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ANALYSIS OF THE DISCONTINUOUS PETROV GALERKIN METHOD AS A TRANSVERSE MODE SOLVER FOR OPTICAL FIBERS IN CONJUNCTION WITH GERNERALIZED POLYNOMIAL CHAOS THEORY

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14. ABSTRACT
This project originally focused on the further developmental of the discontinuous Petrov Galerkin (DPG) Finite Element Method (FEM) for the purpose of modeling optical fiber amplifier guided mode data, and including an uncertainty quantification analysis of this numerically calculated data, first considering the Generalized Polynomial Chaos (gPC) approach. The main thrust of this project later evolved into deleloping the mathematical theory behind, and building a robust and versatile computer model for finding, optical fiber guided modes (and associated mode data) using any suitable finite element discretization in conjunction with the FEAST algorithm (a numerical eigenslolver technique). Furthermore, the project aimed to reolve issues with finding the correct mode loss values when fibers are coiled. Our participation in this effort has always been mostly about consultaion on the mathematical theory and implementaion that eventually leads to eigensolver (optical mode solver) tool.

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TABLE OF CONTENTS

1.0 SUMMARY

This report covers the technical contributions of The University of Texas at Austin (UT) Oden Institute that have supported the development of a computational tool built by the Portland State University (PSU) Mathematics Department for the Air Force Research Laboratory (AFRL), Directed Energy Directorate, Laser Division (RDL). This simulation tool is an improved, accurate, and robust optical fiber transverse guided mode solver to be used for computer modeling applications within RDL.

2.0 INTRODUCTION

This project originally focused on the further development of the discontinuous Petrov Galerkin (DPG) Finite Element Method (FEM) for the purpose of modeling optical fiber amplifier guided modedata, and including an uncertainty quantification analysis of this numerically calculated data, first considering the Generalized Polynomial Chaos (gPC) approach[1,2]. The main thrust of this project later evolved into developing the mathematical theory behind, and building a robust and versatile computer model for finding, optical fiber guided modes (and associated mode data) using any suitable finite element discretization in conjunction with the FEAST (fast eigensolver) algorithm (a numerical eigensolver technique). Furthermore, the project aimed to resolve issues with finding the correct mode loss values for imperfectly guided fibers and/or coiled fibers. The UT participation in this effort has always been mostly about consultation on the mathematical theory and implementation that eventually leads to the eigensolver (optical mode solver) tool.

The transverse modes in optical fibers form a basis for the justification and implementation of AFRL's Coupled Mode Theory (CMT) fiber amplifier models. One major accomplishment from our effort includes the verification of these CMT models and, in part, the numerically calculated fiber modes, using a rigorous but expensive direct numerical simulations (DNS) model based on the solution of time-harmonic Maxwell equations coupled with a transient heat equation. The numerical complexity of the latter calls for a massively parallel implementation. The overall technology is being developed in context of modeling complex transverse geometries of microstructure fibers.

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

The optical mode solver built by the PSU team uses an appropriate finite element (FE) discretization, capable of using curvilinear elements to match any complex boundaries found in all realistic optical fibers, with the FEAST algorithm[3–6]. FEAST is a fast numerical eigensolver that uses an iterative contour integral method. This mode solver will be called the 2D FE-FEAST eigensolver in this report because solving for the optical modes is an eigen problem. Note that the DPG method is a type of FE discretization.

The work of the temam at UT focust on the following technical aspects of the project.

- Development of a unique three dimensional full vectorial finite element Maxwell fiber model based on the DPG method and a state-of-art parallel *hp3D* code[1] (a code that uses

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[1] The *hp3D* code is a software suite that allows for three dimensional (3D) finite element discretizations that can be automatically adapted to best solve the given problem; adaptions: h – element size and p – polynomial order.

three dimensional (3D) finite elements that get automatically adapt the size (h) and polynomial order associated with the elements to best solve the given problem) involving a combined message passing interface (MPI)[2] and message passing (openMP)[3] parallelization. With the possibility of simulating optical fibers with up to 10,000 wavelengths, the code serves as validation tool for developing a simplified CMT model based on eigenmodes obtained with the 2D FE-FEAST eigensolver developed at PSU.

- Development of novel geometry techniques applicable in the fiber model that are important for achieving greater computational performances.
- Development of DPG-based discretization techniques using polygonal elements to facilitate another option in the curvilinear discretization of challenging fiber cross-sectional geometries.
- Theoretical support for the development of the 2D FE-FEAST eigensolver at PSU.
- Conducted mode beating light propagation tests that investigate the origins of mode energy coupling, and important application for the AFRL/RDL team. This effort verifies the importance of obtaining accurate mode data for AFRL's CMT models.

4.0 RESULTS AND DISCUSSION

A unique, parallel MPI/openMP version of the hp3D code has been developed. The code enables discretization of challenging coupled, multi-physics problems with hybrid meshes (hexahedrals, prisms, tetrahedrals, pyramids) with up to 2 billion degrees of freedom. Solutions of the fiber problem with over 10,000 wavelengths have been reported[7,8]. The results include a solid validation of the CMT results obtained at Portland including a study on transverse mode instabilities (TMI). The work constituted the Ph.D. dissertation of Stefan Henneking[8]. The code will be transferred to our Air Force Office of Scientific Research (AFOSR) partners by the end of Summer 2021, along with a documentation[9]. New parametrization techniques to enable the use of prismatic elements has been developed and used in the fiber simulations. The use of prismatic elements called for the accompanying development of fast quadrature rules for the prismatic element reported[10]. The work on the DPG method based on polygonal (2D) and polyhedral (3D) meshes comprised the Ph.D. dissertation of Jaime Mora-Paz[11], and has been (partially) reported in[12,13].

The primary task for the UT team has been to consult and advise the PSU and AFRL teams in the development of this 2D FE-FEAST eigenslover. This occurred through both formal and informal meetings, events, and workshops with these teams. For example the UT team has participated in and attended all of the AFRL mini-workshops on fiber amplifier model development (2017-2021), even offering an in-depth tutorial on the implementation of the DPG method. Moreover, Dr. Grosek was invited to the prestigious Oden Institute Babu.ska Forum to present on the fiber coiling issues that were instrumental in altering the direction of this project as it progressed[14]. Summaries of the main accomplishments of the joint effort have been presented by Drs. Grosek, Gopalakrishnan, and Demkowicz at the annual meetings of the AFOSR Computational Mathematics program in Washington, DC.

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[2] MPI indicates a distributed memory parallelization.

[3] The openMP parallelization indicates that the memory is shared between different processes.

5.0 CONCLUSIONS

The DPG methodology has proved to provide a solid foundation for both targeted applications: computation of transverse eigenmodes and DNS simulations of the optical fibers. The numerical tools developed in the course of this project, along with documentation, will be transferred to the Air Force partners by the end of summer 2021. A major accomplishment of this effort, which will be described in detail in PSU's reports to AFRL, is the fact that the correct mathematical approach for finding mode loss values, especially in regards to coiled fibers, has been discovered and implemented in the new computational tool. If more time and funding were available for this project, returning to the original goal of investigating an uncertainty quantification strategy for the optical mode solver, would be recommended.

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LIST OF ACRONYMS AND ABBREVIATIONS

AFRL	Air Force Research Laboratory
AFOSR	Air Force Office of Scientific Research
CMT	coupled mode theory (optical fiber modeling approach)
DNS	direct numerical simulations
DPG	discontinuous Petrov-Galerkin (finite element discretization approach)
FE	finite element (discretization of a problem domain)
FEAST	fast eigensolver numerical technique
FEM	finite element method
gPC	generalized polynomial chaos (uncertainty quantification approach)
hp3D	Oden Institute's software suite that allows for three dimensional (3D) finite element discretizations that can be automatically adapted to best solve the given problem
MPI	message passing interface (distributed memory parallelization technique)
OpenMP	message passing (shared memory parallelization technique)
PSU	Portland State University
TMI	transverse mode instability (thermally-induced optical fiber nonlinearity)
UT	The University of Texas at Austin (parent organization of the Oden Institute)

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