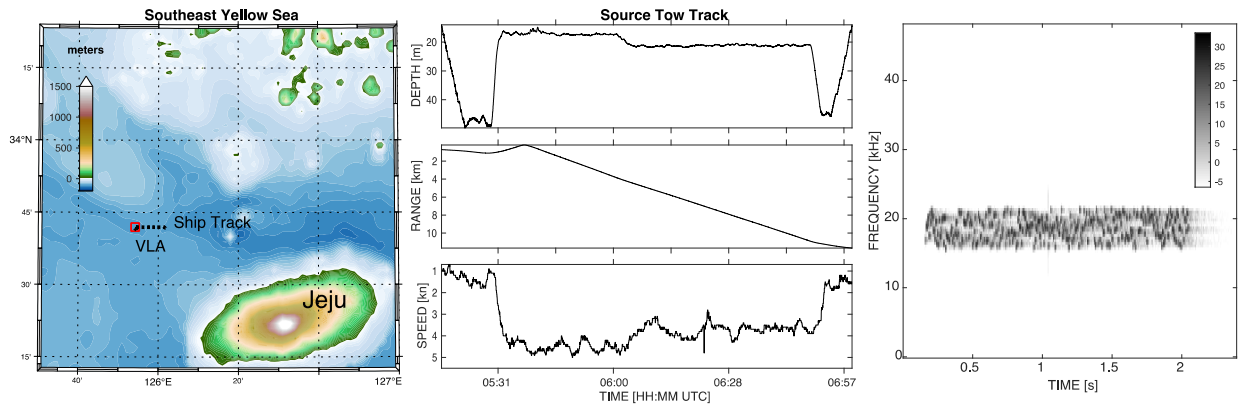


Fig. 3 —The experiment site was in the South China Sea, south of mainland South Korea and to the northwest of the island of Jeju. A vertical line array (VLA) was moored on the bottom. Depicted at center is the relevant source tow metrics during the 2-hour course of FSK transmissions. The source was lowered with a 50 m pay-out while the ship was drifting freely. Once underway, the source rose, reducing the effective receiver level. At a range of about 11 km, the ship slowed and the source was retrieved. The spectrogram at right shows a measured frequency-hopping incoherent FSK transmission made at a short range.

Transmissions were made from a 920 Series ATM-925 MF Teledyne Marine Benthos modem (16 - 21 kHz). All communication waveforms were sent with a center frequency of 18,560 Hz, a bandwidth of 5,120 Hz, and a symbol duration of 6.25 ms. The unencoded data rate was 160 bps. A preamble of 30 symbols was used for synchronization. A measured waveform is shown to the right in Fig. 2. This high SNR reception was made when the ship was drifting at 1-2 kts nearby and is the same waveform that will be used for the calculations depicted later in Fig. 3.

The raw data consisted of 120 bits with an 8-bit cyclic redundancy check. Besides polar, the best nonsystematic rate-1/2 convolutional code with maximal minimum free distance (constraint length 9,  $g(0) = 561$ , and  $g(1) = 753$ ) also was used to encode the data [13]. The encoded data was 256 bits and 272 bits in length for polar and convolutional, respectively. After interleaving, the data was modulated as a frequency-hopping FSK transmission. The available bandwidth was divided into 17 hops to mitigate frequency-selective fading. The signal durations were 1.79 s and 1.89 s for polar and convolutional, respectively.

Noncoherent BFSK has a spectral efficiency of  $1/3$ . Its null-to-null bandwidth is  $3 R_B$  and its 90% signal power bandwidth is  $R_{90\%} = 1.23 R_B$ , where  $R_B = 1/T_S$ . Note that when performing spectrogram-based FSK demodulation, the FFT window size should be on the order of a few times larger than  $R_B$  to achieve desirable energy detection with a balance struck between noise reduction, tone disambiguation, and Doppler tolerance.

The modem was married to a winch that paid out 50 m of cable into the water. Figure 2 contains the practical measurements of time vs. source depth, range, and speed. Deployment began from the R/V Chunghae at a distance of approximately 500 m from the mooring. After drifting freely at about 1 kt, the ship went underway at approximately 4 kt until it was 11.6 km away. Transmissions were made during the 2-hour duration. The increased speed caused the source to rise within the water column to as shallow as 18 m. Due to interaction with the surface, the radiation pattern of a source changes near the surface (dipole), reducing the energy propagating horizontally [13]. Despite the long ranges, transmissions made at the end, while deep and stationary, were received with lower BE than those made nearby, while shallow and underway.

#### 4. RESULTS

Figure 3 depicts the probability of bit error vs.  $E_B/N_0$ . The solid line is the theoretical noncoherent binary frequency shift keying performance in the presence of AWGN from Eq. (1). The (“+”) line depicts unencoded BE. This was estimated by taking a measured at-sea reception of relatively high SNR, and then adding AWGN of various strengths to form a large ensemble from which to estimate average BE. It is apparent that at higher  $E_B/N_0$ , the at-sea data diverges from theoretical expectations. This is because the assumption that only AWGN is present is invalid. In fact, multipath, fading, Doppler, reverberation, and spatially colored noise are present. Simply increasing the source level does not yield significantly improved results. These processes require other methods to mitigate them, such as error-correction encoding as well as interleaving, hopping, pre-whitening, and adaptive gain control. It should be noted that the receiver design utilizes hopping and combines interleaving with the decoding to reduce the effects of fading and burst errors. The convolutional (“O”) and polar (“X”) results shown in Fig. 3 are the BE after error-correction encoding. In the field, polar outperformed convolutional encoding with less overhead. The expected processing gain of polar over convolutional was less than expected in Fig. 1. While both encoding techniques were designed for AWGN channels, the polar method appears more impacted by other sources of error.

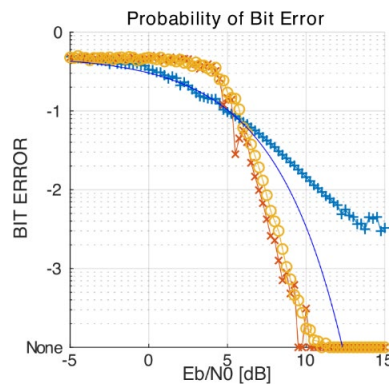


Fig. 4—Probability of bit error (BE) vs. Energy/Noise density per bit. The solid line is the theoretical noncoherent binary frequency shift keying performance in the presence of additive white Gaussian noise. The measured performance (“+”) was based on measured at-sea signals. The convolutional (“O”) and polar (“X”) results are after error correction encoding. In the field, polar outperformed convolutional encoding with less overhead.

Often overlooked in physical layer design, at low  $E_B/N_0$ , error correction encoding can be detrimental to bit error. The theoretical predictions depicted in Fig. 1 show that SNR threshold to be about 1 dB. Below that, BE would be reduced better by simple repetition and averaging of the soft decisions. Though not a conventional metric, rather than averaging to get the probability of bit error, it is possible to use the median of the ensemble to estimate the point at which error-correction encoding is more beneficial than detrimental, on average. That  $E_B/N_0$  threshold was 5.5 dB for polar encoding vs. 7.25 dB for convolutional encoding.

During the communication portion of SACEX, hundreds of FSK transmissions were made. Instead of comparing output BE vs. versus input signal-to-noise ratios, it is also useful to analyze output BE for a given input BE. The probability of decoded BE for polar (left) and convolutional (right) encoding is shown in Fig. 4 for given raw probability of BE. Polar encoding was 1.75 times as likely to produce 0 BE as was

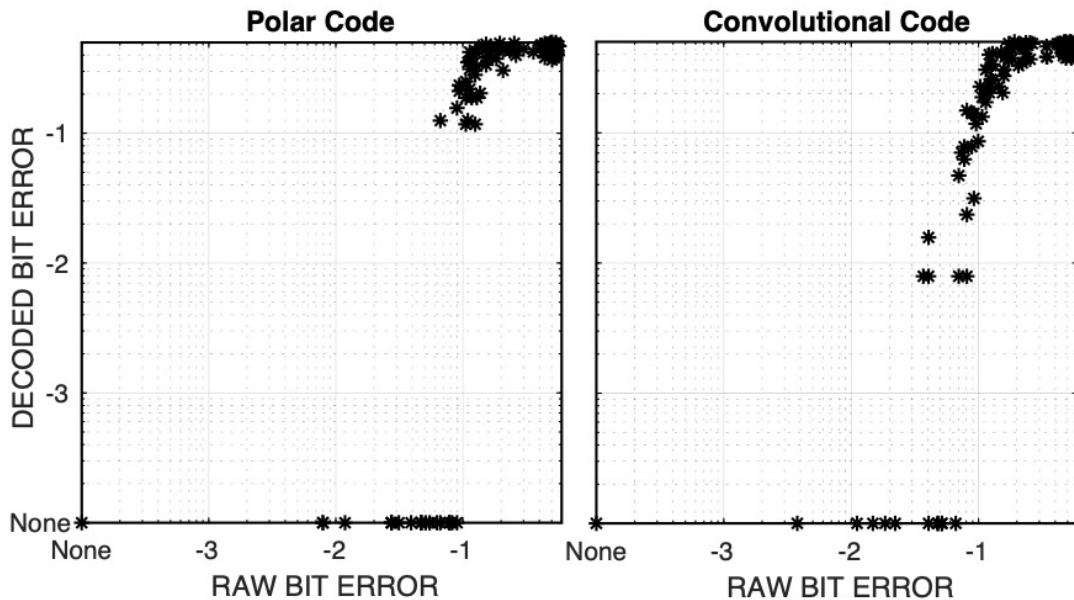


Fig. 5—Probability of the raw bit error versus the probability of decoded bit error for polar (left) and convolutional (right) encoding. Of the hundreds of transmissions, Polar coding was 1.75 times as likely to produce 0 error and only needed an input of about 1/10 errors to consistently produce perfect decoding.

convolutional encoding. Polar only needed an input BE of about 1/10 to consistently produce errorless decoding.

## 5. SUMMARY

Rate-1/2 polar and convolutional error correction encoding were compared and polar was shown to have lower probability of bit error. These methods were demonstrated on measured at-sea acoustic transmissions via noncoherent frequency shift keying demodulation. In addition to having a higher coding gain, it also had less overhead – another critical concern for very short transmission frames. Both of the encoding methods utilized were designed within the presumption of an additive white Gaussian noise channel. While both still performed favorably in the presence of multipath, Doppler, and reverberation, the polar coding gain was more impacted.

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**REFERENCES**

1. M. Stojanovic and J. Preisig, "Underwater acoustic communication channels: Propagation models and statistical characterization," *IEEE Commun. Mag.*, vol. 47, no. 1, pp. 84–89, Jan. 2009, doi: 10.1109/MCOM.2009.4752682.
2. Y. R. Zheng, J. Wu, and C. Xiao, "Turbo equalization for single-carrier underwater acoustic communications," *IEEE Commun. Mag.*, vol. 53, no. 11, pp. 79–87, Nov. 2015, doi: 10.1109/MCOM.2015.7321975.
3. F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski, "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 74–80, Feb. 2014, doi: 10.1109/MCOM.2014.6736746.
4. G. Liva, L. Gaudio, T. Ninacs, and T. Jerkovits, "Code Design for Short Blocks: A Survey," arXiv:1610.00873 [cs, math], Oct. 2016, Accessed: Feb. 02, 2021. [Online]. Available: <http://arxiv.org/abs/1610.00873>.
5. M. Behgam, Y. R. Zheng, and Z. Liu, "Coding for Short Messages in Multipath Underwater Acoustic Communication Channels," in *OCEANS 2018 MTS/IEEE Charleston*, Charleston, SC, Oct. 2018, pp. 1–5, doi: 10.1109/OCEANS.2018.8604711.
6. V. Bioglio, C. Condo, and I. Land, "Design of Polar Codes in 5G New Radio," *IEEE Commun. Surv. Tutorials*, pp. 1–1, 2020, doi: 10.1109/COMST.2020.2967127.
7. E. Arikan, "Channel Polarization: A Method for Constructing Capacity-Achieving Codes for Symmetric Binary-Input Memoryless Channels," *IEEE Trans. Inform. Theory*, vol. 55, no. 7, pp. 3051–3073, Jul. 2009, doi: 10.1109/TIT.2009.2021379.
8. H. Vangala, E. Viterbo, and Y. Hong, "A Comparative Study of Polar Code Constructions for the AWGN Channel," arXiv:1501.02473 [cs, math], Jan. 2015, Accessed: Feb. 02, 2021. [Online]. Available: <http://arxiv.org/abs/1501.02473>.
9. Polar Codes. (2021) [www.polarcodes.com](http://www.polarcodes.com).
10. J. Rice and D. Green, "Underwater Acoustic Communications and Networks for the US Navy's Seaweb Program," in *2008 Second International Conference on Sensor Technologies and Applications (sensorcomm 2008)*, Cap Esterel, France, 2008, pp. 715–722, doi: 10.1109/SENSORCOMM.2008.137.
11. L. Freitag, M. Grund, S. Singh, J. Partan, P. Koski, and K. Ball, "The WHOI Micro-Modem: An Acoustic Communications and Navigation System for Multiple Platforms," in *Proceedings of OCEANS 2005 MTS/IEEE*, Washington, DC, USA, 2005, pp. 1–7, doi: 10.1109/OCEANS.2005.1639901.
12. J. G. Proakis and M. Salehi, *Digital communications*. Boston: McGraw-Hill 2008, Ch. 4.5-2.
13. S. B. Wicker, *Error control systems for digital communication and storage*. Prentice-Hall, Inc. 1995, Ch 11.3.
14. A. Song, M. Badiy, H. C. Song, and W. S. Hodgkiss, "Impact of source depth on coherent underwater acoustic communications," *The Journal of the Acoustical Society of America*, vol. 128, no. 2, pp. 555–558, Aug. 2010, doi: 10.1121/1.3459843.