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Permeable coatings for turbulent drag reduction

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**PERMEABLE COATINGS FOR
TURBULENT DRAG REDUCTION**

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University of Cambridge**

JUNE 2021

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Summary

The reduction of energy consumption and of pollutant emissions are deep and growing concerns for air transport application. This has motivated the investigation of several technologies to reduce turbulent skin friction on vehicle surfaces. This project investigates a recently proposed method for turbulent drag reduction: coatings with anisotropic permeability (i.e. anisotropic permeable substrates). In the first phase of this project, we conduct microscale calculations of the microscopic flow for specific microscale configurations. In the second phase, the interaction between the permeable substrate and the overlying turbulence is studied using fully resolved direct numerical simulation.

In the first phase, a methodology is proposed to characterize anisotropic permeable substrates, to identify the configurations that can offer the best drag reduction properties and understand if Brinkman equation is a good model.

To characterize anisotropic permeable substrates, a sufficiently deep anisotropic permeable substrate is considered, in which the fibers are very small compared to the lengthscales of the flow in the outer region. The Stokes equations are assumed to be the governing equation of the motion within the substrate. The Darcy equation is considered for the macroscopic velocities. From the solutions of these equations with a constant pressure gradient, the permeabilities in the streamwise, spanwise and wall-normal direction are obtained. Different configurations are analyzed in terms of drag reduction properties. As a result, fibers with a cross cross-section are proposed to be the best-suited to achieve a high anisotropy ratio.

A following study is carried out to understand if Brinkman equation is a good model, to find the best fit value for the effective macroscopic viscosity and to get a better understanding of the flow behavior at the substrate-fluid interface. These analyses are limited to permeable substrates with a circular cross-section. The results indicate that the Brinkman equation is valid for the streamwise flow where volume averaging methods can be well defined. The effective viscosity is subsequently calculated.

In the spanwise direction, shear driven flows exhibit regions of flow reversal and recirculation, and therefore do not currently fit with the Brinkman model. A potential self-similar solution is identified in the core of the substrate, which may in future work allow for the

microscale flow to be matched to the Brinkman model. The flow within the substrate is then decomposed into an exponential decay component, and base functions representing the variations in the spanwise velocity and streamfunction due to the fibres. Different spatial averages of the spanwise velocity are then observed. These results support the use of a self-similar solution in the substrate core.

The coupling of the flow field at the interface between the substrate and free flow is explored. It is shown that Brinkman equation overpredicts the slip length up to 50% if shear stress across the interface is assumed continuous. However, fitting the Brinkman solution to the inner flow gives a good estimation of the velocity at the interface but a significant jump in shear. Contributions of pressure and shear to the slip length are examined independently, showing that the effect of pressure is negligible in the external flow. Finally, drag reduction of three different configurations are estimated, based on the slip length.

In the subsequent second phase, turbulent flows over permeable substrates composed of streamwise-aligned fibers are subsequently studied. substrates with higher streamwise permeability are known to have great potential in reducing turbulent drag. This effect is previously modeled by homogeneous descriptions of the substrates. However, there is still a lack of full understanding of the details of the interaction between porous structures and overlying turbulence, which informs the design of an optimal substrate configuration. This study performs direct numerical simulation on fully developed turbulence in a channel bounded by permeable substrates, providing insights into the direct effect of these substrates.

We develop an immersed boundary method to fully resolve the streamwise-elongated fibers with a circular cross-section. The method is capable of accurately representing the micro-scale fibers, while ensuring the stability of the numerical iteration. Together with the immersed boundary method, a pseudo-spectral/finite difference solver is employed to conduct parallel computing.

Parametric simulations are carried out at a friction Reynolds number $Re_\tau \simeq 180$. The substrates with varying permeabilities have constant porosity $\theta = 0.9127$ and anisotropy $R_{xy} = 1.42$. Turbulent structures in the near-wall region are captured. Turbulence dynamics modified by the fibers are analyzed. It is reported that the mean flow is strongly damped in the substrate region, where turbulent fluctuations display wavy distributions. The Reynolds stress also penetrates into the substrate region, yielding non-zero value at the substrate interface. However, the Reynolds stress ceases at a small depth below the interface, indicating that the overlying turbulence is not allowed to penetrate deeply into the substrates. The fibrous substrate produces a drag reduction of 2.688% at a moderate permeability $\sqrt{K_y^+} = 0.479$, and is expected to achieve a higher drag reduction as its permeability further decreases.

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List of Symbols, Abbreviations and Acronyms

Roman Symbols

$\bar{p}$	Macroscopic kinematic pressure
$\bar{S}_x$	Applied shear in the streamwise direction
$\bar{S}_z$	Applied shear in the spanwise direction
$\bar{u}$	Macroscopic streamwise velocity component
$\bar{w}$	Macroscopic spanwise velocity component
l_y	Length of the computational domain in the wall-normal direction
l_z	Length of the computational domain in the spanwise direction
$\mathbf{f}$	Additional force term in the momentum equation
$\mathbf{K}$	Permeability tensor
$\mathbf{u}$	Velocity vector, $\mathbf{u} = (u\mathbf{e}_x, v\mathbf{e}_y, w\mathbf{e}_z)$, where $\mathbf{e}$ is the unit vector.
$\mathbf{u}^*$	Intermediate velocity vector in the fractional step method
$\mathbf{V}$	Desired velocity at forcing point
Re	Reynolds number
Re_τ	Friction Reynolds number
c	Coefficient for linear interpolation
c_f	Friction coefficient

DR	Drag reduction
h	Thickness of substrate
j	Grid point index in wall-normal direction
k	Grid point index in spanwise direction
k	Runge-Kutta sub-step
K_x	Streamwise permeability
K_y	Wall-normal permeability
N	Number of wall-normal grid points in channel region
N_p	Number of wall-normal grid points in porous region
p	Pressure
s	Periodicity of the fibre
t	Time
U	Mean streamwise coordinate
u	Streamwise velocity
U_b	Bulk velocity in channel
u_τ	Friction velocity
v	Wall-normal velocity
w	Spanwise velocity
w	Weighting coefficient for linear interpolation
x	Streamwise coordinate
y	Wall-normal coordinate
z	Spanwise coordinate
D	Discretised divergence operator
G	Discretised gradient operator

L Discretised Laplace operator

Greek Symbols

ψ Stream function

ψ_0 Stream function for the mean flow

ψ_I Stream function for the velocity fluctuations

θ Porosity of the permeable substrates

$\tilde{\nu}_x$ Effective macroscopic viscosity in the streamwise direction

$\tilde{\nu}_z$ Effective macroscopic viscosity in the spanwise direction

ψ_{walli} Stream function at a solid boundary

α_k Runge-Kutta coefficient for the explicit viscous term

β_k Runge-Kutta coefficient for the implicit viscous term

Δt Time-step

ΔU Shift in mean streamwise velocity

Δx Grid resolution in streamwise direction

Δy Grid resolution in wall-normal direction

Δz Grid resolution in spanwise direction

Δ Gradient operator

δ Half channel height

δ_ν Viscous lengthscale

η Kolmogorov length scale

γ_k Runge-Kutta coefficient for the advective term

κ von Kármán constant

λ_x Streamwise wavelength

λ_z Spanwise wavelength

ν	Kinematic viscosity
ϕ	Diameter of the fibre
ρ	Density of fluid
τ_w	Shear stress at wall
θ	Porosity
$\tilde{\nu}$	Effective macroscopic viscosity
$\tilde{\nu}_x$	Streamwise effective macroscopic viscosity
$\tilde{\nu}_z$	Spanwise effective macroscopic viscosity
ξ	Ratio between the molecular and effective macroscopic viscosity of a permeable substrate
ξ_x	Streamwise ratio of the molecular viscosity to the effective viscosity
ξ_z	Spanwise ratio of the molecular viscosity to the effective viscosity
ζ_k	Runge-Kutta coefficient for the advective term

Superscripts

$(\cdot)'$	Fluctuation
$(\cdot)^+$	Variable in wall units
$(\cdot)^k$	Variable at Runge-Kutta sub-step

Subscripts

$(\cdot)_0$	Variable of the smooth channel
$(\cdot)_{max}$	Maximum value
$(\cdot)_{min}$	Minimum value

Other Symbols

ℓ_T	Virtual origin for turbulence
ℓ_u	Streamwise virtual origin

ℓ_w Spanwise virtual origin

$\mathcal{O}(\cdot)$ Order of magnitude

Acronyms / Abbreviations

CFL Courant-Friedrichs-Lewy number

DNS Direct Numerical Simulation

IBM Immersed Boundary Method

r.m.s. Root mean square

Chapter 1

Introduction

1.1 Motivation and background for drag-alternating surfaces

The study of turbulence interacting with textured surfaces is of great interest in terms of making a greener world with less emissions and pollution. It is motivated by the potential of surface texturing to reduce turbulent skin friction on transport vehicles and the need for methodology to maximize this benefit, which is based on the understanding of underlying physics of the interaction.

Surface texturing includes different configurations which are often inspired by their prototypes in the nature world ([Bottaro, 2019](#)). To name a few, there are riblets, superhydrophobic surfaces and anisotropic permeable substrates. For instance, riblets have already demonstrated substantial drag reduction ($\sim 8\%$) in wind tunnel and flight tests on Airbus A320 ([Neittaanmäki et al., 2004](#)). Despite of its considerable potential, concerns are raised on the maintenance of riblets whose performance degrades as their shape erodes and need to be replaced after 2-3 years, reported from an in-service application on an A340 by the Cathay Pacific Airways ([Robert, 1992](#)). Another example of great potential and highlighted in this project is the anisotropic permeable substrates. The idea of this substrate is originated from [Itoh et al. \(2006\)](#) who conducted experiments on turbulence over seal fur. [Itoh et al. \(2006\)](#) demonstrated that seal fur is capable of reducing drag by more than 10% over a wide range of Reynolds number, outperforming riblets. [Abderrahaman-Elena and García-Mayoral \(2017\)](#); [Gómez-de-Segura and García-Mayoral \(2019\)](#) noticed that seal fur, a bundle of hair oriented along a certain direction, is essentially one kind of anisotropic permeable substrates. With streamwise preferential permeable layers, the streamwise component of overlying turbulence experiences less resistance than the transverse or secondary flows, producing drag reductions of 20% – 25% ([Gómez-de-Segura and García-Mayoral, 2019](#)). It is suggested that a substrate made up of streamwise aligned fibers with circular cross-sections would be befitting ([García-Mayoral et al.,](#)

2019). Previous results of anisotropic permeable substrates are obtained by modeling the macroscopic effect of porous media. To what extent is the homogeneous model valid without considering the microscopic flow phenomena, especially close to the porous interface, is still to be answered.

In the first phase of this project, we focus on the precise characterization of different configurations of the permeable substrate, with the aim to understand which one can offer the best properties in terms of drag reduction. Next, we analyze the validity of the modeling equations used to perform the previously mentioned studies, and we try to answer some open questions related to the flow behavior at the substrate-fluid interface.

The subsequent second phase would be developing numerical schemes to fully resolve the micro-structures within fibrous substrates, explicitly examining the interaction between anisotropic permeable substrates and the overlying turbulence. The attempt to fully resolve the permeable substrates imposes challenges in numerical process. For instance, the solid structures within porous substrates should firstly be represented accurately in turbulent flows. Considering the small scale of the fibrous structures, the requirement of local spatial resolutions will increase significantly. Eventually the resolution requirement of fibers prevail that of turbulence, prompting optimization of conventional numerical schemes for better efficiency.

The aim of the second phase is to conduct direct numerical simulations of turbulence over fully resolved fibrous substrates, achieved by modifying the immersed boundary method (Mittal and Iaccarino, 2005) and integrating it into the parallelized, pseudo spectral/finite difference solver for turbulent channel flows (García-Mayoral and Jiménez, 2011). The results provide quantitative analysis of the interaction between turbulence and fibers. Preliminary discussion on drag reduction by the fibrous substrates is carried out. The analysis of the relationship between micro and macroscopic parameter of the substrates will benefit the design of an optimal substrate configuration.

The report is composed of four chapters, including the current one for introducing, followed by reviews the theoretical background of this project. The organization and main content of the rest chapters are summarized as following. Chapter 2 presents the mathematical and numerical methods for the microscale calculations and the fully resolved direct numerical simulations. Chapter 3 and Chapter 4 show the results from the two phases of the project. Discussion and analyzes are given on the microscopic characterize properties of the permeable coatings and their effects on the overlying turbulence. Chapter 5 concludes the project. In addition, we discuss ideas for subsequent work on exploring the drag reduction potential and understanding physics underlying the turbulence/porous interaction.

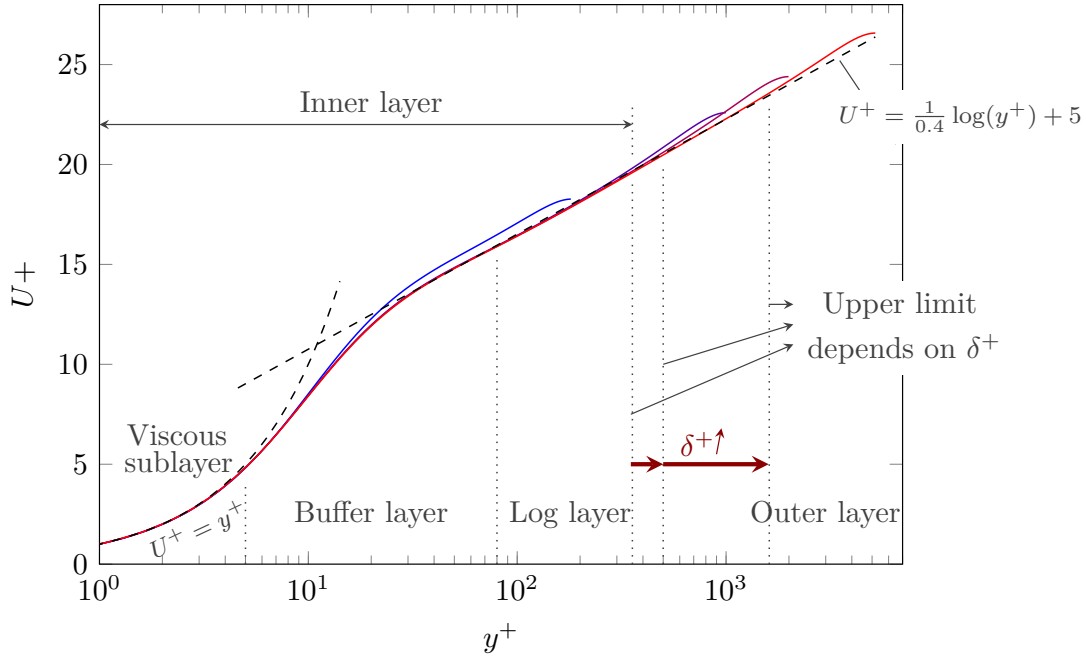


Figure 1.1 The different layers in wall-bounded turbulence. From blue to red, mean velocity profiles in a channel flow for different friction Reynolds numbers $\delta^+ = 180, 1000, 2000$, and 5200 . The dashed lines correspond to $U^+ = y^+$ and $U^+ = \frac{1}{k} \log(y^+) + B$. Data from [Lee and Moser \(2015\)](#). Image from [Gómez-de-Segura and García-Mayoral \(2019\)](#).

1.2 Wall-bounded turbulence

Turbulence over textured surfaces is one kind of wall-bounded turbulent flows. With the presence of solid walls, this kind of turbulence is anisotropic and includes multiple scales to be considered. Particularly, it introduces complex flow phenomena that are related to estimating and manipulating friction drag. Many comprehensive studies unravel the physics of wall-bounded turbulence. Although its physics is commonly investigated and demonstrated with smooth walls, it is universal and applies to more complicated rough surfaces.

Wall-bounded turbulence can be divided into two regions in the wall-normal direction y , according to the relative importance of viscosity ([Tennekes et al., 1972](#)). They are the inner layer and the outer layer. The inner layer is viscosity-dominated and consists of the viscous sub-layer, buffer layer and logarithmic layer. This description is illustrated in figure 1.1. Very close to the wall, viscosity and the viscous stress are important to the dynamics of the flow. The velocities there are adequately scaled with the friction velocity $u_\tau \equiv \sqrt{\tau_w/\rho}$, where τ_w is the wall shear stress and ρ is the density. Based on u_τ , the viscous lengthscale is derived as $\delta_v \equiv \nu/u_\tau$, where ν is the kinematic viscosity. Conventionally, these viscous scales are taken as references to non-dimensionalize variables. Variables are then said to be in wall or viscous units, and are denoted by a superscript ‘+’. The region $0 \leq y^+ \lesssim 5$ is defined as the viscous

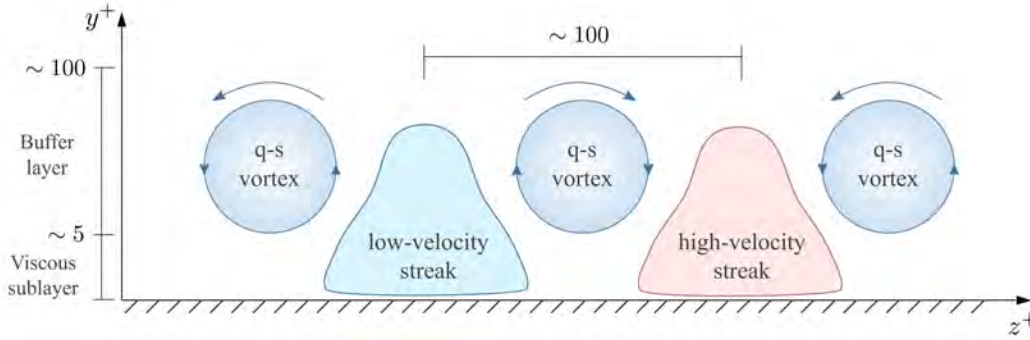


Figure 1.2 Sketch of the near-wall cycle across the span with quasi-streamwise vortexes and low- and high-velocity streaks. Image from [Gómez-de-Segura and García-Mayoral \(2019\)](#).

sub-layer, where viscous stresses dominate. Within the sub-layer, the mean velocity varies linearly with height $U^+ = y^+$. Above this region is the buffer layer, where neither the Reynolds stresses nor the viscous stresses can be neglected. The viscous sub-layer and the buffer layer together form the near-wall region. In the logarithmic layer, the log-law of the wall applies to the mean velocity and is valid from $y^+ \simeq 80$ up to $y/\delta \approx 0.2$. The mean velocity profile is given by

$$U^+ = \frac{1}{\kappa} \log y^+ + B, \quad (1.1)$$

where κ is the von Kármán constant and B is a constant. In this region, the velocities scale with the friction velocity, but appropriate lengthscale becomes the height y . Further away from the wall is the outer layer, where the effect of viscosity is negligible. When the velocities are normalized by u_τ , the reference length adopts a value δ of the same order of magnitude of large turbulent eddies. For example, δ takes the half channel height in channel flows. The friction Reynolds number is defined by $Re_\tau \equiv u_\tau \delta / \nu$, demonstrating the ratio between the lengthscale in the outer region and the viscous lengthscale.

Generally speaking, the near-wall region is where turbulence energy dissipation concentrates. In addition, the effect of textured surfaces is confined there ([García-Mayoral et al., 2019](#)). Hence, the dynamics in the near-wall region need to be understood before thinking about changing drag. In this region, the turbulence is dominated by a self-sustaining cycle that involves coherent structures known as streaks and quasi-streamwise vortexes, as depicted in figure 1.2. Streaks are longitudinal stripes containing either positive or negative streamwise velocity fluctuations. Superposing these fluctuations on the mean profile results in a pair of a low-velocity streak and a high-velocity streak, observed periodically in the spanwise direction. Experiments show that, typically, the length of streaks is $\lambda_x^+ \approx 1000$ and the width is $\lambda_z^+ \approx 100$

(Kim et al., 1971; Smith and Metzler, 1983). Meanwhile, quasi-streamwise vortexes are formed in between streaks, with a shorter length $\lambda_x^+ \approx 200$ (Jeong et al., 1997).

These two structures interact with each other in the self-sustaining process, where the vortexes re-distribute the mean shear to enhance the streaks whose instability leads to the generation of the vortexes, thereby closing the cycle (Swearingen and Blackwelder, 1987; Waleffe, 1997). Driven by the longitudinal vortexes, the slow fluid in the low-velocity streaks is displaced away from the wall, while the fast fluid is moved close to the wall, into the high-velocity streaks. This process produces a net effect that increases the averaged wall stress which causes higher friction at the wall in turbulence compared to laminar flows (Orlandi and Jiménez, 1994). Furthermore, this process is proven to be essentially insensitive to whether there is slip velocity at the wall (Waleffe, 1997) and is independent of the outer flow (Jiménez and Pinelli, 1999).

1.3 Drag reduction by textured surfaces

Surface texturing, belonging to passive flow control strategies, is a powerful and promising tool to reduce skin friction. Initially inspired by flows in the nature, previous studies introduced different textured surfaces, e.g. riblets, superhydrophobic surfaces and anisotropic permeable substrates. Although their properties vary from one to another, their effects on the dynamics of the overlying turbulence can be interpreted in a universal way when the texture size is vanishingly small. In that case, the direct effect of the texture is confined to the near-wall region (García-Mayoral et al., 2019). The overlying flow does not perceive the detail of the texture but experiences a homogenized effect. This homogeneous effect displaces the mean velocity profile but leave the self-similarity in the outer layer unaffected, it is reflected in a constant shift ΔU^+ in the mean velocity profile with respect to the smooth wall. The profile therefore becomes

$$U^+ = \frac{1}{\kappa} \log y^+ + B + \Delta U^+, \quad (1.2)$$

where the constants κ and B of smooth wall flows remain unchanged (Clauser, 1956). Previous studies show that the shift ΔU^+ is independent of the Reynolds number and solely determined by the texture size in wall units (García-Mayoral et al., 2019; Gatti and Quadrio, 2016). García-Mayoral et al. (2019) argued that the shift ΔU^+ provides a universal way to estimate the change in surface drag, since its value obtained at a certain Re_τ can be extrapolated to cases at other Re_τ . The change in drag is typically measured in terms of the drag reduction, DR , which is based on the relative decrease in friction coefficient c_f compared to that for a smooth wall c_{f0} ,

at the same Re_τ . Once the shift ΔU^+ is given, the drag reduction can be obtained as

$$DR = -\frac{c_f - c_{f0}}{c_{f0}} = 1 - \frac{1}{(1 + \Delta U^+ / U_{b0}^+)^2}, \quad (1.3)$$

$$c_f = \frac{2\tau_w}{U_b^2} = \frac{2}{U_b^{+2}} \quad (1.4)$$

where U_b^+ is the bulk velocity in wall units. Additionally, ΔU^+ is the difference of the mean velocity between the smooth and the textured cases, measured at a location away from the wall. This leads to the question of where to set the height origin for flows over textured surfaces.

When the size of the texture is of a few wall units, i.e. the texture does not protrude from the viscous sub-layer, the governing equations of the flow there is simplified to the Stokes equations whose linearity allows us to calculate each velocity component separately. In addition, the velocity profiles are essentially linear. Virtual origins for different velocity components can be used to characterize the homogeneous effect of textured surfaces on the overlying turbulent flow. The concept was first proposed by [Bechert and Bartenwerfer \(1989\)](#) for riblets, quantified as protrusion height ℓ_u^+ . Considering the case in figure 1.3 where the mean streamwise profile is moved downward, the velocity at the interface (e.g. the riblet tip plane) is non-zero and can be linearly extrapolated to zero at a virtual wall that is ℓ_u^+ from the interface. The relationship between the velocity at the interface and the ℓ_u^+ can be expressed by a Navier slip condition

$$u|_{y^+=0} = \ell_u^+ \frac{\partial u}{\partial y} \Big|_{y^+=0}^+. \quad (1.5)$$

The ℓ_u^+ is adopted for superhydrophobic surfaces with the name slip length ([Fairhall et al., 2019](#)) and permeable substrates ([Gómez-de-Segura and García-Mayoral, 2019](#)). In this report, the terminologies 'virtual origin' and 'slip length' are used interchangeably, since they both can describe how deep flows perceive walls. [Luchini et al. \(1991\)](#) suggested that a spanwise virtual origin ℓ_w^+ is necessary to account for the offset of turbulent fluctuations represented by the quasi-streamwise vortexes. The vortexes perceive an origin at $y^+ = -\ell_w^+$ below the interface where a condition similar to equation (1.5) applies

$$w|_{y=0} = \ell_w^+ \frac{\partial w}{\partial y} \Big|_{y=0}^+. \quad (1.6)$$

The homogeneous effect on the turbulence dynamics is making them perceive a smooth wall at their virtual origin. Therefore, the overall effect is that the mean velocity profile is shifted away

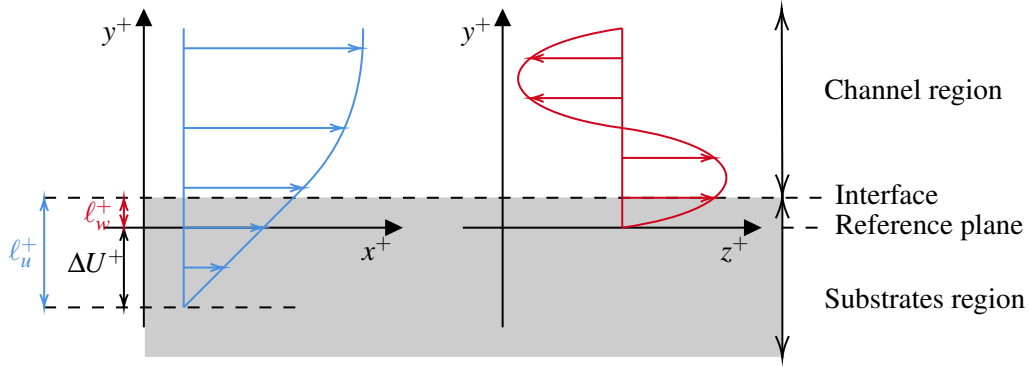


Figure 1.3 Sketch of virtual origins for the streamwise velocity (blue) and the spanwise turbulent fluctuation (red) which is represented by a quasi-streamwise vortex. Figure adapted from (Gómez-de-Segura, 2019).

from that smooth wall by

$$\Delta U^+ = \ell_u^+ - \ell_w^+, \quad (1.7)$$

as illustrated in figure 1.3. The ℓ_u^+ and ℓ_w^+ are also used to describe the slip effect, since they are related to the apparent velocities at the interface. This relationship gives us insights into how skin friction is changed by textured surfaces. If $\ell_u^+ > \ell_w^+$, the quasi-streamwise vortices are pushed away from the virtual wall of the mean flow, undermining their contributions to the turbulent mixing of streamwise momentum, so that the drag is reduced, and vice versa.

In particular, equation (1.7) fails to predict the velocity shift if the spanwise velocity component w and the wall-normal v have different virtual origins, e.g. turbulence over superhydrophobic surfaces. Gómez-de-Segura et al. (2018) proposed that the quasi-streamwise vortices experience an intermediate virtual origin ℓ_T^+ that is determined by ℓ_w^+ and ℓ_v^+ . As a result, equation (1.7) becomes

$$\Delta U^+ = \ell_U^+ - \ell_T^+, \quad (1.8)$$

where ℓ_U^+ locates the virtual origin for the mean flow and $\ell_U^+ \approx \ell_u^+$. The reference plane should then be set at the virtual origin of the turbulence ℓ_T^+ when calculating the friction velocity and measuring the drag reduction. On the other hand, it is argued that $\ell_T^+ \approx \ell_w^+$ when $\ell_v^+ \approx \ell_w^+$ (Fairhall and García-Mayoral, 2018; García-Mayoral et al., 2019; Gómez-de-Segura et al., 2018). However, the generic form of $\ell_T^+ = f(\ell_w^+, \ell_v^+)$ is yet to be studied.

As the texture size in wall units increases from infinitely small, ΔU^+ varies linearly with $\ell_U^+ - \ell_T^+$ until its maximum value for optimal performance. Further increasing the size, the performance degrades and eventually becomes drag increase. The region where the linear relationship validates is called the viscous regime of drag reduction curves. García-Mayoral and Jiménez (2011) demonstrated that the breakdown of the viscous regime is caused by the spanwise Kelvin-Helmholtz rollers, which are likely triggered by the transpiration effect ℓ_v^+ in

wall-normal direction. The onset of the rollers is considered to be a universal mechanism in analyzes of drag reduction, since the rollers are observed in turbulence over riblets, canopies, and porous walls (García-Mayoral and Jiménez, 2011). In summary, textured surfaces that producing positive ΔU^+ reduce friction drag to a certain limit. The principle ideas for design and optimization are maximizing the streamwise slip effect as well as suppressing the slip in spanwise direction and the transpiration in wall-normal direction to delay the breakdown.

1.4 Flow control with anisotropic permeable coatings

As mentioned in the previously, different studies have shown that streamwise-preferential complex surfaces, such as riblets, can reduce drag in turbulent flows (Luchini et al. (1991); Jiménez (1994); Bechert et al. (1997)). The general idea to achieve a drag reduction is that textured surfaces must offer more resistance to the cross-flow than to the streamwise flow. Abderrahaman-Elena and García-Mayoral (2017) suggested that the same mechanism is at the base of the drag reduction ability of anisotropic permeable coatings. Recently, Gómez-de-Segura and García-Mayoral (2019) have shown that streamwise-preferential substrates, similar to riblets, exhibit a linear regime in which the drag reduction is proportional to the difference between the virtual origin perceived by the mean flow, and that perceived by turbulence. They also observed that, as the permeabilities increase, the linear regime eventually breaks down due to the appearance of spanwise-coherent structures, which arise from a Kelvin-Helmholtz-like instability.

1.4.1 Flow over anisotropic permeable coatings

The flow within permeable coatings can be studied with a microscopic or macroscopic approach. The former implicates a detailed representation of the geometry of the substrates and an explicit resolution of the flow within. The latter, in contrast, involves a modernization of the porous medium as a continuum, obtaining an averaged solution of the flow.

We now focus on the most common macroscopic approach used in literature. To characterize the homogenized flow within a permeable substrate, a possible approach is Darcy's equation (Darcy, 1856):

$$\nabla p = -\nu \mathbf{K}^{-1} \mathbf{u}, \quad (1.9)$$

where p is the kinematic pressure, $\mathbf{u}$ the velocity vector, ν the molecular viscosity of the fluid, and $\mathbf{K}$ is the permeability tensor, which measures the ability of the fluid to flow through the permeable substrate. Darcy's equation results from a volume average of the Stokes equation

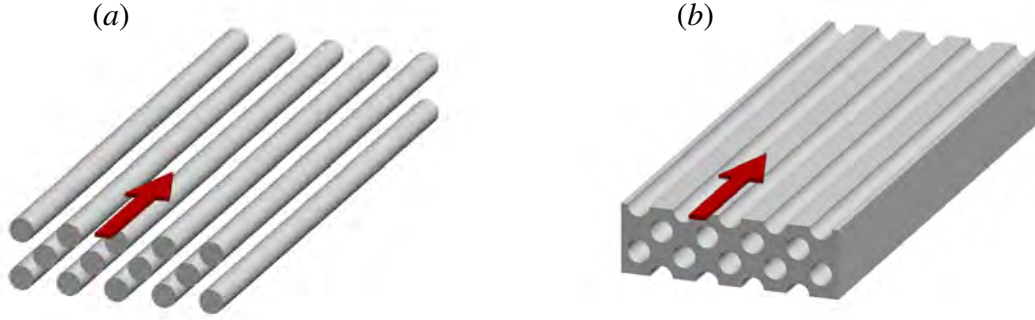


Figure 1.4 Conceptual sketches of (a) a highly-connected material, where the interstitial flow is well interconnected and macroscale diffusive effects can propagate throughout, and (b) a poorly-connected permeable material, where no diffusive effects connect different pores. The red arrow represents the direction of the overlying flow. Image from [Gómez-de-Segura and García-Mayoral \(2019\)](#).

over many pores, and it is a balance between the pressure gradient acting on the porous medium and the viscous drag caused by the pressure of the solid matrix.

The volume average, implicit in Darcy's equation, filters out the diffusive effects acting over large lengthscales, accounting only for the viscous stresses caused by the velocity gradients over lengthscales smaller than the averaging one. When the diffusive effects over larger scales are relevant, they can be considered using the Brinkman equation ([Brinkman, 1949](#)), which include a macroscopic diffusive term:

$$\nabla p = -\nu \mathbf{K}^{-1} \mathbf{u} + \tilde{\nu} \nabla^2 \mathbf{u}. \quad (1.10)$$

The first two terms in the equation (1.10) constitute Darcy's equation, and the last term is the Brinkman term, where $\tilde{\nu}$ is the effective macroscopic viscosity, which is dependent from the microstructure of the porous medium.

Let us now recover a distinction between permeable substrates made by [Abderrahaman-Elena and García-Mayoral \(2017\)](#). They identified substrates made up of open matrices of obstacles as "highly-connected" substrates, such as that in figure 1.4 (a), and substrates made up of micro ducts essentially isolated from each other as "poorly-connected" substrates, as in figure 1.4 (b). They also argued that the former offered better properties for drag reduction.

The choice of which approach to use depends on the microstructure of the porous medium. Brinkman model, for instance, is better suited for "highly-connected" substrates, where diffusion can act efficiently over large scales ([Sangani and Behl, 1989](#)). Conversely, Darcy's equation provides a reasonable model for the flow within "poorly-connected" substrates ([Lévy \(1983\)](#); [Auriault \(2009\)](#)).

Following the work of [Gómez-de-Segura and García-Mayoral \(2019\)](#), we focus on "highly-connected" substrates. We need to point out that the Brinkman equation is also a volume averaging model, so it implicitly assumes the presence of a large number of fibers in any small volume within the substrate. However, on approaching the interface between the substrate and the free flow region, that assumption would eventually cease to hold. In the literature there is no general agreement on how to treat the substrate-fluid interface ([Ochoa-Tapia and Whitaker \(1995\)](#); [Le Bars and Worster \(2006\)](#); [Zampogna and Bottaro \(2016\)](#); [Lācis and Bagheri \(2017\)](#)). Some studies impose a jump condition in the velocity ([Beavers and Joseph \(1967\)](#); [Mikelic and Jäger \(2000\)](#); [Valdés-Parada et al. \(2013\)](#)), others a jump in the shear stress ([Ochoa-Tapia and Whitaker \(1995\)](#); [Hill and Straughan \(2009\)](#); [Minale \(2014\)](#)) or continuity of both velocity and shear stress ([Neale and Nader \(1974\)](#); [Vafai and Kim \(1990\)](#); [Le Bars and Worster \(2006\)](#); [Battiato \(2012\)](#)).

Equation (1.10) shows that flows are balanced by pressure gradient, viscous stresses and a macroscopic diffusive term $\tilde{\nu} \nabla^2 \mathbf{u}$, known as the Brinkman term. For the structure shown in figure 1.4 (a), the fluid is well interconnected so that diffusion effects at scales larger than the fibers' lengthscale are non-trivial and are taken into consideration by the Brinkman term.

The permeability is a macroscopic property, describing the ability of a porous medium to transmit fluids. The permeability tensor $\mathbf{K}$ has components K_x , K_y and K_z which are assumed to be along streamwise, wall-normal and spanwise directions respectively. The matrix of fibers in figure 1.4 (a) has a higher streamwise permeability and smaller, equivalent permeabilities in the other directions along which the structures are similar, $K_x > K_y = K_z$. To iterate, the homogenized effect of the permeability on the turbulence is setting the depth of virtual origins, which can be analytically derived from equation (1.10) as ([Abderrahaman-Elena and García-Mayoral, 2017](#))

$$\ell_u^+ = \xi_x \sqrt{K_x^+} \tanh \left(\frac{h^+}{\sqrt{K_x^+}} \right), \quad (1.11a)$$

$$\ell_w^+ = \xi_z \sqrt{K_z^+} \tanh \left(\frac{h^+}{\sqrt{K_z^+}} \right), \quad (1.11b)$$

where the ratio between the molecular viscosity ν and the effective viscosity $\tilde{\nu}$ of the permeable substrate is defined as

$$\xi = \sqrt{\frac{\nu}{\tilde{\nu}}}. \quad (1.12)$$

For substrates with sufficient thickness, i.e. $h^+ \gtrsim \sqrt{K_x^+}$, the hyperbolic tangent in equation (1.11) tends to 1, resulting in

$$\ell_u^+ \approx \xi_x \sqrt{K_x^+}, \quad \ell_w^+ \approx \xi_z \sqrt{K_z^+}. \quad (1.13)$$

Furthermore, the substrate sketched in figure 1.4 (a) is assumed to have isotropic effective viscosity which approximately equals the molecular viscosity, yielding

$$\ell_u^+ \approx \sqrt{K_x^+}, \quad \ell_w^+ \approx \sqrt{K_z^+}. \quad (1.14)$$

The drag reduction can then be estimated by

$$\Delta U^+ = \ell_U^+ - \ell_T^+ \approx \sqrt{K_x^+} - \sqrt{K_z^+}. \quad (1.15)$$

Note that equation (1.15) is validated only for the viscous regime where the typical length scale of the permeable substrate is vanishingly small. The wall-normal permeability does not appear in the equation (1.15), but as mentioned before, when a textured surface introduces a non zero wall-normal velocity, some degradation effects set in. [Abderrahaman-Elena and García-Mayoral \(2017\)](#) and [Gómez-de-Segura et al. \(2018\)](#) argued that these degradation effects are due to the appearance of spanwise-coherent rollers associated with a Kelvin-Helmholtz-like instability. [Gómez-de-Segura et al. \(2018\)](#), following the work of [Abderrahaman-Elena and García-Mayoral \(2017\)](#), developed a model to characterize the onset of Kelvin-Helmholtz rollers. For sufficiently deep and streamwise-preferential substrates, they argued that the amplification of the instability is essentially governed by K_y^+ .

With a microscopic approach, the permeability of substrates are determined by their porosity θ and typical lengthscale s , which is the periodic spacing of fibers in this study. It is written as

$$\hat{K} = f(\theta) s^2, \quad (1.16)$$

which has units of length square, L^2 , and can be non-dimensionalized by the typical lengthscale s , yielding

$$K = f(\theta). \quad (1.17)$$

The function of porosity in (1.17) is empirical and varies from case to case, depending on the shape and arrangement of the structure. Therefore, it is obtained from experiments or simulations. [Bottaro \(2019\)](#) measured permeabilities of a staggered array of cylinders in a two-dimensional flow. The similarity, between the flow field in [Bottaro \(2019\)](#) and that of the $y-z$ plane of collocated cylinders in figure 1.4 (a), allows the wall-normal permeability K_y in

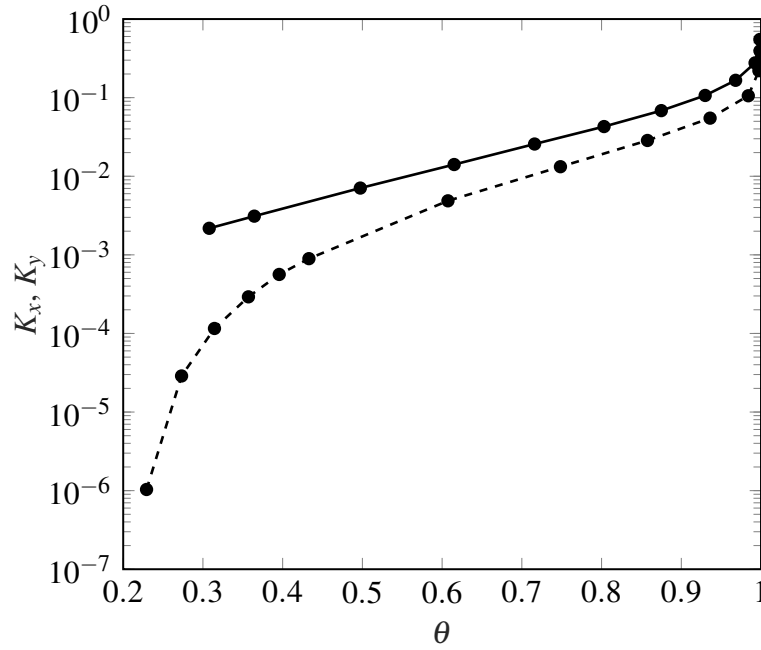


Figure 1.5 The streamwise and wall-normal permeability as a function of porosity for a porous medium formed by collocated cylinders. The streamwise permeability K_x is marked by solid segments, data obtained from [Busetto \(2020\)](#), the wall-normal one K_y is marked by dashed segments, adapted from [Bottaro \(2019\)](#).

this study to be calculated directly from that in [Bottaro \(2019\)](#). The result is shown in figure 1.5, portrayed with dashed segments and dots. The streamwise permeability, marked by solid segments in figure 1.5, is obtained from [Busetto \(2020\)](#), who solves the Stokes equations for pressure driven flows within collocated fibers and obtains the streamwise permeability from the Dracy's equation. It is seen that both permeabilities increase with the porosity, up to unit at $\theta = 1$ when there is no solid obstacle in the flow. For the streamwise-aligned fibers, the streamwise permeability K_x is always greater than the wall-normal permeability at $\theta \in (0.3, 1)$, but the difference between these two reduces as porosity increases.

1.4.2 Drag reduction by anisotropic permeable substrates

[Gómez-de-Segura and García-Mayoral \(2019\)](#) conducted a series of DNS of a channel flow bounded by permeable substrates. For these simulations they considered streamwise-preferential substrates, with equally low spanwise and wall-normal permeability, $K_x^+ > K_z^+ = K_y^+$, and assumed $\xi = 1$. They characterized different substrates configuration in terms of the anisotropy ratio $R_{xy} = \sqrt{K_x/K_y} > 1$ and $R_{zy} = \sqrt{K_z/K_y} = 1$. They obtained drag reduction curves like the ones in figure 1.6, where the linear regime is well identifiable. They also observed that the

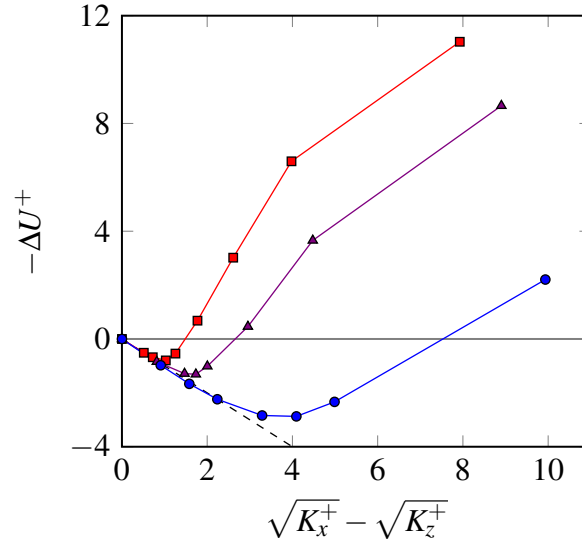


Figure 1.6 Drag reduction curves for substrates with different anisotropy ratios: $R_{xy} \approx 11.4$, $R_{xy} \approx 5.5$, and $R_{xy} \approx 3.6$. Image from [Gómez-de-Segura and García-Mayoral \(2019\)](#)

breakdown was mainly determined by the wall-normal permeability, as predicted by the theoretical model. From the analysis of these curves, [Gómez-de-Segura and García-Mayoral \(2019\)](#) proposed an equation to estimate the maximum ΔU^+ achievable with permeable substrates. They observed that the maximum ΔU^+ occurs for $\sqrt{K_y^+} \approx 0.38$ and is approximately 80% of that estimated by equation (1.15). As a consequence, the maximum ΔU^+ can be calculated from

$$\Delta U_{max}^+ \approx 0.8 \cdot 0.38 \cdot (R_{xy} - R_{zy}), \quad (1.18)$$

where $R_{zy} = 1$ for substrates with equal permeability in the spanwise and wall-normal directions. Moreover, to illustrate how the overlying turbulence is modified at different points along the drag reduction curves, [Gómez-de-Segura and García-Mayoral \(2019\)](#) have shown instantaneous realizations of u and v in an $x-z$ plane immediately above the substrate-channel interface, as portrayed in figure 1.7. The panels (a-b) shows the u -field and the v -field for small $\sqrt{K_y^+}$, where the flow field resembles that observed over a smooth wall. As $\sqrt{K_y^+}$ increases beyond the linear regime, the flow begins to be alternated, as shown in panels (c-d). For high $\sqrt{K_y^+}$, in panels (e-f) the flow becomes strongly spanwise-coherent, and there is a drag-increasing effect.

1.5 Direct numerical simulations of turbulent channel flows

To examine fluid flows within the framework of continuum mechanics, mathematicians and engineers seek solutions to the Navier-Stokes equations, which predicts all the information we

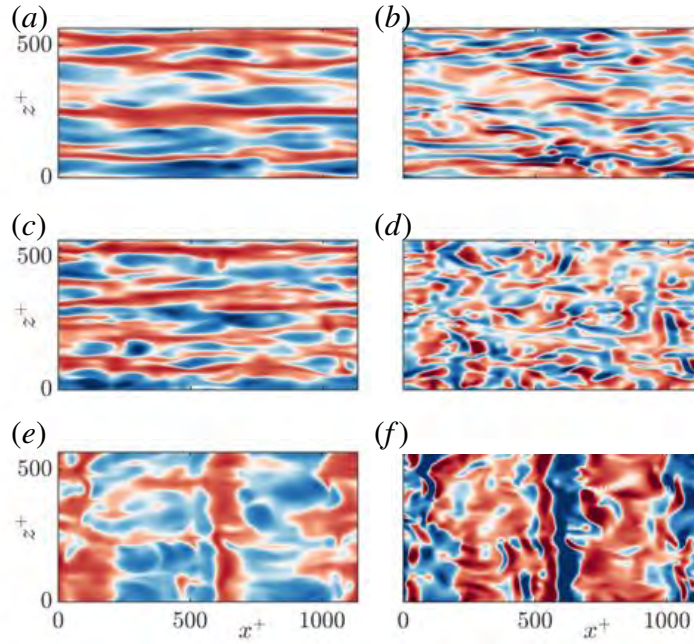


Figure 1.7 Instantaneous realizations of u^+ and v^+ for a smooth channel and for substrates with $R_{xy} = 11.4$ at an $x - z$ plane immediately above the substrate-channel interface. From left to right the columns are u^+ and v^+ . From top to bottom, representing increasing permeabilities. Data from [Gómez-de-Segura and García-Mayoral \(2019\)](#)

need to know about the flow field. The incompressible flow is discussed throughout the study, it is governed by the three-dimensional Navier-Stokes equations

$$\nabla \cdot \mathbf{u} = 0, \quad (1.19)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u}, \quad (1.20)$$

where the pressure p and the velocity vector $\mathbf{u} = (u\mathbf{e}_x, v\mathbf{e}_y, w\mathbf{e}_z)$ are functions of time and spatial coordinates. The flow is characterized by the non-dimensional Reynolds number. Unfortunately, analytical solutions to this set of equations only exist in the laminar regime, leaving turbulence as an unsolved problem. The difficulties are largely due to turbulent flows' chaotic, multi-scale nature ([Moin and Mahesh, 1998](#)). Nonetheless, approximation solutions to fully developed turbulence can still be obtained by alternative methods, e.g. experiments and simulations. One method, that numerically solves the Navier-Stokes equations for instantaneous flow variables, is known as the direct numerical simulation (DNS). DNS has many advantages as a research tool. On one hand, it resolves the smallest eddies in the energy spectrum, allowing researchers to gain insight into the near-wall structures. Without employing turbulent models, it is regarded as the most accurate numerical approximation ([Ferziger et al., 2002](#)). On the other hand, DNS

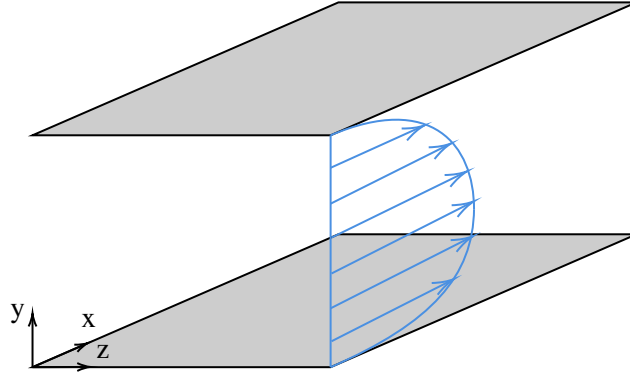


Figure 1.8 Schematic of a plane channel. Blue curve indicates the streamwise mean velocity profile.

prevails over laboratory experiments practically and economically in our study, considering that the substrates composed of fibers are only conceptual at this stage.

Numerical analyzes of wall-bounded turbulence are commonly carried out in a rectangle domain bounded by solid top and bottom wall. Fully developed turbulence is driven through the domain, with a mean velocity in the streamwise direction x . Ever since the first DNS performed in a channel by [Kim et al. \(1987\)](#), this configuration has been proven to be a useful framework to study wall-bounded turbulence ([Moin and Mahesh, 1998](#)). A sketch of a channel is shown in figure 1.8. Within this framework, the Reynolds number in equation 1.20 is $Re \equiv \delta U_b / \nu$, defined based on the kinematic viscosity ν , the half channel height δ and the bulk velocity U_b . Considering that fully developed turbulent channel flow is homogeneous in streamwise and spanwise directions, periodic boundary conditions can be imposed along x and z in the computational domain. The size of the channel should be large enough to sustain turbulence and produce large scale motions ([Jiménez and Moin, 1991](#)), whilst kept as small as possible to reduce computing efforts. In this study where the dynamics in the near-wall region are of most interest, we employ the channel that is of streamwise and spanwise periods $2\pi\delta$ and $\pi\delta$, and wall-normal size 2δ . This domain size is proven to be sufficient for correct one-point statistics ([Lozano-Durán and Jiménez, 2014](#)).

Chapter 2

Methodology

2.1 Methods for microscale calculations

We study the flow within the porous mediums at a microscale level. We focus on sufficiently deep anisotropic permeable substrates made by matrices of fibers, as the ones figure 1.4 (a), with equally low permeabilities in the spanwise and wall-normal direction. Considering that the fibers are very small compared to the lengthscales of the flow in the outer region, the flow within the substrates is predominately viscous, and the effect of the advective terms is negligible. Consequently, we can assume the steady Stokes flow equations as the governing equations for motion within the substrates:

$$\begin{cases} \nabla \cdot \mathbf{u} = 0, \\ \nu \nabla^2 \mathbf{u} = \nabla p. \end{cases} \quad (2.1)$$

Given that Stokes' equations are linear and the flow is invariant in the streamwise direction, we can decouple the equations into two systems, one for the streamwise velocity component, (u), and one for the spanwise and wall-normal components, (w, v).

We now analyze the methodology used to characterize the permeable substrates. Firstly, we solve a flow driven by a constant pressure gradient within the substrate to obtain the permeabilities of different configurations. Secondly, we consider a flow driven by an shear to study the validity of the Brinkman model, to evaluate $\tilde{\nu}$ and to investigate the substrate-fluid interface.

2.1.1 Pressure driven flow

We analyze the flow in the core of the substrates driven by a macroscopic constant pressure gradient. Provided that the substrates are made by matrices of fibers, we can analyze a single texture unit, with periodic boundaries conditions in the spanwise and wall-normal directions. We solve the Stokes equations with a constant pressure gradient for the streamwise and spanwise direction separately. From these solutions, we recover the velocity field. Then, we obtain the macroscopic velocities averaging the velocity field. Given that the macroscopic velocities within the porous medium, far from the substrate-fluid interface, satisfy Darcy's equation (1.9), we finally obtain the permeabilities.

Streamwise flow

As mentioned before, assuming that the flow is invariant in the streamwise direction, we can decouple the Stokes equations. Consequently, in the streamwise direction we solve

$$\nu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \frac{\partial p}{\partial x}, \quad (2.2)$$

with $u = u(y, z)$. In this scenario, we assign a constant pressure gradient in the x-direction. The boundary conditions consist of periodicity in the spanwise and wall-normal direction and no-slip at the solid boundaries of the fibers. From the solution of the equation (2.2), we get the streamwise velocity field in the domain.

To obtain the permeability from Darcy's equation, we do a spatial average of the velocity in the domain, recovering the macroscopic velocity $\bar{u}$. Provided that, we can find the permeability from

$$-\frac{\nu}{K_x} \bar{u} = \frac{\partial \bar{p}}{\partial x}. \quad (2.3)$$

Spanwise flow

For the spanwise flow, the Stokes equations reduce to

$$\begin{cases} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \\ \nu \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \frac{\partial p}{\partial y}, \\ \nu \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \frac{\partial p}{\partial z}, \end{cases} \quad (2.4)$$

with $v = v(y, z)$ and $w = w(y, z)$. These equations allow us to use the stream-function ψ , such that

$$\begin{cases} v = -\frac{\partial \psi}{\partial z}, \\ w = \frac{\partial \psi}{\partial y}. \end{cases} \quad (2.5)$$

We can then manipulate the equations (2.4) to obtain the bi-harmonic equation in ψ ,

$$\nabla^4 \psi = \frac{\partial^4 \psi}{\partial y^4} + 2 \frac{\partial^4 \psi}{\partial y^2 \partial z^2} + \frac{\partial^4 \psi}{\partial z^4} = 0. \quad (2.6)$$

The boundary conditions for ψ are the periodicity in the spanwise and wall-normal direction. In addition, at the boundary of solids we impose the no-slip and no-transpiration conditions

$$\begin{cases} \psi|_{\partial\Omega_i} = \psi_{wall_i} \quad \text{with} \quad \psi_{wall_i} = \text{constant}, \\ \frac{\partial \psi}{\partial n} \Big|_{\partial\Omega_i} = 0. \end{cases} \quad (2.7)$$

Solving the equation (2.6), we obtain ψ , and from the equations (2.5) we recover the velocity field in the domain. Using the velocity field, we calculate the pressure gradient in the spanwise direction with the equations (2.4). Note that in equation (2.6) the pressure terms dropped out. This means that we need to verify *a posteriori* that the pressure gradient is constant in the spanwise direction.

To obtain the macroscopic velocity $\bar{w}$, we do a spatial average of the velocity in the domain. Then, we can find the permeability from Darcy's equation in the spanwise direction:

$$-\frac{v}{K_z} \bar{w} = \frac{\partial \bar{p}}{\partial z}. \quad (2.8)$$

Note that in this study, we restrict to anisotropic permeable substrates with equal permeabilities in the spanwise and wall-normal direction, so that $K_y = K_z$.

2.1.2 Shear driven flow

We study the flow in response to an applied shear or a constant velocity at the substrate-fluid interface, with zero mean pressure gradient in the streamwise and spanwise directions. In this context, the flow is no more periodic in the wall-normal direction, but only in the spanwise one.

At first, to analyze the macroscopic velocity profile within the substrates, far from the interface, we solve the flow over a sufficiently high column of fibers. We use the Stokes

equations to obtain the velocity field, and we do a spatial average in the spanwise direction. Next, we compare the velocity profiles obtained with the analytical solution of Brinkman equation (1.10). Therefore, we see whether the two solutions agree. If so, we get the values of the effective macroscopic viscosity that would give a good match between the two solutions.

Secondly, we analyze the substrate-fluid interface, comparing the solutions of Brinkman equation, using the best fit values for $\tilde{\nu}$, with the velocity profile in the free flow region. To achieve that, we solve the flow on a domain in which the upper half is free-flow, and the lower half is filled with fibers.

Streamwise flow

If the mean pressure gradient is zero in the streamwise direction, the Stokes equations for u reduce to

$$\nu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 0. \quad (2.9)$$

The boundary conditions consist of periodicity in the spanwise direction, no-slip at the bottom boundary of the domain and a constant shear at the top surface so that $\frac{\partial u}{\partial y} = \bar{S}_x$. From the solution of the equation (2.9), we obtain $u(y, z)$ and we recover the mean velocity profile doing an average in the spanwise direction.

In this case, Brinkman equation (1.10) takes the following form:

$$\frac{d^2 \bar{u}}{dy^2} - \frac{\nu}{\tilde{\nu}_x K_x} \bar{u} = 0. \quad (2.10)$$

The analytical solution of this equation is in an exponential form:

$$\bar{u} = C_1 e^{C_2 y}, \quad (2.11)$$

where C_1 is a constant that depends on the condition at the interface and C_2 is:

$$C_2 = \sqrt{\frac{\nu}{\tilde{\nu}_x K_x}}. \quad (2.12)$$

In order to understand if Brinkman equation is a good model, and to find the best fit values for $\tilde{\nu}$, we fit the exponential in the equation (2.11) with the mean velocity profile obtained from the solution of the Stokes equations. Note that we assume the permeability to be that obtained from the solution of pressure driven flow. With this assumption, the only free parameter in the fitting is the effective macroscopic viscosity in the streamwise direction, $\tilde{\nu}_x$.

Once the shear flow within the porous medium is solved, we analyze the substrates-fluid interface. To achieve that, we solve the flow on a domain in which the top half is free-flow, and the bottom half is filled with fibers. The governing equations are the same as for the previous problem, as well as the boundary conditions. We analyze the results in terms of the jump conditions that we would need to impose on the free flow using the Brinkman model in the substrates up until the interface, with the previously obtained values for the permeabilities and $\tilde{v}$. Moreover, looking only at the mean velocity profile obtained with the solution of the Stokes equations, we calculate the slip-lengths, from the following equation:

$$u|_{y=0} = \ell_x \left. \frac{\partial u}{\partial y} \right|_{y=0}. \quad (2.13)$$

Note that in agreement with previous studies ([Abderrahaman-Elena and García-Mayoral \(2017\)](#), [Gómez-de-Segura and García-Mayoral \(2019\)](#)), we set the reference plane for the solid-fluid interface at the tip of the last fiber.

Spanwise flow

For the spanwise flow the Stokes equations are:

$$\begin{cases} \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \\ v \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \frac{\partial p}{\partial y}, \\ v \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \frac{\partial p}{\partial z}. \end{cases} \quad (2.14)$$

Note that in this case, the mean pressure gradient in the z -direction is zero. Following what we have done for the pressure drive flow, with the relation (2.5), we can study the bi-harmonic equation in ψ :

$$\nabla^4 \psi = 0. \quad (2.15)$$

The boundary conditions are periodicity in the spanwise direction, $\psi = \text{constant}$ and $\frac{\partial \psi}{\partial y} = 0$ at the bottom to satisfy no-slip and no-transpiration conditions, $\psi = \text{constant}$ and $\frac{\partial \psi}{\partial y} = \text{constant}$ at the top of the domain to impose a uniform velocity. In addition, at the solid boundaries, we impose the same conditions expressed in the equations (2.7).

From the solution of equations (2.15), we obtain $\psi = \psi(y, z)$. With equations (2.5) we recover the velocity field. Note that we need to verify *a posteriori* that the pressure gradient in

the spanwise direction is zero. Then, averaging the spanwise component of the velocity in the z -direction, we obtain the mean velocity profile.

Following the same procedure proposed for the streamwise flow, we consider Brinkman equation in the spanwise direction

$$\frac{d^2 \bar{w}}{dy^2} - \frac{\nu}{\tilde{\nu}_z K_z} \bar{w} = 0, \quad (2.16)$$

and it's analytical solution, in the following form:

$$\bar{w} = C_3 e^{C_4 y}, \quad (2.17)$$

where C_3 is a constant that depends on the condition at the interface and C_4 is:

$$C_4 = \sqrt{\frac{\nu}{\tilde{\nu}_z K_z}}. \quad (2.18)$$

We fit the exponential in equation (2.17) with the mean velocity profile obtained from the solution of the Stokes equations, analyzing the goodness of the fit and obtaining the best guess for the effective macroscopic viscosity in the spanwise direction, $\tilde{\nu}_z$.

Then, we analyze the substrates-fluid interface, solving the flow on the same domain that we used for the streamwise case. The governing equations are still the same (2.14), as well as the boundary conditions. As before we analyze the results in terms of the jump conditions and slip-lengths, that can be obtained from:

$$w|_{y=0} = \ell_z \left. \frac{\partial w}{\partial y} \right|_{y=0}. \quad (2.19)$$

2.1.3 Numerical methods for microscale calculations

To solve the equations mentioned in the previous section, we choose to use finite difference methods on a Cartesian grid. We use a uniform grid, but we decide to implement the possibility to have different spacing between the points in the two main directions, i.e. spanwise and wall-normal directions. Moreover, we develop specific methods to treat the solid boundaries to obtain a very fast convergence of the results with the refinement of the grid, as we will see in the next sections.

Streamwise flow

As mentioned before, for the streamwise flow, we solve:

$$v \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \frac{\partial p}{\partial x}. \quad (2.20)$$

To estimate the two second-order derivatives at a point, we use a second-order centered finite difference approximation, characterized by a 5-point stencil:

$$\begin{cases} \left(\frac{\partial^2 u}{\partial y^2} \right)_{j,k} = \frac{u_{j-1,k} - 2u_{j,k} + u_{j+1,k}}{dy^2}, \\ \left(\frac{\partial^2 u}{\partial z^2} \right)_{j,k} = \frac{u_{j,k-1} - 2u_{j,k} + u_{j,k+1}}{dz^2}, \end{cases} \quad (2.21)$$

where dy and dz are the distances between two consecutive points of the grid in the y - and z -direction. Combining these equations we obtain:

$$(\nabla^2 u)_{j,k} = - \left(\frac{2}{dy^2} + \frac{2}{dz^2} \right) u_{j,k} + \frac{1}{dy^2} (u_{j-1,k} + u_{j+1,k}) + \frac{1}{dz^2} (u_{j,k-1} + u_{j,k+1}). \quad (2.22)$$

Writing these equations for each point of the grid, equation (2.20) becomes:

$$[A]\mathbf{u} = \mathbf{P}, \quad (2.23)$$

where $[A]$ is the matrix of the coefficients, $\mathbf{u}$ is the unknown velocity and $\mathbf{P}$ is the prescribed constant pressure gradient vector.

Treatment of solid boundaries

To obtain a fast convergence of the results with the refinement of the grid, we use the following methodology, developed by [Leonardi et al. \(2006\)](#). Firstly, we identify all the points outside the solid boundaries that have at least one point of their stencil inside the solids. Secondly, for each of these points, we modify the coefficient in the $[A]$ matrix, and we impose a zero velocity in the intersections between the grid and the solids. Note that now these intersections become points in the stencil and velocity in that points is known and equal to zero. Moreover, the stencil's points are no more equidistant. Therefore, we rewrite the relations (2.21), using

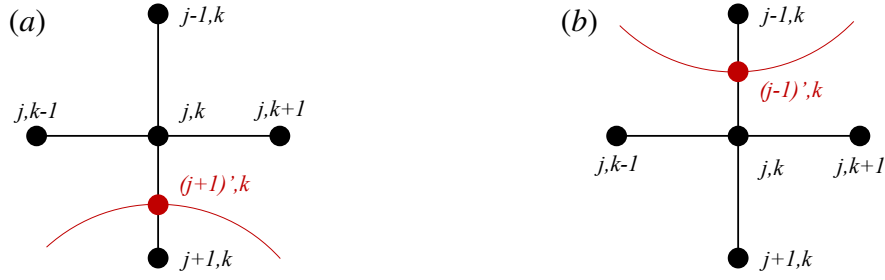


Figure 2.1 Sketch of stencil near the solid boundary when the solid line is (a) between (j,k) and $(j+1,k)$ and (b) between $(j-1,k)$ and (j,k) .

the coefficients for a non-uniform grid:

$$\begin{cases} \left(\frac{\partial^2 u}{\partial y^2} \right)_{j,k} = a_{j1} u_{j-1,k} + a_{j2} u_{j,k} + a_{j3} u_{j+1,k}, \\ \left(\frac{\partial^2 u}{\partial z^2} \right)_{j,k} = a_{k1} u_{j,k-1} + a_{k2} u_{j,k} + a_{k3} u_{j,k+1}, \end{cases} \quad (2.24)$$

with:

$$\begin{cases} a_{j1} = \frac{2}{(y_{j,k} - y_{j-1,k})(y_{j+1,k} - y_{j-1,k})}, \\ a_{j2} = -\frac{2}{(y_{j+1,k} - y_{j,k})(y_{j,k} - y_{j-1,k})}, \\ a_{j3} = \frac{2}{(y_{j+1,k} - y_{j,k})(y_{j+1,k} - y_{j-1,k})}, \end{cases} \quad (2.25)$$

and:

$$\begin{cases} a_{k1} = \frac{2}{(z_{j,k} - z_{j,k-1})(z_{j,k+1} - z_{j,k-1})}, \\ a_{k2} = -\frac{2}{(z_{j,k+1} - z_{j,k})(z_{j,k} - z_{j,k-1})}, \\ a_{k3} = \frac{2}{(z_{j,k+1} - z_{j,k})(z_{j,k+1} - z_{j,k-1})}. \end{cases} \quad (2.26)$$

As mentioned before, we modify the coefficient in the stencil for every relevant point outside the solid. To illustrate the method, we now look at the y -direction, and the generalization to the z -direction is straight forward. We need to consider two different cases, as shown in figure 2.1:

(a) If the solid boundary is between (j,k) and $(j+1,k)$, we proceed as follows:

$$\left(\frac{\partial^2 u}{\partial y^2} \right)_{j,k} = a'_{j1} u_{j-1,k} + a'_{j2} u_{j,k} + a'_{j3} u_{j+1,k}, \quad (2.27)$$

with:

$$\begin{cases} a'_{j1} = \frac{2}{(y_{j,k} - y_{j-1,k})(\mathbf{y}_{(\mathbf{j}+1)',\mathbf{k}} - y_{j-1,k})}, \\ a'_{j2} = -\frac{2}{(\mathbf{y}_{(\mathbf{j}+1)',\mathbf{k}} - y_{j,k})(y_{j,k} - y_{j-1,k})}, \\ a'_{j3} = 0. \end{cases} \quad (2.28)$$

(b) If the solid boundary is between (j-1,k) and (j,k), we write:

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{j,k} = a'_{j1}u_{j-1,k} + a'_{j2}u_{j,k} + a'_{j3}u_{j+1,k}, \quad (2.29)$$

with:

$$\begin{cases} a'_{j1} = 0, \\ a'_{j2} = -\frac{2}{(y_{j+1,k} - y_{j,k})(y_{j,k} - \mathbf{y}_{(\mathbf{j}-1)',\mathbf{k}})}, \\ a'_{j3} = \frac{2}{(y_{j+1,k} - y_{j,k})(y_{j+1,k} - \mathbf{y}_{(\mathbf{j}-1)',\mathbf{k}})}. \end{cases} \quad (2.30)$$

Note that the treatment of the derivative in the spanwise direction is analogous to that in the y-direction, modifying the a_{ki} coefficients instead of the a_{ji} .

Spanwise flow

In the spanwise direction we solve the bi-harmonic equation for ψ :

$$\nabla^4 \psi = \frac{\partial^4 \psi}{\partial y^4} + 2 \frac{\partial^4 \psi}{\partial y^2 \partial z^2} + \frac{\partial^4 \psi}{\partial z^4} = 0. \quad (2.31)$$

The flow satisfies the bi-harmonic equation as well as the following boundary conditions

$$\begin{cases} \frac{\partial \psi}{\partial y} = 0 & \psi = \psi_{\text{bottom}}, & \text{on the bottom of the domain,} \\ \frac{\partial \psi}{\partial n} = 0 & \psi = \psi_{\text{wall}}^i, & \text{on the boundary of the } i\text{-th fiber,} \\ \frac{\partial \psi}{\partial y} = w_{\text{top}} & \psi = \psi_{\text{top}}, & \text{on the top of the domain,} \end{cases} \quad (2.32)$$

where w_{top} denotes the constant velocity at which the top wall moves and ψ_{bottom} , ψ_{wall}^i and ψ_{top} are constant values. These boundary conditions, except for ψ_{bottom} that can be determined *a priori*, are part of the solution to the governing equation (2.6) and must respond to the flow.

We now focus on the discretization of the bi-harmonic equation (2.31). In order to do that, we use a second-order centered finite difference approximation for each fourth-order derivatives in the bi-harmonic equation. The stencil is then made up by 13 points. The equation for the fourth-order derivatives are:

$$\left\{ \begin{aligned} \left(\frac{\partial^4 \psi_I}{\partial y^4} \right)_{j,k} &= \frac{\psi_{I,j-2,k} - 4\psi_{I,j-1,k} + 6\psi_{I,j,k} - 4\psi_{I,j+1,k} + \psi_{I,j+2,k}}{dy^4}, \\ \left(\frac{\partial^4 \psi_I}{\partial z^4} \right)_{j,k} &= \frac{\psi_{I,j,k-2} - 4\psi_{I,j,k-1} + 6\psi_{I,j,k} - 4\psi_{I,j,k+1} + \psi_{I,j,k+2}}{dz^4}, \\ \left(\frac{\partial^4 \psi_I}{\partial y^2 \partial z^2} \right)_{j,k} &= \frac{4\psi_{I,j,k} - 2(\psi_{I,j+1,k} + \psi_{I,j-1,k} + \psi_{I,j,k+1} + \psi_{I,j,k-1})}{dy^2 dz^2} + \\ &\quad + \frac{(\psi_{I,j+1,k+1} + \psi_{I,j-1,k+1} + \psi_{I,j+1,k-1} + \psi_{I,j-1,k-1})}{dy^2 dz^2}. \end{aligned} \right. \quad (2.33)$$

So the discretization of the bi-harmonic operator results in:

$$\begin{aligned} (\nabla^4 \psi_I)_{j,k} &= \left(\frac{6}{dy^4} + \frac{6}{dz^4} + \frac{8}{dy^2 dz^2} \right) \psi_{I,j,k} - \left(\frac{4}{dy^4} + \frac{4}{dy^2 dz^2} \right) (\psi_{I,j+1,k} + \psi_{I,j-1,k}) + \\ &\quad - \left(\frac{4}{dz^4} + \frac{4}{dy^2 dz^2} \right) (\psi_{I,j,k+1} + \psi_{I,j,k-1}) + \\ &\quad + \frac{2}{dy^2 dz^2} (\psi_{I,j+1,k+1} + \psi_{I,j-1,k+1} + \psi_{I,j+1,k-1} + \psi_{I,j-1,k-1}) + \\ &\quad + \frac{1}{dy^4} (\psi_{I,j+2,k} + \psi_{I,j-2,k}) + \frac{1}{dz^4} (\psi_{I,j,k+2} + \psi_{I,j,k-2}). \end{aligned} \quad (2.34)$$

A problem occurs for the grid points next to the bottom and top solid walls, because points $\psi_{j+2,k}$ or $\psi_{j-2,k}$ are outside the domain and therefore not available for completing the stencil. In Busetto (2020), a first-order single-sided difference is used to replace the 13-point stencil and implement the Neumann boundary conditions in (2.32). However, this is a very low order approximation and not compatible with the standard 13-point discretization for those interior grid points. As a result, an obvious jump in the spanwise pressure gradient can be seen near the solid walls. One way to correct it is to approximate the undefined $\psi_{j+2,k}$ and $\psi_{j-2,k}$ involving points inside the domain and accordingly modify the stencil. If the grid spacing in y and z are equal and denoted by h , $\psi_{j-2,k}$ can be obtained by a cubic extrapolation, together with the normal derivative boundary condition, giving

$$\psi_{j-2,k} = -\frac{1}{2}\psi_{j+1,k} + 3\psi_{j,k} - \frac{3}{2}\psi_{j-1,k} - 3h \left. \frac{\partial \psi}{\partial y} \right|_{j-1,k}, \quad (2.35)$$

which is a $\mathcal{O}(h^4)$ accurate boundary approximation (Gupta and Manohar, 1979). Subsequently, the modified discretization stencil for the points next to the bottom wall is given by

$$\nabla^4 \psi = \frac{1}{h^4} \begin{bmatrix} & & 1 & & \\ & 2 & -8.5 & 2 & \\ 1 & -8 & 23 & -8 & 1 \\ & 2 & -9.5 & 2 & \end{bmatrix} \psi - 3h^{-3} \frac{\partial \psi}{\partial y}. \quad (2.36)$$

We have extended this idea to mesh with different grid spacing in y and z as well as stretching grid in y , but the numerical details are omitted here as they are tedious.

Following the same procedure of the previous case, we write this equation for each point of the grid in an algebraic form, so that:

$$[A_b] \psi_{\mathbf{I}} = \mathbf{C}_b, \quad (2.37)$$

where $[A_b]$ is the matrix of the coefficients, $\psi_{\mathbf{I}}$ is the stream function vector and $\mathbf{C}_b$ is a vector made up by zeros, except for the points where we impose the boundary conditions. Then we impose the boundary conditions inside the matrix $[A_b]$ and in the vector $\mathbf{C}_b$. Finally, we obtain the value of the stream function ψ in each point of the grid inverting the equations (2.37).

Treatment of solid boundaries

Starting from the same idea used for the streamwise flow, we expand the method of Leonardi et al. (2006) for a bi-harmonic equation. At first, we identify all the points outside the solid boundaries that have at least one point of their stencil inside the solids, and then we modify the coefficient in the $[A_b]$ matrix. However, in this case, we have to impose multiple conditions at the boundary, i.e. a constant stream function and a zero gradient in the wall-normal direction.

In order to do that, we re-calculate the coefficient for the finite difference approximation, making them dependent from the coordinate of the points:

$$\left\{ \begin{array}{l} \left(\frac{\partial^4 \psi}{\partial y^4} \right)_{j,k} = e_{j1} \psi_{j-2,k} + e_{j2} \psi_{j-1,k} + e_{j3} \psi_{j,k} + e_{j4} \psi_{j+1,k} + e_{j5} \psi_{j+2,k}, \\ \left(\frac{\partial^4 \psi}{\partial z^4} \right)_{j,k} = e_{k1} \psi_{j,k-2} + e_{k2} \psi_{j,k-1} + e_{k3} \psi_{j,k} + e_{k4} \psi_{j,k+1} + e_{k5} \psi_{j,k+2}, \\ \left(\frac{\partial^4 \psi}{\partial y^2 \partial z^2} \right)_{j,k} = e_{jk1} \psi_{j-1,k-1} + e_{jk2} \psi_{j,k-1} + e_{jk3} \psi_{j+1,k-1} + e_{jk4} \psi_{j-1,k} + e_{jk5} \psi_{j,k} + \\ \quad + e_{jk6} \psi_{j+1,k} + e_{jk7} \psi_{j-1,k+1} + e_{jk8} \psi_{j,k+1} + e_{jk9} \psi_{j+1,k+1}, \end{array} \right. \quad (2.38)$$

where the coefficients are:

$$\begin{aligned}
 e_{j1} &= b_{j1}c_{j1}, & e_{j2} &= b_{j1}c_{j2} + b_{j1}b_{j2}, \\
 e_{j3} &= b_{j1}c_{j3} + b_{j2}^2 + d_{j1}b_{j3}, & e_{j4} &= b_{j3}d_{j2} + b_{j2}b_{j3}, \\
 e_{j5} &= b_{j3}d_{j3}, & e_{k1} &= b_{k1}c_{k1}, \\
 e_{k2} &= b_{k1}c_{k2} + b_{k1}b_{k2}, & e_{k3} &= b_{k1}c_{k3} + b_{k2}^2 + d_{k1}b_{k3}, \\
 e_{k4} &= b_{k3}d_{k2} + b_{k2}b_{k3}, & e_{k5} &= b_{k3}d_{k3}, \\
 e_{jk1} &= b_{j1}b_{k1}, & e_{jk2} &= b_{j2}b_{k1}, \\
 e_{jk3} &= b_{j3}b_{k1}, & e_{jk4} &= b_{j1}b_{k2}, \\
 e_{jk5} &= b_{j2}b_{k2}, & e_{jk6} &= b_{j3}b_{k2}, \\
 e_{jk7} &= b_{j1}b_{k3}, & e_{jk8} &= b_{j2}b_{k3}, \\
 e_{jk9} &= b_{j3}b_{k3},
 \end{aligned}$$

with:

$$\begin{cases}
 b_{j1} = \frac{2}{(y_{j,k} - y_{j-1,k})(y_{j+1,k} - y_{j-1,k})}, \\
 b_{j2} = -\frac{2}{(y_{j+1,k} - y_{j,k})(y_{j,k} - y_{j-1,k})}, \\
 b_{j3} = \frac{2}{(y_{j+1,k} - y_{j,k})(y_{j+1,k} - y_{j-1,k})}, \\
 c_{j1} = \frac{2}{(y_{j-1,k} - y_{j-2,k})(y_{j,k} - y_{j-2,k})}, \\
 c_{j2} = -\frac{2}{(y_{j,k} - y_{j-1,k})(y_{j-1,k} - y_{j-2,k})}, \\
 c_{j3} = \frac{2}{(y_{j,k} - y_{j-1,k})(y_{j,k} - y_{j-2,k})}, \\
 d_{j1} = \frac{2}{(y_{j+1,k} - y_{j,k})(y_{j+2,k} - y_{j,k})}, \\
 d_{j2} = -\frac{2}{(y_{j+2,k} - y_{j+1,k})(y_{j+1,k} - y_{j,k})}, \\
 d_{j3} = \frac{2}{(y_{j+2,k} - y_{j+1,k})(y_{j+2,k} - y_{j,k})},
 \end{cases}$$

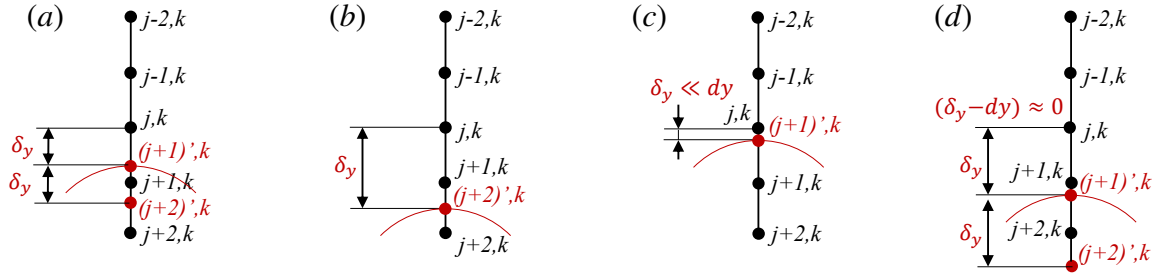


Figure 2.2 Sketch of treatment of the 5-point stencil for the fourth-order derivative in the y-direction near the solid boundary.

$$\begin{cases} b_{k1} = \frac{2}{(z_{j,k} - z_{j,k-1})(z_{j,k+1} - z_{j,k-1})}, \\ b_{k2} = -\frac{2}{(z_{j,k+1} - z_{j,k})(z_{j,k} - z_{j,k-1})}, \\ b_{k3} = \frac{2}{(z_{j,k+1} - z_{j,k})(z_{j,k+1} - z_{j,k-1})}, \end{cases}$$

$$\begin{cases} c_{k1} = \frac{2}{(z_{j,k-1} - z_{j,k-2})(z_{j,k} - z_{j,k-2})}, \\ c_{k2} = -\frac{2}{(z_{j,k} - z_{j,k-1})(z_{j,k-1} - z_{j,k-2})}, \\ c_{k3} = \frac{2}{(z_{j,k} - z_{j,k-1})(z_{j,k} - z_{j,k-2})}, \end{cases}$$

$$\begin{cases} d_{k1} = \frac{2}{(z_{j,k+1} - z_{j,k})(z_{j,k+2} - z_{j,k})}, \\ d_{k2} = -\frac{2}{(z_{j,k+2} - z_{j,k+1})(z_{j,k+1} - z_{j,k})}, \\ d_{k3} = \frac{2}{(z_{j,k+2} - z_{j,k+1})(z_{j,k+2} - z_{j,k})}. \end{cases}$$

We now focus on the different cases that we need to take into account and how to implement the correct boundary conditions. We start analyzing the fourth derivative in the y-direction. The cases that we study are eight, but we now discuss only the ones in which the intersection of the stencil and the solid boundary occurs between the (j, k) and the $(j+2, k)$ points, as shown in figure 2.2. Defining δ_y as the distance between the central point of the stencil and the solid boundary, we have:

(a) If $\delta_y < dy$, we write:

$$\left(\frac{\partial^4 \psi}{\partial y^4}\right)_{j,k} = e'_{j1} \psi_{j-2,k} + e'_{j2} \psi_{j-1,k} + e'_{j3} \psi_{j,k} + e'_{j4} \psi_{(j+1)',k} + e'_{j5} \psi_{(j+2)',k}, \quad (2.39)$$

where the coefficients e'_j are calculated with the new positions of the $((j+1)',k)$ and $((j+2)',k)$ points. We assume that the $((j+2)',k)$ is at a distance equal to δ_y from the intersection of the grid and the solid. Then, we impose the boundary conditions:

$$\begin{cases} \psi_{(j+1)',k} = -\psi_0|_{y_{(j+1)',k}} + \psi_{wall i}, \\ \left(\frac{\partial \psi}{\partial y}\right)_{(j+1)',k} = \frac{\psi_{j,k} - \psi_{(j+2)',k}}{2\delta_y} = -\frac{\partial \psi_0}{\partial y} \Big|_{y_{(j+1)',k}}. \end{cases} \quad (2.40)$$

Consequently, the equation (2.39) results in:

$$\begin{aligned} e'_{j1} \psi_{j-2,k} + e'_{j2} \psi_{j-1,k} + (e'_{j3} + e'_{j5}) \psi_{j,k} = & -e'_{j4} (-\psi_0|_{y_{(j+1)',k}} + \psi_{wall i}) + \\ & -e'_{j5} 2\delta_y \frac{\partial \psi_0}{\partial y} \Big|_{y_{(j+1)',k}}. \end{aligned} \quad (2.41)$$

(b) If $dy < \delta_y < 2dy$, we write again the equation for the fourth derivative in y with new coefficients. Note that in this case only the position of $((j+2)',k)$ changes.

$$\left(\frac{\partial^4 \psi}{\partial y^4}\right)_{j,k} = e'_{j1} \psi_{j-2,k} + e'_{j2} \psi_{j-1,k} + e'_{j3} \psi_{j,k} + e'_{j4} \psi_{j+1,k} + e'_{j5} \psi_{(j+2)',k}. \quad (2.42)$$

Then, we impose only the condition on the value of the stream function at the wall, that is:

$$\psi_{(j+2)',k} = -\psi_0|_{y_{(j+2)',k}} + \psi_{wall i}. \quad (2.43)$$

Provided that, the equation (2.42) results in:

$$e'_{j1} \psi_{j-2,k} + e'_{j2} \psi_{j-1,k} + e'_{j3} \psi_{j,k} + e'_{j4} \psi_{j+1,k} = -e'_{j5} (-\psi_0|_{y_{(j+2)',k}} + \psi_{wall i}). \quad (2.44)$$

(c) This scenario is a particular case of the first one and it occurs when $\delta_y < dy$. Consequently, we cannot use the e'_j coefficients. We opt to obtain the value of $\psi_{j,k}$ from the value of $\psi_{j-1,k}$ and $\psi_{wall i}$:

$$\psi_{j,k} = \frac{\delta_y}{\delta_y + dy} \psi_{j-1,k} + \frac{dy}{\delta_y + dy} (-\psi_0|_{y_{(j+1)',k}} + \psi_{wall i}). \quad (2.45)$$

- (d) This is a particular case of the second one and occurs when $\delta_y > dy$ and $\delta_y - dy \ll dy$. In this scenario, we opt to move the point $(j+1, k)$ to a distance equal to $\delta_y + dy$ from the central point and $(j+2, k)$ to $2(\delta_y + dy)$. Given that, we proceed as for the case (a).

In the cases in which the intersection of the stencil and the solid boundary occurs between (j, k) and $(j-2, k)$, the procedure is analogous.

We now study the fourth derivative in the z -direction. The cases are the same that can occur in the y -direction. Consequently, their treatment is also the same, apart from the fact that the gradient of ψ in the z -direction is zero. Given that, the only equations that change are the ones for case (a) and, consequently, also for case (d). Defining δ_z as the distance between the central point of the stencil and the solid boundary in the z -direction, we have:

$$\left(\frac{\partial^4 \psi}{\partial z^4} \right)_{j,k} = e'_{k1} \psi_{j,k-2} + e'_{k2} \psi_{j,k-1} + e'_{k3} \psi_{j,k} + e'_{k4} \psi_{j,(k+1)'} + e'_{k5} \psi_{j,(k+2)'}, \quad (2.46)$$

where the coefficients e'_k are calculated with the new positions of the $(j, (k+1)')$ and $(j, (k+2)')$ points. As for the previous case, we assume that the $(j', (k+2)')$ is at a distance equal to δ_z from the intersection of the grid and the solid. Then, the boundary conditions are:

$$\begin{cases} \psi_{j,(k+1)'} = -\psi_0|_{y_{j,(k+1)'}} + \psi_{wall i}, \\ \left(\frac{\partial \psi}{\partial z} \right)_{j,(k+2)'} = \frac{\psi_{j,k} - \psi_{j,(k+1)'}}{2\delta_z} = 0. \end{cases} \quad (2.47)$$

Consequently, the equation (2.46) results in:

$$e'_{k1} \psi_{j,k-2} + e'_{k2} \psi_{j,k-1} + (e'_{k3} + e'_{k5}) \psi_{j,k} = e'_{k4} (-\psi_0|_{y_{j,(k+1)'}} + \psi_{wall i}). \quad (2.48)$$

As mentioned before, for all the other cases it is sufficient to change the notation from the one for the y -direction to the one for the z -direction, but the treatment does not change.

We now analyze the cross derivative $\frac{\partial^4 \psi}{\partial y^2 \partial z^2}$. To obtain this derivative we can apply first the second-order derivative in the y -direction on the $(j, k-1)$, (j, k) and $(j, k+1)$ points, and then apply the second-order derivative in the z -direction. Due to the very complicated discretization of this derivative, we opt to implement the information about the intersection between the grid and the solid only in derivative in the y -direction.

We can focus on the second-order derivative in the point $(j, k+1)$, because the treatment of the other two points is the same. For $(j+1, k)$, the cases that can occur are four, two when the solid boundary is beyond the central point and two when it is above, as shown in figure 2.3.

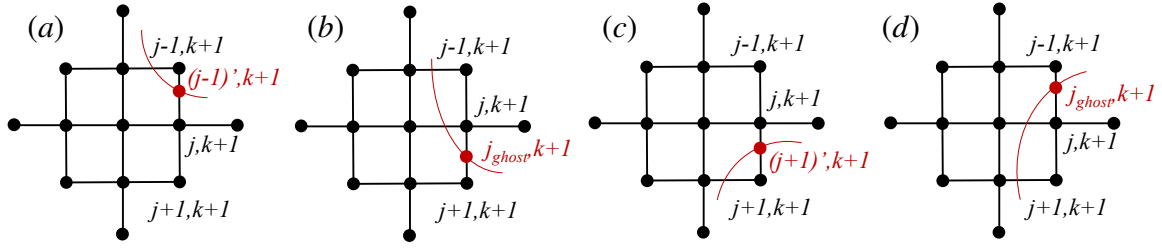


Figure 2.3 Sketch of the treatment of the second-order derivative in $(j, k+1)$ near the solid boundary.

We now discuss only the first two cases, because the strategy for the other two is essentially the same.

- (a) If the intersection of the grid and the solid occurs between $(j-1, k+1)$ and $(j, k+1)$, we use a centered finite difference. Considering also the boundary conditions, we write:

$$\begin{cases} \left(\frac{\partial^2 \psi}{\partial z^2} \right)_{j, k+1} = b'_{j1} \psi_{(j-1)', k+1} + b'_{j2} \psi_{j, k+1} + b'_{j3} \psi_{j+1, k+1}, \\ \psi_{(j-1)', k+1} = -\psi_0|_{y_{(j-1)', k+1}} + \psi_{wall i}. \end{cases} \quad (2.49)$$

- (b) If the intersection occurs between $(j, k+1)$ and $(j+1, k+1)$, we use a sided finite difference to approximate the second derivative on $(j, k+1)$, using the points $(j, k+1)$, $(j+1, k+1)$ and $(j_{ghost}, k+1)$. Consequently, we have:

$$\begin{cases} \left(\frac{\partial^2 \psi}{\partial z^2} \right)_{j, k+1} = b'_{j1} \psi_{j, k+1} + b'_{j2} \psi_{j_{ghost}, k+1} + b'_{j3} \psi_{j+1, k+1}, \\ \psi_{j_{ghost}, k+1} = -\psi_0|_{y_{j_{ghost}, k+1}} + \psi_{wall i}. \end{cases} \quad (2.50)$$

Finally, we briefly discuss how we calculate the values of $\psi_{wall i}$. If we study a domain with a single fiber, we set $\psi_{wall} = 0$ for simplicity. If multiple fibers are present in the domain or if we have to set multiple values for the stream function at the boundaries, we have to assign the correct difference between them.

For pressure driven flows, we analyze cases where there is only two unknown value for $\psi_{wall i}$. Since one of the two is a free parameter and can be imposed equal to zero, we only have to find an unknown value for ψ_{wall} . We can use the linearity of the problem, running two simulations with two different values for the stream function at the wall, ψ_{wall1} and ψ_{wall2} . For each simulation, we measure the pressure gradient in the spanwise direction applied in

two different points of the domain, $\frac{dp}{dz}\big|_{a_1}$, $\frac{dp}{dz}\big|_{b_1}$ and $\frac{dp}{dz}\big|_{a_2}$, $\frac{dp}{dz}\big|_{b_2}$. Consequently, we find the correct value of ψ_{wall} to obtain a constant pressure gradient from the following equation:

$$\psi_{wall} = \psi_{wall1} + \left(\frac{dp}{dz}\bigg|_{b_1} - \frac{dp}{dz}\bigg|_{a_1} \right) \frac{\psi_{wall2} - \psi_{wall1}}{\left(\frac{dp}{dz}\bigg|_{a_2} - \frac{dp}{dz}\bigg|_{a_1} \right) - \left(\frac{dp}{dz}\bigg|_{b_2} - \frac{dp}{dz}\bigg|_{b_1} \right)}. \quad (2.51)$$

For shear driven flows, we analyze domains with multiple fibers, and we also assign a constant value for the stream function on the top and bottom of the domain. Consequently, we develop an iterative method to find the differences in the stream function that give a zero pressure gradient between two consecutive fibers. When the methods converge, we obtain the values of the stream function to have a zero pressure gradient in the spanwise direction everywhere in the domain.

In [Busetto \(2020\)](#), ψ is decomposed into one mean and n perturbation parts

$$\psi = \psi_0 + \sum_{i=1}^n A_i \psi_i, \quad (2.52)$$

where n is the number of fibers in the computational domain, A_i the weighting coefficient of the i -th unit problem. This idea of decomposition is illustrated in figure 2.4 (b). Previously, we assumed that the mean flow was a Couette flow, given the similarity between configurations of these two cases. For a given velocity of the top wall w_{top} , the mean part is simply

$$\psi_0 = w_{top} \frac{y^2}{2l_y}, \quad (2.53)$$

where l_y is the vertical length of the domain. Correspondingly, the stream function at the top wall can be determined as $\psi_{top} = w_{top} l_y / 2$, leaving the system with n unknown boundary conditions. In the i -th unit problem, only the unknown ψ_{wall}^i on that fiber is given a value of 1 as perturbation, whilst other boundary conditions are zeroed out. The perturbed flow also satisfies equation (2.6) and consequently results in a field of ψ_i for the entire domain. The weighting coefficients in the linear combination (2.52) indicate the difference between stream functions of each fiber. They are calculated such that the final solution meets a certain condition which, in this case, forces the mean spanwise pressure gradient to zero.

Unfortunately, we have found spurious fluctuations in the velocity profile obtained from the above-mentioned method. For example, for an exceedingly small fiber radius, the mean velocity was expected to decrease monotonically as the flow goes into the substrate in y , but it actually decreased first and then increased near the stationary bottom solid wall. The problem comes from the assumed Couette-like mean flow ψ_0 and the prescribed stream function at the

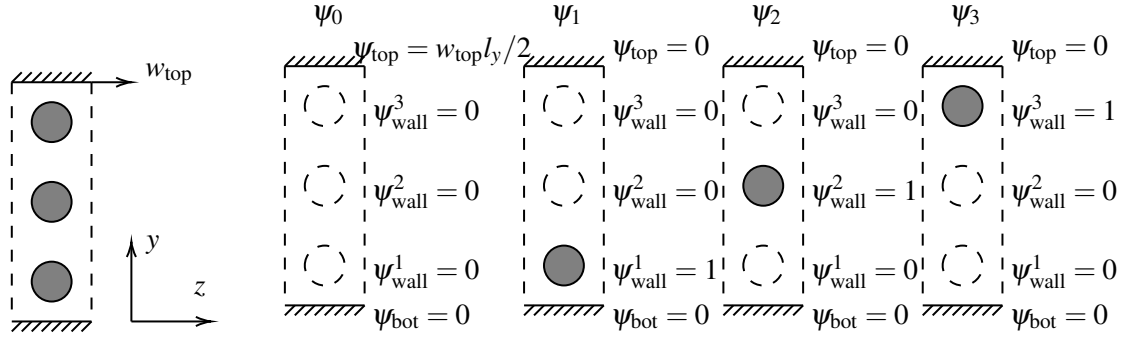


Figure 2.4 (a) Sketch of the computational domain of the cross flow. (b) Four unit problems after decomposing the flow in a channel with three fibers.

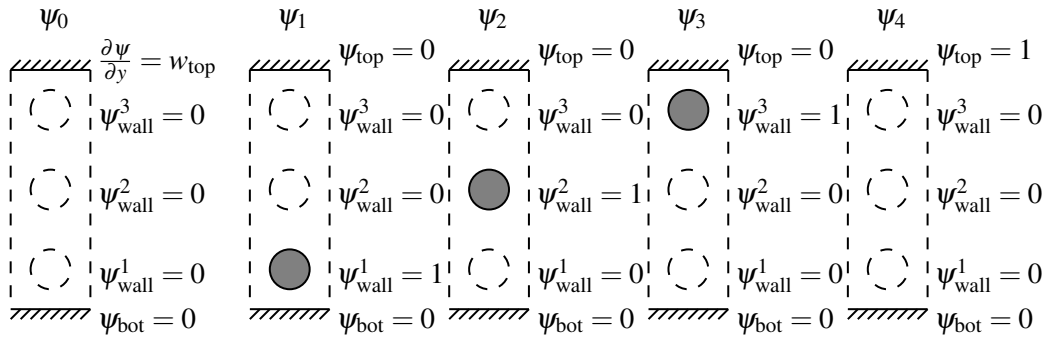


Figure 2.5 Decomposition of the flow into $n + 2$ unit problems.

top wall ψ_{top} . In fact, equation (2.53) is not valid in a channel with solid obstacles and ψ_{top} remains unknown. At the top wall, either a constant velocity $\partial \psi / \partial y|_{\text{top}} = w_{\text{top}}$ or a constant stream function ψ_{top} should be prescribed and let the other one vary according to the flow, otherwise, the whole problem is overdetermined. Therefore, we could decompose the flow of figure 2.4 (a) into $n + 2$ unit problems

$$\psi = \psi_0 + \sum_{i=1}^n A_i \psi_i + A_{n+1} \psi_{n+1}, \quad (2.54)$$

where the variables are the same as those in (2.52). This decomposition and the corresponding implement of boundary conditions are illustrated in figure 2.5. With $n + 1$ known boundary conditions, the system can then be solved with $n + 1$ error functions as in Busetto (2020).

Corrected results compared with previous ones

We now make a qualitative comparison between the old and new solution scheme and code. We take the configuration of figure 2.4 (a) as an example, where fiber radius is $r = 0.03$ and flow is driven by the top wall moving at a constant speed $w_{\text{top}} = 1$. The solid fibers and bottom

wall are stationary and impermeable. The domain is periodic in the spanwise direction. By solving this flow using the old and new code, we show the results in figure 2.6. With the old version code, figure 2.6 (a)-(c) show spanwise velocity without physical meaning. For example, the flow in between the first fiber and the bottom wall is expected to be creeping but shows relatively high intensity. The mean velocity at the top drops to zero suddenly. After the correction, results in figure 2.6 (e)-(g) appear to be more reasonable. The mean velocity profile in figure 2.6 (f) decreases monotonically from unity to zero, starting from the top in y . The mean flow between solid walls displays a linear profile and is therefore locally Couette-like. Because of the resistance of the solid fibers, the mean flow decreases exponentially in y , as can be seen in figure 2.6 (g).

Verification of the Pressure Field

We now look to verify the solution produced by the code for a column of fibers using the pressure field; here, we use the column of eight fibers studied previously. The pressure field is shown in Figure 2.7a; critically, the field is anti-symmetric, as expected for a flow of such low Reynolds number. Of particular note are the pressure lobes on the top fiber, which bear similarity to the positive and negative pressure faces of the inviscid flow around a cylinder. These are shifted towards the top of the fiber, where the spanwise velocity is higher, as a greater pressure is required to redirect the flow around the top of the fiber. The pressure field therefore appears physically sound.

The Laplacian of the pressure, $\nabla^2 p$, can be seen in Figure 2.7b. The single grid spacing around the surface of the fibers where no special treatment of the solid boundary is performed for the first-order derivative matrices can be seen; however, in the remainder of the domain, $\nabla^2 p$ clearly equals zero, thus verifying the pressure field produced. It is interesting to be able to see the direction of numerical integration performed to calculate the pressure field in the solution for $\nabla^2 p$: vertically up the left-hand side of the domain, across from left to right, and then meeting at the fiber centers on the right-hand side.

Figure 2.8 shows the pressure variation along horizontal lines through the domain, at the three-quarter height of each fiber cell; the horizontal lines are shown in the top plots in red, and their corresponding pressure profiles below them. The pressure can be seen to vary sinusoidally across the domain. The pressure variation along the bottom horizontal line does not meet at zero, as the sinusoidal shape is squashed; this is likely due to the order of magnitude of the pressure gradients at this depth being sufficiently small, that machine error is significant. The pressure at this depth is of order 10^{-16} in Figure 2.8.

We can equally look at the Laplacian of the pressure along these same lines; this is shown in Figure 2.9. These lines all give $\nabla^2 p$ of practically zero across the domain. The spikes at the

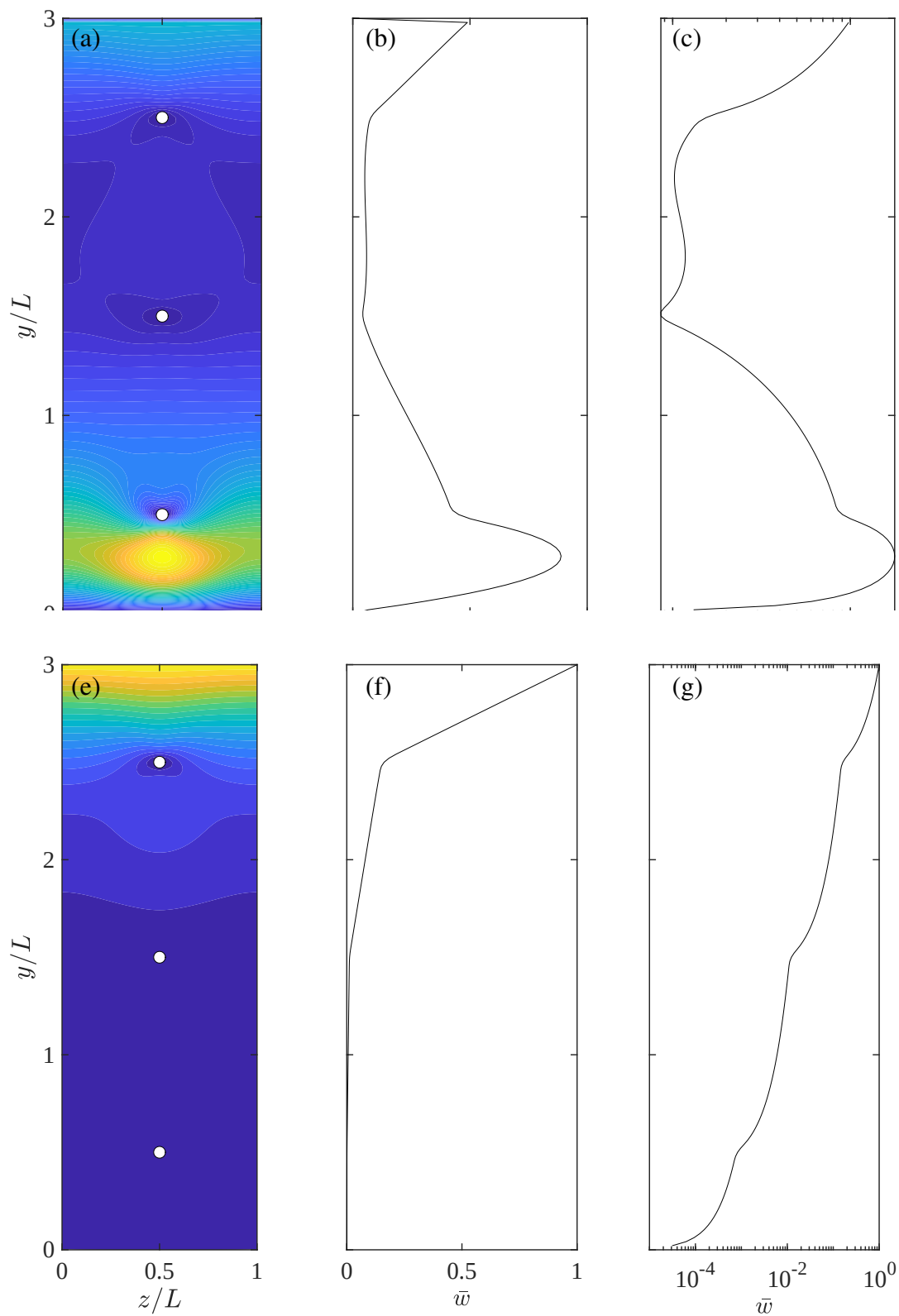


Figure 2.6 The spanwise velocity w . (a)-(c) results from the old method; (e)-(g) corrected results from new method. From left panel to right are contours, mean velocity profiles and profiles in logarithmic scale respectively.

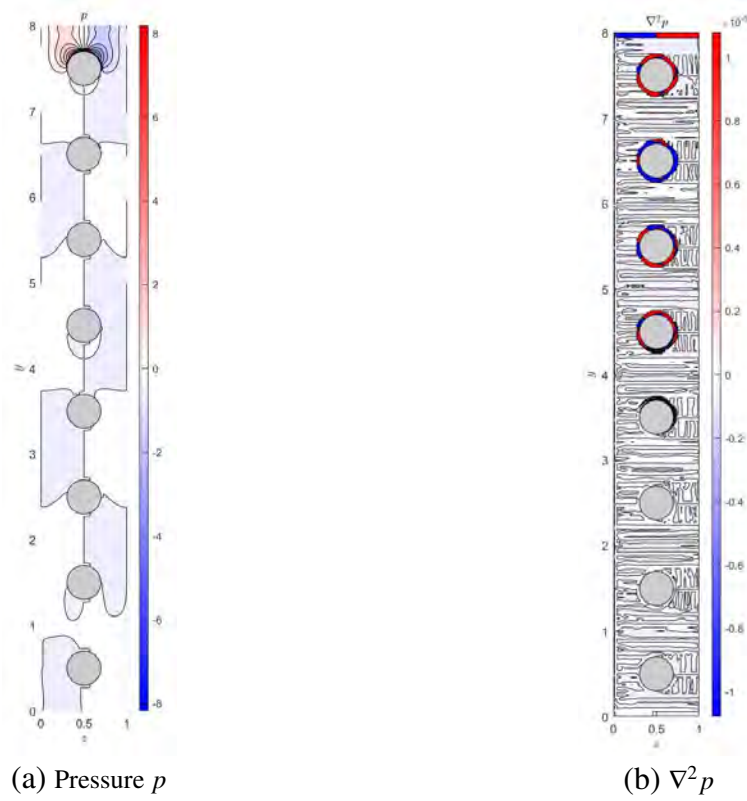


Figure 2.7 The pressure distribution and its Laplacian for the spanwise shear-driven flow through a column of eight fibers, of radius $r = 0.2$.

edges of the domain are likely caused by the addition of the final grid column to the matrix in the post-processing to be displayed, using the periodicity of the solution, and can thus be ignored.

2.1.4 Convergence

To test the grid convergence of our numerical codes, we analyze again a permeable substrate made of aligned fibers with a circular cross-section, and we consider the case of $\theta = 0.80$, that is the porosity of the substrates defined as the ratio between the fluid volume and the total volume of the domain. In figure 2.10 we portray the permeabilities in the streamwise and spanwise direction versus the number of grid points in the y -direction. Then, in figure 2.11 we portray the relative errors:

$$\begin{cases} E_{K_x} = \frac{K_x - K_{x288}}{K_{x288}} \\ E_{K_z} = \frac{K_z - K_{z288}}{K_{z288}} \end{cases} \quad (2.55)$$

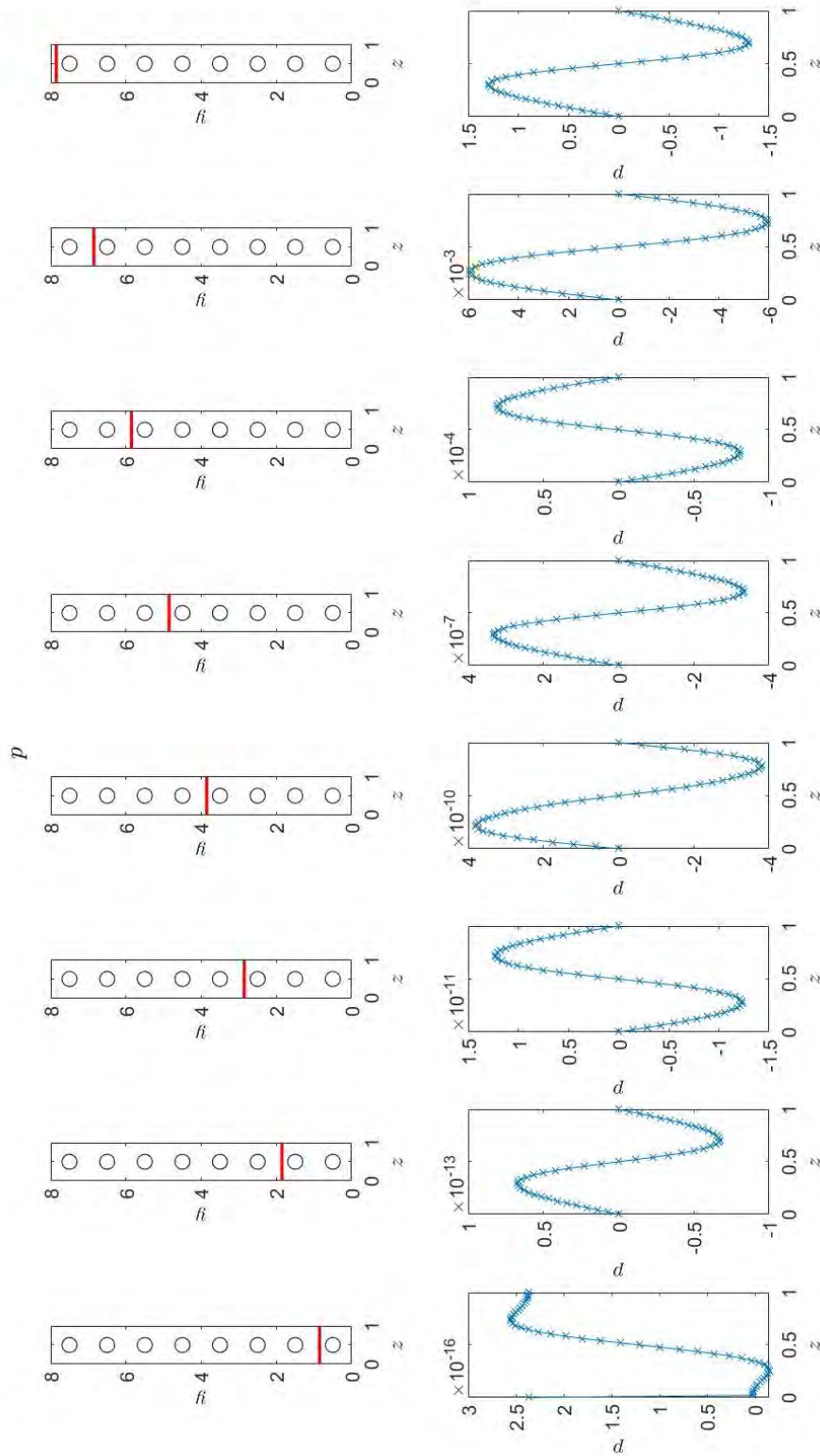


Figure 2.8 Pressure profiles along horizontal lines above each fiber, corresponding to the red lines, for the spanwise shear-driven flow through a column of eight fibers, of radius $r = 0.2$.

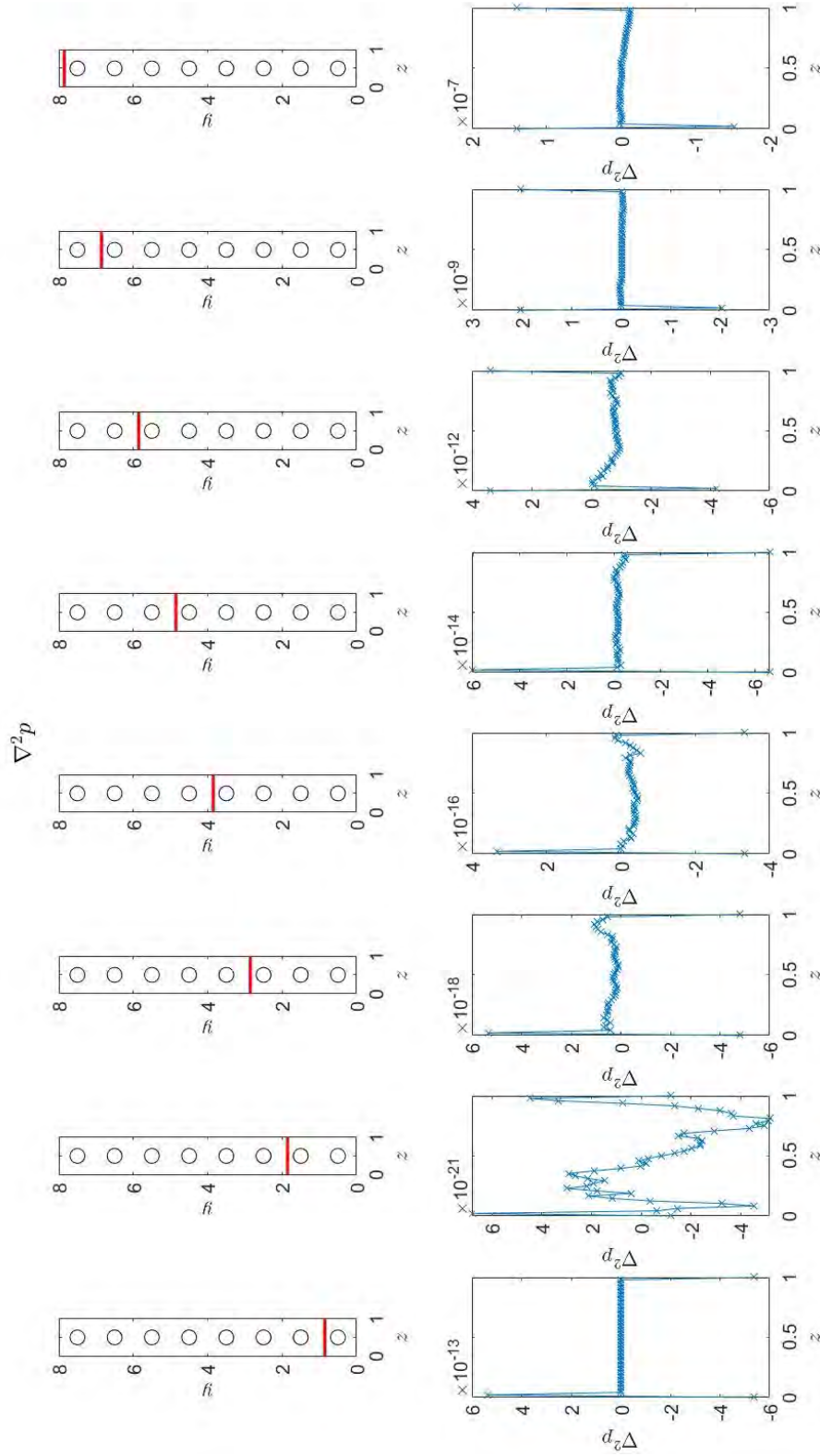


Figure 2.9 Profiles of $\nabla^2 p$ along horizontal lines above each fiber, corresponding to the red lines, for the spanwise shear-driven flow through a column of eight fibers, of radius $r = 0.2$.

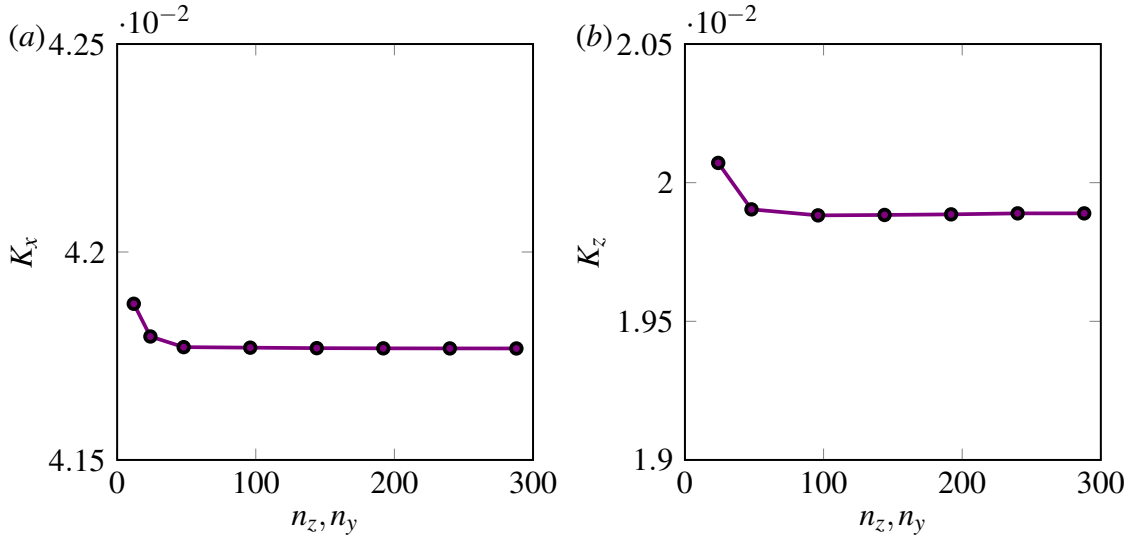


Figure 2.10 Permeability in the streamwise (a) and spanwise (b) direction versus the number of grid points in the y-direction.

From these results, the convergence of the codes is verified. We can observe that the solution becomes essentially grid-independent for N_z and N_y equal to 48.

2.1.5 Validation of the codes

To validate our codes, we consider the well-studied case of a permeable substrate made of aligned fibers with a circular cross-section, as the one shown in figure 1.4 (a). In figure 2.12, the results of our analyzes for the streamwise and spanwise permeability are displayed against the ones obtained by [Zampogna and Bottaro \(2016\)](#). Looking at the figure, we can conclude that our models and codes reproduce well the values for the permeability in both directions studied. Consequently, we also validate our treatment of Laplacian and bi-harmonic operators. Therefore, these results justify the validation of all our codes, even when the flow is driven by shear..

2.2 Methods for fully resolved DNSs

In this section, numerical methods employed for fully resolved DNSs are discussed, presenting the discretized governing equations and the treatment for the permeable substrates. Direct numerical simulations are performed using the parallelized, pseudo spectral/finite difference code of [Fairhall et al. \(2019\)](#); [García-Mayoral and Jiménez \(2011\)](#) and [Sharma and García-Mayoral \(2020\)](#) with an newly developed immersed boundary method ([Mittal and Iaccarino,](#)

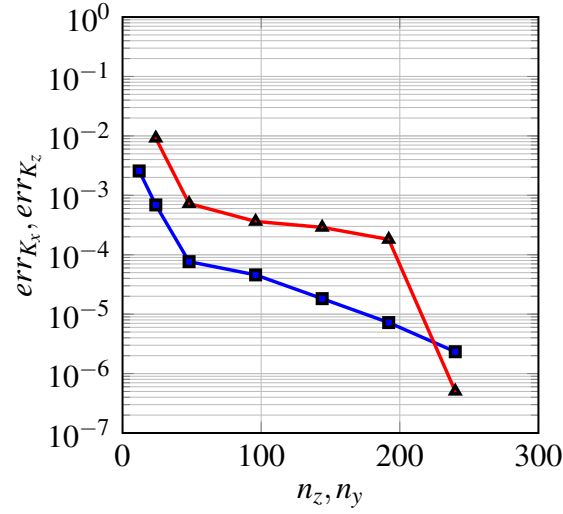


Figure 2.11 Error in the permeabilities results versus the number of grid points in the y-direction. $\blacksquare$, streamwise permeability; $\blacktriangle$, spanwise permeability.

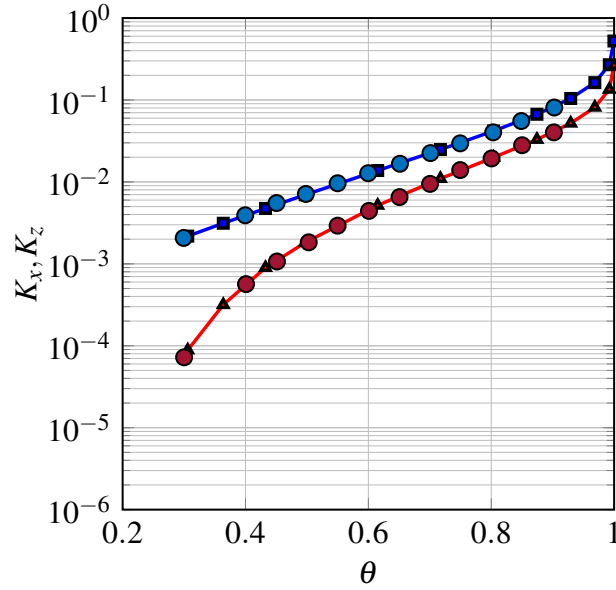


Figure 2.12 Permeabilities versus porosity for a regular array of cylinders. $\blacksquare$, $\blacktriangle$, results from our analyzes for the streamwise and spanwise permeability; $\bullet$, $\circ$, data from Zampogna and Bottaro (2016).

2005). The code is originally for simulating turbulence over complex surfaces in a channel. It is developed here to fully resolve micro-structures within the permeable substrates and the overlying turbulent flows simultaneously.

2.2.1 Discretization schemes

Spatial discretization

The computational domain is meshed with Cartesian grid. The mesh is uniform in directions that are parallel to the wall, x and z , along which flow variables (u , v , w and p) are stored in collocated locations. In the wall-parallel directions, the periodicity of the computational domain and the uniformity of the mesh allow the use of a Fourier spectral discretization. The nonlinear terms in Navier-Stokes equations are calculated with a 2/3 de-aliasing in the physical space to avoid introducing convolution. In the wall-normal direction y , the mesh is staggered and non-uniform, where the wall-normal velocity v is offset from the other three variables and a second-order finite difference scheme is used. This conjunctive design is numerically realized and tested by [García-Mayoral and Jiménez \(2011\)](#); [Gómez-de-Segura and García-Mayoral \(2019\)](#) and [Sharma and García-Mayoral \(2020\)](#), for sufficient spatial accuracy with better efficiency than calculating in full physical space.

The grid size is based on the range of scales of turbulent motions to be resolved. The largest scale is limited by the size of the domain, whereas the smallest scale is determined by the grid. In DNSs, the smallest scale to be solved is of $\mathcal{O}(\eta)$ ([Moin and Mahesh, 1998](#)), where η is the Kolmogorov length scale. Therefore, using the grid spacing to correctly represent those turbulent scales means this spacing would be sufficiently small. It is suggested that, in a smooth channel, adequate resolutions for near-wall turbulence are $\Delta x^+ \lesssim 8$, $\Delta z^+ \lesssim 4$ and $\Delta y_{min}^+ \lesssim 0.3$ ([García-Mayoral and Jiménez, 2011](#); [Moin and Mahesh, 1998](#)). Along the wall-normal direction, grid nodes are distributed by a seventh-order polynomial modified from a fifth-order one by [Sharma \(2020\)](#)

$$y(j) = \begin{cases} pj - 1, & j \in [-N_p, 0) \\ \sum_{i=1}^4 c_i \left(j - \frac{N}{2}\right)^{(2i-1)} + c_0, & j \in [0, N] \\ p(j - N) + 1, & j \in (N, N + N_p] \end{cases}, \quad (2.56)$$

where j is the node index and y is the coordinate in the wall-normal direction. In the channel region there are N grid nodes in y whose coordinates are calculated with coefficients c_i , and in the substrates region there are N_p nodes with coefficient p . The coefficients c_i and p are determined by the desired grid resolutions and the vertical length of the channel and substrates regions. The polynomial (2.56) distributes the grid nodes in the following manner: from the bottom of the computational domain to the interface between the substrate and the channel, the grid spacing in wall units is kept constant and less than 0.3; in the channel region, the mesh is stretched, and the spacing is increased with y going up until reaches its maximum $\Delta y^+ \approx 3$

at the channel center; the grid nodes are symmetric with respect to the channel center and the piecewise-defined function (2.56) is continuous for the whole domain. An example grid is depicted in figure 2.13.

However, the grid resolution is not only dependent on the turbulent scales but also on the length scale of the fibers, which is determinative of the finest resolution in this study. With the aim of achieving optimal drag reduction, the typical length scale of the fiber would be of $\mathcal{O}(10^{-1}) \sim \mathcal{O}(1)$ in wall units (Gómez-de-Segura and García-Mayoral, 2019), in which case the grid resolution in the spanwise and wall-normal directions in the substrate region needs to be significantly increased to fully resolve the geometry. Detailed grid resolutions for each case are listed in Table 4.1.

Temporal discretization

A three-step Runge-Kutta method is used for the time advancement of the Navier-Stokes equations, together with a fractional-step scheme to deal with the coupling between the velocity and the pressure field (Kim and Moin, 1985; Le and Moin, 1991). With the Runge-Kutta scheme, each time step n is split into three sub-steps $k = 1, 2, 3$, where the Navier-Stokes equations in Section 1.5 are discretized as

$$\frac{\mathbf{u}^k - \mathbf{u}^{k-1}}{\Delta t} = \frac{\alpha_k}{\text{Re}} \mathbf{L}(\mathbf{u}^{k-1}) + \frac{\beta_k}{\text{Re}} \mathbf{L}(\mathbf{u}^k) - \gamma_k \mathbf{N}(\mathbf{u}^{k-1}) - \zeta_k \mathbf{N}(\mathbf{u}^{k-2}) - (\alpha_k + \beta_k) \mathbf{G}(p^k) \quad (k = 1, 2, 3), \quad (2.57a)$$

$$\mathbf{D}(\mathbf{u}^k) = 0, \quad (2.57b)$$

which include implicit treatment for the nonlinear terms and explicit for the advection terms. To clarify, $\mathbf{u}^0$ is at the beginning of time step n as well as at the end of $n - 1$, and similarly $\mathbf{u}^3$ is at the beginning of n and end of $n + 1$. $\mathbf{L}$, $\mathbf{N}$, $\mathbf{G}$ and $\mathbf{D}$ represent second-order finite difference approximations to the Laplace, advective, gradient and divergence operators respectively. α_k , β_k , γ_k and ζ_k are Runge-Kutta coefficients whose values are listed in Table 2.1 and predetermined such that the viscous terms are calculated with second-order accuracy and the advection terms with third-order accuracy (Le and Moin, 1991). Rewriting equation (2.57) yields

$$\left[I - \Delta t \frac{\beta_k}{\text{Re}} \mathbf{L} \right] \mathbf{u}^k = \mathbf{u}^{k-1} + \Delta t \left[\frac{\alpha_k}{\text{Re}} \mathbf{L}(\mathbf{u}^{k-1}) - \gamma_k \mathbf{N}(\mathbf{u}^{k-1}) - \zeta_k \mathbf{N}(\mathbf{u}^{k-2}) - (\alpha_k + \beta_k) \mathbf{G}(p^k) \right] \quad (k = 1, 2, 3), \quad (2.58a)$$

$$\mathbf{D}(\mathbf{u}^k) = 0, \quad (2.58b)$$

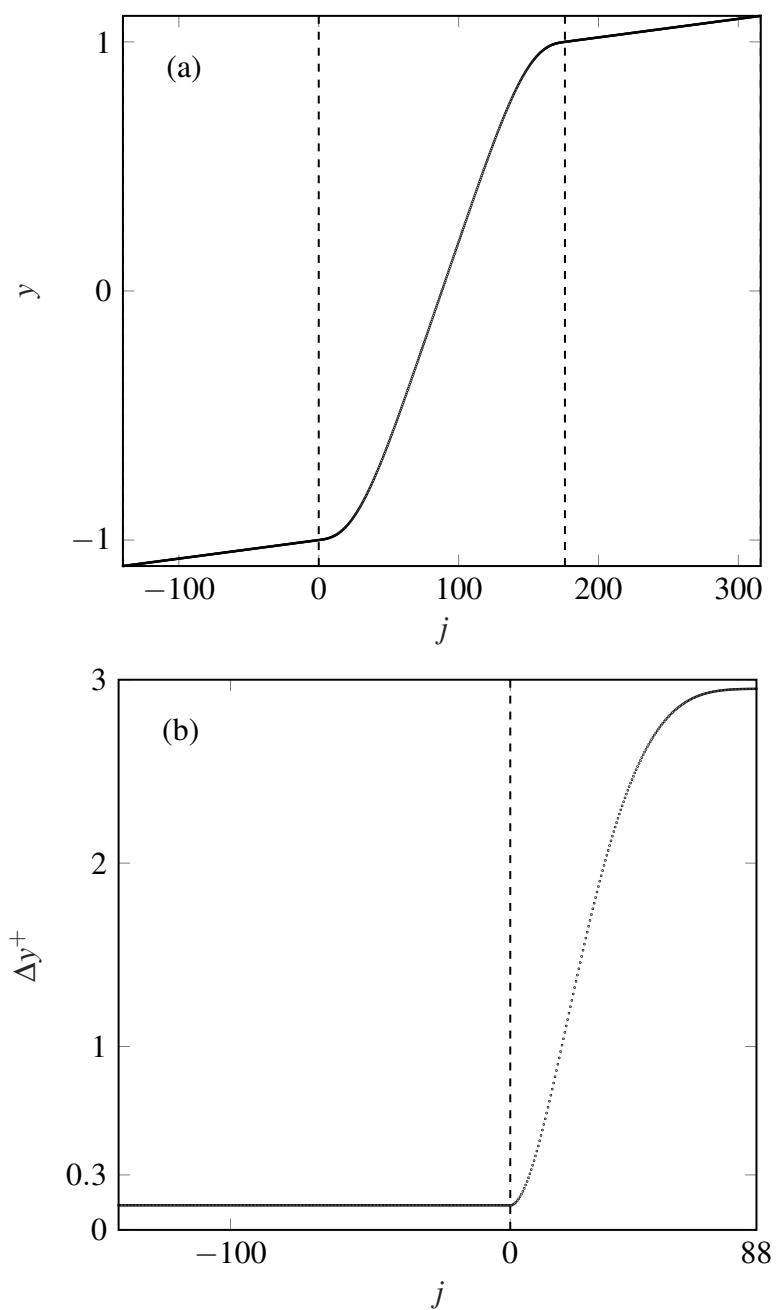


Figure 2.13 Exemplified grid nodes distribution (a) and resolution in half channel (b). Variable y is the coordinate of nodes and j is the index of nodes. The dashed lines mark the interface between the substrate and channel region.

	$k = 1$	$k = 2$	$k = 3$
α_k	4/15	1/15	1/6
β_k	4/15	1/15	1/6
γ_k	8/15	5/12	3/4
ζ_k	0	-17/60	-5/12

Table 2.1 Runge-Kutta coefficients used for each time sub-step, taken from [Le and Moin \(1991\)](#).

so that the velocity at current time step is expressed as a function of velocities at previous steps and pressure at current step. Subsequently, the fractional step method is employed as a solution to the pressure-velocity coupling in equation (2.58). A set of equations used in the numerical solver is derived as

$$\left[\mathbf{I} - \Delta t \frac{\beta_k}{\text{Re}} \mathbf{L} \right] \mathbf{u}^* = \mathbf{u}^{k-1} + \Delta t \left[\frac{\alpha_k}{\text{Re}} \mathbf{L} \left(\mathbf{u}^{k-1} \right) - \gamma_k \mathbf{N} \left(\mathbf{u}^{k-1} \right) - \zeta_k \mathbf{N} \left(\mathbf{u}^{k-2} \right) - (\alpha_k + \beta_k) \mathbf{G} \left(p^{k-1} \right) \right] \quad (k = 1, 2, 3), \quad (2.59a)$$

$$\text{DG}(p^*) = \frac{1}{(\alpha_k + \beta_k) \Delta t} \mathbf{D}(\mathbf{u}^*), \quad (2.59b)$$

$$\mathbf{u}^k = \mathbf{u}^* - (\alpha_k + \beta_k) \Delta t \mathbf{G}(p^*), \quad (2.59c)$$

$$p^k = p^{k-1} + p^*. \quad (2.59d)$$

The procedure is carried out in the following order:

1. Take the pressure field from previous sub-step, p^{k-1} , as initial condition.
2. Solve the momentum equation (2.58a) with p^{k-1} (equation (2.59a)), to obtain the intermediate velocity field $\mathbf{u}^*$.
3. Solve equation (2.59b) to produce the pressure correction p^* .
4. Correct the velocity (equation (2.59c)) to produce the velocity field at current sub-step $\mathbf{u}^k$ which is divergence-free.
5. Correct the pressure field p^k and use it for next sub-step $k + 1$, return to step 2 and iterate until converged.

The time step Δt varies according to the grid resolution and flow variables, and ensures that convective CFL number $\text{CFL}_c \leq 0.7$ and viscous CFL number $\text{CFL}_v \leq 2.5$. It is therefore

determined by (Fairhall and García-Mayoral, 2018)

$$\Delta t = \min \left\{ \text{CFL}_c \left[\frac{\Delta x}{\pi |u|}, \frac{\Delta y}{\pi |v|}, \frac{\Delta z}{\pi |w|} \right], \text{Re CFL}_v \left[\frac{\Delta x^2}{\pi^2}, \frac{\Delta y_{\min}^2}{4}, \frac{\Delta z^2}{\pi^2} \right] \right\}. \quad (2.60)$$

2.2.2 Immersed boundary method

The micro-structure within the permeable substrates presents, in principle, complex geometry which cannot be implemented directly as solid wall boundary conditions (i.e. no-slip and no-penetration conditions) on a Cartesian grid. For the streamwise elongated fibers in this study, their curvilinear solid boundary requires special consideration in its two-dimensional cross sectional plane. The immersed boundary method approximates numerically this curvilinear boundary and solves it without modifying the regular grid mesh. After firstly being proposed for low Reynolds number cardiac flow by Peskin (1977), this method was further developed to deal with flows governed by the full Navier-Stokes equations, using a discrete forcing approach (Iaccarino and Verzicco, 2003; Mittal and Iaccarino, 2005; Mohd-Yusof, 1997).

Mathematical and numerical formulation

The above-mentioned immersed boundary method could be realized by adding an additional force term $\mathbf{f}$ to the discretized momentum equation. The additional force $\mathbf{f}$ drives velocities on certain grid points to desired values $\mathbf{V}$. The solid phase can then be represented by a region where the fluid is at rest in the physical domain. In Mohd-Yusof (1997), the time-discretized momentum equation is written as

$$\frac{\mathbf{u}^k - \mathbf{u}^{k-1}}{\Delta t} = \left(-\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{f} \right)^{k-1}, \quad (2.61)$$

where the force $\mathbf{f}$ depends on the position and the velocity field. For grid points on the solid boundary and in the solid region, their velocities are expected to be driven to 0. However, when the boundary does not conform the mesh and intersects the cell face between two nodes, a non-zero velocity on one of the nodes needs to be obtained from its neighboring nodes. In general, the relationship between the force term $\mathbf{f}$ and the desired velocity $\mathbf{V}$ at time step k could be interpreted as

$$\mathbf{f}^{k-1} = \frac{\mathbf{V} - \mathbf{u}^{k-1}}{\Delta t} - \left(-\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} \right)^{k-1}. \quad (2.62)$$

By substituting equation (2.62) into (2.61), the right hand side of momentum equation (2.57a) is replaced by a new term involving the desired velocity

$$\frac{\mathbf{u}^k - \mathbf{u}^{k-1}}{\Delta t} = \frac{\mathbf{V} - \mathbf{u}^{k-1}}{\Delta t}, \quad (2.63)$$

which needs to be updated every time step to enforce the immersed boundary, leaving the left hand side of the momentum equation unchanged. In other words, the immersed boundary implement could be simplified as

$$\mathbf{u}^k = \mathbf{V}. \quad (2.64)$$

It should be noted here that equations (2.61) to (2.64) are not the exact discretization scheme embedded in the numerical solver, but present a illustration of it.

Numerically, $\mathbf{u}^k$ is approximated as a function of variables at previous steps, as shown in equation (2.59a). It brings the term $(-\Delta t \beta_k \text{Re}) \mathbf{L}(\mathbf{u}^k)$ into the left hand side, which could be accounted for by that term from its last step (García-Mayoral and Jiménez, 2011). The condition of equation (2.64) can then be incorporated into the time discrete equation (2.59a), yielding

$$\left[\mathbf{I} - \Delta t \frac{\beta_k}{\text{Re}} \mathbf{L} \right] \mathbf{u}^* = \mathbf{V} - \Delta t \frac{\beta_k}{\text{Re}} \mathbf{L}(\mathbf{u}^{k-1}), \quad (2.65)$$

which is implemented in the code as

$$\left[\mathbf{I} - \Delta t \frac{\beta_k}{\text{Re}} \mathbf{L} \right] \mathbf{u}^* = (\mathbf{V} - \mathbf{u}^{k-1}) + \left[\mathbf{I} - \Delta t \frac{\beta_k}{\text{Re}} \mathbf{L} \right] \mathbf{u}^{k-1}. \quad (2.66)$$

It is worthwhile to note that the boundary of a streamwise elongated fiber is a function of y and z , and is identical along the streamwise direction, i.e. independent of x . This feature allows equation (2.66) to be applied for each streamwise Fourier mode in physical $y - z$ plane, instead of in three-dimensional physical space. As a result, the implementation has spectral resolution in x . Additionally, by conducting Fourier transformation only in spanwise direction, the calculation of the immersed boundary requires less memory and computing time.

Calculation of desired velocity field

The basic idea and the goal of the immersed boundary method is to reconstruct a velocity field $\mathbf{V}$, where the fluid in solid regions is at rest due to additional forces meanwhile that in fluid regions is driven by the Navier-Stokes equations. From a numerical perspective, the desired velocity $\mathbf{V}$ is assigned on nodes associated with the solid region, enforcing the velocity on the boundary to become zero. How is a solid region with curvilinear boundary incorporated into the Cartesian mesh is illustrated in figure 2.14 (a).

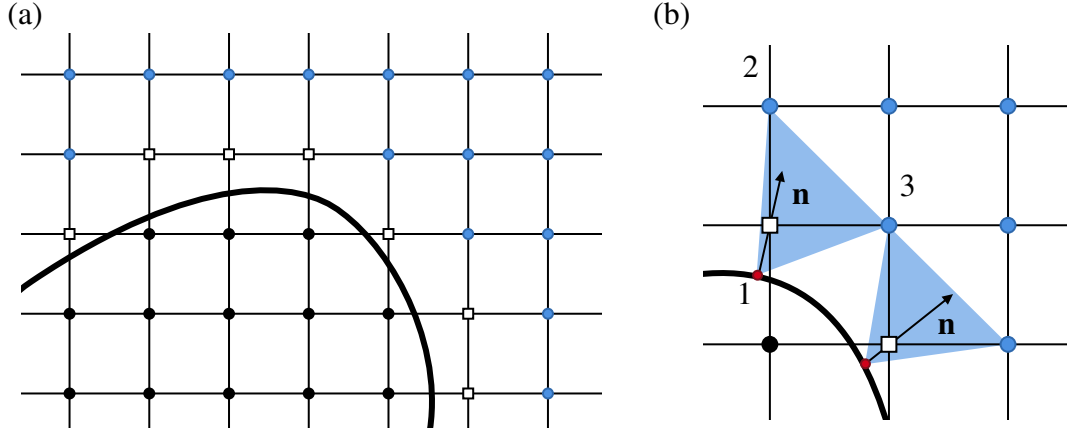


Figure 2.14 Schematics in the $y-z$ plane of (a) mesh node types; (b) interpolation stencils of the linear reconstruction. ●, fluid nodes; ●, solid nodes; □, boundary nodes where desired velocities are imposed. ●, points where solid boundary conditions are expected to be satisfied.

Firstly, we need to identify grid points according to their location with respect to the solid body. As shown in figure 2.14, in the computational domain, the discrete mesh nodes are subdivided into three catalogues, fluid nodes, solid nodes and boundary nodes depending on whether they are outside, inside, or in the periphery but having at least one neighboring node inside the immersed boundary, respectively. In the following discussion, the solid boundary is defined by a shape function $\Gamma(y, z) = 0$, and the mesh node is labeled by $P_{j,k}$, where j and k indicate its discrete indexes in y and z . Assigning nodes into groups is achieved by the following searching scheme:

1. if $\Gamma(j, k) > 0$, then $P_{j,k}$ is a fluid node;
2. for this fluid node, if $\{\Gamma(j, k-1), \Gamma(j-1, k), \Gamma(j, k+1), \Gamma(j+1, k)\} \cap \mathbb{R}_-^* \neq \emptyset$, where $\mathbb{R}_-^*$ is the real negative set;
3. if $\Gamma(j, k) = 0$, then $P_{j,k}$ is a boundary node;
4. else $P_{j,k}$ is a solid node.

The subsequent step is to assign points on the solid surface, not necessarily belonging to the grid, where we intend the boundary conditions to be satisfied. Each of these points is assigned to a boundary node, such that the two are aligned along the direction normal to the boundary, as illustrated in figure 2.14 (b).

Linear interpolation is used to calculate the desired velocity field $\mathbf{V}$ in this study. In a two-dimensional plane, a triangular stencil is formed covering a boundary node, correlating

the corresponding point on the border and two surrounding fluid nodes. Taking one velocity component as example, the local velocity field obtained with a linear interpolation is written as

$$u(y, z) \approx c_1 y + c_2 z + c_3. \quad (2.67)$$

The three vertexes, labeled by 1, 2 and 3 in figure 2.14 (b) for illustration, have their known velocity values $u_1(y_1, z_1)$, $u_2(y_2, z_2)$, $u_3(y_3, z_3)$, which can either be given values or obtained from previous time step. Substituting these three velocities into equation (2.67), we obtain a set of equations that can be written in matrix form as

$$\begin{bmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad (2.68)$$

which can be solved for c_1 , c_2 and c_3 .

$$\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \left(\begin{bmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{bmatrix}^{-1} \right)^T. \quad (2.69)$$

Alternatively, the velocity V could be formulated in terms of nodal values and weighting coefficients

$$V(y, z) \approx w_1 u_1 + w_2 u_2 + w_3 u_3. \quad (2.70)$$

In essence, equation (2.70) is equivalent to equation (2.67) of V , yielding

$$V(y, z) = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} y \\ z \\ 1 \end{bmatrix}, \quad (2.71)$$

where the weighting coefficients w depend solely on the coordinates of the node or the point, as can be seen after substituting equation (2.69) into equation (2.71). Given the above, for a boundary node at (y, z) , the coefficients are obtained by solving

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \left(\begin{bmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{bmatrix}^{-1} \right)^T \begin{bmatrix} y \\ z \\ 1 \end{bmatrix}. \quad (2.72)$$

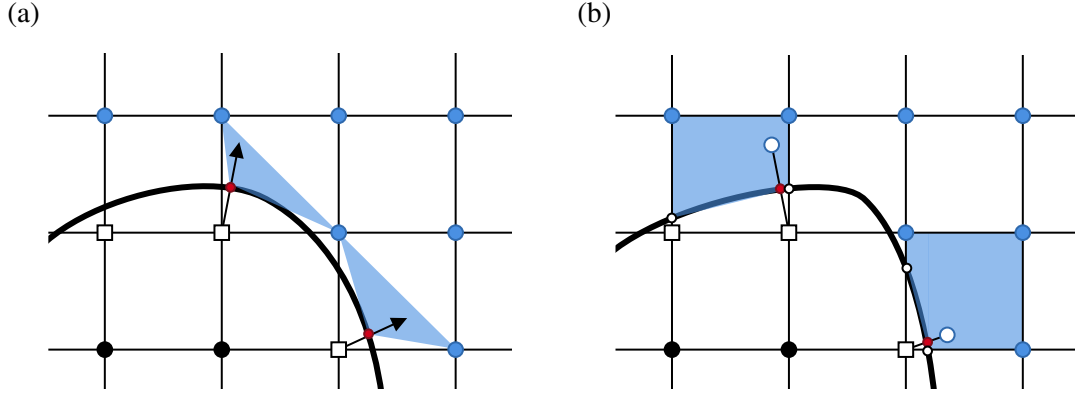


Figure 2.15 Alternative interpolation schemes: (a) Linear extrapolation in triangular domain, with internal boundary nodes; (b) Bilinear interpolation in quadrilateral domain, where $\circ$ is the image point and $\bullet$ is the wall intersection point.

Finally, the velocity distribution to be met in equation (2.66) is

$$\mathbf{V} = \begin{cases} w_2 \mathbf{u}_2 + w_3 \mathbf{u}_3 & , \text{ on boundary nodes,} \\ 0 & , \text{ on solid nodes,} \end{cases} \quad (2.73)$$

given that the solid body is stationary in our study. It is straightforward to extend this scheme for a staggered grid, requiring that the assigning and interpolation are carried out on each grid separately, resulting in one set of weighting coefficients for each velocity component. Furthermore, the evaluation of the desired velocity distribution $\mathbf{V}$ depends on the unsteady velocity field and the geometric information, i.e. the boundary shape and the location of targeting points. While the former is involved in the calculation, the latter is independent of the flow and invariant once the stationary solid obstacles are created. Therefore, the geometric details, i.e. shape function of the solid boundary and weighting coefficients, can be obtained in the pre-process and taken into the temporal integration of (2.59a) as input parameters.

How to formulate the reconstruction scheme is a vital factor, determining the efficiency and stability of the numerical process. The interpolation scheme used here minimizes the probability of numerical instability, as the weighting coefficients in (2.73) are positive and less than 1. On the contrary, if a boundary node is defined within a solid region, the desired velocity thereof needs to be extrapolated from its three corresponding points, as shown in figure 2.15 (a). When this boundary node is far away from the boundary compared to its fluid counterpart, a large negative weighting coefficient will be generated, resulting in a reverse flow with large velocity inside the solid body. This produces sharp velocity gradients, which could lead to serious spurious pressure fluctuations and cause the iteration to diverge (Tseng and Ferziger, 2003).

Although the current method is more stable with inherently positive weighting coefficients, it is not unconditionally preferred, because it implements additional forces in the fluid region. Here we give a brief discussion of an alternative interpolation scheme that shall be tested in the next phase of this project. It is practicable to replace the current method by one that only forces inside the solid phase while does not produce large negative coefficients, e.g. by introducing the image point and the bilinear interpolation in figure 2.15 (b) (Majumdar et al., 2001). Under this scheme, each boundary node (also referred to as ghost point) corresponds to a image point identified in the fluid region. The image point is located along the normal to the boundary through the ghost point, and both points have the same distance to the boundary. Then a quadrilateral domain is formed and bounded by fluid nodes and wall intersection points, to evaluate the velocity on the image point. Assuming that the velocity varies linearly along the normal and the boundary is stationary, the desired velocity to be imposed on the ghost point is $V = -u_{image}$.

Validation

To validate the immersed boundary method with the linear interpolation, we conducted DNS on a test case of fibrous substrates. As shown in figure 2.16, the immersed boundary scheme introduces nearly zero-velocity regions in the flow field and these regions are confined within the solid boundary as expected. Inevitably, the scheme brings penetration error, i.e. non-zero velocities inside the solid phase. This could be explained by the pressure field fluctuates in solid regions due to the pressure correction step (2.59d), causing the non-zero velocity field (Fadlun et al., 2000). Nonetheless, figure 2.17 shows that the penetration velocity in wall units of our immersed boundary scheme is of $\mathcal{O}(10^{-2}) - \mathcal{O}(10^{-3})$, which is reasonably small compared to the surrounding velocity of $\mathcal{O}(1) - \mathcal{O}(10)$. Even for the worst scenario that happens in the first layer of fibers next to the interface $y^+ = 0$, the error is still negligible.

The scheme is validated when the grid mesh over the fibers is fine enough, because the linear interpolation is based on assuming the velocity varies linearly along the normal to the solid boundary, which is reasonable only in the vicinity of the boundary. The substrate region in the test case has resolution $\Delta z^+ \approx 0.8$, which is greater than any of that in our DNSs of use cases. We therefore consider current immersed boundary method working for simulations in Table 4.1, even if their grid meshes are different from case to case.

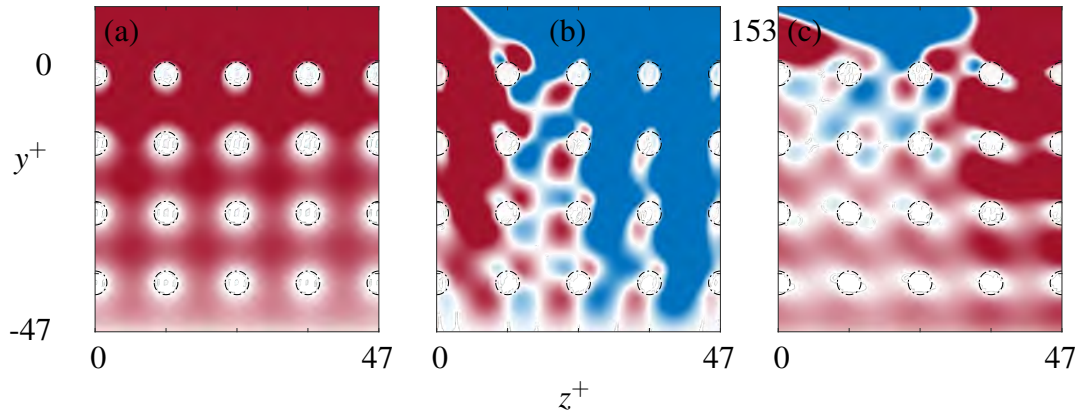


Figure 2.16 Instantaneous visualisations of velocity components: (a) streamwise $u^+ \in [-0.05, 0.25]$, (b) wall-normal $v^+ \in [-0.1, 0.1]$ and (c) spanwise $w^+ \in [-0.1, 0.1]$. The contours are colored with red, white and blue at their minimum, 0 and maximum values correspondingly, in between filled with color intensity gradient. Solid-dot lines are depicted overlying the flow field to mark the designed solid boundary. These snapshots are taken at a random time step and shows a portion of the whole bottom substrates in a cross-streamwise plane.

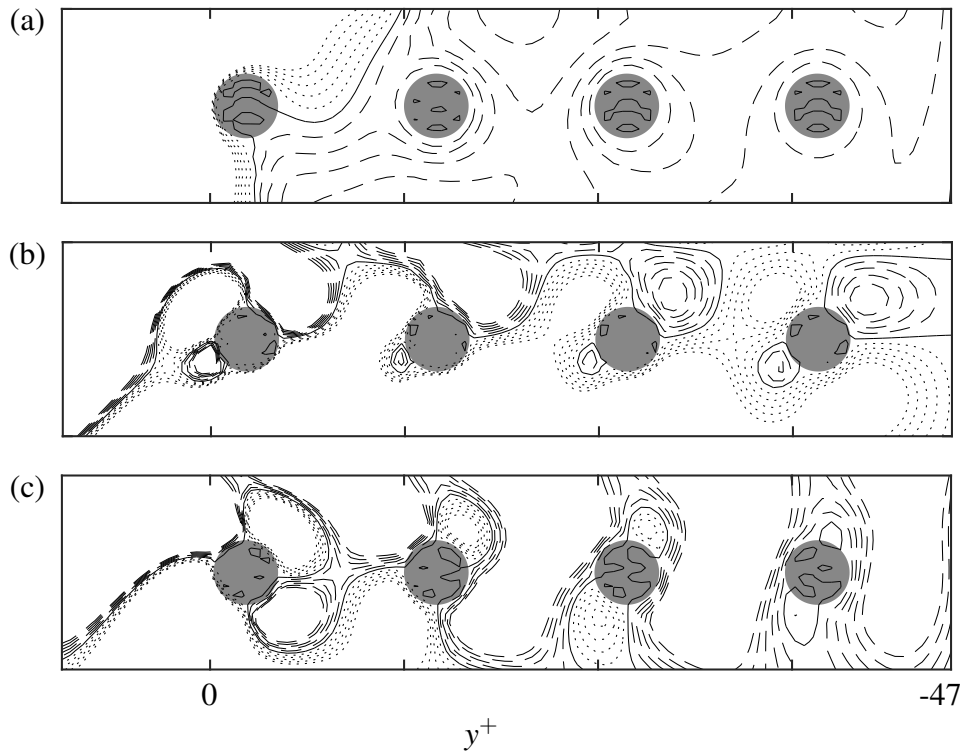


Figure 2.17 Line contours of velocity components: (a) streamwise u^+ , (b) wall-normal v^+ and (c) spanwise w^+ , scaled in viscous units. Zero, positive, negative values are depicted using solid, dotted, dashed lines respectively, equally spaced by 0.1 for u^+ , w^+ and 0.2 for v^+ . These snapshots are taken at a random time step and show a column of fibers in substrates in a cross-streamwise plane.

Chapter 3

Results of microscale calculations

We now discuss the results of the analyzes proposed in section 2.1. Firstly, we focus on pressure-driven flows, showing the results in terms of the velocity field, permeabilities and anisotropy ratio. Secondly, we analyze the shear driven flows within the substrate and at the substrate-free flow interface.

3.1 Permeability

Following the methodology proposed microscale calculations, we analyze different configurations of the permeable substrates, with the aim of understanding which of them can offer better properties in terms of drag reduction. From all the configurations considered, we report only those that help to get a better insight into the present work and those that give a high anisotropy ratio.

3.1.1 Fibers with a circular cross-section

The first configuration we examine is composed of fibers with a circular section, as it is one of the most studied in the literature. We first study the permeabilities in the streamwise and wall-normal directions. To do that, we solve the flow driven by a constant pressure gradient, obtaining the streamwise velocity component from equation (2.2), and the stream function and the spanwise velocity component from equations (2.6) and (2.4), as shown in figures 3.1 (a), (b) and (c). Then, we do a spatial average of the velocity components to find the macroscopic velocities within the substrate. Given that, we obtain the permeabilities, and we calculate the anisotropy ratio. We analyze how these parameters vary with θ , the porosity of the substrates defined as the ratio between the fluid volume and the total volume of the domain. Note that we solve for the length of domain sides equal to unity, so the permeabilities obtained are

dimensionless. We obtain the curves of figures 3.1 (d) and (e). For a high value of the porosity, i.e. a small radius of the circular cross-section, the anisotropy ratio is low, and it increases as porosity is reduced. This data suggests that configuration with lower porosity could be more suited for drag reduction applications. This evidence drives us in the development of cross-section that can offer better properties in terms of drag reduction.

3.1.2 Fibers with a squared cross-section

The second configuration that we analyze is another well-studied case. Following the same procedure of the previous case, we obtain the streamwise velocity component, the stream function and the spanwise velocity component, as shown in figure 3.2 (a), (b) and (c). Then, we portray in figure 3.2 (d) and (e) the permeabilities in the two directions and the anisotropy ratio obtained as a function of the porosity. Surprisingly, we can observe that the anisotropy ratio is small and almost constant with the variation of the porosity. Considering that the drag reduction is a function of the anisotropy ratio, this data suggests that the configuration under examination is not well suited for drag reduction application.

3.1.3 Fibers with a diamond cross-section

Subsequently, we decide to modify the previous configuration, changing the arrangement of the squared fibers, rotating the cross section of 90° degrees. The aim is to understand how this modification can effects the drag reduction properties of the substrate. Again, we obtain the velocity field, figure 3.3 (a), (b) and (c), and the curves in figure 3.3 (d) and (e). We achieve a better configuration in terms of drag reduction properties, with a much higher anisotropy ratio for low values of the porosity. These results are due to the fact that the diamond cross-section strangles the flow in the spanwise direction while leaving significant gaps for the streamwise flow.

3.1.4 Fibers with a tilted elliptical cross-section

From what we have learned in the previous cases, we decide to study some configuration that could potentially lead to a higher value of the anisotropy ratio. We focus now on fibers with elliptical cross-section and the axes tilted with respect to the wall-normal direction. We decide to study different cases, changing the characteristic dimensions of the cross-section and the angle of tilting. In figure 3.4 (a), (b) and (c) we report results for the case of fibers with elliptical cross-section with a semi-major axis equal to 0.470, a semi-minor axis equal to 0.025 and an angle of tilting, α , equal to $\pi/4$. Then in figure 3.4 (d) and (e) we plot the

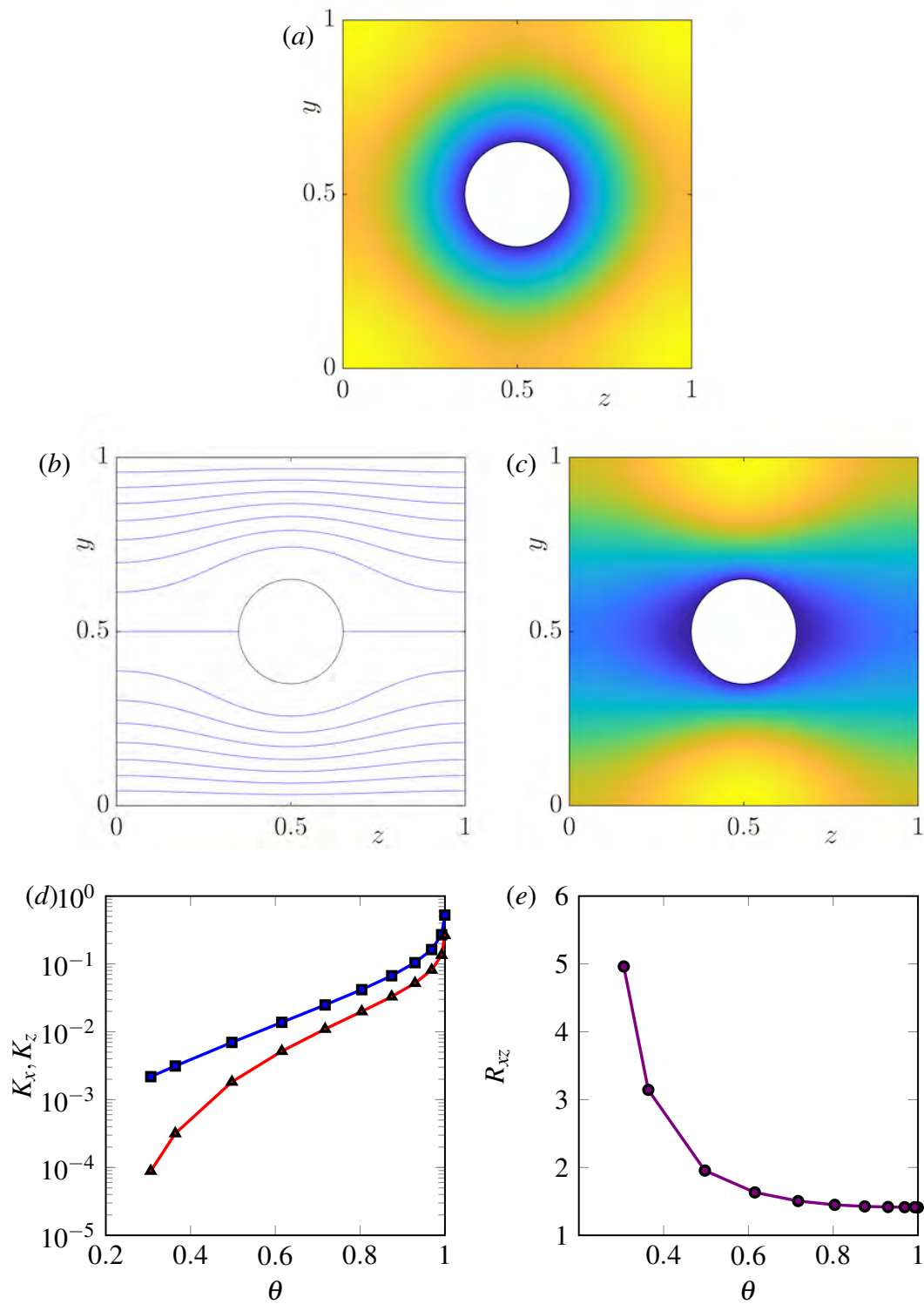


Figure 3.1 Flow within fibers with a circular cross-section: (a) streamwise velocity component, (b) stream function for the spanwise flow, (c) spanwise velocity component. Permeabilities versus porosity: (d), $\blacksquare$, streamwise permeability K_x , $\blacktriangle$, spanwise permeability K_z ; (e), $\bullet$, anisotropy ratio R_{xz} .

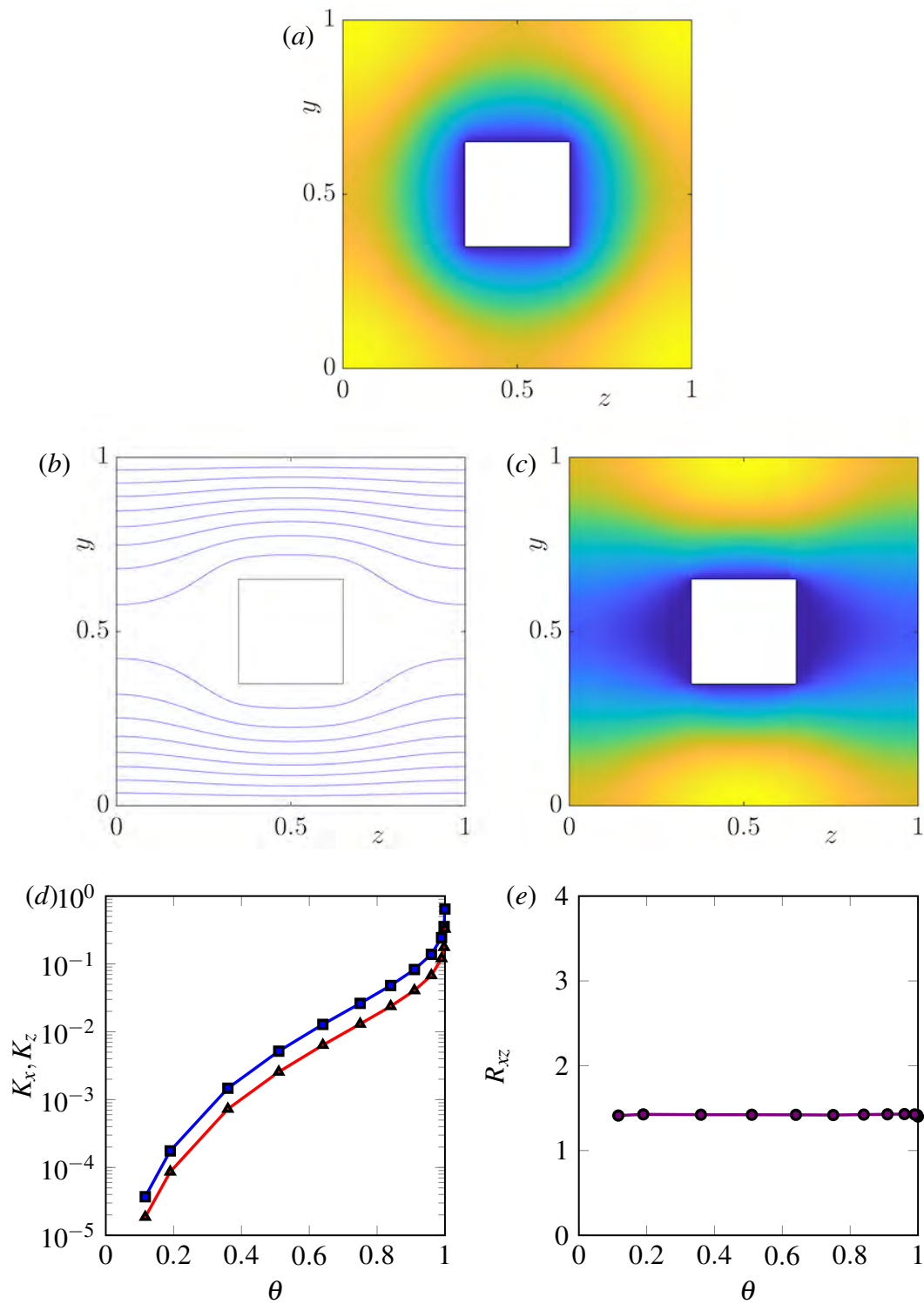


Figure 3.2 Flow within fibers with a squared cross-section: (a) streamwise velocity component, (b) stream function for the spanwise flow, (c) spanwise velocity component. Permeabilities versus porosity: (d), $\blacksquare$, streamwise permeability K_x , $\blacktriangle$, spanwise permeability K_z ; (e), $\bullet$, anisotropy ratio R_{xz} .

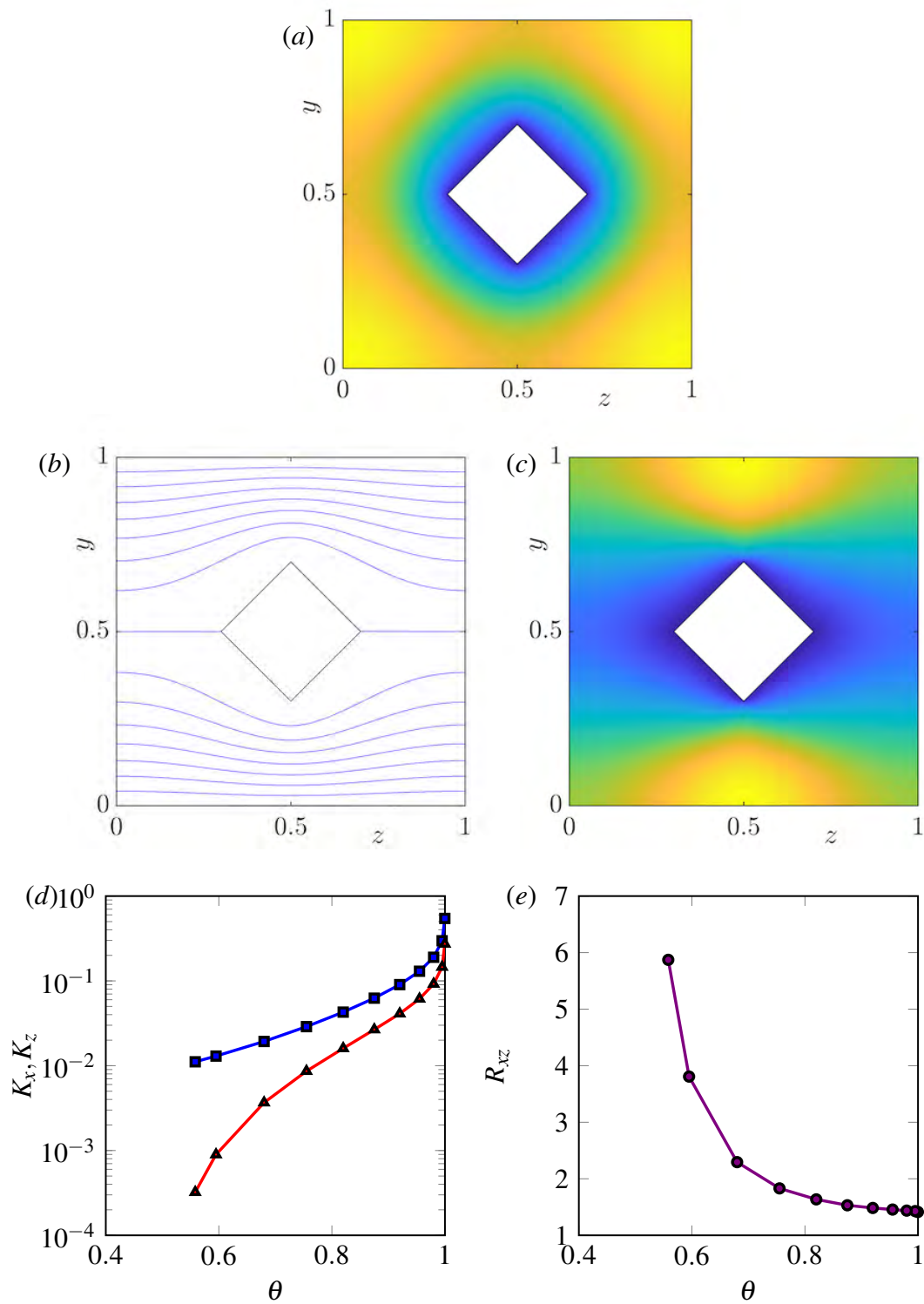


Figure 3.3 Flow within fibers with a diamond cross-section: (a) streamwise velocity component, (b) stream function for the spanwise flow, (c) spanwise velocity component. Permeabilities versus porosity: (d), $\blacksquare$, streamwise permeability K_x , $\blacktriangle$, spanwise permeability K_z ; (e), $\bullet$, anisotropy ratio R_{xz} .

curves of the permeabilities and the anisotropy ratio as a function of the porosity. Note that in order to change the porosity, we decide to modify the semi-major axis without changing the semi-minor axis. From these results, we observe a significant increase in the maximum achievable anisotropy ratio for low porosity configurations. This evidence reinforces the idea that we need to seek for configurations that offer high resistance to the flow in the spanwise direction, without strongly modifying the flow in the streamwise direction.

We then analyze how the permeabilities and the anisotropy ratios change with the angle of tilting. The configuration under examination is the same as the previous case. We report the results of this further analysis in figure 3.5. We note that the permeability in the streamwise direction does not significantly change with α . Conversely, as we could expect, the others permeabilities and the anisotropy ratios show a relevant variation with α . In particular, R_{xy} shows a peak for $\alpha = \pi/2$, and R_{xz} for $\alpha = 0$. These results lead us to the next configurations, where we design a more complicated arrangement of the fibers to achieve better drag reduction properties.

Array of fibers with a tilted elliptical cross-section in a closed arrangement

From the observations made in the previous case, we understood the needed of developing fibers that do not strongly modify the flow in the streamwise direction and at the same time can achieve high resistance to the cross-flow. In this framework, we analyze the following two configurations. The first one, is made by fibers with elliptical cross-section organized in a closed arrangement, as shown in figure 3.6. To obtain the curves of the permeabilities and the anisotropy ratio, we decide to take a constant semi-minor axis for all the ellipses, equal to 0.01, and let the semi-major axis discretely vary from 0.05 to 0.33. Looking at the curves in figure 3.6 (d) and (e), we observe that the anisotropy ratio reaches a peak value close to 12, much higher than all the others configurations previously investigated. This result means that the arrangement under examination can potentially offer much better properties in terms of drag reduction.

3.1.5 Fibers with an "X" cross-section

Following the same idea of the previous case, we now study a particular case in which the cross-section of the fibers is a cross, as shown in figure 3.7. To obtain the curves of figures 3.7 (d) and (e), we use a constant thickness equal to 0.0075, and we vary the semi-axis from 0.05 to 0.48. These curves clearly show the validity of our previous observations, supporting the necessity of designs that allow the fluid to flow easier in the streamwise direction than in the spanwise and wall-normal ones. In particular, to obtain a high anisotropy ratio, the

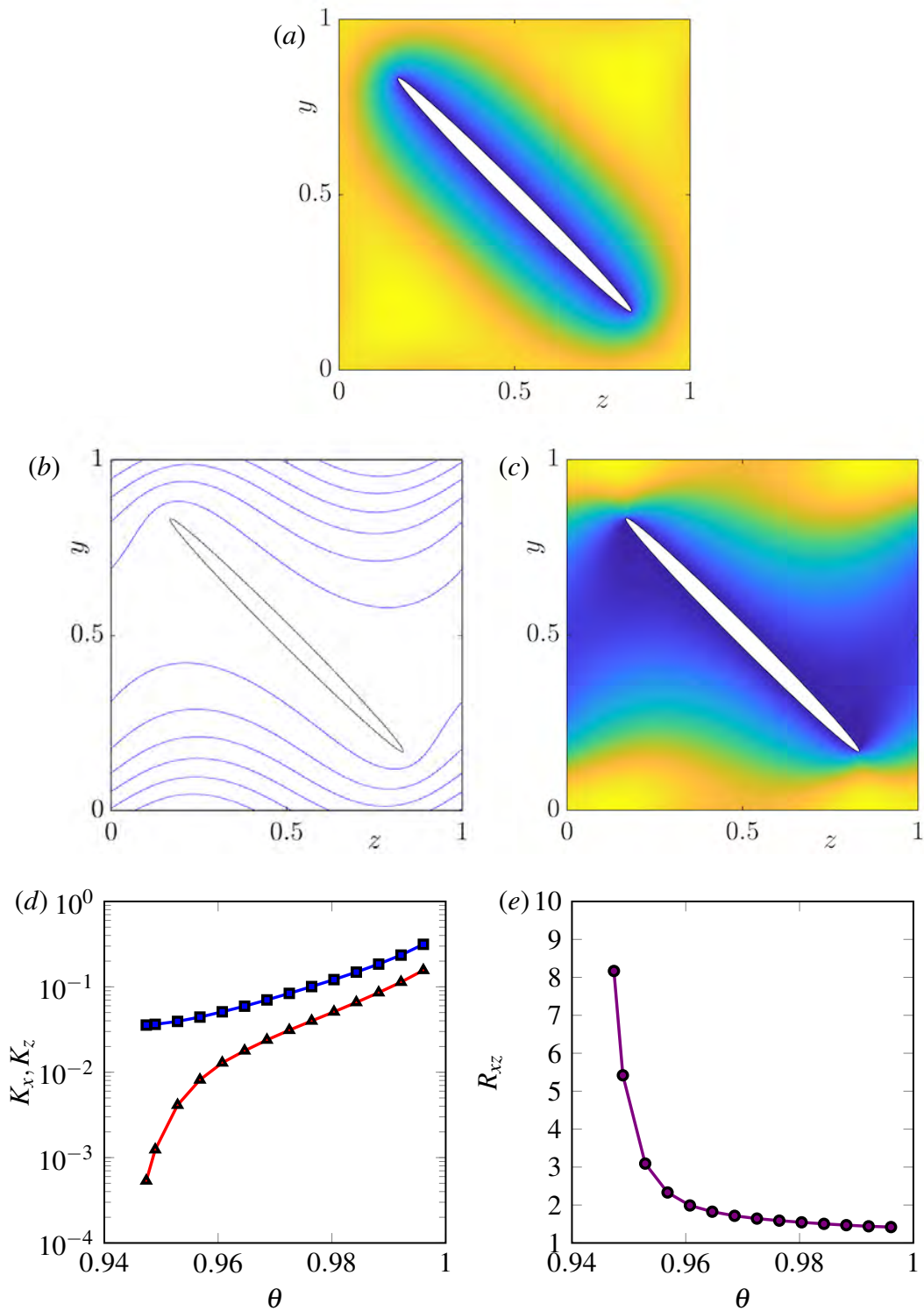


Figure 3.4 Flow within a permeable substrates made by fibers with a tilted elliptical cross-section: (a) streamwise velocity component, (b) stream function for the spanwise flow, (c) spanwise velocity component. Permeabilities versus porosity: (d), $\blacksquare$, streamwise permeability K_x , $\blacktriangle$, spanwise permeability K_z ; (e), $\bullet$, anisotropy ratio R_{xz} .

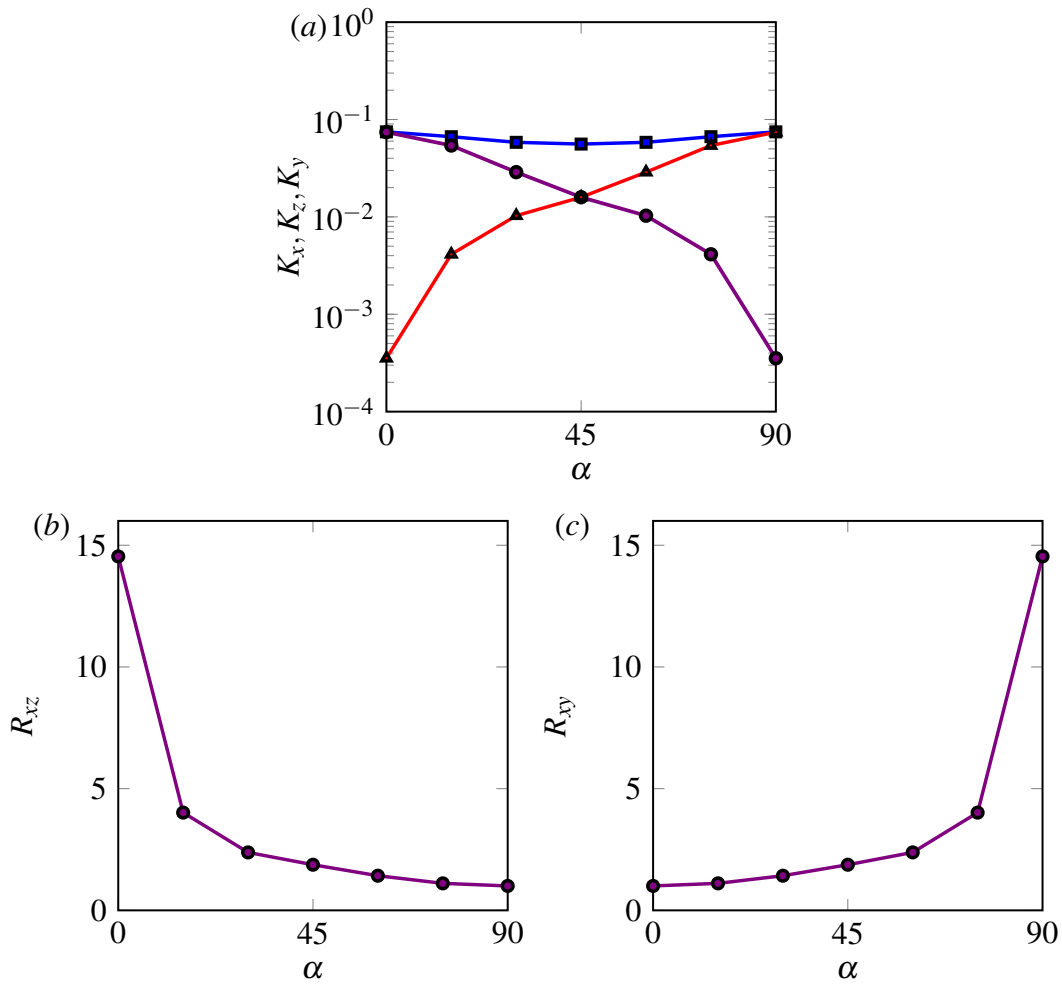


Figure 3.5 Permeabilities for an array of fibers with tilted elliptical cross-section versus angle of tilting: (a), $\blacksquare$, streamwise permeability K_x , $\blacktriangle$, spanwise permeability K_z , $\bullet$, wall-normal permeability K_y ; (b) anisotropy ratio R_{xz} ; (c) anisotropy ratio R_{xy} .

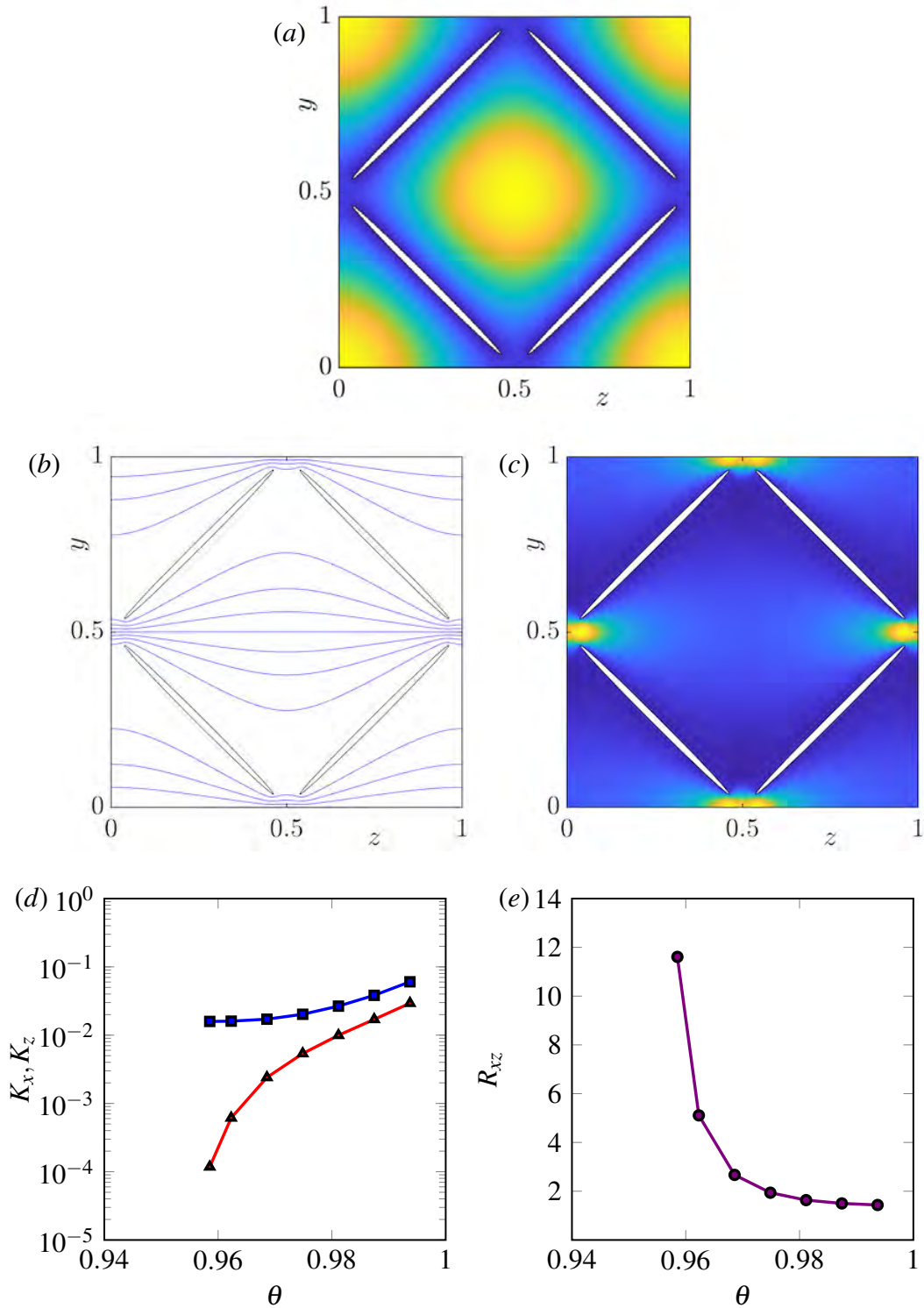


Figure 3.6 Flow within fibers with a tilted elliptical cross-section in a closed arrangement: (a) streamwise velocity component, (b) stream function for the spanwise flow, (c) spanwise velocity component. Permeabilities versus porosity: (d), $\blacksquare$, streamwise permeability K_x , $\blacktriangle$, spanwise permeability K_z ; (e), $\bullet$, anisotropy ratio R_{xz} .

configurations have to strangle the flow in the spanwise direction while leaving significant gaps for the streamwise flow. Moreover, the results show that it is possible to achieve an anisotropy ratio close to 15, higher than all the others case analyzed. As a consequence, this configuration appears to be the best suited for drag reduction application. However, it is also evident that this arrangement of the fibers is one of the most complicated ones, and it could be expensive to produce. Nevertheless, considering the goodness of the theoretical results, further studies on this kind of configurations would be worthwhile and could lead to the development of new arrays of fibers with higher potential to reduce the skin-friction drag.

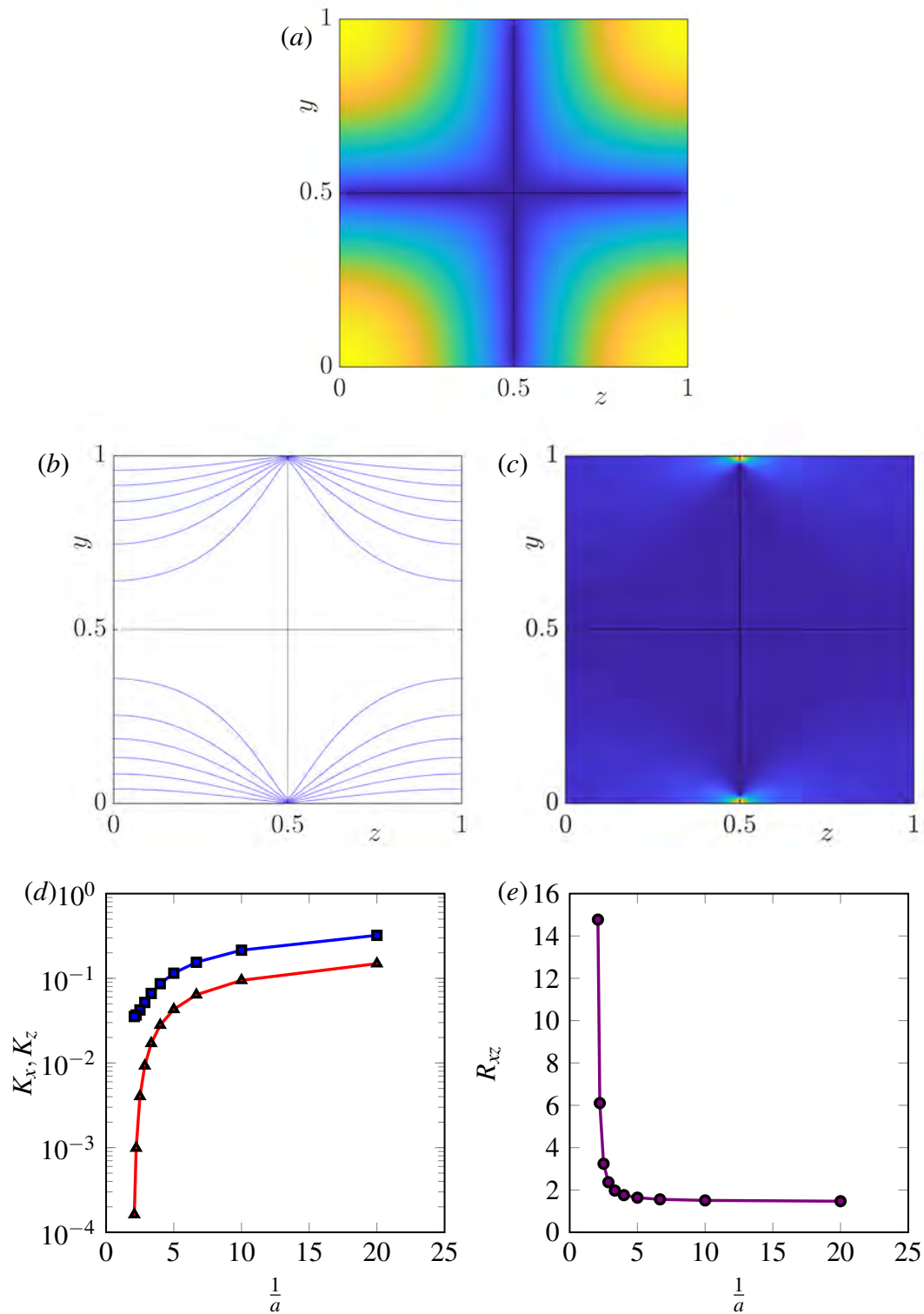


Figure 3.7 Flow within fibers with an "X" cross-section: (a) streamwise velocity component, (b) stream function for the spanwise flow, (c) spanwise velocity component. Permeabilities versus the inverse of semi-major axis: (d), $\blacksquare$, streamwise permeability K_x , $\blacktriangle$, spanwise permeability K_z ; (e), $\bullet$, anisotropy ratio R_{xz} .

3.2 Effective macroscopic viscosity

Following the methodology proposed in section 2.1, we now examine the flow in response to an applied shear at the substrate-fluid interface, with zero mean pressure gradient in the streamwise and spanwise directions, over a sufficiently high column of fibers. These analyzes aim to understand if Brinkman equation is a good model and if yes, to find the best fit value for the effective macroscopic viscosity. In this project, we investigate only the case of fibers with circular cross-section. However, further work is required to understand better the behaviour of shear-driven flows over the other configurations presented in the previous section.

3.2.1 Streamwise flow

We start from the analysis of the flow in the streamwise direction. As already mentioned, we consider as computation domain a sufficiently high column of fibers with a circular cross-section. We solve the Stokes equation (2.9) with a constant shear applied at the top surface. Consequently, we obtain the streamwise velocity component $u(y,z)$, as shown in figure 3.8 (a). Then, we do a spatial average in the spanwise direction, recovering the mean velocity profile. To compare our results with the analytical solution of Brinkman equation (2.11), we look at a region in the core of the flow, so that is not strongly influenced by the boundary conditions details. This region is visible in figure 3.9, where we portray the analytical solution of Brinkman equation with the best fit value for the macroscopic viscosity only in the core of the substrate over the mean velocity profile in logarithmic scale. We repeat this procedure for different values of fibers radius, and for each of them we try to obtain curves like the ones shown in figure 3.8 (b) and a value for the effective macroscopic viscosity. Doing this, we observe that our results show a good fit with the analytical solution of Brinkman equation only for a very high value of porosity (i.e. low value of radius). Conversely, for configurations with an intermediate or low value of porosity, the mean velocity profile cannot be fitted to the analytical solution without a high error. So, we argue that Brinkman equation is not a good model for the macroscopic velocities when the porosity is low. Focusing only on the configurations with a high value of porosity, we report in figure 3.10 the best fit values for the macroscopic viscosity in the streamwise direction. Surprisingly, we observe that the value of $\tilde{\nu}_x$ is very close to unity, and that is almost constant for all the substrates under consideration. So, we can note that it is reasonable assuming a macroscopic viscosity equal to one in the streamwise direction.

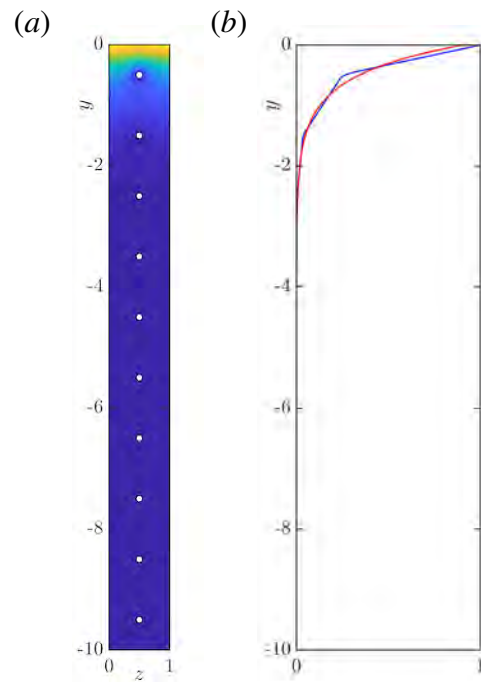


Figure 3.8 Flow within a column of fibers with a circular cross-section of radius $r = 0.05$ in response to an applied shear in the streamwise direction: (a) streamwise velocity component; (b), —, mean velocity profile in the streamwise direction, —, analytical solution of Brinkman equation with the best fit value of the effective macroscopic viscosity.

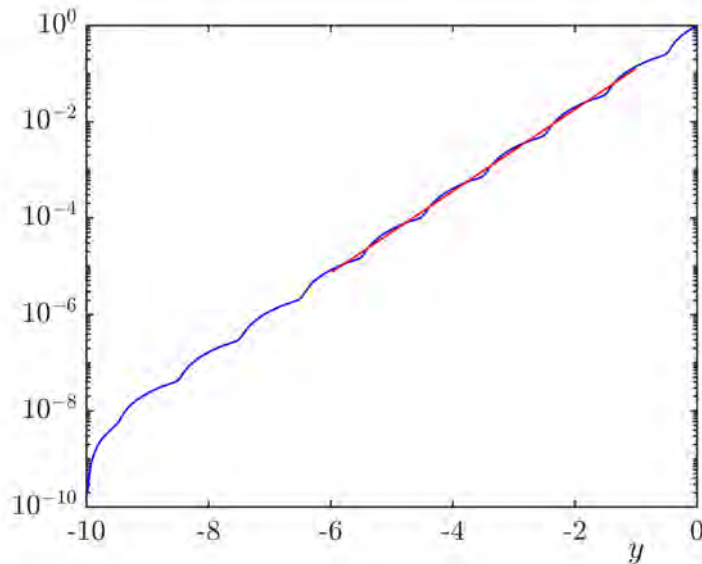


Figure 3.9 Flow within a column of fibers with a circular cross-section of radius $r = 0.05$: —, mean velocity profile in the streamwise direction, —, analytical solution of Brinkman equation. Ordinate in logarithmic scale.

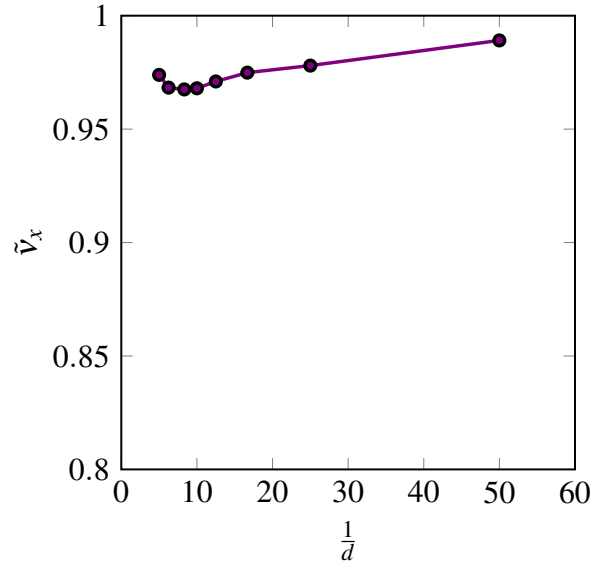


Figure 3.10 Effective macroscopic viscosity in the streamwise direction versus the inverse of the diameter of the circular cross-section.

3.2.2 Spanwise flow

We now investigate a shear driven flow in the spanwise direction over the same domain of the previous case. We solve the Stokes equation (2.14) and we obtain the spanwise velocity component $w(y, z)$ and the mean velocity profile, as shown in figure 3.11. Then, we follow the same procedure previously proposed to understand if Brinkman equation is a good model and if yes, to find the best fit value for the effective macroscopic viscosity in the spanwise direction. Again, we find that the analytical solution of Brinkman equation shows good agreement with our results only for substrates with a high value of the porosity. For those configurations that show a good fit with the analytical solution, we report the values of $\tilde{\nu}_z$ in figure 3.13. Firstly, we can note that the values of the effective macroscopic viscosity in the spanwise direction are different from the ones for the streamwise direction. Secondly, we observe that the macroscopic viscosity tends to one only for extremely small fibers, so we can state that the assumption of a macroscopic viscosity equal to one does not hold for spanwise flows. From these results, we can conclude that Brinkman model can be applied only to permeable substrates with very high porosity, and even then, the effective macroscopic viscosity needs tensional treatment. Finally, we point out the necessity of further studies to develop a better-suited model for shear driven flows over poorly connected substrates.

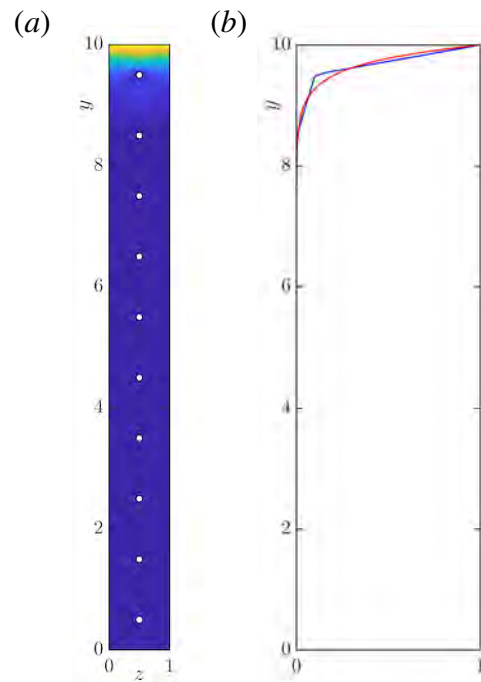


Figure 3.11 Flow within a column of fibers with a circular cross-section of radius $r = 0.05$ in response to an applied shear in the spanwise direction: (a) spanwise velocity component; (b), —, mean velocity profile in the spanwise direction, —, analytical solution of Brinkman equation with the best fit value of the effective macroscopic viscosity.

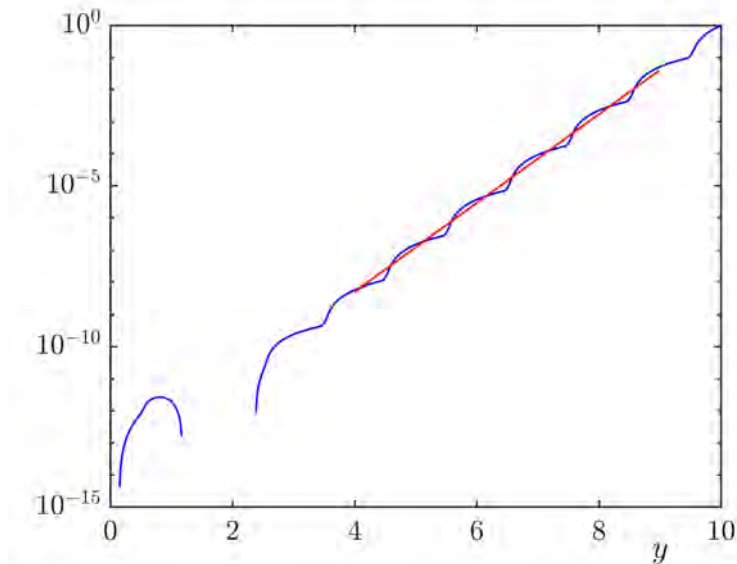


Figure 3.12 Flow within a column of fibers with a circular cross-section of radius $r = 0.05$: —, mean velocity profile in the spanwise direction, —, analytical solution of Brinkman equation. Ordinate in logarithmic scale.

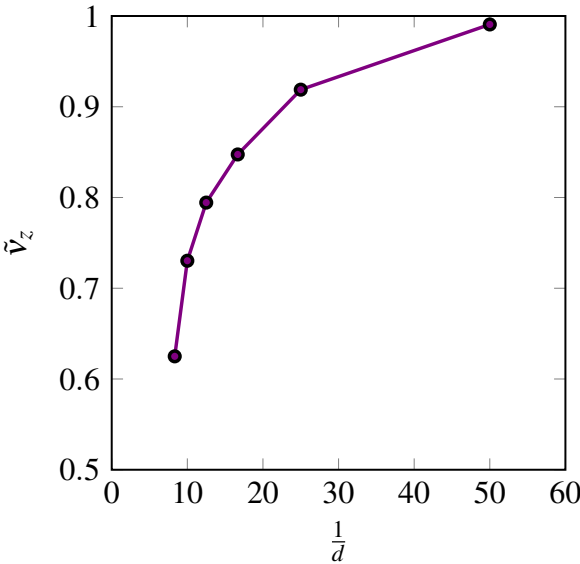


Figure 3.13 Effective macroscopic viscosity in the spanwise direction versus the inverse of the diameter of the circular cross-section.

3.3 Topology of the micro-scale spanwise flow

In a shear-driven flow through circular fibers of large radius, we find that the micro-scale flows display recirculating eddies, organized in a self-similar way along the wall-normal direction. Figure 3.14 shows the spanwise flow in three cases of fiber's radius of $r = 0.05$, 0.15 and 0.25 , labeled by (a), (b) and (c) respectively. There exists a critical value of radius, $r_{cr} \approx 0.06$, below which the flow is streamlined, e.g. in figure 3.14 (a1)-(a3). However, recirculation emerges once the radius exceeds r_{cr} and can be seen in the streamline contours of figure 3.14 (b1)-(b3) and (c1)-(c3). The recirculation is sensitive to the geometric parameter of the fiber, which is the radius in this case. Its vertical length scale decreases as the radius increases. For $r = 0.15$ in figure 3.14 (b), one recirculating zone spans four fibers, but for $r = 0.25$ in (c), the zone reduces to cover about two fibers. The recirculation results in flow reversal, whose absolute profiles are shown in blue curves in figure 3.14. The opposite flows could impose difficulties when applying volume averaging. Together with the corresponding mean spanwise velocity profile, we can also see that the intensity of the recirculation decreases exponentially in y , a general feature for small radii and streamwise cases. Additionally, the red-blue patterns also repeat themselves in y , suggesting the existence of a possible self-similar solution, which will be discussed in the next section.

For validation, those recirculation patterns are previously reported in numerical (James and Davis, 2001; Larson and Higdon, 1987) and experimental study (Tachie et al., 2003), but only for a limited range of fiber radii.

3.3.1 Self-similarity

Following the identification of the recirculating patterns, we now explore the self-similarity in the spanwise flow. Let the radius of the fiber be 0.2 . The streamlines are shown in Figure 3.15a, using a contour plot of the streamfunction ψ . Above the first fiber, a layer resembling a shear flow can be seen. In the core of the substrate, away from the boundaries at the top and bottom of the domain, the recirculation pattern observed in Figure 3.14 can be seen. This pattern appears to repeat every three fibers.

Figure 3.15d clearly shows the flow reversal by plotting the log of the magnitude of the spanwise-averaged velocity w , $\log |\bar{w}|$, against y : by showing the negative elements, in red, we can again see this repeating flow pattern over a range of three fibers. This suggests the possibility of a self-similar solution, which is important for the assumption that the permeability obtained using the pressure driven flow can be used. This permeability could then be applied for some volume average possibly over multiple fibers, in this case three, rather than a single fiber cell. This in turn may allow for the calculation of the effective macroscopic viscosity

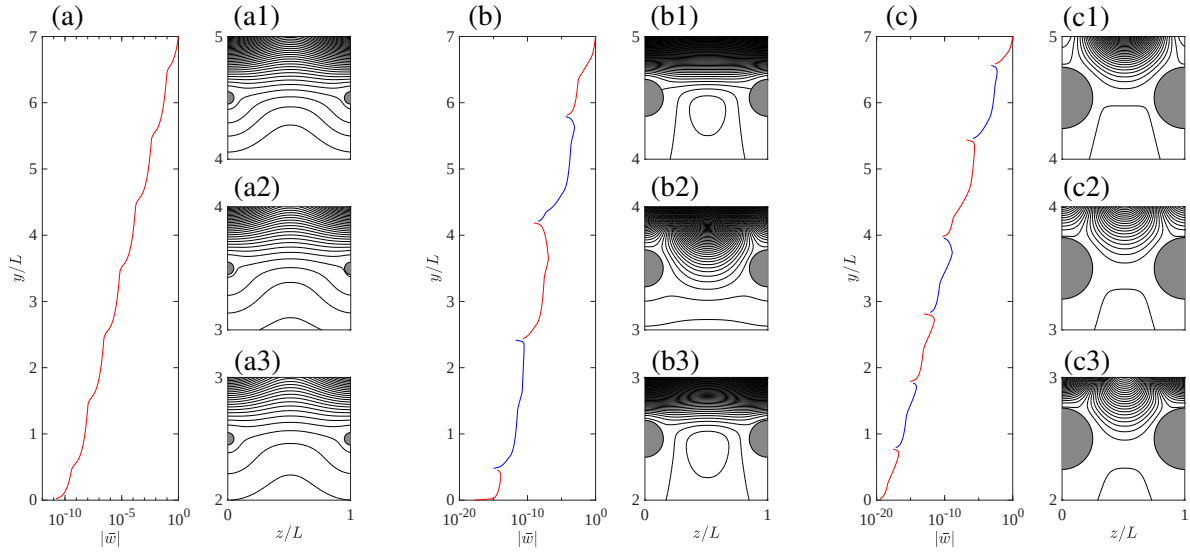
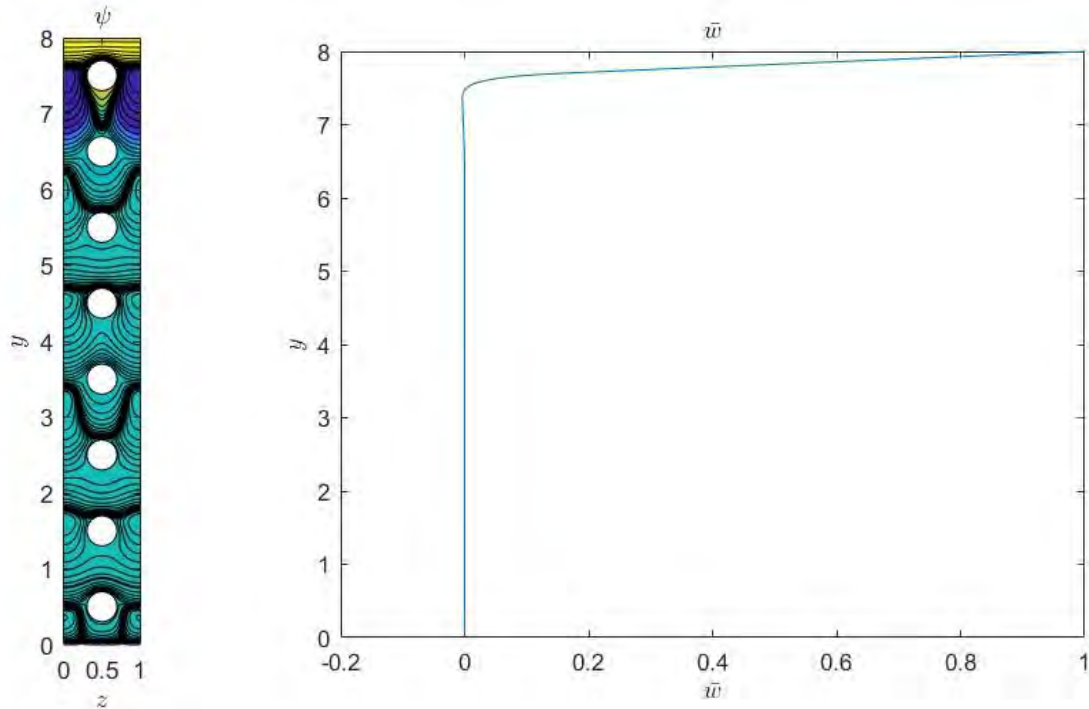


Figure 3.14 Micro-scale flow patterns for (a) $r = 0.05$, (b) $r = 0.15$ and (c) $r = 0.25$. In each case, the absolute value of the mean profile is depicted on the left and three sections are chosen to show the local streamlines. Streamlines in each unit cell are rescaled to show the details. The red curves indicate that the flow is along the same direction as the driving lid, while the blue curves indicate the opposite direction, where they are shown.

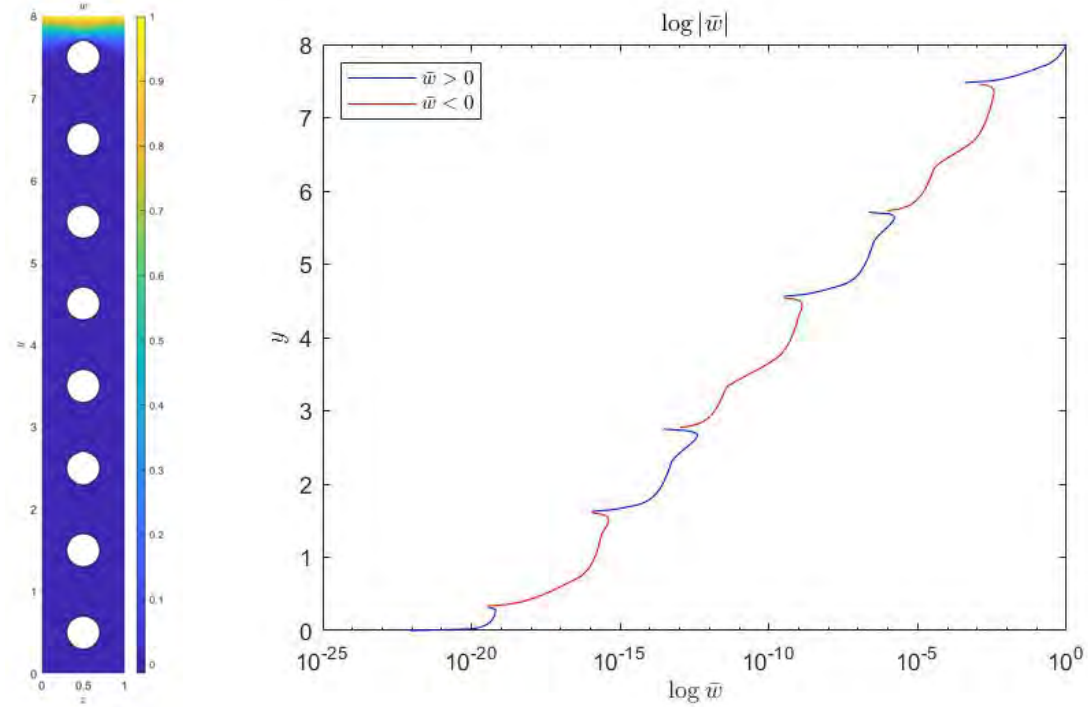
$\tilde{V}_z$, using the exponential decay of the self-similar solution. The boundary conditions would then be set by the streamfunction of the top and bottom fiber cells, which do not follow the self-similarity of the core.

Similar results can be obtained for a depth of eleven fibers, as seen in Figure 3.16; the same pattern is repeated in the streamlines, although the pattern is not visibly maintained deep into the plot of $\log \bar{w}$, as velocities are of the order of machine error.

Having observed this potential self-similarity, the flow field in the core of the substrate was studied using a column of three fibers, in order to emulate one repeat unit of three fibers. To perform this simulation, the boundary conditions at the top and bottom of the domain were replaced with quasi-periodic boundary conditions, such that the streamfunction decays by a pre-determined exponential between the top and bottom of the domain. This exponential decay is calculated in Section 3.3.1. The flow can be seen in Figure 3.17. The streamlines shown in Figure 3.17a closely resemble those of the columns of eight and eleven fibers; likewise, the variation of $\log \bar{w}$ with depth shown in Figure 3.17d appears to match with the cores of Figures 3.15d and 3.16d, reinforcing the idea that a self-similar solution is present.



(a) Streamlines

(b) Spanwise average of the spanwise velocity, $\bar{w}$ (c) Spanwise velocity w (d) $\log |\bar{w}|$ **Figure 3.15** Spanwise shear-driven flow through a column of eight fibers, of radius $r = 0.2$.

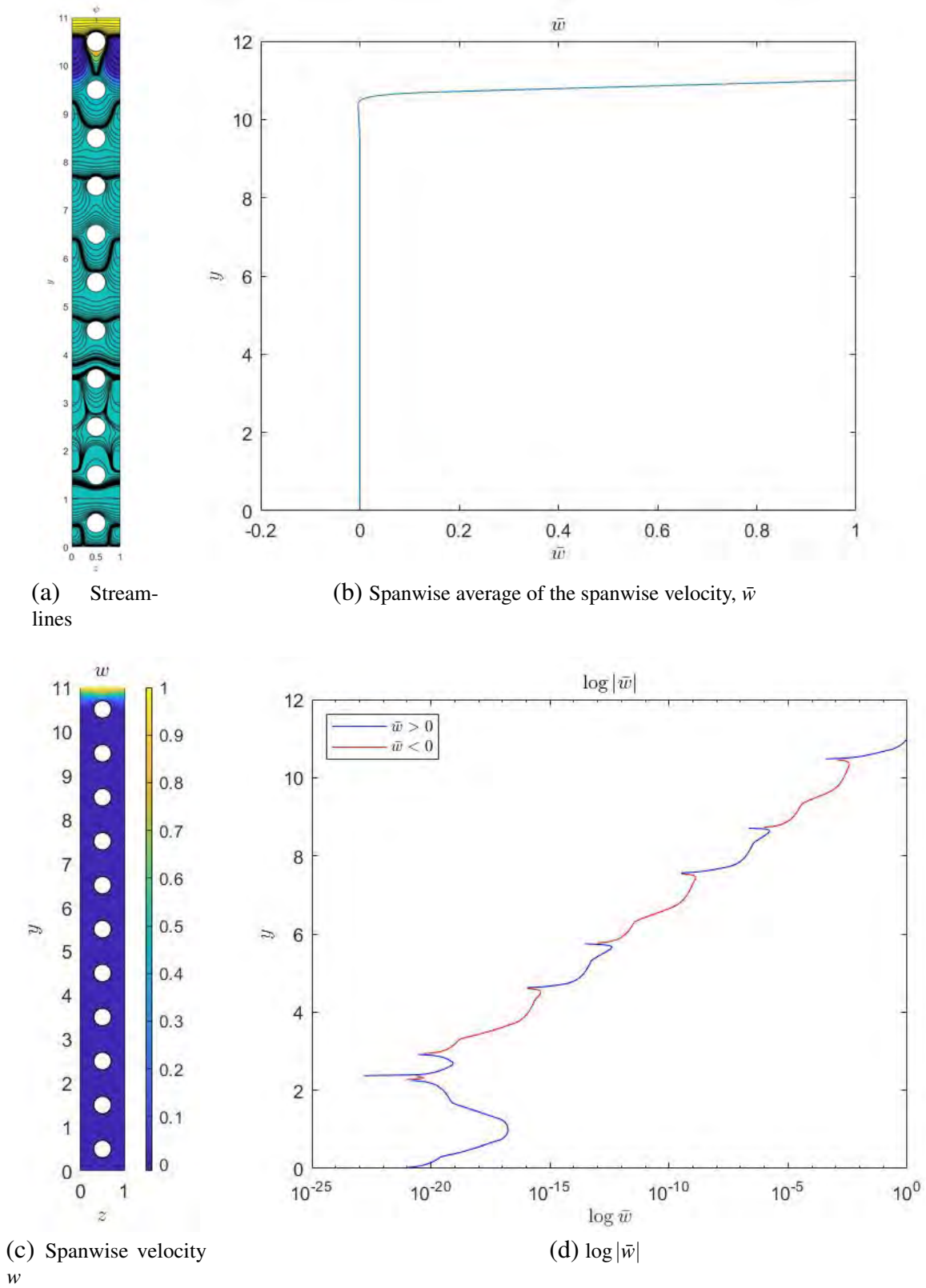


Figure 3.16 Spanwise shear-driven flow through a column of eleven fibres, of radius $r = 0.2$.

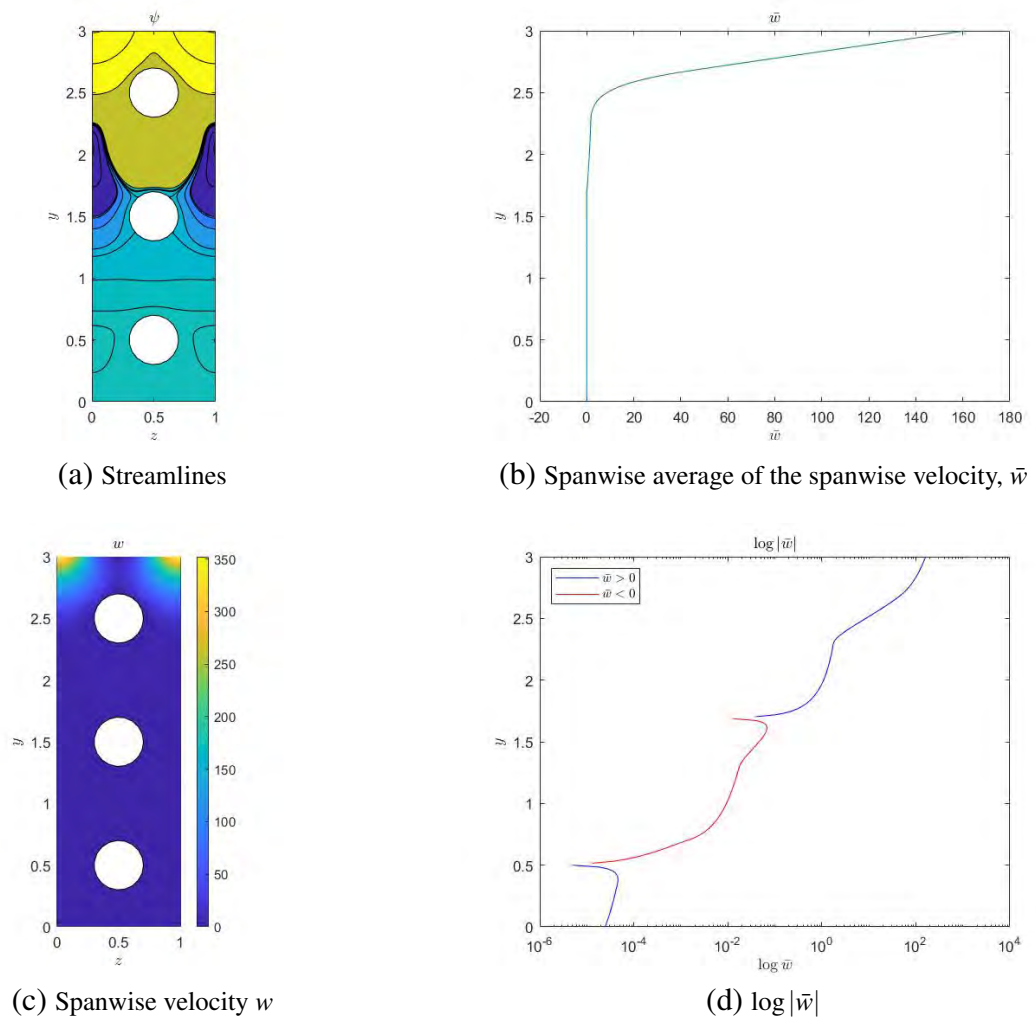


Figure 3.17 Spanwise shear-driven flow through a three fiber unit in the core of a substrate, of radius $r = 0.2$.

Velocity fluctuation about an exponential decay

We now look to decompose the solution for the column of eight fibers seen in Figure 3.15 into an exponential decay component, and a base function describing the variation of the streamfunction and spanwise velocity due to the fibers. This would allow for the simulation of much deeper substrates, where machine error would otherwise become significant in the calculation of the streamfunction, by repeating units. The spanwise averaged velocity therefore takes the form

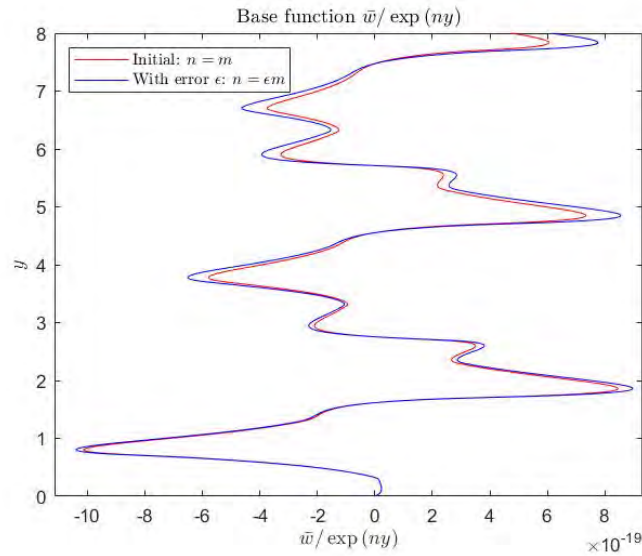
$$\bar{w} = \bar{w}_{base} \exp(ny) \quad (3.1)$$

where n is the rate of exponential decay.

We begin by calculating an initial estimate of the rate of exponential decay. We divide the magnitude of the spanwise-averaged velocity $\bar{w}$ at the top of the eighth fiber by that at the top of the second fiber, and take the log to obtain the value of n ; we call this initial rate m . We can then divide $\bar{w}$ by the exponential to obtain the base function $\bar{w}_{base}$, as shown by the red line in Figure 3.18a. The base function here appears close to a repeating unit, with two almost matching sections: one between $y = 1$ and $y = 4$, and one between $y = 4$ and $y = 7$.

We can introduce an error ε into the decay rate to try to correct for the differences between the two units, such that $n = \varepsilon m$; this is shown for values of ε between 0.99 and 1.0 in Figure 3.18b, where $\varepsilon = 1.0$ gives the initial estimate m . The optimal value of ε can be calculated by taking the root mean square of the difference between the two segments for each value of ε . The root mean square values are shown in Figure 3.19, giving an optimal error at the value where the rms is a minimum, around $\varepsilon = 0.994$. Using this to calculate the base function results in the blue line in Figure 3.18a, which has a closer match between repeating sections. The value obtained for the decay rate n is 5.2419, which was subsequently used to give the repeat unit shown in Figure 3.17.

Base functions for the streamfunction ψ and spanwise velocity w can be calculated similarly to the spanwise-averaged velocity $\bar{w}$; these are shown in Figure 3.20. These display the repeating pattern expected with self-similarity, and more clearly resolve values deep into the substrate, where these are too small to distinguish between in the full solution of Figure 3.15. In particular, the recirculation regions are visible in the streamlines of Figure 3.20a, whilst the flow reversal regions are more clearly displayed in Figure 3.20b, as values of w are of the same order of magnitude for each repeat unit.



(a) Initial base function, and base function corrected using the rms

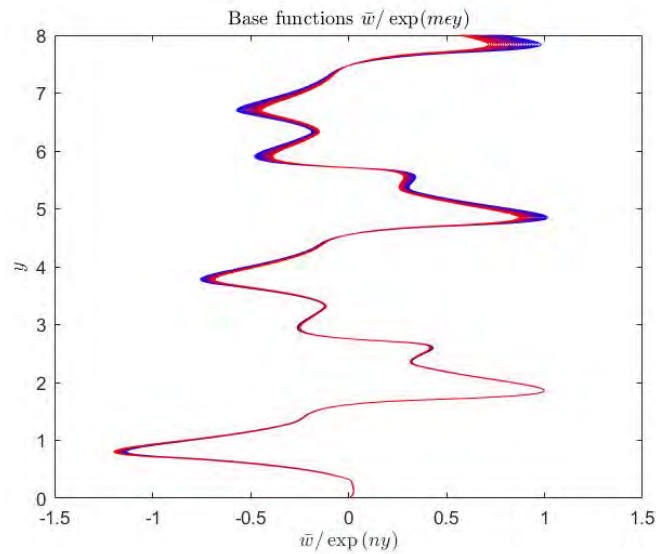
(b) Base functions for error ϵ varying between 0.99 and 1.0 (blue to red)

Figure 3.18 Decomposition of $\bar{w}$ into an exponential component, and a base function describing the shape of the distribution, for the spanwise shear-driven flow through a column of eight fibers, of radius $r = 0.2$.

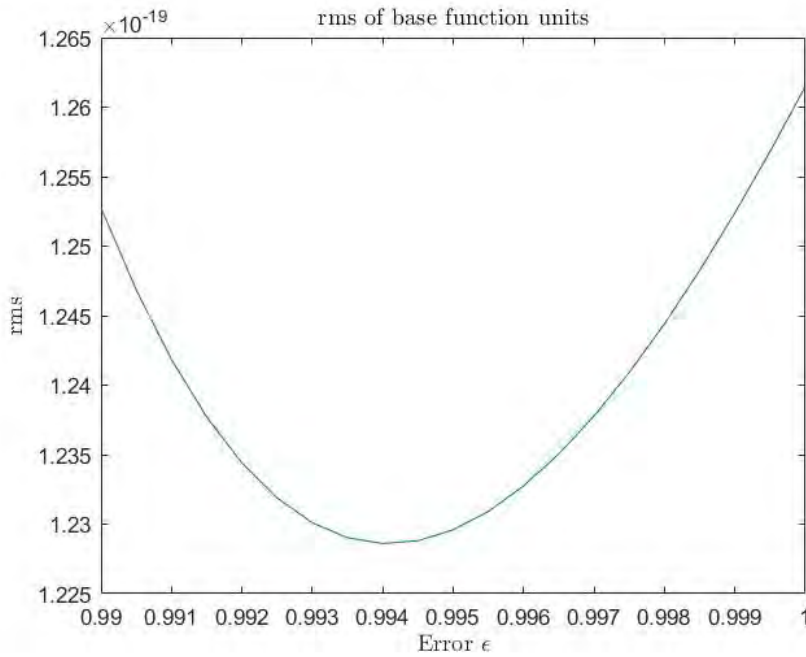


Figure 3.19 Root mean square (rms) values of the difference in w between the two repeat units present in the base function, for varying error ϵ

3.4 Coupling the inner and outer solution

In section 1.4.1, we pointed out that Brinkman equation is a volume averaging model, so it implicitly assumes the presence of a large number of fibers in any small volume within the substrate. However, approaching the interface between the substrate and the free flow region, that assumption would eventually cease to hold. Moreover, we reported that in the literature there is no general agreement on how to treat the substrate-fluid interface.

To get a better understanding of this problem, we now analyze the substrate-fluid interface, comparing the solutions of Brinkman equation, using the best fit values for $\tilde{v}$ presented in the previous sections, with the velocity profiles solution of Stokes' equations. Note that we limit our studies to cases in which the analytical solution of Brinkman equation shows a good agreement with the mean velocity profiles obtained from the solution of the Stokes equations, i.e. configurations with a high value of porosity.

3.4.1 Streamwise flow

To study the substrate-fluid interface in the streamwise direction, we solve the Stokes equations (2.9) on a domain in which half is free-flow, and half is filled with fibers. We obtain the streamwise velocity component $u(y, z)$, as shown in figure 3.21 (a). We do a spatial average in

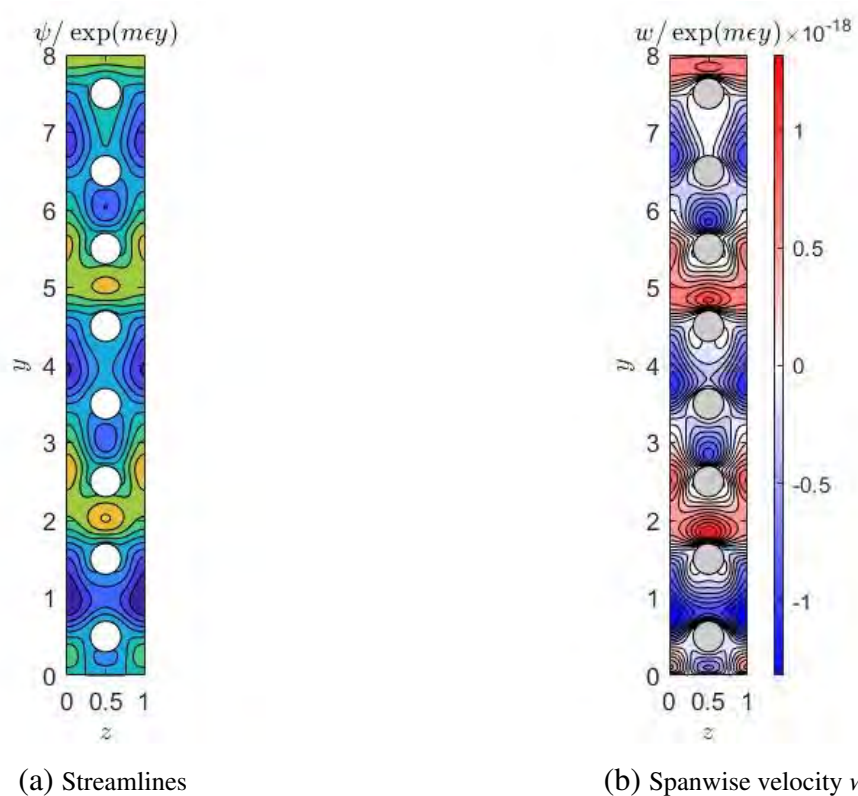


Figure 3.20 Streamlines and spanwise velocity w of the base function for the spanwise shear-driven flow through a column of eight fibers, of radius $r = 0.2$.

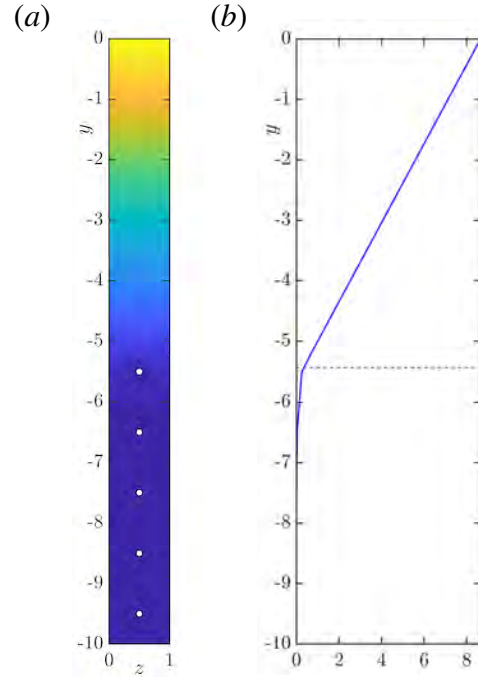


Figure 3.21 Flow over a column of fibers with a circular cross-section: (a) streamwise velocity component; (b) mean velocity profile in the streamwise direction.

the spanwise direction, recovering the mean velocity profile (figure 3.21 (b)). In agreement with the studies of [Abderrahaman-Elena and García-Mayoral \(2017\)](#) and [Gómez-de-Segura and García-Mayoral \(2019\)](#), we set the reference plane for the solid-fluid interface at the tip of last fiber. We assume that the analytical solution of Brinkman equation holds up to the reference plane, and we compare the two profiles at the reference plane, as shown in figure 3.21. Firstly, we analyze the results in terms of jump conditions that we would need to impose on the free flow using the Brinkman model in the substrates up until the interface. We define the jump in the velocity and the shear as follow:

$$\begin{cases} \Delta u|_{interface} = \bar{u}|_{interface} - \bar{u}_B|_{interface}, \\ \Delta S_x|_{interface} = \frac{d\bar{u}}{dy}|_{interface} - \frac{d\bar{u}_B}{dy}|_{interface}, \end{cases} \quad (3.2)$$

where $\bar{u}$ is the mean velocity component obtained from the solution of the Stokes equation and $\bar{u}_B$ is the analytical solution of Brinkman equation. So, we get the values reported in figure 3.23 and table 3.1.

From the values reported in the table, we can note that there is no continuity in the velocity neither in the shear, so we would need to impose jump conditions for both of them. Moreover, the analytical solution seems to over predict the slip velocity at the interface and to underestimate the shear. We have already quantified the shear and velocity jump at the

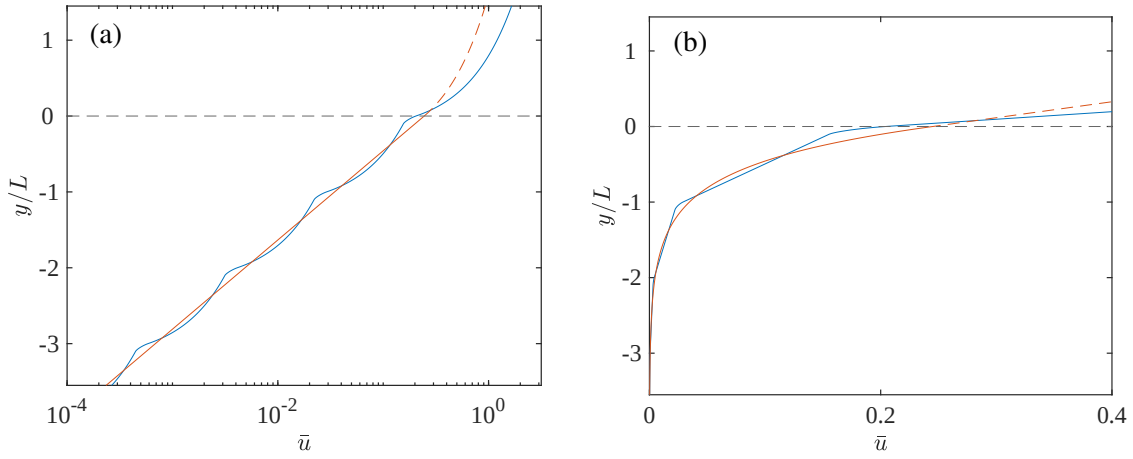


Figure 3.22 Spanwise-averaged velocity profile (blue) near the interface (dashed line), compared with the resulting macro-scale Darcy-Brinkman profile (red).

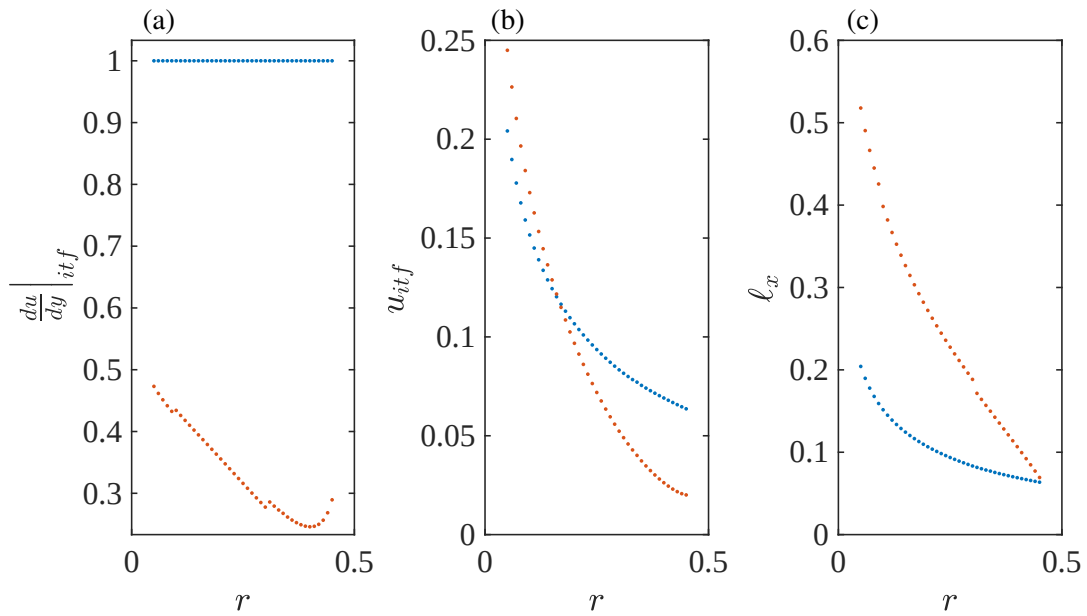


Figure 3.23 Interfacial jump conditions. Blue, fully resolved simulation; red, Brinkman model.

R	Δu	ΔS_x
0.06	-0.0521	0.8285
0.05	-0.0503	0.7980
0.04	-0.0647	0.7949
0.03	-0.0628	0.7596
0.02	-0.0908	0.7108
0.01	-0.0708	0.6928

Table 3.1 Jump conditions at the substrate fluid interface in the streamwise direction.

interface for circular fibers, but for its explanation, we may need to take the interior flow into consideration. Figure 3.24 (a) maps the ratio of shear below and above one fiber, i.e. $\mathbb{S} = du/dy|_{\text{below}}/du/dy|_{\text{above}}$, for different row of fibers and different radius. For one radius, i.e. one curve in the figure, the ratio is constant starting from the row below the one next to the interface, from $y/L = -1$ to $y/L = -5$, supporting the idea that Brinkman reflects fairly the flow in that region. However, the ratio across the outermost row or the interface (at $y/L = 0$) differs from its inner values. Since Brinkman equation accurately describes the flow in the middle of the substrate, we assume it also predicts correctly the ratio of shear across fibers there and takes it as reference. Normalize the ratio at the interface by the reference value producing a law for the jump condition of shear, depicted in figure 3.24 (b). It allows us to correct the Brinkman solution without running a fully resolved simulation. It should be noted that this law is only appropriate for circular fibers.

Now we examine the slip length in accordance with its analytical form derived in Gómez-de-Segura and García-Mayoral (2019). Consider a channel partially filled with an array of fibers, above which is the free-flow region. The flow is driven by the top wall moving at a constant speed. A nominal interface is defined at the tip of the outermost layer of fibers (dashed line in figure 3.25). Note that the position of the nominal interface is defined in this way as a convention, but we shall see later that the position does not affect our analysis on the jump conditions. The fully resolved solution of the streamwise flow is shown in the blue curve in figure 3.25. Since we can obtain the permeability and effective viscosity for the same substrate, we subsequently calculate the solution to the Brinkman equation up until the interface, assuming this solution couples the overlying flow with the same amount of shear stress. The resulting Brinkman solution is sketched in the red curve in figure 3.25. As observed, the slip length is over-predicted by the Brinkman solution. It deviates from the micro-scale solution near the interface, roughly over a depth of order L , resulting in the apparent differences in slip velocity and slip length reported from these two solutions.

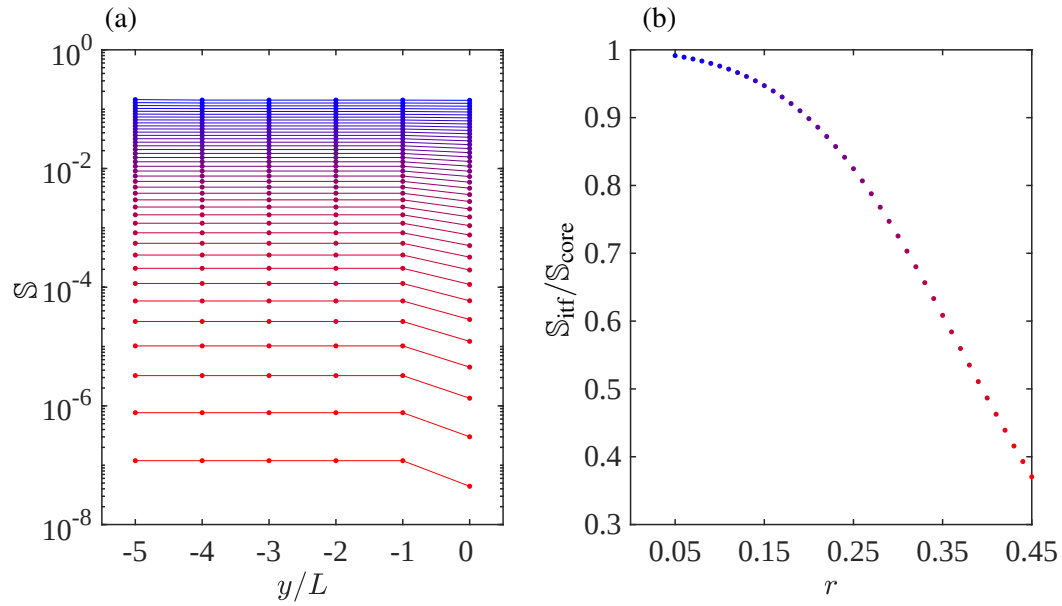


Figure 3.24 (a) Ratio of shear below and above each row of fibers. The data for fibers next to the solid bottom walls are excluded. (b) Ratio of shear at the interface normalized by the ratio in the middle of the substrate. From blue to red, the fiber radius increases.

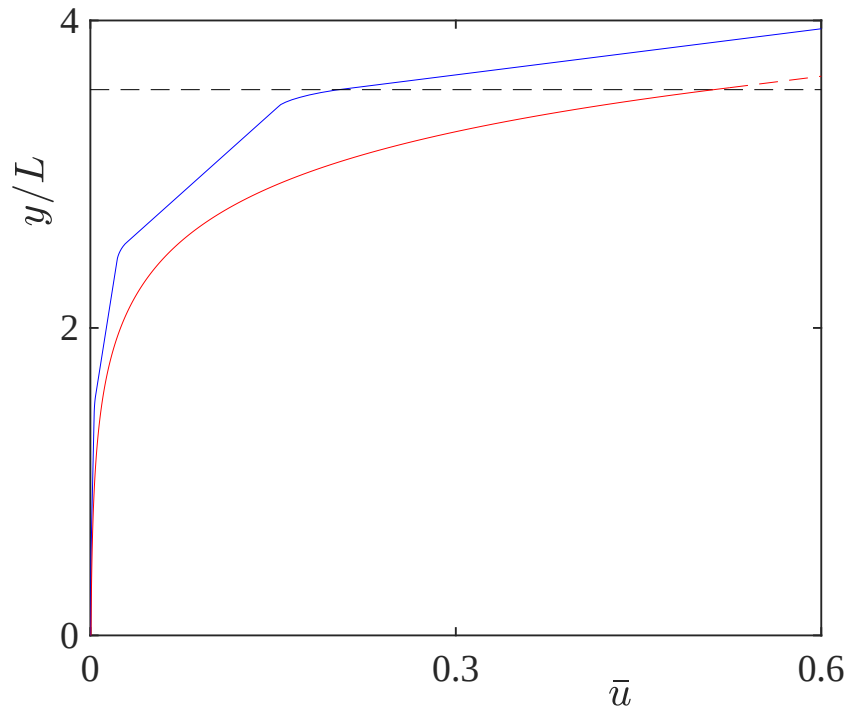


Figure 3.25 Spanwise-averaged velocity profile (blue) near the interface (dashed line), compared with the resulting macroscale Darcy-Brinkman profile (red). fiber radius $r = 0.05$.

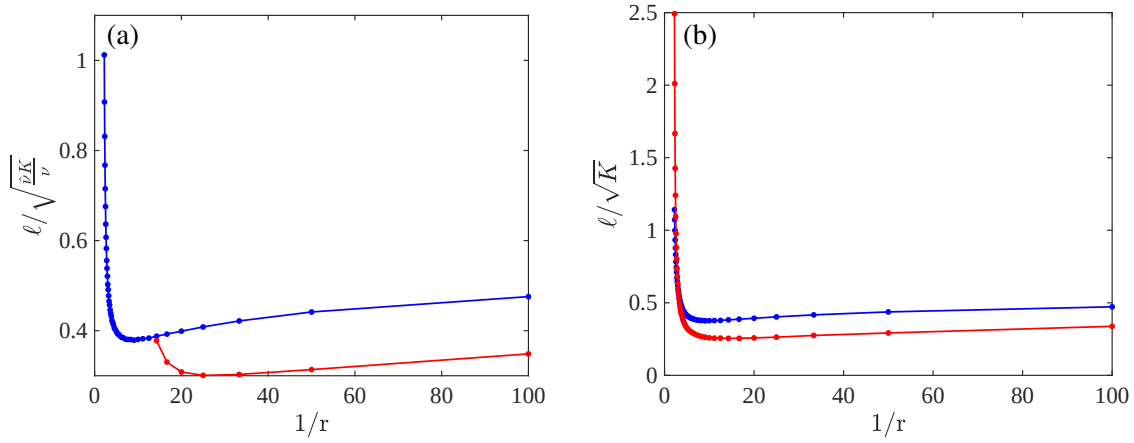


Figure 3.26 (a) Ratio of the actual slip length to the Brinkman prediction. (b) Ratio of slip length assuming $\tilde{\nu} \approx \nu$ (Abderrahaman-Elena and García-Mayoral, 2017). Blue, streamwise; red, spanwise.

We conducted a parametric study for fiber of radius ranging from 0.01 to 0.48. The ratio of the actual slip length to the Brinkman prediction are summarized in figure 3.26 (a). For fine and moderate fibers, the streamwise slip length is approximately 50% of that produced by Brinkman equation. The spanwise ratio for small fiber, $r < 0.07$, appears to be less than the streamwise one, below 40%. Note that in the spanwise direction, we cannot formally obtain the effective viscosity due to the presence of recirculation, so that the curve is not complete. Assuming that $\tilde{\nu} \approx \nu$, the ratio for spanwise direction can then be recovered for the whole range, as shown in figure 3.26 (b).

3.4.2 Spanwise flow

We now analyze the substrate-fluid interface in the spanwise direction. We follow the same procedure presented in the previous section. We obtain the spanwise velocity component and the mean velocity profile, shown in figure 3.27, and we compare it with the Brinkman solution in figure 3.28. Then, we define:

$$\begin{cases} \Delta w|_{interface} = \bar{w}|_{interface} - \bar{w}_B|_{interface}, \\ \Delta S_z|_{interface} = \frac{d\bar{w}}{dy}|_{interface} - \frac{d\bar{w}_B}{dy}|_{interface}, \end{cases} \quad (3.3)$$

where $\bar{w}$ is the mean velocity component obtained from the solution of Stokes' equation and $\bar{w}_B$ is the analytical solution of Brinkman equation. Consequently, we obtain the values reported in table 3.1. Once again, we see that there is no continuity in velocity neither in shear and that the analytical solution over predicts the slip velocity at the interface, meanwhile it underestimate

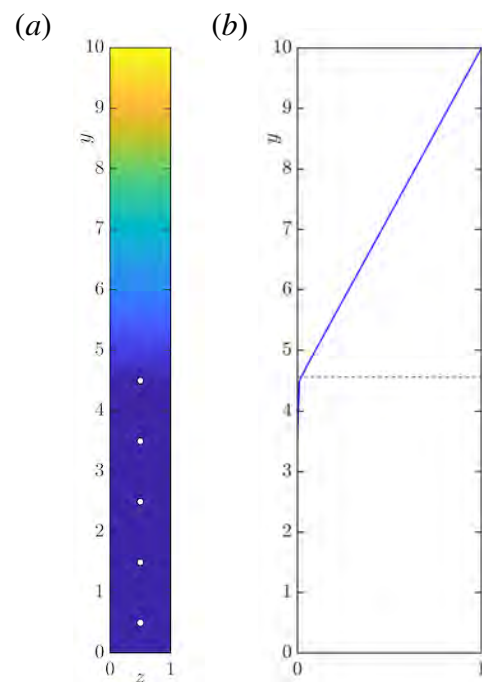


Figure 3.27 Flow over a column of fibers with a circular cross-section: (a) spanwise velocity component; (b) mean velocity profile in the spanwise direction.

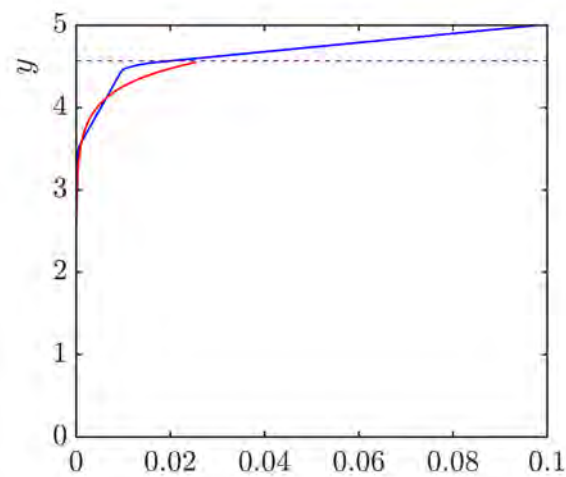


Figure 3.28 Detail of velocity profiles up to the substrate-fluid interface: —, mean velocity profile in the spanwise direction, —, analytical solution of Brinkman equation.

R	Δw	ΔS_z
0.06	-0.0126	0.0608
0.05	-0.0083	0.0941
0.04	-0.0112	0.0791
0.03	-0.0103	0.0701
0.02	-0.0105	0.0891
0.01	-0.0110	0.0560

Table 3.2 Jump conditions at the substrate fluid interface in the spanwise direction.

R	ℓ_z	$\ell_z/\sqrt{K_z}$
0.06	0.0918	0.2644
0.05	0.0991	0.2700
0.04	0.1117	0.2863
0.03	0.1305	0.3121
0.02	0.1444	0.3171
0.01	0.2240	0.4360

Table 3.3 Slip length in the spanwise direction.

the shear. We can then conclude that the jump conditions are needed for all the component of velocity and shear.

Finally, we look at the mean spanwise velocity component alone, and we determine the slip length and the ratio between ℓ_z and the square root of permeability, $\sqrt{K_z}$. We obtain the quantities presented in table 3.3 and in figure 3.29. We find again that all the parameters increase as the radius of fibers decreases. Besides, we find that the ratio between the slip length and the permeability in the spanwise direction is always less than 0.5 and, most importantly, it is always lower than the one in the streamwise direction. This last evidence leads to a small increase in the maximum achievable drag reduction compared to that suggested by the analysis of the streamwise case alone.

3.5 Contributions to the slip length

The linear relation of slip lengths and ΔU^+ (1.15) is valid for flows with zero or mild pressure gradient, such that the flow near the wall is essentially driven by the overlying shear and the effect of the mean pressure gradient within the permeable substrate is negligible (Gómez-de-Segura and García-Mayoral, 2019). This is usually considered true for boundary layer flows over flat surfaces, where our prediction of ΔU^+ is found applicable. However, the effect

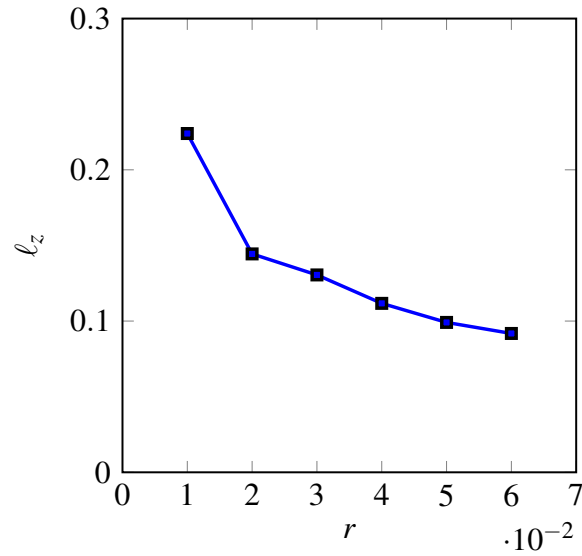


Figure 3.29 Analysis of the substrate-fluid interface in the spanwise direction: $\blacksquare$, slip length.

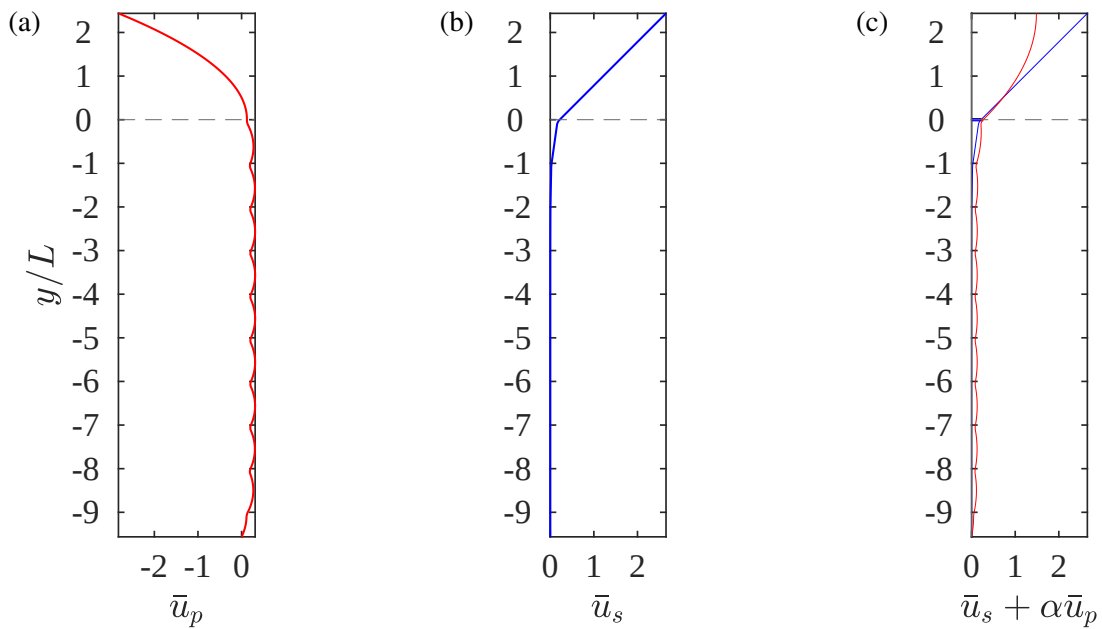


Figure 3.30 Decomposition of flow into pressure and shear driven parts. fiber radius $r = 0.05$.

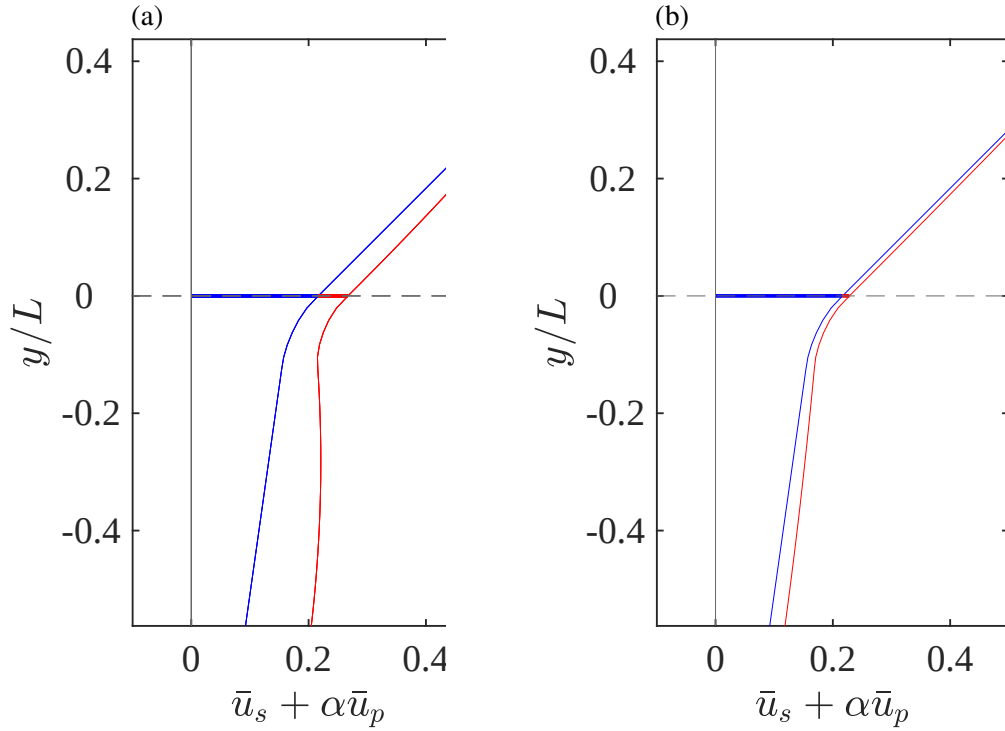


Figure 3.31 Contributions of shear (blue) and pressure (red) to the slip length. (a) Height of the free-flow region is 20% of the substrate. (b) Height of the free-flow region is 100% of the substrate.

of pressure gradient could become important in some internal scenarios, for example pipe flow. To better understand to what extent does the pressure gradient act on external flows and consequently affect the slip length, we consider separating the pressure part from a linear problem, leaving another part containing only the effect of shear. Let the domain be an open channel bounded with the permeable substrate at the bottom wall. The streamwise flow is described by the Stokes equation (2.1) with a free slip boundary condition at the top

$$\tilde{u}_y|_{\text{top}} = 0. \quad (3.4)$$

We decompose the flow into a shear-driven and a pressure-driven part

$$\nu \nabla^2 u = 0, \quad u_y|_{\text{top}} = S, \quad (3.5a)$$

$$\nu \nabla^2 u = p_x, \quad u_y|_{\text{top}} = 0. \quad (3.5b)$$

Since we want to fully decouple their effects on the slip length, the pressure-driven component (3.5b) should have zero shear at the interface. Meanwhile the shear-driven part (3.5a) should have zero pressure gradient at the interface which is automatically satisfied, as shown in figure

3.30 (b). To generate a flow that meets (3.5b), we consider the following two cases

$$\nu \nabla^2 u_1 = \tilde{p}_x, \quad u_y|_{\text{top}} = S_1, \quad (3.6a)$$

$$\nu \nabla^2 u_2 = \tilde{p}_x, \quad u_y|_{\text{top}} = S_2. \quad (3.6b)$$

Solving for their corresponding shear at the interface yields

$$u_y|_{\text{itf}} = \tilde{S}_1, \quad u_y|_{\text{itf}} = \tilde{S}_2. \quad (3.7)$$

Define a linear combination of u_1 and u_2 by

$$u_0 = u_1 - \frac{\tilde{S}_1}{\tilde{S}_2} u_2, \quad (3.8)$$

such that the zero pressure gradient condition at the interface, $u_{0y}|_{\text{itf}} = 0$, is recovered. Consequently, the pressure-driven flow is described by

$$\nu \nabla^2 u_0 = p_x, \quad u_{0y}|_{\text{top}} = \frac{S_1 - \frac{\tilde{S}_1}{\tilde{S}_2} S_2}{1 - \frac{\tilde{S}_1}{\tilde{S}_2}}, \quad (3.9)$$

as shown in figure 3.30 (a). Then adding flows in figure 3.30 (a) and (b) with a weighting coefficient α to recover the condition (3.4) results in the final solution, shown as the red curve in figure 3.30 (c). Figure 3.31 shows the decomposed slip length of two cases: (a) a small free-flow region compared to the substrate region; (b) a moderate one which has the same height as the substrate. It can be seen that the contribution of pressure is relatively small compared to the shear and becomes insignificant with the growth of thickness of the boundary layer. In a large scale domain with very a thin layer of substrate, the pressure contribution is likely to be smaller than the shear part even by one order of magnitude. These results are in agreement with the theoretical analysis of Bottaro (2019).

3.6 Drag reduction estimation

Recall that, for vanishingly small permeabilities, the drag reduction can be estimated by a linear relation

$$\Delta U^+ = \ell_x^+ - \ell_z^+, \quad (3.10)$$

where ℓ_x^+ and ℓ_z^+ are the slip lengths in streamwise and spanwise direction respectively (García-Mayoral et al., 2019; Luchini et al., 1991). It suggests that the drag reduction is determined

by the difference between slip lengths in two directions. In the linear regime, drag reduction increases with ΔU^+ until the breakdown point. [Gómez-de-Segura and García-Mayoral \(2019\)](#) reported that the breakdown depends solely on the wall-normal permeability, which is universally $\sqrt{K_y^+}|_{\text{opt}} = 0.38$ for all permeable substrates. The maximum ΔU^+ can be estimated by

$$\Delta U_{\max}^+ = 0.8 \sqrt{K_y^+}|_{\text{opt}} \frac{\ell_x - \ell_z}{\sqrt{K_y}}, \quad (3.11)$$

where the coefficient 0.8 corrects this linear prediction to match the DNS results ([Gómez-de-Segura and García-Mayoral, 2019](#)). [Abderrahaman-Elena and García-Mayoral \(2017\)](#) derived the slip lengths of porous materials by analytically solving the Brinkman equation. Based on the assumption of the deep substrate and unitary viscosity ratio, the slip lengths are related to the permeabilities by

$$\ell_x^+ = \sqrt{K_x^+} \quad \text{and} \quad \ell_z^+ = \sqrt{K_z^+}. \quad (3.12)$$

Hence,

$$\Delta U^+ = \sqrt{K_x^+} - \sqrt{K_z^+}. \quad (3.13)$$

Since we have the streamwise and spanwise slip lengths and their differences from previous sections, we could calculate the maximum drag reduction the substrate could achieve at $Re_\tau = 180$. As shown in black in figure 3.32, the drag reduction is exceptionally small, of only 1% for fine fibers, which may be a result of inherently low anisotropic ratio of the circular fibers. For large radius, the anisotropic ratio increases rapidly and the maximum drag reduction is relatively large. However, the porosity decreases quadratically with radius increasing, indicating that the flow perceives a lumpy solid wall just below the interface for large radii. Hence, the actual slip lengths and the corresponding difference caused by the substrate is much less than the linear prediction of (3.13), as shown in the red dot in figure 3.32. In this case, the substrate is actually similar to riblets with semicircular cross-sections. Modeling the substrate as riblets avoids singularity due to extremely small $\sqrt{K_y}$ and predicts the drag reduction via

$$\Delta U_{\max}^+ = 0.8 \sqrt{A_g^+}|_{\text{opt}} \frac{\ell_x - \ell_z}{\sqrt{A_g}}, \quad (3.14)$$

where $\sqrt{A_g^+}$ is the groove area and $\sqrt{A_g^+}|_{\text{opt}} = 11$ ([García-Mayoral and Jiménez, 2011](#)). We revised the drag reduction results for thick fibers, $r \geq 0.4$, shown as the empty circles in figure 3.32. Although its academically arranged configuration unravels intriguing recirculating patterns and could help to better understand flows associated with porous media, such a substrate is probably of little interest to engineering.

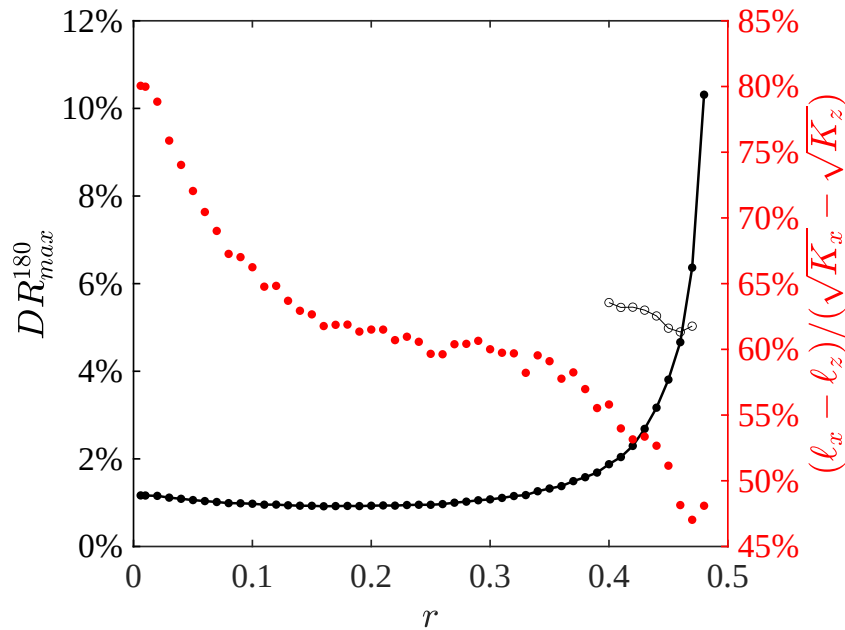


Figure 3.32 Drag reduction for round fibers.

The results shown previously suggest that, in order to increase drag reduction, a desirable substrate should have a high anisotropic ratio, $\sqrt{K_x/K_z}$, as well as low porosity, θ . The former allows divergence between streamwise and spanwise slip length, ℓ_x and ℓ_z , while the latter makes sure that the magnitude of their difference is significant for meaningful applications. In other words, the ideal substrate should be a blockage only to the spanwise flow, guiding mean stream to move in the streamwise direction. We let the substrate be composed of streamwise-aligned fibers as before, but we redesign the cross-section of the fiber. Two types of fiber are shown in figure 3.33 along with their corresponding estimations of drag performance. The 'cross' shape looks promising at this stage. From an engineering perspective, fibers with cross-section elongated in one direction may be of interest. They could be naturally more streamwise-preferential than cylinders or could form 'cross' fibers or channels if arranged carefully, and they are easy to manufacture.

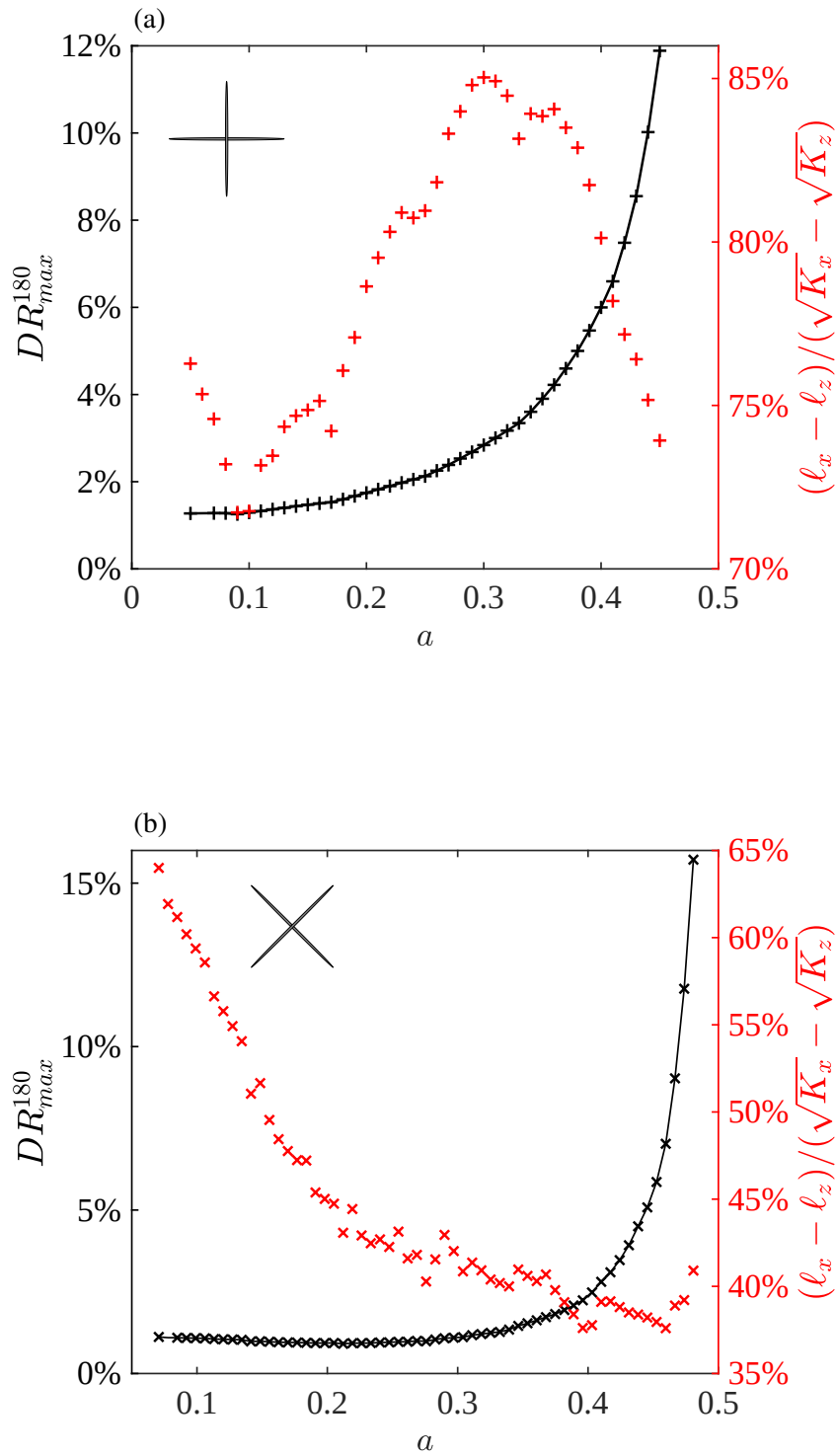


Figure 3.33 Drag reduction for (a) cross fibers; (b) tilted cross fibers.

Chapter 4

Results of fully resolved DNSs

This section presents results from DNSs. Visualized instantaneous flow fields are shown in the first place, illustrating flows within fully resolved permeable structures and turbulent structures in the channel region. Streamwise elongated streaks are observed in small permeability cases, resembling flows over a smooth wall. Spanwise coherence emerges as permeability increases. Turbulence statistics are then given to discuss quantitatively the interaction between turbulence and the porous wall. Drag reduction analysis is also conducted. For the fibrous substrate with porosity $\theta = 0.9127$, it can achieve a 2.688% drag reduction at $\sqrt{K_y^+} = 0.4786$. In the final part of this section, the macroscopic effective viscosity is analyzed.

4.1 Simulation set-up and results

A series of DNSs are conducted to study the interaction between porous substrates and the overlying turbulence, by varying the porosity and the fiber size of the substrates.

The simulations are performed in a rectangular channel, bounded by layers of permeable material at the bottom and top walls. The channel has dimensions of $2\pi\delta \times \pi\delta \times 2(\delta + h)$ in x , z and y , where $\delta = 1$ is the half channel height, h is the thickness of the permeable region. figure 4.1 (b) shows a schematic of this computational domain. The incompressible fluid within the channel and the porous regions has kinematic viscosity $\nu = 1/2870.5$, and is driven by a constant pressure gradient at friction Reynolds number $Re_\tau = 180$.

As mentioned before, streamwise aligned fibers are considered in the present study. In particular, with the aim of exploring the optimal drag-reduction performance, we focus on fibers with circular cross-section. The whole permeable region could be subdivided into cuboids, whose cross section by a unit cell is sketched in figure 4.1 (a). The spacing between fibers, s , and the diameter of one fiber, ϕ , are defined in the cross sectional plane as microscopic characteristic lengths. Subsequently, a macroscopic property, porosity θ , is defined as the ratio

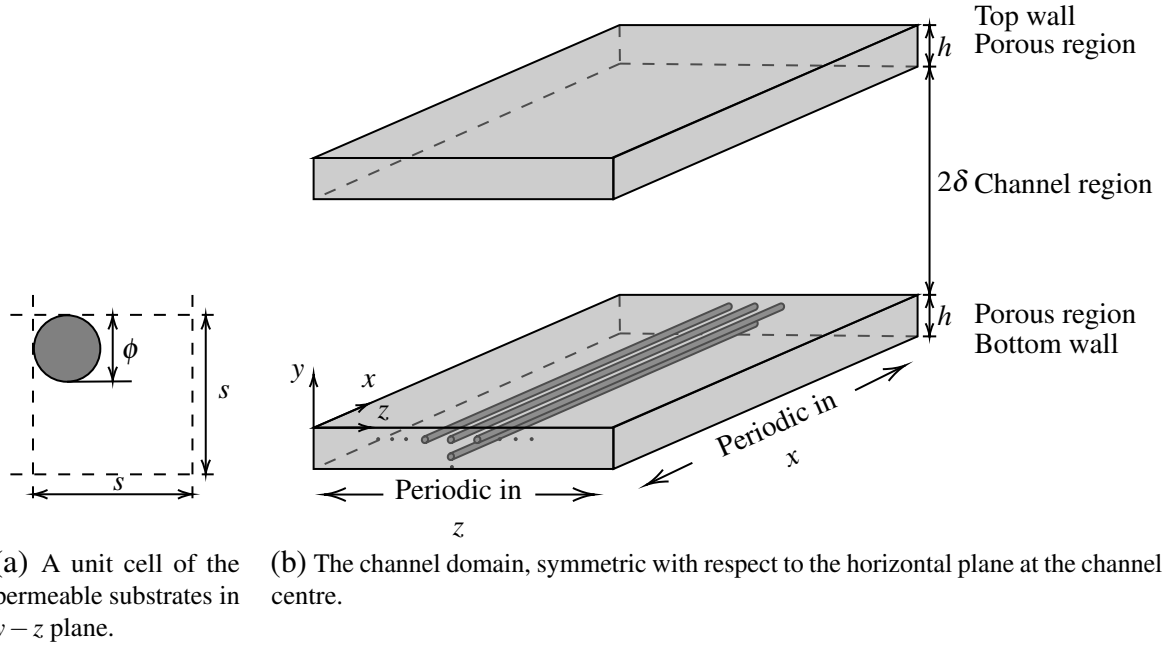


Figure 4.1 Schematic of the computational domain and the substrates configuration.

of volume of fluid within the porous region to that of the entire porous region,

$$\theta = 1 - \frac{\pi \phi^2}{4s^2}. \quad (4.1)$$

The unit cells are collocated, forming a $N_z \times N_y$ matrix in the cross-sectional plane, where N_z is the number of columns of unit cells and N_y is the number of rows of unit cells. The configuration in DNSs has $N_y = 4$ which is deep enough to minimize the effect of the plane solid walls on the overlying turbulence.

The parameters of each simulation are listed in Table 4.1. Cases are named with their value of N_z . In this series of cases, the ratio between ϕ^+ and s^+ are constant, and consequently the porosity is also a constant $\theta = 0.9127$. As a result, the anisotropy ratio, defined as the square root of the streamwise permeability to the wall-normal permeability, is given by $R_{xy} = \sqrt{K_x/K_y} = 1.42$. By changing N_z , we obtain a range of permeability, which is expected to cover different regions in terms of changing in drag. The streamwise and wall-normal permeability are interpolated from figure 1.5 and scaled to be in wall units. By taking the square root of the permeability, $\sqrt{K_x^+}$ and $\sqrt{K_y^+}$ are regarded as the permeable lengthscales, which characterize the macroscopic effect of the substrate. A fully developed turbulent smooth channel is taken as reference.

	Case	N_z	Δz^+	h^+	ϕ^+	s^+	$\sqrt{K_x^+}$	$\sqrt{K_y^+}$
Smooth	SC							
	P256	256	0.1052	8.8357	0.7362	2.2087	0.6791	0.4790
	P192	192	0.1402	11.781	0.9816	2.9449	0.9201	0.6387
Permeable substrates	P168	168	0.1603	13.464	1.1219	3.3656	1.0438	0.7299
	P144	144	0.1851	15.708	1.2954	3.8861	1.2215	0.8428
	P120	120	0.2240	18.850	1.5683	4.7048	1.4477	1.0203
	P96	96	0.2853	23.562	1.9972	5.9917	1.8438	1.2994

Table 4.1 Parameters of DNSs. Variables are scaled in wall units when assuming $\text{Re}_\tau = 180$. The streamwise and wall-normal permeability are interpolated from figure 1.5 and scaled to be in wall units.

The results are enumerated in Table 4.2. In each case, the friction velocity u_τ is calculated at its corresponding virtual origin for turbulence. The depth of the virtual origin for turbulence ℓ_T^+ is measured by aligning the Reynolds stress profile of each case with that of the smooth wall case, given that we assume the turbulent dynamics remain unchanged with small scale fibers and hence the Reynolds stress becomes zero when turbulence approach a actual or virtual wall. The virtual origin for the mean flow ℓ_U^+ is calculated from the mean velocity profile, through the Navier slip conditions (1.5). After displacing the mean velocity profile by ℓ_T^+ , its shift, ΔU^+ , is measured directly as the difference between a permeable case and the smooth wall reference. Simulations are initialized with the fully developed smooth channel turbulence files, labeled by 'SC' in table 4.1. In order to exclude the transitional effect which flows experience as the wall changes from smooth to porous, the first 10 eddy-turnover times of data are discarded. For the statistics in this report, at least 30 eddy-turnover times of data are collected to avoid uncertainty.

Case	u_τ	Re_τ	ℓ_U^+	ℓ_T^+	ΔU^+	DR
P256	0.0627	179.8	0.3173	0.0719	0.2151	2.688%
P192	0.0637	182.7	0.4252	0.3251	0.0481	0.611%
P168	0.0632	181.4	0.4892	0.6717	-0.1997	-2.595%
P144	0.0634	181.9	0.5739	0.9475	-0.3385	-4.459%
P120	0.0626	179.7	0.6833	1.3508	-0.7104	-9.712%
P96	0.0638	183.1	0.8768	1.7443	-1.9316	-30.06%

Table 4.2 Results from DNSs. The friction velocity u_τ in each case is obtained at its corresponding virtual origins for turbulence $y^+ = -\ell_T$. Wall-normal permeability $\sqrt{K_y^+}$ is updated to be normalized by its corresponding viscous scales. The virtual origin for the mean flow ℓ_U^+ is calculated by the Navier slip conditions (1.5), while that for the turbulence ℓ_T^+ is measured from the Reynolds stress profile. ΔU^+ is the shift of the velocity profile measured in the logarithmic region at $y^+ = 150$. DR is obtained with equation (1.3).

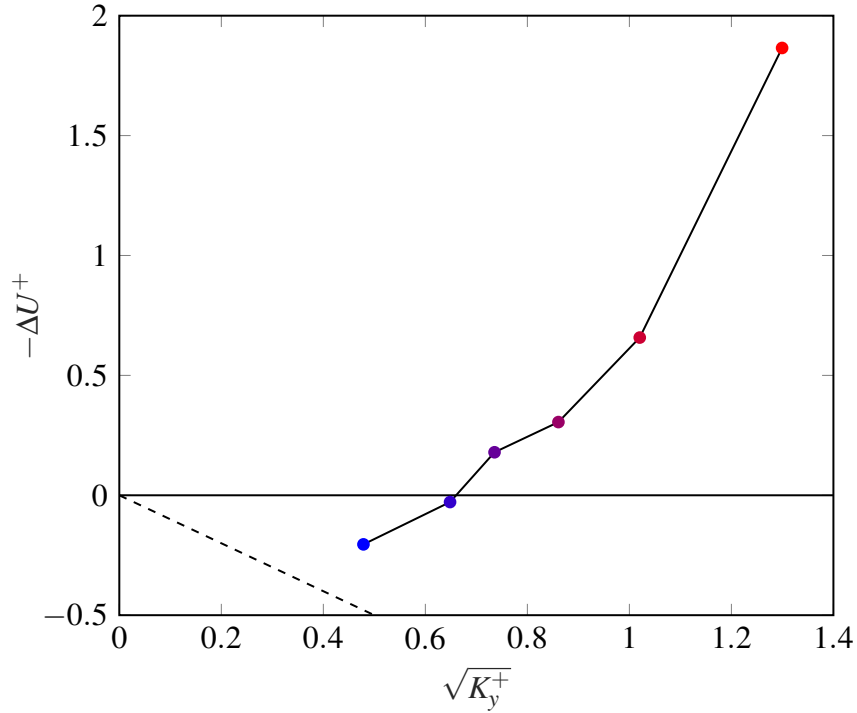


Figure 4.2 Drag reduction curves in terms of mean velocity shift ΔU^+ , where case P256 to P96 are marked with blue to red as permeability increases. The dashed line is produced by the linear prediction $\Delta U^+ \approx \sqrt{K_x^+} - \sqrt{K_y^+}$, which marks the expected viscous regime when permeability further decreases.

It should be noticed that the simulations in the study have moderate and high permeabilities, as listed in table 4.1. Although they are designed to achieve the possible maximum drag reduction, their permeabilities are still beyond the point where the performance is optimal, which is expected to be of $\sqrt{K_y^+} \approx 0.38$ (Gómez-de-Segura and García-Mayoral, 2019). Nonetheless, our simulation set demonstrates the capability of the newly developed numerical scheme, and allows further simulations on fibrous substrates with small permeabilities to be carried out.

4.2 Drag reduction curve

A drag reduction curve produced by the results of DNSs is shown in figure 4.2. The shift in the mean velocity profile ΔU^+ is portrayed against the square root of wall-normal permeability $\sqrt{K_y^+}$, which is used as a characteristic length scale of the substrates. The resulting drag reduction curve has its most part falls within the drag increasing region above the horizontal line, whereas two cases are drag reducing. For the cases in our DNSs, the drag decreases as $\sqrt{K_y^+}$ reduces for smaller permeabilities. When $\sqrt{K_y^+} = 0.4786$, the substrate produces a 2.688% drag reduction, compared to a smooth wall case. In case P192 of $\sqrt{K_y^+} = 0.6484$,

the drag reduction is 0.611%, where the drag is approximately equal to that at a smooth wall. It can be inferred that the drag becomes greater than that for a smooth wall when $\sqrt{K_y^+} \gtrsim 0.65$ for a substrate of porosity $\theta = 0.9127$. The critical point $\sqrt{K_y^+} \simeq 0.65$ found in fully resolved DNSs is slightly higher than that from homogeneous model by [Gómez-de-Segura and García-Mayoral \(2019\)](#), where they predict the critical value would be $\sqrt{K_y^+} \simeq 0.6$. In this regime, where drag reduction begins to degrade, ΔU^+ deviates from the linear prediction of $\Delta U^+ = \ell_U^+ - \ell_T^+ \approx \sqrt{K_x^+} - \sqrt{K_y^+}$. This could be explained by that the Kelvin-Helmholtz-like instability emerges and the virtual origin theory eventually ceases to hold because turbulence dynamics are modified by the fibers and are therefore no longer smooth-wall-like. The evidence is given by visualized flow fields and statistics, which are discussed in the following section.

As the drag reduction still tends to increase with smaller permeability, it is suggested that the permeable substrates tested in this study lie in the region where drag reduction degrades, without covering the whole drag reduction domain. Therefore, the linear prediction of ΔU^+ of equation (1.15) does not apply to current cases, which can also be proved from table 4.1 and 4.2, where the measured ΔU^+ is unequal to the difference between the virtual origin depth or that between permeability lengths. It is expected that as the permeability keeps decreases with finer fibers, the drag reduction will further increases before culminating, and then decreases linearly in the region where the theoretical prediction (1.7) holds.

4.3 Flow fields and turbulent structures

In this study, the flows within the permeable substrates and the overlying turbulence are resolved simultaneously, taking into account the direct effect of the interaction between the turbulence and the microscopic fibers. Instantaneous cross-sectional views from case P256 and P96 are portrayed in figure 4.3 (a) and (b) for illustration. It is seen that the turbulence is allowed to penetrate into the permeable region, though its intensity is significantly damped. In the flow region, the streamwise velocity grows from small values (blue) to relatively high (red), as wall-normal distance increases from $y^+ = 0$, the interface, to $y^+ = 180$, the channel center. Above the interface $y^+ = 0$, turbulent structures discussed in Section 1.3 are observed in the flow region. The vectors illustrate the quasi-streamwise vortexes, they appear close to the interface and one clockwise vortex is neighbored with counter-clockwise ones. These counter-rotating vortexes lift the slow fluid (blue) and bring down the fast fluid (red), feeding the low- and high-velocity streaks. Compared to P96 in figure 4.3 (b), P256 in (a) is obtained from a denser array of fibers with lower permeability where the overlying flow is expected to resemble flow over a smooth wall. Hence the streaks are better organized and easier to be identified in figure

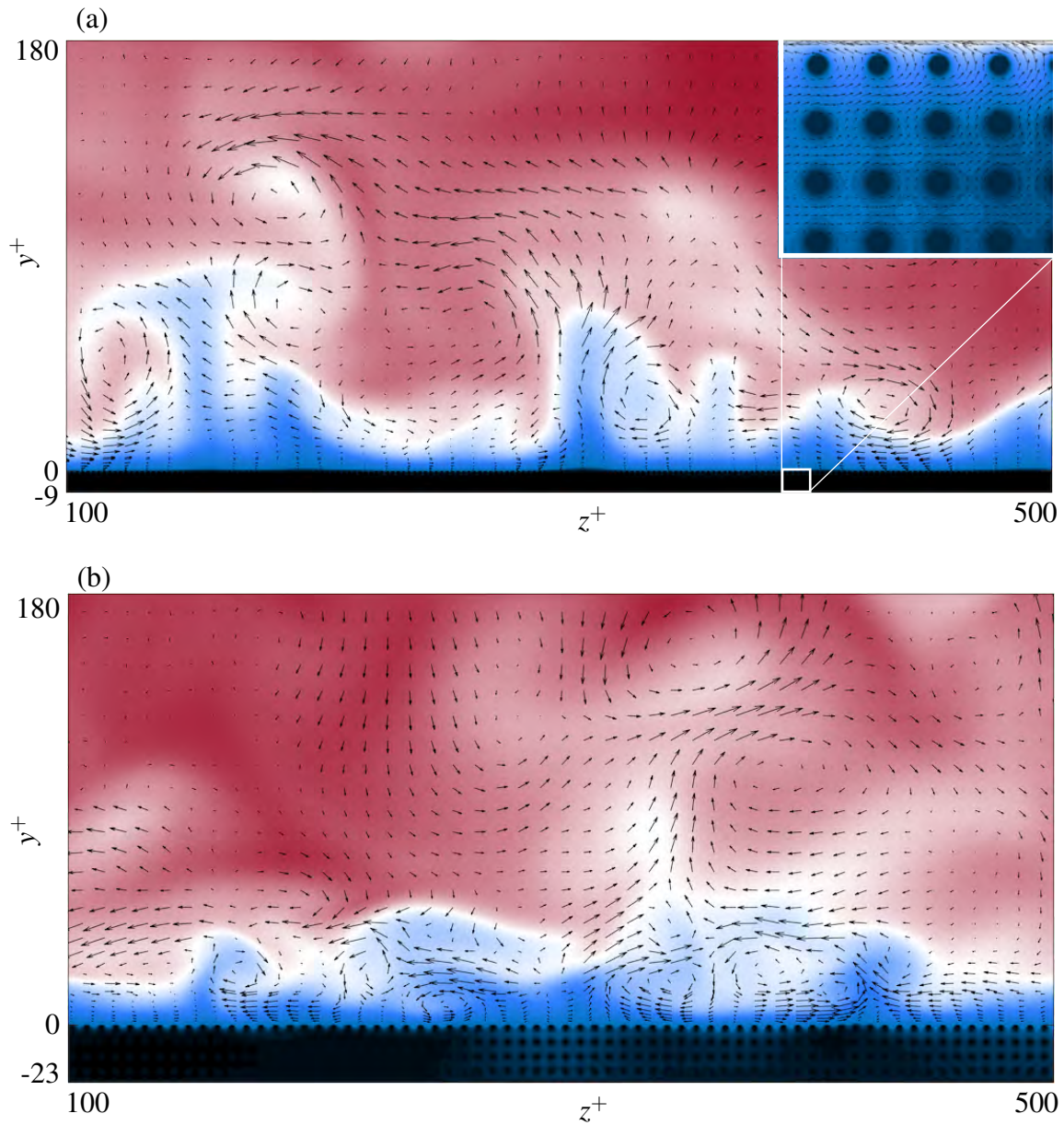


Figure 4.3 A portion of bottom half streamwise and secondary flow field in $y-z$ plane of case: (a) P144, (b) P96. The streamwise velocity $u^+ \in [0, 18]$, represented by contours from black to red. The magnified inset in (a) is recolored for $u^+ \in [0, 0.163]$ from black to white. Above the interface, secondary flows v^+ and w^+ are shown by vectors whose values are from 0 to 0.075 in wall units.

4.3 (a). A magnified inset in 4.3 (a) demonstrates the flow deep within the permeable region. The flow motions there are not strong turbulent, but mild responses to the overlying turbulence.

The contours from left to right columns in figure 4.4 visualize the instantaneous streamwise velocity u^+ , wall-normal velocity v^+ and spanwise velocity w^+ immediately above the flow/substrate interface, showing how the overlying turbulence is modified by the fibers. For small permeability cases, the direct effect of the fibers is negligible and the flow fields are

smooth wall-like. With the size of the fibers ϕ^+ further increasing, as in case P192 where $\sqrt{K_y^+} = 0.6484$, the granular effects of fibers begin to appear in the flow fields. Streamwise thin lines corresponding to locations of fibers can be seen in three velocity components fields, and become obvious in P96. It is found that the fibers have a predominant effect on the wall-normal velocity rather than wall-parallel velocities. As shown in the second row for P256 in figure 4.4, the wall-normal velocity field is altered and displays disordered assemblies of negative and positive velocities without the trace of streamwise-coherence, even though the permeability $\sqrt{K_y^+} = 0.4786$ is small, while u^+ and w^+ are similar to the smooth wall case. From row P256 to row P96 as permeability increases, the wall-parallel velocities gradually lose their strips elongated in streamwise direction and show evidence of spanwise coherence, but w^+ experiences this change earlier than u^+ . On the other hand, v^+ forms small patches which distributed in a relatively random manner. The disordered wall-normal velocity fields in the vicinity of the interface show trace of the Kelvin-Helmholtz spanwise rollers. [Sharma and García-Mayoral \(2020\)](#) observed very similar flow patterns in canopy flows, which they proved to be highly associated with the Kelvin-Helmholtz instability. They suggested the Kelvin-Helmholtz instability becomes obvious because the background turbulence is weakened and not allowed to penetrate deeply into the canopy, which is also applied to our permeable substrates. From the middle column of P256 to P96, it suggests that Kelvin-Helmholtz rollers exist in all those cases.

Slightly away from the interface, the contours at $y^+ \approx 10$ in figure 4.5 provide clear illustrations for the turbulence structures in the near-wall cycle. The first thing to notice is that there is no sign of the granularity from the fibers, confirming our assumption that their direct effect is confined in the vicinity of the interface. Furthermore, a general trend that streamwise streaks decompose into small patch of fluid is observed in column u^+ , from row P256 to row P96. The quasi-streamwise vortexes displayed in column v^+ and w^+ see the same trend. Case P256 shows flow fields that are smooth-wall-like. In cases of higher $\sqrt{K_y^+}$, spanwise coherent structures emerge, which eventually become prevalent. [Gómez-de-Segura and García-Mayoral \(2019\)](#) reported prevalent spanwise structures, i.e. the Kelvin-Helmholtz rollers, at $\sqrt{K_y^+} = 0.96$. However, it is observed from our fully resolved DNSs that these spanwise structures are generated at a higher $\sqrt{K_y^+}$, compared to that predicted by the homogeneous model in [Gómez-de-Segura and García-Mayoral \(2019\)](#). The difference is likely a consequence of the presence of fibers.

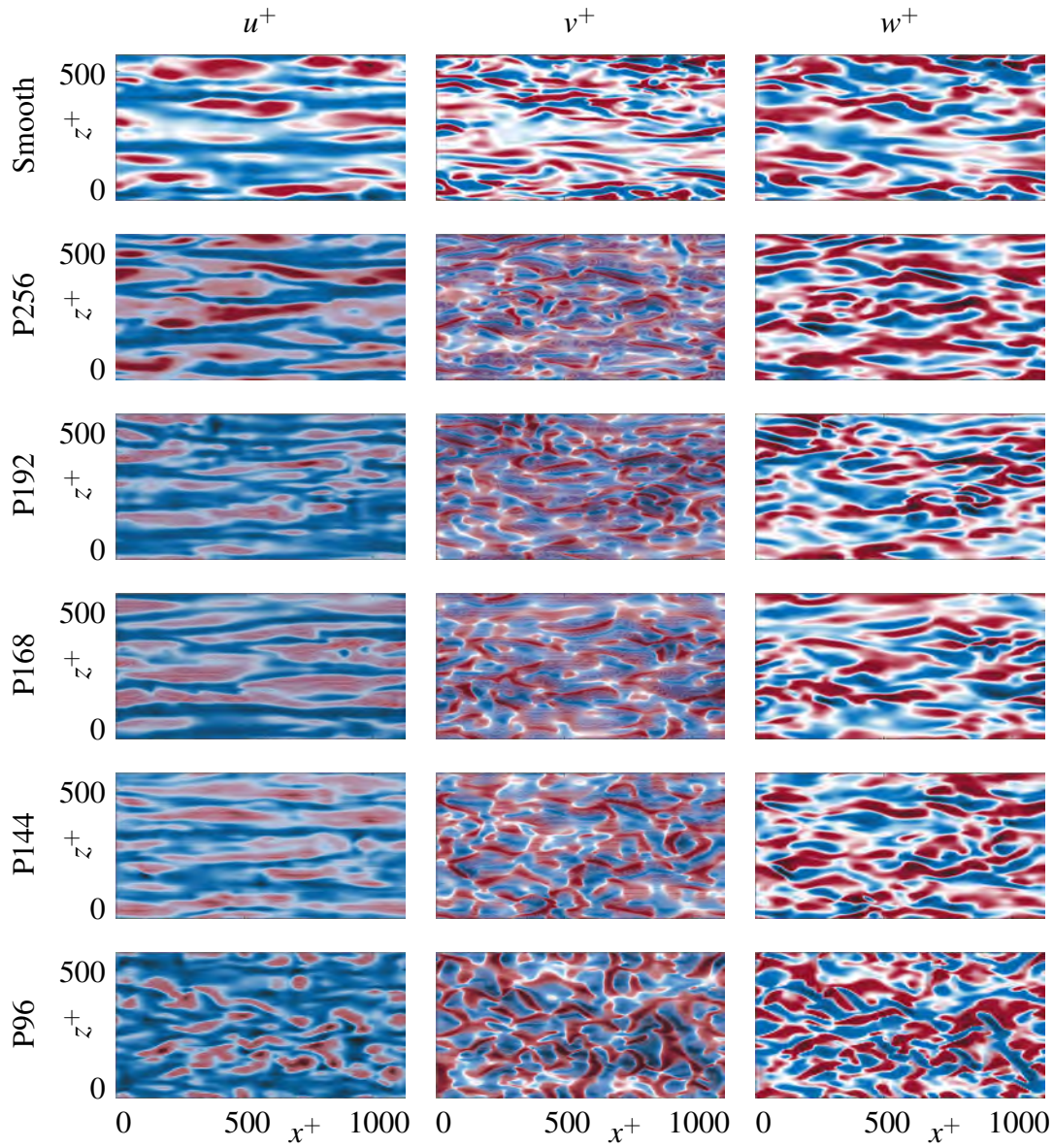


Figure 4.4 From left to right, instantaneous flow contours of u^+ , v^+ and w^+ for a smooth channel and for substrates at a $x-z$ plane $y^+ \approx 1$. Blue to red corresponds to $[0, 0.55 + 3\ell_u^+]$ for u^+ , $(0.01 + 0.5\ell_T^+) [-1, 1]$ for v^+ and $(0.25 + 0.25\ell_T^+) [-1, 1]$ for w^+ .

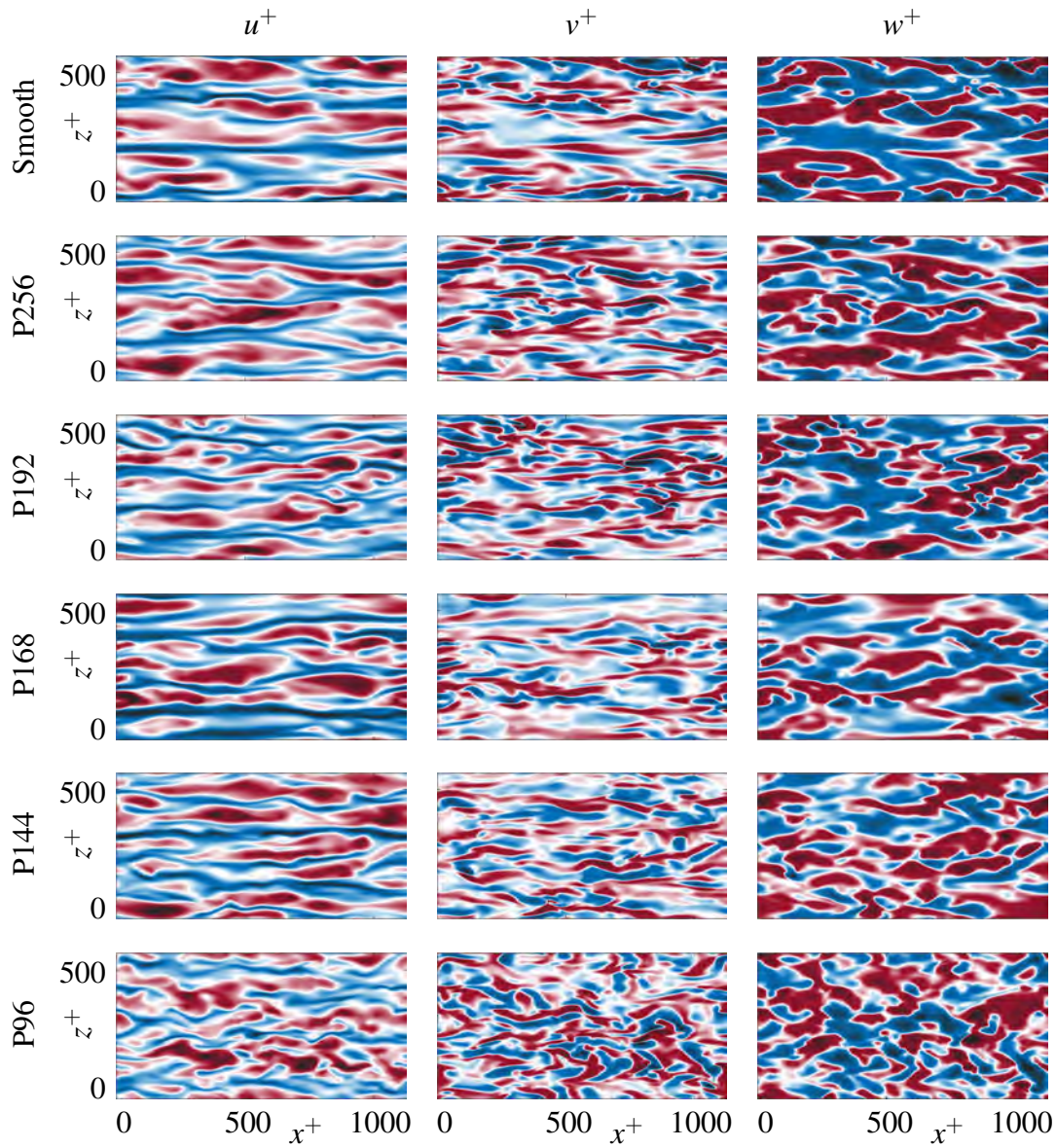


Figure 4.5 From left to right, instantaneous flow contours of u^+ , v^+ and w^+ for a smooth channel and for substrates at a $x-z$ plane $y^+ \approx 10$. Blue to red corresponds to $[2, 16 + 2\ell_u^+]$ for u^+ , $(1.5 + \ell_T^+) [-1, 1]$ for v^+ and $(3 + \ell_T^+) [-1, 1]$ for w^+ .

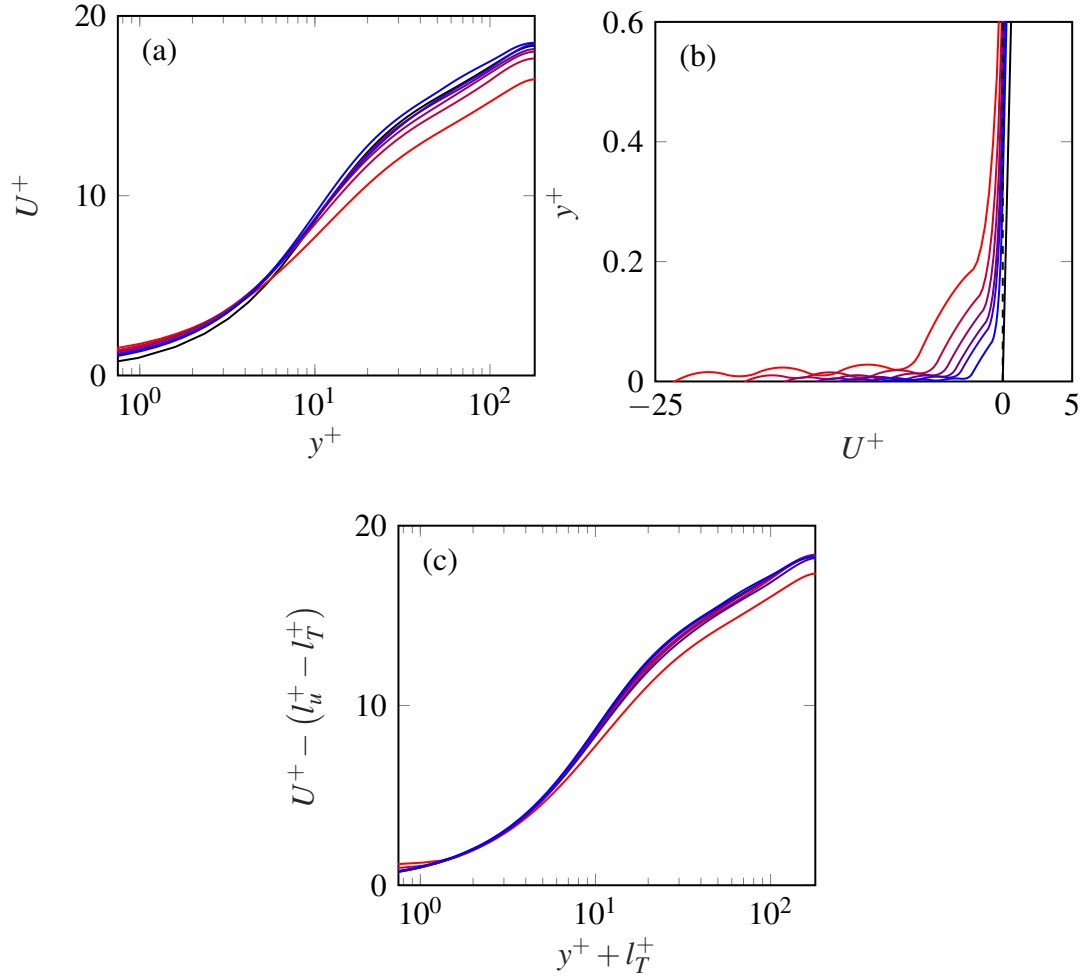


Figure 4.6 (a) Mean velocity profiles; (b) Mean velocity profiles at the interface (marked by dashed line at $y^+ = 0$) and inside the porous region. Variables in (a) and (b) are scaled with u_τ calculated at the interface $y^+ = 0$. (c) Modified profiles, scaled with u_τ at their corresponding virtual origins for turbulence $y^+ = -\ell_T$ and shifted by ℓ_T . From blue to red, curves represent cases with increasing permeability, whereas the black curve is the smooth channel case.

4.4 Turbulence statistics

Turbulence statistics are presented to estimate quantitatively the flow within and outside the substrate region. Figure 4.6 (a) and (b) show mean velocity profiles in the channel and porous region. In the channel region, the mean profiles deviate from that of the smooth wall case, especially obvious in cases with high permeability. Close to the interface, the slip velocity results in accelerated mean flows. On the hand, in the outer layer, profiles are shifted according to the homogeneous effect of fibers given by ℓ_U^+ and ℓ_T^+ . Below the interface, the mean velocity is damped steeply in the porous region. The mean flows show wavy distributions

with decreasing peak magnitudes from the interface to the solid walls, which come from the co-action of streamwise pressure gradient and overlying turbulent shear. These distributions have similarity, irrespective of the substrate depths h^+ which is sufficient deep, excluding the effect of the bottom solid wall.

As suggested in [García-Mayoral et al. \(2019\)](#), depicting the profiles by aligning their virtual origins for turbulence ℓ_T^+ provides a convenient way to assess different contributions to modifying the flow. If the turbulence remains smooth wall-like, its mean velocity and r.m.s. fluctuation profiles are expected to be identical to that of the smooth wall except for a displacement of $\Delta U^+ = \ell_U^+ - \ell_T^+$, which is uniform along the wall-normal direction. Therefore, the permeable profile should coincide with the smooth wall one after subtracting ΔU^+ from U^+ . However, this prediction does not apply to cases with high permeability, as seen in figure 4.6 (c). Profiles for P96 are deviated apparently from the smooth wall profile, suggesting the overlying turbulence is modified at $\sqrt{K_y^+} = 1.2994$. Profiles for other cases are collapsed and coincide with that for the smooth wall.

The flows within the permeable region and the effect on the turbulence dynamics are also observed in the r.m.s velocity fluctuations and Reynolds stress in figure 4.7. In the porous region, the r.m.s. fluctuations in figure 4.7 (a), (b) and (c) also display wavy distributions throughout the substrates. However, flows within the permeable region are rather laminar than turbulence because they are significantly damped, seen in figure 4.3. It is argued that the wavy distributions are not the footprint of penetrating turbulence, but simply coherent flows in between the solid fibers, interacting with each other. While the streamwise and spanwise fluctuations are damped sharply below the interface, the magnitude of wall-normal fluctuation is reduced progressively. Similarly, profiles in figure 4.7 (d) show that the Reynolds stress also penetrates into the fibers, but goes to zero approximately between the first and second row of fibers, counted from the interface. This indicates that the overlying turbulence is not allowed to penetrate deeply into the substrates. Nonetheless, non-zero Reynolds stress is produced at the porous interface, as a result of the offset effect of the virtual origin. Those observations are consistent with the results from experiments of turbulence over arrays of rods by [Suga et al. \(2020\)](#). When permeabilities are varied, the profiles keep the abovementioned features and the only change is in their magnitude. Within the permeable region, all three velocity components increase in intensity as permeabilities go up.

The r.m.s. fluctuations above the interface show deviation from smooth-wall profiles. Compared to profiles of the smooth wall case, the presence of permeable structures results in an increase in all r.m.s fluctuations at the porous interface, which is also the case in DNS of grid of cubes by [Breugem and Boersma \(2005\)](#). Furthermore, [Breugem and Boersma \(2005\)](#) argued that the peak of the streamwise r.m.s. velocity is associated with the low- and high-velocity

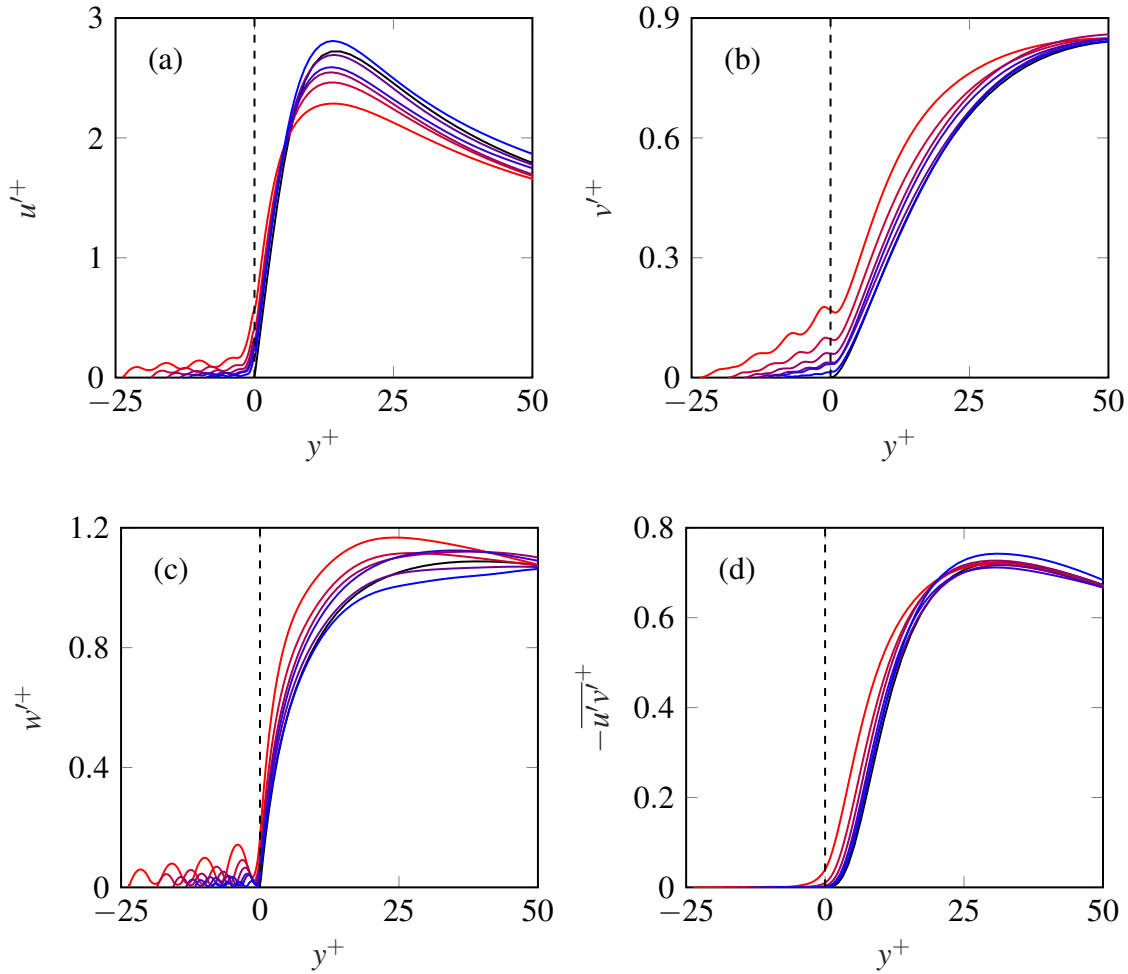


Figure 4.7 Turbulence quantities of the smooth channel (black) and substrates (permeabilities increase from blue to red): (a) - (c) r.m.s. fluctuations of streamwise, wall-normal and spanwise velocities respectively; (d) Reynolds shear stress. Profiles of substrates are scaled with u_τ at their corresponding virtual origins for turbulence $y^+ = -\ell_T$. Dashed lines mark the location of the porous interface.

streaks in the near-wall region and the peak decreases once the streaks disappear over a highly permeable wall. It explains what we see in figure 4.7 (a) where profiles of high permeability cases fall behind the smooth wall curve. We also report that from case P256 to P96 with increasing permeability, fluctuations of the streamwise velocity decrease in intensity and those of the secondary velocity are increased. This agrees with the results reported in the DNSs with a homogeneous model for the permeable layers by Gómez-de-Segura and García-Mayoral (2019). It should also be noticed that both the streamwise velocity in figure 4.6 (b) and figure 4.7 (a) and the spanwise r.m.s. velocity in figure 4.7 (c) experience a downward shift, conforming the

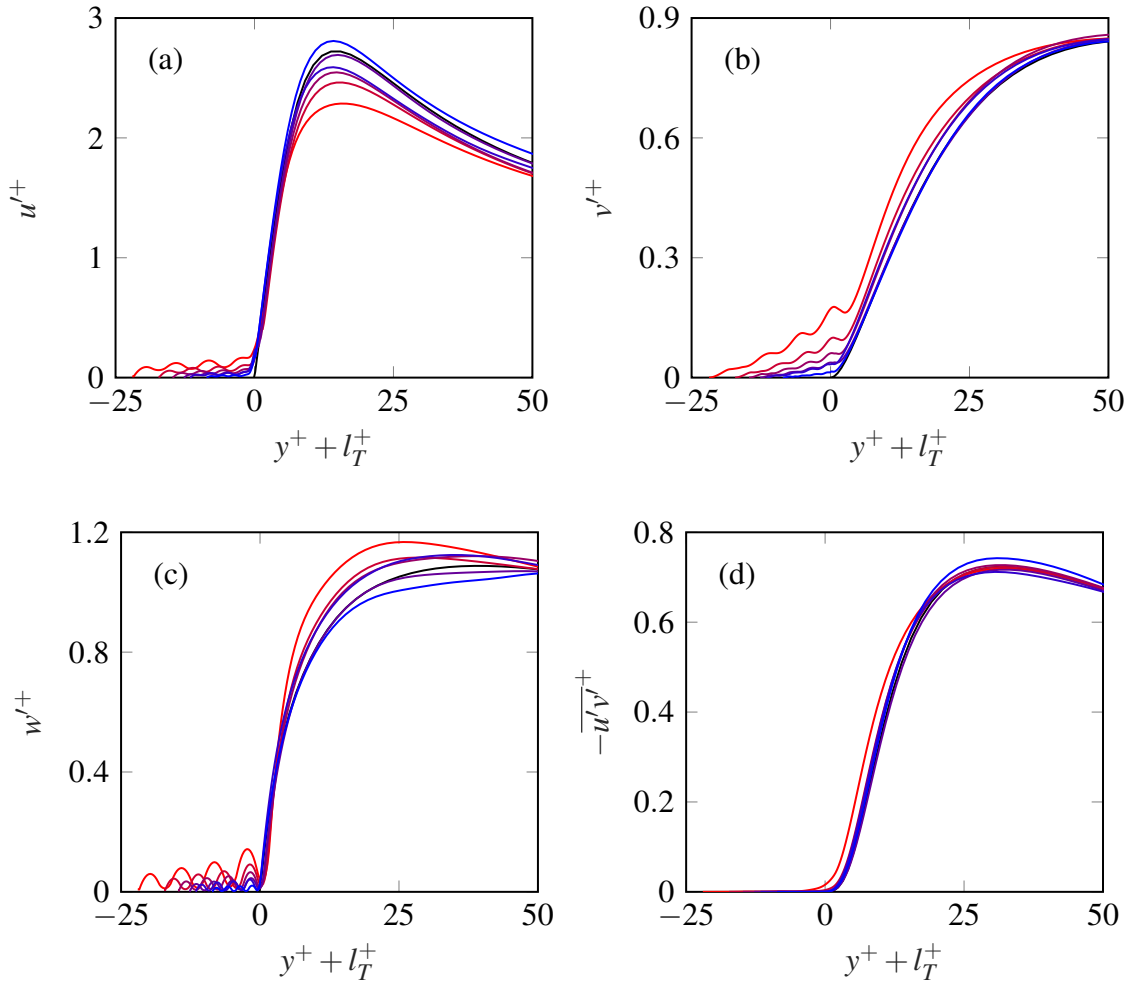


Figure 4.8 Turbulence quantities of the smooth channel (black) and substrates (permeabilities increase from blue to red): (a) - (c) r.m.s. fluctuations of streamwise, wall-normal and spanwise velocities respectively; (d) Reynolds shear stress. Profiles of substrates are scaled with u_τ at their corresponding virtual origins for turbulence $y^+ = -\ell_T$. They are shifted by moving their virtual origins for turbulence to the same interface. Dashed lines mark the location of the porous interface.

virtual origin theory that the mean flow and the turbulence perceive a virtual wall beneath the interface.

Figure 4.8 shows profiles of the r.m.s. fluctuations and the Reynolds stress that are shifted by ℓ_T^+ , i.e. aligning their virtual origins for turbulence with the substrate interface where is the actual wall for smooth-wall turbulence. For the r.m.s. velocities in figure 4.8 (a), (b) and (c), the curves collapse in the near-wall region, except for the wall-normal fluctuation v'^+ of P96. However, the peaks of u'^+ and w'^+ do not coincide with their corresponding smooth wall

profiles, as seen in (a) and (c). The collapse particularly works for the profiles of the Reynolds stress in (d).

4.5 Reassessment of slip lengths using DNSs data

The DNSs results show that the Brinkman equation used in the homogeneous model (1.10) fails when flows within porous region are fully interacting with the overlying turbulence. As in table 4.3, the ratio of the molecular viscosity ν to the effective viscosity $\tilde{\nu}$ cannot be assumed to equal 1, in both the streamwise and spanwise direction. Consequently, using permeable lengths to predict slip lengths and ΔU^+ , as in equation (1.15), is not valid. The simulations on shear driven flows within arrays of fibers in section 3 suggests that the Brinkman equation provides an accurate model for the flow deep inside the porous region, but the analytical results from the Brinkman equation do not agree with the overlying flow at the porous interface. The results of fully resolved DNSs confirm this disagreement and suggest jump conditions are necessary at the interface to compensate the change in the effective viscosity.

Case	ℓ_u^+	ℓ_w^+	$\sqrt{K_x^+}$	$\sqrt{K_z^+}$	ξ_x	ξ_z
P256	0.6791	0.4790	0.3173	0.2342	0.4674	0.4895
P192	0.9201	0.6387	0.4252	0.3218	0.4678	0.5025
P168	1.0438	0.7299	0.4892	0.3724	0.4683	0.5059
P144	1.2215	0.8428	0.5739	0.4454	0.4691	0.5167
P120	1.4477	1.0203	0.6833	0.5405	0.4709	0.5286
P96	1.8438	1.2994	0.8768	0.7044	0.4742	0.5406

Table 4.3 The ratios of streamwise and spanwise effective viscosities with respect to the molecular viscosity, ξ_x and ξ_z , calculated by equation (1.13). Streamwise slip length $\ell_u^+ \approx \ell_U^+$, and spanwise slip length ℓ_w^+ is calculated from the spanwise velocity profile.

The viscosity ratio ξ is found to be depend on the geometric parameters of the fibers. For fibrous substrates of $\theta = 0.9127$, the permeability length is approximately half the slip length, $\xi = \ell^+/\sqrt{K^+} \approx 0.5$, but the ratio in the streamwise direction ξ_x is higher than that in the spanwise direction. As in figure 4.9, both streamwise and spanwise ratios grow with the permeability, but the latter has a larger increase than the former. It is concluded that ξ_x and ξ_z are sensitive to the geometry. A general description of ξ would be of interest in terms of developing appropriate jump conditions for the homogeneous model.

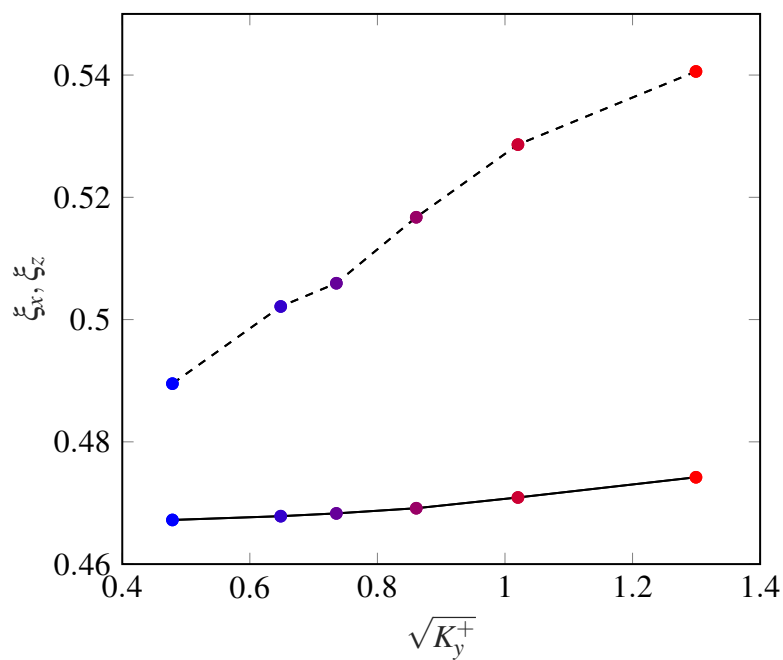


Figure 4.9 The ratio between the molecular and effective viscosities of the permeable substrate. Solid line, the streamwise ratio ξ_x ; dashed line, the spanwise ratio ξ_z . Case P256 to P96 are marked from blue to red.

Chapter 5

Conclusions

This project studies anisotropic permeable coatings for turbulent drag reduction, focusing on the microscopic flow within the coating and its interaction with the overlying turbulence.

In the first phase of this project, a study on the characterization of anisotropic permeable substrates for turbulent drag reduction has been conducted.

Firstly, we analyzed different configurations of the substrate, with the aim to identify the ones that offer the best properties in terms of drag reduction. To maximize these properties, [Abderrahaman-Elena and García-Mayoral \(2017\)](#) and [Gómez-de-Segura and García-Mayoral \(2019\)](#) have suggested the necessity to seek highly anisotropic substrates, maximizing the streamwise permeability, K_x , while minimizing the spanwise one, K_z . We focused on sufficiently deep anisotropic permeable substrates made by matrices of fibers, and considering that the obstacles are very small compared to the lengthscales of the flow in the outer region, we assumed the steady Stokes equations as the governing equations for the motion within the substrates. We solved the Stokes equations with a constant pressure gradient for the streamwise and spanwise direction. From these solutions, we got the velocity field. Then, we recovered the macroscopic velocities averaging the velocity field. Given that the macroscopic velocities within the porous medium, far from the substrate-fluid interface, satisfy Darcy's equation, we finally obtain the permeabilities. Following this procedure, we examined different configurations, starting from the case of a circular cross-section, which is frequently studied in the literature, and then moving forward to some new ones. From the first configurations analyzed, we learned that to obtain substrates with high anisotropy ratio, we had to develop configurations that strangles the flow in the spanwise direction while leaving significant gaps for the streamwise flow. This idea led us to develop substrates in which the fibers' cross-section is made by tilted ellipses in a closed arrangement or by crosses. In particular, in the last case, we found that it is possible to achieve an anisotropy ratio close to $\sqrt{K_x/K_z} \approx 15$, higher than all the other cases studied.

Secondly, we considered a flow driven by a shear. [Sangani and Behl \(1989\)](#) have suggested that Brinkman model is well suited for "highly-connected" substrates, where diffusion can act efficiently over large scales. Starting from this consideration, we analyzed if Brinkman equation is a good model for the macroscopic velocities in the core of the substrates, and we developed a method to evaluate the effective macroscopic viscosity. Moreover, Brinkman equation is a volume averaging model, so it implicitly assumes the presence of a large number of fibers in any small volume within the substrate. However, approaching the interface between the substrate and the free flow region, that assumption would eventually cease to hold, and in the literature there is no general agreement on how to treat the substrate-fluid interface ([Ochoa-Tapia and Whitaker \(1995\)](#); [Le Bars and Worster \(2006\)](#); [Zampogna and Bottaro \(2016\)](#); [Lācis and Bagheri \(2017\)](#)). To answer these open questions, we first solved the Stokes equation over a sufficiently high column of fibers, recovering the velocity field. Then, we compared the velocity profiles, obtained from a spatial average of the velocity field, with the analytical solution of Brinkman equation. As a result, we got the best fit values for the effective macroscopic viscosity, within the substrates in the streamwise and spanwise direction, as well as the limit of validity for the model. We found that our results showed a good fit with the analytical solution of Brinkman equation only for streamwise flows and spanwise ones of a very high value of porosity, where volume averaging can be well defined. We also observed that the effective macroscopic viscosity assumes different values in the streamwise and spanwise directions, and we noticed that the value of $\tilde{\nu}_x$ is very close to unity. As a consequence, we suggested the possibility to assume an effective macroscopic viscosity equal to one in the streamwise direction. Conversely, we found that the same assumption does not hold in the spanwise direction. In fact, $\tilde{\nu}_z$ tends to one only for extremely small fibers.

We identified the recirculation eddies in the spanwise flow. The recirculation zones emerge for moderate and large radii fibers and are sensitive to the geometry. However, they are organized in a self-similar way that allows us to decompose the mean velocity profile into an alternating function times an exponential decay. This finding suggests the possibility of applying volume averaging to the spanwise direction for calculating effective viscosity. In that sense, Brinkman is no longer a faithful model for the flow deep within the substrate since it cannot capture the flow reversal, but only a superficial expression for the exponential decay in the absolute mean profile.

Next, we solved the Stokes equations on a domain in which the top half is free-flow, and the bottom half is filled with fibers. Following a similar procedure to the previous problem, we obtained the mean velocity profiles, and we compared them with the analytical solution of Brinkman equation at the reference plane. We found that at the interface, there is no continuity in the velocity neither in the shear, so we argued the necessity to impose jump conditions for

both of them. Moreover, we observed that the analytical solution seems to over predict the slip velocity at the interface and to underestimate the shear.

Finally, considering only the mean velocity profile solution of the Stokes equation, we calculated the slip lengths and the ratio between the slip lengths and the square root of the permeabilities. We found that all the parameters increase as the radius of fibres decreases. In addition, the ratio between the slip lengths and the permeabilities gives us an idea of the maximum achievable drag reduction. From the results, we argued that the maximum drag reduction is significantly lower than the one predicted from the theoretical models.

Following the preliminary results from the microscale calculations, a numerical study of turbulence over one kind of anisotropic permeable substrates is conducted, using parametric direct numerical simulations that fully resolve streamwise-aligned fibers. The simulations use a modified immersed boundary scheme, and explicitly resolve the interaction between the fibers and overlying turbulence, providing insights into the drag reduction performance of the fibrous substrates.

A substantial portion of the second part of the project is dedicated to developing an immersed boundary method and integrates it into a pseudo-spectral solver for turbulent channel flows. The immersed boundary method is capable of resolving the micro-scale fiber with curved boundary in its cross-sectional plane. It is shown that the method generates desired solid bodies in the turbulent flow at $Re \simeq 180$, with negligible penetration error. To drive the velocity on the solid boundary to zero, a linear interpolation scheme with a triangular stencil is used. This scheme imposes artificial forces on grid points immediately outside the solid boundary, ensuring numerical stability of the solver. Nonetheless, this interpolation manipulates the flow in the vicinity of the solid body, undermining the reliability of direct results of turbulence from the Navier-Stokes equations. This drawback could be addressed by introducing a scheme that forces inside the solid. Several attempts are made in this project. The linear interpolation with a triangular stencil is confirmed to be inefficacious for forcing inside, because it inherently generates large opposite velocity within the very limited solid region and leads to spurious gradients of flow variables (Tseng and Ferziger, 2003). In the following study, we suggest introducing the image point (Majumdar et al., 2001) or high-order interpolations.

The algorithm developed is subsequently used to perform direct numerical simulations on fully developed channel turbulence at $Re_\tau \simeq 180$, over permeable substrates composed of streamwise-aligned fibers with porosity $\theta = 0.9127$ and anisotropy $R_{xy} = 1.42$. The number of columns of fibers in z is varied from 256 to 96, resulting in permeabilities $\sqrt{K_x^+} \in [0.68, 1.84]$ and $\sqrt{K_y^+} \in [0.48, 1.30]$. These cases lie in the drag degradation region where the linear prediction is inapplicable. The mean flow is significantly damped in the porous region, where the flow is not fully turbulent. Turbulent fluctuations is observed to penetrate deeply, showing

wavy distributions throughout the substrates. These fluctuations have their magnitudes positively correlated with the porosity. The penetration depth for Reynolds stress is limited within the first layer of fibers, indicating the overlying turbulence cannot penetrate deeply into the porous region. Nonetheless, the Reynolds stress is increased at the interface as a effect of the offset by the virtual origin. In the channel region, over substrates with small permeabilities, turbulence remains smooth-wall-like and experiences a homogeneous shift in the mean velocity profile, which can be expressed by the virtual origins produced by the fibers. On the other hand, over highly permeable substrates, turbulent dynamics are modified by the substrates, where the streaks and quasi-streamwise vortexes in the near-wall cycle are contaminated. As most of our simulation cases are of high permeability, the results differ from the theoretical, linear prediction. As permeability increases, the intensity of spanwise and wall-normal r.m.s. velocities increases, whereas that of streamwise r.m.s. velocity decreases. Associated with suppressing drag reduction, the Kelvin-Helmholtz rollers are observed in all permeable cases. The Kelvin-Helmholtz rollers leave trace at the interface or within the porous region where turbulence is not strong. How to quantify this direct effect from the fibers is still need to be investigated based on the fully resolved data.

It is confirmed that, within the limited scope of permeability in this report, permeable substrates composed of streamwise-aligned fibers demonstrate great potential in reducing friction drag. The fibrous substrate can reduce the turbulent drag by 2.688% at a moderate permeability $\sqrt{K_y^+} = 0.479$. Under its porosity $\theta = 0.9127$, the substrate is expected to produce a greater drag reduction, as the permeability further decreases. A following step is to continue simulations of smaller permeability, to extend the drag reduction curve into the viscous regime and compare those results with the theoretical prediction. In order to obtain an optimal substrate configuration in terms of reducing drag, the porosity needs to be considered in addition to the permeability. The porosity is proportional to the ratio of the fiber's diameter to its periodicity. Increasing the porosity means the spacing between fibers will be enlarged, offering more slip velocity at the interface and a higher ΔU^+ in consequence. However, the anisotropy ratio will be decreased in this way, which tends to undermine ΔU^+ . On the other hand, reducing the porosity is beneficial for gaining large anisotropy ratio, but the slip at the interface will cease eventually. Under which porosity condition will the positive effect of substrates prevail over the adverse effect and to what extent the net effect contributes to drag reduction are still to be answered.

Finally, the slip lengths associated with the effective viscosity of fluid within the substrates is examined. The slip lengths are unequal to the permeability lengths in both the streamwise and spanwise direction. This result indicates the interaction of permeable substrates and turbulence cannot be accurately represented by the Brinkman equation, which has been proved to be

a sufficient model for the flow deep within the porous region. For the fibrous substrates of $\theta = 0.9127$ and $R_{xy} = 1.42$, the ratio of the molecular viscosity to the effective viscosity is about 0.5, varying according to parameters of the fibers. The effective viscosity is a macroscopic property based on microscopic parameters like the porosity whose effect is still need further investigation.

Chapter 6

Publications under grant

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1. Gómez-de-Segura, G. & García-Mayoral, R. (2019, November). Drag-reduction curves for anisotropic permeable substrates. In *APS Division of Fluid Dynamics Meeting Abstracts* (pp. C26-006).
2. García-Mayoral, R. (2020, February). Turbulence over drag-reducing, anisotropically permeable substrates. In *CUED Fluid Seminars*.
3. Lu, Z., Busetto, D., Harwood, D. & García-Mayoral, R. Assessment of Darcy/Brinkman models for the characterisation of flows in anisotropic permeable substrates. *In preparation*.

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