

Final Technical Report

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**“Optimization under Uncertainty and Conflict:
Algorithms for Heterogenous Quadratic Programs”**

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Abstract

A theory and algorithms for quadratic programs (QPs) are developed. QPs are formulated with different types of variables (continuous, integer, or mixed) and objective and constraint functions (convex, nonconvex). They have single or multiple objectives modeling conflict and may carry multiple parameters of two types: one type models unknown or uncertain data while the other is required by a proposed methodology. The theoretical results include derivation of strong polyhedral relaxations and valid inequalities for nonconvex quadratic functions, small-sized relaxations for bilinear functions, and bounds on errors in convexifying general polynomial functions. The algorithms designed for different types of QPs rely on a foundational algorithm for solving the multiparametric linear complementarity problem with sufficient matrices and parameters in general positions. This algorithm appears to be the first one ever proposed for this class of problems and solves previously unsolved multiparametric problems with parameters in general positions including single and multiobjective, linear and convex quadratic programs. Applications in the area of portfolio optimization and statistics are included.

Parametric optimization is related to robust multiobjective optimization, which is another way to model conflict and uncertainty. Properties of different types of robust Pareto solutions are developed and a notion of multiobjective robustness gap is proposed and examined. Multiobjective optimization is extended to model decision problems whose complexity is reflected in the interaction among the subproblems, a feature that has not been addressed before in a multiobjective setting.

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Introduction

This Final Technical Report consists of nine parts and has been prepared based on the instructions available at <https://www.onr.navy.mil/work-with-us/manage-your-award/manage-grant-award>. In Part I, we present the highlights of our accomplishments, and in Part II we give an overview of our research objectives. We describe our accomplishments in detail in Part III, present training opportunities in Part IV and list different modes of dissemination of the results of this project in Part V. The awards received by the PI and her students during the funding period are presented in Part VI. The project participants and products including books, papers, and software are listed in Parts VII and VIII, accordingly. In Part IX, we give additional information on professional service activities undertaken by the PIs during the funding period. We close with a list of articles from the optimization literature we refer to.

I. Highlights

Our most significant accomplishments include:

1. Algorithm for a multiparametric linear complementarity problem (Adelgren, 2021)

Modeling unknown or uncertain data in the linear complementarity problem (LCP) with a (vector) parameter leads to a multiparametric LCP (mpLCP). We develop and implement an algorithm for solving mpLCPs with sufficient matrices and parameters in general positions. This algorithm appears to be the first one ever proposed for this class of problems and solves previously unsolved multiparametric problems with parameters in general positions including single and multiobjective, linear and convex quadratic programs. The algorithm development is contained in a 63-page research monograph published in the *SpringerBriefs in Optimization* series (Adelgren, 2021).

2. Convexification techniques for mixed-integer and nonconvex optimization

Mixed-integer nonconvex quadratically constrained quadratic programs (MIQCQPs) are known to be theoretically and computationally challenging problems. Solving them to global optimality requires strong and efficiently computable convex relaxations of the feasible region. We derive strong polyhedral relaxations for a mixed 0/1 set appearing in constraints of an MIQCQP, new small-sized relaxations of bilinear functions, and gave bounds on errors arising from standard approaches for relaxing polynomial functions. We prove combinatorial properties of discrete sets arising from reformulations of integer variables in an MIQCQP.

3. The Gold Medal Award for the PI

In recognition of her lifetime research, in 2019 the PI was presented the Gold Medal award by the International Society on Multiple Criteria Decision Making (MCDM). The MCDM Gold Medal is the highest honor that this Society bestows upon a scholar who, over a distinguished career, has markedly contributed to the theory, methodology, practice and professional development of MCDM. The Society, having about 2700 members in 96 countries, is the leading

worldwide organizations for academicians and practitioners interested in MCDM. The PI has joined the past recipients of the MCDM Gold Medal, including the prominent U.S. researchers such as R. L. Keeney, H. Raiffa, T. Saaty, and R. Steuer.

<https://www.mcdmsociety.org/content/awards>

<https://newsstand.clemson.edu/mathematician-wiecek-earns-gold-medal-from-prestigious-international-society/>

4. **One research monograph and twelve articles** published in refereed journals (including one being in print); **fifty-one presentations** delivered locally, nationwide, and internationally, or submitted online.

II. Major goals

The major objectives of this project include:

- (i) Develop new polyhedral and nonlinear convex relaxations for the bounded feasible set of nonconvex mixed-integer quadratically constrained quadratic programs.
- (ii) Develop and implement algorithms for the following classes of quadratic programs
 - a) multiparametric quadratic programs,
 - b) biobjective quadratic programs,
 - c) biobjective mixed-integer quadratic programs,
 - d) single-parametric mixed-binary quadratically constrained quadratic programs.
- (iii) Apply the algorithms to real-life decision-making problems.
- (iv) Perform research that is tangible to the main objectives of this project.

Regarding (iv), new research goals have naturally emerged as by-products of the work conducted for the originally proposed objectives. These new goals refer to a theory and algorithms for multiobjective optimization under uncertainty.

III. Accomplishments

We present our accomplishments in four sections with each section addressing one of the four research goals given in Part II. For each accomplishment we give a title and a relevant reference we have authored indicating whether the paper has been published, submitted, or is in preparation. Each paper we have authored is also listed in Part VIII of this report. Whenever suitable, we refer to bibliographical sources that are included in the References at the end of this report.

The project lasted five years. The work proceeded through various stages and the overall research goals have not been changed. However, in some situations specific research tasks had to be modified due to discovered mathematical features that required to accommodate new assumptions. Additionally, some research goals have been extended because of new research ideas or collaborations. Certain tasks were accomplished quickly while some others required more time and effort than expected. We address all these cases below.

1. Convexification and relaxation techniques for mixed-integer and nonconvex optimization.

1.1 Polyhedral relaxations and valid inequalities for quadratic functions (Gupte et al., 2017)

This research was performed before this grant was funded but is reported here because it gave the co-PI a foundation for the development of the proposal leading to this funded project.

The pooling problem, a folklore NP-hard global optimization problem, is formulated as a quadratically constrained quadratic program (QCQP) that features nonconvex quadratic objective and constraint functions and finds applications in industries such as petrochemical refining, wastewater treatment and mining. We assimilate the vast literature on QCQPs that was dispersed over different areas and give new insights on prevalent techniques. We also present new ideas for computing dual bounds on the global optimum by solving high-dimensional linear programs. Finally, we propose discretization methods for inner approximating the feasible region and obtaining good primal bounds. Valid inequalities are derived for the discretized models, which are formulated as mixed integer linear programs. The strength of our relaxations and usefulness of our discretizations is empirically validated on random test instances. We report best known primal bounds on some of the large-scale instances.

1.2. Combinatorial properties of discrete sets arising from reformulations (Gupte and Poznanović, 2018)

A common way of reformulating the general integer variables in a mixed-integer problem is to use their bit representation so that all integer variables are binaries. We analyze combinatorial properties of the polytopes arising from this binarization and the computational complexity of optimizing over such sets. We construct two families of Dantzig figures, which are d -dimensional polytopes with $2d$ facets and an antipodal vertex pair, from convex hulls of initial subsets for the graded lexicographic and graded reverse lexicographic orders. These two polytopes have the same number of vertices, $O(d^2)$, and the same number of edges, $O(d^3)$, but are not combinatorially equivalent. We provide an explicit description of the vertices and the facets for both families and describe their graphs along with analyzing their basic properties such as the radius, diameter, existence of Hamiltonian circuits, and chromatic number. Moreover, we also analyze the edge expansions of these graphs.

1.3. Bounding errors in convexifying general polynomial functions (Adams et al., 2018)

Convex hulls of monomials have been widely studied in the literature, and monomial convexifications are implemented in global optimization software for relaxing polynomials. However, there has been no study of the error in the global optimum from such approaches. We give bounds on the worst-case error for convexifying a monomial over subsets of $[0,1]^n$. This implies additive error bounds for relaxing a polynomial optimization problem by convexifying each monomial separately. Our main error bounds depend primarily on the degree of the monomial, making them easy to compute. Since monomial convexification studies depend on the bounds on the associated variables, in the second part, we conduct an error analysis for a

multilinear monomial over two different types of box constraints. As part of this analysis, we also derived the convex hull of a multilinear monomial over $[0,1]^n$.

1.4. Small-sized relaxations of bilinear functions (Gupte et al., 2020)

We consider the problem of characterizing the convex hull of the graph of a bilinear function f on the n -dimensional unit cube $[0,1]^n$. Extended formulations for this convex hull are obtained by taking subsets of the facets of the Boolean Quadric Polytope (BQP). Extending existing results, we propose a systematic study of properties of f that guarantee that certain classes of BQP facets are sufficient for an extended formulation. We use a modification of Zuckerberg’s geometric method for proving convex hull characterizations (Zuckerberg, 2016) to prove some results in this direction. In particular, we provide small-sized extended formulations for bilinear functions whose corresponding graph is either a cycle with arbitrary edge weights or a clique or an almost clique with unit edge weights. On the computational side, we apply the strong inequalities to literature instances of QCQP to find good lower bounds fast.

1.5 Polyhedral analysis of symmetric multilinear polynomials over box constraints (Xu et al., submitted, 2021)

We derive polyhedral results for the convex hull of symmetric multilinear polynomials (SMPs) taken over a box domain. Exponential-sized extended formulations of general multilinear polynomials are available via the reformulation-linearization-technique (RLT), but symmetry and disjunctive programming enable a quadratic-sized extended formulation. The goal is to obtain the convex hulls in the original variable spaces. Instead of adopting the tedious method of projecting the extended formulations, our approach is more elegant in the sense that we directly exploit the problem structure to define special core facets, by which all facets can be characterized, and to then devise necessary and/or sufficient conditions on these facets. Whereas much of the theory is applicable to general SMPs over box constraints, we focus attention on two special problem classes: general supermodular (submodular) functions, and monomials having the variable lower bound equal to the negative of the upper bound. For each class, we use the necessary conditions to motivate families of core facets, and then prove that no other such facets can exist. Our derivations of these convex hulls provide alternate proofs to those in literature. For both classes, we use RLT results to characterize for each facet the set of all points at which the inequality is satisfied exactly.

1.6. Application of mixed-integer nonlinear programs to transportation network protection against extreme events (Lu et al., 2018)

We focus on transportation network protection to hedge against extreme events such as earthquakes. Traditional two-stage stochastic programming has been widely adopted to obtain solutions under a risk-neutral preference by using expectations in the recourse function. In reality, decision makers hold different risk preferences. We develop a mean-risk two-stage stochastic programming model that allows for greater flexibility in handling risk preferences when allocating limited resources. In particular, the first stage minimizes the retrofitting cost by making strategic retrofit decisions whereas the second stage minimizes the travel cost. The conditional value-at-risk (CVaR) is included as the risk measure for the total system cost. The

two-stage model is equivalent to a nonconvex mixed integer nonlinear program. To solve this model using the generalized Benders decomposition method, we derive a convex reformulation of the second-stage problem to overcome algorithmic challenges embedded in the nonconvexity, nonlinearity, and nonseparability of first- and second-stage variables. The model is used for developing retrofit strategies for networked highway bridges, which is one of the research areas that can significantly benefit from mean-risk models. We first justify the model using a hypothetical nine-node network. Then we evaluate our decomposition algorithm by applying the model to the Sioux Falls network, which is a large-scale benchmark network in the transportation research community. The effects of the chosen risk measure and critical parameters on optimal solutions are empirically explored.

1.7. Integer programming techniques for the envy-free pricing problem (Nazari and Gupte, in preparation)

We propose new mixed-integer programming formulations and develop new polyhedral and nonlinear relaxations for the envy-free pricing problems. We also characterize some properties of the optimal solutions; so far, in literature, different properties have been found under certain (strong) assumptions on the data. Our research on this problem makes very mild assumptions on the data, and hence has wider applicability than much of the existing literature. This is currently work-in-progress with a graduate student.

2. Collection of algorithms

2.1 Algorithm 1 for the multiparametric linear complementarity problem (mpLCP) with parameters in the right-hand-side vector (Adelgren and Wiecek, 2016)

This research was performed before this grant was funded but is reported here because it gave the PI a foundation for the development of the proposal leading to this funded project. Furthermore, contrasting Algorithm 1 with Algorithm 2, that is described below, is useful since the latter is also listed in Section I Highlights on page 4 as the first most important achievement of this project.

The solution to multiparametric programs comes in the form of (i) exact representations of the optimal solutions as functions of the parameters, and (ii) partitioning of the parameter space into invariancy regions (IRs) for which these function representations are valid. For the mpLCP, the IRs have the form of polyhedral subsets of the feasible parameter set while the optimal solutions are affine functions of the parameters. The IRs making up the solution to the mpLCP are at least $k-1$ dimensional, where k is the dimension of the parameter space. Given a current IR, we determine necessary and sufficient conditions for identifying the set of feasible bases which have IRs that are adjacent to the current one and at least $k-1$ dimensional. This is done by finding the set of all facets of the current IR and then, for each facet, determining all bases which have IRs that are adjacent to the current one along that facet. The adjacency and appropriate dimensionality are determined by means of linear programs (LPs) whose optimal value must be strictly positive or negative. For these LPs, we only seek a feasible solution assuming a strictly positive (or negative) objective value.

Algorithm 1 is a two-phase algorithm in tableau format that solves the auxiliary LPs in

order to solve the mpLCP with sufficient matrices. Phase 1 is used for initialization and provides an answer to a very important question that no other work we are aware of has considered, the question of how one can determine an initial complementary feasible solution (c.f.s.) for the mpLCP. In all works we know of it is simply assumed that such a solution is available. When an initial c.f.s. and the corresponding IR have been discovered in Phase 1, Phase 2 is used to partition the parameter space. Phase 2 is inspired by literature but has an improved worst-case complexity.

We implement Algorithm 1 in C using CPLEX to solve all auxiliary LPs. We compare its performance with the multiparametric toolbox (MPT) in MATLAB) that utilizes an implementation of the mpLCP solver proposed in literature (Herceg et al., 2013). We solve test instances using Algorithm 1 and the MPT. Algorithm 1 is superior to MPT in terms of average CPU time and the number of instances solved.

2.2 Algorithm 2 for the multiparametric linear complementarity problem (gen-mpLCP) with parameters in general locations (Adelgren, 2021; Jayasekara et al., 2019)

The gen-mpLCP has parameters in general locations and encompasses three important classes of problems. Multiparametric linear programs and multiparametric convex quadratic programs with parameters in general locations, and multiobjective convex quadratic programs can all be reformulated into gen-mpLCPs. Algorithm 2 is capable to solve all these problems. Applications of these classes of problems are widespread in business, management, and engineering. Prior to our work, no solution procedure existed for these general classes of problems except for the recently proposed algorithms for linear programs with parameters contained a bounded hypercube and placed in general locations (Charitopoulos et al., 2017; Kolev & Skalna, 2018).

In the gen-mpLCP, the IRs regions have the form of semi-algebraic sets, which may be nonconvex, while the optimal solutions are polynomial functions of the parameters. To establish properties of these IRs, we first perform a decomposition of the parameter space into two subspaces, which carries over to the IRs since their defining inequalities are polynomial with respect to the elements of one subspace and linear with respect to other. We then establish necessary and sufficient conditions for the semi-algebraic sets to have dimension at least $k-1$. This is an extremely challenging task that has taken more time than we anticipated. It required a merger of several concepts from the fields of real algebraic geometry, algebraic topology, and optimization. The conditions give a foundation for the development of an algorithm for solving the gen-mpLCP.

Algorithm 2 is a two-phase algorithm in tableau format that solves auxiliary nonlinear programs (NLPs) in order to solve the gen-mpLCP with sufficient matrices. This algorithm is an extension of Algorithm 1 because Phase 1 and 2 play the same role. Phase 1 answers the previously unanswered question of how one can determine an initial full dimensional IR that can be used as a starting point in the process of partitioning the parameter space. The partition of the parameter space is carried out in Phase 2. In both phases, Algorithm 2 relies on the auxiliary NLPs that have linear objective functions and polynomial constraints. Similar to the auxiliary LPs in Algorithm 1, we only seek certain feasible solutions to the auxiliary NLPs.

We implement Algorithm 2 using MATLAB and the code for this implementation is made freely available at https://github.com/Nadelgren/mpLCP_solver. Within this implementation, all auxiliary NLPs are solved using the “fmincon” function within MATLAB.

Since this version of mpLCP was previously unsolved, there is no other method with which we can compare. For our experiment we randomly generate 105 instances and observe a positive correlation between the instance size, the number of iterations and regions, and the average CPU time spent in each iteration. Because Algorithm 2 solves optimization problems with polynomial constraints, it encounters numerical difficulties or returns local rather than global optimal solutions. To make Algorithm 2 efficient, future work is needed to develop a more robust optimizer that can handle optimization problems with polynomial constraints.

The results related to the theoretical development and implementation of Algorithm 2 are contained in a 115-page long initial manuscript that has been published as a 63-page book in the *SpringerBriefs in Optimization* series (Adelgren, 2021). Additionally, in (Jayasekara et al., 2019) we present a detailed study of methods for solving multiparametric convex quadratic programs and ascertain that Algorithm 2 is the state-of-the-art algorithm.

2.3 Algorithm 3 for biobjective quadratic programs (BOQPs) (Jayasekara et al., submitted, 2021)

Algorithm 3 for solving BOQPs is a foundation for Algorithm 4 for solving bi-objective mixed-integer quadratic programs (BOMIQPs) in Section 2.6. Algorithm 3 is implemented in two ways. One way is a straightforward application of Algorithm 2. Taking the weighted-sum of the objective functions, the BOQP can be reformulated into a single-parametric quadratic program (spQP) that can be solved with Algorithm 2. The other way to solve BOQPs is an algorithm in (Väliäho, 1994) that we unexpectedly discovered during the first year of this project and that has never been implemented. Because Algorithm 2 does symbolic computation to produce explicit solutions functions, it is expected to be slow when used as a subroutine in a procedure. Väliäho's algorithm solves LCPs for specific values of the parameter and therefore is expected to have much better run times than Algorithm 2. However, it is applicable only to a class of BOQPs that fulfill very specific assumptions. Recognizing the tradeoff between the two algorithms, we implement Väliäho's algorithm in MATLAB and compare the efficiency of both algorithms in (Jayasekara et al., submitted, 2021). Despite performing symbolic computations, Algorithm 2 turns out to be more efficient than Väliäho's algorithm and is used as a subroutine in Algorithm 4.

2.4 Algorithm 4 for biobjective mixed-integer quadratic programs (BOMIQPs) (Jayasekara and Wiecek, in preparation)

Algorithm 4 is a BB algorithm for finding all Pareto solutions to the BOMIQP. This is a global optimization problem for which a solution method has not been proposed and Algorithm 4 is the first ever algorithm to solve this problem. The algorithm consists of the following modules: (i) computing a subset of the Pareto set as an initial solution set, (ii) solving node problems, (iii) bounding, (iv) fathoming, (v) branching, (vi) updating the current nondominated set. During the algorithm, when the node problems are solved, the initial Pareto subset evolves towards the Pareto set of the BOMIQP by adding Pareto sets and points of node BOQPs and discarding dominated sets and points. We prove that Algorithm 4 obtains the complete Pareto set for the BOMIQP. This research task has been very challenging and has taken much longer time than expected.

At initialization, an initial subset of the Pareto set of the BOMIQP is computed by solving the weighted sum problem for fixed weights. Algorithm 3 is used to solve BOQPs, or equivalently spQPs, at the BB nodes. The bounding rules operate on lower and upper bound sets in R^2 and are developed based on the prior research of the PIs reported in Belotti et al. (2016) and Adalgren et al. (2018). Since the solutions of the spQPs come in the form of Pareto sets on the invariancy intervals, branching has to be done with respect to these intervals. Because of branching, spQPs with additional constraints in which integer variables are fixed at some feasible values must be solved. While these constraints are very simple, adjusting Algorithm 3 to handle such spQPs has been difficult.

Updating the current nondominated set with the newly discovered Pareto sets and points from solving the spQPs turned out to be the most challenging part. A new set domination procedure is developed to compute the nondominated set of two strictly convex curves in R^2 whose only end points are known. It turns out that such curves may have many mutual locations and may intersect at points that must be computed in order to find the resulting nondominated set. These points are computed by solving two polynomial equations that are known only after the subsets of the two curves that are likely to contain the points have been identified.

The theoretical work on this task is completed and all six parts of Algorithm 4 have been coded in the form of MATLAB modules. We are currently integrating the modules into a final algorithm that will be tested on instances of the BOMIQP. This is currently work-in-progress with a graduate student who is going to graduate in December 2021.

2.5 Algorithm 5 for biobjective mixed-integer linear programs (BOMILPs) (Adalgren & Gupte, in print, 2021).

Algorithm 5 is a BB algorithm for finding all Pareto solutions to BOMILPs. This algorithm was not part of the original proposal but followed naturally from the theory earlier developed by the co-PI on fathoming and branching rules for BOMILPs (Adalgren et al., 2018). Comparing Algorithm 5 to the other BB algorithms described in (Gadegaard et al., 2019, Table 1) there are several features the other algorithms do not have. Algorithm 5 seems to be the only full implementation so far that handles the mixed-integer case, whereas all others are either pure binary or mixed-binary. Branching is not simply a matter of variable-fixing but a scoring scheme motivated by reliability branching in the single-objective case is developed. Additional features include extensive presolve and preprocessing at the root node; probing before branching which provides a huge speedup; using dual information to obtain tighter bounds; a new optimality gap measure, that is an approximate Hausdorff metric in the objective space, to allow for early termination of the algorithm once a prescribed gap has been attained. Algorithm 5 outperforms the triangle splitting method in (Boland et al., 2016) on instances from literature, and performs comparably on large, challenging instances that were developed by the authors.

Despite some similarity between BOMILPs and BOMIQPs (e.g., convexity of the objective and constraint functions) the findings contributing to Algorithm 5 could not be applied to support the work on Algorithm 4.

2.6 Algorithm 6 for single-parametric mixed-binary quadratically constrained quadratic programs (spM0/1QCQPs) (Pangia and Wiecek, in preparation)

The location of the scalar parameter in the left-hand-side of the quadratic constraint in

spM0/1QCQP makes this problem challenging. Algorithm 6 is also a BB algorithm that relies on two relaxations of spM0/1QCQP. Lagrangian relaxation is applied to the convex quadratic constraints and yields a single parametric mixed 0/1 biconvex quadratic program (spM0/1bcQP) with linear constraints. The biconvexity shows up in the objective function with respect to the continuous variables and the single parameter. Relaxing the integrality constraints in the spM0/1bcQP, we obtain an spbcQP. In the proposal, the biconvexity was not considered and a multiparametric subgradient algorithm (mpSubA) (Leverenz et al., submitted, 2020) for convex spQPs was proposed as a node solver in Algorithm 6. The mpSubA uses a multiparametric approximate simplex algorithm (mpASA) (Bemporad & Filipi, 2006) as a subroutine. This algorithm assumes joint convexity rather than the biconvexity and relies on linear interpolation of the objective function. As a result, a new and challenging research task emerged to modify the mpASA to handle the biconvexity.

We first improve the preliminary implementations of the mpASA and mpSubA, and then experiment with the mpASA by applying it to instances of biconvex spQCQP and mpQCQP. This work was not contained in the proposal. We observe that the relaxation of joint convexity to biconvexity does not adversely affect this algorithm's performance. Although the mpASA cannot find true solution sets, it is capable to approximate them and in the absence of an exact method, it is a useful solution tool. These numerical results are reported in (Jayasekara et al., submitted, 2021).

Despite the good performance of the mpASA, we have decided to advance the theory behind this algorithm and are currently working on its suitable modification that will account for the biconvexity. We are in process of finding new error bounds resulting from the biconvexity and proving the convergence. We also design the parts of Algorithm 6. At every node of the BB tree, an spbcQP is solved with the mpSubA. Because the fathoming rules act on the obtained optimal solutions and objective values that are functions of the parameter, we develop new data structures for storing these parametric functions. We are ready to start a MATLAB implementation of Algorithm 6 as soon as we complete the modification of the mpASA. This is currently work-in-progress with a graduate student who is going to graduate in 2022.

3. Application of algorithms to real-life decision problems

3.1. Portfolio optimization (Jayasekara et al., 2019)

In portfolio optimization, quadratic functions typically model the risk associated with the investment while linear functions model the return on the investment. While it has been of interest to solve multiobjective portfolio optimization problems with two or more quadratic objective functions (MOPOPs), this task had not been possible due to lack of a suitable algorithm. We apply Algorithm 2 to solve the MOPOP and show the real-life context benefits.

We solve the MOPOP in two ways. The first approach is traditional and aggregates all objectives by a weighted-sum, leading to an mpQP which is solved as a gen-mpLCP by Algorithm 2. In the second approach, we propose a new modified hybrid method, a new scalarization of the MOPOP which makes use of two independent parameter vectors. One of them serves as a vector of weights to aggregate the quadratic risk functions, while the other vector determines the expected levels of return on the investment. The resulting single objective problem is again of the form of an mpQP which is solved with Algorithm 2.

In both approaches, Algorithm 2 yields Pareto solutions to the MOPOP as functions of the parameter but the modified hybrid method allows for a more insightful analysis. The DM can choose any positive weights for the risk functions and calculate the Pareto portfolios (along with their associated return and risk) among which there is a portfolio minimizing the weighted risk. Alternatively, the DM can choose any level of the expected return and calculate Pareto portfolios that generate the lowest weighted risk.

We intended to apply Algorithm 2 to portfolio optimization with mixed-binary variables. In some cases, one may want to control the number of assets in a portfolio due to limited cognitive capacities of DMs or limited operational capacities of financial institutions. Under such settings, the DM may need to introduce constraints on selecting individual assets into the portfolio, which gives rise to mixed-binary variables. However, since these variables show up in the constraints but not in the objective functions, the resulting model is not a good application for Algorithm 4.

3.2 Machine learning (Jayasekara et al., submitted, 2021)

The elastic net problem, a practical application problem of high importance in machine learning, includes three limit cases: (i) least shrinkage and selection operator (LASSO), (ii) ridge regression, and (iii) ordinary least squares, and can be formulated as a triobjective quadratic optimization problem (TQOP). We apply Algorithm 2 to solve the TQOP to demonstrate its effectiveness and show the real-life context benefits.

3.3 Engineering design

As explained in our 2019 interim report, collaborating with colleagues in the Mechanical Engineering Department at Clemson University, we intended to apply Algorithm 4 to a routing problem associated with the vehicle underhood design. This routing problem is a location problem seeking shortest paths from the origin to multiple destinations in the presence of (convex or nonconvex) barriers and subject to the requirement that the shortest paths, starting from the origin, overlap before they split to reach each destination so that the total length of all paths is the shortest. Unfortunately, the final model of this engineering problem (Masoudi et al., 2019) is in the form of a biobjective mixed-binary polynomial optimization problem rather than the BOMIQP and Algorithm 2 cannot be used.

3.4 Radiation therapy

We apply Algorithm 3 to a radiation therapy problem modeled as an unconstrained BOQP with identity matrices in quadratic terms. While this application is of high importance, the simple quadratic terms and the lack of constraints make this application numerically too simple and does not allow us to show the effectiveness of Algorithm 3.

4. Research in robust multiobjective optimization

Parametric optimization, a main theme in this project can be applied to model uncertainty because parameters can represent unknown or uncertain data, and a parametric Pareto solution is

the decision to make for all uncertainty realizations. Under certain assumptions, parametric optimization can be transformed into robust optimization in which a robust Pareto solution is the decision to make in a worst-case realization or other specific realizations. Our work on parametric optimization and multiobjective optimization has naturally led us to research on robust multiobjective optimization, which we have conducted in parallel to the research proposed for this project.

4.1 Min-max robust efficiency (Doolittle et al., 2018)

We provide a theoretical study of robust multiobjective optimization by verifying that the techniques of robust single objective optimization remain valid in the case of a vector-valued objective function. For the column-wise uncertainty of (Soyster, 1973), we obtain an immediate generalization to the multiobjective case. For the row-wise uncertainty model of (Ben-Tal and Nemirovski, 1999) and under some conditions, we show that min-max robust Pareto solutions to the original uncertain problem are also Pareto to specific instances of this problem. We develop an algorithm to compute these solutions and apply it to an intradomain biobjective routing problem with polyhedral traffic uncertainty. We use objective functions corresponding to link utilizations, present computational results for the Abilene network, and analyze their meaning in the context of the application.

4.2. Highly robust efficiency (Dranichak and Wiecek, 2018a, 2018b)

We present the first in-depth analysis of highly robust efficient (HRE) solutions to uncertain multiobjective linear programs (UMOLPs) with uncertainty in the objective function coefficients. We first develop properties of the HRE set. Under a condition of acuteness, we provide a characterization of the HRE set by means of the normal cone and the strict polar of the closed cone of improving directions associated with every instance of the UMOLP. The acuteness of the closed cone of improving directions also leads to a lower bound set on the HRE set and guarantees that the HRE set is nonempty. Furthermore, the convexity and acuteness of the latter cone together lead to an MOLP that is a robust counterpart for the UMOLP. The convexity of the cone may be verified and its algebraic representation may be computed by an existing algorithm. The acuteness of the cone may be checked by either of two proposed methods both of which are easily performed using readily available software.

We provide methods for checking whether or not the HRE set is empty, for computing HRE points, and for determining whether a given feasible solution is HRE. The study is applied to an UMOLP modeling a bank management application.

4.3. Multiobjective robustness gap (Krüger et al., submitted, 2020)

In robust single-objective optimization, the robustness gap is a measure of the distance between the robust optimal objective value and the optimal objective values of the scenarios. While robust multiobjective optimization is a growing field of study, no notion of a robustness gap has been proposed. We introduce a new concept of a robustness gap for uncertain multiobjective optimization problems (MOPs) and define it as the minimal distance between the robust Pareto set and the Pareto sets of the realizations.

We relate the proposed gap to the classical robustness gap concept in the single-objective case. The well-known result about the single-objective zero robustness gap is generalized for a convex multiobjective setting, i.e., the multiobjective robustness gap is zero whenever the uncertainty is constraint-wise and objective-wise. Moreover, we show that for every element in the robust Pareto set, there exists a scenario whose Pareto set is arbitrarily close to that specific robust Pareto objective vector. Because Pareto sets are typically nonconvex even for linear programs and computing a minimal distance between them is hard, lower and upper bounds for the robustness gap are derived for convex MOPs.

We present more theoretical results and the first ever computation of the gap and bounds for the linear case, and also include practical observations on the use of the bounds to support decision making.

4.4. Preserving efficiency under uncertainty (Kerivin et al., submitted, 2021)

We examine the dependence of the Pareto set of a general (finite) MOP with respect to two types changes in the objective vector. The first type results from using vector- or scalar-valued operators to modify the original finite set of objectives into a new, possibly infinite, set of objectives. The other type modifies the original objectives infinitely many times to yield another (infinite) MOP (the all-in-one (AiO) model) or an infinite collection of (finite) MOPs (the all-at-once (AaO) model). The two types lead to four cases for which the obtained theoretical results rely on monotonicity of the operators but otherwise hold for general functions without any convexity assumptions.

Preservation of efficiency is next understood as robustness of the Pareto set under function uncertainty proposed for the first time and modeled by augmentation or reduction of the set of objectives. The four cases lead to four models of UMOPs with different types of uncertainty sets that are made up of general functions. Special results are obtained for uncertainty sets built with linear operators. The AiO model allows to preserve efficiency and make it “robust” under fewer assumptions than the AaO-approach. Taking that the former is arguably a more realistic model than the latter, makes the former more attractive. An uncertain biobjective engineering design problem motivating the entire study is solved for illustration.

4.5. Multiobjective complex systems (Dietz et al., 2020)

Complex systems are very common in the modern world. Financial markets, social networks, communication systems, public health providers, cyber-security systems, global corporations, educational organizations are all examples of complex systems that are composed of multiple but dissimilar parts in the form of subsystems that give rise to the collective behavior of the overall system. On one hand, these subsystems are subject to multiple objectives, local constraints as well as local variables, and they are associated with an own, subsystem-dependent decision maker. On the other hand, these subsystems are interconnected to each other by global variables or linking constraints. Due to these interdependencies, it is in general not possible to simply optimize each subsystem individually to improve the performance of the overall system.

We extend the state-of-the-art theory of multiobjective optimization to model a system whose complexity is reflected in the interaction among the subsystems, a feature that has not been addressed before in a multiobjective setting. The complexity requires that domination-based

efficiency, which recognizes conflict between the objective functions, be lifted to the new concept of system-domination-based superiority, which accounts for conflict between subsystems as well as their objective functions. The goal of decision making is to find superior solutions for the complex system without ever dealing with this system in its entirety. The complex systems for which this task is easily achieved are identified. Hierarchical and scalarization based algorithms are designed to compute superior solutions. In this context, ε -superiority is defined to approximate the set of superior solutions. When superior solutions are unattainable, methods for finding compromise solutions are proposed.

IV. Training Opportunities

The Ph.D. students, who have worked on various parts of this project, benefited from training opportunities in the following areas:

1. Optimization theories and methodologies: the linear complementarity problem (LCP) and multiparametric LCP; multiparametric linear, quadratic, and nonlinear programs, each class with continuous or mixed-integer variables; multiobjective portfolio selection; multiobjective branch and bound; convexification theory for global optimization; cutting planes and polyhedral relaxations for MIQCQPs;
2. Optimization software: Multiparametric and Optimization toolboxes for MATLAB, the AMPL modeling language, and the ILOG CPLEX Optimization Studio for mixed-integer optimization.
3. Real Algebraic Geometry: Properties of semi-algebraic sets: dimension, ambient isotopic sets, semi-algebraically (path) connected sets.

Below for each PhD student we give the name, title of the doctoral thesis, advisor's name, date of graduation and the first job (if applicable).

Nathan Adelgren, *Solution Techniques for Classes of Biobjective and Parametric Programs*, co-advised by both PIs, graduated in August 2016, first job: Department of Mathematics and Computer Science, Edinboro University, Edinboro, PA. Since May 2021, Associate Research Scholar at Andlinger Center for Energy and the Environment, Princeton University.

Michael Eldredge, advised by the co-PI, took a leave of absence in 2020, and then quit Clemson University.

Pubudu L.W. Jayasekara, *A Branch-and Bound Algorithm for Biobjective Mixed-Integer Quadratic Programs*, advised by the PI, expects to graduate in December 2021.

Hamid Nazari, advised by the co-PI but switched to another faculty member when the co-PI left Clemson University.

Andrew Pangia, *On Parametric Quadratically Constrained Quadratic Programs*, advised by the PI, expects to graduate in 2022.

Additionally, two Ph.D. students have been advised by the PIs on topics related to this project:

Garrett Dranichak, *Highly Robust Solutions for Multiobjective Optimization*, advised by the PI, graduated in May 2018, first job: Sandia National Lab, Albuquerque, New Mexico.

Yibo Xu, *Convex Hulls, Relaxations, and Approximations of General Monomials and Multilinear Functions*, co-advised by the co-PI and Dr. Warren Adams, graduated in May 2018, first job: postdoctoral researcher, Rensselaer Polytechnic Institute, Troy, NY.

V. Dissemination of Results

We use three modes for disseminating the results of this project and related findings.

1. Journal Papers: Some preprints of manuscripts submitted to journals for publication are made available on the internet on preprint websites such as arXiv and Optimization Online. In part VIII. Products, a list of publications is provided. This list includes the papers that are directly related to our accomplishments as well as the papers on related topics that emerged as a by-product of this project.

2. Conference & Seminar Presentations: only presentations by the PIs and the students involved in this project are listed below (talks by co-authors are not listed).

3. Mini-symposia and conference sessions organized by the PIs are also reported below.

PRESENTATIONS given by M.M. Wiecek

Invited

“Robust Multiobjective Optimization for Decision Making under Conflict and Uncertainty”, CREDO Seminar, Department of Mechanical Engineering, Clemson University, October 24, 2016.

“Robust Multiobjective Optimization for Decision Making under Conflict and Uncertainty”, 75-minute INFORMS Tutorial, INFORMS Annual Meeting, Nashville, November 15, 2016.

“Robust Multiobjective Optimization for Decision Making under Conflict and Uncertainty”, Management Science Seminar, Lancaster University, United Kingdom, February 8, 2017. Colloquium.

“Robust Multiobjective Optimization for Decision Making under Conflict and Uncertainty”, Decision and Cognitive Sciences Seminar Series, Alliance Manchester Business School, The University of Manchester, Manchester, United Kingdom, March 15, 2017. Colloquium.

“Multiobjective Complex Systems: Decomposition, Coordination, and Applications in Engineering Design”, Department of Mathematics, University of Wuppertal, Wuppertal, Germany, May 30, 2017. Colloquium.

Multiobjective Complex Systems: Decomposition, Coordination, and Applications in Engineering Design”, Department of Mathematics, University of Kaiserslautern, Kaiserslautern, Germany, June 6, 2017. Colloquium.

“On the Robustness Gap in Uncertain Multiobjective Optimization“, INFORMS Annual Meeting, Houston, October 23, 2017.

“Introduction to Personalized Multiobjective Optimization – An Analytics Perspective”, Dagstuhl, Germany, January 15, 2018.

“On Highly Robust Efficient Solutions to Uncertain Multiobjective Linear Programs”, Industrial Optimization Group Seminar, Department of Mathematical Information Technology, University of Jyväskylä, Finland, October 16, 2018. Colloquium.

“On Highly Robust Efficient Solutions to Uncertain Multiobjective Linear Programs”, INFORMS Annual Meeting, Phoenix, November 5, 2018.

“Optimization under Uncertainty and Conflict: Algorithms for Heterogeneous Quadratic Programs”, Optimization and Discrete Mathematics Program and Peer Review, Office of Naval Research, Washington DC, December 7, 2018.

“On Robust Solutions to Uncertain Multiobjective Linear Programs”, Department of Mathematics, University of Tennessee at Chattanooga, April 8, 2019. Colloquium.

“The Pareto Set: An Evolving Challenge in Multiobjective Optimization,” 25th International Conference on MCDM, Istanbul, Turkey, June 21, 2019.

“Portfolio Optimization with Multiple Quadratic Functions”, INFORMS Annual Meeting, Seattle, October 20, 2019.

M.M. Wiecek, “Toward a Branch-and-Bound Algorithm for Biobjective Mixed-Integer Quadratic Programs”, EUROPT 2021, Toulouse, France, July 9, 2021 (online).

Contributed

“On Convex Multiobjective Programs with Application to Portfolio Optimization,” MCDM 2019, Istanbul, Turkey, June 19, 2019.

“On Highly Robust Efficient Solutions to Uncertain Multiobjective Linear Programs”, MOPGP 2019, Marrakech, Morocco, October 29, 2019.

“Revealing Tradeoffs in Multiobjective Optimization with Applications in Engineering Design,” CEDAR Seminar Series, Department of Mechanical Engineering, Clemson University, Clemson, July 10, 2020.

PRESENTATIONS given by A. Gupte

Invited

“The Graded Lexicographic Polytope and Dantzig Figures,” SIAM Conference on Discrete Mathematics, Atlanta, June 2016.

“Approximation Guarantees for Monomial Convexification in Polynomial Optimization,” MOPTA 2016, Lehigh University, August 2016.

“Analysis of Strength of Convex Relaxations of Monomials,” INFORMS Annual Meeting, Nashville, November 2016.

“Errors from Monomial Convexification in Polynomial Optimization,” in Mini-symposium on Convexifications in Mixed Integer Nonlinear Optimization, SIAM Conference on Optimization, Vancouver, May 2017.

“Ordered Sets in Discrete Optimization,” Mini-symposium on Algebraic and Geometric Methods in Discrete Optimization, SIAM Conference on Applied Algebraic Geometry, Atlanta, July 2017.

“Disjunctive Cuts and Extended Formulations for Some Bilinear Functions,” INFORMS Annual Meeting, Houston, October 2017.

“Lexicographic Approximations of Integer Programs,” INFORMS Annual Meeting, Houston, October 2017.

“Disjunctive Cuts and Extended Formulations for Some Bilinear Functions,” Dagstuhl Seminar on Mixed Integer Nonlinear Optimization, Schloss Dagstuhl, Germany, February 2018.

“Discrete Structure in Nonconvex Optimization,” Department of IE Seminar, University of Pittsburgh, March 2018.

“Novel Formulations and Relaxations for MINLP,” School of ORIE Colloquium, Cornell University, March 2018.

“Strong Formulations for Integer Programs and Nonconvex Quadratics,” Department of ISE Seminar, Virginia Tech, April 2018.

“Ordered Sets and Their Polytopes in Mixed-integer Optimization,” Optimization and Discrete Geometry: Theory and Practice, Tel Aviv University, Israel, April 2018.

“Polyhedral Relaxations for Nonconvex Quadratic Functions,” Mini-symposium on Mixed Integer Nonlinear Programming, ISMP 2018, Bordeaux, July 2018.

“A New Extended Formulation for Unstructured Integer Programs,” INFORMS Annual Meeting, Phoenix, November 2018.

Presentations given by the graduate students

M. Eldredge, “Some Lexicographic Perspectives on Valid Inequalities”, INFORMS Annual Meeting, Nashville, November 15, 2016.

Y. Xu, “On the Strength of Linear Approximations for Multilinear Monomials,” INFORMS Annual Meeting, Nashville, November 14-17, 2016.

M. Eldredge, “A Two-term Disjunction for Mixed-integer Programming”, Poster session at Mixed Integer Programming (MIP) Workshop, Montreal, Canada, June 19, 2017.

N. Adelgren, “New Algorithms for the Multiparametric Linear Complementarity Problem,” INFORMS Annual Meeting, Houston, October 23, 2017.

M. Eldredge, “A Two-Term Disjunction for MIP,” INFORMS Annual Meeting, Houston, October 22-25, 2017.

P.L.W. Jayasekara, “Multiobjective Quadratic Programs: State of the Art,” INFORMS Annual Meeting, Houston, October 23, 2017.

H. Nazari, “Integer Programming for Equilibrium Pricing,” Workshop on Mixed-Integer Programming (MIP 2018), Greenville, SC, June 18, 2018.

P.L.W. Jayasekara, “A Modified Hybrid Method for Multiobjective Portfolio Optimization Problem,” Graduate Student Seminar, Clemson University, Clemson, October 22, 2018.

H. Nazari, “Optimizing a Bundle Pricing Problem,” INFORMS Annual Meeting, Phoenix, November 6, 2018.

P.L.W. Jayasekara, “A Smart Way to Make your Money Grow,” Graduate Student Seminar, Clemson University, Clemson, February 27, 2019.

P.L.W. Jayasekara, “A Smart Way to Make your Money Grow,” Clemson University, poster presentation at “GRADS 2019,” April 2019. Recipient of the First Place Award.

H. Nazari, “Integer Programming Approaches for Envy-Free Equilibrium Pricing,” INFORMS Annual Meeting, Seattle, October 23, 2019.

P.L.W. Jayasekara, “Biobjective Set Domination under Limited Information,” OR Seminar - School of Mathematical and Statistical Sciences, Clemson University, Clemson, October 5, 2020 (online).

P.L.W. Jayasekara, “Biobjective Set Domination under Limited Information, INFORMS Annual Meeting, November 13, 2020 (online).

P.L.W. Jayasekara, “Biobjective Mixed Integer Quadratic Programs,” Lecture for MATH 8190 - School of Mathematical and Statistical Sciences, Clemson University, Clemson, December 1, 2020 (online).

P.L.W. Jayasekara, “Domination between Two Pareto Sets under Limited Information,” Southeast Joint SIAM Student Conference, April 3, 2021 (online).

A. Pangia, “On Solving Parametric Multiobjective Quadratic Programs with Parameters in General Locations,” Southeast Joint SIAM Student Conference, April 3, 2021 (online).

P.L.W. Jayasekara, “A Branch-and-Bound Algorithm for Biobjective Mixed-Integer Biobjective Quadratic Programs,” recorded & submitted to INFORMS Annual Meeting, Anaheim, October 24-27, 2021 (online).

A. Pangia, “On Solving Parametric Multiobjective Quadratic Programs with Parameters in General Locations,” recorded & submitted to INFORMS Annual Meeting, Anaheim, October 24-27, 2021 (online).

Conference Organization by M.M. Wiecek

MCDM Cluster Chair, INFORMS National Meeting, Nashville, November 13-16, 2016.
Organized 16 sessions.

MCDM Cluster Chair, INFORMS National Meeting, Houston, October 22-25, 2017.
Organized a session entitled “Parametric LCP and Multiobjective Optimization” in the MCDM Cluster.

Session Organizer and Chair: “MCDM Awards” and “Theory and Applications of Multiobjective Optimization”, INFORMS Annual Meeting, Phoenix, November 4-7, 2018.

Session Organizer: “MCDM Awards” and “Theory and Applications of Multiobjective Optimization”, INFORMS Annual Meeting, Seattle, October 20-23, 2019.

Session Organizer and Chair: “Advancing Theory and Methodology of Multiobjective Optimization”, INFORMS Annual Meeting, Anaheim, October 24-27, 2021.

Conference Organization by A. Gupte

INFORMS Annual Meeting, Nashville, November 2016, chaired two sessions with four talks in each: “Advances in Mixed-Integer Polynomial Optimization” in Global Optimization Cluster, and “New Results on Polyhedral Relaxations” in Integer Programming Cluster.

SIAM Conference on Optimization, Vancouver, May 2017, organized and chaired mini-symposium “Convexification in Mixed-Integer Nonlinear Optimization” with four talks.

INFORMS Annual Meeting, Houston, October 2017, organized two sessions: “Mixed-Integer Polynomial Optimization” in Global Optimization Cluster with four talks and “Advances in Integer Programming” in Integer Programming Cluster with four talks.

15th Mixed Integer Programming Workshop MIP 2018 at Clemson University, Organizing Committee Chair.

23rd International Symposium on Math Programming (ISMP 2018), Bordeaux, France, July 2018, organized mini-symposium “Convexification and more” in the MINLP stream.

VI. Awards received by M.M. Wiecek and her advisees

MCDM Gold Medal presented by the International Society on Multicriteria Decision Making, 2019. This is the highest honor that this Society bestows upon a scholar who, over a distinguished career, has markedly contributed to the theory, methodology, practice and professional development of MCDM.

<https://www.mcdmsociety.org/content/awards>

<https://newsstand.clemson.edu/mathematician-wiecek-earns-gold-medal-from-prestigious-international-society/>

Excellence in Discovery Faculty Award presented by the College of Science, Clemson University, 2020, <https://www.youtube.com/watch?v=DUgHygiFI5g>

Invited to join the Dagstuhl Seminar Committee for the 2018, 2020, 2023 Seminars on MCDM. Dagstuhl seminars (similar to Oberwolfach seminars) in Germany are in the group of the most prestigious professional events in the world in the area of mathematics and computer science. These events are closed to the general academic community. For each seminar, the program is prepared and the attendees are invited by the Committee that consists of top-level experts in the field.

<https://www.dagstuhl.de/en/program/calendar/semhp/?semnr=18031>

<https://www.dagstuhl.de/en/program/calendar/semhp/?semnr=20031>

Finalist for the 2020 MCDM Junior Researcher Best Paper Award

Paper entitled “On highly robust efficient solution for multiobjective linear programs”, *European Journal of Operational Research*, 273 (1): 20–30, 2018, co-authored by graduate student G. Dranichak and the PI is one of the three finalists for the MCDM Junior Researcher Best Paper Award, INFORMS Annual Meeting, Washington, 2020.

<https://www.informs.org/Recognizing-Excellence/Award-Recipients/Garrett-Dranichak>

Winner of a Poster Competition P.L.W. Jayasekara is the Recipient of the First Place Award in the Poster Competition GRADS 2019 at Clemson University, April 2019.

<https://gsg.people.clemson.edu/initiatives/grads.php>

VII. Participants

Role	Name	Man-months
PI	Margaret M. Wiecek	3
Co-PI	Akshay Gupte	3
PhD students	Nathan Adelgren	3
	Michael Eldredge	5.5
	Pubudu L.W. Jayasekara	20
	Hamid Nazari	22
	Andrew Pangia	12

VIII. Products

PUBLICATIONS

Books

N. Adelgren, *Advancing Parametric Optimization: Theory and Solution Methodology for Multiparametric Linear Complementarity Problems with Parameters in General Locations*, SpringerBriefs on Optimization Series, 2021.

<https://www.springer.com/gp/book/9783030618209>

Refereed journal papers published

W. Adams, A. Gupte, and Y. Xu, Error bounds for monomial convexification in polynomial optimization, *Mathematical Programming Series A*, 175: 355–393, 2018.

<https://link.springer.com/content/pdf/10.1007%2Fs10107-018-1246-8.pdf>

N. Adelgren and A. Gupte, Branch-and-bound for biobjective mixed-integer linear programming, *INFORMS Journal on Computing*, accepted, in print, 2021.

http://www.optimization-online.org/DB_HTML/2016/10/5676.html

N. Adelgren and M.M. Wiecek, A two-phase algorithm for the multiparametric linear complementarity problem, *European Journal on Operational Research*, 254 (3), 715–738, 2016.

<https://www.sciencedirect.com/science/article/pii/S0377221716302806>

T. Dietz, K. Klamroth, K. Kraus, S. Ruzika, L. Schäfer, B. Schulze, M. Stiglmayr, and M. M. Wiecek, Introducing multiobjective complex systems, *European Journal of Operational Research*, 280: 581–596, 2020.

<https://www.sciencedirect.com/science/article/pii/S0377221719305910>

E. Doolittle, H. Kerivin, and M.M. Wiecek, Robust multiobjective optimization problem with application to internet routing, *Annals of Operations Research*, 271: 487–525, 2018.
<https://link.springer.com/article/10.1007/s10479-017-2751-5?shared-article-renderer>

G.M. Dranichak and M.M. Wiecek, On highly robust efficient solution for multiobjective linear programs, *European Journal of Operational Research*, 273 (1): 20–30, 2018a.
<https://www.sciencedirect.com/science/article/pii/S037722171830657X?via%3Dihub>

G.M. Dranichak and M.M. Wiecek, On computing highly robust efficient solutions, *Journal of Multi-Criteria Decision Analysis*, 25 (5-6): 177–188, 2018b.
<https://onlinelibrary.wiley.com/doi/abs/10.1002/mcda.1653>

A. Gupte, S. Ahmed, S. Dey, M.M. Cheon, Relaxations and discretizations for the pooling problem, *Journal of Global Optimization*, 67 (3): 631–669, 2017.
<https://link.springer.com/article/10.1007/s10898-016-0434-4>

A. Gupte, T. Kalinowski, F. Rigterink, H. Waterer, Extended formulations for convex hulls of some bilinear functions, *Discrete Optimization*, 36, 34 p., 100569, 2020.
<https://www.sciencedirect.com/science/article/pii/S1572528620300037>

A. Gupte, S. Poznanović, On Dantzig figures from graded lexicographic orders, *Discrete Mathematics*, 341 (6): 1534–1554, 2018.
<https://www.sciencedirect.com/science/article/pii/S0012365X18300554>

P. L.W. Jayasekara, N. Adelgren, and M. M. Wiecek, On convex multiobjective programs with an application to portfolio optimization, *Journal of Multi-Criteria Decision Analysis*, 27 (3-4) 189–202, 2019. <https://onlinelibrary.wiley.com/doi/abs/10.1002/mcda.1690>

J. Lu, A. Gupte, Y. Huang, A mean-risk MINLP for transportation network protection, *European Journal of Operational Research*, 265 (1): 277–289, 2018.
<https://www.sciencedirect.com/science/article/pii/S0377221717306525>

PH.D. THESES

N. Adelgren, “Solution Techniques for Classes of Biobjective and Parametric Programs,” 2016.
https://tigerprints.clemson.edu/cgi/viewcontent.cgi?article=2755&context=all_dissertations

G. Dranichak, “Highly Robust Solutions for Multiobjective Optimization,” 2018.
https://tigerprints.clemson.edu/cgi/viewcontent.cgi?article=3157&context=all_dissertations

Y. Xu, “Convex Hulls, Relaxations, and Approximations of General Monomials and Multilinear Functions,” 2018. https://tigerprints.clemson.edu/all_dissertations/2094/

P. L.W. Jayasekara, “A Branch-and Bound Algorithm for Biobjective Mixed-Integer Quadratic Programs,” in preparation, is going to graduate in December 2021.

A. Pangia, “On Parametric Quadratically Constrained Quadratic Programs,” in preparation, is going to graduate in 2022.

SOFTWARE

N. Adelgren, MATLAB code for solving multiparametric LCP with parameters in general locations, https://github.com/Nadelgren/mpLCP_solver, posted in 2019.

SUBMITTED PAPERS

P. L.W. Jayasekara, A. Pangia, and M.M. Wiecek, On solving parametric multiobjective quadratic programs with parameters in general locations, submitted to *Annals of Operations Research*, 2021.

H. Kerivin, M.M. Wiecek, R. Sadiwala, and E. Doolittle, Preserving efficiency in multiobjective optimization under uncertainty, submitted to *INFORMS Journal on Optimization*, 2021.

C. Krüger, A. Schöbel, and M.M. Wiecek, The robustness gap in uncertain multiobjective optimization, revised & resubmitted to *Mathematical Programming Series A*, 2020.

J. Leverenz, H. Lee, and M.M. Wiecek, A subgradient algorithm for multiparametric optimization, revised & submitted to *ESAIM: Control, Optimisation and Calculus of Variations*, 2020.

Y. Xu, W. Adams, and A. Gupte, Polyhedral analysis of symmetric multilinear polynomials over box constraints, 2021.

WORKING PAPERS

M. Eldridge and A. Gupte, On lexicographic approximations of integer programs.
<https://arxiv.org/abs/1610.06470>

PAPERS IN PREPARATION

P. L.W. Jayasekara and M.M. Wiecek, A branch-and-bound algorithm for biobjective mixed-integer quadratic programs.

H. Nazari and A. Gupte, Integer programming for envy-free equilibrium pricing.

A. Pangia and M.M. Wiecek, An algorithm for parametric mixed-binary quadratically constrained quadratic programs.

IX. Professional Service

M.M. Wiecek

Professional Societies

INFORMS, MCDM Section

President (2016 – 2017): Established the MCDM Junior Researcher Best Paper Award
Board member and Award Coordinator (2018 – 2019)

International Society on MCDM, Executive Committee member (2016 – 2019)

Guest co-editor

Annals of Operations Research: special volume “Theory, Computation, and Practice of Multiobjective Optimization”, to appear in 2022, with M. Ehrgott, and A. Engau.

<https://connect.informs.org/communities/community-home/digestviewer/viewthread?MessageKey=daf1fdc3-cc8d-47a5-a811-8e31a72e30fd&CommunityKey=1d5653fa-85c8-46b3-8176-869b140e5e3c&tab=digestviewer>

Computers and Operations Research: special volume “Modern Trends in Multiobjective Optimization”, to appear in 2022, with C. Fonseca and K. Klamroth.

<https://www.sciencedirect.com/journal/computers-and-operations-research/special-issue/106PR6N55D3>

Editorial Board member

Invited, *Journal of Optimization Theory and Applications*, 2021

Ph.D. Reviewer in Other Countries

Y. Zhou-Kangas, “Interactive Methods for Multiobjective Robust Optimization” (October 2018)
University of Jyväskylä, Finland.

https://jyx.jyu.fi/bitstream/handle/123456789/59712/978-951-39-7549-4_vaitos15102018.pdf?sequence=1&isAllowed=y

M. Milano, “Solution Approaches for Multiobjective Convex and Nonlinear Optimization Problems” (July 2019) Bergische Universität Wuppertal, Germany.

<http://elpub.bib.uni-wuppertal.de/servlets/DerivateServlet/Derivate-9209/dc1926.pdf>

Dagstuhl Seminar Leader

Seminar “Personalized Multiobjective Optimization: An Analytics Perspective”, Dagstuhl, Germany, January 14-19, 2018.

<https://www.dagstuhl.de/en/program/calendar/semhp/?semnr=18031>

Seminar “Scalability in Multiobjective Optimization”, Dagstuhl, Germany, January 12-17, 2020.
<https://www.dagstuhl.de/en/program/calendar/semhp/?semnr=20031>

Seminar “Multiobjective Optimization in Dynamic and Uncertain Environments”, proposal submitted to Dagstuhl, Germany, April 2021.

External Evaluator for Vienna Graduate School on Computational Optimization

Participated in the hearing in Vienna, Austria, October 8-9, 2019.

Conference Organization

Scientific Program Committee Member: MCDM 2019, Istanbul, Turkey, June 16-21, 2019.
Reviewed 4 papers.

Scientific Committee Member and Session Chair: MOPGP 2019, Marrakech, Morocco, October 28-31, 2019. Reviewed 3 papers.

Session Chair: Recent Advances in Multiobjective Optimization (RAMOO 2020) Linz, Austria September 17, 2020 (online).

A. Gupte

Professional Societies

INFORMS Optimization Society, Vice-Chair (2018-2020) and Vice-Chair of the Integer and Discrete Optimization cluster (2018-2019)

INFORMS Computing Society, Vice-Chair (2020-2021) and Chair (2022-2023)

Editor

Operations Research Forum (Springer), Associate Editor, 2019 – present

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- A. Bemporad and C. Filipi, An algorithm for approximate multiparametric convex programming, *Computational Optimization and Applications*, 35 (1): 87–108, 2006.
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- H. Bonnel and Ch. Schneider, Post-Pareto analysis and a new algorithm for the optimal parameter tuning of the elastic net, *Journal of Optimization Theory and Applications*, 183: 993–1027, 2019.
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- S. L. Gadegaard, L. R. Nielsen, and M. Ehrgott, Bi-objective branch-and-cut algorithms based on LP relaxation and bound sets, *INFORMS Journal on Computing*, 31(4): 790–804, 2019.
- M. Herceg, M. Kvasnica, C.N. Jones, and M. Morari. Multi-Parametric Toolbox 3.0. In *Proc. of the European Control Conference*, pages 502–510, Zurich, Switzerland, July 17–19 2013.
<http://control.ee.ethz.ch/~mpt>
- L. Kolev and I. Skalna, Exact solution to parametric linear programming problem, *Numerical Algorithms*, 78 (4): 1183–1194, 2018.
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