



AFRL-AFOSR-JP-TR-2021-0004

**Systems Theory Approach to Modeling and Robust Design of Quantum
Information Processing Devices**

James, Matthew
AUSTRALIAN NATIONAL UNIVERSITY RESEARCH OFFICE ACTON (AUSTRALIA)
10C EAST RD
ACTON, ,
AU

07/25/2021
Final Technical Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
Air Force Office of Scientific Research
Asian Office of Aerospace Research and Development
Unit 45002, APO AP 96338-5002

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE (DD-MM-YYYY) 25-07-2021		2. REPORT TYPE Final		3. DATES COVERED (From - To) 30 Sep 2016 - 29 Feb 2020	
4. TITLE AND SUBTITLE Systems Theory Approach to Modeling and Robust Design of Quantum Information Processing Devices				5a. CONTRACT NUMBER FA2386-16-1-4065	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER 61102F	
6. AUTHOR(S) Matthew James				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) AUSTRALIAN NATIONAL UNIVERSITY RESEARCH OFFICE ACTON (AUSTRALIA) 10C EAST RD ACTON, AU				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AOARD UNIT 45002 APO AP 96338-5002				10. SPONSOR/MONITOR'S ACRONYM(S) AFRL/AFOSR IOA	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-AFOSR-JP-TR-2021-0004	
12. DISTRIBUTION/AVAILABILITY STATEMENT A Distribution Unlimited: PB Public Release					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT See attached report					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON CHRISTOPHER VERGIEN
a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (Include area code) 315-227-7002
U	U	U	SAR	17	

Final Report for AOARD Grant FA2386-16-1-4065

Linear Quantum Systems: Non-Classical
States and Robust Stability

Matthew R. James
and
Ian R. Petersen

Research School of Electrical, Energy and Materials Engineering
Australian National University

Period of Performance: 2015 – 2020

Summary:

This project will develop new quantum linear systems concepts and methods for application to the design of quantum information processing devices so that robust performance can be achieved. Feedback control will feature prominently in this research because of feedback's inherent robustness characteristics. Linear quantum stochastic models will be employed to capture functional dynamical behavior at a level between abstract qubits and very low-level physical detail. In this manner new approaches to the modelling and design of quantum information devices will be achieved. This project will advance systems and control theory through this focus on key engineering issues for quantum technology.

Introduction:

Linear systems theory. Classical linear systems theory has a history going back some 50 years, to the birth of modern control theory with Kalman's foundational work on filtering and LQG optimal control. However, there are new imperatives arising in emerging quantum technologies that demand new concepts and tools from control theory. This has motivated recent work in developing aspects of a new quantum linear systems theory intended to provide a framework for exploiting the special features of quantum mechanics. This recent work includes coherent quantum H-infinity and LQG control, physical realization, entanglement generation limitations, passivity characterizations, response to single photon fields, coherent observer design, robust stability theory, quantum memories, non-classical and pure Gaussian state generation, and synchronization of quantum nano-mechanical systems. The work to date in this area motivates new research concerning the development and application of systems theory approaches to the modelling and design of quantum devices engineered for quantum information processing. This in turn will lead to new concepts and methods in quantum feedback control theory.

Quantum Information Processing. Quantum computing and quantum information are two emerging technologies with significant security implications. These technologies are among the first of a new generation of technologies that by exploiting the unique resources of quantum mechanics, promise significantly faster computing and secure communication. In order to realize such technologies, significant challenges exist in the design and fabrication of the constituent components. These components include quantum memories, quantum repeaters, quantum amplifiers, and quantum state sources. Quantum memories are used for storing quantum states, and additionally, may be used for processing the stored states. Due to signal loss over distance, some form of regeneration or repeater systems for quantum states is needed for long distance transmission of quantum information. The quantum states needed for computing and communication, such as pure Gaussian, single photons and Schrodinger cat states, need to be created on-demand (not randomly). One key characteristic of classical analog and digital electronic systems is the robustness that has been enhanced by their design, for example, through feedback control (analog amplifier stabilization, digital flip flop latching, etc).

The primary goal of this research is to develop linear systems concepts and methods. These would be applied to the design of quantum information processing devices so that robust performance can be achieved. In these methods, the use of feedback will play a prominent role.

Experiment: N/A

Results and Discussion:

Linear systems theory.

Significant progress has been made on the analysis and design of quantum feedback systems using the risk-sensitive criterion with AFOSR-funded Research Fellow Dr Igor Vladimirov.

New results have been obtained that resolve a long-standing open question concerning the robustness properties for optimal risk-sensitive controllers for quantum systems. The risk-sensitive optimal control problem for quantum systems was formulated and solved in 2004 and 2005 using a time-ordered exponential criterion. However, the question of robustness was unresolved. Fundamental robustness inequalities for a direct non time-ordered exponential criterion have been derived. The time-ordered criterion can be transformed to an equivalent direct exponential, thereby resolving robustness properties. Robustness is important due to the presence of uncertainty, noise and disturbances. Quantum systems differ from classical systems because of their non-commutative structure, and our results give important new insights into the fundamental nature of quantum control systems.

New frequency-domain methods have been developed to compute the risk-sensitive quadratic cost criterion. We use an integral operator representation to compute its asymptotic infinite-horizon growth rate in the invariant Gaussian state when the stable system is driven by vacuum input fields. The resulting frequency-domain formulas express the growth rate in terms of two spectral functions associated with the real and imaginary parts of the quantum covariance kernel of the system variables. These results may be applied for robust performance analysis as well as for feedback system design.

Also, important results have been obtained on the physical realizability of linear quantum systems, a Kalman decomposition for linear quantum systems and pure Gaussian state covariance assignment.

Quantum Information Processing.

Single photon pulse shape is important for a number of reasons in quantum information processing. For instance, efficient photo absorption requires that the pulse shape is matched to the characteristics of the receiving device. This project has made a number of important advances concerning the generation and absorption of photons.

A scheme to generate arbitrarily shaped single photons has been developed and analyzed. This scheme consists of two coupled cavities, a memory cavity and a shutter cavity, containing a second-order optical nonlinearity and electro-optic modulator (EOM), respectively. Photodetection events of the shutter cavity output herald preparation of a single photon in the memory cavity, which may be stored by immediately changing the optical length of the shutter cavity with the EOM after detection. On-demand readout of the photon, with arbitrary shaping, can be achieved through modulation of the EOM.

We showed how a perfect single device absorber of arbitrary traveling single photon fields can be constructed using an ideal tunable input-output open quantum system, realizable as either a single qubit or single resonator system, which can perfectly absorb traveling single photon states of arbitrary temporal profiles. This tunability allows for photon pulse shape matching.

Refereed Journal Papers:

Guofeng Zhang, Ian R. Petersen, and Jinghao Li. *Structural characterization of linear quantum systems with application to back-action evading measurement*. IEEE Transactions on Automatic Control, 65(7):3157–3163, 2020.

The purpose of this article is to study the structure of quantum linear systems in terms of their Kalman canonical form, which was proposed in a recent paper. The spectral structure of quantum linear systems is explored, which indicates that a quantum linear system is both controllable and observable provided that it is Hurwitz stable. A new parameterization method for quantum linear systems is proposed. This parameterization is designed for the Kalman canonical form directly. Consequently, the parameters involved are in a blockwise form in correspondence with the blockwise structure of the Kalman canonical form. This parameter structure can be used to simplify various quantum control design problems. For example, necessary and sufficient conditions for the realization of quantum back-action evading (BAE) measurements are given in terms of these new parameters. Due to their blockwise nature, a small number of parameters are required for realizing quantum BAE measurements.

Igor G Vladimirov, Matthew R James, Ian R Petersen, *Measurement-based feedback control of linear quantum stochastic systems with quadratic-exponential criteria*, arXiv:1912.11687, 2019.

This paper is concerned with a risk-sensitive optimal control problem for a feedback connection of a quantum plant with a measurement-based classical controller. The plant is a multimode open quantum harmonic oscillator driven by a multichannel quantum Wiener process, and the controller is a linear time invariant system governed by a stochastic differential equation. The control objective is to stabilize the closed-loop system and minimize the infinite-horizon asymptotic growth rate of a quadratic-exponential functional (QEF) which penalizes the plant variables and the controller output. We combine a frequency-domain representation of the QEF growth rate, obtained recently, with variational techniques and establish first-order necessary conditions of optimality for the state-space matrices of the controller.

Igor G Vladimirov, Ian R Petersen, Matthew R James, *Frequency-domain computation of quadratic-exponential cost functionals for linear quantum stochastic systems*, arXiv:1911.03031, 2019.

This paper is concerned with quadratic-exponential functionals (QEFs) as risk-sensitive performance criteria for linear quantum stochastic systems driven by multichannel bosonic fields. Such costs impose an exponential penalty on quadratic functions of the quantum system variables over a bounded time interval, and their minimization secures a number of robustness properties for the system. We use an integral operator representation of the QEF, obtained recently, in order to compute its asymptotic infinite-horizon growth rate in the invariant Gaussian state when the stable system is driven by vacuum input fields. The resulting frequency-domain formulas express the QEF growth rate in terms of two spectral functions associated with the real and imaginary parts of the quantum covariance kernel of the system variables. We also discuss the computation of the QEF growth rate using homotopy and contour integration techniques and provide two illustrations including a numerical example with a two-mode oscillator.

Igor G Vladimirov, Ian R Petersen, Matthew R James, *A Girsanov type representation of quadratic-exponential cost functionals for linear quantum stochastic systems*, arXiv:1911.01539, 2019.

This paper is concerned with multimode open quantum harmonic oscillators and quadratic-exponential functionals (QEFs) as quantum risk-sensitive performance criteria. Such systems are described by linear quantum stochastic differential equations driven by multichannel bosonic fields. We develop a finite-horizon expansion for the system variables using the eigenbasis of their two-point commutator kernel with noncommuting position-momentum pairs as coefficients. This quantum Karhunen-Loeve expansion is used in order to obtain a Girsanov type representation for the quadratic-exponential functions of the system variables. This representation is valid regardless of a particular system-field state and employs the averaging over an auxiliary classical Gaussian random process whose covariance operator is defined in

terms of the quantum commutator kernel. We use this representation in order to relate the QEF to the moment-generating functional of the system variables. This result is also specified for the invariant multipoint Gaussian quantum state when the oscillator is driven by vacuum fields.

Igor G Vladimirov, Ian R Petersen, Matthew R James, *A quantum Karhunen-Loeve expansion and quadratic-exponential functionals for linear quantum stochastic systems*, arXiv:1904.03265, 2019.

This paper extends the Karhunen-Loeve representation from classical Gaussian random processes to quantum Wiener processes which model external bosonic fields for open quantum systems. The resulting expansion of the quantum Wiener process in the vacuum state is organised as a series of sinusoidal functions on a bounded time interval with statistically independent coefficients consisting of noncommuting position and momentum operators in a Gaussian quantum state. A similar representation is obtained for the solution of a linear quantum stochastic differential equation which governs the system variables of an open quantum harmonic oscillator. This expansion is applied to computing a quadratic-exponential functional arising as a performance criterion in the framework of risk-sensitive control for this class of open quantum systems.

Shibei Xue, Thien Nguyen, Matthew R. James, Alireza Shabani, Valery Ugrinovskii, and Ian R. Petersen. *Modelling for non-Markovian quantum systems*. IEEE Transactions on Control Systems Technology, 2019. Early access, DOI: 10.1109/TCST.2019.2935421.

This brief presents an augmented Markovian system framework that can be applied to model non-Markovian quantum systems. In this augmented system model, ancillary systems are designed to play the role of internal modes of the non-Markovian environment converting white noise to colored noise. To capture the non-classical mutual influence between a quantum system and non-Markovian environments, direct interactions between a principal system and the ancillary system are introduced. Consequently, we showed that the augmented Markovian formulation can be used to theoretically model the environment for observed non-Markovian behavior in a recent experiment on quantum dots.

Qing Gao, Daoyi Dong, Ian R. Petersen, and Steven Ding. *Design of a discrete-time fault tolerant quantum filter and fault detector*. IEEE Transactions on Cybernetics, 2019. Early access DOI: 10.1109/TCYB.2019.2899877.

This paper solves the problem of discrete-time fault-tolerant quantum filtering for a class of laser-atom open quantum systems subject to the stochastic faults. We show that by using the discrete-time quantum measurements, optimal estimates of both the atomic observables and the classical fault process can be simultaneously determined in terms of recursive quantum stochastic difference equations. A dispersive interaction quantum system example is used to demonstrate the proposed filtering approach.

Igor G. Vladimirov, Ian R. Petersen, and Matthew R. James. *Parametric randomization, complex symplectic factorizations, and quadratic-exponential functionals for Gaussian quantum states*. Infinite Dimensional Analysis, Quantum Probability and Related Topics, vol. 22, no. 3, pp 1950020, 2019.

This paper combines probabilistic and algebraic techniques for computing quantum expectations of operator exponentials (and their products) of quadratic forms of quantum variables in Gaussian states. Such quadratic-exponential functionals (QEFs) resemble quantum statistical mechanical partition functions with quadratic Hamiltonians and are also used as performance criteria in quantum risk-sensitive filtering and control problems for linear quantum stochastic systems. We employ a Lie-algebraic correspondence between complex symplectic matrices and quadratic-exponential functions of system variables of a quantum harmonic oscillator. The complex symplectic factorizations are used together with a parametric randomization of the quasi-characteristic or moment-generating functions according to an auxiliary classical Gaussian distribution. This reduces the QEF to an exponential moment of a quadratic form of classical Gaussian random variables with a complex symmetric matrix and is applicable to recursive computation of such moments.

Igor G Vladimirov, Ian R Petersen, Matthew R James, Effects of parametric uncertainties in cascaded open quantum harmonic oscillators and robust generation of Gaussian invariant states, *SIAM Journal on Control and Optimization*, 57(3), 1597-1628, 2019. doi.org/10.1137/17M1135050

This paper is concerned with the generation of Gaussian invariant states in cascades of open quantum harmonic oscillators governed by linear quantum stochastic differential equations. We carry out infinitesimal perturbation analysis of the covariance matrix for the invariant Gaussian state of such a system and the related purity functional subject to inaccuracies in the energy and coupling matrices of the subsystems. This leads to the problem of balancing the state-space realizations of the component oscillators through symplectic similarity transformations in order to minimize the mean square sensitivity of the purity functional to small random perturbations of the parameters. This results in a quadratic optimization problem with an effective solution in the case of cascaded one-mode oscillators, which is demonstrated by a numerical example. We also discuss a connection of the sensitivity index with classical statistical distances and outline infinitesimal perturbation analysis for translation invariant cascades of identical oscillators. The findings of the paper are applicable to robust state generation in quantum stochastic networks.

Yihong Qi, Chuan cun Shu, Daoyi Dong, Ian R. Petersen, Kurt Jacobs, and Shangqing Gong. *Fast quantum state transfer in hybrid quantum dot-metal nanoparticle systems by shaping ultrafast laser pulses*. *Journal of Physics D: Applied Physics*, 52(42):425101, 2019.

Semiconductor quantum dots (SQDs) have relatively short decoherence times, but have the potential for extremely fast operations implemented by femto-second laser pulses, making them a possible candidate for very high speed quantum information processing. These operations are enhanced by positioning a metal nano-particle (MNP) near to the SQD which significantly increases the strength of electric fields applied to it. Here we consider an SQD with three-levels in a Lambda configuration, and explore the requirements for implementing a transfer from one ground state to the other via the upper level. A recently developed optimization method allows us to fix the energy of the laser pulse while satisfying the requirement that the pulse has no DC component. We show that the use of carefully shaped pulses significantly reduces the required laser power. We also determine the dependence of the pulse power on the distance between the SQD and the MNP, and explore the degree to which the process follows the pulse area theorem for first-order perturbative processes.

Yanan Liu, Daoyi Dong, Ian R. Petersen, and Hidehiro Yonezawa. *Filter-based feedback control for a class of Markovian open quantum systems*. *IEEE Control Systems Letters*, 3(3):565–570, 2019.

This letter considers target state preparation for Markovian open quantum systems subject to continuous measurement. Conditions on invariant and attractive subspaces are investigated, which ensure the stabilization of the target state/subspace. For a class of open quantum systems with time delay in the feedback loop, a bang-bang-like control law is proposed, and the stability of the feedback control strategy is proved. An example of four-level Markovian open quantum systems is presented to demonstrate the effectiveness of the proposed control strategy.

Qi Yu, Daoyi Dong, Ian R. Petersen, Muhammad Emzir, and Qing Gao. *Filtering for a class of quantum systems with classical stochastic disturbances*. *IEEE Transactions on Control Systems Technology*, 27(6):2774–2780, 2019.

We present filtering equations for a class of hybrid quantum-classical systems where a quantum optical cavity system is continuously disturbed by a classical linear stochastic process. Another quantum cavity system is employed to describe the classical stochastic process as an analog. The initial hybrid quantum-classical system consisting of both a classical signal and a quantum cavity system is then modeled by a combined quantum system with two quantum cavity systems. The stochastic master equations, or filtering equations, are then derived using quantum filtering theory. A quantum extended Kalman filter method is adopted as a convenient solution to this filtering problem. The effectiveness of both methods is illustrated by numerical examples.

Shibei Xue, Jun Zhang, and Ian R. Petersen. *Identification of non-Markovian environments for spin chains*. IEEE Transactions on Control Systems Technology, 27(6):2574–2580, 2019.

Correlations of an environment are crucial for the dynamics of non-Markovian quantum systems, which may not be known in advance. In this brief, we propose a gradient algorithm for identifying the correlations in terms of the time-varying damping rate functions in a time-convolution-less master equation for spin chains. By measuring time trace observables of the system, the identification procedure can be formulated as an optimization problem. The gradient algorithm is designed based on a calculation of the derivative of an objective function with respect to the damping rate functions, whose effectiveness is shown in a comparison with a differential approach for a two-qubit spin chain.

Qing Gao, Guofeng Zhang, and Ian R Petersen. *An exponential quantum projection filter for open quantum systems*. Automatica, 99:59–68, 2019.

An approximate exponential quantum projection filtering scheme is developed for a class of open quantum systems described by Hudson–Parthasarathy quantum stochastic differential equations, aiming to reduce the computational burden associated with online calculation of the quantum filter. By using a differential geometric approach, the quantum trajectory is constrained in a finite-dimensional differentiable manifold consisting of an unnormalized exponential family of quantum density operators, and an exponential quantum projection filter is then formulated as a number of stochastic differential equations satisfied by the finite-dimensional coordinate system of this manifold. A convenient design of the differentiable manifold is also presented through reduction of the local approximation errors, which yields a simplification of the quantum projection filter equations. It is shown that the computational cost can be significantly reduced by using the quantum projection filter instead of the quantum filter. It is also shown that when the quantum projection filtering approach is applied to a class of open quantum systems that asymptotically converge to a pure state, the input-to-state stability of the corresponding exponential quantum projection filter can be established. Simulation results from an atomic ensemble system example are provided to illustrate the performance of the projection filtering scheme. It is expected that the proposed approach can be used in developing more efficient quantum control methods.

Yuanlong Wang, Qi Yinand, Daoyi Dong, Bo Qi, Ian R. Petersen, Zhibo Hou, Hidehiro Yonezawa, and Guo-Yong Xiang. *Quantum gate identification: error analysis, numerical results and optical experiment*. Automatica, 101:269–279, 2019.

The identification of an unknown quantum gate is a significant issue in quantum technology. In this paper, we propose a quantum gate identification method within the framework of quantum process tomography. In this method, a series of pure states are applied to the gate and then a fast state tomography on the output states is performed and the data are used to reconstruct the quantum gate. The algorithm has computational complexity $O(d^3)$ with the system dimension d . The identification approach is compared with the maximum likelihood estimation method for the running time, which shows an efficiency advantage of our method. An error upper bound is established for the identification algorithm and the robustness of the algorithm against impurities in the input states is also tested. We perform a quantum optical experiment on a single-qubit Hadamard gate to verify the effectiveness of the identification algorithm.

Igor G Vladimirov, Ian R Petersen, Matthew R James, *Parametric randomization, complex symplectic factorizations, and quadratic-exponential functionals for Gaussian quantum states*, arXiv:1809.06842, 2018.

This paper combines probabilistic and algebraic techniques for computing quantum expectations of operator exponentials (and their products) of quadratic forms of quantum variables in Gaussian states. Such quadratic-exponential functionals (QEFs) resemble quantum statistical mechanical partition functions with quadratic Hamiltonians and are also used as performance criteria in quantum risk-sensitive filtering and control problems for linear quantum stochastic systems. We employ a Lie-algebraic correspondence between complex symplectic matrices and quadratic-exponential functions of system variables of a quantum harmonic oscillator. The complex symplectic factorizations are used together with a parametric randomization of the quasi-characteristic or moment-generating functions according to an auxiliary classical Gaussian distribution. This reduces the QEF to an exponential moment of a quadratic form of

classical Gaussian random variables with a complex symmetric matrix and is applicable to recursive computation of such moments.

Igor G Vladimirov, Ian R Petersen, Matthew R James, *Multi-point Gaussian states, quadratic exponential cost functionals, and large deviations estimates for linear quantum stochastic systems*, Applied Mathematics and Optimization, 1-55, doi.org/10.1007/s00245-018-9512-y, 2018.

This paper is concerned with risk-sensitive performance analysis for linear quantum stochastic systems interacting with external bosonic fields. We consider a cost functional in the form of the exponential moment of the integral of a quadratic polynomial of the system variables over a bounded time interval. An integro-differential equation is obtained for the time evolution of this quadratic-exponential functional, which is compared with the original quantum risk-sensitive performance criterion employed previously for measurement-based quantum control and filtering problems. Using multi-point Gaussian quantum states for the past history of the system variables and their first four moments, we discuss a quartic approximation of the cost functional and its infinite-horizon asymptotic behaviour. The computation of the asymptotic growth rate of this approximation is reduced to solving two algebraic Lyapunov equations. We also outline further approximations of the cost functional, based on higher-order cumulants and their growth rates, together with large deviations estimates. For comparison, an auxiliary classical Gaussian Markov diffusion process is considered in a complex Euclidean space which reproduces the quantum system variables at the level of covariances but has different higher-order moments relevant to the risk-sensitive criteria. The results of the paper are also demonstrated by a numerical example and may find applications to coherent quantum risk-sensitive control problems, where the plant and controller form a fully quantum closed-loop system, and other settings with nonquadratic cost functionals.

Igor G. Vladimirov, Ian R. Petersen, Matthew R. James, *Risk-sensitive performance criteria and robustness of quantum systems with a relative entropy description of state uncertainty*, arXiv:1802.00250 2018.

This paper considers links between the original risk-sensitive performance criterion for quantum control systems and its recent quadratic-exponential counterpart. We discuss a connection between the minimization of these cost functionals and robustness with respect to uncertainty in system-environment quantum states whose deviation from a nominal state is described in terms of the quantum relative entropy. These relations are similar to those in minimax LQG control for classical systems. The results of the paper can be of use in providing a rational choice of the risk-sensitivity parameter in the context of robust quantum control with entropy theoretic quantification of statistical uncertainty in the system-field state.

G. Zhang, S. Grivopoulos, I. R. Petersen, and J. E. Gough. *The Kalman decomposition for linear quantum systems*. IEEE Transactions on Automatic Control, 63(2):331–346, 2018.

This paper studies the Kalman decomposition for linear quantum systems. Contrary to the classical case, the coordinate transformation used for the decomposition must belong to a specific class of transformations as a consequence of the laws of quantum mechanics. We propose a construction method for such transformations that put the system in a Kalman canonical form. Furthermore, we uncover an interesting structure for the obtained decomposition. In the case of passive systems, it is shown that there exist only controllable/observable and uncontrollable/unobservable subsystems. In the general case, controllable/unobservable and uncontrollable/observable subsystems may also be present, but their respective system variables must be conjugate variables of each other. This decomposition naturally exposes decoherence-free modes, quantum-nondemolition modes, quantum-mechanics-free subsystems, and back-action evasion measurements in the quantum system, which are useful resources for quantum information processing, and quantum measurements. The theory developed is applied to physical examples.

M. F. Emzir, M. Woolley, and I. R Petersen. *On physical realizability of nonlinear quantum stochastic differential equations*. Automatica, 95:254–265, 2018.

In this article we study physical realizability for a class of nonlinear quantum stochastic differential equations (QSDEs). Physical realizability is a property in which a QSDE corresponds to the dynamics of an open quantum system. We derive a sufficient and necessary condition for a nonlinear QSDE to be

physically realizable.

S. Ma, M. Woolley, I. R. Petersen, and N. Yamamoto. *Cascade and locally dissipative realizations of linear quantum systems for pure Gaussian state covariance assignment*. *Automatica*, 90:263–270, 2018.

This paper presents two realizations of linear quantum systems for covariance assignment corresponding to pure Gaussian states. The first one is called a cascade realization; given any covariance matrix corresponding to a pure Gaussian state, we can construct a cascaded quantum system generating that state. The second one is called a locally dissipative realization; given a covariance matrix corresponding to a pure Gaussian state, if it satisfies certain conditions, we can construct a linear quantum system that has only local interactions with its environment and achieves the assigned covariance matrix. Both realizations are illustrated by examples from quantum optics.

Yuanlong Wang, Daoyi Dong, Bo Qi, Jun Zhang, Ian R. Petersen, and Hidehiro Yonezawa. *A quantum Hamiltonian identification algorithm: Computational complexity and error analysis*. *IEEE Transactions on Automatic Control*, 63(5):1388–1403, 2018.

Quantum Hamiltonian identification (QHI) is important for characterizing the dynamics of quantum systems, calibrating quantum devices, and achieving precise quantum control. In this paper, an effective two-step optimization (TSO) QHI algorithm is developed within the framework of quantum process tomography. In the identification method, different probe states are input into quantum systems and the output states are estimated using the quantum state tomography protocol via linear regression estimation. The time-independent system Hamiltonian is reconstructed based on the experimental data for the output states. The Hamiltonian identification method has computational complexity $O(d^6)$, where d is the dimension of the system Hamiltonian. An error upper bound $O(d^3/\sqrt{N})$ is also established, where N is the resource number for the tomography of each output state, and several numerical examples demonstrate the effectiveness of the proposed TSO Hamiltonian identification method.

Ruvindha L Lecamwasam, Michael R Hush, Matthew R James, Andre RR Carvalho, *Measurement-based generation of shaped single photons and coherent state superpositions in optical cavities*, *Physical Review A*, 95(1), 013828, 2017.

We propose related schemes to generate arbitrarily shaped single photons, i.e., photons with an arbitrary temporal profile, and coherent state superpositions using simple optical elements. The first system consists of two coupled cavities, a memory cavity and a shutter cavity, containing a second-order optical nonlinearity and electro-optic modulator (EOM), respectively. Photodetection events of the shutter cavity output herald preparation of a single photon in the memory cavity, which may be stored by immediately changing the optical length of the shutter cavity with the EOM after detection. On-demand readout of the photon, with arbitrary shaping, can be achieved through modulation of the EOM. The second scheme consists of a memory cavity with two outputs, which are interfered, phase shifted, and measured. States that closely approximate a coherent state superposition can be produced through post selection for sequences of detection events, with more photon detection events leading to a larger superposition. We furthermore demonstrate that no-knowledge feedback can be easily implemented in this system and used to preserve the superposition state, as well as provide an extra control mechanism for state generation.

A. Khodaparastsichani and I. R. Petersen. *A modified frequency domain condition for the physical realizability of linear quantum stochastic systems*. *IEEE Transactions on Automatic Control*, 63(1):277–282, 2018.

This note is concerned with a modified version of the frequency domain physical realizability (PR) condition for linear quantum systems. We consider open quantum systems whose dynamic variables satisfy the canonical commutation relations of an open quantum harmonic oscillator and are governed by linear quantum stochastic differential equations (QSDEs). In order to correspond to physical quantum systems, these QSDEs must satisfy PR conditions. We provide a relatively simple proof that the PR

condition is equivalent to the frequency domain (J;J)-unitarity of the input-output transfer function and orthogonality of the feedthrough matrix of the system without the technical spectral assumptions required in previous work. We also show that the poles and transmission zeros associated with the transfer function of PR linear quantum systems are the mirror reflections of each other about the imaginary axis. An example is provided to illustrate the results.

S. L. Vuglar and I. R. Petersen. *Quantum noises, physical realizability and coherent quantum feedback control*. IEEE Transactions on Automatic Control, 62(2):998–1003, 2017.

Physical Realizability addresses the question of whether it is possible to implement a given linear time invariant (LTI) system as a quantum system. A given synthesized quantum controller described by a set of stochastic differential equations does not necessarily correspond to a physically meaningful quantum system. However, if additional quantum noises are permitted in the implementation, it is always possible to implement an arbitrary LTI system as a quantum system. In this paper, we give an expression for the number of introduced noise channels required to implement a given LTI system as a quantum system. We then consider the special case where only the transfer function to be implemented is of interest. We give results showing when it is possible to implement a transfer function as a quantum system by introducing the same number of quantum noises as there are system outputs. Finally, we demonstrate the utility of these results by providing an algorithm for obtaining a suboptimal solution to a coherent quantum LQG control problem.

Shibdas Roy, Dominic W. Berry, Ian R. Petersen, and Elanor H. Huntington. *Robust guaranteed-cost adaptive quantum phase estimation*. Physical Review A, 95(5):052322, 2017.

Quantum parameter estimation plays a key role in many fields like quantum computation, communication, and metrology. Optimal estimation allows one to achieve the most precise parameter estimates, but requires accurate knowledge of the model. Any inevitable uncertainty in the model parameters may heavily degrade the quality of the estimate. It is therefore desired to make the estimation process robust to such uncertainties. Robust estimation was previously studied for a varying phase, where the goal was to estimate the phase at some time in the past, using the measurement results from both before and after that time within a fixed time interval up to current time. Here, we consider a robust guaranteed-cost filter yielding robust estimates of a varying phase in real time, where the current phase is estimated using only past measurements. Our filter minimizes the largest (worst-case) variance in the allowable range of the uncertain model parameter(s) and this determines its guaranteed cost. It outperforms in the worst case the optimal Kalman filter designed for the model with no uncertainty, which corresponds to the center of the possible range of the uncertain parameter(s). Moreover, unlike the Kalman filter, our filter in the worst case always performs better than the best achievable variance for heterodyne measurements, which we consider as the tolerable threshold for our system. Furthermore, we consider effective quantum efficiency and effective noise power, and show that our filter provides the best results by these measures in the worst case.

Ian R. Petersen. *Quantum Popov robust stability analysis of an optical cavity containing a saturated Kerr medium*. Quantum Science and Technology, 2(3):034009, 2017.

This paper applies results on the robust stability of nonlinear quantum systems to a system consisting of an optical cavity containing a saturated Kerr medium. The system is characterized by a Hamiltonian operator which contains a non-quadratic term involving a quartic function of the annihilation and creation operators. A saturated version of the Kerr nonlinearity leads to a sector bounded nonlinearity which enables a quantum small gain theorem to be applied to this system in order to analyze its stability. Also, a non-quadratic version of a quantum Popov stability criterion is presented and applied to analyze the stability of this system.

Arash Khodaparastsichani and Ian R. Petersen. *Quantum linear coherent controller synthesis: A linear fractional representations approach*. Automatica, 85:264–271, 2017.

This paper is concerned with a linear fractional representation approach to the synthesis of linear coherent quantum controllers for a given linear quantum plant. The plant and controller represent open quantum

harmonic oscillators and are modelled by linear quantum stochastic differential equations. The feedback interconnections between the plant and the controller are assumed to be established through quantum bosonic fields. In this framework, conditions for the stabilization of a given linear quantum plant via linear coherent quantum feedback are addressed using a stable factorization approach. The class of all stabilizing quantum controllers is parameterized in the frequency domain. Coherent quantum weighted H_2 and H_∞ control problems for linear quantum systems are formulated in the frequency domain. Finally, a projected gradient descent scheme is outlined for the coherent quantum weighted H_2 control problem.

Arash Khodaparastsichani, Igor G. Vladimirov, and Ian R. Petersen. *A numerical approach to optimal coherent quantum LQG controller design using gradient descent*. *Automatica*, 85:314–326, 2017.

This paper is concerned with the Coherent Quantum Linear Quadratic Gaussian (CQLQG) control problem of finding a stabilizing measurement-free quantum controller for a quantum plant so as to minimize an infinite-horizon mean square performance index for the fully quantum closed-loop system. In comparison with the observation-actuation structure of classical controllers, the coherent quantum feedback is less invasive to the quantum dynamics and quantum information. Both the plant and the controller are open quantum systems whose dynamic variables satisfy the canonical commutation relations (CCRs) of a quantum harmonic oscillator and are governed by linear quantum stochastic differential equations (QSDEs). In order to correspond to such oscillators, these QSDEs must satisfy physical realizability (PR) conditions, which are organised as quadratic constraints on the controller matrices and reflect the preservation of CCRs in time. The CQLQG problem is a constrained optimization problem for the steady-state quantum covariance matrix of the plant-controller system satisfying an algebraic Lyapunov equation. We propose a gradient descent algorithm equipped with adaptive stepsize selection for the numerical solution of the problem. The algorithm minimizes the LQG cost over the parameters of the Hamiltonian and coupling operators of a stabilizing PR quantum controller, thus taking the PR constraints into account. A convergence analysis of the proposed algorithm is presented. A numerical example of optimal CQLQG controller design is provided to demonstrate the algorithm performance.

I. R. Petersen. *Time averaged consensus in a direct coupled coherent quantum observer network*. *Journal of Control Theory and Technology*, 15(3):163–176, 2017.

This paper considers the problem of constructing a direct coupling quantum observer for a closed linear quantum system. The proposed distributed observer consists of a network of quantum harmonic oscillators and it is shown that the observer network converges to a consensus in a time averaged sense in which each element of the observer estimates the specified output of the quantum plant. An example and simulations are included to illustrate the properties of the observer network.

Shibdas Roy, Ian R. Petersen, and Elanor Huntington. *Coherent-classical estimation for linear quantum systems*. *Automatica*, 82:109–117, 2017.

We study a coherent-classical estimation scheme for a class of linear quantum systems, where the estimator is a mixed quantum–classical system that may or may not involve coherent feedback. We show that when the quantum plant or the quantum part of the estimator (coherent controller) is an annihilation operator only system, coherent-classical estimation without coherent feedback can provide no improvement over purely-classical estimation. Otherwise, coherent-classical estimation without feedback can be better than classical-only estimation for certain homodyne detector angles, although the former is inferior to the latter for the best choice of homodyne detector angle. Moreover, we show that coherent-classical estimation with coherent feedback is no better than classical-only estimation, when both the plant and the coherent controller are annihilation operator only systems. Otherwise, coherent-classical estimation with coherent feedback can be superior to purely-classical estimation, and in this case, the former is better than the latter for the optimal choice of homodyne detector angle.

G. Shi, S. Fu, and I. R. Petersen. *Consensus of quantum networks with directed interactions: Fixed and switching structures*. *IEEE Transactions on Automatic Control*, 62(4):2014–2019, 2017.

We study consensus seeking of quantum networks under interactions defined by a set of permutation operators among a network of qubits. The state evolution of the quantum network is described by a continuous-time master equation. We show that the network state always converges along the equation, and the convergence limit is determined by the generating subgroup of the permutation operators. We also give a tight graphical criterion regarding when such limit admits a reduced-state consensus. Quantum consensus conditions under switching interactions are also established with the help of cut-balanced consensus processes in the classical setting.

Fu S, Shi G, Petersen IR, James MR. *Entropy Evolution in Consensus Networks*. Scientific Reports. 2017, 7:1520. doi:10.1038/s41598-017-01615-5.

We investigate the evolution of the network entropy for consensus dynamics in classical and quantum networks. We show that in the classical case, the network differential entropy is monotonically non-increasing if the node initial values are continuous random variables. While for quantum consensus dynamics, the network's von Neumann entropy is in contrast non-decreasing. In light of this inconsistency, we compare several distributed algorithms with random or deterministic coefficients for classical or quantum networks, and show that quantum algorithms with deterministic coefficients are physically related to classical algorithms with random coefficients.

L. A. Duffaut Espinosa, Z. Miao, I. R. Petersen, V. Ugrinovskii, and M. R. James. *Physical realizability and preservation of commutation and anticommutation relations for n -level quantum systems*. SIAM Journal on Control and Optimization, 54(2):632–661, 2016.

The purpose of this paper is to address the problem of physical realizability for n -level quantum systems. We provide necessary and sufficient conditions for quantum stochastic differential equations that ensure the existence of physical parameters characterizing the unitary evolution required by the laws of quantum mechanics. Also, these conditions guarantee the preservation of the commutation and anticommutation relations of the underlying Lie algebra $\mathfrak{su}(n)$.

Zibo Miao, Matthew R James, Ian R Petersen, *Coherent observers for linear quantum stochastic systems*, Automatica, 71, 264-271, 2016.

The theory of observers is a basic part of classical linear system theory. The purpose of this paper is to develop a theory of coherent observers for linear quantum systems. We provide a class of coherent quantum observers, which track the observables of a linear quantum stochastic system in the sense of mean values, independent of any additional quantum noise in the observer. We prove that there always exists such a coherent quantum observer described by quantum stochastic differential equations in the Heisenberg picture, and show how it can be designed to be consistent with the laws of quantum mechanics. We also find a lower bound for the mean squared estimation error due to the uncertainty principle. In addition, we explore the quantum correlations between a linear quantum plant and the corresponding coherent observer. It is shown that considering a joint plant-observer Gaussian quantum system, entanglement can be generated under the condition that appropriate coefficients of the coherent quantum observer are chosen, and this issue is illustrated in an example. These results pave the way towards observer-based quantum control.

Yu Pan, Guofeng Zhang, Matthew R James, *Analysis and control of quantum finite-level systems driven by single-photon input states*, Automatica, 69, 18-23, 2016.

Single-photon states, which carry quantum information and coherently interact with quantum systems, are vital to the realization of all-optical engineered quantum networks. In this paper we derive the analytical form of the output field state for a large class of quantum finite-level systems driven by single-photon input field states using a transfer function approach. Single-photon pulse shaping via coherent feedback is also studied.

Shuangshuang Fu, Andre RR Carvalho, Michael R Hush, Matthew R James, *Cross-phase modulation and entanglement in a compound gradient echo memory*, Physical Review A, 93(2), 023809, 2016.

We present a theoretical model for a Kerr-like interaction between two modes (or registers) of a compound gradient echo memory (GEM). This type of interaction is known to generate cross-phase modulation (XPM) between optical fields, an effect that is limited by the typically small values of non-linearities in crystals. Here we show that in GEM systems the phase shift increases linearly with the interaction time and quadratically with the strength of the field. Increasing storage (interaction) times would then lead to stronger XPM effects even with fields with very low intensity. This interaction also generates two other effects: entanglement between the registers, which depends on the strength of the interaction and its spatial profile, and an interaction-induced gradient. We show that the latter produces leakage during the storage stage depending on the shape of the stored pulses, an undesirable consequence that can be minimised by carefully designing the temporal profile of the input fields.

Re-Bing Wu, Constantin Brif, Matthew R. James and Hershel Rabitz, *Limits of optimal control yields achievable with quantum controllers*, Physical Review A 91(4), 042327, 2015.

In quantum optimal control theory, kinematic bounds are the minimum and maximum values of the control objective achievable for any physically realizable system dynamics. For a given initial state of the system, these bounds depend on the nature and state of the controller. We consider a general situation where the controlled quantum system is coupled to both an external classical field (referred to as a classical controller) and an auxiliary quantum system (referred to as a quantum controller). In this general situation, the kinematic bound is between the classical kinematic bound (CKB), corresponding to the case where only the classical controller is available, and the quantum kinematic bound (QKB), corresponding to the ultimate physical limit of the objective's value. Specifically, when the control objective is the expectation value of a quantum observable (a Hermitian operator on the system's Hilbert space), the QKBs are the minimum and maximum eigenvalues of this operator. We present, both qualitatively and quantitatively, the necessary and sufficient conditions for surpassing the CKB and reaching the QKB, through the use of a quantum controller. The general conditions are illustrated by examples in which the system and controller are initially in thermal states. The obtained results provide a basis for the design of quantum controllers capable of maximizing the control yield and reaching the ultimate physical limit.

Kurt Jacobs, Hendra I Nurdin, Frederick W Strauch, Matthew James, *Comparing resolved-sideband cooling and measurement-based feedback cooling on an equal footing: Analytical results in the regime of ground-state cooling*, Physical Review A 91(4), 043812, 2015.

We show that in the regime of ground-state cooling, simple expressions can be derived for the performance of resolved-sideband cooling—an example of coherent feedback control—and optimal linear measurement-based feedback cooling for a harmonic oscillator. These results are valid to leading order in the small parameters that define this regime. They provide insight into the origins of the limitations of coherent and measurement-based feedback for linear systems, and the relationship between them. These limitations are not fundamental bounds imposed by quantum mechanics, but are due to the fact that both cooling methods are restricted to use only a linear interaction with the resonator. We compare the performance of the two methods on an equal footing—that is, for the same interaction strength—and confirm that coherent feedback is able to make much better use of the linear interaction than measurement-based feedback. We find that this performance gap is caused not by the back-action noise of the measurement but by the projection noise. We also obtain simple expressions for the maximal cooling that can be obtained by both methods in this regime, optimized over the interaction strength.

Refereed Conference Papers:

Igor G Vladimirov, Matthew R James, Ian R Petersen, *A Karhunen-Loeve expansion for one-mode open quantum harmonic oscillators using the eigenbasis of the two-point commutator kernel*, 2019 Australian and New Zealand

This paper considers one-mode open quantum harmonic oscillators with a pair of conjugate position and momentum variables driven by vacuum bosonic fields according to a linear quantum stochastic differential equation. Such systems model cavity resonators in quantum optical experiments. Assuming that the quadratic Hamiltonian of the oscillator is specified by a positive definite energy matrix, we consider a modified version of the quantum Karhunen-Loeve expansion of the system variables proposed recently. The expansion employs eigenvalues and eigenfunctions of the two-point commutator kernel for linearly transformed system variables. We take advantage of the specific structure of this eigenbasis in the one-mode case (including its connection with the classical Ornstein-Uhlenbeck process). These results are applied to computing quadratic-exponential cost functionals which provide robust performance criteria for risk sensitive control of open quantum systems.

Igor G. Vladimirov, Ian R. Petersen, and Matthew R. James. A quantum Karhunen-Loeve expansion and quadratic-exponential functionals for linear quantum stochastic systems. In Proceedings of the 58th IEEE Conference on Decision and Control, Nice, France, Dec 2019.

This paper extends the Karhunen-Loeve representation from classical Gaussian random processes to quantum Wiener processes which model external bosonic fields for open quantum systems. The resulting expansion of the quantum Wiener process in the vacuum state is organised as a series of sinusoidal functions on a bounded time interval with statistically independent coefficients consisting of noncommuting position and momentum operators in a Gaussian quantum state. A similar representation is obtained for the solution of a linear quantum stochastic differential equation which governs the system variables of an open quantum harmonic oscillator. This expansion is applied to computing a quadratic-exponential functional arising as a performance criterion in the framework of risk-sensitive control for this class of open quantum systems.

Ian R. Petersen, Matthew R. James, Valery Ugrinovskii, and Naoki Yamamoto. *A systems theory approach to the synthesis of non-reciprocal phase-insensitive quantum amplifiers*. In Proceedings of the AIAA Science and Technology Forum and Exposition, San Diego, January 2019.

We present a synthesis procedure for constructing a non-reciprocal phase-insensitive quantum amplifier using quantum optics. An expression is obtained for the corresponding level of noise by this amplifier. This noise level is greater than the level required in a previous paper which did not impose the non-reciprocal requirement. The synthesis procedure uses a singularly perturbed quantum system and leads to an amplifier involving three squeezers and a number of beamsplitters.

Yanan Liu, Daoyi Dong, Sen Kuang, Ian R. Petersen, and Hidehiro Yonezawa. *Feedback preparation of Bell states for two-qubit systems with time delay*. In Proceedings of the 2019 American Control Conference, Philadelphia, PA, USA, 2019.

Bell states are the maximally entangled states in two-qubit quantum systems, and play a significant role in quantum computation and quantum communication. Feedback control of stochastic quantum systems usually suffers from time-delay problems caused by the computation time for filter states and the control input. This paper addresses the preparation of Bell states in two-qubit systems with a constant delay time. A Lyapunov method is used to design a switching control law and a constant is introduced to compensate for the computation time of the estimated states and the feedback control input, thereby stabilizing the target Bell state globally. Numerical results show the effectiveness of the proposed control law.

Shan Ma, Matthew J. Woolley, and Ian R. Petersen. *A derivation of moment evolution equations for linear open quantum systems*. In Proceedings of the Youth Academic Annual Conference of Chinese Association of Automation, Nanjing, China, May 2018.

Given a linear open quantum system which is described by a Lindblad master equation, we detail the calculation of the moment evolution equations from this master equation. We stress that the moment evolution equations are well-known, but their explicit derivation from the master equation cannot be found

in the literature to the best of our knowledge, and so we provide this derivation for the interested reader.

Ian R. Petersen, Matthew R. James, Valery Ugrinovskii, and Naoki Yamamoto. *A systems theory approach to the synthesis of minimum noise phase-insensitive quantum amplifiers*. In Proceedings of the 2018 European Control Conference, pages 1–6, Limassol, Cypress, June 2018.

We present a systems theory approach to the proof of a result bounding the required level of added quantum noise in a phase-insensitive quantum amplifier. We also present a synthesis procedure for constructing a quantum optical phase-insensitive quantum amplifier which adds the minimum level of quantum noise and achieves a required gain and bandwidth. This synthesis procedure is based on a singularly perturbed quantum system and leads to an amplifier involving two squeezers and two beamsplitters.

Ian R. Petersen and Elanor Huntington. *Implementation of a distributed coherent quantum observer*. In Proceedings of the 2017 American Control Conference, Seattle, WA, May 2017.

This paper considers the problem of implementing a previously proposed distributed direct coupling quantum observer for a closed linear quantum system. By modifying the form of the previously proposed observer, the paper proposes a possible experimental implementation of the observer plant system using a non-degenerate parametric amplifier and a chain of optical cavities which are coupled together via optical interconnections. It is shown that the distributed observer converges to a consensus in a time averaged sense in which an output of each element of the observer estimates the specified output of the quantum plant.

Ian R Petersen and Elanor Huntington. *A direct coupling coherent quantum observer for a qubit, including observer measurements*. In Proceedings of the 20th IFAC World Congress, Toulouse, France, July 2017.

This paper proposes a direct coupling coherent quantum observer for a quantum plant which consists of a two level quantum system. The quantum observer, which is a quantum harmonic oscillator, includes homodyne detection measurements. It is shown that the observer can be designed so that it does not affect the quantum variable of interest in the quantum plant and that measured output converges in a given sense to the plant variable of interest. Also, the plant variable of interest-observer system can be described by a set of linear quantum stochastic differential equations. A minimum variance unbiased estimator form of the Kalman filter is derived for linear quantum systems and applied to the direct coupled coherent quantum observer.

Qi Yu, Daoyi Dong, Ian R Petersen, and Qing Gao. *Hybrid filtering for a class of quantum systems with classical disturbances*. In Proceedings of the 20th IFAC World Congress, Toulouse, France, July 2017.

A filtering problem for a class of quantum systems disturbed by a classical stochastic process is investigated in this paper. The classical disturbance process, which is assumed to be described by a linear stochastic differential equation, is modeled by a quantum cavity model. Then the hybrid quantum-classical system is described by a combined quantum system consisting of two quantum cavity subsystems. Quantum filtering theory and a quantum extended Kalman filter method are employed to estimate the states of the combined quantum system. An estimate of the classical stochastic process is derived from the estimate of the combined quantum system.

Yuanlong Wang, Daoyi Dong, Ian R Petersen, and Jun Zhang. *An approximate algorithm for quantum Hamiltonian identification with complexity analysis*. In Proceedings of the 20th IFAC World Congress, Toulouse, France, July 2017.

Identification of the Hamiltonian is vital for characterizing the dynamical evolution of a quantum system. The dimension of a multi-qubit system increases exponentially with the qubit number, which usually leads to daunting computational complexity for general Hamiltonian identification algorithms. In this paper, we design an efficient quantum Hamiltonian identification method based on periodical sampling. The computational complexity is $O(M^2 + MN^2)$, where M is the number of unknown parameters to be identified

in the Hamiltonian and N is the length of the sampling data. Numerical results with different data lengths demonstrate the effectiveness of the proposed identification algorithm.

Ian R. Petersen. *A survey of Riccati equation results in negative imaginary systems theory and quantum control theory*. In Proceedings of the 20th IFAC World Congress, Toulouse, France, July 2017.

This paper presents a survey of some new applications of algebraic Riccati equations. In particular, the paper surveys some recent results on the use of algebraic Riccati equations in testing whether a system is negative imaginary and in synthesizing state feedback controllers which make the closed loop system negative imaginary. The paper also surveys the use of Riccati equation methods in the control of quantum linear systems including coherent H -infinity control.

Ian R. Petersen. *A direct coupling coherent quantum observer for an oscillatory quantum plant*. In Proceedings of the 2017 IEEE Conference on Control Technology and Applications, Kohala Coast, Hawai'i, USA, August 2017.

A direct coupling coherent observer is constructed for a linear quantum plant which has oscillatory solutions. It is shown that a finite time moving average of the quantum observer output can provide an estimate of the quantum plant output without disturbing this plant signal. By choosing a sufficiently small averaging time and a sufficiently large observer gain, the observer tracking error can be made arbitrarily small.

Shibei Xue, Ian R Petersen, Valery Ugrinovskii, and Matthew R. James. *H -infinity filtering for an optical cavity system disturbed by Lorentzian quantum noise*. In Proceedings of the 20th IFAC World Congress, Toulouse, France, July 2017.

In this paper, we present a H^∞ filter for estimating dynamics of a Lorentzian noise-disturbed non-Markovian cavity system with disturbances. The non-Markovian cavity system can be represented by an augmented system, where the Lorentzian quantum noise is parameterized by an ancillary system. This ancillary system is directly coupled to a principal system of interest and induces the non-Markovian dynamics of the principal system. The noise channels in this augmented model carry disturbances. To reduce the effect of the disturbances on the estimation, we design a H^∞ filter which can be obtained by solving a H^∞ control problem.

Shibei Xue, Min Jiang, Dewei Li, Jun Zhang, and Ian R. Petersen. *Identifying a damping rate function for a non-Markovian single qubit system*. In 2017 IEEE International Conference on Systems, Man, and Cybernetics, Banff, Canada, October 2017.

In this paper, we present a gradient algorithm to identify a damping rate function for a non-Markovian single qubit system. The dynamics of the single qubit system in a non-Markovian environment are assumed to obey a time convolutionless master equation, where all the non-Markovian effects of the environment are combined in the unknown damping rate function. To identify the damping rate function, we measure time trace observables of the qubit such that we can formulate the identification procedure as an optimization problem. Thus, we design a gradient algorithm to optimally reveal the damping rate function.

Muhammad Fuady Emzir, Ian R. Petersen, and Matthew James Woolley. *Lyapunov stability analysis for invariant states of quantum systems*. In Proceedings of the 56th IEEE Conference on Decision and Control, Melbourne, Australia, Dec 2017.

In this article, we propose a Lyapunov stability approach to analyze the convergence of the density operator of a quantum system. In contrast to many previously studied convergence analysis methods for invariant density operators which use weak convergence, in this article we analyze the convergence of density operators by considering the set of density operators as a Banach space. We show that the set of invariant

density operators is both closed and convex, which implies the impossibility of having multiple isolated invariant density operators. We then show how to analyze the stability of this set via a candidate Lyapunov operator.

Yuanlong Wang, Daoyi Dong, and Ian R. Petersen. *An approximate quantum Hamiltonian identification algorithm using a Taylor expansion of the matrix exponential function*. In Proceedings of the 56th IEEE Conference on Decision and Control, Melbourne, Australia, Dec 2017.

An approximate quantum Hamiltonian identification algorithm is presented with the assumption that the system initial state and observation matrix can be set appropriately. We sample the system with a fixed period and using the sampled data we estimate the Hamiltonian based on a Taylor expansion of the matrix exponential function. We prove the estimation error is linear in the variance of the additive Gaussian noise. We also propose a heuristic formula to find the order of magnitude of the optimal sampling period. Two numerical examples are presented to validate the theoretical results.

Shibe Xue, Thien Nguyen, and Ian R. Petersen. *Feedback control of a non-Markovian single qubit system*. In Proceedings of 2017 Asian Control Conference, Gold Coast, Australia, December 2017.

In this paper, we utilize a feedback strategy to stabilize a single qubit in a non-Markovian environment with a Lorentzian spectrum. The non-Markovian single qubit is represented in an augmented system model, where an ancillary system is introduced as the internal mode of the environment to generate Lorentzian noise. The density matrix of the single qubit can be estimated by using an augmented-system-based quantum stochastic master equation. Then a feedback controller can be driven by the estimates to stabilize the qubit in a target state. Simulations show the feedback strategy can provide effective control of the non-Markovian single qubit.

Yu Pan, Thien Nguyen, Zibo Miao, Nina Amini, Valeri Ugrinovski, Matthew James, *Coherent observer engineering for protecting quantum information*, 35th Chinese Control Conference (CCC), 9139-9144, 2016.

In this paper, we extend the applicability of coherent quantum observers to enable the generation of decoherence free subsystem which can be used for quantum information storage. In particular, a general quantum linear passive system can be equipped with a decoherence free subsystem by placing some of its poles on the imaginary axis via engineered observer-based controller. The controller design for the case of optical cavity systems is illustrated in the context of examples.

Hendra I Nurdin, Matthew R James, N Yamamoto, *Perfect single device absorber of arbitrary traveling single photon fields with a tunable coupling parameter: A QSDE approach*, IEEE 55th Conference on Decision and Control (CDC), 2513-2518, 2016.

We describe an ideal tuneable input-output open quantum system, realizable as either a single qubit or single resonator system, which can perfectly absorb traveling single photon states of arbitrary temporal profiles. That is, the system has a time-dependent coupling parameter to an external field that can be modulated according to the temporal profile of a single photon. We obtain an explicit analytical form for the modulation function that enables perfect absorption of single photon fields with arbitrary temporal profiles by solving the QSDE describing the system being driven by a traveling single photon field. However, a practical limit on the coupling strength prohibits a faithful physical realization of the absorber, and a numerical example is included to illustrate the absorber's performance under constrained coupling strengths.

Naoki Yamamoto, Hendra I Nurdin, Matthew R James, *Quantum state transfer for multi-input linear quantum systems*, IEEE 55th Conference on Decision and Control (CDC), 2535-2538, 2016.

Effective state transfer is one of the most important problems in quantum information processing. Typically, a quantum information device is composed of many subsystems with multi-input ports. In this paper, we develop a general theory describing the condition for perfect state transfer from the multi-input ports to the internal system components, for general passive linear quantum systems. The key notion used is the zero of

the transfer function matrix. Application to entanglement generation and distribution in a quantum network is also discussed.

Arash Khodaparastsichani, Ian R. Petersen, and Igor G. Vladimirov. *Parameterization of stabilizing linear coherent quantum controllers*. In Proceedings of the 10th ASIAN CONTROL CONFERENCE 2015, Kota Kinabalu, Malaysia, May 2015.

This paper is concerned with application of the classical Youla-Kucera parameterization to finding a set of linear coherent quantum controllers that stabilize a linear quantum plant. The plant and controller are assumed to represent open quantum harmonic oscillators modelled by linear quantum stochastic differential equations. The interconnections between the plant and the controller are assumed to be established through quantum bosonic fields. In this framework, conditions for the stabilization of a given linear quantum plant via linear coherent quantum feedback are addressed using a stable factorization approach. The class of stabilizing quantum controllers is parameterized in the frequency domain. Also, this approach is used in order to formulate coherent quantum weighted H_2 and H_∞ control problems for linear quantum systems in the frequency domain. Finally, a projected gradient descent scheme is proposed to solve the coherent quantum weighted H_2 control problem.

Shuangshuang Fu, Guodong Shi, Alexandre Proutiere, Matthew R James, *Feedback design for quantum state manipulation by measurements*, IEEE American Control Conference, 2015 104-107

In this paper, we propose feedback designs for manipulating a quantum state to a target state by performing sequential measurements. In light of Belavkin's quantum feedback control theory, for a given set of (projective or non-projective) measurements and a given time horizon, we show that finding the measurement selection policy that maximizes the successful manipulation is an optimal control problem for a controlled Markovian process. The optimal policy is Markovian and can be solved by dynamical programming. Numerical examples indicate that making use of feedback information significantly improves the success probability compared to classical scheme without taking feedback.