AFRL-AFOSR-VA-TR-2021-0069

Canonical Duality Theory and Algorithm for Solving NP-Hard Problems in Decision Science and Complex Systems

Gao, David
FEDERATION UNIVERSITY AUSTRALIA
UNIVERSITY DRIVE
MT HELEN, VIC, 3350
AUS

07/07/2021
Final Technical Report

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Air Force Research Laboratory
Air Force Office of Scientific Research
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

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6. AUTHOR(S)
David Gao

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5e. TASK NUMBER

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
FEDERATION UNIVERSITY AUSTRALIA
UNIVERSITY DRIVE
MT HELEN, VIC 3350
AUS

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875 N. Randolph St. Room 3112
Arlington, VA 22203

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12. DISTRIBUTION/AVAILABILITY STATEMENT
A Distribution Unlimited: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Supported by this AFOSR grant, the PI and his Co-PI, student, post-doctoral fellow and coworkers have successfully applied the canonical duality theory and its associated algorithms for solving a large class of challenging problems in global optimization and decision science. His group has published 2 books and about 26 papers. The most significant achievements of this project are the analytical solution to the linear knapsack problem and its applications to structural topology optimization. The knapsack problems are well-known “NP-Hard” problems in computer science and global optimization. Due to the integer constraint, traditional theories and methods cannot be applied to obtain global optimal solution in polynomial time. Even the simplest linear knapsack problem is listed as one of Karp’s 21 NP-complete problems. However, by using the canonical duality theory, the integer constrained global optimization problems in n-dimensional space can be equivalently converted to a unified concave maximization problem in ONE dimensional continuous space, which can be solved to obtain global optimal solution in polynomial time. Particularly, the linear knapsack problem can be solved analytically. Results show that as long as the knapsack problem has a unique solution, it is not NP-hard and its solution can be obtained analytically by the canonical duality theory. This research reveals, for the first time, the reason for NP-Hardness, i.e. the so-called NP-hardness is mainly due to certain symmetry such that the problem may have multiple solutions. Therefore, most of NP-hard problems are artificially proposed since the perfect symmetry does not exist in real world. Applications to topology optimization leads to a correct mathematical model and a powerful algorithm for 3-D structural optimal design. A powerful deterministic method and algorithm have been developed, which can be used for lightweight design of aircrafts and armored vehicles. The main results have been published in Computational Methods in Applied Mechanics and Engineering, the top journal in computational mechanics and engineering sciences, and two papers have been accepted by the top rank IEEE journals. Additionally, a series of challenging problems have been solved, such as the well-known NP-hard sensor location problem in network optimization, a complete set of analytical solutions to nonconvex mechanics, and nonconvex/mixed integer optimization problems in chaotic dynamics, computational biology, decision science, machine learning, neural networks, post-buckling of large deformed structures, industrial and systems engineering, etc. A powerful deterministic method and algorithm have been developed, which can be used for solving efficiently large-scale non-convex/non-smooth/discrete optimization problems.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			HAL GREENWALD
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Response Summary:

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Q1. Award Number (Federal Award Identification Number XXXXXX-XX-X-XXXX)
FA9550-17-1-0151

Q3.
Our system shows you have the following report due:
Please confirm the appropriate report below.

- Final Performance

Q4. Principal Investigator
David Gao

Q5. Principal Investigator Email
d.gao@federation.edu.au

Q6. Principal Investigator Phone
+61457987344

Q7. Project Title
Canonical Duality Theory and Algorithm for Solving 'NP-Hard Problems' in Decision Science and Complex Systems

Q8. Recipient Organization
FEDERATION UNIVERSITY AUSTRALIA

Q310. Business Office Email
research.funding@federation.edu.au

Q9. Report Due Date
03/31/2021

Q10. Report Period Start Date
02/01/2017

Q11. Report Period End Date
12/31/2020

Q297. Current Program Officer
Hal S. Greenwald

Q298. Please list any other Co-Program Officers (if applicable)
James Lawton

Q395. Please confirm the report type you are submitting is: Final Performance
- Yes, that is correct.

Q12. Is this survey being submitted by someone other than the Principal Investigator?
- No

Q407. How many participants worked on the grant during this period of performance?
This number includes all PIs and each person who worked, and was funded by the project during this reporting period.

You will be asked to provide the following information for: (1) PDs/PIs; and (2) each person who worked, and was funded by the project, during this reporting period. Please note that such reporting does not constitute a formal institutional report of effort on the project, but rather is used by agency program staff to evaluate the progress of the project during a given reporting period.
(Max 20 participants)
3

Q401#1. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Name

	Last Name, First Name
Participant 1	Gao, David
Participant 2	Ruan, N
Participant 3	Latorre, V.
Participant 4	N/A
Participant 5	N/A
Participant 6	N/A
Participant 7	N/A
Participant 8	N/A
Participant 9	N/A
Participant 10	N/A
Participant 11	N/A
Participant 12	N/A
Participant 13	N/A
Participant 14	N/A
Participant 15	N/A
Participant 16	N/A
Participant 17	N/A
Participant 18	N/A
Participant 19	N/A
Participant 20	N/A

Q401#2. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Months Worked

	#
Participant 1	36
Participant 2	31
Participant 3	12
Participant 4	N/A
Participant 5	N/A
Participant 6	N/A
Participant 7	N/A
Participant 8	N/A
Participant 9	N/A
Participant 10	N/A
Participant 11	N/A
Participant 12	N/A
Participant 13	N/A
Participant 14	N/A
Participant 15	N/A
Participant 16	N/A
Participant 17	N/A
Participant 18	N/A
Participant 19	N/A
Participant 20	N/A

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Q401#3. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Describe briefly how this person contributed to the project

Participant 1	responsible for the whole project: theory, method, algorithms, and applications
Participant 2	Responsible for the first key research area, i.e. mathematical theory algorithm and software development
Participant 3	Responsible for the second key research area, i.e. the dynamical traveling salesman problem and network optimization problems.
Participant 4	N/A
Participant 5	N/A
Participant 6	N/A
Participant 7	N/A
Participant 8	N/A
Participant 9	N/A
Participant 10	N/A
Participant 11	N/A
Participant 12	N/A
Participant 13	N/A
Participant 14	N/A
Participant 15	N/A
Participant 16	N/A
Participant 17	N/A
Participant 18	N/A
Participant 19	N/A
Participant 20	N/A

Q401#4. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Project Role

Participant 1	Principal Investigator
Participant 2	Co-Investigator
Participant 3	Postdoctoral (scholar, fellow, or other)

Q401#5. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- International Business during Reporting Period. 1) Did the individual collaborate with individuals located in a foreign country? 2) Did this individual travel to a foreign country as part of the collaboration?

N/A

Q401#6. Please answer the following for each participant.
(Currently our system has a maximum allowable entry of 20 participants)
- Add'l Funding Source(s)

Participant 1	No
Participant 2	No
Participant 3	No

Q403. Please confirm whether any of your participants had any international business associated with this grant during this reporting period.
(If you input any information on the above International Business question, you should select yes.)

- No, none of the participants had international business

Q353. Archival Publications (published) during reporting period: State "Nothing to Report" if nothing to report

Books Published:
1. Gao, DY, Latorre, V. and Ruan, N. Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, July, Springer. 2017, 377p.
2. V.K. Singh, D.Y. Gao, and A. Fisher (2019). Advances in Mathematical Methods and High Performance Computing, Advances in Mechanics and Mathematics 41, Springer Nature Switzerland AG 2019, 503p.

Papers Published:
1. Gao, D.Y. (2019). Canonical Duality Theory and Algorithm for Solving Bilevel Knapsack Problems with Applications, IEEE Transactions on Systems, Man, and Cybernetics: Systems (published online first at IEEE, Impact Factor: 7.351) , IEEE Transactions on Systems, Man, and Cybernetics: Systems (Volume: 51, Issue: 2, Feb. 2021) DOI: 10.1109/TSMC.2018.2882792
2. Latorre, V. and Gao, D.Y. (2019). Efficient Deterministic Algorithm for Huge-Sized Noisy Sensor Localization Problems via Canonical Duality Theory, IEEE Transaction on Cybernetics. (published online first) DOI: 10.1109/TCYB.2019.2891112 Impact Factor: 11.079
3. Gao, D.Y. (2018). On Topology Optimization and Canonical Duality Method, Computer Methods in Applied Mechanics and Engineering, Volume 341, 1 November 2018, Pages 249-277. https://doi.org/10.1016/j.cma.2018.06.027,
4. Ali, E. and Gao, DY (2018). On SDP Method for Solving Canonical Dual Problem in Post Buckling of Large Deformed Elastic Beam. Communications in mathematical sciences 16(5), 2018, DOI: 10.4310/CMS.2018.v16.n5.a3
5. Ruan, N. Gao, D.Y. (2018). On Modelling and Complete Solutions to General Fixpoint Problems in Multi-Scale Systems with Applications, J. Fixed Point Theory, Springer 23(1):23. DOI: 10.1186/s13663-018-0648-x
6. Gao, DY, Neff, P., Roventa, I., Thiel, C. (2017) On the convexity of nonlinear elastic energies in the right Cauchy-Green tensor, Journal of Elasticity, April 2017, Volume 127, Issue 2, pp 303–308
7. Jin Z., Gao D.Y. (2017), On modeling and global solutions for d.c. optimization problems by canonical duality theory. Applied Mathematics and Computation, vol. 296, pp. 168-181,10.1016/j.amc.2016.10.010
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10. Gao, DY and Elaf Jaafar Ali (2019). A Novel Canonical Duality Theory for Solving 3-D Topology Optimization Problems, in V.K. Singh et al (eds.) Advances in Mathematical Methods and High Performance Computing, Advances in Mechanics and Mathematics 41, https://doi.org/10.1007/978-3-030-02487-1_13 Springer Nature Switzerland AG 2019, pp.209-246
11. Gao, D.Y., Ruan, N., and Latorre, V. (2017). Canonical duality-triality: Bridge between nonconvex analysis/mechanics and global optimization, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 1-48.
12. Gao, D.Y. (2017). Canonical Duality Theory for Topology Optimization, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 263-276.
13. Gao, D.Y. (2017). Remarks on Analytic Solutions and Ellipticity in Anti-plane Shear Problems of Nonlinear Elasticity, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 89-104.
14. Gao, D.Y. (2017). Analytic Solutions to Large Deformation Problems Governed by Generalized Neo-Hookean Model, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 49-68.
15. Gao, DY and Hajilarov, E (2017). Analytic Solutions to 3-D Finite Deformation Problems Governed by St Venant–Kirchhoff Material, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 69-88.
16. Gao, D.Y. and Wu, C. (2017). Triality Theory for General Unconstrained Global Optimization Problems, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 127-154.
17. Latorre, V., Sagratella, S., Gao, DY (2017) Canonical Dual Approach for Contact Mechanics Problems with Friction, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 173-186. http://arxiv.org/abs/1402.6909
18. Wu, C. and Gao, DY (2017). Canonical Primal-Dual Method for Solving Non-convex Minimization Problems, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp.223-248,

http://arxiv.org/abs/1212.6492
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20. Morales-Silva, D. and Gao, DY (2017). On Minimal Distance Between Two Surfaces, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp.359-372.
21. Ruan, N. and Gao, D.Y. (2017). Global Optimal Solution Computation of a Quadratic Integer Programming Problem with Linear Inequality Constraints, in D.Y. Gao et al. (eds.), Canonical Duality-Triality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 315-338. http://arxiv.org/abs/1205.0856
22. Jin, Z. and Gao, DY (2017). On D.C. Optimization Problems, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 203-222.
23. Chen, Y. and Gao, DY (2017). Global Solutions to Spherically Constrained Quadratic Minimization via Canonical Duality Theory, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 291-314.
24. Ruan, N. and Gao, DY (2017). Canonical Duality Theory for Solving Nonconvex/Discrete Constrained Global Optimization Problems, in D.Y. Gao et al. (eds.), Canonical Duality-Triality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 187-202.
25. Lu, X.J. and Gao, D.Y. (2017). Canonical Duality Method for Solving Kantorovich Mass Transfer Problem, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 105-126.
26. Wu, C. and Gao, DY (2017). Canonical Primal–Dual Method for Solving Nonconvex Minimization Problems, in D.Y. Gao et al. (eds.), Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, Springer, pp. 223-248

Q354. New discoveries, inventions, or patent disclosures to report for this period? This question is required.

- Yes

Q355. Changes in research objectives (if any):

N/A

Q356. Change in AFOSR Program Officer, if any:

Dr. James Lawton retired at the end of April, 2021, Dr. Hal S. Greenwald is the new Program Office.

Q357. Extensions granted or milestones slipped, if any:

The PI's school was reformed in 2018, the PI was forced to take unexpected teaching duty by the new dean in 2019 (which violets the project contact). Also due to the impact of the Covid-19, the PI's request for extension was denied by the dean in 2020, the third milestone, i.e. Task 3 in the project, is slipped.

Q412. Abstract
Please submit your report abstract below.

Supported by this AFOSR grant, the PI and his co-PI, post-doctor and co-workers have successfully developed a breakthrough canonical duality theory and its associated algorithms for solving a large class of challenging problems in global optimization and decision science. Within three years, his team has published 2 books and about 19 research papers.
The most significant achievement of this project is the solution to the well-known NP-complete knapsack problem. Application of this solution to engineering science leads to a correct math model and a powerful algorithm for structural topology optimization.
Additional achievements include a new global optimal method for solving chaotic dynamical systems, an efficient deterministic algorithm for solving huge-sized sensor network problems, a unified modelling and complete solutions to d.c. programming and general fixpoint problems in multi-scale systems. The relation between chaos in complex systems and NP-hardness in computer science is discovered for the first time.
However, due to the facts that the PI was forced to take unexpected teaching duty in 2019 (which violets the program contract), and his request for the program extension was denied by the new school dean in 2020, the PI is not able to complete the project for the first time.

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Q394. Appendix Documents (Upload any additional documentation to support Appendix A and B as specified in AFOSR Instruction 61-7)
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AFOSR Grant FA9550-17-1-0151 Final Report
February 1, 2017 – December 31, 2020

PI: David Y Gao, Co-PI: N. Ruan, *Federation University Australia*
Title: Canonical Duality Theory and Algorithm for Solving 'NP-Hard Problems' in Decision Science and Complex Systems

Personnel Supported:
Post-doctor: Vittorio Latorre
Visiting Scientists:
1. Professor Peter Olver, University of Minnesota
2. Professor Z.C. Cai, Purdue University
3. Professor Zhi-Li Liao, Hong Kong Baptist University
4. Professor Jun Wang, Hong Kong City University
5. A/Professor Justin Wang, Latrobe University

Other Senior Collaborators:
1. Dr. Eldar Hajilarov, Federation University

Accomplishments/New Findings:
Research and Education Activities
Supported by this AFOSR grant**,** the PI and his Co-PI, student, post-doctor and co-workers have successfully applied the canonical duality theory and its associated algorithms for solving a large class of challenging problems in global optimization and decision science. He's group has published **2** books and about **26** papers. The most significant achievements of this project are the analytical solution to the linear knapsack problem and its applications to structural topology optimization. The knapsack problems are well-known "NP-Hard" problems in computer science and global optimization. Due to the integer constraint, traditional theories and methods can't be applied to obtain global optimal solution in polynomial time. Even the simplest linear knapsack problem is listed as one of Karp's 21 NP-complete problems. However, by using the canonical duality theory, the integer constrained global optimization problems in n-dimensional space can be equivalently converted to a unified concave maximization problem in ONE dimensional continuous space, which can be solved to obtain global optimal solution in polynomial time. Particularly, the linear knapsack problem can be solved analytically. Results show that as long as the knapsack problem has a unique solution, it is not NP-hard and its solution can be obtained analytically by the canonical duality theory.
This research reveals, for the first time, the reason for NP-Hardness, i.e. the so-called NP-hardness is mainly due to certain symmetry such that the problem may have multiple solutions. **Therefore, most of NP-hard problems are artificially proposed since the perfect symmetry does not exist in real world.** Applications to topology optimization leads to a correct mathematical model and a powerful algorithm for 3-D structural optimal design. A powerful deterministic method and algorithm have been developed, which can be used for lightweight design of aircrafts and armored vehicles.

The main results have been published in *Computational Methods in Applied Mechanics and Engineering*, the top journal in computational mechanics and engineering sciences, and two papers have been accepted by the top rank IEEE journals. Additionally, a series of challenging problems have been solved, such as the well-known NP-hard sensor location problem in network optimization, a complete set of analytical solutions to nonconvex mechanics, and nonconvex/mixed integer optimization problems in chaotic dynamics, computational biology, decision science, machine learning, neural networks, post-buckling of large deformed structures, industrial and systems engineering, etc. A powerful deterministic method and algorithm have been developed, which can be used for solving efficiently large scale nonconvex/nonsmooth/discrete optimization problems. The first two milestones, i.e. Task 1 and 2, proposed in the proposal are accomplished.

However, since the PI's school was reformed again in 2018, the PI was forced to take unexpected teaching duty by the new school dean in 2019, this violets the program contract listed in Budget Justification (the PI has reported this case to Program Office James Lawton via email in 2019). Also the school failed to provide the promised scholarships for two PhD students to support this program, the new dean of the school denied the PI's requests for the continuation of the Co-PI's appointment and the extension for the program, the third milestone, i.e. the Task 3 in the program, is slipped. This is the first time in PI's career that he could not finish his granted project.

Contributions:

Contributions within Discipline:

The canonical duality theory and algorithm can be used for solving a large class of challenging problems in decision science and complex systems.

Contributions to Other Disciplines:

Canonical duality theory has been used successfully in neural networks optimization, sensor communication systems, filter design, signal processing, machine learning, chaotic dynamical systems, decision making, supply chain, scheduling problems, and computational mechanics, etc.

Impacts to the Communities:

Supported by grants from AFOSR since 2010, the PI's Applied Mathematics research program at Federation University has been ranked a perfect 5 (well above world standard) in 2018 ERA outcomes by Australian Research Council (ARC), which is even higher than those top universities such as Univ. Melbourne and Univ. Sydney (both of them are only ranked as 4, https://dataportal.arc.gov.au/ERA/Web/Outcomes#/for/0102).
Since the PI arrived this university in 2010, the ranking of this applied math program has been increased from NA (Not assessed due to low volume) in 2010 to 2 in 2012, 3 in 2015 and jumped to 5 in 2018.
Also, due to the PI and Co-PI's breakthrough contributions in structural mechanics and topology optimal design, the ERA ranking of the Civil Engineering program at Federation University has jumped from 3 in 2015 to 5 in 2018,
see https://dataportal.arc.gov.au/ERA/Web/Outcomes#/for/0905.
The continuous support from AFOSR played an important role for this key achievement!

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The following international conference was organized successfully.

1. Chair, International Symposium of Control and Optimization for Multidisciplinary Studies, February 6-9, 2017, Federation University Australia

Distinguished, Plenary, and Invited Lectures at International Conferences.

1. Invited speaker, Symposium of Defense Industry and Security, December 4-5, 2019, Deakin University. Title: Decision making for multi-target, multi-level complex systems.
2. IEEE SMC Special Lecturer, Deakin University, Thursday, 9 May 2019. Title: Triality: God Image for Natural Phenomena and Solutions to NP-Hard Knapsack Problems
3. Distinguished Lecture, 2nd International Conference on Modern Mathematical Methods and High Performance Computing in Science and Technology, 4-6 January, 2018, New Delhi, India.
4. Plenary Lecture, The 5th IFAC Workshop on Mining, Mineral and Metal Processing, Shanghai, China, August 23-25, 2018.
5. Plenary Lecture, International Conference in Mathematics Trends and Developments 2017 (ICMTD-17), 28 – 30 Dec. 2017, Cairo, Egypt.
6. Plenary Lecture, Frontier Forum on Intelligent Control and Decision Optimization, 23-25 August, 2017, Central South University, Changsha, China
7. Invited Lecture, Symposia on Intelligent Technologies for Advancing and Safeguarding Australia. 15 August 2017, Deakin University, Geelong, Australia.
8. Invited Lecture, 12th International Symposium on Health Informatics and Management, 2-3 June, 2017, National Chiao Tung University, Taiwan.

Books Published:

1. Gao, DY, Latorre, V. and Ruan, N. Canonical Duality Theory: Unified Methodology for Multidisciplinary Study, July, Springer. 2017, 377p.
2. V.K. Singh, D.Y. Gao, and A. Fisher (2019). Advances in Mathematical Methods and High Performance Computing, Advances in Mechanics and Mathematics 41, Springer Nature Switzerland AG 2019, 503p.

Papers Published:

1. Gao, D.Y. (2019). Canonical Duality Theory and Algorithm for Solving Bilevel Knapsack Problems with Applications, IEEE Transactions on Systems, Man, and Cybernetics: Systems (published online first at IEEE, Impact Factor: 7.351), IEEE Transactions on Systems, Man, and Cybernetics: Systems (Volume: 51, Issue: 2, Feb. 2021) DOI:10.1109/TSMC.2018.2882792
2. Latorre, V. and Gao, D.Y. (2019). Efficient Deterministic Algorithm for Huge-Sized Noisy Sensor Localization Problems via Canonical Duality Theory, IEEE Transaction on Cybernetics. (published online first) DOI: 10.1109/TCYB.2019.2891112 Impact Factor: 11.079
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4. Ali, E. and Gao, DY (2018). On SDP Method for Solving Canonical Dual Problem in Post Buckling of Large Deformed Elastic Beam. Communications in mathematical sciences 16(5), 2018, DOI:10.4310/CMS.2018.v16.n5.a3
5. Ruan, N. Gao, D.Y. (2018). On Modelling and Complete Solutions to General Fixpoint Problems in Multi-Scale Systems with Applications, J. Fixed Point Theory, Springer 23(1):23. DOI: 10.1186/s13663-018-0648-x
6. Gao, DY, Neff, P., Roventa, I., Thiel, C. (2017) On the convexity of nonlinear elastic energies in the right Cauchy-Green tensor, Journal of Elasticity, April 2017, Volume 127, Issue 2, pp 303–308
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AFOSR Grant FA9550-17-1-0151 Final Report
February 1, 2017 – December 31, 2020

PI: David Y Gao, Co-PI: N. Ruan, *Federation University Australia*
Title: Canonical Duality Theory and Algorithm for Solving 'NP-Hard Problems' in Decision Science and Complex Systems

Personnel Supported:
Post-doctor: Vittorio Latorre
Visiting Scientists:
1. Professor Peter Olver, University of Minnesota
2. Professor Z.C. Cai, Purdue University
3. Professor Zhi-Li Liao, Hong Kong Baptist University
4. Professor Jun Wang, Hong Kong City University
5. A/Professor Justin Wang, Latrobe University

Other Senior Collaborators:
1. Dr. Eldar Hajilarov, Federation University

Accomplishments/New Findings:

Research and Education Activities

Supported by this AFOSR grant**,** the PI and his Co-PI, student, post-doctor and co-workers have successfully applied the canonical duality theory and its associated algorithms for solving a large class of challenging problems in global optimization and decision science. He's group has published **2** books and about **26** papers. The most significant achievements of this project are the analytical solution to the linear knapsack problem and its applications to structural topology optimization. The knapsack problems are well-known "NP-Hard" problems in computer science and global optimization. Due to the integer constraint, traditional theories and methods can't be applied to obtain global optimal solution in polynomial time. Even the simplest linear knapsack problem is listed as one of Karp's 21 NP-complete problems. However, by using the canonical duality theory, the integer constrained global optimization problems in n-dimensional space can be equivalently converted to a unified concave maximization problem in ONE dimensional continuous space, which can be solved to obtain global optimal solution in polynomial time. Particularly, the linear knapsack problem can be solved analytically. Results show that as long as the knapsack problem has a unique solution, it is not NP-hard and its solution can be obtained analytically by the canonical duality theory.

This research reveals, for the first time, the reason for NP-Hardness, i.e. the so-called NP-hardness is mainly due to certain symmetry such that the problem may have multiple solutions. **Therefore, most of NP-hard problems are artificially proposed since the perfect symmetry does not exist in real world.** Applications to topology optimization leads to a correct mathematical model and a powerful algorithm for 3-D structural optimal design. A powerful deterministic method and algorithm have been developed, which can be used for lightweight design of aircrafts and armored vehicles.

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The main results have been published in *Computational Methods in Applied Mechanics and Engineering*, the top journal in computational mechanics and engineering sciences, and two papers have been accepted by the top rank IEEE journals. Additionally, a series of challenging problems have been solved, such as the well-known NP-hard sensor location problem in network optimization, a complete set of analytical solutions to nonconvex mechanics, and nonconvex/mixed integer optimization problems in chaotic dynamics, computational biology, decision science, machine learning, neural networks, post-buckling of large deformed structures, industrial and systems engineering, etc. A powerful deterministic method and algorithm have been developed, which can be used for solving efficiently large scale nonconvex/nonsmooth/discrete optimization problems. The first two milestones, i.e. Task 1 and 2, proposed in the proposal are accomplished.

However, since the PI's school was reformed again in 2018, the PI was forced to take unexpected teaching duty by the new school dean in 2019, this violets the program contract listed in Budget Justification (the PI has reported this case to Program Office James Lawton via email in 2019). Also the school failed to provide the promised scholarships for two PhD students to support this program, the new dean of the school denied the PI's requests for the continuation of the Co-PI's appointment and the extension for the program, the third milestone, i.e. the Task 3 in the program, is slipped. This is the first time in PI's career that he could not finish his granted project.

Contributions:

Contributions within Discipline:

The canonical duality theory and algorithm can be used for solving a large class of challenging problems in decision science and complex systems.

Contributions to Other Disciplines:

Canonical duality theory has been used successfully in neural networks optimization, sensor communication systems, filter design, signal processing, machine learning, chaotic dynamical systems, decision making, supply chain, scheduling problems, and computational mechanics, etc.

Impacts to the Communities:

Supported by grants from AFOSR since 2010, the PI's Applied Mathematics research program at Federation University has been ranked a perfect 5 (well above world standard) in 2018 ERA outcomes by Australian Research Council (ARC), which is even higher than those top universities such as Univ. Melbourne and Univ. Sydney (both of them are only ranked as 4, https://dataportal.arc.gov.au/ERA/Web/Outcomes#/for/0102).
Since the PI arrived this university in 2010, the ranking of this applied math program has been increased from NA (Not assessed due to low volume) in 2010 to 2 in 2012, 3 in 2015 and jumped to 5 in 2018.
Also, due to the PI and Co-PI's breakthrough contributions in structural mechanics and topology optimal design, the ERA ranking of the Civil Engineering program at Federation University has jumped from 3 in 2015 to 5 in 2018,
see https://dataportal.arc.gov.au/ERA/Web/Outcomes#/for/0905.
The continuous support from AFOSR played an important role for this key achievement!

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The following international conference was organized successfully.

1. Chair, International Symposium of Control and Optimization for Multidisciplinary Studies, February 6-9, 2017, Federation University Australia

Distinguished, Plenary, and Invited Lectures at International Conferences.

1. Invited speaker, Symposium of Defense Industry and Security, December 4-5, 2019, Deakin University. Title: Decision making for multi-target, multi-level complex systems.
2. IEEE SMC Special Lecturer, Deakin University, Thursday, 9 May 2019. Title: Triality: God Image for Natural Phenomena and Solutions to NP-Hard Knapsack Problems
3. Distinguished Lecture, 2nd International Conference on Modern Mathematical Methods and High Performance Computing in Science and Technology, 4-6 January, 2018, New Delhi, India.
4. Plenary Lecture, The 5th IFAC Workshop on Mining, Mineral and Metal Processing, Shanghai, China, August 23-25, 2018.
5. Plenary Lecture, International Conference in Mathematics Trends and Developments 2017 (ICMTD-17), 28 – 30 Dec. 2017, Cairo, Egypt.
6. Plenary Lecture, Frontier Forum on Intelligent Control and Decision Optimization, 23-25 August, 2017, Central South University, Changsha, China
7. Invited Lecture, Symposia on Intelligent Technologies for Advancing and Safeguarding Australia. 15 August 2017, Deakin University, Geelong, Australia.
8. Invited Lecture, 12th International Symposium on Health Informatics and Management, 2-3 June, 2017, National Chiao Tung University, Taiwan.