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# Simple Model for Scattering of Acoustic Signals from Rough Surfaces

by Geoffrey H Goldman

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# Simple Model for Scattering of Acoustic Signals from Rough Surfaces

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| <b>14. ABSTRACT</b><br>Models based on a simplified Kirchhoff approximation and locally reactive impedance are used to characterize the acoustic signal reflected by the ground from an elevated source in the far field and received by an array of microphones. The surface of the ground is modeled as mostly smooth, but with small variations in the surface height. The height of the ground is modeled using two 1-D Gauss–Markov processes. The signal propagation is modeled using geometric optics in a homogeneous atmosphere with the angle of incidence equal to the angle of reflectance. The ground impedance is modeled as locally reactive. For relatively smooth surfaces, these models result in the addition of phase noise with a multivariate Gaussian distribution incorporated into the reflected signal. For rougher surfaces, the effect of the ground results in phase noise that is multiplicative with a multivariate Gaussian distribution. These results can be incorporated into acoustic simulation software and the design and evaluation of beam-forming algorithms. |                                    |   |   |  |  |
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## 1. Introduction

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Scattering from rough surfaces has been extensively studied (Darmon 2020). The Kirchhoff approximation is one of the most cited models for characterizing scattering from rough surfaces. It assumes each point on the surface belongs to an infinite tangent plane (Meecham 1956). This results in the angle of incidence for a ray scattering off the ground to equal the angle of reflectance. However, even with this simplification, the integral to estimate the scattered field is complex, and it is difficult to calculate closed form solutions for the statistical moments.

For many acoustic applications, incorporating models for rough surface scattering into the signal processing algorithms should improve performance and testing. However, the prior information on the environment and the selected placement of the microphone array on the ground by the user are dependent upon numerous factors, which are often difficult to estimate. This high uncertainty suggests only simple rough surface scattering models are needed for many acoustic applications.

Digital elevation maps (DEMs) that characterize the height of the ground over a grid of sample points have been generated using numerous sensors for locations throughout the world. The height of the ground at other points within the grid can be estimated using interpolation. However, for many applications, the resolution of the DEM is too low, and additional models are needed to characterize the surface.

Gauss–Markov processes (GMPs) are often used to model the height of the ground due to their simple mathematical description and computational efficiency. GMPs combine the properties of each process. A Gaussian process sampled at various spatial or time samples will have a multivariate Gaussian distribution that is completely described by its mean and covariance matrix. A Markov process is a stochastic process with the property that its future state is only dependent upon the current state. The previous states add no new information. For applications that use a large DEM, a Gaussian Markov random field (GMRF) is often used to model the heights of the ground (Aguilar 2016). The statistics associated with the random field are dependent upon a neighborhood of points, not a single point. However, for applications that are focused on small areas, this model is not appropriate. A GMP with an exponential decaying autocorrelation is often used to model a smaller area of interest that is mostly homogeneous.

For most acoustic signal processing applications, propagation is modeled using geometric optics with straight-line propagation that is independent of frequency. In addition, the source is modeled as a point radiator in the far field, and the atmosphere is assumed to be transparent. These assumptions generate planar

wavefronts at the receiver. For many applications, these assumptions are appropriate and are made in this report.

## 2. Modeling

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A model is developed to statistically describe scattering from the ground of a narrowband acoustic source in the far field that is received by an array of microphones near the ground. The analysis makes several approximations and leverages results from other models. The atmosphere is assumed to be transparent with the signals propagating along straight paths, and the impedance of the ground is assumed to be locally reactive and independent of aspect angle (Attenborough et al. 2007).

The complex reflection coefficient of the ground is calculated using

$$\rho(\Theta, \omega) = \frac{Z(\omega)\cos(\Theta)-1}{Z(\omega)\cos(\Theta)+1} \quad (1)$$

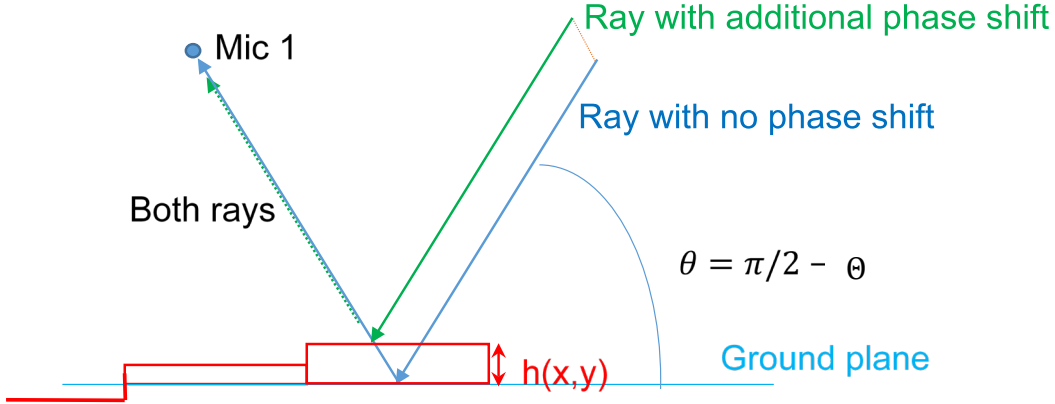
where  $Z(\omega)$  is the normalized acoustic impedance,  $\omega$  is the frequency of the signal, and  $\Theta$  is the angle of incidence for a ray (Salomons 2001).

The signals received from the direct and indirect path can be combined using the principle of superposition. The pressure of the complex envelope of a narrowband signal received near the ground is modeled using

$$P(\Theta) = p_c(\Theta) + p_c(\Theta)\rho e^{j(\varphi+\zeta)} = p_c(\Theta)(1 + \rho e^{j(\varphi+\zeta)}) \quad (2)$$

where  $p_c(\Theta)$  is the complex pressure from the direct path signal,  $\varphi$  is the deterministic phase difference between the direct path and indirect path,  $\zeta$  is a random variable describing the phase shift caused by variations in the height of the ground, and  $j$  is a complex number (Salomons 2001).

The model developed in this report for the rough surface scattering is illustrated in Fig. 1. Similar to the Kirchhoff approximation, the model assumes that each point on the ground reflects the signal as if it belonged to an infinite tangent plane. However, an additional simplification is made relative to the Kirchhoff approximation. The slope of the ground is assumed to be zero as illustrated by the horizontal red lines at the bottom of Fig. 1, where the height of the ground for the ray of interest is given by  $h(x,y)$ . The rays from the source, illustrated by the blue and green lines, are assumed to be parallel. The difference in spherical attenuation between the two rays shown in Fig. 1 is assumed to be small and ignored.



**Fig. 1 Geometric model for the reflection of a ray by a rough surface**

The height of the ground is modeled by combining two independent 1-D GMPs. Conceptually, the location of the desired sample points of the GMPs corresponds to the location on the ground of a ray that intersects a microphone above the ground. In Fig. 1, it corresponds to the location of the arrow of the blue ray on the ground plane. The autocorrelation of the GMP is given by

$$R(x) = \sigma^2 e^{-\beta|x|} \quad (3)$$

where  $x$  is the distance between two sample points,  $\sigma^2$  is the variance of  $x$  in units of meters squared and  $1/\beta$  is a spatial distance constant. The random variable associated with the height of the ground for a reflection at the coordinate  $(x,y)$  is modeled using

$$h(x, y) = z_x(x) + z_y(y) \quad (4)$$

where  $h(x)$  and  $z_y(y)$  are independent and identical GMPs.

Using geometry, the additional phase shifts due to variations in the height of the ground at  $(x,y)$  for a signal of interest are calculated using

$$\zeta(x, y) = \frac{-h(x,y) 2\pi}{\sin(\theta) \lambda} = \frac{-(z_x(x)+z_y(y)) 2\pi}{\sin(\theta) \lambda} \quad (5)$$

where  $\lambda$  is the wavelength and  $\theta = \pi/2 - \Theta$  is the elevation angle as shown in Fig. 1.

For this model to be valid, the probability of a ray reflecting off a vertical red line in Fig. 1 should be much smaller than the probability of a ray reflecting off a horizontal red line. Based on geometry and the constants in Eq. 3, this model

requires that  $\tan(\theta) \gg \frac{\sigma}{L}$  where  $L=1/B$  is the characteristic length associated with the GMPs. This model is not valid at grazing angles where  $\theta$  is close to zero.

The 1-D distribution of the additional phase shifts for a signal of interest at a given location is given by

$$\zeta \sim N(0, C_0) \quad (6)$$

where  $N$  denotes normal or Gaussian distribution and the covariance matrix is  $C_0 = 2 \left( \frac{\sigma 2\pi}{\sin(\theta)\lambda} \right)^2$ . Since  $z_x(x)$  and  $z_y(y)$  are independent, their individual covariance matrices are added, and the scaling factor in Eq. 5 is squared to calculate  $C_0$ .

The covariance of the phase for the  $(i, k)$  microphones is given by

$$C_\zeta(i, k) = E(\zeta(x_i, y_i)\zeta(x_k, y_k)^*) \quad (7)$$

where  $E$  denotes expected value and  $*$  denotes complex conjugate. It is evaluated using Eqs. 3–5 to obtain

$$C_\zeta(i, k) = \frac{C_0}{2} (e^{-(|x_i-x_k|)\beta} + e^{-(|y_i-y_k|)\beta}) \quad (8)$$

As expected, when  $i=k$ ,  $C_\zeta(k, k) = C_0$ .

Using Eq. 2, the pressure of the complex envelope of the indirect or reflected signal for element  $i$  is given by

$$P_r(i) = p_c(\Theta)\rho e^{j\varphi_i} e^{j\zeta_i} \quad (9)$$

The mean of  $P_r(i)$  is calculated using

$$u_{Pr} = K_{Pr} \int_{-\infty}^{\infty} e^{j\zeta} f_\zeta d\zeta \quad (10)$$

where  $K_{Pr}(i) = p_c(\Theta)\rho e^{j\varphi_i}$  and  $f_\zeta$  is the probability distribution function (pdf) of the normal distribution described in Eq. 6. Using Euler's formula, the integral is rewritten as

$$E(e^{j\zeta}) = \int_{-\infty}^{\infty} \cos(\zeta) f_\zeta(\zeta) d\zeta + j \int_{-\infty}^{\infty} \sin(\zeta) f_\zeta(\zeta) d\zeta \quad (11)$$

Now, the previous integral can be solved using the standard integral formulas

$$K \int_{-\infty}^{\infty} \cos(kx) e^{-ax^2} = K \sqrt{\frac{\pi}{a}} e^{-\frac{k^2}{4a}} \quad (12)$$

$$\int_{-\infty}^{\infty} \sin(kx) e^{-ax^2} = 0 \quad (13)$$

to obtain

$$u_{Pr}(i) = K_{Pr}(i)e^{-\frac{C_0}{2}}. \quad (14)$$

As expected, as the ground becomes smoother,  $\sigma^2 \rightarrow 0$ , which implies  $C_0 \rightarrow 0$ ,  $u_{Pr} \rightarrow K_{Pr}$ , and there is no change in the average pressure value. As the ground becomes rougher, the average pressure value of the complex envelope is zero since the phase is random. However, for any particular realization, the amplitude is not changed.

The covariance of the pressure of the indirect signal at microphone elements  $(i, k)$  is given by

$$C_{Pr}(i, k) = E\left((P_r(i) - u_{Pr}(i))(P_r(k) - u_{Pr}(k))^*\right) \quad (15)$$

Substituting Eqs. 9 and 14 into 15 results in

$$C_{Pr}(i, k) = K_{P2} E\left(e^{j(\zeta_i - \zeta_k)} + 1 - e^{-j\zeta_k} e^{-\frac{C_0}{2}} - e^{j\zeta_i} e^{-\frac{C_0}{2}}\right) \quad (16)$$

where  $K_{P2} = |p_c(\Theta)\rho|^2 e^{j(\varphi_i - \varphi_k)}$ .

The evaluation of the expected value of the first term in Eq. 16 can be simplified by setting  $\Psi_{ik} = \zeta_i - \zeta_k$ . The pdf of  $\Psi_{ik}$  can be easily calculated once we realize that it is a linear transformation of the 2-D Gaussian distribution with a covariance matrix (described in Eq. 8) to a 1-D Gaussian distribution with a mean of zero and variance of

$$C_{\Psi_{ik}} = A\Gamma_{ik}A^T \quad (17)$$

where  $\Gamma_{ik} = \begin{bmatrix} C_\zeta(i, i) & C_\zeta(i, k) \\ C_\zeta(k, i) & C_\zeta(k, k) \end{bmatrix}$

and  $A = [1 \quad -1]$ . From Eq. 8,  $C_\zeta(i, i) = C_\zeta(j, j)$  and  $C_\zeta(i, j) = C_\zeta(j, i)$ . Applying these equalities to Eq. 17 results in

$$C_{\Psi_{ik}} = 2\left(C_\zeta(i, i) - C_\zeta(i, k)\right) = C_0\left(2 - e^{-|x_i - x_k|\beta} - e^{-|y_i - y_k|\beta}\right) \quad (18)$$

As expected, if  $i=k$ , then  $C_{\Psi_{ii}} = 0$ , since the covariance of two identical signals is zero.

Now, the expected value of the first term in Eq. 16 is equal to

$$E\left(e^{j(\zeta_i - \zeta_k)}\right) = \int_{-\infty}^{\infty} e^{j\Psi} f_{\Psi_{ik}} d\Psi \quad (19)$$

where  $f_{\Psi_{ik}}$  is the pdf of  $\Psi_{ik}$ . This integral can be evaluated using Eqs. 12 and 13 with  $k=1$ ,  $a = \frac{1}{2C_{\Psi_{ik}}}$ , and  $K = \frac{K_{P2}}{\sqrt{2\pi C_{\Psi_{ik}}}}$  to obtain

$$K_{P2}E(e^{j(\zeta_i - \zeta_k)}) = K_{P2}e^{-\frac{C_{\Psi_{ik}}}{2}} \quad (20)$$

The expected value of the other phasor terms in Eq. 16 is calculated using the same procedure. The integration is over two dimensions, but since the term of interest is only a function of a single variable, integration over the second variable with any pdf is equal to 1. Therefore, the calculation can be performed over a single variable.

This integral of the third term is also solved using Eqs. 12 and 13, with  $k = 1$ ,  $a = \frac{1}{2C_0}$  and  $K = \frac{K_{P2}e^{-\frac{C_0}{2}}}{\sqrt{2\pi C_0}}$  to

$$C_{i,k}^3 = K_{P2}e^{-\frac{C_0}{2}}. \quad (21)$$

The integrals of the third and fourth terms in Eq. 16 are equal because cosine is an even function, therefore  $\cos(-x) = \cos(x)$ . The integral of the second term, which is a constant, is the same constant, since the integral of a pdf is always equal to 1. Combining all the terms in Eq. 16 results in

$$C_{P_r}(i, k) = K_{P2} \left( e^{-\frac{C_{\Psi_{ik}}}{2}} - e^{-C_0} \right) \quad (22)$$

where  $C_{\Psi_{ik}} = C_0 \left( 2 - e^{-|x_i - x_k|/\beta} - e^{-|y_i - y_k|/\beta} \right)$  as shown in Eq. 8.

For the case when the ground is perfectly flat,  $\sigma^2 = 0$ , which implies  $\Psi_{ik} = C_0 = 0$ . As expected, when there is no random variations in the surface height, Eq. 22 reduces to  $C_{P_r}(i, k) = 0$ . Equations 10 and 22 were validated using the Monte Carlo simulation shown in Appendix A. As the number of samples increased, the error between the predicted and simulated mean and covariance matrices converged to zero.

For small variations in the ground height relative to a wavelength, the effect of the rough surface can be approximated using a first-order Taylor series approximation. The first-order Taylor series approximation on Eq. 2 results in a linear dependency on  $\zeta$  as shown here:

$$p_c(\Theta)(1 + \rho e^{j(\varphi + \zeta)}) \approx p_c(\Theta) \left( 1 + \rho(1 + j(\varphi + \zeta)) \right). \quad (23)$$

This approximation is valid when the second-order term of the Taylor series approximation of the *exponential* function is small. This requires that  $\frac{\zeta^2}{2} \ll 1$ , which can be expressed using Eq. 6 as  $\left(\frac{\sigma 2\pi}{\sin(\theta)\lambda}\right)^2 \ll 1$ . Since  $\zeta$  is Gaussian distributed, the pdf of the combined direct and indirect signal in Eq. 23 is also Gaussian distributed when  $p_c(\Theta)$ ,  $\varphi$  and  $\rho$  are constants. The random variable  $\zeta$  can be modeled as additive Gaussian noise with respect to the additional phase term  $\varphi$ , but both terms are multiplied by the amplitude of the signal,  $p_c(\Theta)$ . As the random variations in the phase become larger, the Taylor series approximation is not valid and the phasor transformation becomes nonlinear. For this region, the effect of the surface roughness corresponds to phase noise that is multiplicative on the indirect signal. This case is of less interest for many applications, since it is difficult to solve inverse problems for signals with large random multiplicative noise.

### 3. Simulation

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These models are implemented in MATLAB to simulate scattered rays by a rough surface and received by an array of microphones. Since the distribution for both cases is based on Gaussian statistics, data can be simulated using a mean and covariance matrix calculated from input parameters. An alternative approach that is conceptually appealing, but computationally expensive, is to simulate the height of ground over a grid, then interpolate the height at the desired ground reflection location. Data with Gaussian distributed phases with a known mean and covariance matrix can be generated using standard techniques as shown in the MATLAB code in Appendix B. First, the covariance matrix is calculated using Eq. 22, then random numbers are generated using the MATLAB function `mvnrnd`. A second approach that uses a Cholesky decomposition on the covariance matrix is also shown. The input to the model is an array of (x,y) values corresponding to the location of the ground bounce for the signal of interest, root mean square (RMS) value of the ground height, a spatial length associated with the ground decorrelation, incident angle, and frequency, amplitude, and phase of the signal. The output of the code is the received signal from the indirect path for an array of microphones. The statistics associated with the data are in agreement with the predicted results.

Data can also be generated by simulating the height of the ground using two GMPs, interpolating to the desired (x,y) locations of the ground reflection, then converting the heights to phases. A GMP can be digitally simulated using a single pole infinite impulse response (IIR) filter driven by white Gaussian noise as shown here:

$$x(n + 1) = ax(n) + bz(n) \quad (24)$$

where  $a$  and  $b$  are constants,  $z(n)$  are samples from a Gaussian distribution, and  $n$  is an integer. The coefficients in the filter can be calculated from the constants in the autocorrelation function of the GMP in Eq. 3. The decay value is calculated using

$$a = d = e^{-\frac{dL}{L}} \quad (25)$$

where  $d$  is the decay value which is set equal to the  $a$  coefficient in the IIR filter,  $L$  is the spatial length constant, and  $dL$  is the spatial sample rate. These coefficients result in an autocorrelation function value of  $R(L/dL) = e^{-1} \approx 0.37$  and a slope of the natural log of the autocorrelation function equal to approximately  $d - 1$ . A typical value of  $d$  is 0.99. Using conservation of energy, the  $b$  coefficient in Eq. 24 is equal to

$$b = \sqrt{\sigma^2(1 - a^2)} \quad (26)$$

Data can be generated from simulated height data using the code in Appendix C. The height of the ground was simulated in MATLAB for a surface roughness of  $\sigma = 0.01$  m,  $L = 20$  m, and  $dL = 0.1$  m. The results for a typical realization of ground heights are shown in Fig. 2.

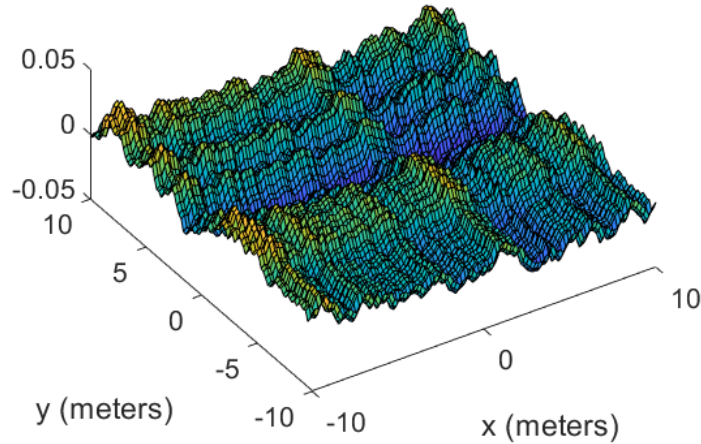


Fig. 2 Simulated height of the ground

## 4. Conclusion

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Models based on a simplified Kirchhoff approximation, locally reactive impedance, and a single multipath return are used to characterize the acoustic signal reflected by the ground and received by an array of microphones for elevated sources in the

far field. The models are used to characterize the mean and covariance of the complex envelope of the signal. The absolute value of the amplitude of the reflected signal for a given realization is not attenuated due to surface roughness; however, the mean value is attenuated due to random phase shifts. For surfaces that are relatively smooth, the average amplitude of the received signal is slightly attenuated due to surface roughness, and each realization is perturbed with additive multivariate Gaussian noise. For rougher surfaces, the average value of the received signal is significantly attenuated due to surface roughness, and each realization is perturbed with multiplicative phase noise that is Gaussian distributed. The models are implemented in MATLAB and used to generate simulated data. The simulated and theoretical results are in good agreement. These models can be incorporated into acoustics simulation software and used to help the design and evaluation of beamforming algorithms.

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## **Appendix A. MATLAB Code to Validate Mathematical Equations**

```

% check ground reflection model equations
% simulate signal reflecting off the ground and measured with two
microphones
% predicted and estimated mean and covariance matrices are in
good agreement

% written by Geoffrey Goldman, 3/22/2021

% input parameters

mu=[1; 2]; % mean phase for two microphones

rho=0.5; % correlation between two microphones
S=0.3; % power of measured signal at each mic, sigma^2

Sigma=S*[1 rho; % covariance matrix of phase
         rho 1]

N=1e5; % number of data samples

% calculations

Cpsi=[1 -1]*Sigma*[1;-1] % covariance of phase difference, C12

x = mvnrnd(mu,Sigma,N); % generate phase values

mean_x=mean(x); % check
cov_x=cov(x);

% convert x to phasor

phase2=exp(j*x); % phasor, no correction
ave_phase2_no_correction=mean(phase2); % not mc
est_phase=exp(-S/2); % mc

ex_phase=exp(j*mu); % exact_phase
ex_phase_no_correct=ex_phase*exp(-S/2); % correct for surface
roughness

error_ave_phase = ave_phase2_no_correction-ex_phase_no_correct.'
% correct, small numbers

cor_phase2=cov(phase2)

cov_ex_phase21=exp(j*(mu(1)-mu(2)))*exp(-Cpsi/2) - exp(j*(mu(1)-
mu(2)))*exp(-S)
cov_ex_phase11=1 - exp(-S);

cov21_error=cor_phase2(2,1)-cov_ex_phase21 % correct, small
numbers
cov11_error=cor_phase2(1,1)-cov_ex_phase11 % correct, small
numbers

```

## **Appendix B. MATLAB Code to Simulate the Signal Reflected by a Rough Surface**

---

```

% generate random phases using a Gauss-Markov processes to model
the height of the ground

% input parameters

Rough_surface_type=1; % 0=smooth surface, 1= rough surface
Data_gen_alg=1;      % 1=use matlab's mvnrnd, 2=use own
algorithm
RMS=.01; % Root mean square variation of ground height in x or y
LT=2.0; % spatial time constant in meters
el_angle=pi/2; % angle of incident, pi/2 is overhead, 0=grazing
lambda=0.3; % wavelength in meters
ground_loc_array=[0 0; 2 2; 4 4]'; % (x,y,0) point of interest
on the ground associated with each microphone
[two2,N]=size(ground_loc_array); % N = number of microphones
Pc0=1*exp(j*0); % pressure of signal near array
rel_phase_of_signal=exp(j*randn(1,N)*2*pi); % phase at each mic,
should be based on geometry
Khp=(2*pi/(lambda*sin(el_angle))) % scale factor to convert
height to phase

cov0=zeros(N,N); % covariance matrix of phase, mean is zero

for n=1:N
    for m=1:N
        % covariance of height
        cov0(m,n)=exp(-abs(ground_loc_array(1,m)-
ground_loc_array(1,n))/LT) + ...
            exp(-abs(ground_loc_array(2,m)-
ground_loc_array(2,n))/LT);
    end
end

cov1=(RMS*Khp)^2*cov0; % covariance of phase

if (Data_gen_alg==1)
    random_phase_array=mvnrnd(zeros(1,N),cov1)
else
    cov_upper_tri=cholcov(cov1); % factor to obtain upper
triangular
    random_phase_array=(randn(1,N)*cov_upper_tri)'; % generate
data
end

if (Rough_surface_type==0) % smooth surface
    data=Pc0*rel_phase_of_signal.*(ones(1,N) +
j*random_phase_array);
else % not smooth surface
    data=Pc0*rel_phase_of_signal.*exp(j*random_phase_array);
end

if (0) % test

```

```

cov0=[2    j -j/2;
      -j   2  1;
      j/2  1  2];
cov_upper_tri=cholcov(cov0); % factor to obtain upper
triangular

cov1=zeros(N);
KK=1e4; % number of trials
for k=1:KK
    data2=(randn(1,3)*cov_upper_tri)';
    cov1=cov1 + data2*data2'/KK; % not correct for complex
cov
end
random_phase_array=mvnrnd(zeros(N,1),cov0,KK);
cov2=cov(random_phase_array);

end

```

## **Appendix C. MATLAB Code to Simulate the Height of the Ground**

```

% simulate Gauss-Markov process to model the height of the ground

%  $y(n) = a*y(n-1) + b*x(n)$ 

% input parameters

RMS=.01; % Root mean square variation of ground height in x or y
LT=2.0; % spatial time constant in meters
L=20; % length of square of coverage area
res = 0.2 ; % x and y sample resolution in meters
N_time_const=LT/res; % number of elements associated spatial
constant
N=round(L/res)+1; % number of x and y values in arrays
NAUTO=60; % number autocorrelation values to plot
el_angle=pi/2; % angle of incident, pi/2 is overhead, 0=grazing
lambda=0.3; % wavelength in meters

plot_data=1;
plot_results=1;

x_array=-L/2:res:L/2; % x-axis in meters note, y=x

d=exp(-1/((LT/res))); % d= decay value , typical 0.99
alpha=d; % filter coefficient
slope=alpha-1; % slope of log of autocorrelation function

b=RMS*(1-alpha^2)^0.5; % filter coefficient
power=b^2/(1-alpha^2); % check == RMS^2

mean_reversion=1-d;
Ninit=round(1.2/mean_reversion); % start of autocorrelation
calculation after Ninit

NT=N+Ninit; % total number of samples per trial
th1=alpha^2/(1-(1-mean_reversion)^2);

K=10000; % number of trials

datax0=zeros(1,NT+1); % zx data arrays
datay0=zeros(1,NT+1); % zy data arrays

ave_power_zx=0;

data00_array=zeros(1,K); % data at (0,0)
dataNN_array=zeros(1,K); % data at (LT,LT) R=e-1

for k=1:K

    for n=1:NT % generate height data for a trial
        X=randn(1);
        Y=randn(1);
    end
end

```

```

        datax0(1,n+1)=alpha*datax0(1,n) + X*b;           % iir
filter0
        datay0(1,n+1)=alpha*datay0(1,n) + Y*b;           % iir
filter0
    end

    data00_array(1,k)=datax0(1,((N+1)/2)+Ninit) +
datay0(1,((N+1)/2)+Ninit); % save height data at xy=(0,0)
meters
    dataNN_array(1,k)=datax0(1,((N+1)/2)+Ninit+N_time_const) +
datay0(1,((N+1)/2)+Ninit+N_time_const); % save data at
xy=(LT,LT)

    ave_power_zx=ave_power_zx + (datax0(1,end).^2); % get last
value, its at steady state

    if (plot_data & k==1)

        figure('DefaultAxesFontSize',16)
        plot(datax0,'r')
        hold on
        plot(datay0,'k')

        figure('DefaultAxesFontSize',16)
        plot(x_array,datax0(1+Ninit:NT),'r')
        hold on
        plot(x_array,datay0(1+Ninit:NT),'k')

        xlabel('grid (m)')
        ylabel('amplitude')
        legend('x','y')

        figure
        plot(datax0,'r')
        [Xgrid,Ygrid]=meshgrid(x_array,x_array);
        Zx=ones(N,1)*datax0(1+Ninit:NT);
        Zy=(ones(N,1)*datay0(1+Ninit:NT))';

        Z=Zx+Zy; % height array, add heights at x and y

        figure('DefaultAxesFontSize',16)
        surface(Xgrid,Ygrid,Z)
        view(3)
        xlabel('x (meters)')
        ylabel('y (meters)')
        zlabel('z (meters)')
    end

    if (k==1)
        auto_array=xcorr(datax0(1+Ninit:NT),datax0(1+Ninit:NT));
    else

```

```

        auto_array=auto_array +
xcorr(datax0(1+Ninit:NT),datax0(1+Ninit:NT));
    end
end

% analysis of height data

mean_data00_array=mean(data00_array); % check = 0
var_data00_array=var(data00_array); % check variance = var_pred
mean_dataNN_array=mean(dataNN_array); % check = 0
var_dataNN_array=var(dataNN_array);
var_pred=2*RMS^2; % predicted variance

cov_0_N=mean((data00_array.*dataNN_array)); % real arrays
cov_pred=2*RMS^2*exp(-1); % check
error_var00=cov_0_N-cov_pred; % cov error, should be 0

Khp=(2*pi/(lambda*sin(el_angle))); % constant to covert height
to phase

phasor_data00=exp(j*data00_array*Khp); % generate phase data at
(0,0)
phasor_dataNN=exp(j*dataNN_array*Khp); % generate phase data at
(LT,LT)

mean_data00_array=mean(phasor_data00);
cov_data00_array=cov(phasor_data00);
mean_dataNN_array=mean(phasor_dataNN);
cov_dataNN_array=cov(phasor_dataNN);

C0=2*(RMS*Khp)^2;
mean_pred_phasor=exp(-C0/2); % checked, correct
ratio_error=mean_dataNN_array/mean_pred_phasor -1; % should be
zero

cov12=((phasor_data00-
(ones(1,K)*mean_pred_phasor))*(phasor_dataNN-
(ones(1,K)*mean_pred_phasor))')/K;

Sigma=C0*[1 exp(-1);
exp(-1) 1];

Cpsi=[1 -1]*Sigma*[1;-1] % covariance of phase difference, C12

cov12_pred_phasor=exp(-Cpsi/2)-exp(-C0); % checked, correct

cov12_error=cov12-cov12_pred_phasor;

sim_power_zx=ave_power_zx/K

theoretical_power_mean_reversion1=RMS^2*(alpha)^2*(1-b^(2*N))/(1-
b^2)

```

```

theoretical_power_mean_reversion_asym=RMS^2*(alpha)^2/(1-b^2)

th1=RMS^2*alpha^2/(1-b^2)
th3=RMS^2*mean_reversion^2/(1-b^2)

if (1)
  figure('DefaultAxesFontSize',16)
  plot((auto_array/max(auto_array)), 'k.')

  figure('DefaultAxesFontSize',16)
  plot((auto3_array(N-NAUTO-NSS:N+NAUTO-NSS)/auto3_array(N-
NSS+2)), '.')
end

figure('DefaultAxesFontSize',16)
plot(log(auto_array(round(end/2)-
20:(round(end/2)+20))/max(auto3_array)), '.')
xlabel('Lag + offset')
ylabel('Log of autocorrelation')
slope=-0.01

```

## List of Symbols, Abbreviations, and Acronyms

---

|      |                                   |
|------|-----------------------------------|
| 1-D  | one-dimensional                   |
| 2-D  | two-dimensional                   |
| DEM  | digital elevation map             |
| GMP  | Gauss–Markov processes            |
| GMRF | Gaussian Markov random field      |
| IIR  | infinite impulse response         |
| pdf  | probability distribution function |
| RMS  | root mean square                  |

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